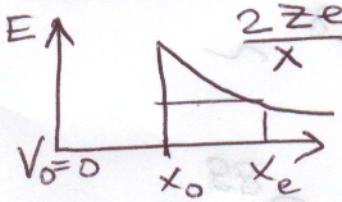
Homework #4
PH206

16.2.4



$E = \frac{2Ze^2}{x_e}$; $x_e = \frac{2Ze^2}{E}$.

$$\gamma = \frac{2}{\hbar}\int_{x_0}^{x_e}\sqrt{2m(V(x)-E)}\,dx = \frac{2\sqrt{2m}}{\hbar}\int_{x_0}^{x_e}\sqrt{\frac{2Ze^2}{x}-\frac{2Ze^2}{x_e}}\,dx =$$

$$= \frac{2\sqrt{2m}\sqrt{2Ze^2}}{\hbar}\int_{x_0}^{x_e}\sqrt{\frac{1}{x}-\frac{1}{x_e}}\,dx = \frac{2\sqrt{2m\,2Ze^2}}{\hbar}\int_{x_0}^{x_e}\sqrt{1-\frac{x}{x_e}}\,\frac{dx}{\sqrt{x}} =$$

$$= \frac{4\sqrt{2m\,2Ze^2}}{\hbar}\int_{x_0}^{x_e}\sqrt{1-\frac{x}{x_e}}\,d\sqrt{x} = \frac{4\sqrt{2m\cdot 2Ze^2\cdot x_e}}{\hbar}\int_{\sqrt{\frac{x_0}{x_e}}}^{1}\sqrt{1-t^2}\,dt =$$

$$= \frac{4(2Ze^2)}{\hbar}\sqrt{\frac{2m}{E}}\int_{0}^{\cos^{-1}\sqrt{\frac{x_0}{x_e}}}\sin\upsilon\,d(-\cos\upsilon) =$$

$$= \frac{16Ze^2}{\hbar\upsilon}\int_{0}^{\cos^{-1}\sqrt{\frac{x_0}{x_e}}}\sin^2\upsilon\,d\upsilon = \frac{8Ze^2}{\hbar\upsilon}\int_{0}^{\cos^{-1}\sqrt{\frac{x_0}{x_e}}}(1-\cos 2\upsilon)\,d\upsilon =$$

$$= \frac{8Ze^2}{\hbar\upsilon}\left[\cos^{-1}\sqrt{\frac{x_0}{x_e}} - \sqrt{\frac{x_0}{x_e}}\sqrt{1-\frac{x_0}{x_e}}\right].$$

$$\left[\frac{8Ze^2}{\hbar\upsilon}\right] = \left[\frac{e^2}{\hbar c}\right] = [1].$$

$$\frac{8Ze^2}{\hbar\upsilon} = 8\cdot 90\,\frac{e^2}{\hbar c}\,\frac{c}{\upsilon} = \frac{720}{137}\sqrt{\frac{mc^2}{2\,\frac{m\upsilon^2}{2}}} =$$

$$= \frac{720}{137}\sqrt{\frac{2m_pc^2}{E}} = \frac{720}{137}\sqrt{2\,\frac{938\,MeV}{4.2\,MeV}} = 111.$$

$$y = \frac{x_0}{x_e} = \frac{x_0E}{2Ze^2} = \frac{10^{-12}\,cm\cdot 4.2\cdot 10^6\cdot 1.16\cdot 10^4\cdot 1.38\cdot 10^{-16}\,erg}{1.8\cdot 10^2\cdot (4.8)^2\cdot 10^{-20}\,cgs^2}$$

$$= \frac{10^{-18}\cdot 4.2\cdot 1.16\cdot 1.38}{10^{-18}\cdot 1.8\cdot (4.8)^2} = \frac{4.2\cdot 1.16\cdot 1.38}{1.8\cdot (4.8)^2} = 0.16 \ll 1$$

$\sqrt{y} = 0.4$

$$\cos^{-1}\sqrt{y} = \frac{\pi}{2} - \sin^{-1}\sqrt{y} = \frac{\pi}{2} - \sqrt{y} - \frac{y^{3/2}}{6} \approx 1.57 - 0.40 - 0.01 \approx$$

≈ 1.16

$$\left[\cos^{-1}\sqrt{y} - \sqrt{y}\sqrt{1-y}\right] \approx 1.16 - 0.37 \approx 0.8.$$

$\gamma \approx 89.$

$$\tau = \frac{1}{R} \sim \frac{2m x_0}{\sqrt{2mE}} e^{\gamma} = \frac{x_0}{\sqrt{\frac{E}{2m}}} e^{\gamma} = x_0 \sqrt{\frac{2m}{E}}\, e^{\gamma} =$$

$$= 10^{-12}\,\text{cm} \sqrt{\frac{2 \cdot 4 \cdot 1.6 \cdot 10^{-24}\,\text{g}}{4.2 \cdot 10^{6} \cdot 1.16 \cdot 10^{4} \cdot 1.38 \cdot 10^{-16}\,\text{erg}}}\, e^{\gamma} \sim$$

$$\sim 10^{-12}\,\text{cm} \sqrt{\frac{10^{-24}}{10^{-6}}\,\frac{\text{g}}{\text{cm}}}\, e^{\gamma} = 10^{-12} \cdot 10^{-9}\,\text{s}\, e^{89} =$$

$$= 10^{-21} e^{89}\,\text{s} \sim \underline{10^{18}\,\text{s}} \left(\sim 10^{10}\,\text{years}\right)$$

16.2.5. $\mu = \frac{m^2}{2m} = m/2.$

$$p = \sqrt{2\mu(E - V(z))} = \sqrt{2\mu k (z_{max} - z)}.$$

$$\int_0^{z_{max}} p(z)\,dz = \sqrt{2\mu k} \int_0^{z_{max}} (z_{max} - z)^{1/2}\,dz = \frac{2}{3}\sqrt{2\mu k}\, z_{max}^{3/2} =$$

$$= \frac{2}{3} \frac{\sqrt{2\mu k}}{k^{3/2}} (k z_{max})^{3/2} = \frac{2}{3} \frac{\sqrt{2\mu}}{k} (E - V_0)^{3/2} = \left(n + \frac{3}{4}\right)\pi\hbar$$

$$\frac{(E_0 - V_0)^{3/2}}{(E_1 - V_0)^{3/2}} = \frac{3/4}{1 + 3/4} = \frac{3}{7}$$

$$\frac{E_0 - V_0}{E_1 - V_0} = \left(\frac{3}{7}\right)^{2/3}; \quad E_0 - V_0 = \left(\frac{3}{7}\right)^{2/3} (E_1 - V_0);$$

$$V_0\left(1 - \left(\frac{3}{7}\right)^{2/3}\right) = E_0 - \left(\frac{3}{7}\right)^{2/3} E_1$$

$$V_0 = \frac{E_0 - \left(\frac{3}{7}\right)^{2/3} E_1}{1 - \left(\frac{3}{7}\right)^{2/3}} \approx \frac{E_0 - 0.75\,E_1}{0.25} = 4E_0 - 3E_1$$

$$= (12.4 - 11.1)\,\text{eV} = \underline{1.3\,\text{eV}}$$

$$\frac{(E_1 - V_0)^{3/2}}{(E_2 - V_0)^{3/2}} = \frac{7/4}{11/4} = \frac{7}{11}$$

$$\frac{E_1 - V_0}{E_2 - V_0} = \left(\frac{7}{11}\right)^{2/3} \approx 0.86;$$

$$E_1 - V_0 = 0.86\,E_2 - V_0\,0.86;$$

$$0.14\,V_0 = E_1 - 0.86\,E_2; \quad E_2 = \frac{E_1 - 0.14\,V_0}{0.86} = \underline{4.1}$$

[redacted]

Hw 4-2

16.2.7 $E = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}$

$$\left(n+\tfrac{1}{2}\right)\hbar\pi = \int_{-x_{max}}^{x_{max}} p\,dx = \int_{-x_{max}}^{x_{max}} \sqrt{2m\left(E - \frac{m\omega^2 x^2}{2}\right)}\,dx =$$

$$= \sqrt{2mE}\; 2\int_0^{x_{max}} \sqrt{1 - \frac{m\omega^2 x^2}{2E}}\,dx = 2\sqrt{2mE}\int_0^{x_{max}} \sqrt{1 - \frac{x^2}{x_{max}^2}}\,dx =$$

$$= 2\sqrt{2mE}\; x_{max}\int_0^1 \sqrt{1-z^2}\,dz = 2\sqrt{2mE}\; x_{max}\int_0^{\frac{\pi}{2}} \cos\upsilon\, d\sin\upsilon =$$

$$= 2\sqrt{2mE}\; x_{max}\frac{\pi}{4} = \frac{\pi}{2}\sqrt{2mE}\; x_{max}.$$

$$\frac{m\omega^2 x_{max}^2}{2} = E; \quad x_{max} = \sqrt{\frac{2E}{m}}\,\frac{1}{\omega}.$$

$$\hbar\pi\left(n+\tfrac{1}{2}\right) = \frac{\pi}{2}\omega\sqrt{4E^2}; \quad E = \hbar\omega\left(n+\tfrac{1}{2}\right).$$

Last problem:

$$T \sim \exp\left[-\frac{2}{\hbar}\int_0^{x_{max}} \sqrt{2m(U(x)-E)}\,dx\right]$$

$$U_0 - F x_{max} = E; \quad x_{max} = \frac{U_0 - E}{F}$$

$$T \sim \exp\left[-\frac{2\sqrt{2m}}{\hbar}\int_0^{x_{max}} \sqrt{F(x_{max}-x)}\,dx\right] =$$

$$= \exp\left[-\frac{2\sqrt{2m}}{\hbar F}\int_0^{Fx_{max}} \sqrt{F(x_{max}-x)}\,d(y=Fx)\right] =$$

$$= \exp\left[-\frac{2\sqrt{2m}}{\hbar F}\int_0^{U_0-E} \sqrt{y}\,dy\right] = \exp\left[-\frac{4}{3}\frac{\sqrt{2m}}{\hbar F}(U_0-E)^{3/2}\right].$$