Quantum optics + Atom optics

Introduction:

Light $\longleftrightarrow$ Atom

(light up the beginning of life)

(Smallest particle)

Understanding

+ exploring light

(thermal light $\longrightarrow$ Laser)

The most important two aspects in modern physics

Classical optics $\longrightarrow$ QED $\longrightarrow$ Quantum optics

What ? → Study quantum statistics

(EM radiation (light) quantum nature)
+ Interaction of EM with matter

Why ? → After laser was invented, scientists faced these questions:

What is the nature of radiation (light)?

How many types of states does light have?

How does light ↔ matter?

How ? → I hope that the course will tell you something!

Theoretical methods

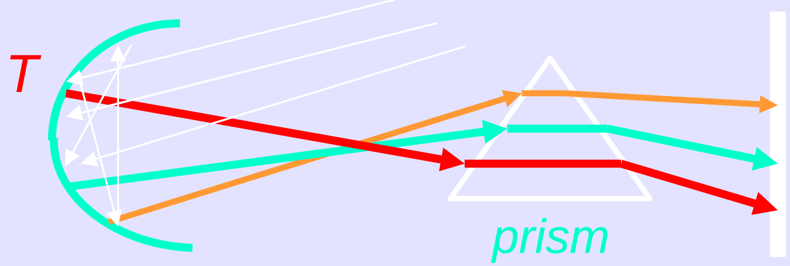
Experimental study

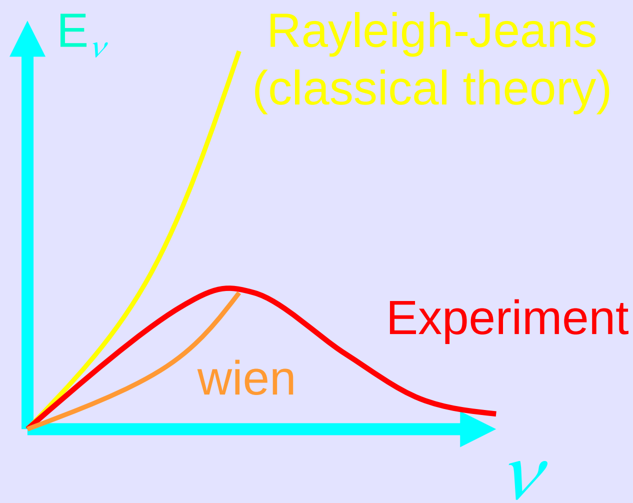
Where to start ?

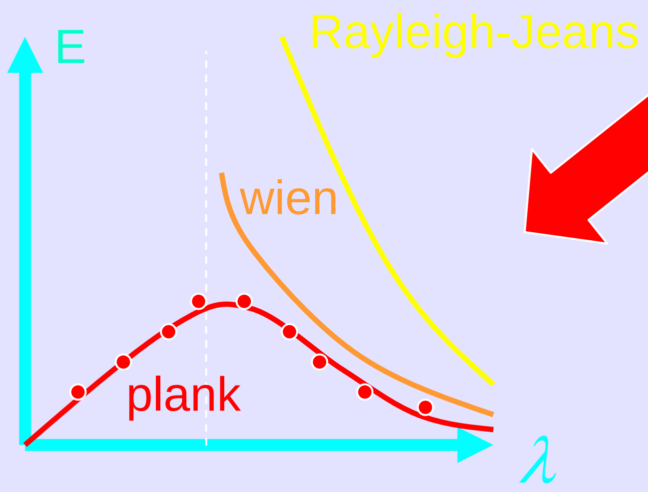
Back to 1900-1930 (golden time for physics)

<u>Several problems haunting around physics:</u>

1) Blackbody radiation spectrum (1889-1901?)







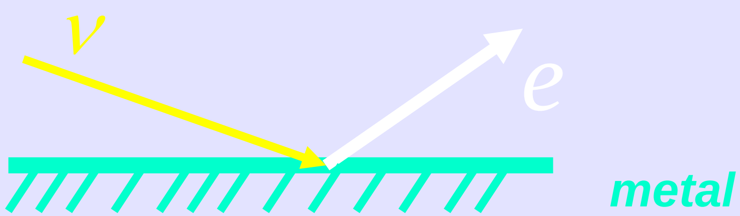
Classical

Theory

Fails !

2) Photoelectric effect

Experiment (H. Hertz, 1888)

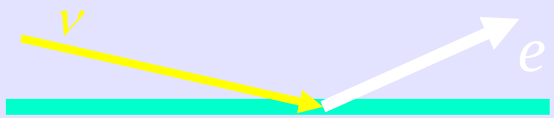


Results:

a) Given metal → Critical frequency ν_0

$\nu < \nu_0$

$\nu > \nu_0$



b) $E_e \infty \nu$ **Independent of light intensity I**

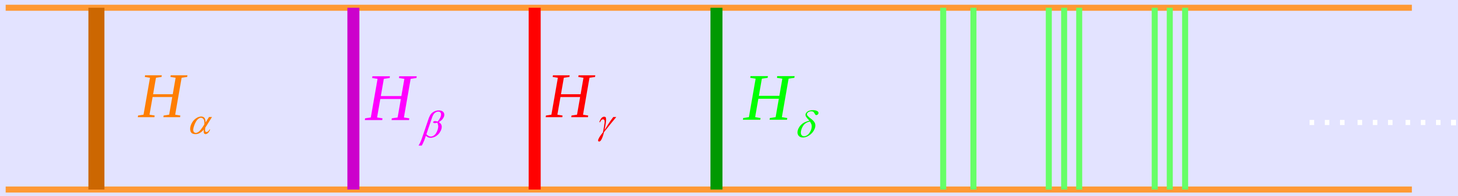
c) $\nu > \nu_0$ **Immediately emits electron** $(10^{-9} s)$

<u>Classical theory fails to explain !</u>

3) Atomic “discrete” spectral lines

1885 Balmer $\longrightarrow$ Hydrogen atom

$$\nu = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad n = 3, 4, 5\ldots$$

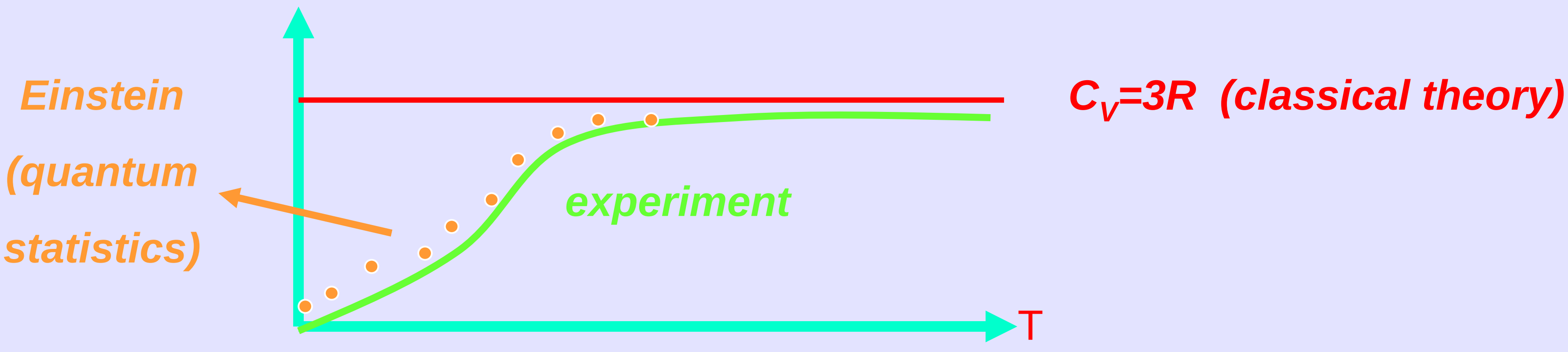


Why discrete ? ***Classical theory fails***

4) Other problems:

Stability of atoms or atomic nucleus;

Molar specific heat at constant V



Quantum theory (natural laws are not continuous) must come up (microscopic world)

Historical events of QT development:

1901 M. PLANCK $\longrightarrow$ quanta (EM)

1905 A. Einstein $\longrightarrow$ photon

light $\longleftrightarrow$ ***"particle"***

1913 Bohr $\longrightarrow$ Atomic theory $\longrightarrow$ H-atom discrete spectral lines

(exploring period --- → searching for correct directions)

1923 Compton scattering $\longrightarrow$ Confirms "photon"

1924 De brőglie

Matter particle $\longrightarrow$ "wave" ***"wave-particle duality"***

1923-1927 ***Quantum Mechanics***

Later developments:

P. Dirac → generalization

photon ⟷ ***electron***

interaction

Feymann et. al ⟷ QED built up

1940s'-1960' ***World War II*** → **Radar**

Enhance signal → ***MASER***

Thinking : ***<u>How does atom emit radiation ?</u>***

Difference: ***Classical antenna*** ⟷ ***MASER***

Wigner-Weiskoff radiation theory

vacuum fluctuations *(QED-key)*

Einstein (phenomenological) theory:

Spontaneous Emission

Stimulated Emission

MASER → *LASER*

W. Lamb

(UA)

H. Haken

(German)

M. Lax

W. H. Louisell

Forming 3 Laser schools:

Lamb school: { W. Lamb, M. Sargent III, M. O. Scully } → Semiclassical theory

Haken school { R. Graham, H. walther, F. Hakke } → Full quantum theory

Lax school { W. Louisell, M. Lax } → Mixing type (QT ⟷ ST)

→ R. Glauber (Harvard) (coherent state) → D. F. Walls (PHD with Glauber; postdoc with Haken)

D. F. Walls

Interference of photons (1973)

photon bunching, anti bunching (1976)

Squeezed state of light (1983)

EXP:

IBM	***U of Texas, Austin***	***M. Planck QO Inst.***
(M. Lewenson)	***J. Kimble***	***H. walther***

nonclassical light field

Applications → Carl. Caves (gravitational wave detection)

→ ***Einstein models*** → ***Quantum optics***

Quantum optics: *new development of optics*

- nature of light
- light ⟷ matter in quantum level
- Control (manipulate) light ⟶ different states
- ⟶ applications

Australiasia:

D. F. Walls

G. J. Milburn

P. D. Drummond

G. Gardiner

H. Carmicheal

……

USA:

M. Scully

J. Kimble

M. Lewenson

C. Caves

Pierre Meystre

……

Euroup:

C. Chen Tanminji

S. Harose

P. Zoller

H. Walther

W. Schleich

P. Knight

R. loudon

……

India

Egypt

South Africa

Brazil

China

……

Another direction:

Laser → *Trapping particles (Askin-1976?)*

Laser cooling → *Schanlow Hänsch Letkhor (1976-1977)*

↓

Laser manipulation of Atoms

↓ Balykin, Minogin, letkhov

Quantization of C of M motion

↓

Atom optics (1985-1995)

↓

Splitting of QO community *(-1990)*

QO

Applications of nonclassical light

Quantum information

Quantum computation

Atom optics:

Coherent matter wave

Cold atoms

BEC

coherence

New combination

(2002, 2003)

What is next ?

Your task

Teaching approach :

1. contents: *QO (3/4) + AO (1/4)*

2. Methods : *Teaching + discussions*

3. exam : *exercise, reading papers, final exam*

4. emphasis : *research (principle, methods, examples)*

Introduction of frontier

Current state

5. Textbook :

QO → ***M Scully***, ***M. Zubairy***

AO → ***References + teaching materials*** → ***Papers***

Reference :

1. M. O. Scully and M. S. Zubairy 《Quantum Optics》
2. P. Meystre and M. Sargent Ⅲ 《Elements of Quantum Optics》
3. D. F. Walls and G. J. Milburn 《 Quantum Optics》
4. L. Mandel and E. Wolf 《Optical Coherence and Quantum Optics》
5. Hans-A. Bachor 《A Guide to Experiments in Quantum Optics》
6. R. R. Puri 《Mathematical Methods of Quantum Optics》
7. P. Meystre 《Atom Optics》
8. 谭维翰 《非线性与量子光学》
9. 彭金生 《近代量子光学导论》
10. 郭光灿 《量子光学》

Chap. 1 Quantum theory of radiation

Classical Maxwell-Eq

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \quad (\vec{J} = 0, \text{ no current})$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho \quad (\rho = 0, \text{ no charge})$$

$$\nabla \cdot \vec{B} = 0$$

Physical Observables for EM:

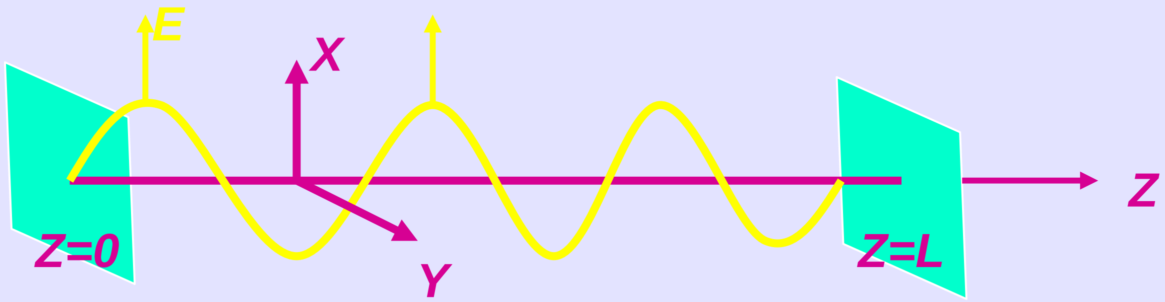
$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \quad (\vec{P} \neq 0 \text{ medium})$$

$$\vec{B} = \mu_0 \vec{H}$$

(EM) classical Energy

$$H = \frac{1}{2}\int [\frac{1}{\varepsilon_0}\vec{D}^2(\vec{r}\,',t) + \frac{1}{\mu_0}\vec{B}^2(\vec{r}\,',t)]d^3\vec{r}\,'$$

e. g. ID EM field in a cavity



Expansion : $\vec{E}(\vec{z}\,',t)=\sum_j q_j(t)U_j(\vec{z})\vec{x}$

$$\nabla\times\vec{H}=-\frac{\partial\vec{D}}{\partial t}$$

$$\vec{H}(\vec{z}\,',t)=\sum_j\frac{\varepsilon_0\dot{q}_j}{k_j}A_j\cos k_j\vec{z}$$

$$H=\frac{1}{2}\sum_j(m_j\dot{q}_j{}^2+m_j\nu_j^2q_j)=\sum_j(\frac{P_j^2}{2m_j}+\frac{1}{2}m_j\nu_j^2q_j)$$

Quantization :

$$\left[\hat{q}_j, \hat{p}_{j'}\right] = i\hbar\delta_{jj'}$$

$$\left[\hat{q}_j, \hat{q}_{j'}\right] = \left[\hat{p}_j, \hat{p}_{j'}\right] = 0$$

For convenience , introduce canonical transformation:

$$e^{-i\nu_j t}\hat{a}_j = \frac{1}{\sqrt{2m_j\hbar\nu_j}}(m_j\nu_j\hat{q}_j + i\hat{p}_j) \rightarrow \text{annihilation}$$

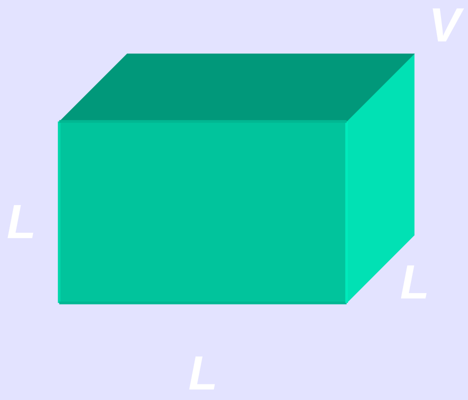
$$e^{i\nu_j t}\hat{a}_j^+ = \frac{1}{\sqrt{2m_j\hbar\nu_j}}(m_j\nu_j\hat{q}_j - i\hat{p}_j) \rightarrow \text{creation}$$

$$\Rightarrow \quad \left[\hat{a}_j, \hat{a}_{j'}^+\right] = \delta_{jj'}$$

$$\left[\hat{a}_j, \hat{a}_{j'}\right] = \left[\hat{a}_j^+, \hat{a}_{j'}^+\right] = 0$$

$$H = \sum_j \hbar\nu_j(\hat{a}_j^+\hat{a}_j + 1/2)$$

3D generalization :



$L \to \infty$ **Free space**

$k_j \to \vec{k}$

$$\begin{cases} \sin \vec{k}\cdot\vec{r} \\ \cos \vec{k}\cdot\vec{r} \end{cases} \Rightarrow L \to \infty$$ **(Apply periodic boundary)**

$$\vec{E}(\vec{r},t) = \sum_{\vec{k}} \varepsilon_{\vec{k}} \vec{e}_{\vec{k}} \hat{a}_{\vec{k}} e^{-i\nu_k t + i\vec{k}\cdot\vec{r}} + H.C.$$

$$\vec{H}(\vec{r},t) = \sum_{\vec{k}} \frac{1}{\mu_0} \frac{\vec{k}\times\vec{e}_{\vec{k}}}{\nu_{\vec{k}}} \varepsilon_{\vec{k}} \hat{a}_{\vec{k}} e^{-i\nu_k t + i\vec{k}\cdot\vec{r}} + H.C$$

$$\nabla\cdot\vec{D} = 0 \Rightarrow \vec{k}\cdot\vec{e}_{\vec{k}} = 0 \quad \text{(transverse wave)}$$

$$\vec{k} = \left(\frac{2\pi n_x}{L}, \frac{2\pi n_y}{L}, \frac{2\pi n_z}{L}\right)$$

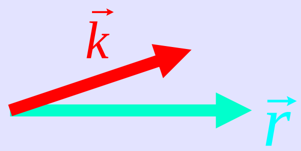
Mode density $\nu \to \nu + d\nu \quad (\mathrm{L} \to \infty)$

$$\sum_{\vec{k}} \to 2\left(\frac{L}{2\pi}\right)^3 \int d^3k$$

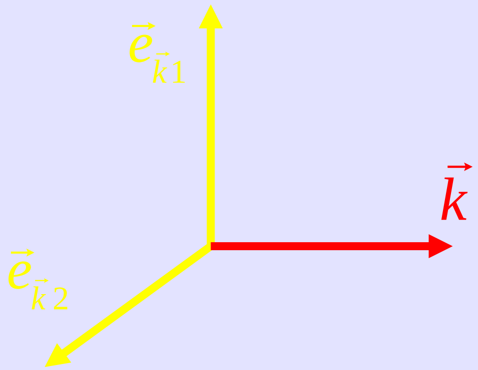
In future application $\vec{E}(\vec{r},t) = \vec{E}^{(+)}(\vec{r},t) + \vec{E}^{(-)}(\vec{r},t)$

Because: $\hat{E}^{(-)+}(\vec{r},t) = \hat{E}^{(+)}(\vec{r},t)$

$$\hat{E}^{(+)}(\vec{r},t) = \sum_{\vec{k}} \varepsilon_{\vec{k}} \hat{a}_{\vec{k}} e^{-i\nu_k t + i\vec{k}\cdot\vec{r}}$$



Including all two independent polarization



$$\hat{E}^{(+)}(\vec{r},t) = \sum_{\vec{k}\,\lambda=1,2} \vec{e}_{\vec{k}\lambda} \varepsilon_{\vec{k}} \hat{a}_{\vec{k}\lambda} e^{-i\nu_k t + i\vec{k}\cdot\vec{r}}$$

$$\vec{H}^{(+)}(\vec{r},t) = \frac{1}{\mu_0} \sum_{\vec{k}\lambda} \frac{\vec{k} x \vec{e}_{\vec{k}\lambda}}{\nu_k} \varepsilon_{\vec{k}} \hat{a}_{\vec{k}\lambda} e^{-i\nu_k t + i\vec{k}\cdot\vec{r}}$$

$$\left[\hat{a}_{\vec{k}\lambda}, \hat{a}_{\vec{k}'\lambda'}\right] = \left[\hat{a}_{\vec{k}\lambda}^{+}, \hat{a}_{\vec{k}'\lambda'}^{+}\right] = 0$$

$$\left[\hat{a}_{\vec{k}\lambda}, \hat{a}_{\vec{k}'\lambda'}^{+}\right] = \delta_{\vec{k}\vec{k}'} \delta_{\lambda\lambda'}$$

General commutation relations (equal time)

$$\left[\vec{E}_j(\vec{r},t),\vec{H}_j(\vec{r},t)\right]=0 \quad (j=x,y,z)$$

$$\left[\vec{E}_j(\vec{r},t),\vec{H}_k(\vec{r},t)\right]=-i\hbar c^2\frac{\partial}{\partial x_l}\delta^{(3)}(\vec{r}-\vec{r}\,') \quad (j,k,l) \text{ cyclic permutation}$$

$$\delta^{(3)}(\vec{r}-\vec{r}\,')=\frac{1}{\nu}\sum_{\vec{k}}e^{i\vec{k}(\vec{r}-\vec{r}\,')}\rightarrow\frac{1}{(2\pi)^3}\int d^3k e^{i\vec{k}(\vec{r}-\vec{r}\,')}$$

EM $\longrightarrow$ ***Parallel components*** $\longrightarrow$ ***commute***

Perpendicular $\longrightarrow$ ***not***

(E_x,H_x) $\longrightarrow$ ***Simutaneously measurable !***

(E_x,H_y) $\longrightarrow$ ***Not !***

1.2 Fock 0r (particle) number state (of EM)

Single-mode EM field with frequency ν

$$\hat{H} = \hbar\nu(\hat{a}^{+}\hat{a} + \frac{1}{2})$$

Eigenstates $|E\rangle$

$$\begin{cases} H|E\rangle = \hbar\nu(\hat{a}^{+}\hat{a} + 1/2)|E\rangle = E|E\rangle \\ [\hat{a}, \hat{a}^{+}] = 1 \end{cases}$$

Defining: $\hat{N} = \hat{a}^{+}\hat{a}$ **Let** $\hat{N}|n\rangle = n|n\rangle$

$$\hat{N}\hat{a}|n\rangle = (n-1)\hat{a}|n\rangle$$

$$\hat{N}\hat{a}^{+}|n\rangle = (n+1)\hat{a}^{+}|n\rangle$$

Prove n $\longrightarrow$ *Only "0" or "integer" (positive)*

Properties $|n\rangle$

$$\hat{a}|n\rangle \Rightarrow |n-1\rangle$$
$$\hat{a}^+|n\rangle \Rightarrow |n+1\rangle$$

$$\hat{a}\,|n\rangle = C_n\,|n-1\rangle$$
$$\langle n\,\hat{a}^+\hat{a}\,|n\rangle = \langle n-1|C_n^* C_n\,|n-1\rangle$$
$$n\,\underbrace{\langle n|n\rangle}_{=1} = |C_n|^2\,\underbrace{\langle n-1|n-1\rangle}_{=1\ \text{normalized}}$$

$$|C_n|^2 = n$$
$$C_n(real) = \sqrt{n}$$

Summary:

$$\hat{N}\,|n\rangle = n\,|n\rangle$$
$$\hat{a}\,|0\rangle = 0$$
$$\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle$$
$$\hat{a}^+\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

Fock states

(number states)

$$|n\rangle = \frac{(\hat{a}^+)^2}{\sqrt{n!}}|0\rangle$$

$\hat{N} \longrightarrow$ ***(photon) number operator !***

Fock states are complete set of quantum states

$$\sum_{n=0}^{\infty} |n\rangle\langle n| = \mathrm{I}$$

$$\langle n | n' \rangle = \delta_{nn'} \quad \rightarrow \quad \text{orthognal}$$

Physics:

$$H\,|E\rangle = \hbar\nu\,(\hat{a}^{+}\hat{a} + 1/2)\,|E\rangle$$

$$[H, N] = 0$$

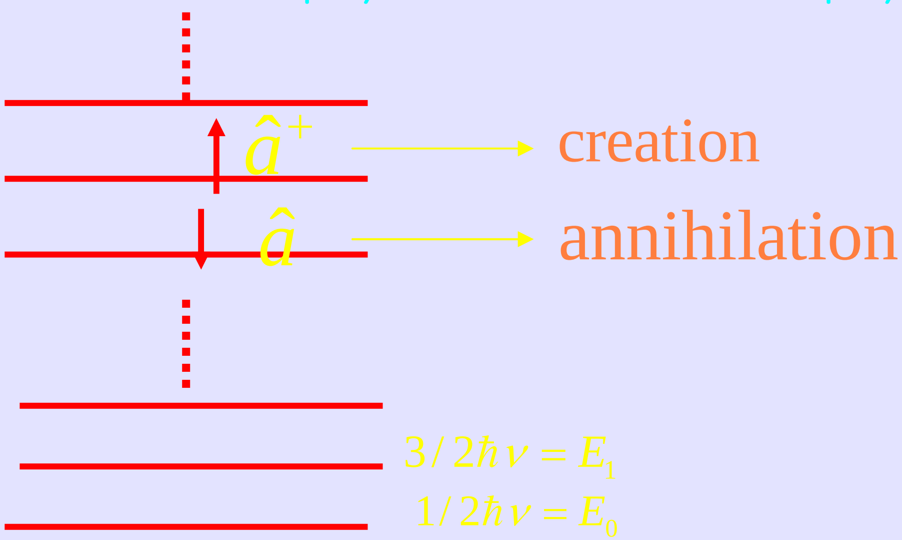
$$|E\rangle \rightarrow |n\rangle$$

$$H\,|n\rangle = \hbar\nu\,(n + 1/2)\,|n\rangle$$

Simutaneouly

Have the common

Eigenstates



$$|0\rangle \rightarrow E_0 = \frac{1}{2}\hbar\nu$$

“empty” is not empty

“nothing” contains “something”

$$|0\rangle \Rightarrow n = 0$$

but $$E_0 = \frac{1}{2}\hbar\nu \neq 0$$

Further see E-field

Single-mode

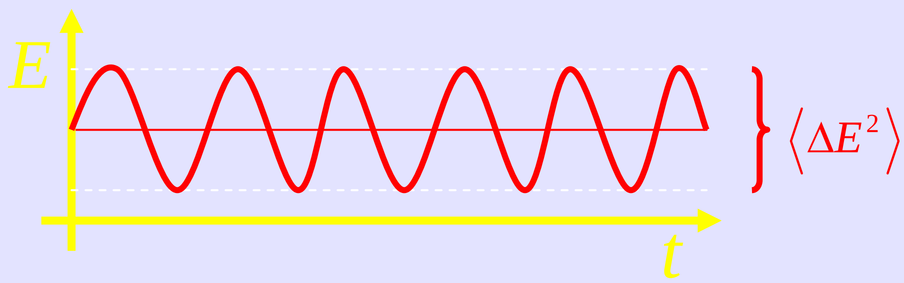
$$E(\vec{r},t) = \underbrace{\varepsilon \hat{a} e^{-i\nu t + i\vec{k}\cdot\vec{r}}}_{\hat{E}^{(+)}} + \underbrace{\hat{E}^{(-)}}_{H.C.}$$

In Fock state:

$$\langle E\rangle = \langle n|E|n\rangle = 0$$

$$\langle E^2\rangle = \langle n|E^2|n\rangle = 2|\varepsilon|^2(n+1/2)$$

$$\langle \Delta E^2\rangle = \langle E^2\rangle - \langle E\rangle^2 = 2|\varepsilon|^2(n+1/2)$$



Multi-mode case

$$\hat{H} = \sum_{\vec{k}\lambda} H_{\vec{k}\lambda}$$

$$H_{\vec{k}\lambda} = \hbar\nu_{\vec{k}}(\hat{a}_{\vec{k}\lambda}^{+}\hat{a}_{\vec{k}\lambda} + 1/2)$$

$$H_{\vec{k}\lambda}\left|n_{\vec{k}\lambda}\right\rangle = \hbar\nu_{\vec{k}}(n_{\vec{k}\lambda} + 1/2)\left|n_{\vec{k}\lambda}\right\rangle$$

$$\left|n_{\vec{k}_1\lambda}, n_{\vec{k}_2\lambda}\ldots\ldots n_{\vec{k}_l\lambda}\ldots\ldots\right\rangle \equiv \left|\{n_{\vec{k}\lambda}\}\right\rangle$$

$$\hat{a}_{\vec{k}_l\lambda}\left|\{n_{\vec{k}_l\lambda}\}\right\rangle = \sqrt{n_{\vec{k}_l\lambda}}\left|n_{\vec{k}_1\lambda}, n_{\vec{k}_2\lambda}\ldots\ldots n_{\vec{k}_l\lambda}\ldots\ldots\right\rangle$$

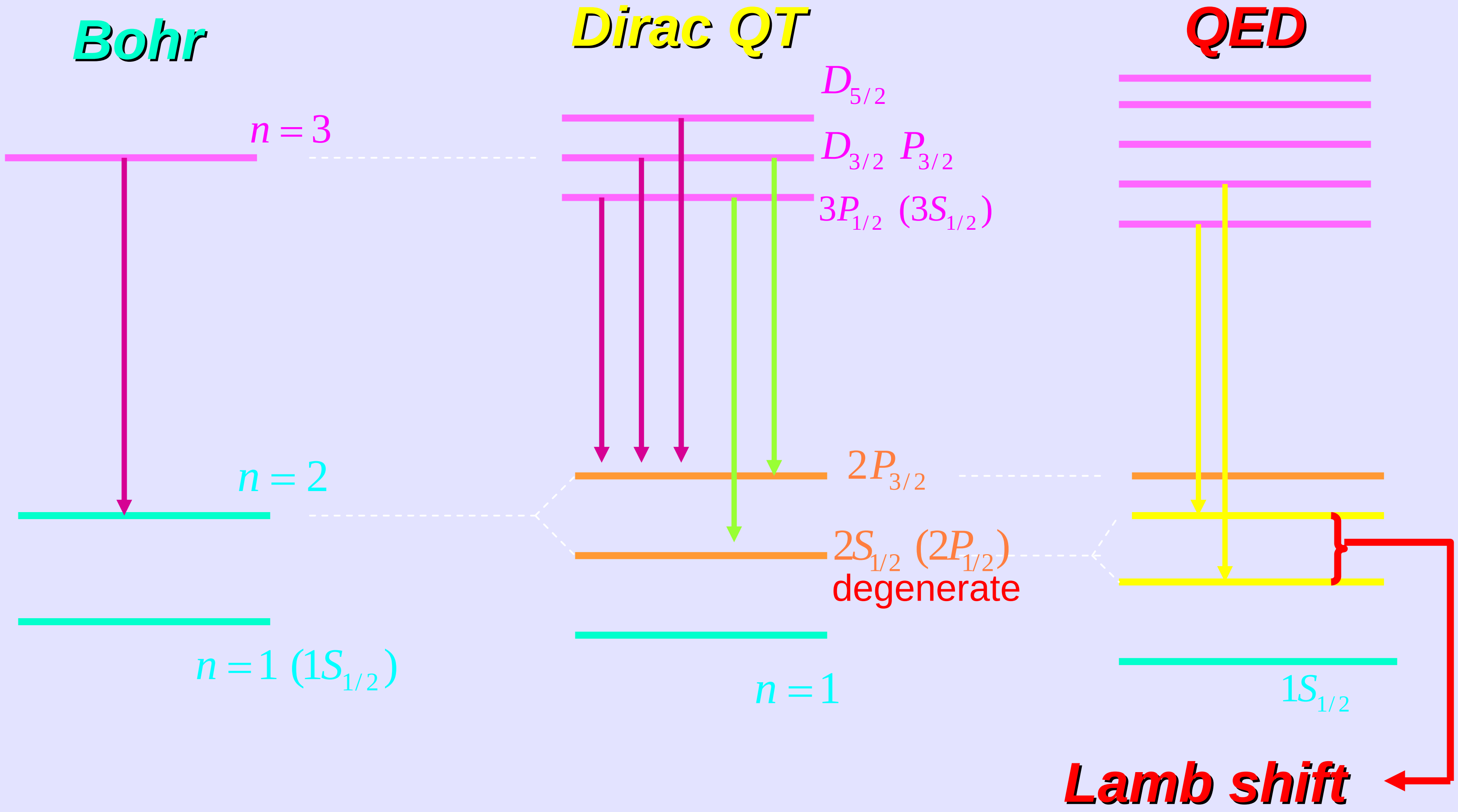
General state

$$\left|\psi\right\rangle = \sum_{n_{\vec{k}_1\lambda}, n_{\vec{k}_2\lambda}\ldots\ldots n_{\vec{k}_l\lambda}\ldots\ldots} C_{n_{\vec{k}_1\lambda}, n_{\vec{k}_2\lambda}\ldots\ldots n_{\vec{k}_l\lambda}\ldots\ldots}\left|n_{\vec{k}_1\lambda}, n_{\vec{k}_2\lambda}\ldots\ldots n_{\vec{k}_l\lambda}\ldots\ldots\right\rangle$$

$$= \sum_{\{n_{\vec{k}\lambda}\}} C_{\{n_{\vec{k}\lambda}\}}\left|\{n_{\vec{k}\lambda}\}\right\rangle$$

Lamb shift: → *First example of vacuum fluctuation effect !*

H-Atom



Theory (QED)

Bethe → *Simple nonrelativistic calculation*

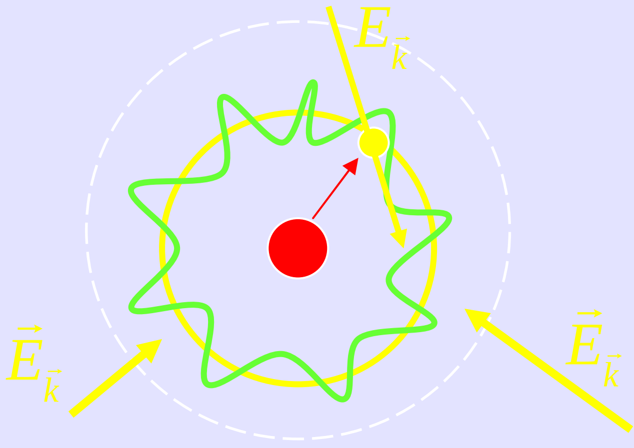
Kramers, Schwinger, Weisskopf → *Simple "substracting off"*

Schwinger, Feynman, Dyson → *QED*

Need "long time" to understand formal calculation in QED

We use simple method (melton)

H-Atom



$$\left[-\frac{\hbar^2\nabla^2}{2m}+V(\vec{r})\right]\varphi_{\vec{n}}(\vec{r})=E_{\vec{n}}\varphi_{\vec{n}}(\vec{r})$$

$$\Delta V = V(\vec{r} + \delta\vec{r}(t)) - V(\vec{r})$$

$$= \delta\vec{r} \cdot \nabla V + \frac{1}{2}(\delta\vec{r} \cdot \nabla)^2 V(\vec{r}) + ...$$

$$\longrightarrow \quad E_n \rightarrow E_n^0 + \Delta E_n$$

$$\Delta E_n \propto \Delta V \qquad \text{(ignore } \Delta(\frac{P^2}{2m}))$$

Level shift (average)

$$\langle \Delta \mathrm{E}_n \rangle = \langle \Delta V \rangle = \langle \delta\vec{r} \cdot \nabla V \rangle + \frac{1}{2}\langle (\delta\vec{r} \cdot \nabla)^2 V(\vec{r}) \rangle$$

$$\langle \delta\vec{r} \rangle = 0 \rightarrow \text{fluctuation isotropic}$$

$$\langle (\delta\vec{r} \cdot \nabla)^2 V(\vec{r}) \rangle_{Vac,Atom} = \left\langle \left(\delta x \cdot \frac{\partial}{\partial x} + \delta y \cdot \frac{\partial}{\partial y} + \delta z \cdot \frac{\partial}{\partial z} \right) \right\rangle_{Vac}$$

$$= \frac{1}{3}\langle \delta\vec{r} \rangle_{Vac} \langle \nabla^2 V \rangle_{Atom}$$

Quantum beats: *Another example using Quantized EM field*

Note: *Plank "quanta"* → *Einstein's "photon"*

→ *Quantization of EM (light)* → *Vacuum fluctuations*

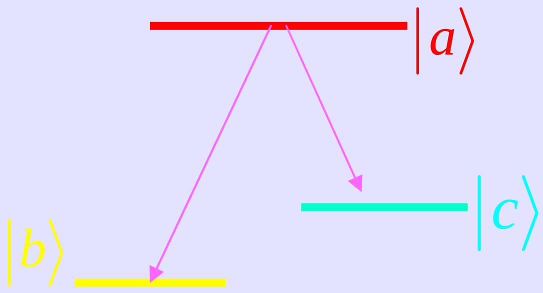
EM: *Classical Maxwell Eq.* + *introducing "vacuum fluctuation"*

Matter (Atom) → *Schrődinger QM*

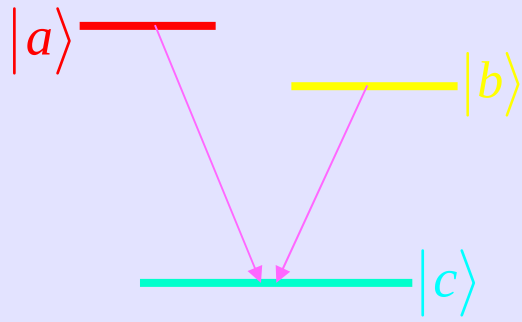
→ ***SCT (semiclassical theory)*** → ***Lamb school***

SCT:

Three level atom



" Λ " type



"v" type

Prepared in coherent superposition of states

$$|\varphi\rangle = C_a e^{-i\omega_a t}|a\rangle + C_b e^{-i\omega_b t}|b\rangle + C_c e^{-i\omega_c t}|c\rangle$$

$$\left\langle \vec{u}_{\Lambda}^{(+)} \right\rangle = \vec{u}_{\Lambda}^{ab} c_a^* c_b e^{i(\omega_a - \omega_b)t} + \vec{u}_{\Lambda}^{ac} c_a^* c_c e^{i(\omega_a - \omega_c)t} + c.c$$

$$\left\langle \vec{u}_{V}^{(-)} \right\rangle = \vec{u}_{V}^{ac} c_a^* c_c e^{i(\omega_a - \omega_b)t} + \vec{u}_{V}^{bc} c_b^* c_c e^{i(\omega_b - \omega_c)t} + c.c$$

$$\text{Defecter} \propto I = \left|E^+\right|^2 = |\varepsilon_1|^2 + |\varepsilon_2|^2 + \underbrace{\left\{\varepsilon_1^* \varepsilon_2 e^{i(\nu_1 - \nu_2)t} + c.c\right\}}_{\text{Beat term}}$$

$$\left.\begin{aligned} \nu_1 &= \omega_a - \omega_b \\ \nu_2 &= \omega_a - \omega_c \end{aligned}\right\} \Lambda$$

$$\left.\begin{aligned} \nu_1 &= \omega_a - \omega_c \\ \nu_2 &= \omega_b - \omega_c \end{aligned}\right\} V$$

Interference term ← **Beat term**

Both Λ-type and v-type atoms → **Interference beat**

Full quantum theory:

$$\left|\varphi\right\rangle = (C_a\left|a\right\rangle + C_b\left|b\right\rangle + C_c\left|c\right\rangle) \times \left|\{0\}\right\rangle_{v_1 v_2}$$

$$E_1^{(+)} \propto \hat{a}_1 e^{-iv_1 t} \quad E_2^{(+)} \propto \hat{a}_2 e^{-iv_2 t} \quad E^{(+)} = E_1^{(+)} + E_2^{(+)}$$

$$\text{Detector} \propto I = \left\langle E^{(+)*} E^{(+)} \right\rangle = \left\langle E_1^{(-)} E_1^{(+)} \right\rangle + \left\langle E_2^{(-)} E_2^{(+)} \right\rangle + \underline{\left\langle E_1^{(-)} E_2^{(+)} \right\rangle} + c.c$$

Interference term

$$\left\langle E_1^{(-)} E_2^{(+)} \right\rangle_V \propto \left\langle \varphi_V \left| \hat{a}_1^+ \hat{a}_2 \right| \varphi_V \right\rangle e^{i(v_1 - v_2)t} \neq 0$$

$$\left\langle E_1^{(-)} E_2^{(+)} \right\rangle_\Lambda \propto \left\langle \varphi_\Lambda \left| \hat{a}_1^+ \hat{a}_2 \right| \varphi_\Lambda \right\rangle e^{i(v_1 - v_2)t} = 0$$

No beat for Λ-type atom, beat only for v-type atom

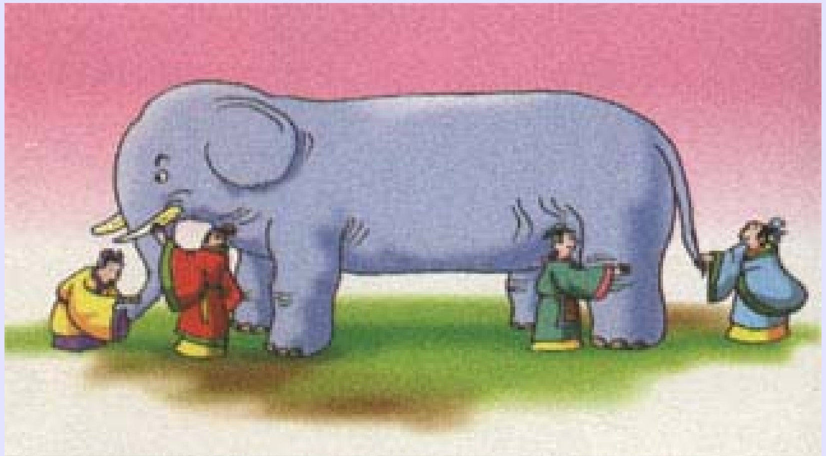
SCT cannot explain

Lead to quantum measurement theory

Similar exam: $\left\{ \begin{array}{l} \textbf{Young's double-slit interference} \\ \textbf{Quantum beat} \end{array} \right.$

Philosophical summary

What is light ? → photon concept



Maxwell: light → EM wave

Planck: radiation → Use "quanta"

Einstein: photoemission → introduce "photon"

QED: quantization of EM → "wave-particle" duality

Chapter 2. states of radiation field:
Coherence states + squeezed states

Quantization of EM field → *State of EM field*

Laser appears → *Confirm existence of different "state" of light*

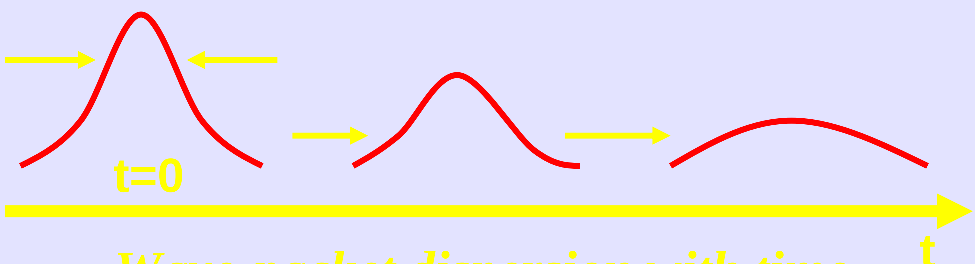
Other states: squeezed states, nonclassical states

2.1 coherent state

Quantum mechanics

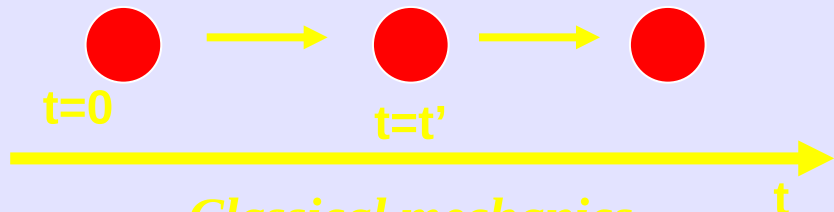
<u>Uncertainty of motion</u> ↔ *Quantum fluctuation*

(Heisenberg's principle)



Wave-packet dispersion with time

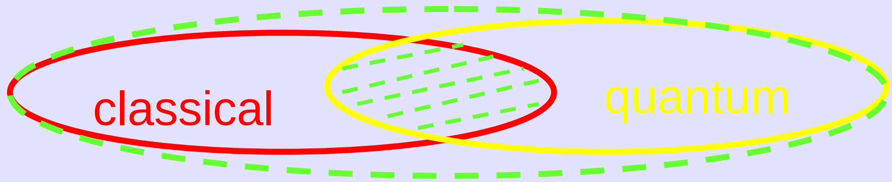
Point particle



Classical mechanics

Seeking for states in quantum (permitted) region

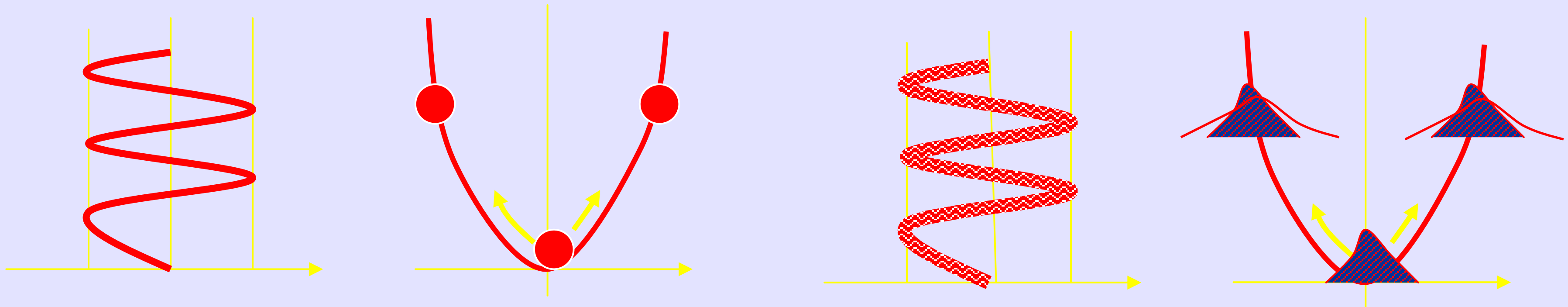
→ *Close to classical case*



Single-mode light field In a cavity ⟷ ***Particle in harmonic potential***

$$E(\vec{r},t) \infty \hat{q}(t)$$

$$H = (p^2/2m) + (1/2) m v^2 q^2$$



Find a state close to classical case

Radiation from a classical current:

Classical current $\vec{J}(\vec{r},t)$ (not operator)

$$\nabla\times\vec{H}=\frac{\partial\vec{D}}{\partial t}+\vec{J}(\vec{r},t)\qquad \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

For convenience: introduce vector potential

$$\vec{B}=\nabla\times\vec{A}\qquad \vec{E}=\frac{\partial\vec{A}}{\partial t}\qquad \nabla^2\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}=-u_0\vec{J}$$

$$\hat{H}=\frac{1}{2}\int[\varepsilon_0(\frac{\partial\vec{A}}{\partial t})^2+\frac{1}{u_0}(\nabla\times\vec{A})^2]d^3\vec{r}-\int\vec{J}\cdot\vec{A}\,d^3\vec{r}$$

$$\vec{A}(\vec{r},t)=(\int\vec{E}(t)dt)=-i\sum_{\vec{k}}\frac{1}{\nu_k}\vec{e}_{\vec{k}}\varepsilon_{\vec{k}}\hat{a}_{\vec{k}}e^{-i\nu_k t+i\vec{k}\cdot\vec{r}}+H.C.$$

$$\hat{H}_I=\sum_{\vec{k}}\hbar\nu_{\vec{k}}(\hat{a}^+_{\vec{k}}\hat{a}_{\vec{k}}+1/2)+\hat{V}_I(t)$$

$$\hat{V}_I(t)=-\int\vec{J}(\vec{r},t)\cdot\vec{A}(\vec{v},t)d^3\vec{r}$$

$$=\sum_{\vec{k}}(i\frac{1}{\nu_k}\varepsilon_{\vec{k}}\int\vec{J}(\vec{r},t)\cdot\vec{e}_{\vec{k}}d^3\vec{r})\hat{a}_{\vec{k}}e^{-i\nu_k t+i\vec{k}\cdot\vec{r}}+H.C$$

In interaction picture :

$$i\hbar\frac{\partial|\psi\rangle}{\partial t}=V_I(t)|\psi\rangle \qquad |\psi(t)\rangle=\underbrace{\exp[-\frac{i}{\hbar}\int_0^t V(t')dt']}_{\prod_{\vec{k}}\exp(\alpha_{\vec{k}}\hat{a}_{\vec{k}}^{+}-\alpha_{\vec{k}}^{*}\hat{a}_{\vec{k}})}|\psi(0)\rangle$$

$$\alpha_{\vec{k}}=\frac{-1}{\hbar\nu_k}\varepsilon_{\vec{k}}\int_0^t dt'\int d^3\vec{r}\vec{e}_{\vec{k}}\cdot\vec{J}(\vec{r},t)e^{-i\nu_k t'+i\vec{k}\cdot\vec{r}}$$

$$|\{\alpha_{\vec{k}}\}\rangle=\prod_{\vec{k}}|\alpha_{\vec{k}}\rangle$$ **Single-mode:** $$|\alpha\rangle=e^{\alpha\hat{a}^{+}-\alpha^{*}\hat{a}}|0\rangle$$

Prove:

$|\alpha\rangle$ Is the eigenstate of the annihilation operator $\hat{a}$ (or it is the eigenstate of the electric field $E^{(+)}$)

Coherent state $\longrightarrow$ a "classical"

amplitude of EM $\longrightarrow$ close to "classical"

$$\hat{a}|\alpha\rangle = \hat{a}e^{\alpha\hat{a}^{+}-\alpha^{*}\hat{a}}|0\rangle = \alpha|\alpha\rangle$$

Defining displacement operator

$$\hat{D}(\alpha) = \hat{a}e^{\alpha\hat{a}^{+}-\alpha^{*}\hat{a}}(= e^{-|\alpha^{2}|/2}e^{\alpha\hat{a}^{+}}e^{-\alpha^{*}\hat{a}})$$

$$\hat{D}^{+}(\alpha) = \hat{D}(-\alpha) = \left[\hat{D}(\alpha)\right]^{-1}$$

$$\left.\begin{aligned} &\hat{D}^{-1}(\alpha)\hat{a}\hat{D}(\alpha) = \hat{a}+\alpha \\ &\hat{D}^{-1}(\alpha)\hat{a}^{+}\hat{D}(\alpha) = \hat{a}^{+}+\alpha^{*} \end{aligned}\right\} \quad |\alpha\rangle \equiv \hat{D}(\alpha)|0\rangle$$

$$\hat{D}^{-1}(\alpha)\hat{a}\hat{D}(\alpha)|0\rangle = (\hat{a}+\alpha)|0\rangle$$

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \rightarrow \underbrace{|\alpha\rangle = \hat{D}(\alpha)|0\rangle}_{\text{Coherent state}} \xrightarrow{?} |\alpha\rangle = e^{-|\alpha^{2}|/2}\sum_{n=0}^{\infty}\frac{\alpha^{n}}{\sqrt{n!}}|n\rangle$$

Coherent state $\longrightarrow |0\rangle$ ***State is displaced***

What means "coherence" ?

Cohere $\longrightarrow$ **stick together**

$\Downarrow$

Useful (understanding) approach

Particle $\longrightarrow$ **harmonic oscillator analogy**

Number state

$$|n\rangle \to \phi_n(q) = \langle q|n\rangle \qquad \hat{a} = \frac{1}{\sqrt{2m\hbar\nu}}(m\nu\hat{q} + i\hat{p})$$

$$\text{in } |q\rangle \to q - \text{representation} \quad \hat{p} = -i\hbar\frac{\partial}{\partial q}$$

$$\langle q|\hat{a}|0\rangle = \langle q|a\int dq|q'\rangle\langle q'|0\rangle = \int dq'\underbrace{\langle q|\hat{a}|q'\rangle}_{\left\{\frac{1}{\sqrt{2m\hbar\nu}}(m\nu\hat{q}+\hbar\frac{\partial}{\partial q})\delta(q-q')\right\}}\phi_0(q') = 0$$

$$\phi_0(q) = \left(\frac{\nu}{\pi\hbar}\right)^{1/4}\exp\left(-\frac{\nu q^2}{2\hbar}\right) \qquad (m=1)$$

$$\phi_n(q) = \langle q|\frac{(a^+)^n}{\sqrt{n!}}|0\rangle = \frac{1}{\sqrt{n!}}\frac{1}{(2\hbar\nu)^{n/2}}\left(\nu q - \hbar\frac{\partial}{\partial q}\right)^n\phi_0(q) = \frac{1}{(2^n n!)^{1/2}}H_n\left(\sqrt{\frac{\nu}{\hbar}}\right)\phi_0(q)$$

$\phi_n(q) \rightarrow$ harmonic oscillator eigenfuctions

$$\langle \mathrm{q} \rangle = \int_{-\infty}^{\infty} \Phi_n^*(q) q \Phi_n(q) dq = 0 \quad \langle p \rangle = 0 \quad \langle p^2 \rangle = \hbar \nu (n + \frac{1}{2}) \quad \langle q^2 \rangle = \frac{\hbar}{\nu}(n + \frac{1}{2})$$

$\Rightarrow$ $\Delta p \Delta q = (n + \frac{1}{2})\hbar$ **n=0 (ground state of harmonic oscillator)** $\Delta p \Delta q = \frac{\hbar}{2}$

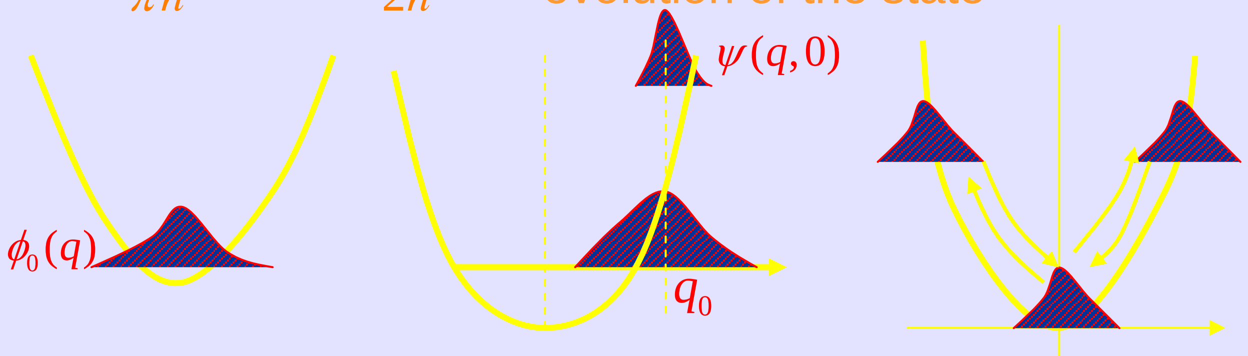
Minimum uncertainty state $\xrightarrow{\Delta q = 0, \Delta p = 0}$ **Close to classical**

(in general state $\Delta p \Delta q \geq \frac{\hbar}{2}$ **)**

$$\phi_0(q) = (\frac{\nu}{\pi\hbar})^{1/4} \exp(-\frac{\nu q^2}{2\hbar})$$

We now see the time evolution of the state

$$\psi(q,0) = (\frac{\nu}{\pi\hbar})^{1/4} \exp[-\frac{\nu}{2\hbar}(q - q_0)^2]$$



Shape keeps unchanged cohere !!!

$$|\psi(q,t)|^2 = (\frac{\nu}{\pi\hbar})^{1/2} \exp[-\frac{\nu}{\hbar}(q - q_0 \cos \nu t)^2]$$

$$|\psi(q,0)|^2 = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \langle q | n \rangle = \langle q | \alpha \rangle$$

Coherent state* $\longrightarrow$ *minimum-uncertainty wavepacket in q-representation c0ordination

Some properties of coherent states:

1) Mean photon number

$\bar{n} = \langle \alpha | \hat{a}^{+}\hat{a} | \alpha \rangle = |\alpha|^2$ probability finding n photon in $|\alpha\rangle$ $P(n) = \left| \langle n | \alpha \rangle \right|^2 = \dfrac{|\alpha|^{2n} e^{-\langle n \rangle}}{n!} = \dfrac{\bar{n}^n e^{-\bar{n}}}{n!}$

2) Coherent state ⟶ minimum uncertainty state ⟶ close to classical

$$\Delta p \, \Delta q = \frac{\hbar}{2}$$

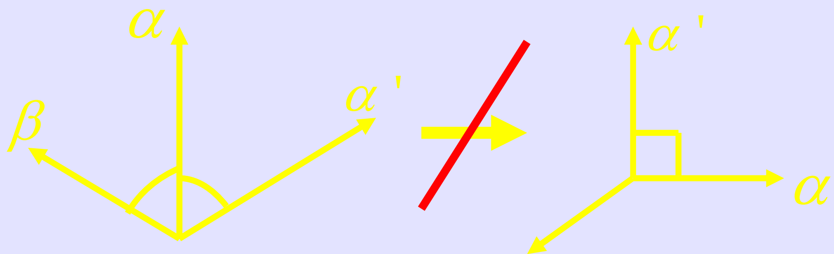
3) $\{|\alpha\rangle\}$ ⟶ Complete set

$$\int |\alpha\rangle\langle\alpha| \, d^2\alpha = \prod \sum_n |n\rangle\langle n| = \prod \hat{I}$$

4) $|\alpha\rangle$, $|\beta\rangle$ is not orthogonal

$$\langle \alpha | \beta \rangle = e^{-|\alpha|^2/2} e^{-|\beta|^2/2} \sum_{n=0} \sum_{m=0} \frac{\alpha^{*n}}{\sqrt{n!}} \frac{\beta^m}{\sqrt{m!}} \delta_{nm} = e^{-|\alpha|^2/2 - |\beta|^2/2 + \alpha^* \beta} \neq 0$$

$$\left| \langle \alpha | \beta \rangle \right|^2 = e^{-|\alpha - \alpha'|^2} \xrightarrow{|\alpha - \alpha'| \gg 1} 0$$



$|\alpha\rangle = \dfrac{1}{\pi} \int d^2\alpha' \langle \alpha' | \alpha \rangle |\alpha'\rangle$ overcomplete *laser* $\xrightarrow{?}$ $|\alpha\rangle$

Theory + experiment $\longrightarrow$ *phase of laser is uncertain*

$\big| |\alpha| e^{i\varphi} \big\rangle \longrightarrow$ φ *Fluctuates* $\longrightarrow$ *make laser is a mixing state*

$$\hat{e} = \int_0^{2\pi} \big| |\alpha| e^{i\varphi} \big\rangle \big\langle e^{i\varphi} |\alpha| \big| d\varphi$$

$$= e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{(|\alpha|^2)^n}{n!} |n\rangle\langle n|$$

$$= \sum_{n=0}^{\infty} P(n) |n\rangle\langle n|$$

ANU Teleportation experiment claims successful

But theoretical explanation is wrong

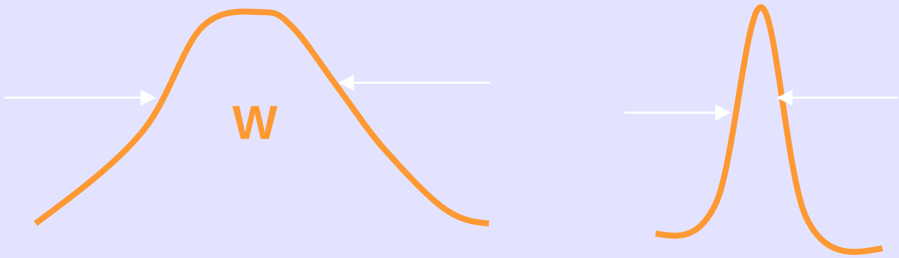
Teleportation needs pure coherent state

Squeezed state physics

Gravitational wave detection

Harmonic oscillator

Can gravitational wave affect it ?



$W \gg \Delta a$(given by gravitation wave)

Squeeze “W”

Example : wavepacket in harmonic potential

$$i\hbar\frac{\partial}{\partial t}\psi = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}mv^2x^2\right)\psi$$

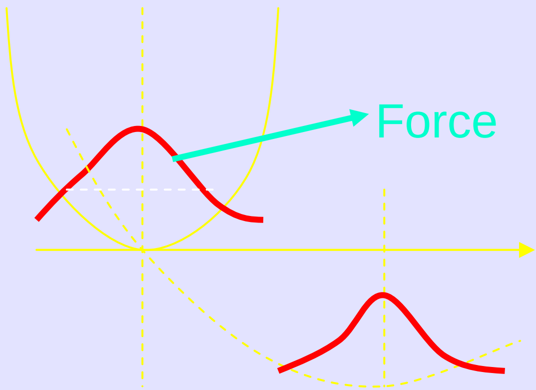
$$\psi(x,t) = \int dx' G(x,x',t)\psi(x,0)$$

$$G(x,x',t) = \int dx' e^{-i\frac{\hbar t}{2m}\frac{\partial^2}{\partial x^2} + \frac{m}{2i\hbar}v^2x^2}\delta(x-x')$$

$$\begin{cases} t=0 & \delta-\text{function wave packet : } \psi(x',0)=\delta(x'-x_0) \\ t=\dfrac{\pi}{2v} & \text{plane wave } \sqrt{\dfrac{mv}{2\pi\hbar}}e^{i\frac{mvx_0}{\hbar}x} \\ t=\dfrac{\pi}{2} & \delta(x-x_0) \end{cases}$$

Gedanken experiment: preparing squeezed state

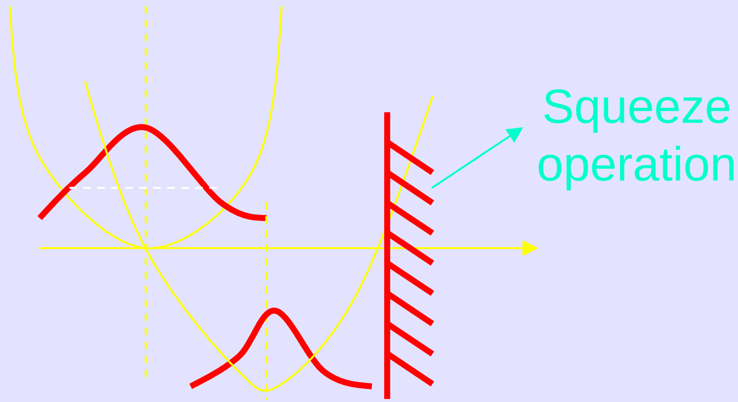
Coherent state



Displaced the ground state by a force F

$$H = \frac{P^2}{2m} + \frac{1}{2}kx^2 - eE_0 x$$

Squeezed state



$$H = \frac{P^2}{2m} + \frac{1}{2}kx^2 - eE_0(ax - bx^2)$$

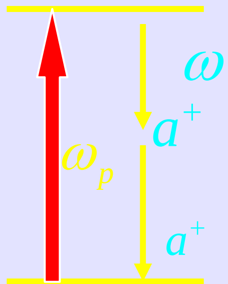
$$H = \frac{P^2}{2m} + \frac{1}{2}(k + 2ebE_0)x^2 - eaE_0 x$$

Light field $H = \dfrac{P^2}{2m} + \dfrac{1}{2}m\nu^2 q^2$

$$H = \frac{P^2}{2m} + \frac{1}{2}m\nu^2 q^2 - \alpha(q - \beta q^2) \qquad H_I = (a^+)^2, a^2$$

Generation of squeezed state:

Parametric process and squeezed



two photon

$$\omega_p = \omega_1 + \omega_2 \qquad \vec{k}_p = \vec{k}_1 + \vec{k}_2$$

$$H_I = i\hbar[g(\hat{a}^+)^2 - g^*\hat{a}^2]$$

degenerate $\omega_1 = \omega_2 \qquad \omega_p = 2\omega$

$$|\psi(t)\rangle = e^{g(\hat{a}^+)^2 - g^*\hat{a}^2}|0\rangle \qquad g = \frac{1}{2}\xi = \frac{1}{2}re^{i\theta} \qquad |0,\xi\rangle = \hat{S}(\xi)|0\rangle$$

Squeezed vacuum state

$$\hat{S}(\xi) = \exp(\frac{1}{2}\xi^*\hat{a}^2 - \frac{1}{2}\xi(\hat{a}^+)^2) \qquad \hat{S}^+(\xi) = S(-\xi) = S^{-1}(\xi)$$

$$\hat{S}^+(\xi)\hat{a}\hat{S}(\xi) = \hat{a}\cosh r - \hat{a}^+e^{i\theta}\sinh r$$

$$\hat{S}^+(\xi)\hat{a}^+\hat{S}(\xi) = \hat{a}^+\cosh r - \hat{a}e^{-i\theta}\sinh r$$

$$(e^ABe^{-A} = B + [A,B] + \frac{1}{2}[A,[AB]]......)$$

Squeezed state and uncertainty relation

$[\hat{A}, \hat{B}] = i\hat{c}$ $\quad \Delta A \Delta B \geq \frac{1}{2}|\hat{c}|$ Squeezed state $\longrightarrow$ $\Delta A = \sqrt{(\Delta A)^2}$ $\longrightarrow$ $(\Delta B^2) \gg \frac{1}{2}|c|$

$\Delta A^2 < \frac{1}{2}|c|$

Ideal case: $\Delta A \Delta B = \frac{1}{2}|c|$ ***Minimum uncertainty state (coherent state)***

Squeezed coherent state $\longrightarrow$ ***ideal squeezed state***

Example:

$$\vec{E} = \varepsilon \vec{e}(\hat{a} e^{-ivt} + \hat{a}^+ e^{ivt}) \qquad [\hat{a}, \hat{a}^+] = 1$$

$$\hat{x} = \sqrt{\frac{2\hbar / mv}{4}}(\hat{a} + \hat{a}^+) \qquad [\hat{x}, \hat{p}] = i\hbar$$

$$\hat{p} = \frac{\sqrt{2m\hbar v}}{2i}(\hat{a} - \hat{a}^+)$$

Normalize $\hat{x}$ $\hat{p}$ $\longrightarrow$ ***dimensionless physical variable***

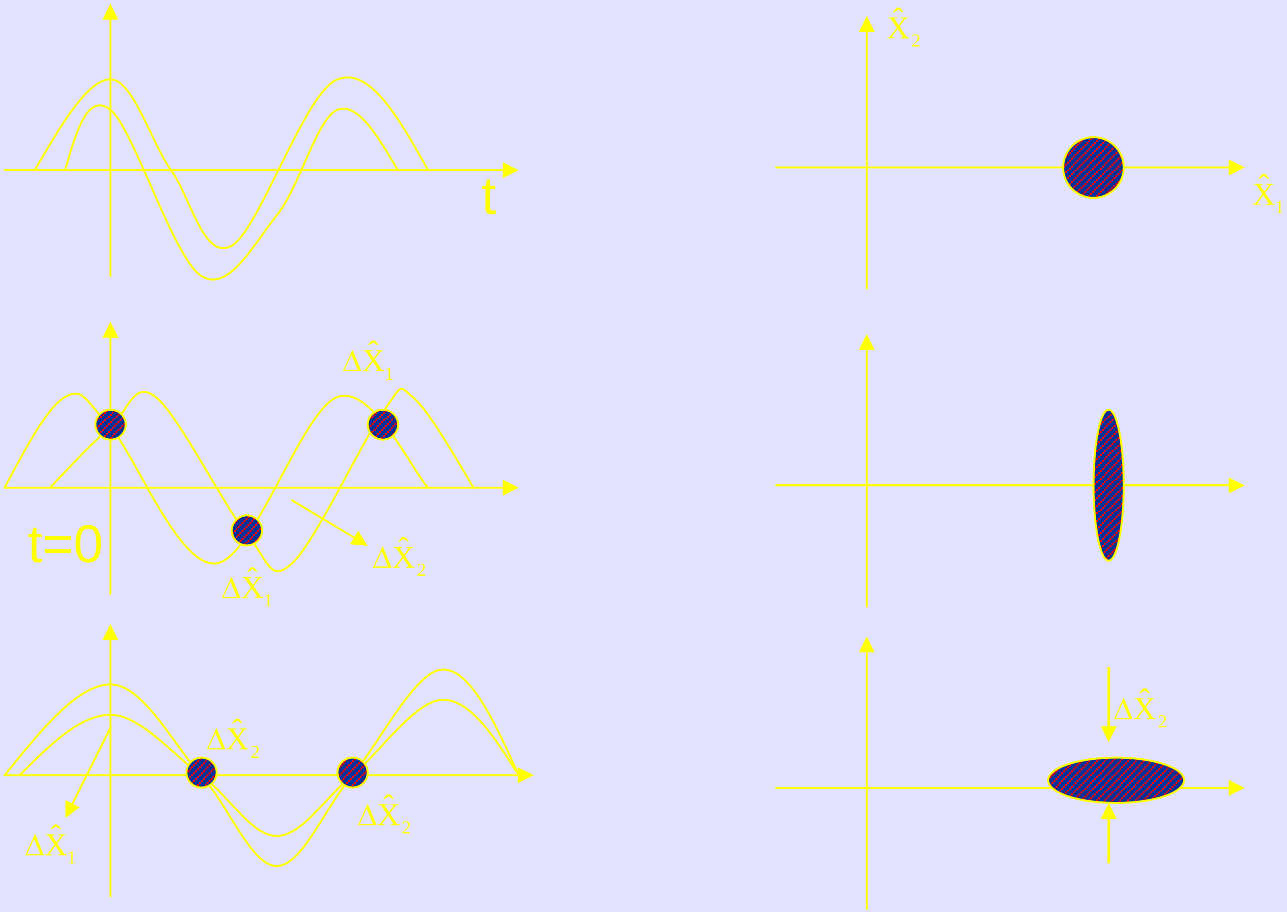
$$\hat{X}_1 \to \hat{x} \Rightarrow \frac{1}{2}(\hat{a} + \hat{a}^+) \quad \hat{X}_2 \to \hat{p} \Rightarrow \frac{1}{2i}(\hat{a} + \hat{a}^+) \quad \left[\hat{X}_1, \hat{X}_2\right] = \frac{i}{2}$$

$\longrightarrow$ $\vec{E} = 2\varepsilon \vec{e}(\hat{X}_1 \cos vt + \hat{X}_2 \sin vt)$

In general $\Delta \hat{X}_1 \Delta \hat{X}_2 \geq \frac{1}{4}$

Squeezed if $(\Delta \hat{X}_i)^2 < \frac{1}{4}$

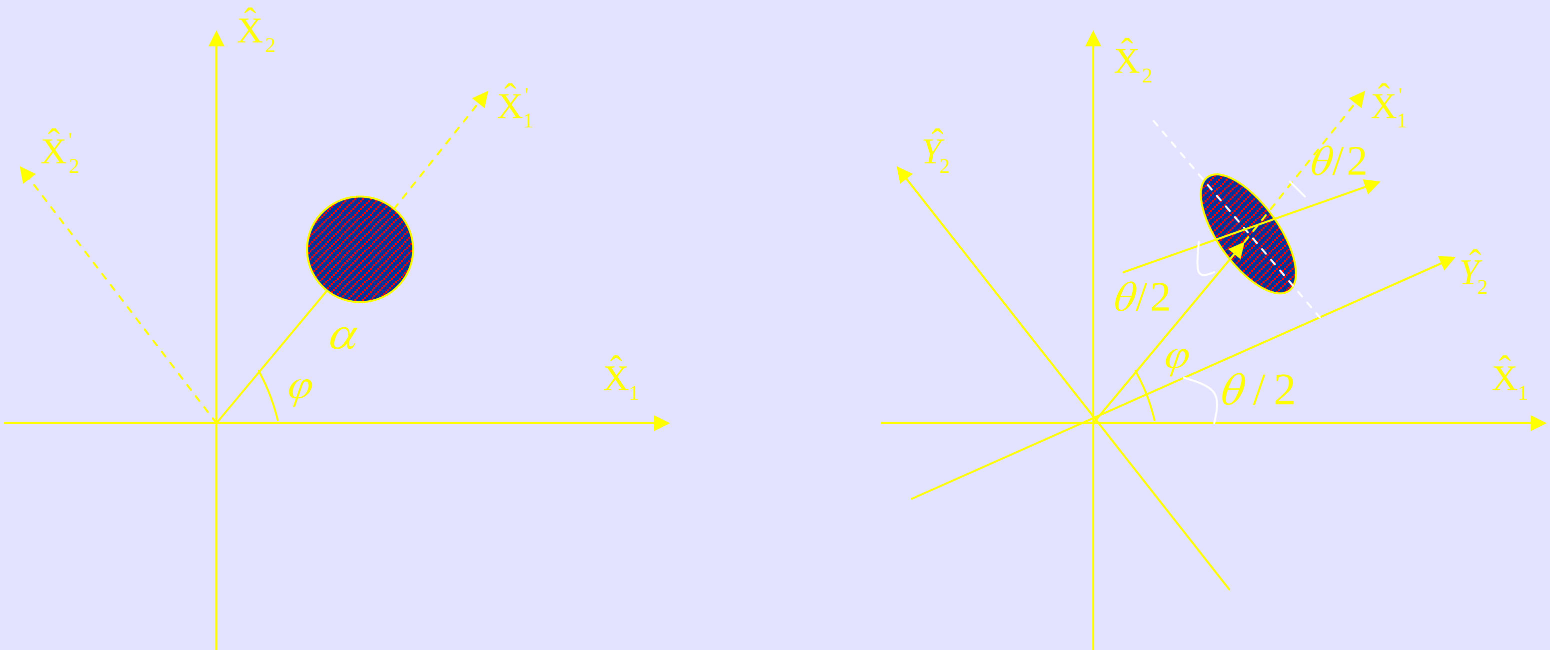
ideal $\Delta \hat{X}_1 \Delta \hat{X}_2 = \frac{1}{4}$



t
$\hat{X}_2$
$\hat{X}_1$
$\Delta\hat{X}_1$
t=0
$\Delta\hat{X}_2$
$\Delta\hat{X}_1$
$\Delta\hat{X}_2$
$\Delta\hat{X}_2$
$\Delta\hat{X}_2$
$\Delta\hat{X}_1$

Quadrature variance of squeezed coherent state

$$\alpha = |\alpha| e^{i\varphi} \quad \xi = re^{i\theta} \quad |\alpha,\xi\rangle = \hat{S}(\xi)\hat{D}(\alpha)|0\rangle$$



Define: $$\hat{Y}_1 + i\hat{Y}_2 = (\hat{X}_1 + i\hat{X}_2)e^{-i\theta/2}$$

$$\Rightarrow \quad \hat{S}^+(\xi)(\hat{Y}_1 + i\hat{Y}_2)\hat{S}(\xi) = \hat{Y}e^{-r} + i\hat{Y}e^{r}$$

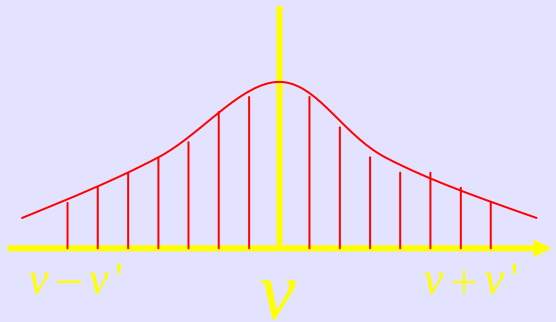
Multi-mode squeezing

$$S_{\nu\nu'}(\xi) = e^{\xi^* \hat{a}_{\nu+\nu'}\hat{a}_{\nu-\nu'} - \xi a^+_{\nu+\nu'} a^+_{\nu-\nu'}}$$

Multi-mode :

$$\hat{S}[\xi(\nu)] = \int_\Delta \frac{d\nu'}{2\pi} \exp[\xi'(\nu')\hat{a}_{\nu+\nu'}\hat{a}_{\nu-\nu'} - \xi(\nu')a^+_{\nu+\nu'}a^+_{\nu-\nu'}]$$

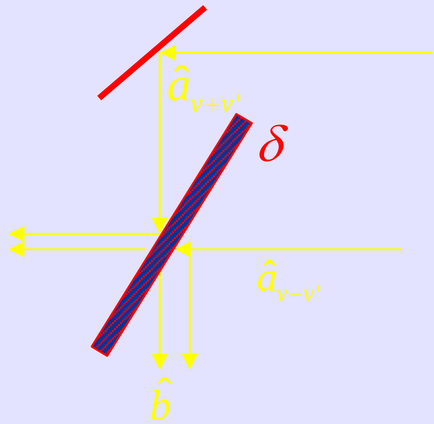
$$|\alpha(\nu), \xi(\nu)\rangle = \hat{S}(\xi(\nu))\hat{D}(\alpha(\nu))\left|\{0\}_\nu\right\rangle$$



Two-mode quadrature:

$$\hat{b}_1 = \frac{1}{2}(\hat{b} + \hat{b}^+) \qquad \hat{b}^+ = \frac{1}{\sqrt{2}}[\hat{a}^+_{\nu+\nu'} + e^{i\delta}\hat{a}^+_{\nu-\nu'}]$$

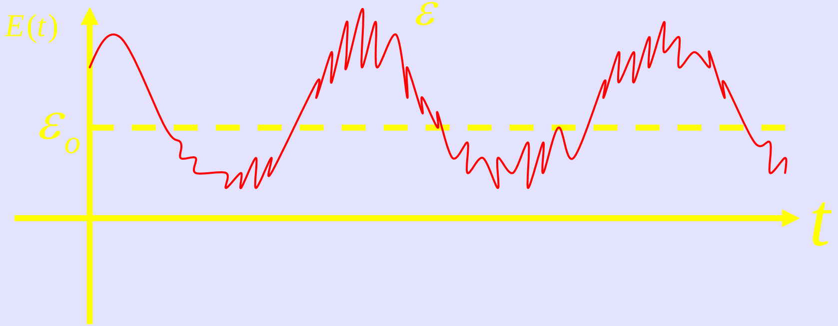
$$\hat{b}_2 = \frac{1}{2i}(\hat{b} - \hat{b}^+) \qquad \hat{b} = \frac{1}{\sqrt{2}}[\hat{a}_{\nu+\nu'} + e^{-i\delta}\hat{a}_{\nu-\nu'}]$$

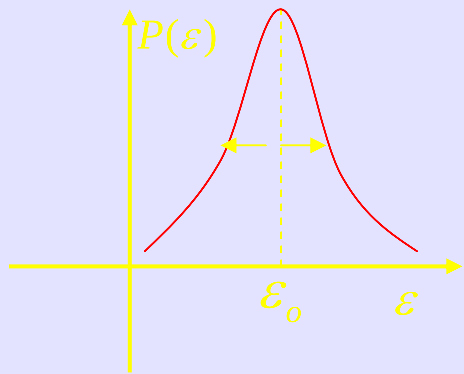


Quantum distribution theory and partially coherent radiation

Q pure state

- Coherent state $|\alpha\rangle$
- Number state $|n\rangle$
- Squeezed state $|\alpha,\xi\rangle$
- Vacuum state $|0\rangle$





Distribution theory is desirable

(one of method) dealing with quantum fluctuations

Wigner-distribution

P-distribution → Coherent state → representation

Q-distribution

positive-P

Coherent-state representation

Ideal:

$$|n\rangle \rightarrow H(\hat{N} = a^{+}a)$$

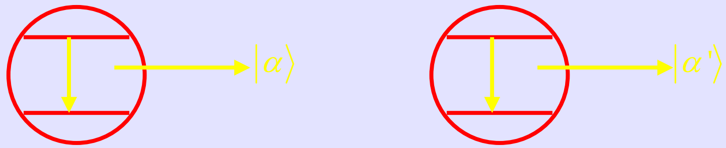
$$|\alpha\rangle \rightarrow \hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

eigenstate

Reality

laser $\overline{|\alpha\rangle\langle\alpha|} \Rightarrow \sum_{n=0}^{\infty} P(n)|n\rangle\langle n|$ mixture state

Reason :



Atom 1 Atom 2 …… The initial phase is random

Mixture state $\longrightarrow$ must use density operator

$$\langle \hat{0} \rangle_{QM} = \langle \psi | \hat{0} | \psi \rangle \rightarrow QM$$

But the quantum system can experience all different $|\psi\rangle$ $|\psi'\rangle$ **…….**

Quantum statistics

$$\left\langle \left\langle \hat{0} \right\rangle_{QM} \right\rangle_{QS} = \sum_{\psi} P_{\psi} \left\langle \psi \right| \hat{0} \left| \psi \right\rangle$$

$$= \sum_{\psi} P_{\psi} \left\langle \psi \right| \sum_{n} \left| n \right\rangle \left\langle n \right| \hat{0} \left| \psi \right\rangle$$

$$= \sum_{n} \left\langle n \right| \hat{0} \sum_{\psi} P_{\psi} \left| \psi \right\rangle \left\langle \psi \right| \left| n \right\rangle$$

$$= \sum_{n} \left\langle n \right| \hat{0} \hat{\rho} \left| n \right\rangle = Tr(\hat{O}\rho) = Tr(\rho O)$$

$\hat{\rho}$ ⟶ How to describe it ? ⟶ distribution theory (representation)

↓

Number state $\left| n \right\rangle$ $\quad \hat{\rho} = \sum_{n} \left| n \right\rangle \left\langle n \right| \hat{\rho} \sum_{m} \left| m \right\rangle \left\langle m \right| = \sum_{n} \sum_{m} \hat{\rho}_{nm} \left| n \right\rangle \left\langle m \right|$

$(n = m)$ $\quad \hat{\rho}_{nm} \rightarrow$ probabilty occupying $\left| n \right\rangle$ state

$(n \neq m)$ $\quad \hat{\rho}_{nm}$ $\quad$ cross (nondiagonal) term

$n \rightleftharpoons m$ transition "probability"

Example:

$$\hat{\rho}=\frac{1}{\pi}\int d^2\alpha\left|\alpha\right\rangle\left\langle\alpha\right|\hat{\rho}\frac{1}{\pi}\int d^2\beta\left|\beta\right\rangle\left\langle\beta\right|$$

$$=\int d^2\alpha\int d^2\beta\rho_{\alpha^*\beta}\left|\alpha\right\rangle\left\langle\beta\right|=\int\frac{d^2\alpha}{\pi}\int\frac{d^2\beta}{\pi}R(\alpha^*,\beta)e^{-\frac{1}{2}(|\alpha|^2+|\beta|^2)}\left|\alpha\right\rangle\left\langle\beta\right|$$

Define $\quad R(\alpha^*,\beta)\equiv\left\langle\alpha\right|\hat{\rho}\left|\beta\right\rangle e^{-\frac{1}{2}(|\alpha|^2+|\beta|^2)}$

$$\text{If }\ \beta=\alpha \quad\Rightarrow\quad R(\alpha^*,\alpha)\quad\Rightarrow\quad P(\varepsilon)$$

Coherent-state representation:

Consider an operator (EM)

$\hat{O}$ ⟶ function $\hat{a}\ \ \hat{a}^+$ ⟶ Have two forms $\begin{cases}\text{Normal order}\\ \text{Anti-normal order}\end{cases}$

Normal order:

$$\hat{O}(\hat{a},\hat{a}^{+}) \Rightarrow \hat{O}_N(\hat{a},\hat{a}^{+}) = \sum_n \sum_m C_{nm} (\hat{a}^{+})^n \hat{a}^m$$

Example:

$$\hat{O} = sim(\hat{a}^{+}\hat{a}\hat{a}^{+})$$

$$= \sum_{l=0}^{\infty} (-1)^l \frac{(\hat{a}^{+}\hat{a}\hat{a}^{+})^{2l+1}}{(2l+1)!}$$

$$= \sum_{l=0}^{\infty} (-1)^l \frac{1}{(2l+1)!} \overbrace{(\hat{a}^{+}\hat{a}\hat{a}^{+})(\hat{a}^{+}\hat{a}\hat{a}^{+}) \quad \ldots\ldots \quad (\hat{a}^{+}\hat{a}\hat{a}^{+})}^{(2l+1)}$$

$$\hat{a}\hat{a}^{+} - \hat{a}^{+}\hat{a} = 1$$

$$= \sum_l \sum_m C_{lm} (\hat{a}^{+})^l \hat{a}^m$$

Average value: $\langle \hat{O}_N \rangle = Tr(\hat{e}\hat{O}_N(\hat{a},\hat{a}^{+})) = \sum_n \sum_n C_{nm} Tr[\hat{\rho}(\hat{a}^{+})^n \hat{a}^m]$

$$Tr[\hat{\rho}(\hat{a}^{+})^n \hat{a}^m] \xrightarrow{\hat{a}^{+}\to\alpha^{*},\hat{a}\to\alpha} Tr[\int d^2\alpha \hat{P}\delta(\alpha^{*}-\hat{a}^{+})\delta(\alpha-\hat{a})(\alpha^{*})^n \alpha^m]$$

Define:

$$P(\alpha,\alpha^{*}) = Tr(\hat{\rho}\delta(\alpha^{*}-\hat{a}^{+})\delta(\alpha-\hat{a}))$$

$$\langle O_N(\hat{a},\hat{a}^{+}) \rangle = \int d^2\alpha P(\alpha,\alpha^{*}) O_N(\hat{a},\hat{a}^{+})$$

$$\begin{cases} Tr(\hat{\rho}) = 1 \to \int d^2\alpha P(\alpha,\alpha^{*}) = 1 \\ \hat{\rho}^{+} = \hat{\rho} \to P(\alpha,\alpha^{*}) \text{ is real number} \end{cases}$$

Prove:

$$\hat{\rho} = \int P(\alpha,\alpha^*)\left|\alpha\right\rangle\left\langle\alpha\right| d^2\alpha$$

$$P(\alpha,\alpha^*) = Tr(\hat{\rho}\delta(\alpha^* - \hat{a}^+)\delta(\alpha - \hat{a}))$$

$$P(\alpha,\alpha^*) \Rightarrow Tr\left\{\int P(\alpha',\alpha'^*)\left|\alpha'\right\rangle\left\langle\alpha'\right|\delta(\alpha^* - \hat{a}^+)\delta(\alpha - \hat{a})\right\}$$

$$= \int d^2\alpha' P(\alpha',\alpha'^*) Tr\left\{\delta(\alpha - \hat{a})\left|\alpha'\right\rangle\left\langle\alpha'\right|\delta(\alpha^* - \hat{a}^+)\right\}$$

$$= \int d^2\alpha' P(\alpha',\alpha'^*) \underbrace{Tr\left\{\left|\alpha'\right\rangle\left\langle\alpha'\right|\right\}}_{=1} \delta(\alpha - \alpha')\alpha(\alpha^* - \alpha'^*)$$

$$\Rightarrow P(\alpha,\alpha^*)$$

Example:

$$\langle -\beta|\hat{\rho}|\beta\rangle = \int P(\alpha,\alpha^*)\langle -\beta|\alpha\rangle\langle\alpha|\beta\rangle d^2\alpha$$

$$= e^{-|\beta|^2}\int\left(P(\alpha,\alpha^*)e^{-|\alpha|^2}\right)e^{\beta\alpha^*-\beta^*\alpha}d^2\alpha$$

$$\alpha = x_\alpha + iy_\alpha$$

$$\beta = x_\beta + iy_\beta$$

$$\langle -\beta|\hat{\rho}|\beta\rangle e^{-|\beta|^2} = \iint dx_\alpha dy_\alpha\left[P(x_\alpha,y_\alpha)e^{-(x_\alpha^2+y_\alpha^2)}\right]e^{2i(y_\beta x_\alpha - x_\beta y_\alpha)}$$

Fourier transfer

$$P(x_\alpha,y_\alpha)e^{-(x_\alpha^2+y_\alpha^2)} = \frac{1}{\pi^2}\iint\langle -\beta|\hat{\rho}|\beta\rangle e^{|\beta|^2}e^{-2i(y_\beta x_\alpha - x_\beta y_\alpha)}dx_\beta dy_\beta$$

$$P(\alpha,\alpha^*) = \frac{e^{|\alpha|^2}}{\pi^2}\iint\langle -\beta|\hat{\rho}|\beta\rangle e^{|\beta|^2}e^{-\beta\alpha^*+\beta^*\alpha}d\beta^2$$

Example 1:

$$\hat{\rho} = \frac{\exp(-\hat{H}/k_B T)}{Tr\left[\exp(-\hat{H}/k_B T)\right]}$$

热场统计分布

$$\hat{H} = \hbar\nu(a^+ a + \frac{1}{2})$$

Single mode

粒子数表象

$$\hat{\rho} = \sum_{nm} \langle n | \frac{\exp(-\hat{H}/k_B T)}{Tr\left[\exp(-\hat{H}/k_B T)\right]} | m \rangle | n \rangle \langle m |$$

$$= \sum_n \frac{e^{-\frac{n\hbar\nu}{k_B T}}}{\underline{Tr\left[\exp(-\hat{H}/k_B T)\right]}} | n \rangle \langle n |$$

$$\sum_n (e^{-\frac{n\hbar\nu}{k_B T}})^n = \frac{1}{1-\exp(-\frac{\hbar\nu}{k_B T})}$$

$$\hat{\rho} = \sum_n \frac{\langle \hat{n} \rangle^n}{(1+\langle \hat{n} \rangle)^{n+1}} | n \rangle \langle n |$$

$$\langle n \rangle = Tr(\hat{a}^+ \hat{a} \hat{\rho}) = \frac{1}{\exp(\frac{\hbar\nu}{k_B T})-1}$$

P(n) $\longrightarrow$ Bose-Einstein distribution

coherent state representation

$$\langle -\beta | \beta \rangle = \sum_n \frac{\langle \hat{n} \rangle^n}{(1+\langle \hat{n} \rangle)^{n+1}} \underline{\langle -\beta | n \rangle \langle n | \beta \rangle}$$

$$\frac{e^{-|\beta|^2/2}(-\beta^*)^n}{\sqrt{n!}} \frac{e^{-|\beta|^2/2}(\beta)^n}{\sqrt{n!}}$$

$$= \frac{e^{-|\beta|^2}}{1+\langle n \rangle} \sum_{n=0}^{\infty} \frac{(-|\beta|^2)^n}{n!} \left(\frac{\langle n \rangle}{1+\langle n \rangle} \right)^n$$

$$= \frac{e^{-|\beta|^2}}{1+\langle n \rangle} e^{-|\beta|^2/(1+\frac{1}{\langle n \rangle})}$$

$$P(\alpha, \alpha^*) = \frac{e^{|\alpha|^2}}{\pi^2 (1+\langle n \rangle)} \int e^{-|\beta|^2/(1+\frac{1}{\langle n \rangle})} e^{-\beta \alpha^* + \beta^* \alpha} d\beta^2$$

$$= \frac{1}{\pi \langle n \rangle} e^{-|\alpha|^2 / \langle n \rangle} \longrightarrow$$ Gaussian distribution

Example 2:

Pure coherent state $|\alpha_0\rangle \to \hat{\rho} = |\alpha_0\rangle\langle\alpha_0|$

$$\langle -\beta|\hat{\rho}|\beta\rangle = \langle -\beta|\alpha_0\rangle\langle\alpha_0|\beta\rangle$$

$$= \exp\left(-|\alpha|^2 - |\beta|^2 - \alpha_0\beta^* + \beta\alpha_0^*\right)$$

$$P(\alpha,\alpha^*) = \delta^{(2)}(\alpha - \alpha_0)$$

$$P(\alpha,\alpha^*) = \frac{(-1)^n e^{|\alpha|^2}}{\pi^2 n!} \int |\beta|^{2n} e^{-\beta\alpha^* + \beta^*\alpha} d^2\beta$$

$$= \frac{e^{|\alpha|^2}}{\pi^2 n!} \frac{\partial^{2n}}{\partial\alpha^n \partial(\alpha^*)^n} \int e^{-\beta\alpha^* + \beta^*\alpha} d^2\beta$$

$$= \frac{e^{|\alpha|^2}}{\pi^2 n!} \frac{\partial^{2n}}{\partial\alpha^n \partial(\alpha^*)^n} \delta^{(2)}(\alpha)$$

$n > 0$ $\quad P(\alpha,\alpha^*)$ Not nonnegative definite function

number state $|n\rangle$ Not well-defined coherent state representation

P-representation $\longrightarrow$ normal ordering

"close to classical field"

Q-representation $\longrightarrow$ antinormal ordering

"nonclassical field"

$$\hat{O}(\hat{a},\hat{a}^{+}) \Rightarrow O_A(\hat{a},\hat{a}^{+}) = \sum_n \sum_m d_{nm} \hat{a}^n (\hat{a}^{+})^m$$

$$\downarrow$$

$$\left\langle O_A(\hat{a},\hat{a}^{+}) \right\rangle \Rightarrow Tr\left[O_A \hat{\rho}\right]$$

$$= \sum_n \sum_m d_{nm} Tr(\hat{a}^n (\hat{a}^{+})^m \hat{\rho})$$

$$\swarrow$$

$$\int Q(\alpha,\alpha^*) O_A(\alpha,\alpha^*) d^2\alpha \longrightarrow Q(\alpha,\alpha^*) \equiv Tr\left(\hat{\rho}\delta(\alpha-\hat{a})\delta(\alpha^*-\hat{a}^{+})\right)$$

$$Q(\alpha,\alpha^*)=\frac{1}{\pi}Tr(\int d^2\alpha'\left[\hat{\rho}\delta(\alpha-\hat{a})|\alpha'\rangle\langle\alpha'|\delta(\alpha^*-\hat{a}^+)\right])$$

$$=\frac{1}{\pi}Tr(\int d^2\alpha'\hat{\rho}\delta(\alpha-\alpha')|\alpha'\rangle\langle\alpha'|\delta(\alpha^*-\alpha'^*))$$

$$=\frac{1}{\pi}Tr(\hat{\rho}|\alpha\rangle\langle\alpha|)=\frac{1}{\pi}\langle\alpha|\hat{\rho}|\alpha\rangle$$

$$Tr(\hat{\rho})=1\Rightarrow\int Q(\alpha,\alpha^*)d^2\alpha=1$$

$$Q(\alpha,\alpha^*)=\frac{1}{\pi}\int P(\alpha',\alpha^*)e^{-|\alpha-\alpha'|^2}d^2\alpha'$$

$$Q(\alpha,\alpha^*)=\frac{1}{\pi}\langle\alpha|\int P(\alpha',\alpha^*)|\alpha'\rangle\langle\alpha'|d^2\alpha'|\alpha\rangle$$

$Q(\alpha,\alpha^*)$ for $|n\rangle$ state

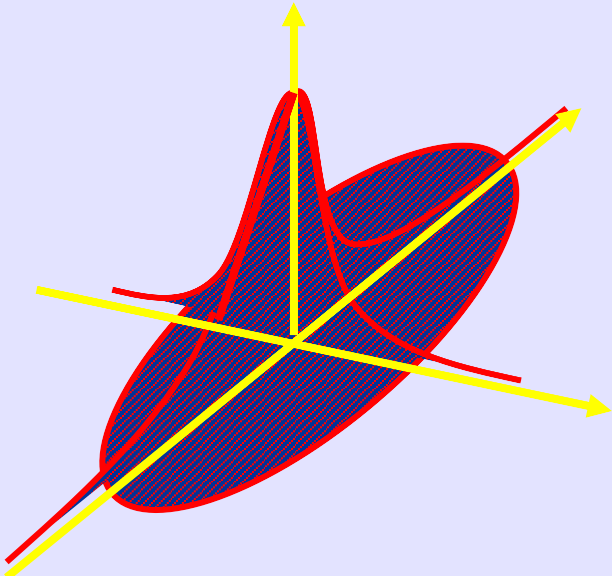
$$Q(\alpha,\alpha^*)=\frac{1}{\pi}|\langle\alpha|n\rangle|^2=\frac{e^{-|\alpha|^2}|\alpha|^{2n}}{\pi n!}\longrightarrow \text{positive}$$

Example:

Squeezed coherent state $\longrightarrow$ Q-representation

$$
\begin{aligned}
Q(\alpha,\alpha^*) &= \frac{1}{\pi}\langle\alpha|\hat{\rho}|\alpha\rangle = \frac{1}{\pi}\langle\alpha|\alpha,\xi\rangle\langle\alpha,\xi|\alpha\rangle \\
&= \frac{1}{\pi}\left|\langle\alpha|\alpha,\xi\rangle\right|^2 = \frac{1}{\pi}\left|\langle\alpha|\hat{S}(\xi)\hat{D}(\beta)|0\rangle\right|^2 \\
&= \left|\langle\alpha|\hat{S}(\xi)|\beta\rangle\right|^2
\end{aligned}
$$

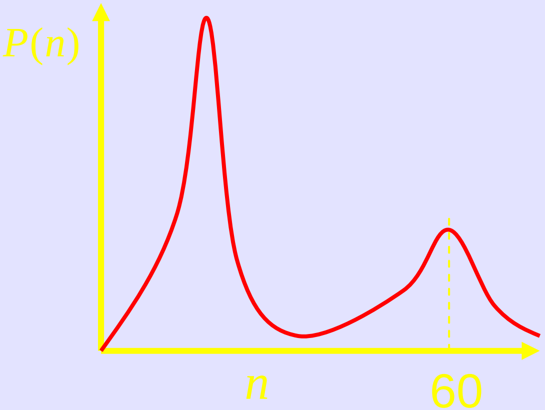
$$
\begin{aligned}
Q(\alpha,\alpha^*) = \frac{\operatorname{sech} r}{\pi}\exp\{&-(|\alpha|^2+|\beta|^2)+(\alpha^*\beta+\beta^*\alpha)\operatorname{sech} r \\
&-\frac{1}{2}[e^{i\theta}(\alpha^{*2}-\beta^{*2})+e^{-i\theta}(\alpha^2-\beta^2)]\tanh r\}
\end{aligned}
$$

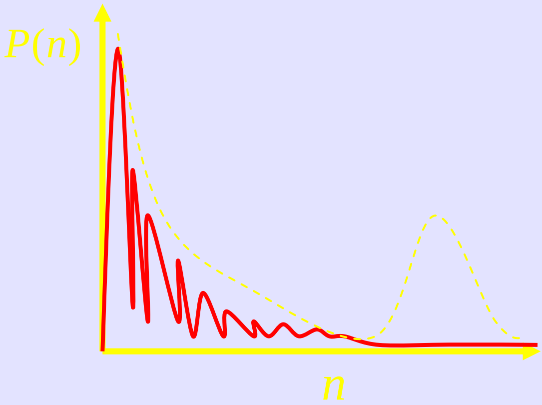


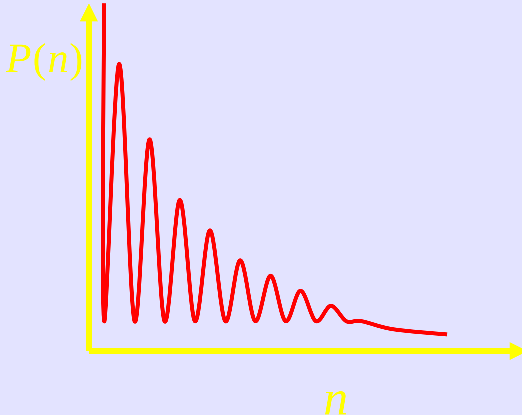
Photon distribution function

$$P(n) = \left|\langle n|\beta,\xi\rangle\right|^2 = \frac{(\tanh r)^n}{2^n n!\cosh r}\exp\left\{-|\beta|^2 + \frac{1}{2}\left[e^{-i\theta}\beta^2 + e^{i\theta}\beta^{*2}\right]\tanh r\right\}$$
$$\times\left|H_n\left(\frac{\beta e^{-i\theta/2}}{\sqrt{2\cosh r\sinh r}}\right)\right|^2$$

$|\beta|^2 \gg \sinh^2 r$ | $|\beta|^2 \ll \sinh^2 r$ | $|\beta| = 0$ squeezed vacuum







$$P(2n) = (\cosh r)^{-1}\frac{(2n)!}{(n!)^2}\left(\frac{1}{2}\tanh r\right)^{2n}$$

$$P(2n+1) = 0$$

Chap .4 field-field and photon-photon interferometry

Interferometry → ***key technique in modern optics basis***

↓

Modern metrology

Laser (invented)

↓ revolution

↓ Coherence theory development !

Two type of interferometers:

1) Field (amplitude) correlation interferometry
2) Photon (intensity) correlation interferometry

Interferometry subjects: (application region)

micro —— macrocosmos

↓ ↓

Quantum mechanics General relativity

4.1 the interferometer as a cosmic probe

Foundation of physics

(unchored)

(bedrock) curved space (minkowski)

Physical events (emission of a photon, e.g.)

+

Time arrow

Including gravitation → 4D space (time) bending → curved

Einstein theory of general relativity:

Gravity arising from curvature of 4D space

This curvature (itself)

produced by massive bodies in the universe

Theorists regard → General theory of relativity → The most beautiful

Experimentalist → Feel very scared to test

Misner, Thorne, Wheeler

"for the first half century of its life, general relativity was a theorist's paradise but an experimentalist's hell "

However, modern Laser optics changed the case

4.1.1 Michelson interferometry and general relativity

(today, a type of Michelson interferometer is being built to detect gravity waves)

Time-independent (Newtonian) gravity, the (scalar) potential Φ

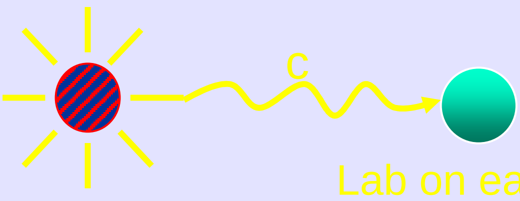
$$\nabla^2\Phi = 0$$

Time-independent (Einsteinian) metric gravity, the tensor

field $\Phi_{uv}(\vec{r},t) \quad (u,v=1,...4)$

$$(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\Phi_{uv}(\vec{r},t) = 0$$

Effects of gravity propagates with the speed of light



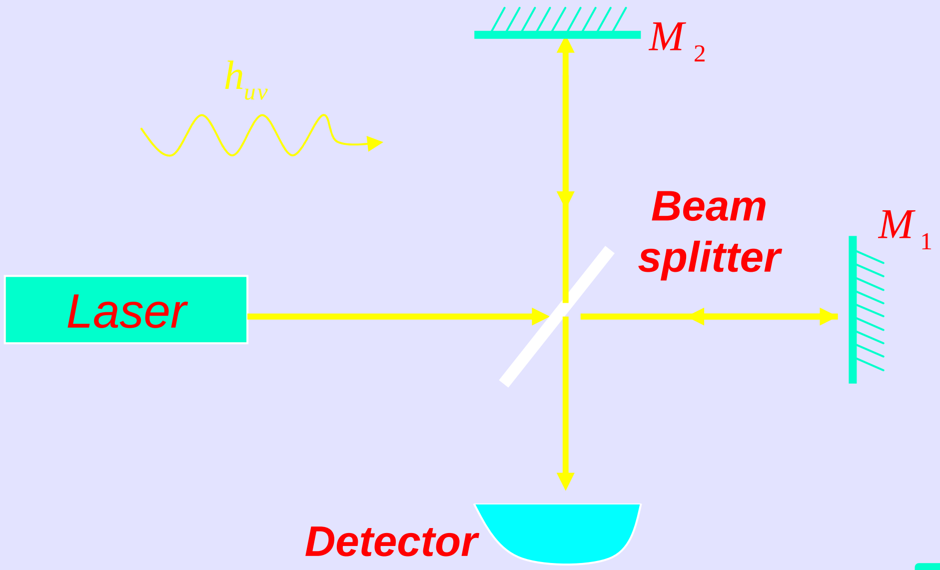
Origin (binary stars….)

GW frequency ν_g

$$L_x = L(1 + h_0 \cos \nu_g t)$$

h_0 (gravitatinal wave amplitude) $\leq 10^{-21}$

Gravitational wave → tiny oscillation of instruments

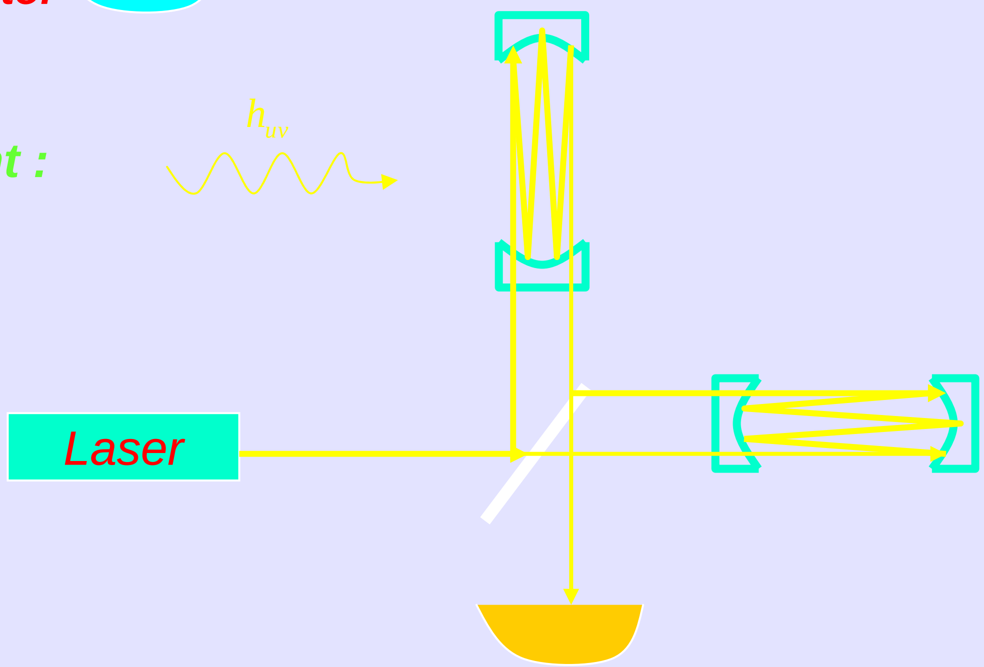


Phase shift: due to gravitational wave

$$\delta = k(L_x - L_y) = kL(1 + h_0 \cos \nu_g t) - L$$
$$= kLh_0 \cos \nu_g t$$

$$I_D = \frac{1}{2} I_0 (1 + \cos \delta)$$

Real experiment :



Fundamental quantum limit : (what can bury the signal)

(photon — shot — noise)

- *Laser average photon number* $\bar{n}$
- *Power at detector P*
- *Quantum efficiency* $\eta = 1$

Phase uncertainty due to shot noise :

$$\Delta\theta_n \approx \frac{1}{\sqrt{\bar{n}}} \approx \sqrt{\frac{1}{\frac{Pt_m}{\hbar\nu}}}$$

$\langle \Delta\bar{n} \rangle = \sqrt{\bar{n}}$ coherent state laser

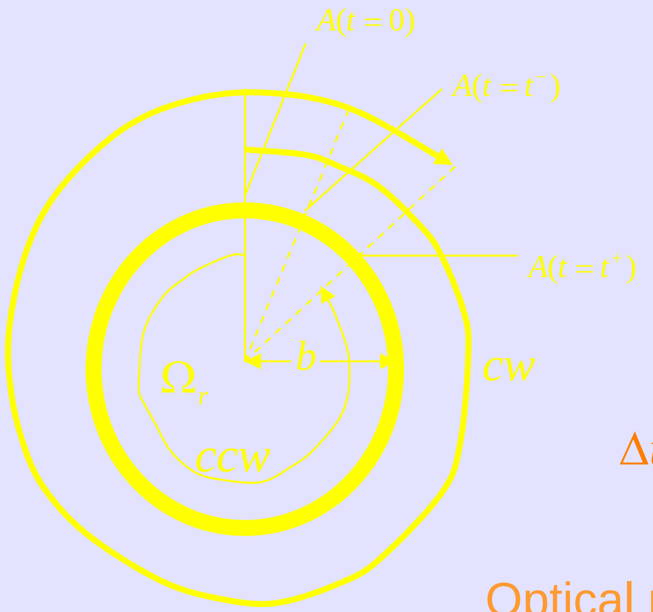
$\bar{P}t_m = \bar{n} = \bar{P}'t_m'$

minimum measurable signal $\Delta\theta^{(p)} \geq \Delta\theta_n$

$$\nu\tilde{L}h_0 / c \approx \sqrt{\frac{\hbar\nu}{Pt_m}} \qquad h_{\min} \sim \frac{c}{\nu\tilde{L}}\sqrt{\frac{\hbar\nu}{Pt_m}}$$

4.1.2 The Sagnac ring interferometer

(1913, Sagnac) ⟶ phase drift due rotating frame!



cw beam $$t^{(+)} = \frac{2\pi b + b\Omega_r t^{(+)}}{c}$$

ccw beam $$t^{(-)} = \frac{2\pi b - b\Omega_r t^{(-)}}{c}$$

$$\Delta t = t^{(+)} - t^{(-)} = \frac{4\pi b^2\Omega_r}{c^2 - b^2\Omega_r^2} \approx \frac{4\pi b^2\Omega_r}{c^2} \quad (b^2\Omega_r \ll c^2)$$

Optical path difference $$\Delta L = c\Delta t = \frac{4\pi b^2\Omega_r}{c^2}$$

Phase shift after one round trip $$\Delta\theta = k\Delta L = \frac{2\pi\Delta L}{\lambda} = \frac{8\pi^2 b^2\Omega_r}{c\lambda} = k\frac{A\Omega_r}{c}$$

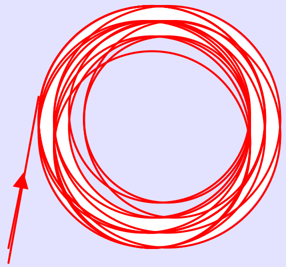
$b = 1m, \quad \Omega_r = 10\,\mathrm{deg}/h$

$\Delta L = 4 \times 10^{-12}\,m$ → Seem too small

↓ $\Delta\theta$ ~ very small

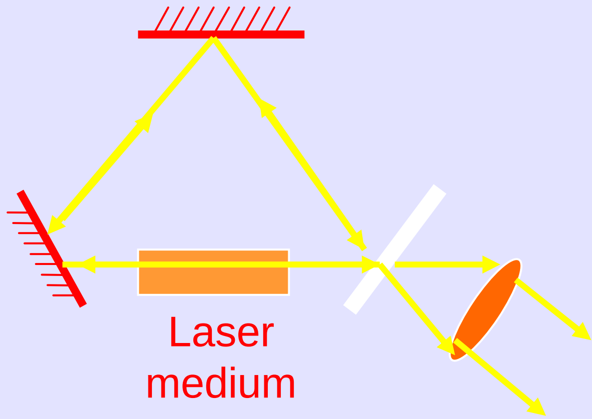
2 scheme increase the sensitivity of Laser gyros

1) 1-kilometer optical fiber



N furns $\quad \Delta L \rightarrow N\Delta L \quad \Delta\theta = \dfrac{8\pi^2 b^2 N\Omega_r}{c\lambda}$

2) Increase signal (using ring Laser)



cw beam

ccw beam

$$\left.\right\} \Omega_r = 0 \qquad \nu_{cw} = \nu_{ccw} = \nu$$

$$\Omega_r \neq 0$$

Cavity resonance condition

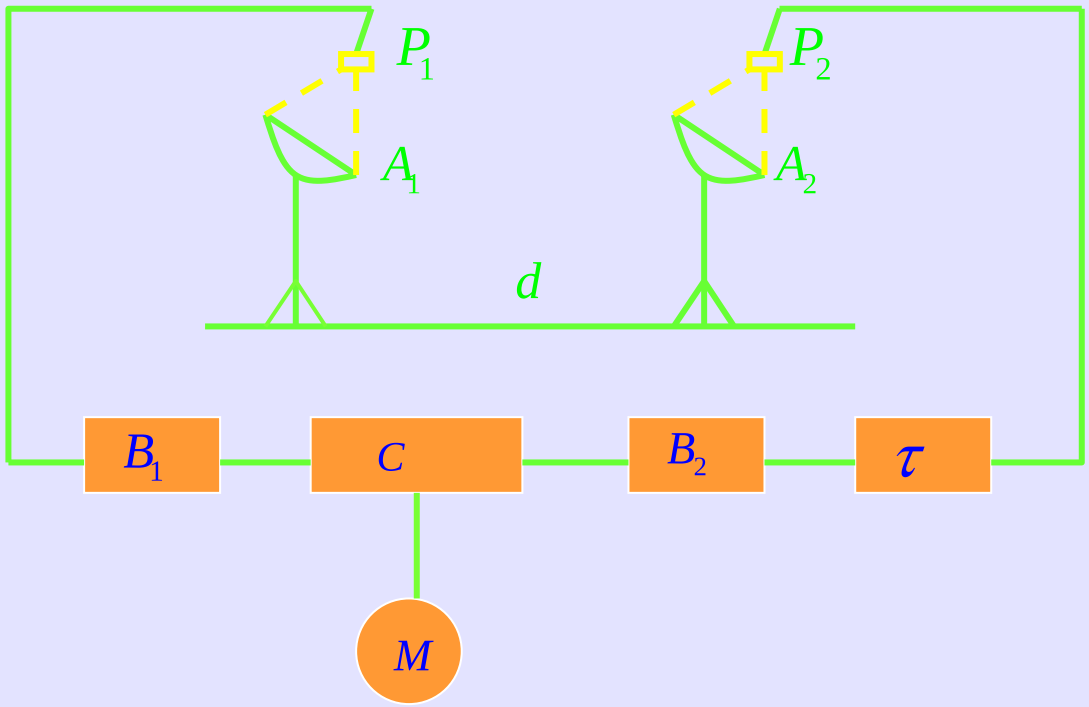
$$\begin{cases} \nu_{cw} = \dfrac{m\pi c}{L_+} \\ \nu_{ccw} = \dfrac{m\pi c}{L_-} \end{cases}$$

$$L_\pm = L(1 \pm \frac{b\Omega_r}{c}) \qquad (\Delta L = L_+ - L_-)$$

$$\Delta\nu = \nu_- - \nu_+ = \frac{m\pi c \Delta L}{L^2} = \nu \frac{\Delta L}{L} \qquad (L_+ L_- \approx L)$$

$$\Delta\nu = \nu \frac{2b\Omega_r}{c}$$

Hanbury - Brown - Twiss interferometer



$$I(\vec{r}_i, t) = \kappa \left\{ \left|E_{\vec{k}}\right|^2 + \left|E_{\vec{k}'}\right|^2 + [E_{\vec{k}} E^*_{\vec{k}'} e^{i(\vec{k}-\vec{k}')\cdot\vec{r}_i} + c.c] \right\} \qquad (i = 1, 2)$$

$$\left\langle I(\vec{r}_1, t) I(\vec{r}_2, t) \right\rangle = \kappa^2 \left\{ \left\langle \left(\left|E_{\vec{k}}\right|^2 + \left|E_{\vec{k}'}\right|^2 \right)^2 \right\rangle + \left\langle \left|E_{\vec{k}}\right|^2 \right\rangle \left\langle \left|E_{\vec{k}'}\right|^2 \right\rangle \left[e^{i(\vec{k}-\vec{k}')\cdot(\vec{r}_1-\vec{r}_2)} + c.c \right] \right\}$$

4.2 photon detection and quantum coherence

$$\vec{E}(\vec{r},t) = \vec{E}^{(+)}(\vec{r},t) + \vec{E}^{(-)}(\vec{r},t)$$

$$\sum_{\vec{k}} \hat{\in}_{\vec{k}} \ \varepsilon_{\vec{k}} a_{\vec{k}} e^{-i\nu_k t + i\vec{k}\cdot\vec{r}}$$

Linearly polarized

Probability $\longrightarrow$ ***detector atom for absorbing a photon from the field at position r between t*** $\rightarrow$ ***t + dt***

$$W_1(\vec{r},t) \propto \left| \langle f \,|\, \vec{E}^{(+)}(\vec{r},t) \,|\, i \rangle \right|^2$$

$|i\rangle$ $\longrightarrow$ Initial state of the field (before detection)

$\langle f|$ $\longrightarrow$ final state after detection

Never measured

$$W_1(\vec{r},t) \propto \left| \langle f | \vec{E}^{(+)}(\vec{r},t) | i \rangle \right|^2$$

$$W_1(\vec{r},t) = \sum_f \left| \langle f | \vec{E}^{(+)}(\vec{r},t) | i \rangle \right|^2$$

$$= \sum_f \langle i | \vec{E}^{(-)}(\vec{r},t) | f \rangle \langle f | \vec{E}^{(+)}(\vec{r},t) | i \rangle$$

$$= \langle i | \vec{E}^{(-)}(\vec{r},t) \vec{E}^{(+)}(\vec{r},t) | i \rangle$$

In reality, we have no precise knowledge of the field

$\longrightarrow$ ***statistical average***

$$W_1(\vec{r},t) = \sum_i P_i \langle i | \vec{E}^{(-)}(\vec{r},t) \vec{E}^{(+)}(\vec{r},t) | i \rangle$$

Introduce : $\hat{\rho} = \sum_i P_i | i \rangle \langle i |$

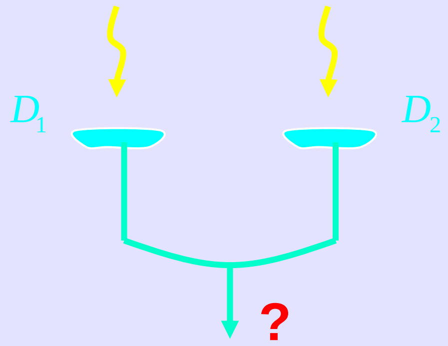
$$W_1(\vec{r},t) = Tr[\hat{\rho} \vec{E}^{(-)}(\vec{r},t) \vec{E}^{(+)}(\vec{r},t)]$$

define : 1st order correlation function

$$W_1(\vec{r},t) = Tr[\hat{\rho} E^{(-)}(\vec{r},t) E^{(+)}(\vec{r},t)]$$

$$G^{(1)}(\vec{r}_1,\vec{r}_2,t_1,t_2) = \left\langle E^{(-)}(\vec{r}_1,t_1) E^{(+)}(\vec{r}_2,t_2) \right\rangle = Tr(\hat{\rho} E^{(-)}(\vec{r}_1,t_1) E^{(+)}(\vec{r}_2,t_2))$$

Two photo detectors at $\vec{r}_1, \vec{r}_2 \longrightarrow$ *joint counting*



Observing 1 photoionization at $\vec{r}_2$ between $t_2 \rightarrow t_2 + dt_2$ and another one at $\vec{r}_1, t_1 \rightarrow t_1 + dt_1$

$$W_2(\vec{r}_1, t_1, \vec{r}_2, t_2) = \left| \langle f \,|\, E^{(+)}(\vec{r}_2, t_2) E^{(+)}(\vec{r}_1, t_1) \,|\, i \rangle \right|^2$$

$$W_2(\vec{r}_1, t_1, \vec{r}_2, t_2) = Tr(\hat{\rho} E^{(-)}(\vec{r}_1, t_1) E^{(-)}(\vec{r}_2, t_2) E^{(+)}(\vec{r}_2, t_2) E^{(+)}(\vec{r}_1, t_1))$$

Second-order quantum (mechanical) correlation function

$$G^{(2)}(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4, t_1, t_2, t_3, t_4) = Tr(\hat{\rho} E^{(-)}(\vec{r}_1, t_1) E^{(-)}(\vec{r}_2, t_2) E^{(+)}(\vec{r}_3, t_3) E^{(+)}(\vec{r}_4, t_4))$$

nth-order:

$$G^{(n)}(\vec{r}_1 \ldots \vec{r}_{n,} t_1 \ldots t_n) = Tr(\hat{\rho} E^{(-)}(\vec{r}_1, t_1) \ldots E^{(-)}(\vec{r}_n, t_n) E^{(+)}(\vec{r}_{n+1}, t_{n+1}) \ldots E^{(+)}(\vec{r}_{2n}, t_{2n}))$$

Any photon detection (based in multi-photon effect)

Normal order (operator)

e.g.

$$I(\vec{r},t) = \left\langle \vec{E}^{(-)}(\vec{r},t) \vec{E}^{(+)}(\vec{r},t) \right\rangle$$

$$= G^{(1)}(\vec{r},\vec{r},0) \left\langle \underbrace{E^{(-)}(\vec{r},t)}_{\hat{a}^{+}} E^{(-)}(\vec{r},t) \underbrace{E^{(+)}(\vec{r},t)}_{\hat{a}} E^{(+)}(\vec{r},t) \right\rangle$$

Differ from $\left\langle \hat{I}(\vec{r},t) \hat{I}(\vec{r},t) \right\rangle$

1st-degree of coherence at $\vec{r}$,2nd …

$$g^{(1)}(\vec{r},t)=\frac{\left\langle E^{(-)}(\vec{r},t)E^{(+)}(\vec{r},t+\tau)\right\rangle}{\sqrt{\left\langle E^{(-)}(\vec{r},t)E^{(+)}(\vec{r},t)\right\rangle\left\langle E^{(-)}(\vec{r},t+\tau)E^{(+)}(\vec{r},t+\tau)\right\rangle}}$$

$$g^{(2)}(\vec{r},t)=\frac{\left\langle E^{(-)}(\vec{r},t)E^{(-)}(\vec{r},t+\tau)E^{(+)}(\vec{r},t+\tau)E^{(+)}(\vec{r},t)\right\rangle}{\left\langle E^{(-)}(\vec{r},t)E^{(+)}(\vec{r},t)\right\rangle\left\langle E^{(-)}(\vec{r},t+\tau)E^{(+)}(\vec{r},t+\tau)\right\rangle}$$

1) ***Normal ordering (operator)***

2) ***Time ordering***

Single-mode:

$$g^{(1)}(\tau)=\frac{\left\langle \hat{a}^{+}(t)\hat{a}(t+\tau)\right\rangle}{\left\langle \hat{a}^{+}\hat{a}\right\rangle}$$

$$g^{(2)}(\tau)=\frac{\left\langle \hat{a}^{+}(t)\hat{a}^{+}(t+\tau)\hat{a}(t+\tau)\hat{a}(t)\right\rangle}{\left\langle \hat{a}^{+}\hat{a}\right\rangle^{2}}$$

Normal-ordered correlation function $\longrightarrow$ ***P-representation***

Classical coherence $\longleftrightarrow$ *quantum coherence*

$P(\alpha,\alpha^*)$

Thermal coherent fields

$$P(\alpha,\alpha^*) = \frac{1}{\pi\langle n\rangle}\exp(-|\alpha|^2/\langle n\rangle)$$

$$\Downarrow$$

$$g^{(2)}(0) = \frac{\int P(\alpha,\alpha^*)|\alpha|^4 d^2\alpha}{\left[\int P(\alpha,\alpha^*)|\alpha|^2 d^2\alpha\right]^2} = 2$$

Laser above threshold $\sim\ |\alpha_0\rangle$ $\quad P(\alpha,\alpha^*) = \delta^{(2)}(\alpha-\alpha_0)$

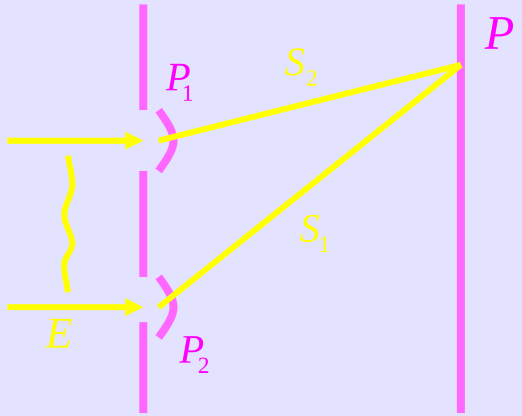
$$g^{(2)}(0) = 1$$

Coherent state $\quad g^{(1)} = g^{(2)} = \cdots = g^{(n)}$

4.3 first-order coherence and Young-type double-source experiments

Young's double-slit experiment

Classic experiments → ***1st-order coherence***



$$E^{+}_{(p)}(\vec{r},t) = K_1 E^{(+)}(\vec{r}_1, t-t_1) + K_2 E^{(+)}(\vec{r}_2, t-t_2)$$

Transmit coefficient

(depends on the geometry and size of the slits)

Photodetector at point P

$$\left\langle I(\vec{r},t)\right\rangle = Tr\left[\hat{\rho}E_P^{(-)}(\vec{r},t)E_P^{(+)}(\vec{r},t)\right]$$

Statistically

stationary field

$$\left\langle I(\vec{r},t)\right\rangle = |K_1|^2 G^{(1)}(\vec{r}_1,\vec{r}_1,0) + |K_2|^2 G^{(1)}(\vec{r}_2,\vec{r}_2,0) + 2\,\mathrm{Re}[K_1^* K_2 G^{(1)}(\vec{r}_1,\vec{r}_2,\tau)]$$

$$\tau = t_1 - t_2 = \frac{s_1 - s_2}{c}$$

define

$$g^{(1)}(\vec{r}_1,\vec{r}_2,\tau) = \frac{G^{(1)}(\vec{r}_1,\vec{r}_2,\tau)}{\sqrt{G^{(1)}(\vec{r}_1,\vec{r}_1,\tau)G^{(1)}(\vec{r}_2,\vec{r}_2,\tau)}}$$

$$\left\langle I^{(n)}(\vec{r})\right\rangle = |K_i|^2 G^{(1)}(\vec{r}_i,\vec{r}_i,0)$$

$$\left\langle I(\vec{r},t)\right\rangle = \left\langle I^{(1)}(\vec{r})\right\rangle + \left\langle I^{(2)}(\vec{r})\right\rangle + 2\left[\left\langle I^{(1)}(\vec{r})\right\rangle\left\langle I^{(2)}(\vec{r})\right\rangle\right]^{1/2} \mathrm{Re}\left[g^{(1)}(\vec{r}_1,\vec{r}_2,\tau)\right]$$

if $\quad g^{(1)}(\vec{r}_1,\vec{r}_2,\tau) = \left|g^{(1)}(\vec{r}_1,\vec{r}_2,\tau)\right| e^{i\alpha(\vec{r}_1,\vec{r}_2,\tau) - i\nu_0\tau} \qquad \alpha(\vec{r}_1,\vec{r}_2,\tau) = \arg(g^{(1)}) + \nu_0\tau$

$$\left\langle \hat{I}(\vec{r},t)\right\rangle = \left\langle I^{(1)}(\vec{r})\right\rangle + \left\langle I^{(2)}(\vec{r})\right\rangle + 2\left[\left\langle I^{(1)}(\vec{r})\right\rangle\left\langle I^{(2)}(\vec{r})\right\rangle\right]^{1/2} \left|g^{(1)}(\vec{r}_1,\vec{r}_2,\tau)\right| \cos\left[\alpha(\vec{r}_1,\vec{r}_2,\tau) - \nu_0\tau\right]$$