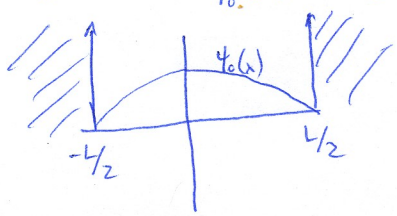
PHYSICS 235 HW7 SOLUTIONS

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18.2.3 The ground state is $\psi_0(x) = \sqrt{\frac{2}{L}} \cos\frac{\pi x}{L}$



Its energy is:

$$E_{ground} = \frac{\pi^2\hbar^2}{2mL}$$, which is all kinetic, because $V=0$.

Semiclassically, then, the particle has velocity $v = \sqrt{\frac{2E_{ground}}{m}} = \frac{\pi\hbar}{Lm}$

So it can travel from one wall to the other in

$$t = L/v = \frac{L^2 m}{\pi\hbar}$$

which is the characteristic time for the system.

When the potential suddenly changes, $\psi(x)$ is unchanged because the perturbation is sudden. The new ground state is

$$\psi^{new}_{ground}(x) = \sqrt{\frac{1}{L}} \cos\frac{\pi x}{2L}$$

Then $P(\text{finding particle in } \psi^{new}) = |\langle \psi^{new} | \psi_0 \rangle|$

$$= \int_{-L}^{L} \sqrt{\frac{2}{L}} \sqrt{\frac{1}{L}} \cos\frac{\pi x}{2L} \cos\frac{\pi x}{L}\, dx$$

$$= \boxed{\frac{4\sqrt{2}}{3\pi}}$$

18.2.4 The characteristic time of the e^- still bound to the atom is semiclassically approximated as $\frac{(\text{size of orbit})}{(\text{velocity of } e^-)}$. Using the Virial Thm, $-\frac{1}{2}mv^2 = \frac{-e^2Z^2}{2a_0n^2}$ where $Z=1$ for H^3. Using $\alpha = \frac{\hbar}{mca_0} = \frac{e^2}{\hbar c}$, we can solve this for $v = c\alpha Z$. The Bohr radius is a_0/Z, so $t_{\text{characteristic}} = \frac{a_0/Z}{c\alpha Z} = \frac{a_0}{Z^2\alpha c} \simeq \frac{137a_0}{c}$.

Now the emitted e^- is travelling at $v = \sqrt{\frac{2\cdot 16\,\text{keV}}{m_e}} = \sqrt{\frac{32}{511}}\,c \simeq \frac{c}{4}$

So it takes $\simeq 4a_0/c$ to exit the atom.

$\frac{137a_0}{c} \gg \frac{4a_0}{c}$, so the sudden perturbation approximation applies.

The Z for $(He^3)^+$ is 2, so the ground state wavefunction in the hydrogenic approximation is given by $\psi^0_{He^{3+}} = \sqrt{\frac{Z^3}{\pi a_0^3}}\, e^{-2r/a_0}$

The overlap with $\psi^0_{H^3}$ is

$$\frac{\sqrt{8}}{\pi a_0^3}\int_0^\infty 4\pi r^2 dr\, e^{-2r/a_0} e^{-r/a_0} = \boxed{\frac{16\sqrt{2}}{27}}.$$

The probability to be in $|n=16, \ell=3, m=0\rangle$ for He^{3+} is **zero**, because the two states are of opposite parity ($(-1)^{\ell=0} = 1$, $(-1)^{\ell=3} = -1$.) Equivalently, because they have different total orbital A.M. ℓ, and such states are orthogonal.

18.3.1 The propagator U_S is defined so that in the Schrödinger picture, $|\psi_S(t)\rangle = U_S(t,t_0)|\psi_S(t_0)\rangle$. In the Heisenberg picture, $|\psi_H(t)\rangle \equiv |\psi_H(t_0)\rangle = |\psi_S(t_0)\rangle$
$= U^\dagger |\psi_S(t)\rangle$,

since U is unitary.

$$\Rightarrow i\hbar\partial_t|\psi_S(t)\rangle = i\hbar\partial_t U_S|\psi_S(t_0)\rangle = i\hbar(\partial_t U_S)|\psi_S(t_0)\rangle$$
$$= H_S|\psi_S(t)\rangle = H_S U_S|\psi_S(t_0)\rangle$$

$$\Rightarrow i\hbar\partial_t U_S = H_S U_S$$

similarly, $-i\hbar\partial_t U_S^\dagger = U_S^\dagger H_S$ (just take adjoint of both sides.)

Heisenberg operators are defined by their Schrödinger-picture counterparts as $\Omega_H(t) = U_S^\dagger \Omega_S U_S$ (this follows from $|\psi_H\rangle = U^\dagger|\psi_S\rangle$.)

Taking $i\hbar\partial_t$ of both sides,

$$i\hbar\partial_t \Omega_H(t) = i\hbar\left[(\partial_t U_S^\dagger)\Omega_S U_S + U_S^\dagger \Omega_S(\partial_t U_S)\right]$$
$$= i\hbar\left[-U_S^\dagger H_S \Omega_S U_S + U_S^\dagger \Omega_S H_S U_S\right]$$
$$= i\hbar U_S^\dagger[\Omega_S H_S - H_S\Omega_S]U_S \times \frac{1}{i\hbar}$$
$$= \cancel{i\hbar} U_S^\dagger[\Omega_S, H_S]\frac{1}{\cancel{i\hbar}}U_S$$
$$= [\Omega_H, H_H] \;//$$