Chapter 12

Superstring Interactions

Following *"String Theory"* by J. Polchinski, Vol.II.
Notes written by students (work still in progress).
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Low Energy Physics

This low energy means compared it to Plank's constant or $E << M_{pl}$, massless modes dominate.

Type IIA

$(NS+, NS+)$ $(R+, NS+)$ $(NS+, R-)$ $(R+, R-)$
The state in $NS+$ sector can be writen in the form of $A_{\mu\nu}\psi^{\mu}_{-1/2}\psi^{\nu}_{-1/2}|0;k>$ where the subscription μ and ν represent

$$8 \times 8 = 1 + 28 + 35.$$

1, 28 and 35 represent a scalar, antisymmetric tensors, $B_{\mu\nu}$ and symmetric tensors, $g_{\mu\nu}$ respectly. The metric tensors $g_{\mu\nu}$ are calculated from an action

$$S_g = \frac{1}{4\pi G_{10}} \int d^{10}x \sqrt{-detg_{\mu\nu}}\ R. \tag{12.1}$$

The antisymmetric tensor $B_{\mu\nu}$ can be considered as a field stregth, eg .$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ with $F^2 = F_{\mu\nu}F^{\mu\nu}$. For rank three antisymmetric tensor, $H_{\mu\nu\rho}$ can be writen in term of $B_{\mu\nu}$ as

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} - \partial_\nu B_{\mu\rho} + \partial_\rho B_{\mu\nu} \tag{12.2}$$

and it is calculted from an action

$$S_B = -\frac{1}{8\pi G_{10}} \int d^{10}x \sqrt{-detg_{\mu\nu}} H_{\mu\nu\rho}H^{\mu\nu\rho} \tag{12.3}$$

and the transformation is

$$H^{\mu\nu\rho} = g^{\mu\mu'}g^{\nu\nu'}g^{\rho\rho'}H_{\mu'\nu'\rho'}. \tag{12.4}$$

For $(R+, R-)$, the state vector can be writen in term of $|\vec{S}, \vec{S}'; k>$, eg. in ground state $|k>$

$$\psi^{\nu}_m\ (\psi^{\nu}_0|k>) = 0, \quad m > 0.$$

That is one can introduce $\psi_0^\mu\psi_0^\nu|k>$ or $\psi_0^\mu\psi_0^\nu\psi_0^\rho|k>$ as ground states and contract with tensors by

$$\begin{array}{ccc} C_\mu\psi_0^\mu|k> & & C_{\mu\nu\rho}\psi_0^{[\mu,}\psi_0^{[\nu,}\psi_0^{\rho]]}|k> \\ 8 & + & 56(irreducible) = 8\times 8 = 64. \end{array}$$

ψ_0^μ, ψ_0^ν and ψ_0^ρ transform among themselves. For R+ sector let

$$F_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu \tag{12.5}$$
$$F_{\mu\nu\rho\sigma} = \partial_\mu C_{\nu\rho\sigma} + \partial_\nu C_{\rho\sigma\mu} + \partial_\rho C_{\sigma\mu\nu} + \partial_\sigma C_{\mu\nu\rho} \tag{12.6}$$
$$\tilde{F}_{\mu\nu\rho\sigma} = F_{\mu\nu\rho\sigma} - \frac{1}{4}\left(C_\mu H_{\nu\rho\sigma} + C_\nu H_{\rho\sigma\mu} + C_\rho H_{\sigma\mu\nu} + C_\sigma H_{\mu\nu\rho}\right) \tag{12.7}$$

and the action for R sector and action for $Chern-Simons$ term(p-form potentials or topolgical action no dependence on the local geometry) are

$$S_R = -\frac{1}{8\pi G_{10}}\int d^{10}x\sqrt{-detg_{\mu\nu}}\left(F_{\mu\nu}F^{\mu\nu} + \tilde{F}_{\mu\nu\rho\sigma}\tilde{F}^{\mu\nu\rho\sigma}\right) \tag{12.8}$$
$$S_{CS} = -\frac{1}{8\pi G_{10}}\int d^{10}x\epsilon^{\mu_1\ldots\mu_{10}}B_{\mu_1\mu_2}F_{\mu_3\mu_4\mu_5\mu_6}F_{\mu_7\mu_8\mu_9\mu_{10}} \tag{12.9}$$

and the total action for R-sector is

$$S = S_g + S_B + S_R + R_{CS} + (\text{super partner}). \tag{12.10}$$

An example in electromagnetic case with field streght $F_{\mu\nu}$ is

$$\begin{aligned} \tilde{F}^{\rho\sigma} &= \epsilon^{\mu\nu\rho\sigma}F_{\mu\nu} \\ F^{\mu\nu}\tilde{F}_{\mu\nu} &= 2\vec{E}\cdot\vec{B} \\ S_{CS} &= \int d^4x\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \\ S &= \int d^4x\tilde{F}^{\rho\sigma}\tilde{F}_{\rho\sigma}. \end{aligned}$$

Supergravity

Superstrings give 10-dimensioal theories, however if considers Kaluza-Klein theory, it is possible to add another dimension to the theories with the same idea before and it becomes 11-dimensioal theories, Supergravity. This still means that nothing depends on 11^{th} dimension and the duality still applies, R(radius of 11^{th}D) $\to 0 \sim R \to \infty$. This is the fundamental theory in 11D, called M-theory. The action for supergravity can be writen as

$$\begin{aligned} S_{11}^{SUGRA} = & \frac{1}{4\pi G_{11}}\int d^{11}x\sqrt{-detG_{MN}}\left(R^{11} - \frac{1}{2}F_{MNQR}F^{MNQR}\right) \\ & -\frac{1}{24\pi G_{11}}\int d^{11}x\epsilon^{M_1,\ldots,M_{11}}A_{M_1M_2M_3}F_{M_4M_5M_6M_7}F_{M_8M_9M_{10}M_{11}} \\ & +(\text{super partner}) \end{aligned} \tag{12.11}$$

where M, $N = 0, 1, \ldots, 10$, μ, $\nu = 0, 1, \ldots, 9$ and

$$F_{MNQR} = \partial_M A_{NQR} + \partial_N A_{QRM} + \partial_Q A_{RMN} + \partial_R A_{MNQ} + (\text{super partner})$$

where $\delta A_{MNQ} = \partial_M \lambda_{NQ}$ and $\delta F_{MNQR} = 0$. The metric can be writen in term of $x^M = (x^\mu, u)$ with nothing depending on 11^{th}, $x^{10} = u$

$$\begin{aligned} ds^2 &= G_{MN}(x^\mu) dx^M dx^N && (12.12)\\ &= G_{\mu\nu} dx^\mu dx^\nu + 2G_{\mu u} dx^\mu du + G_{uu} du^2. && (12.13) \end{aligned}$$

If set $G_{uu} = 1$ for convenient, and call

$$\begin{aligned} A_\mu &= G_{\mu u} \\ g_{\mu\nu} &= G_{\mu\nu} - A_\mu A_\nu, \quad \text{then} \\ ds^2 &= g_{\mu\nu} dx^\mu dx^\nu + (du + A_\mu dx^\mu)^2. && (12.14) \end{aligned}$$

If fix one of the index with u in a 3-rank tensor, then

$$A_{\mu\nu u} = A_{\mu\nu},$$

it can be considered as 2-rank tensor. The same thing can also happen to higher rank tensors. From Kaluza-Klein theory the curvature and those tensors in 11D can be writen in form of those in 10D , ie.

$$\begin{aligned} R^{(11)} &= R^{(10)} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} && (12.15)\\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \\ F_{MNQR} F^{MNQR} &= F_{\mu\nu\rho} F^{\mu\nu\rho} + \tilde{F}_{\mu\nu\rho} \tilde{F}^{\mu\nu\rho} && (12.16)\\ F_{\mu\nu\rho} &= \partial_\mu A_{\nu\rho} + ... && (12.17)\\ F_{\mu\nu\rho\sigma} &= \partial_\mu A_{\nu\rho\sigma} + ... && (12.18)\\ \tilde{F}_{\mu\nu\rho\sigma} &= F_{\mu\nu\rho\sigma} - A_\mu F_{\nu\rho\sigma} && (12.19)\\ \epsilon^{M_1 ... M_{11}} A_{M_1 M_2 M_3} F_{M_4 M_5 M_6 M_7} F_{M_8 M_9 M_{10} M_{11}} &= \epsilon^{\mu_1 ... \mu_{10}} A_{\mu_1 \mu_2 \mu_3} F_{\mu_4 \mu_5 \mu_6} F_{\mu_7 \mu_8 \mu_9 \mu_{10}} && (12.20)\\ &= \epsilon^{\mu_1 ... \mu_{10}} A_{\mu_1} A_{\mu_2} F_{\mu_3 \mu_4 \mu_5 \mu_6} F_{\mu_7 \mu_8 \mu_9 \mu_{10}} \\ &\quad + \text{total derivative, and} && (12.21)\\ det(G_{MN}) &= det(g_{\mu\nu}) && (12.22) \end{aligned}$$

and the action for supergravity is, with $G_{10} = 2\pi R G_{11}$

$$\begin{aligned} S_{11}^{SUGRA} &= \frac{1}{4\pi G_{11}} \int d^{10}x \sqrt{-det\ g_{\mu\nu}} \Big(R^{(10)} - \frac{1}{2} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} F_{\mu\nu\rho} F^{\mu\nu\rho} - \frac{1}{2} \tilde{F}_{\mu\nu\rho\sigma} \tilde{F}^{\mu\nu\rho\sigma} \\ &\quad - \frac{1}{2} \epsilon^{\mu_1 ... \mu_{10}} A_{\mu_1} A_{\mu_2} F_{\mu_3 \mu_4 \mu_5 \mu_6} F_{\mu_7 \mu_8 \mu_9 \mu_{10}} \Big) && (12.23) \end{aligned}$$

SUGRA	type IIA
$G_{MN} \to g_{\mu\nu}$, A_μ	$g_{\mu\nu}$, C_μ
$A_{MNQ} \to A_{\mu\nu\rho}$, $A_{\mu\nu}$	$C_{\mu\nu\rho}$, $B_{\mu\nu}$.