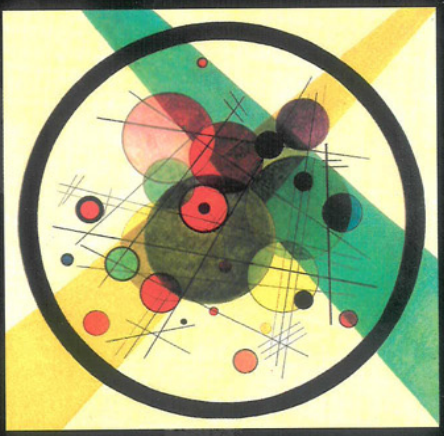

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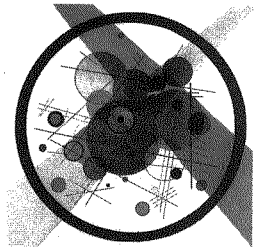

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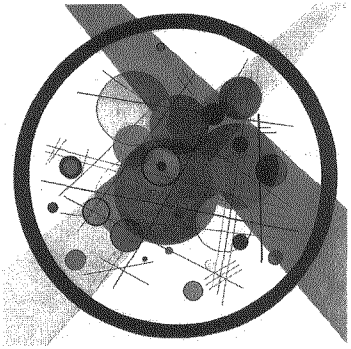

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Wassily Kandinsky
"Circles in the circle"
The Philadelphia Museum of Art

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A Gianna, Armando e Daniele (VB)

To Sandra, Alberto and Alice (PGR)

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Preface

This book is devoted to the theory and phenomenology of transverse-spin effects in high-energy hadronic physics. Contrary to common past belief, it has now become rather clear that such effects are far from irrelevant. A decade or so of intense theoretical work has shed much light on the subject and brought to surface an entire class of possible new phenomena, which now await thorough experimental investigation. Over the next few years a number of experiments worldwide (at BNL, CERN, DESY and JLAB) are due to run with transversely polarised beams and targets, providing data that will enrich our knowledge of the transverse-spin structure of hadrons. We therefore feel that now is the right moment to assess the state of the art and this will be the principal aim of the volume.

The outline is as follows. After a few introductory remarks in Chap. 1, attention is directed in Chap. 2 to polarised deeply-inelastic scattering (DIS), in particular to DIS on transversely polarised targets, which probes the transverse-spin structure function g_2. The existing data are reviewed and discussed (for completeness and comparison, a brief presentation of longitudinally polarised DIS and of the helicity structure of the proton is also provided). In Chap. 3 we illustrate the transverse-spin structure of the proton in some detail: the leading-twist and twist-three distribution functions, including $\Delta_T f$ (the transversity distribution) and g_T (the parton distribution contributing to g_2), are introduced. Model calculations of these quantities are also presented. In Chap. 4, after some general considerations on the renormalisation-group equation and the operator-product expansion, we discuss the evolution of transversity at leading and next-to-leading order in quantum chromodynamics (QCD). In Chap. 5 we study the

g_2 structure function and its related sum rules, within the framework of perturbative QCD. The last three chapters are devoted to the phenomenology of transversity, in the context of Drell–Yan processes (Chap. 6), inclusive leptoproduction (Chap. 7) and inclusive hadroproduction (Chap. 8). The interpretation of some recent single-spin asymmetry data is discussed and finally the prospects for future measurements are also reviewed.

This book is partially based on an earlier review paper written in collaboration with Alessandro Drago, to whom we are extremely grateful for a long and fruitful collaboration on the subject of transversity.

It is a pleasure to thank all the friends and colleagues who have shared with us their knowledge of high-energy spin phenomena in the course of the years. In particular, we should like to mention: Mauro Anselmino, Johannes Blümlein, Daniël Boer, Sigfrido Boffi, Elena Boglione, Alessandro Bacchetta, Umberto D'Alesio, Stefano Forte, Paolo Gambino, Bob Jaffe, Xiangdong Ji, Elliot Leader, Bo-Qiang Ma, Piet Mulders, Francesco Murgia, Enrico Predazzi, Marco Radici, Giovanni Ridolfi, Jacques Soffer, Oleg Teryaev, Werner Vogelsang, Fabian Zomer.

Torino and Como,
July 2002

Vincenzo Barone
Philip G. Ratcliffe

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Chapter 1

Introduction

The idea that transverse-spin effects should be suppressed at high energies has been a tenacious prejudice in hadronic physics over the last thirty years or so. While there is some basis to such a belief, it is far from the entire truth and is certainly misleading as a general statement.

The first point to bear in mind is the distinction between transverse polarisation itself and its measurable effects. As well-known, for example, even the ultra-relativistic electrons and positrons of the LEP storage ring are significantly polarised in the transverse plane (Knudsen *et al.*, 1991) owing to the Sokolov–Ternov effect (Sokolov and Ternov, 1964). The real problem is to identify processes sensitive to such polarisation; while this is not always easy, it is certainly not impossible.

The spin (or polarisation) vector s^μ of a high-energy particle may be decomposed into a longitudinal component $s^\mu_\parallel$, parallel to the particle momentum p^μ, and a transverse component $s^\mu_\perp$, perpendicular to p^μ:

$$s^\mu = s^\mu_\parallel + s^\mu_\perp = \frac{\lambda}{m} p^\mu + s^\mu_\perp \,. \tag{1.1}$$

Here λ is twice the helicity of the particle and m is its mass. It is clear that, since $s^2 = \lambda^2 + s^2_\perp = -1$, in a frame where the particle is moving very fast the transverse spin components are suppressed with respect to the longitudinal component by a factor m/E (E being the particle energy). This explains, for example, why the effects of the transverse polarisation of the lepton beam in deeply-inelastic scattering (DIS) are unobservable. For the same reason, the contribution of the transverse-spin structure function g_2 to the DIS cross-section is of order xM/Q (where M is the target mass

and Q^2 is the four-momentum transfer squared).

The above argument should not, however, lead to the false impression that *all* transverse-polarisation phenomena are subdominant. In fact, some of them – those related to the so-called "transversity" of quarks – are neither kinematically nor dynamically suppressed and represent the leading contribution to certain hadronic processes.

Historically, the first extensive discussion of transverse-spin effects in high-energy hadronic physics followed the discovery in 1976 that Λ^0 hyperons produced in pN interactions, even at relatively high p_T, exhibit an anomalously large transverse polarisation (Bunce *et al.*, 1976). Such a result requires a non-zero imaginary part in the off-diagonal elements of the fragmentation matrix of quarks into Λ^0 hyperons. It was soon pointed out that this is forbidden at leading-twist in QCD and may arise only as an $\mathcal{O}(1/p_T)$ effect (Kane, Pumplin and Repko, 1978; Efremov and Teryaev, 1982, 1985). It thus took a while to fully realise that transverse-polarisation phenomena are indeed sometimes unsuppressed. This was shown in the pioneering paper of Ralston and Soper (1979) on longitudinally and transversely polarised Drell–Yan processes, but the idea remained almost unnoticed for a decade.

An issue related to hadronic transverse spin, and investigated theoretically in the same period, is the g_2 spin structure function (Hey and Mandula, 1972; Heimann, 1974). After some early attempts by Feynman (1972) to incorporate g_2 into the parton model, it was realised that g_2 contains a higher-twist component that does not admit the usual probabilistic interpretation of quark distributions (for the history and a critical review of this issue, see Jaffe, 1990).

1.1 The transverse-spin structure function and the transversity distributions

Phenomena involving transverse spin or polarisation in high-energy physics are mostly related to one or other two separate quantities:

- the g_2 structure function of polarised deeply-inelastic scattering, the so-called *transverse-spin structure function.*
- the transverse-polarisation distributions of quarks $\Delta_T f$, the so-called *transversity distributions.*

To avoid any confusion and misunderstanding (unfortunately common in this particular field), some crucial distinctions are now in order.

First of all, it is important to stress that g_2 is a *structure function* whereas $\Delta_T f$ is a leading-twist (*i.e.*, twist-two) *distribution function*. This means that while g_2 is a physical observable (essentially, a cross-section), $\Delta_T f$ is not and beyond leading order in QCD its definition depends on the factorisation scheme.

The g_2 structure function incorporates a distribution function called g_T. Differently from $\Delta_T f$, this is a twist-three quantity involving quark–quark–gluon correlators. The connection between $\Delta_T f$ and g_T is quite subtle, as will be explained in Sec. 2.7.3.

Another crucial distinction to be kept in mind concerns the physical content of the two quantities under discussion. The g_2 structure function is related to the *transverse spin of the target nucleon*. The transverse-spin operator is

$$\Sigma_\perp = \gamma_5 \gamma_0 \gamma_\perp \,, \tag{1.2}$$

and one immediately sees that it does not commute with the free-quark Hamiltonian, $H_0 = \alpha_z p_z$. Thus, there are no common quark eigenstates of $\Sigma_\perp$ and H_0: stated otherwise, in a transversely polarised nucleon free quarks do not have a definite *transverse-spin* state. As a consequence, g_2 reflects complicated quark–gluon dynamics with no partonic interpretation.

On the other hand, the transversity distribution $\Delta_T f$ carries information about the *transverse polarisation of quarks* inside a transversely polarised nucleon. The transverse-polarisation operator is (see App. A.3)

$$\Pi_\perp = \tfrac{1}{2} \gamma_0 \Sigma_\perp \,, \tag{1.3}$$

and commutes with H_0, owing to the presence of an extra γ_0. Therefore, in a transversely polarised nucleon free quarks may exist in a definite *transverse-polarisation* state and a simple partonic picture holds: $\Delta_T f$ measures the transverse-polarisation asymmetry of quarks inside a transversely polarised nucleon.

1.2 A first look at g_2

Polarised deeply-inelastic scattering (DIS) mediated by photon exchange probes two spin structure functions: $g_1(x, Q^2)$ and $g_2(x, Q^2)$. If the target

is longitudinally polarised the dominant contribution comes from g_1 and the term containing g_2 is suppressed by a factor x^2M^2/Q^2. When the target is transversely polarised, the entire cross-section is of order xM/Q, but the g_2 and g_1 contributions are on the same footing and what is measured is a combination of the two structure functions.

DIS is related, via the optical theorem, to the forward Compton scattering of virtual photons. By virtue of this relationship, g_2 is essentially the imaginary part of the amplitude for the process

$$\gamma^*(+1) + N(1/2) \to \gamma^*(0) + N(-1/2)\,, \tag{1.4}$$

where the bracketed quantities are the helicities. As one can see, process (1.4) involves a t-channel angular-momentum exchange. Since massless quarks cannot undergo helicity flip in perturbative QCD, in order to have a forward Compton amplitude non-diagonal in the helicity basis, one needs either finite quark transverse momentum (*i.e.*, an orbital angular-momentum contribution), and/or interactions between the struck quark and the gluons from the target. Thus, g_2 arises from higher-twist effects.

In the quark–parton model g_2 is given by

$$g_2(x) = -g_1(x) + \tfrac{1}{2}\sum_a e_a^2\, g_T^a(x)\,, \tag{1.5}$$

where $g_T(x)$ is a twist-three distribution function defined as (n is a Sudakov vector, see App. B)

$$g_T(x) = \int \frac{\mathrm{d}\tau}{4\pi M}\, \mathrm{e}^{\mathrm{i}\tau x} \langle PS|\bar\psi(0)\gamma_\perp\gamma_5\psi(\tau n)|PS\rangle\,. \tag{1.6}$$

A detailed analysis shows that g_T may be decomposed as

$$g_T = g_T^{WW} + g_T^m + \widetilde{g}_T\,, \tag{1.7}$$

where the first term (the so-called "Wandzura–Wilczek term") is a twist-two component, the second term is a quark-mass contribution and the third term is the genuine twist-three component arising from quark–gluon interactions. If we retain only the leading-twist contribution, g_2 is constrained by the Wandzura–Wilczek relation (Wandzura and Wilczek, 1977)

$$g_2(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \mathrm{d}y\, \frac{g_1(y,Q^2)}{y}\,. \tag{1.8}$$

Higher-twist effects thus induce departures from (1.8). The presently available data, however, show that the Wandzura–Wilczek relation is approximately fulfilled.

The first measurements of g_2 date back to less than a decade ago (Anthony *et al.*, 1993; Adams *et al.*, 1994). More accurate results have been reported in the last few years (Abe *et al.*, 1996, 1998, 1997; Anthony *et al.*, 1999, 2002), thus allowing some phenomenological studies. In the near future, further analyses of the existing data are expected. On the theoretical side, g_2 is known at the one-loop level in perturbative QCD.

1.3 A prelude to transversity

The transversity distributions $\Delta_T f(x)$ were first introduced in 1979 by Ralston and Soper in their seminal work on Drell–Yan production with polarised beams (Ralston and Soper, 1979). In that paper $\Delta_T f(x)$ was called $h_T(x)$. This quantity was apparently forgotten until the beginning of 90's, when it was rediscovered by Artru and Mekhfi (1990), who called it $\Delta_1 q(x)$ and studied its QCD evolution, and by Jaffe and Ji (1991b, 1992), who renamed it $h_1(x)$ in the framework of a general classification of all leading-twist and higher-twist parton distribution functions. At about the same time, other important studies of the transverse-polarisation distributions exploring the possibility of measuring them in hadron–hadron or lepton-hadron collisions were carried out by Cortes, Pire and Ralston (1992) and by Ji (1992a).

The last decade or so has witnessed a great interest in the transverse-polarisation distributions (for a review, see Barone, Drago and Ratcliffe, 2002). A major effort has been devoted to investigating their structure using evermore sophisticated model calculations and other non-perturbative tools (QCD sum rules, lattice QCD, *etc.*). Their QCD evolution has been calculated up to next-to-leading order (NLO). The related phenomenology has been explored in detail: many suggestions for measuring (or at least detecting) transverse-polarisation distributions have been put forward and a number of predictions for observables containing $\Delta_T f$ are now available. We can say that our theoretical knowledge of the transversity distributions is at this point nearly comparable to that of the helicity distributions. What is really called for is an experimental study of the subject.

The reason transversity has escaped the attention of physicists for so

many years and is still elusive and difficult to observe is that it is a chirally-odd quantity that is not probed in the cleanest hard process, namely DIS. Let us recall the physical meaning and the main properties of $\Delta_T f(x)$.

At leading-twist level, the quark structure of hadrons is described by three distribution functions: the number density, or unpolarised distribution, $f(x)$; the longitudinal polarisation, or helicity, distribution $\Delta f(x)$; and the transverse polarisation, or transversity, distribution $\Delta_T f(x)$.

The first two are well-known quantities: $f(x)$ is the probability of finding a quark with a fraction x of the longitudinal momentum of the parent hadron, regardless of its spin orientation; $\Delta f(x)$ measures the net helicity of a quark in a longitudinally polarised hadron, that is, the number density of quarks with momentum fraction x and spin parallel to that of the hadron minus the number density of quarks with the same momentum fraction but spin antiparallel. If we call $f_\pm(x)$ the number densities of quarks with helicity ± 1 inside a positive helicity hadron, then we have

$$f(x) = f_+(x) + f_-(x)\,, \tag{1.9a}$$

$$\Delta f(x) = f_+(x) - f_-(x)\,. \tag{1.9b}$$

The third, less familiar, distribution function, $\Delta_T f(x)$, also has a very simple meaning. In a transversely polarised hadron $\Delta_T f(x)$ is the number density of quarks with momentum fraction x and polarisation parallel to that of the hadron, minus the number density of quarks with the same momentum fraction and antiparallel polarisation, *i.e.*,

$$\Delta_T f(x) = f_\uparrow(x) - f_\downarrow(x)\,. \tag{1.10}$$

In a basis of transverse-polarisation states $\Delta_T f$ too has a probabilistic interpretation. In the helicity basis, in contrast, it has no simple meaning, being related to an off-diagonal quark–hadron amplitude.

Formally, quark distribution functions are light-cone Fourier transforms of connected matrix elements of certain quark-field bilinears. In particular, $\Delta_T f$ is given by (P is the momentum of the target and S is its spin)

$$\Delta_T f(x) = \int \frac{\mathrm{d}\tau}{4\pi P^+}\, \mathrm{e}^{\mathrm{i}\tau x} \langle PS|\bar\psi(0)\gamma^+\gamma_\perp\gamma_5\psi(\tau n)|PS\rangle\,. \tag{1.11}$$

In the parton model the quark fields appearing in (1.11) are free fields. In QCD they must be renormalised. This introduces a renormalisation-scale

dependence into the parton distributions, which is governed by the DGLAP equations.

It is important to appreciate some properties of $\Delta_T f$. First of all, $\Delta_T f$ is a *leading-twist* quantity. Hence, it enjoys the same status as $f(x)$ and $\Delta f(x)$ and, *a priori*, there is no reason that it should be substantially smaller than its helicity counterpart (model calculations, in fact, show that $\Delta_T f(x)$ and $\Delta f(x)$ are typically of the same order of magnitude, at least at low Q^2, where model pictures hold). The QCD evolution of $\Delta_T f(x)$ and $\Delta f(x)$ is, however, quite different. In particular, at low x, $\Delta_T f(x)$ turns out to be suppressed with respect to $\Delta f(x)$. As we shall see, this behaviour has important consequences for some observables.

Another crucial peculiarity of $\Delta_T f(x)$ is that it has no gluonic counterpart (in spin-half hadrons): gluon transversity distributions for nucleons do not exist. Moreover, $\Delta_T f(x)$ does not mix with gluons in its evolution, and therefore evolves as a non-singlet quantity.

Examining the operator structure in (1.11) one can see that $\Delta_T f(x)$, in contrast to $f(x)$ and $\Delta f(x)$, which contain γ^+ and $\gamma^+\gamma_5$ instead of $\gamma^+\gamma_\perp\gamma_5$, is a *chirally-odd* quantity (see Fig. 1.1a). Now, fully inclusive DIS

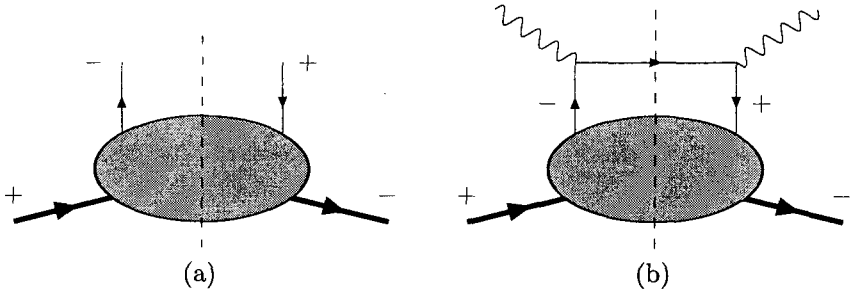

Fig. 1.1 (a) A representation of the chirally-odd distribution $\Delta_T f(x)$ and (b) a handbag diagram forbidden by chirality conservation.

proceeds via the so-called handbag diagram, which cannot flip the chirality of the probed quark (see Fig. 1.1b). Thus, transversity distributions are not observable in DIS. In order to measure $\Delta_T f$, chirality must be flipped twice, so one needs either two hadrons in the initial state (hadron–hadron collisions), or one hadron in the initial state and one in the final state (semi-inclusive leptoproduction), and at least one of these two hadrons must be transversely polarised.

So far we have only discussed $f(x)$, $\Delta f(x)$ and $\Delta_T f(x)$. If the quarks are

perfectly collinear with the parent hadron, these three quantities exhaust the information on the internal dynamics of hadrons. If instead we admit finite quark transverse momentum $\boldsymbol{k}_\perp$, the number of distribution functions increases. At leading twist, assuming time-reversal invariance, there are six $\boldsymbol{k}_\perp$-dependent distributions. Three of them, called in the Jaffe–Ji–Mulders classification scheme (Jaffe and Ji, 1992; Mulders and Tangerman, 1996) $f_1(x, \boldsymbol{k}_\perp^2)$, $g_{1L}(x, \boldsymbol{k}_\perp^2)$ and $h_1(x, \boldsymbol{k}_\perp^2)$, upon integration over $\boldsymbol{k}_\perp^2$, yield $f(x)$, $\Delta f(x)$ and $\Delta_T f(x)$, respectively. The remaining three distributions are new and disappear when the hadronic tensor is integrated over $\boldsymbol{k}_\perp$, as is the case in DIS. Mulders has called them $g_{1T}(x, \boldsymbol{k}_\perp^2)$, $h_{1L}^\perp(x, \boldsymbol{k}_\perp^2)$ and $h_{1T}^\perp(x, \boldsymbol{k}_\perp^2)$. At higher twist the proliferation of distribution functions continues (for example, see Jaffe and Ji, 1992; Mulders and Tangerman, 1996).

In hadron production processes, which, as mentioned above, play an important rôle in the study of transversity, there appear other dynamical quantities: fragmentation functions. These are, in a sense, specular to distribution functions and represent the probability for a quark in a given polarisation state to fragment into a hadron carrying some momentum fraction z. When the quark is transversely polarised and so too is the produced hadron, the process is described by the leading-twist fragmentation function $\Delta_T D(z)$, which is the analogue of $\Delta_T f(x)$. A "time-reversal-odd" fragmentation function, usually called $H_1^\perp(z)$, describes instead the production of unpolarised (or spinless) hadrons from transversely polarised quarks, and can couple to $\Delta_T f(x)$ in certain semi-inclusive processes of great relevance for the phenomenology of transversity. The emergence of $\Delta_T f$ via its coupling to $H_1^\perp$ is known as the Collins effect (Collins, 1993b).

Probing the transverse polarisation of quarks is among the goals of a number of ongoing or future experiments. At the Relativistic Heavy Ion Collider (RHIC) at BNL, $\Delta_T f$ can be extracted from a measurement of the double transverse-spin asymmetry in Drell–Yan dimuon production with two transversely polarised hadron beams (Bunce *et al.*, 2000). Another important class of reactions that can probe transversity distributions is semi-inclusive DIS. The HERMES collaboration at DESY (Airapetian *et al.*, 2000) and the SMC at CERN (Bravar, 1999) have recently presented results on single transverse-spin asymmetries, which might be related to the transverse-polarisation distributions via the hypothetical Collins mechanism (Collins, 1993b). The study of transversity in semi-inclusive DIS is one of the aims of the upgraded HERMES experiment and of the COMPASS experiment (Baum *et al.*, 1996) at the CERN SPS collider, which

started taking data in 2002. We may therefore say that the experimental study of transverse-polarisation distributions, which is at present only in its infancy, promises to have a very interesting future.

1.4 Notation and terminology

While the two polarised structure functions of electromagnetic DIS are universally called g_1 and g_2, the terminology concerning the transverse polarisation of quarks is still unsettled and rather confused.

First of all, the name *transversity*, as a synonym for *transverse polarisation*, was proposed by Jaffe and Ji (1991b). Some authors (Cortes, Pire and Ralston, 1991; Anselmino, Efremov and Leader, 1995) have noted that "transversity" is a pre-existing term in spin physics, with a rather different meaning, and its use in another context might cause some confusion. At this point, however, "transversity" is a widely adopted term and in this book we shall talk indifferently of "quark transverse polarisation" or "quark transversity".

The various notation that has been used in the past for the transversity distribution comprises:

$$\begin{aligned} h_T(x) &\quad \text{(Ralston and Soper)}, \\ \Delta_1 q(x) &\quad \text{(Artru and Mekhfi)}, \\ h_1(x) &\quad \text{(Jaffe and Ji)}. \end{aligned}$$

The first two forms are now obsolete[a] while the third is still widely employed. This last was introduced by Jaffe and Ji in their classification of all twist-two, twist-three and twist-four parton distribution functions. In the Jaffe–Ji scheme, $f_1(x)$, $g_1(x)$ and $h_1(x)$ are the unpolarised, longitudinally polarised and transversely polarised distribution functions, respectively, with the subscript 1 denoting leading-twist quantities. The main disadvantage of such nomenclature is the use of g_1 to denote a *leading-twist distribution function* whereas the same notation is adopted for one of the two polarised DIS *structure functions*. This is a serious source of confusion. In the most recent literature the transverse-polarisation distributions are

[a]Notice, however, that in some experimental papers on g_2, a function $h_T(x)$ is introduced, which corresponds in our notation to $h_T(x) = \frac{1}{2}\sum_a e_a^2 \Delta_T f_a(x)$. In Mulders' classification of distribution functions (Mulders and Tangerman, 1996), the name h_T is instead reserved for a twist-three distribution.

usually called either

$$\delta q(x) \quad \text{or} \quad \Delta_T q(x)\,.$$

Both forms appear quite natural as they emphasise the parallel between the longitudinal and the transverse-polarisation distributions. In this book we shall use $\Delta_T f$, or $\Delta_T q$, to denote the transversity distributions, reserving δq for the *tensor charge* (the first moment of $\Delta_T q$).

The Jaffe–Ji classification scheme has been extended by Mulders and collaborators (Mulders and Tangerman, 1996; Boer and Mulders, 1998) to all twist-two and twist-three $\boldsymbol{k}_\perp$-dependent distribution functions. The letters f, g and h denote unpolarised, longitudinally polarised, and transversely polarised quark distributions, respectively. A subscript 1 labels the leading-twist quantities. Subscripts L and T indicate that the parent hadron is longitudinally or transversely polarised. Finally, a superscript $\perp$ signals the presence of transverse momenta with uncontracted Lorentz indices.

Here we adopt a hybrid terminology. We use the traditional notation for the $\boldsymbol{k}_\perp$-integrated distribution functions: $f(x)$, or $q(x)$, for the number density, $\Delta f(x)$, or $\Delta q(x)$, for the helicity distributions, $\Delta_T f(x)$, or $\Delta_T q(x)$, for the transverse-polarisation distributions, and Mulders' notation for the additional $\boldsymbol{k}_\perp$-dependent distribution functions.

We make the same choice for the fragmentation functions. We call the $\boldsymbol{\kappa}_\perp$-integrated fragmentation functions: $D(z)$ (unpolarised), $\Delta D(z)$ (longitudinally polarised) and $\Delta_T D(z)$ (transversely polarised). Finally, for the $\boldsymbol{\kappa}_\perp$-dependent functions we again use Mulders' terminology.

Occasionally, other notation will be introduced for the sake of transparency. In particular, we shall follow these rules:

- the *subscripts* $0, L$ and T in the distribution and fragmentation functions denote the polarisation state of the *quark* (0 indicates unpolarised, and the subscript L is actually omitted in the familiar helicity distribution and fragmentation functions);
- the *superscripts* $0, L$ and T denote the polarisation state of the parent or off-spring *hadron* (the superscript is omitted when it is equal to the subscript).

Thus, for instance, $\Delta_T^L f$ represents the distribution function of transversely polarised quarks in a longitudinally polarised hadron (it is related to Mulders' $h_{1L}^\perp$). A comparison of the Jaffe–Ji–Mulders terminology and ours

is shown in Table 1.1. The correspondence with other differing notation

Table 1.1 Notation for the distribution and the fragmentation functions (JJM denotes the Jaffe–Ji–Mulders classification).

Distribution functions	
JJM	This book
f_1	f
g_1	Δf
h_1	$\Delta_T f$
g_{1T}	g_{1T}, $\Delta_L^T f$
$h_{1L}^\perp$	$h_{1L}^\perp$, $\Delta_T^L f$
$h_{1T}^\perp$	$h_{1T}^\perp$, $\widetilde{\Delta}_T f$
$f_{1T}^\perp$	$f_{1T}^\perp$, $\Delta_0^T f$
$h_1^\perp$	$h_1^\perp$, $\Delta_T^0 f$

Fragmentation functions	
JJM	This book
D_1	D
G_1	ΔD
H_1	$\Delta_T D$
G_{1T}	G_{1T}, $\Delta_L^T D$
$H_{1L}^\perp$	$H_{1L}^\perp$, $\Delta_T^L D$
$H_{1T}^\perp$	$H_{1T}^\perp$, $\widetilde{\Delta}_T D$
$D_{1T}^\perp$	$D_{1T}^\perp$, $\Delta_0^T D$
$H_1^\perp$	$H_1^\perp$, $\Delta_T^0 D$

also encountered in the literature (Anselmino, Boglione and Murgia, 1999; Anselmino *et al.*, 2000b) is

$$\begin{aligned}
\Delta^N f_{q/N^\uparrow} &\equiv \Delta_0^T f\,, \\
\Delta^N f_{q^\uparrow/N} &\equiv \Delta_T^0 f\,, \\
\Delta^N D_{h/q^\uparrow} &\equiv 2\,\Delta_T^0 D_{h/q}\,.
\end{aligned}$$

1.5 Conventions

We now list some conventions adopted throughout the book. The metric tensor is defined as

$$g^{\mu\nu} = g_{\mu\nu} = \mathrm{diag}\,(+1,-1,-1,-1)\,. \tag{1.12}$$

The totally antisymmetric tensor $\varepsilon^{\mu\nu\rho\sigma}$ is normalised so that

$$\varepsilon^{0123} = -\varepsilon_{0123} = +1\,. \tag{1.13}$$

A generic four-vector A^μ is written, in Cartesian contravariant components, as

$$A^\mu = (A^0, A^1, A^2, A^3) = (A^0, \boldsymbol{A})\,. \tag{1.14}$$

The light-cone components of A^μ are defined as

$$A^\pm = \tfrac{1}{\sqrt{2}}\,(A^0 \pm A^3)\,, \tag{1.15}$$

and in these components A^μ is written as

$$A^\mu = (A^+, A^-, \boldsymbol{A}_\perp)\,. \tag{1.16}$$

The norm of A^μ is given by

$$A^2 = (A^0)^2 - \boldsymbol{A}^2 = 2A^+A^- - \boldsymbol{A}_\perp^2\,, \tag{1.17}$$

and the scalar product of two four-vectors A^μ and B^μ is

$$A{\cdot}B = A^0B^0 - \boldsymbol{A}{\cdot}\boldsymbol{B} = A^+B^- + A^-B^+ - \boldsymbol{A}_\perp{\cdot}\boldsymbol{B}_\perp\,. \tag{1.18}$$

Our fermionic states are normalised as

$$\begin{aligned}\langle p|p'\rangle &= (2\pi)^3\, 2E\, \delta^3(\boldsymbol{p}-\boldsymbol{p}') \\ &= (2\pi)^3\, 2p^+\, \delta(p^+ - p'^+)\, \delta^2(\boldsymbol{p}_\perp - \boldsymbol{p}'_\perp)\,,\end{aligned} \tag{1.19}$$

$$\bar{u}(p,s)\,\gamma^\mu\,u(p,s') = 2p^\mu\,\delta_{ss'}\,, \tag{1.20}$$

with $E = (\boldsymbol{p}^2 + m^2)^{1/2}$. The creation and annihilation operators satisfy the anticommutator relations

$$\begin{aligned}\big\{b(p,s), b^\dagger(p',s')\big\} &= \big\{d(p,s), d^\dagger(p',s')\big\} \\ &= (2\pi)^3\, 2E\, \delta_{ss'}\, \delta^3(\boldsymbol{p}-\boldsymbol{p}')\,.\end{aligned} \tag{1.21}$$

Chapter 2

Polarised deeply-inelastic scattering

Since the celebrated SLAC–Yale experiments in the mid-seventies (Alguard *et al.*, 1976; Baum *et al.*, 1980) polarised deeply-inelastic scattering (DIS) has played a key rôle in providing information about the spin structure of nucleons. In 1988 the European Muon Collaboration reported the results of a measurement of the polarised structure function g_1 (Ashman *et al.*, 1988, 1989) that were interpreted, in the framework of the naïve quark–parton model, as implying that the total spin carried by the quarks inside the proton is closer to zero rather than to 1/2, as commonly expected. This surprising result, which became popularly known as the "spin crisis" (Leader and Anselmino, 1988), triggered further experimental work and more sophisticated theoretical studies. A decade of intense research has led to a fairly good knowledge of g_1 (at least for the proton and the deuteron), to a rather accurate test of the Bjorken sum rule (Bjorken, 1966), a fundamental relation of hadronic physics, and to a better understanding of the helicity structure of the proton; although with some uncertainties on the gluon polarisation and the individual quark distributions, which ongoing and future experiments promise to dissipate.

Our attention in this book, as far as DIS is concerned, will be mostly directed towards the "transverse-spin structure function" g_2. The experimental investigation of g_2 is a more recent achievement and a challenging enterprise, owing to certain kinematic difficulties. The g_2 contribution to DIS is, in fact, suppressed by a factor xM/Q, where M is the nucleon mass and Q is the momentum transfer. The extraction of g_2 is therefore an arduous task and it is not surprising that g_2 is presently known with much poorer accuracy than is g_1.

In this chapter we shall review some general notions and results concerning polarised DIS. The main focus will be on the case of a transversely polarised target, but for the sake of completeness we shall also briefly discuss the phenomenology of g_1 (for an exhaustive treatment of the subject see, *e.g.*, Anselmino, Efremov and Leader, 1995; Leader and Predazzi, 1996; Lampe and Reya, 2000; Filippone and Ji, 2001; Leader, 2001).

2.1 Basics of DIS

Consider inclusive lepton–nucleon scattering (see Fig. 2.1, where the dominance of one-photon exchange is assumed)

$$l(\ell) + N(P) \to l'(\ell') + X(P_X)\,, \tag{2.1}$$

where X is an undetected hadronic system (inside the brackets we put the four-momenta of the particles). Our notation is as follows: M is the nucleon mass, m_ℓ the lepton mass, s_ℓ (s'_ℓ) the spin four-vector of the incoming (outgoing) lepton, S the spin four-vector of the nucleon, while $\ell = (E, \boldsymbol{\ell})$, and $\ell' = (E', \boldsymbol{\ell}')$ are the lepton four-momenta.

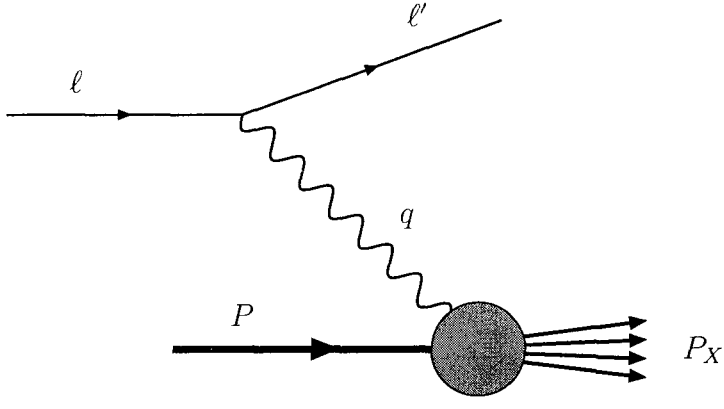

Fig. 2.1 Deeply-inelastic scattering.

Two kinematic variables (besides the centre-of-mass energy $s = (\ell + P)^2$, or, alternatively, the lepton beam energy E) are needed to describe reaction (2.1). They can be chosen among the following invariants (unless otherwise

stated, we neglect lepton masses):

$$
\begin{aligned}
q^2 &= (\ell - \ell')^2 = -2EE'(1-\cos\vartheta)\,, \\
\nu &= \frac{P{\cdot}q}{M} && \text{(the laboratory-frame photon energy)}, \\
x &= \frac{Q^2}{2P{\cdot}q} = \frac{Q^2}{2M\nu} && \text{(the Bjorken scaling variable)}, \\
y &= \frac{P{\cdot}q}{P{\cdot}\ell} && \text{(the inelasticity)},
\end{aligned}
$$

where ϑ is the scattering angle. The photon momentum q is a space-like four-vector and one usually introduces the positive quantity $Q^2 \equiv -q^2$. Both the Bjorken variable x and the inelasticity y take on values between 0 and 1. They are related to Q^2 by $xy = Q^2/(s - M^2)$. The *deeply-inelastic scattering* regime, or *Bjorken limit*, corresponds to

$$
\nu, Q^2 \to \infty\,, \quad \text{with} \quad x = \frac{Q^2}{2M\nu} \quad \text{fixed}. \tag{2.2}
$$

The DIS cross-section is

$$
\mathrm{d}^3\sigma = \frac{1}{4P{\cdot}\ell}\,\frac{e^4}{Q^4}\,L_{\mu\nu}\,W^{\mu\nu}\,2\pi\,\frac{\mathrm{d}^3\boldsymbol{\ell}'}{(2\pi)^3\,2E'}\,, \tag{2.3}
$$

where the leptonic tensor $L_{\mu\nu}$ is defined as (lepton masses are briefly retained here)

$$
\begin{aligned}
L_{\mu\nu} &= \sum_{s_{l'}} \left[\bar{u}_{l'}(\ell', s_{l'})\gamma_\mu u_l(\ell, s_l)\right]^* \left[\bar{u}_{l'}(\ell', s_{l'})\gamma_\nu u_l(\ell, s_l)\right] \\
&= \mathrm{Tr}\left[(\not{\ell} + m_l)\,\tfrac{1}{2}(1+\gamma_5 \not{s}_l)\,\gamma_\mu\,(\not{\ell}' + m_l)\,\gamma_\nu\right].
\end{aligned} \tag{2.4}
$$

and the hadronic tensor $W^{\mu\nu}$ is

$$
\begin{aligned}
W^{\mu\nu} = \frac{1}{2\pi}\sum_X \int \frac{\mathrm{d}^3\boldsymbol{P}_X}{(2\pi)^3\,2E_X}\,(2\pi)^4\delta^4(P+q-P_X) \\
\times \langle PS|J^\mu(0)|X\rangle\langle X|J^\nu(0)|PS\rangle\,.
\end{aligned} \tag{2.5}
$$

Using translational invariance this can be also written as

$$
W^{\mu\nu} = \frac{1}{2\pi}\int \mathrm{d}^4\xi\; \mathrm{e}^{\mathrm{i}q\cdot\xi}\,\langle PS|J^\mu(\xi)J^\nu(0)|PS\rangle\,. \tag{2.6}
$$

It is important to recall that the matrix elements in (2.6) are connected. Therefore, vacuum transitions of the form $\langle 0|J^\mu(\xi)J^\nu(0)|0\rangle\,\langle PS|PS\rangle$ are excluded.

Note that in (2.4) and (2.5) we summed over the final lepton spin but did not average over the initial lepton spin, nor sum over the hadron spin. Thus, we are describing, in general, the scattering of polarised leptons on a polarised target, with no measurement of the outgoing lepton polarisation.

In the target rest frame, where $P{\cdot}\ell = ME$, (2.3) reads

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}E'\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{2MQ^4}\,\frac{E'}{E}\,L_{\mu\nu}W^{\mu\nu}\,, \tag{2.7}$$

where $\mathrm{d}\Omega = \mathrm{d}\cos\vartheta\,\mathrm{d}\varphi$. In terms of the invariants defined above, the cross-section takes on the form

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\varphi} = \frac{\alpha_{\mathrm{em}}^2 y}{2Q^4}\,L_{\mu\nu}W^{\mu\nu}\,. \tag{2.8}$$

The leptonic tensor $L_{\mu\nu}$ can be decomposed into a symmetric and an antisymmetric part under $\mu \leftrightarrow \nu$ interchange

$$L_{\mu\nu} = L_{\mu\nu}^{(\mathrm{S})}(\ell,\ell') + \mathrm{i}\,L_{\mu\nu}^{(\mathrm{A})}(\ell, s_l; \ell')\,, \tag{2.9}$$

and, computing the trace in (2.4), we obtain (retaining lepton masses)

$$L_{\mu\nu}^{(\mathrm{S})} = 2\left[\ell_\mu\ell'_\nu + \ell_\nu\ell'_\mu - g_{\mu\nu}\,(\ell{\cdot}\ell' - m_l^2)\right], \tag{2.10a}$$

$$L_{\mu\nu}^{(\mathrm{A})} = 2m_l\,\varepsilon_{\mu\nu\rho\sigma}s_\ell^\rho(\ell-\ell')^\sigma\,. \tag{2.10b}$$

If the incoming lepton is longitudinally polarised, its spin vector is

$$s_l^\mu = \frac{\lambda_l}{m_l}\,\ell^\mu\,, \quad \lambda_l = \pm 1\,, \tag{2.11}$$

and (2.10b) becomes

$$L_{\mu\nu}^{(\mathrm{A})} = 2\lambda_l\,\varepsilon_{\mu\nu\rho\sigma}\ell^\rho(\ell-\ell')^\sigma = 2\lambda_l\,\varepsilon_{\mu\nu\rho\sigma}\ell^\rho q^\sigma\,. \tag{2.12}$$

Note that the lepton mass m_l appearing in (2.10b) has been cancelled by the denominator in (2.11). In contrast, if the lepton is transversely polarised, that is, $s_l^\mu = s_{l\perp}^\mu$, no such cancellation occurs and the contribution is suppressed by a factor m_l/E. This observation exemplifies the distinction to be made between the existence of transverse spin and its measurable effects. In what follows we shall consider only unpolarised or longitudinally

polarised lepton beams. Neglecting, as usual, lepton masses, the leptonic tensor then reads

$$\begin{aligned} L_{\mu\nu} &= L^{(\mathrm{S})}_{\mu\nu} + \mathrm{i} L^{(\mathrm{A})}_{\mu\nu} \\ &= 2(\ell_\mu \ell'_\nu + \ell_\nu \ell'_\mu - g_{\mu\nu}\, \ell\cdot\ell') - 2\mathrm{i}\lambda_l\, \varepsilon_{\mu\nu\rho\sigma} \ell^\rho \ell'^\sigma \,. \end{aligned} \tag{2.13}$$

The hadronic tensor $W_{\mu\nu}$ admits a similar decomposition

$$W_{\mu\nu} = W^{(\mathrm{S})}_{\mu\nu}(q,P) + \mathrm{i}\, W^{(\mathrm{A})}_{\mu\nu}(q;P,S)\,, \tag{2.14}$$

into a symmetric and an antisymmetric part, which are expressed in terms of two pairs of structure functions, F_1, F_2 and G_1, G_2, as

$$\begin{aligned} \frac{1}{2M} W^{(\mathrm{S})}_{\mu\nu} &= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(P\cdot q, q^2) \\ &\quad + \frac{1}{M^2}\left(P_\mu - \frac{P\cdot q}{q^2}\, q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}\, q_\nu\right) W_2(P\cdot q, q^2)\,, \end{aligned} \tag{2.15a}$$

$$\begin{aligned} \frac{1}{2M} W^{(\mathrm{A})}_{\mu\nu} &= \varepsilon_{\mu\nu\rho\sigma}\, q^\rho \left\{ M S^\sigma\, G_1(P\cdot q, q^2) \right. \\ &\quad \left. + \frac{1}{M}\left[P\cdot q\, S^\sigma - S\cdot q\, P^\sigma\right] G_2(P\cdot q, q^2)\right\}. \end{aligned} \tag{2.15b}$$

Expressions (2.15a, b) are the most general decompositions compatible with the requirement of gauge invariance, which implies

$$q_\mu W^{\mu\nu} = 0 = q_\nu W^{\mu\nu}. \tag{2.16}$$

Using (2.9, 2.14) the cross-section (2.7) becomes

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}E'\,\mathrm{d}\Omega} = \frac{\alpha^2_{\mathrm{em}}}{2MQ^4}\,\frac{E'}{E}\left[L^{(\mathrm{S})}_{\mu\nu} W^{\mu\nu\,(\mathrm{S})} - L^{(\mathrm{A})}_{\mu\nu} W^{\mu\nu\,(\mathrm{A})}\right]. \tag{2.17}$$

2.2 The unpolarised cross-section

The unpolarised cross-section is obtained from (2.17) by averaging over the spins of the incoming lepton (s_l) and of the nucleon (S), and reads

$$\frac{\mathrm{d}^2\sigma^{\mathrm{unp}}}{\mathrm{d}E'\,\mathrm{d}\Omega} = \frac{1}{2}\sum_{s_l}\frac{1}{2}\sum_{S}\frac{\mathrm{d}^2\sigma(s_l,S)}{\mathrm{d}E'\,\mathrm{d}\Omega} = \frac{\alpha^2_{\mathrm{em}}}{2MQ^4}\,\frac{E'}{E}\, L^{(\mathrm{S})}_{\mu\nu} W^{\mu\nu\,(\mathrm{S})}\,. \tag{2.18}$$

Inserting Eqs. (2.10a) and (2.15a) into (2.18) one obtains the well-known expression

$$\frac{\mathrm{d}^2\sigma^{\mathrm{unp}}}{\mathrm{d}E'\,\mathrm{d}\Omega} = \frac{4\alpha_{\mathrm{em}}^2 E'^2}{Q^4}\left\{2W_1\,\sin^2\frac{\vartheta}{2} + W_2\,\cos^2\frac{\vartheta}{2}\right\}. \tag{2.19}$$

It is customary to introduce the dimensionless structure functions

$$\begin{aligned} F_1(x,Q^2) &\equiv M\,W_1(\nu,Q^2)\,, \\ F_2(x,Q^2) &\equiv \nu\,W_2(\nu,Q^2)\,. \end{aligned} \tag{2.20}$$

In the Bjorken limit (2.2) F_1 and F_2 are expected to scale approximately, that is, to depend only on x. In terms of F_1 and F_2 the symmetric part of the hadronic tensor reads

$$\begin{aligned} W_{\mu\nu}^{(\mathrm{S})} = {} & 2\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)F_1(x,Q^2) \\ & + \frac{2}{P{\cdot}q}\left(P_\mu - \frac{P{\cdot}q}{q^2}\,q_\mu\right)\left(P_\nu - \frac{P{\cdot}q}{q^2}\,q_\nu\right)F_2(x,Q^2)\,. \end{aligned} \tag{2.21}$$

Note that, owing to current conservation, we have $L_{\mu\nu}q^\mu = L_{\mu\nu}q^\nu = 0$ and the terms proportional to q^μ and q^ν in (2.21) do not contribute when contracted with the leptonic tensor. Thus, we can rewrite $W_{\mu\nu}^{(\mathrm{S})}$ simply as

$$W_{\mu\nu}^{(\mathrm{S})} = -2g_{\mu\nu}F_1 + \frac{2P_\mu P_\nu}{P{\cdot}q}\,F_2\,. \tag{2.22}$$

Thus, expressed in terms of F_1 and F_2, and as a function of x and y, the unpolarised cross-section is

$$\frac{\mathrm{d}^2\sigma^{\mathrm{unp}}}{\mathrm{d}x\,\mathrm{d}y} = \frac{4\pi\alpha_{\mathrm{em}}^2}{Q^2xy}\left\{xy^2\,F_1(x,Q^2) + \left(1 - y - \frac{\gamma^2y^2}{4}\right)F_2(x,Q^2)\right\}, \tag{2.23}$$

where

$$\gamma \equiv \frac{2Mx}{Q} \tag{2.24}$$

is an $\mathcal{O}(1/Q)$ quantity that will often appear in the formulæ below.

2.3 Polarised cross-sections

Differences of cross-sections with opposite target spin probe the antisymmetric part of the leptonic and hadronic tensors:

$$\frac{\mathrm{d}^2\sigma(s_l,+S)}{\mathrm{d}E'\,\mathrm{d}\Omega} - \frac{\mathrm{d}^2\sigma(s_l,-S)}{\mathrm{d}E'\,\mathrm{d}\Omega} = -\frac{\alpha_{\mathrm{em}}^2}{2MQ^4}\,\frac{E'}{E}\,2L_{\mu\nu}^{(\mathrm{A})}W^{\mu\nu\,(\mathrm{A})}\,. \tag{2.25}$$

In the target rest frame the spin of the nucleon can be parametrised as (assuming $|\boldsymbol{S}| = 1$)

$$S^\mu = (0,\boldsymbol{S}) = (0,\,\sin\alpha\cos\beta,\,\sin\alpha\sin\beta,\,\cos\alpha)\,. \tag{2.26}$$

Taking the direction of the incoming lepton to be the z-axis, we have (see Fig. 2.2)

$$\begin{aligned} \ell^\mu &= E(1,0,0,1)\,,\\ \ell'^\mu &= E'(1,\,\sin\vartheta\cos\varphi,\,\sin\vartheta\sin\varphi,\,\cos\vartheta)\,. \end{aligned} \tag{2.27}$$

We also assume that the incoming lepton is polarised antiparallel to its

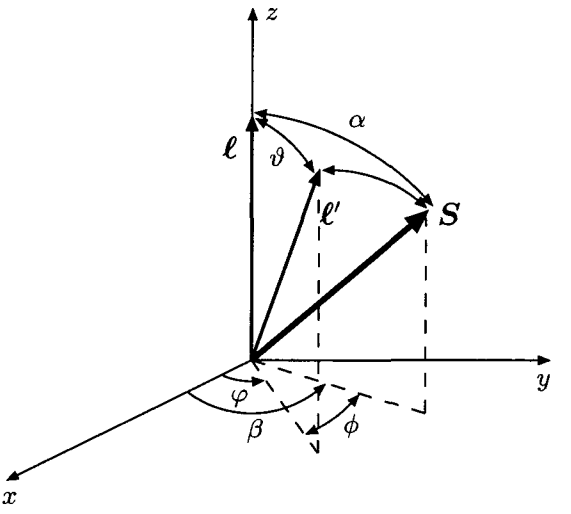

Fig. 2.2 The target spin and the lepton momenta.

direction of motion, *i.e.*, $\lambda_l = -1$ and

$$s_l^\mu = -\frac{\ell^\mu}{m_l}\,. \tag{2.28}$$

Inserting (2.10b) and (2.15b) in Eq. (2.25), with the above parametrisations for the spin and the momentum four-vectors, we obtain for the cross-section difference in the target rest frame[a]

$$\begin{aligned}\frac{\mathrm{d}^2\sigma(+S)}{\mathrm{d}E'\,\mathrm{d}\Omega} - \frac{\mathrm{d}^2\sigma(-S)}{\mathrm{d}E'\,\mathrm{d}\Omega} &= \frac{4\alpha_{\mathrm{em}}^2}{Q^2}\,\frac{E'}{E} \\ &\times \Big\{\Big[E\cos\alpha + E'(\sin\vartheta\sin\alpha\cos\phi + \cos\vartheta\cos\alpha)\Big]\,MG_1 \\ &\quad + 2EE'\Big[\sin\vartheta\sin\alpha\cos\phi + \cos\vartheta\cos\alpha - \cos\alpha\Big]\,G_2\Big\}\,, \end{aligned} \tag{2.29}$$

where $\phi = \beta - \varphi$ is the azimuthal angle between the lepton plane and the $(\hat{\boldsymbol{\ell}}, \hat{\boldsymbol{S}})$ plane (see Fig. 2.3).

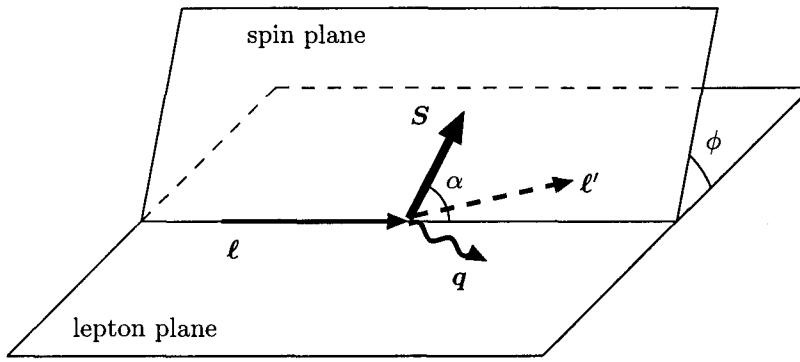

Fig. 2.3 The lepton and spin planes.

In particular, when the target nucleon is *longitudinally polarised* (that is, *polarised along the incoming lepton direction*), one has $\alpha = 0$ and the (2.29) becomes (with the superscript $\leftarrow$ for $\lambda_l = -1$ and $\rightarrow$ for $\lambda_l = +1$)

$$\frac{\mathrm{d}^2\sigma^{\leftarrow\Rightarrow}}{\mathrm{d}E'\,\mathrm{d}\Omega} - \frac{\mathrm{d}^2\sigma^{\leftarrow\Leftarrow}}{\mathrm{d}E'\,\mathrm{d}\Omega} = \frac{4\alpha_{\mathrm{em}}^2 E'}{Q^2 E}\left[(E + E'\cos\vartheta)\,M\,G_1 - Q^2\,G_2\right]. \tag{2.30a}$$

[a]Hereafter, we omit the argument s_l in the cross-sections.

When the target nucleon is *transversely polarised* (that is, *polarised orthogonally to the incoming lepton direction*), one has $\alpha = \pi/2$ and therefore

$$\frac{\mathrm{d}^2\sigma^{\leftarrow\Uparrow}}{\mathrm{d}E'\,\mathrm{d}\Omega} - \frac{\mathrm{d}^2\sigma^{\leftarrow\Downarrow}}{\mathrm{d}E'\,\mathrm{d}\Omega} = \frac{4\alpha_{\mathrm{em}}^2 E'^2}{Q^2 E}\left(M\,G_1 + 2E\,G_2\right)\sin\vartheta\cos\phi\,. \qquad (2.30\mathrm{b})$$

Introducing the dimensionless structure functions

$$\begin{aligned} g_1(x,Q^2) &\equiv M^2\nu\,G_1(\nu,Q^2)\,,\\ g_2(x,Q^2) &\equiv M\nu^2\,G_2(\nu,Q^2)\,, \end{aligned} \qquad (2.31)$$

which approximately scale in the Bjorken limit, the antisymmetric part of the hadronic tensor may be written as

$$W_{\mu\nu}^{(\mathrm{A})} = \frac{2M\,\varepsilon_{\mu\nu\rho\sigma}\,q^\rho}{P{\cdot}q}\left\{S^\sigma\,g_1(x,Q^2) + \left[S^\sigma - \frac{S{\cdot}q}{P{\cdot}q}\,P^\sigma\right]g_2(x,Q^2)\right\}. \qquad (2.32)$$

Equation (2.29), in terms of g_1 and g_2, becomes

$$\begin{aligned} &\frac{\mathrm{d}^3\sigma(+S)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi} - \frac{\mathrm{d}^3\sigma(-S)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi}\\ &\quad= \frac{4\alpha_{\mathrm{em}}^2}{Q^2}\left\{\left[\left(2-y-\frac{\gamma^2y^2}{2}\right)g_1(x,Q^2) - \gamma^2 y\,g_2(x,Q^2)\right]\cos\alpha\right.\\ &\qquad\quad+\gamma\sqrt{1-y-\frac{1}{4}\gamma^2y^2}\left[y\,g_1(x,Q^2)+2g_2(x,Q^2)\right]\\ &\qquad\qquad\left.\times\ \sin\alpha\,\cos\phi\right\}, \end{aligned} \qquad (2.33)$$

Note that the terms containing g_2 are suppressed by at least a factor γ, *i.e.*, by $1/Q$. This renders the measurement of g_2 quite a difficult task.

From (2.33) we get, for $\alpha = 0$ (*i.e.*, for a target longitudinally polarised with respect to the beam direction)

$$\begin{aligned} &\frac{\mathrm{d}^3\sigma^{\leftarrow\Rightarrow}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi} - \frac{\mathrm{d}^3\sigma^{\leftarrow\Leftarrow}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi}\\ &\quad= \frac{4\alpha_{\mathrm{em}}^2}{Q^2}\left\{\left(2-y-\frac{\gamma^2y^2}{2}\right)g_1(x,Q^2) - \gamma^2 y g_2(x,Q^2)\right\}, \end{aligned} \qquad (2.34)$$

and for $\alpha = \pi/2$ (*i.e.*, for a target transversely polarised with respect to

the beam direction)

$$\frac{\mathrm{d}^3\sigma^{\leftarrow\Uparrow}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi} - \frac{\mathrm{d}^3\sigma^{\leftarrow\Downarrow}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi} = \frac{4\alpha_{\mathrm{em}}^2}{Q^2}\,\gamma\sqrt{1-y-\gamma^2y^2/4} \times \Big\{ y\,g_1(x,Q^2) + 2g_2(x,Q^2) \Big\} \cos\phi\,. \quad (2.35)$$

2.4 Target polarisation

A remark on the definition of the target polarisation is in order here. The terms "longitudinal" and "transverse" are somewhat ambiguous, insofar as a reference axis is not specified. From an experimental point of view, the "longitudinal" and "transverse" polarisations of the nucleon are in reference to the lepton beam axis. Thus, "longitudinal" ("transverse") indicates parallel (orthogonal) to this axis. We use the large arrows $\Rightarrow$ and $\Uparrow$ to denote these two cases respectively. From a theoretical point of view, it is simpler to refer to the direction of motion of the virtual photon. One then speaks of the "longitudinal" ($S_\parallel$) and "transverse" ($S_\perp$) spin of the nucleon, meaning by this spin parallel and perpendicular, respectively, to the photon axis. When the target is "longitudinally" or "transversely" polarised in this sense, we shall make explicit reference to $S_\parallel$ and $S_\perp$ in the cross-section. We shall now show how to pass from $\mathrm{d}\sigma^{\Rightarrow} - \mathrm{d}\sigma^{\Leftarrow}$ and $\mathrm{d}\sigma^{\Uparrow} - \mathrm{d}\sigma^{\Downarrow}$ to $\mathrm{d}\sigma(+S_\parallel) - \mathrm{d}\sigma(-S_\parallel)$ and $\mathrm{d}\sigma(+S_\perp) - \mathrm{d}\sigma(-S_\perp)$. Note that, in general, $\mathrm{d}\sigma^{\Rightarrow}$ and $\mathrm{d}\sigma^{\Leftarrow}$ are combinations of $\mathrm{d}\sigma(S_\parallel)$ and $\mathrm{d}\sigma(S_\perp)$.

Let us first of all parametrise the photon momentum $\boldsymbol{q} = \boldsymbol{l} - \boldsymbol{l}'$ as

$$\boldsymbol{q} = |\boldsymbol{q}|\,\big(\sin\vartheta_\gamma\cos(\varphi+\pi),\,\sin\vartheta_\gamma\sin(\varphi+\pi),\,\cos\vartheta_\gamma\big)\,. \quad (2.36)$$

DIS kinematics gives

$$\sin\vartheta_\gamma = \gamma\sqrt{1-y} + \mathcal{O}(\gamma^3)\,, \quad (2.37a)$$

$$\cos\vartheta_\gamma = 1 - \tfrac{1}{2}\gamma^2(1-y) + \mathcal{O}(\gamma^4)\,. \quad (2.37b)$$

The angle between the spin of the nucleon $\boldsymbol{S}$ and $\boldsymbol{q}$, which we call Θ, is given by

$$\begin{aligned}\cos\Theta &= -\sin\vartheta_\gamma\cos\varphi\sin\alpha\cos\beta - \sin\vartheta_\gamma\sin\varphi\sin\alpha\sin\beta + \cos\vartheta_\gamma\cos\alpha \\ &= -\sin\vartheta_\gamma\sin\alpha\cos\phi + \cos\vartheta_\gamma\cos\alpha\,, \end{aligned} \quad (2.38)$$

that is, using Eqs. (2.37a, b),

$$\cos\Theta = \cos\alpha - \gamma\sqrt{1-y}\sin\alpha\cos\phi + \mathcal{O}(\gamma^2)\,. \tag{2.39a}$$

From this we get

$$\sin\Theta = \sin\alpha + \gamma\sqrt{1-y}\cos\alpha\cos\phi + \mathcal{O}(\gamma^2)\,. \tag{2.39b}$$

Inverting (2.39a, b) yields, up to $\mathcal{O}(\gamma)$,

$$\cos\alpha \simeq \cos\Theta + \gamma\sqrt{1-y}\cos\phi\sin\Theta\,, \tag{2.40a}$$

$$\sin\alpha \simeq \sin\Theta - \frac{2Mx}{Q}\sqrt{1-y}\cos\phi\cos\Theta\,, \tag{2.40b}$$

and hence we obtain for the cross-section difference (2.33), ignoring $\mathcal{O}(\gamma^2)$ terms,

$$\frac{\mathrm{d}^3\sigma(+S)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi} - \frac{\mathrm{d}^3\sigma(-S)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi} = \frac{4\alpha_{\mathrm{em}}^2}{Q^2}\Big[(2-y)\,g_1\cos\Theta + 2\gamma\sqrt{1-y}\,(g_1+g_2)\sin\Theta\cos\phi\Big]\,. \tag{2.41}$$

This expression shows that when the target spin is perpendicular to the photon momentum ($\Theta = \pi/2$) DIS probes the combination $g_1 + g_2$; and

$$\frac{\mathrm{d}^3\sigma(+S_\perp)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi} - \frac{\mathrm{d}^3\sigma(-S_\perp)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi} = \frac{8\alpha_{\mathrm{em}}^2}{Q^2}\gamma\sqrt{1-y}\,(g_1+g_2)\cos\phi\,. \tag{2.42}$$

The same result can be obtained in another, more direct, manner. Splitting the spin vector of the nucleon into a longitudinal and transverse part with respect to the photon axis:

$$S^\mu = S_\parallel^\mu + S_\perp^\mu = \frac{\lambda_N}{M}P^\mu + S_\perp^\mu\,, \tag{2.43}$$

where $\lambda_N = \pm 1$ is (twice) the helicity of the nucleon, the antisymmetric part of the hadronic tensor becomes

$$W_{\mu\nu}^{(\mathrm{A})} = \frac{2M\,\varepsilon_{\mu\nu\rho\sigma}\,q^\rho}{P{\cdot}q}\Big[S_\parallel^\sigma\,g_1 + S_\perp^\sigma\,(g_1+g_2)\Big]\,. \tag{2.44}$$

Thus, if the nucleon is longitudinally polarised with respect to the photon axis, the DIS cross-section depends only on g_1; if it is transversely polarised with respect to the photon axis, what is measured is the sum of g_1 and g_2. We shall use expression (2.44) when studying the quark content of structure functions in the parton model.

Note that from (2.39a, b) one finds, for $\alpha = 0$ and $\phi = 0$ (*i.e.*, for a target polarised longitudinally relative to the beam axis)

$$\begin{aligned} &\cos\Theta \simeq 1\,, \\ &\sin\Theta \simeq \gamma\sqrt{1-y}\,. \end{aligned} \tag{2.45}$$

This means that the target spin $\boldsymbol{S}$ has a non-zero transverse component with respect to the photon axis, suppressed by a factor $1/Q$

$$|\boldsymbol{S}_\perp| = \sin\Theta\,|\boldsymbol{S}| \simeq \frac{2Mx}{Q}\sqrt{1-y}\,|\boldsymbol{S}|\,. \tag{2.46}$$

For $\alpha = \pi/2$ (*i.e.*, for a target polarised transversely relative to the beam axis) one has

$$\begin{aligned} &\cos\Theta \simeq -\gamma\sqrt{1-y}\cos\phi\,, \\ &\sin\Theta \simeq 1\,, \end{aligned} \tag{2.47}$$

and therefore

$$|\boldsymbol{S}_\perp| \simeq |\boldsymbol{S}|\,. \tag{2.48}$$

In this case, neglecting $1/Q^2$ kinematical effects, the target is also transversely polarised with respect to the photon axis.

2.5 Forward virtual Compton scattering

The hadronic tensor is related to the forward virtual Compton scattering amplitude,

$$T_{\mu\nu} = \mathrm{i}\int \mathrm{d}^4\xi\ \mathrm{e}^{\mathrm{i}q\cdot\xi}\,\langle PS|T(J_\mu(\xi)J_\nu(0))|PS\rangle\,, \tag{2.49}$$

by

$$W_{\mu\nu} = \frac{1}{2\pi}\,\mathrm{Im}\,T_{\mu\nu}\,. \tag{2.50}$$

In the helicity basis there are four independent virtual Compton amplitudes $T(h, H; h', H')$, see Fig. 2.4 for notation,

$$\begin{aligned} &T(1,\ \tfrac{1}{2};\ 1,\ \tfrac{1}{2})\,, \\ &T(1,-\tfrac{1}{2};\ 1,-\tfrac{1}{2})\,, \\ &T(0,\ \tfrac{1}{2};\ 0,\ \tfrac{1}{2})\,, \\ &T(0,\ \tfrac{1}{2};\ 1,-\tfrac{1}{2})\,, \end{aligned} \tag{2.51}$$

which combine to give the unpolarised and polarised structure functions

$$W_1 \equiv W_T \sim \tfrac{1}{2}\,\mathrm{Im}\big[T(1,\tfrac{1}{2};1,\tfrac{1}{2}) + T(1,-\tfrac{1}{2};1,-\tfrac{1}{2})\big]\,, \tag{2.52a}$$

$$\left(1+\frac{\nu^2}{Q^2}\right) W_2 - W_1 \equiv W_L \sim \mathrm{Im}\,T(0,\tfrac{1}{2};0,\tfrac{1}{2})\,, \tag{2.52b}$$

$$M\nu G_1 - Q^2 G_2 \sim \tfrac{1}{2}\,\mathrm{Im}\big[T(1,\tfrac{1}{2};1,\tfrac{1}{2}) - T(1,-\tfrac{1}{2};1,-\tfrac{1}{2})\big]\,, \tag{2.52c}$$

$$Q(MG_1 + \nu G_2) \sim \mathrm{Im}\,T(0,\tfrac{1}{2};1,-\tfrac{1}{2})\,. \tag{2.52d}$$

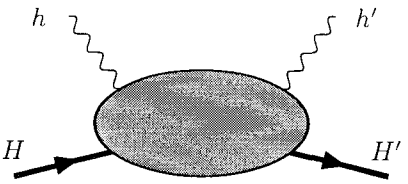

Fig. 2.4 The forward Compton scattering amplitude (h, h', H and H' are the helicities).

Since the imaginary part of the forward virtual Compton amplitude is proportional (via the optical theorem) to the virtual photo-absorption cross-sections, the following relations hold

$$W_1 \propto \sigma^T \equiv \tfrac{1}{2}(\sigma^T_{3/2} + \sigma^T_{1/2})\,, \tag{2.53a}$$

$$\left(1+\frac{\nu^2}{Q^2}\right) W_2 - W_1 \propto \sigma^L \equiv \sigma^L_{1/2}\,, \tag{2.53b}$$

$$M\nu G_1 - Q^2 G_2 \propto \tfrac{1}{2}(\sigma^T_{3/2} - \sigma^T_{1/2})\,, \tag{2.53c}$$

$$Q(MG_1 + \nu G_2) \propto \sigma^{LT}_{1/2}\,. \tag{2.53d}$$

Here T stands for transverse photons, L for longitudinal photons and $\sigma^{LT}_{1/2}$ is the longitudinal–transverse cross-section proportional to the imaginary part of the non-diagonal (*i.e.*, helicity-flip) amplitude $T(0,\frac{1}{2};1,-\frac{1}{2})$. In

terms of the structure functions F_1, F_2, g_1 and g_2 one has

$$F_1 \propto \sigma^T \equiv \tfrac{1}{2}(\sigma^T_{3/2} + \sigma^T_{1/2})\,, \tag{2.54a}$$

$$(1+\gamma^2)\frac{F_2}{2x} - F_1 \propto \sigma^L \equiv \sigma^L_{1/2}\,, \tag{2.54b}$$

$$g_1 - \gamma^2 g_2 \propto \tfrac{1}{2}(\sigma^T_{3/2} - \sigma^T_{1/2})\,, \tag{2.54c}$$

$$\gamma(g_1 + g_2) \propto \sigma^{LT}_{1/2}\,. \tag{2.54d}$$

2.6 Spin asymmetries

The quantities actually measured in the experiments are the *longitudinal spin asymmetry*

$$A_{\text{long}} = \frac{\mathrm{d}\sigma^{\leftarrow\Rightarrow} - \mathrm{d}\sigma^{\leftarrow\Leftarrow}}{\mathrm{d}\sigma^{\leftarrow\Rightarrow} + \mathrm{d}\sigma^{\leftarrow\Leftarrow}} = \frac{\mathrm{d}\sigma^{\leftarrow\Rightarrow} - \mathrm{d}\sigma^{\leftarrow\Leftarrow}}{2\,\mathrm{d}\sigma^{\text{unp}}}\,, \tag{2.55a}$$

and the *transverse-spin asymmetry*

$$A_{\text{transv}} = \frac{\mathrm{d}\sigma^{\leftarrow\Uparrow} - \mathrm{d}\sigma^{\leftarrow\Downarrow}}{\mathrm{d}\sigma^{\leftarrow\Uparrow} + \mathrm{d}\sigma^{\leftarrow\Downarrow}} = \frac{\mathrm{d}\sigma^{\leftarrow\Uparrow} - \mathrm{d}\sigma^{\leftarrow\Downarrow}}{2\,\mathrm{d}\sigma^{\text{unp}}}\,. \tag{2.55b}$$

Using (2.19) and (2.30a), or (2.23) and (2.34), the explicit expression of the longitudinal asymmetry is

$$\begin{aligned} A_{\text{long}} &= \frac{Q^2\left[(E + E'\cos\vartheta)MG_1 - Q^2 G_2\right]}{2EE'\left[2W_1\,\sin^2(\vartheta/2) + W_2\,\cos^2(\vartheta/2)\right]} \\ &= \frac{xy\left[(2 - y - \gamma^2 y^2/2)\,g_1 - \gamma^2 y\,g_2\right]}{xy^2\,F_1 + (1 - y - \gamma^2 y^2/4)\,F_2}\,. \end{aligned} \tag{2.56}$$

Notice that a measurement of A_{long} does not provide information on g_1 alone, but on a combination of g_1 and g_2. The coefficient in front of g_2, however, is $\mathcal{O}(x^2M^2/Q^2)$ and hence is very small. Therefore, to a good approximation, A_{long} is proportional to g_1,

$$A_{\text{long}} \simeq \frac{xy(2-y)\,g_1}{xy^2\,F_1 + (1-y)\,F_2}\,. \tag{2.57}$$

By means of (2.19) and (2.30b), or (2.23) and (2.35), we obtain the

expression of the transverse asymmetry in terms of the structure functions,

$$A_{\text{transv}} = \frac{Q^2\,(MG_1 + 2EG_2)\,\sin\vartheta}{2E\,[2W_1\,\sin^2(\vartheta/2) + W_2\,\cos^2(\vartheta/2)]}\,\cos\phi$$
$$= \frac{\gamma xy\,\sqrt{1-y-\gamma^2y^2/4}\,(yg_1 + 2g_2)}{xy^2\,F_1 + (1-y-\gamma^2y^2/4)\,F_2}\,\cos\phi\,. \tag{2.58}$$

Here one notices the factor $\gamma \equiv 2xM/Q$, which suppresses A_{transv} with respect to A_{long}. Since both g_1 and g_2 appear in (2.58), and the coefficient of g_1 is not negligible, in order to extract g_2 from A_{transv} one needs an independent determination of g_1. Equivalently, g_2 may be obtained from a combination of A_{long} and A_{transv}.

In principle, one could also obtain g_2 from a measurement of A_{long} alone, if this were performed at several different beam energies (Leader and Anselmino, 1988). Inverting (2.56) gives

$$g_1 - \frac{\gamma^2 y}{2-y-\gamma^2y^2/2}\,g_2 = \frac{Q^2}{2\alpha_{\text{em}}^2(2-y-\gamma^2y^2/2)}\,A_{\text{long}}\,\mathrm{d}\sigma^{\text{unp}}\,, \tag{2.59}$$

and we see that at fixed x and Q^2 the only dependence on the beam energy, (*i.e.*, on y) is in the coefficient of g_2. Thus, a study of the energy dependence of the r.h.s. of (2.59) allows extracting g_2. Again, the main difficulty arises from the smallness of the coefficient of g_2, which is $\mathcal{O}(x^2M^2/Q^2)$.

The longitudinal and transverse asymmetries are usually re-expressed in terms of the virtual Compton scattering asymmetries, defined as

$$A_1 \equiv \frac{\sigma_{1/2}^T - \sigma_{3/2}^T}{\sigma_{1/2}^T + \sigma_{3/2}^T}\,, \tag{2.60a}$$

$$A_2 \equiv \frac{2\,\sigma_{1/2}^{LT}}{\sigma_{1/2}^T + \sigma_{3/2}^T}\,. \tag{2.60b}$$

In view of (2.54a–d), the virtual photon asymmetries A_1, A_2 are given by

$$A_1 = \frac{g_1 - \gamma^2 g_2}{F_1}\,, \tag{2.61a}$$

$$A_2 = \frac{\gamma(g_1 + g_2)}{F_1}\,. \tag{2.61b}$$

With a little algebra the relations between A_{long}, A_{transv} and A_1, A_2

are found to be

$$A_{\text{long}} = D(A_1 + \eta A_2)\,, \tag{2.62a}$$

$$A_{\text{transv}} = d(A_2 - \xi A_1)\,, \tag{2.62b}$$

where the coefficients are

$$D = \frac{y(2-y)}{y^2 + 2(1-y)(1+R)}\,, \qquad \eta = 2\gamma\,\frac{1-y}{2-y}\,, \tag{2.63a}$$

$$d = D\sqrt{\frac{2\epsilon}{1+\epsilon}}\,, \qquad \xi = \frac{\eta(1+\epsilon)}{2\epsilon}\,, \tag{2.63b}$$

$$\epsilon = \frac{1-y}{1-y+y^2/2}\,. \tag{2.63c}$$

Note that η and ξ are of order xM/Q. The quantity R appearing in (2.63a) is the ratio of longitudinal to transverse virtual photo-absorption cross-sections

$$R = \frac{\sigma^L}{\sigma^T} = \frac{F_2}{2xF_1}\,(1+\gamma^2) - 1\,. \tag{2.64}$$

It can be shown (Doncel and De Rafael, 1971) that hermiticity of the electromagnetic current leads to the following constraints on the virtual-photon asymmetries

$$|A_1| \leq 1\,, \qquad |A_2| \leq \sqrt{R}\,. \tag{2.65}$$

2.7 The partonic content of structure functions

In the quark–parton model the virtual photon is assumed to scatter incoherently off the constituents of the nucleon (quarks and antiquarks). Currents are treated as in free-field theory and any interaction between the struck quark and the target remnant is ignored. The hadronic tensor $W^{\mu\nu}$ is then represented by the handbag diagram shown in Fig. 2.5 and reads (to simplify the presentation, for the moment we consider only quarks, the

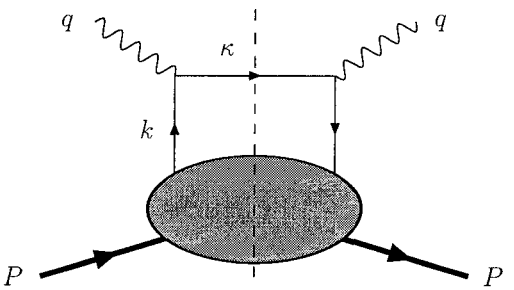

Fig. 2.5 The so-called handbag diagram of deeply-inelastic scattering.

extension to antiquarks being rather straight-forward)

$$W^{\mu\nu} = \frac{1}{2\pi} \sum_a e_a^2 \sum_X \int \frac{\mathrm{d}^3 \boldsymbol{P}_X}{(2\pi)^3\, 2E_X} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \int \frac{\mathrm{d}^4 \kappa}{(2\pi)^4}\, \delta(\kappa^2) \times \left[\bar{u}(\kappa)\gamma^\mu \phi(k;P,S)\right]^* \left[\bar{u}(\kappa)\gamma^\nu \phi(k;P,S)\right] \times (2\pi)^4 \delta^4(P-k-P_X)\, (2\pi)^4 \delta^4(k+q-\kappa)\,, \quad (2.66)$$

where $\sum_a$ is a sum over the flavours, e_a is the quark charge in units of e, and we have introduced the matrix elements of the quark field between the nucleon and its remnant (i is a Dirac index):

$$\phi_i(k,P,S) = \langle X|\psi_i(0)|PS\rangle\,. \quad (2.67)$$

We define the quark–quark correlation matrix $\Phi_{ij}(k,P,S)$ as

$$\Phi_{ij}(k,P,S) = \sum_X \int \frac{\mathrm{d}^3 \boldsymbol{P}_X}{(2\pi)^3\, 2E_X} (2\pi)^4 \delta^4(P-k-P_X) \times \langle PS|\psi_j(0)|X\rangle\langle X|\psi_i(0)|PS\rangle\,. \quad (2.68)$$

Using translational invariance and the completeness of the $|X\rangle$ states this matrix can be re-expressed in the more synthetic form

$$\Phi_{ij}(k,P,S) = \int \mathrm{d}^4\xi\ \mathrm{e}^{\mathrm{i}k\cdot\xi}\, \langle PS|\bar{\psi}_j(0)\psi_i(\xi)|PS\rangle\,. \quad (2.69)$$

With definition (2.68) the hadronic tensor becomes

$$W^{\mu\nu} = \sum_a e_a^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int \frac{\mathrm{d}^4\kappa}{(2\pi)^4}\, \delta(\kappa^2)\,(2\pi)^4\delta^4(k+q-\kappa)\ \mathrm{Tr}\left[\Phi\gamma^\mu \not{\kappa}\gamma^\nu\right]$$
$$= \sum_a e_a^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4}\, \delta\big((k+q)^2\big)\,\mathrm{Tr}\left[\Phi\gamma^\mu(\not{k}+\not{q})\gamma^\nu\right]. \qquad (2.70)$$

In order to calculate $W^{\mu\nu}$, it is convenient to use a Sudakov parametrisation of the four-momenta at hand (the Sudakov decomposition of vectors is described in App. B). We thus introduce the null vectors p^μ and n^μ, satisfying

$$p^2 = 0 = n^2\,, \qquad p{\cdot}n = 1\,, \qquad n^+ = 0 = p^-\,, \qquad (2.71)$$

and work in a frame where the virtual photon and the proton are collinear. As is customary, the proton is taken to be directed along the positive z direction (see Fig. 2.6). In terms of p^μ and n^μ the proton momentum can

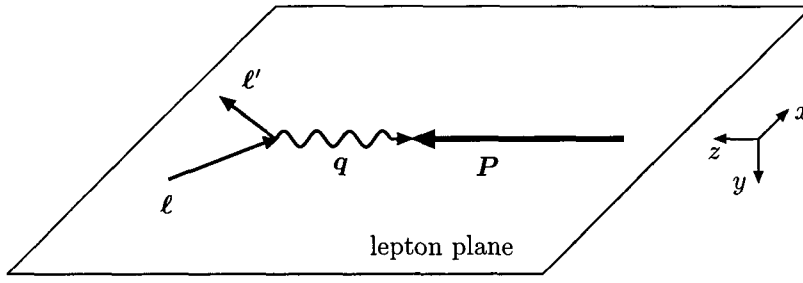

Fig. 2.6 The $\gamma^* N$ collinear frame (note our convention for the axes).

be parametrised as

$$P^\mu = p^\mu + \tfrac{1}{2}M^2 n^\mu \simeq p^\mu\,. \qquad (2.72)$$

Note that, neglecting the mass M, P^μ coincides with the Sudakov vector p^μ. The momentum q^μ of the virtual photon can be written as

$$q^\mu \simeq P{\cdot}q\, n^\mu - x\, p^\mu\,, \qquad (2.73)$$

where $\mathcal{O}(M^2/Q^2)$ terms are implicitly ignored. Finally, the Sudakov decomposition of the quark momentum is

$$k^\mu = \alpha\, p^\mu + \frac{(k^2 + \boldsymbol{k}_\perp^2)}{2\alpha}\, n^\mu + k_\perp^\mu \,. \tag{2.74}$$

In the parton model one assumes that the handbag-diagram contribution to the hadronic tensor is dominated by small values of k^2 and $\boldsymbol{k}_\perp^2$. This means that we can write k^μ approximately as

$$k^\mu \simeq \alpha\, p^\mu \,. \tag{2.75}$$

The on-shell condition of the outgoing quark then implies

$$\delta\big((k+q)^2\big) \simeq \delta(-Q^2 + 2\alpha\, P{\cdot}q) = \frac{1}{2P{\cdot}q}\,\delta(\alpha - x)\,, \tag{2.76}$$

that is, $k^\mu \simeq x\, P^\mu$. Thus, the Bjorken variable $x \equiv Q^2/(2P{\cdot}q)$ is the fraction of the longitudinal momentum of the nucleon carried by the struck quark: $x = k^+/P^+$. (In the following we shall also consider the possibility of retaining the quark transverse momentum; in this case (2.74) will be approximated as $k^\mu \simeq xP^\mu + k_\perp^\mu$.)

Returning to the hadronic tensor (2.70), the identity

$$\gamma^\mu\gamma^\rho\gamma^\nu = \big(g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\nu}g^{\rho\sigma} + \mathrm{i}\varepsilon^{\mu\rho\nu\sigma}\gamma_5\big)\,\gamma_\sigma \,, \tag{2.77}$$

allows us to split $W^{\mu\nu}$ into symmetric (S) and antisymmetric (A) parts under $\mu \leftrightarrow \nu$ interchange.

2.7.1 *Unpolarised structure functions*

Let us first consider $W_{\mu\nu}^{(\mathrm{S})}$ (*i.e.*, unpolarised DIS):

$$\begin{aligned} W_{\mu\nu}^{(\mathrm{S})} = \frac{1}{2P{\cdot}q} \sum_a e_a^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4}\, \delta(x - k^+/P^+) \\ \times \Big[(k_\mu + q_\mu)\,\mathrm{Tr}(\Phi\gamma_\nu) + (k_\nu + q_\nu)\,\mathrm{Tr}(\Phi\gamma_\mu) \\ - g_{\mu\nu}(k^\rho + q^\rho)\,\mathrm{Tr}(\Phi\gamma_\rho)\Big] \,. \end{aligned} \tag{2.78}$$

From (2.73) and (2.74) we have $k_\mu + q_\mu \simeq (P{\cdot}q)\, n_\mu$ and (2.78) becomes

$$W_{\mu\nu}^{(\mathrm{S})} = \frac{1}{2} \sum_a e_a^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\, \delta(x - k^+/P^+) \times \Big[n_\mu \,\mathrm{Tr}(\Phi\gamma_\nu) + n_\nu \,\mathrm{Tr}(\Phi\gamma_\mu) - g_{\mu\nu} n^\rho \,\mathrm{Tr}(\Phi\gamma_\rho) \Big] . \tag{2.79}$$

Introducing the notation

$$\begin{aligned} \langle \Gamma \rangle &\equiv \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\, \delta(x - k^+/P^+)\, \mathrm{Tr}(\Gamma\, \Phi) \\ &= P^+ \int \frac{\mathrm{d}\xi^-}{2\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-}\, \langle PS|\bar\psi(0)\, \Gamma\, \psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle \\ &= \int \frac{\mathrm{d}\tau}{2\pi}\, \mathrm{e}^{\mathrm{i}\tau x}\, \langle PS|\bar\psi(0)\, \Gamma\, \psi(\tau n)|PS\rangle , \end{aligned} \tag{2.80}$$

where Γ represents a generic Dirac matrix, $W_{\mu\nu}^{(\mathrm{S})}$ is written as

$$W_{\mu\nu}^{(\mathrm{S})} = \tfrac{1}{2} \sum_a e_a^2 \left[n_\mu \langle \gamma_\nu \rangle + n_\nu \langle \gamma_\mu \rangle - g_{\mu\nu}\, n^\rho \langle \gamma_\rho \rangle \right] . \tag{2.81}$$

We have now to parametrise $\langle \gamma^\mu \rangle$, which is a vector quantity containing information on the quark dynamics. At leading twist, *i.e.*, considering contributions $\mathcal{O}(P^+)$ in the infinite-momentum frame, the only vector at our disposal is $p^\mu \simeq P^\mu$ (recall that $n^\mu = \mathcal{O}(1/P^+)$ and $k^\mu \simeq xP^\mu$). Thus, we can write

$$\begin{aligned} \langle \gamma^\mu \rangle &\equiv \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\, \delta(x - k^+/P^+)\, \mathrm{Tr}(\gamma^\mu\, \Phi) \\ &= \int \frac{\mathrm{d}\tau}{2\pi}\, \mathrm{e}^{\mathrm{i}\tau x}\, \langle PS|\bar\psi(0)\, \gamma^\mu\, \psi(\tau n)|PS\rangle = 2f(x)\, P^\mu , \end{aligned} \tag{2.82}$$

where the coefficient of P^μ, which we have already called $f(x)$, is the *quark number density*, as will become clear later on (see Secs. 3.2 and 3.3). From (2.82) we obtain the following expression for $f(x)$

$$f(x) = \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-}\, \langle PS|\bar\psi(0)\gamma^+\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle . \tag{2.83}$$

Inserting (2.82) into (2.81) yields

$$W_{\mu\nu}^{(\mathrm{S})} = \sum_a e_a^2\, (n_\mu P_\nu + n_\nu P_\mu - g_{\mu\nu})\, f_a(x) . \tag{2.84}$$

The structure functions F_1 and F_2 can be extracted from $W_{\mu\nu}$ by means of the projectors defined in App. C. Neglecting terms of relative order M^2/Q^2 one has

$$F_1 = \mathcal{P}_1^{\mu\nu} W_{\mu\nu} = \frac{1}{4}\left(\frac{4x^2}{Q^2}\, P^\mu P^\nu - g^{\mu\nu}\right) W_{\mu\nu}\,, \tag{2.85a}$$

$$F_2 = \mathcal{P}_2^{\mu\nu} W_{\mu\nu} = \frac{x}{2}\left(\frac{12x^2}{Q^2}\, P^\mu P^\nu - g^{\mu\nu}\right) W_{\mu\nu}\,. \tag{2.85b}$$

Since $(P^\mu P^\nu/Q^2)\, W_{\mu\nu} = \mathcal{O}(M^2/Q^2)$, we find that F_1 and F_2 are proportional to each other, the so-called Callan–Gross relation (Callan and Gross, 1969), and are given by

$$F_2(x) = 2x\, F_1(x) = -\frac{x}{2}\, g^{\mu\nu} W_{\mu\nu}^{(\mathrm{S})} = \sum_a e_a^2\, x\, f_a(x)\,, \tag{2.86}$$

which is the well-known parton-model expression for the unpolarised structure functions, restricted to quarks. To obtain the full expressions for F_1 and F_2, one must simply add to (2.85b) the antiquark distributions $\bar{f}_a$, which were left aside in the above discussion. They read (the rôles of ψ and $\bar{\psi}$ are interchanged with respect to the quark distributions; see Sec. 3.2 for a detailed discussion)

$$\bar{f}(x) = \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-}\, \langle PS|\, \mathrm{Tr}\left[\gamma^+ \psi(0)\bar{\psi}(0,\xi^-,\mathbf{0}_\perp)\right] |PS\rangle\,, \tag{2.87}$$

and the structure functions F_1 and F_2 are

$$F_2(x) = 2x\, F_1(x) = \sum_a e_a^2\, x\, \left[f_a(x) + \bar{f}_a(x)\right]. \tag{2.88}$$

2.7.2 *The longitudinal spin structure function*

Turning now to polarised DIS, the parton-model expression for the antisymmetric part of the hadronic tensor is

$$W_{\mu\nu}^{(\mathrm{A})} = \frac{1}{2P\cdot q} \sum_a e_a^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4}\, \delta(x - k^+/P^+) \times \varepsilon_{\mu\nu\rho\sigma}(k+q)^\rho\, \mathrm{Tr}(\gamma^\sigma\gamma_5\Phi)\,. \tag{2.89}$$

With $k^\mu = xP^\mu$ this becomes, using the notation (2.80)

$$W^{(\mathrm{A})}_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma}\, n^\rho \sum_a \frac{e_a^2}{2} \langle \gamma^\sigma \gamma_5 \rangle \,. \tag{2.90}$$

At leading twist the only pseudovector at hand is $S^\mu_\parallel$ (recall that $S^\mu_\parallel = \mathcal{O}(P^+)$ and $S^\mu_\perp = \mathcal{O}(1)$) and thus $\langle \gamma^\sigma \gamma_5 \rangle$ is parametrised as (a factor M is inserted for dimensional reasons)

$$\langle \gamma^\sigma \gamma_5 \rangle = 2M\, \Delta f(x)\, S^\sigma_\parallel = 2\lambda_N\, \Delta f(x)\, P^\sigma \,. \tag{2.91}$$

Here $\Delta f(x)$, given explicitly by

$$\Delta f(x) = \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-}\, \langle PS|\bar\psi(0)\gamma^+\gamma_5\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle \,, \tag{2.92}$$

is the longitudinal polarisation (*i.e.*, helicity) distribution of quarks. In fact, inserting (2.91) in (2.90), we find

$$W^{(\mathrm{A})}_{\mu\nu} = 2\lambda_N\, \varepsilon_{\mu\nu\rho\sigma}\, n^\rho p^\sigma \sum_a \frac{e_a^2}{2}\, \Delta f_a(x) \,. \tag{2.93}$$

Comparing this with the longitudinal part of the hadronic tensor (2.44), which can be rewritten as

$$W^{(\mathrm{A})}_{\mu\nu,\,\mathrm{long}} = 2\lambda_N\, \varepsilon_{\mu\nu\rho\sigma}\, n^\rho p^\sigma\, g_1 \,, \tag{2.94}$$

we obtain the usual parton-model expression for the polarised structure function g_1

$$g_1(x) = \tfrac{1}{2} \sum_a e_a^2\, \Delta f_a(x) \,. \tag{2.95}$$

Again, antiquark distributions $\Delta f_{\bar q}$ should be added to (2.95) to obtain the full parton-model expression for g_1

$$g_1(x) = \tfrac{1}{2} \sum_a e_a^2 \left[\Delta f_a(x) + \Delta \bar f_a(x)\right] . \tag{2.96}$$

2.7.3 *The transverse-spin structure function*

Since $S_\perp$ is suppressed by a power of P^+ with respect to its longitudinal counterpart $S_\parallel$, transverse-polarisation effects in DIS first manifest

themselves at the twist-three level. Including subdominant contributions, Eq. (2.91) becomes

$$\langle \gamma^\sigma \gamma_5 \rangle = 2M\, \Delta f(x)\, S_\parallel^\sigma + 2M\, g_T(x)\, S_\perp^\sigma \,, \tag{2.97}$$

where we have introduced a new, twist-three, distribution function g_T, defined as ($\gamma_\perp$ is either γ^1 or γ^2, according to the orientation of the nucleon spin)

$$\begin{aligned} g_T(x) &= \frac{P^+}{2M} \langle \gamma_\perp \gamma_5 \rangle \\ &= \frac{P^+}{M} \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-} \langle PS | \bar\psi(0)\, \gamma_\perp \gamma_5\, \psi(0,\xi^-,\mathbf{0}_\perp) | PS \rangle \,, \end{aligned} \tag{2.98}$$

As we are working at twist three (that is, with quantities suppressed by $1/P^+$), we must take into account the transverse components of the quark momentum, $k^\mu \simeq xp^\mu + k_\perp^\mu$. Moreover, quark mass terms cannot be ignored. Reinstating these terms into the hadronic tensor, we have

$$\begin{aligned} W_{\mu\nu}^{(\mathrm{A})} = \frac{1}{2P{\cdot}q} \sum_a e_a^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\, \delta(x - k^+/P^+) \\ \times \varepsilon_{\mu\nu\rho\sigma} \left\{ (k+q)^\rho\, \mathrm{Tr}\left[\gamma^\sigma \gamma_5 \Phi\right] - \frac{1}{2}\, m_q\, \mathrm{Tr}\left[\mathrm{i}\, \sigma^{\rho\sigma}\, \gamma_5\, \Phi\right] \right\}. \end{aligned} \tag{2.99}$$

Notice that now we cannot simply set $k^\rho + q^\rho \simeq P{\cdot}q\, n^\mu$, as we did in the case of longitudinal polarisation. Let us rewrite Eq. (2.99) as

$$W_{\mu\nu}^{(\mathrm{A})} = \frac{1}{2P{\cdot}q}\, \varepsilon_{\mu\nu\rho\sigma}\, q^\rho \sum_a e_a^2 \langle \gamma^\sigma \gamma_5 \rangle + \Delta W_{\mu\nu}^{(\mathrm{A})} \,, \tag{2.100}$$

where

$$\Delta W_{\mu\nu}^{(\mathrm{A})} = \frac{1}{2P{\cdot}q}\, \varepsilon_{\mu\nu\rho\sigma} \sum_a e_a^2 \left\{ \langle \mathrm{i} \gamma^\sigma \gamma_5 \partial^\rho \rangle - \frac{1}{2} m_q \langle \mathrm{i} \sigma^{\rho\sigma} \gamma_5 \rangle \right\}. \tag{2.101}$$

If we could neglect the term $\Delta W_{\mu\nu}^{(\mathrm{A})}$ then, for a transversely polarised target, we should have, using Eq. (2.97)

$$W_{\mu\nu}^{(\mathrm{A})} = \frac{2M \varepsilon_{\mu\nu\rho\sigma} q^\rho\, S_\perp^\sigma}{P{\cdot}q} \sum_a \frac{e_a^2}{2}\, g_T^a(x) \,. \tag{2.102}$$

Comparing with Eq. (2.44) yields the parton-model expression for the polarised structure function combination $g_1 + g_2$:

$$g_1(x) + g_2(x) = \tfrac{1}{2} \sum_a e_a^2 \, g_T^a(x) \,, \tag{2.103}$$

This result has been obtained by ignoring the term $\Delta W_{\mu\nu}^{(\mathrm{A})}$ in the hadronic tensor—rather a strong assumption, which might seem lacking in justification. Surprisingly enough, however, Eq. (2.101) turns out to be correct. The reason is that at twist three one has to add an extra term $W_{\mu\nu}^{(\mathrm{A})g}$ into (2.100), arising from non-handbag diagrams with gluon exchange (see Fig. 2.7) and which exactly cancel $\Delta W_{\mu\nu}^{(\mathrm{A})}$. Referring the reader to the

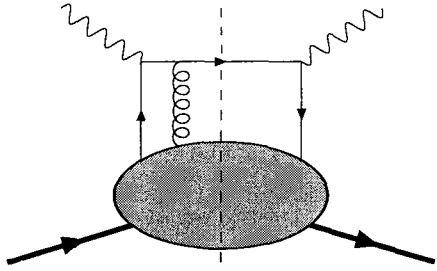

Fig. 2.7 A higher-twist contribution to DIS involving quark–quark–gluon correlation.

original papers (Efremov and Teryaev, 1984; Ratcliffe, 1986) for detailed proof, we limit ourselves to presenting the main steps.

For the sum $\Delta W_{\mu\nu}^{(\mathrm{A})} + W_{\mu\nu}^{(\mathrm{A})g}$ one obtains

$$\begin{aligned} &\Delta W_{\mu\nu}^{(\mathrm{A})} + W_{\mu\nu}^{(\mathrm{A})g} \\ &\quad = \frac{1}{4P{\cdot}q} \sum_a e_a^2 \left\{ \varepsilon_{\mu\nu\rho\sigma} \left(\langle \mathrm{i}\gamma^\sigma \gamma_5 D^\rho(\tau n) \rangle - \frac{1}{2} m_q \langle \mathrm{i}\sigma^{\rho\sigma}\gamma_5 \rangle \right) \right. \\ &\qquad \left. - \langle \gamma_\mu D_\nu(\tau n) - \gamma_\nu D_\mu(\tau n) \rangle + \ldots \right\}, \end{aligned} \tag{2.104}$$

where $D_\mu = \partial_\mu - \mathrm{i}g\,A_\mu$ and the ellipsis denotes terms with the covariant derivative acting to the left and the gluon field evaluated at the space–time

point 0. We now exploit the identity

$$\tfrac{1}{2}\,\varepsilon_{\mu\nu\tau\sigma}\sigma^{\tau\sigma}\gamma_5\gamma_\rho = g_{\rho\nu}\gamma_\mu - g_{\mu\rho}\gamma_\nu - \mathrm{i}\varepsilon_{\mu\nu\rho\sigma}\gamma^\sigma\gamma_5\,. \tag{2.105}$$

Contracting with D^ρ and using the equations of motion, $(\mathrm{i}\not{D} - m_q)\psi = 0$, we ultimately obtain

$$\tfrac{1}{2}\,m_q\,\varepsilon_{\mu\nu\tau\sigma}\langle \mathrm{i}\sigma^{\tau\sigma}\gamma_5\rangle = \langle\gamma_\nu D_\mu\rangle - \langle\gamma_\mu D_\nu\rangle + \varepsilon_{\mu\nu\rho\sigma}\langle \mathrm{i}\gamma^\sigma\gamma_5 D^\rho\rangle\,, \tag{2.106}$$

which implies the vanishing of (2.104).

Concluding, DIS with transversely polarised nucleons (where transverse refers to the photon axis) probes a twist-three distribution function, $g_T(x)$, which, as we shall see, has no probabilistic meaning and is *not* related in a simple manner to the transverse polarisation of quarks.

The parton-model description of g_2 has been the object of much discussion in the past (for a critical assessment of this issue, see Jaffe, 1990). Consider DIS on a free quark. The antisymmetric part of the hadronic tensor is then

$$w_{\mu\nu}^{(\mathrm{A})} = \frac{2m_q\varepsilon_{\mu\nu\rho\sigma}q^\rho s^\sigma}{k\cdot q}\,\dot{\delta}\!\left(1 - \frac{Q^2}{2k\cdot q}\right), \tag{2.107}$$

where k and s are the quark momentum and spin, respectively. Comparing (2.107) with (2.32 or 2.44)—with the replacements $M \to m_q$, $P \to k$ and $S \to s$, we notice first of all that the quark must be kept massive, otherwise the factor multiplying g_2 vanishes. The massless limit is to be taken afterwards. The free-quark structure functions read off from (2.107) are

$$g_1(x) = \delta(x-1)\,, \tag{2.108a}$$

$$g_2(x) = 0\,. \tag{2.108b}$$

Thus, in the naïve parton model, in which quarks are free objects, g_2 (or g_T) vanishes (for a discussion of this point, see also Anselmino and Leader, 1992). In order to have g_2 non-zero, one needs to consider the off-shellness of partons and their transverse motion (Jackson, Ross and Roberts, 1989).

In order to understand the structure of g_T, let us focus on the quark mass term appearing in the antisymmetric hadronic tensor—see Eqs. (2.101) and (2.104). We have just shown that it cancels out and does not contribute to DIS. Now we shall see that it contains the transverse-polarisation distribution of quarks $\Delta_T f$ (the decoupling of the quark mass term thus entails the explicit absence of $\Delta_T f$ from DIS, even at higher-twist level).

The matrix element $\langle \mathrm{i}\sigma^{\rho\sigma}\gamma_5\rangle$ admits a unique leading-twist parametrisation in terms of a tensor structure containing the transverse-spin vector of the target $S_\perp^\mu$ and the dominant Sudakov vector p^μ

$$\langle \mathrm{i}\sigma^{\rho\sigma}\gamma_5\rangle = 2(p^\sigma S_\perp^\rho - p^\rho S_\perp^\sigma)\,\Delta_T f(x)\,. \tag{2.109}$$

The coefficient $\Delta_T f(x)$ is indeed the transversity distribution. It can be singled out by contracting (2.109) with n_ρ, which gives (for definiteness, we take the spin vector directed along x)

$$\begin{aligned}\Delta_T f(x) &= \tfrac{1}{2}\,\langle \mathrm{i}n_\rho\sigma^{1\rho}\gamma_5\rangle \\ &= \int\frac{\mathrm{d}\xi^-}{4\pi}\,\mathrm{e}^{\mathrm{i}xP^+\xi^-}\,\langle PS|\bar\psi(0)\mathrm{i}\sigma^{1+}\gamma_5\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle\,.\end{aligned} \tag{2.110}$$

Equation (2.106) can be cast into the form of a constraint between $\Delta_T f(x)$ and other twist-three distributions embodied in $\langle\gamma_\mu D_\nu\rangle$ and $\langle \mathrm{i}\gamma^\sigma\gamma_5 D^\rho\rangle$. Let us consider the partonic content of these last two quantities. The gluonic (non-handbag) contribution $W_{\mu\nu}^{(\mathrm{A})g}$ to the hadronic tensor involves traces of a quark–quark–gluon correlation matrix. We introduce the following two quantities:

$$\begin{aligned}\langle\!\langle\gamma^\mu D^\nu(\tau_2 n)\rangle\!\rangle \equiv{}& \int\frac{\mathrm{d}\tau_1}{2\pi}\int\frac{\mathrm{d}\tau_2}{2\pi}\,\mathrm{e}^{\mathrm{i}\tau_1 x_2}\,\mathrm{e}^{\mathrm{i}\tau_2(x_1-x_2)} \\ &\times\langle PS|\bar\psi(0)\gamma^\mu D^\nu(\tau_2 n)\,\psi(\tau_1 n)|PS\rangle\,,\end{aligned} \tag{2.111a}$$

$$\begin{aligned}\langle\!\langle\mathrm{i}\gamma^\mu\gamma_5 D^\nu(\tau_2 n)\rangle\!\rangle \equiv{}& \int\frac{\mathrm{d}\tau_1}{2\pi}\int\frac{\mathrm{d}\tau_2}{2\pi}\,\mathrm{e}^{\mathrm{i}\tau_1 x_2}\,\mathrm{e}^{\mathrm{i}\tau_2(x_1-x_2)} \\ &\times\langle PS|\bar\psi(0)\mathrm{i}\gamma^\mu\gamma_5 D^\nu(\tau_2 n)\,\psi(\tau_1 n)|PS\rangle\,,\end{aligned} \tag{2.111b}$$

which are related to the matrix elements in (2.104) by

$$\int\mathrm{d}x_2\,\langle\!\langle\gamma^\mu D^\nu(\tau_2 n)\rangle\!\rangle = \langle\gamma^\mu D^\nu(\tau_2 n)\rangle\,, \tag{2.112a}$$

$$\int\mathrm{d}x_2\,\langle\!\langle\mathrm{i}\gamma^\mu\gamma_5 D^\nu(\tau_1 n)\rangle\!\rangle = \langle\mathrm{i}\gamma^\mu\gamma_5 D^\nu(\tau_1 n)\rangle\,. \tag{2.112b}$$

Keeping only the relevant terms, $\langle\!\langle\gamma^\mu D^\nu\rangle\!\rangle$ and $\langle\!\langle\mathrm{i}\gamma^\nu\gamma_5 D^\nu\rangle\!\rangle$ are parametrised as

$$\langle\!\langle\gamma^\mu D^\nu\rangle\!\rangle = 2M\,G_D(x_1,x_2)\,p^\mu\,\varepsilon^{\nu\alpha\beta\rho}\,p_\alpha\,n_\beta\,S_{\perp\rho}\,, \tag{2.113a}$$

$$\langle\!\langle\mathrm{i}\gamma^\mu\gamma_5 D^\nu\rangle\!\rangle = 2M\,\widetilde{G}_D(x_1,x_2)\,p^\mu S_\perp^\nu\,. \tag{2.113b}$$

Here two multiparton distributions, $G_D(x_1, x_2)$ and $\widetilde{G}_D(x_1, x_2)$, have been introduced. As we shall see in Sec. 3.8, Eq. (2.106) translates into the following relation between quark distribution and quark–quark–gluon correlation functions,

$$x\, g_T(x) = \int \mathrm{d}y \left[G_D(x, y) + \widetilde{G}_D(x, y) \right] + \frac{m_q}{M}\, \Delta_T f(x)\,. \tag{2.114}$$

Thus, the g_T distribution admits a decomposition into a quark mass term and an interaction-dependent part. Had we taken into account quark transverse motion, a further term would have appeared in (2.114). The transversity distributions, which do not emerge explicitly in DIS, are hidden into the quark mass term of g_T, as a consequence of the QCD equations of motion. We shall come back to this point in Sec. 3.8.

2.8 Mellin moments of polarised structure functions

The operator-product expansion (OPE—Wilson, 1969) of the hadronic tensor allows us to write the *moments* of g_1 and g_2, that is, their Mellin transforms with respect to x, as

$$\int_0^1 \mathrm{d}x\, x^{n-1}\, g_1(x, Q^2) = \frac{1}{2} a_n(Q^2)\,, \qquad n = 1, 3, 5, \ldots\,, \tag{2.115a}$$

$$\int_0^1 \mathrm{d}x\, x^{n-1}\, g_2(x, Q^2) = \frac{1}{2} \frac{n-1}{n} \left[d_n(Q^2) - a_n(Q^2) \right], \qquad n = 3, 5, 7, \ldots\,, \tag{2.115b}$$

where a_n and d_n are the matrix elements of the n-th operators of twist two and three, respectively, see Eq.(5.9a–e) for their definitions[b]. While g_1 is determined solely by leading-twist (*i.e.*, twist-two) operators, g_2 gets a twist-three contribution. Note that the OPE predicts only the *odd* moments. Moreover, there is no relation for the first moment of g_2 since no twist-three operator exists for $n = 1$.

[b]We warn the reader that in the literature a_n and d_n are sometimes defined differently and correspond to our a_{n+1} and d_{n+1}.

2.8.1 *The first moment of g_1*

In leading-order QCD, the first moment of g_1, that is,

$$\Gamma_1(Q^2) \equiv \int_0^1 \mathrm{d}x\, g_1(x, Q^2)\,, \tag{2.116}$$

admits the decomposition (*e.g.*, see Lampe and Reya, 2000)

$$\Gamma_1^{p,n} = \pm\frac{1}{12} A_3 + \frac{1}{36} A_8 + \frac{1}{9} A_0\,, \tag{2.117}$$

where the plus sign refers to a proton target (p), the minus sign to a neutron (n), and the A_i's are the matrix elements of the axial-vector quark currents (λ_3 and λ_8 are Gell-Mann flavour matrices):

$$\langle PS|\bar{\psi}\gamma_\mu\gamma_5\frac{\lambda_3}{2}\psi|PS\rangle = MA_3S_\mu\,, \tag{2.118a}$$

$$\langle PS|\bar{\psi}\gamma_\mu\gamma_5\frac{\lambda_8}{2}\psi|PS\rangle = MA_8S_\mu\,, \tag{2.118b}$$

$$\langle PS|\bar{\psi}\gamma_\mu\gamma_5\psi|PS\rangle = 2MA_0S_\mu\,. \tag{2.118c}$$

In terms of the first moments of the quark and antiquark distributions (which we call Δq and $\Delta\bar{q}$, respectively), A_3, A_8 and A_0 are given by

$$A_3 = \Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d}\,, \tag{2.119a}$$

$$A_8 = \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} - 2(\Delta s + \Delta\bar{s})\,, \tag{2.119b}$$

$$A_0 = \Delta\Sigma \equiv \sum_q(\Delta q + \Delta\bar{q}) = A_8 + 3(\Delta s + \Delta\bar{s})\,. \tag{2.119c}$$

At leading order (LO), both the non-singlet (NS) combinations A_3, A_8 and the singlet (S) A_0 are independent of Q^2, and $\Gamma_1^{p,n}$ is therefore constant. Beyond LO, A_0 is generally Q^2-dependent, according to the particular factorisation scheme adopted. The A_3 and A_8 matrix elements are conserved quantities related to the F and D parameters of hyperon β-decays (for example, see García and Kielanowski, 1985) by

$$A_3 = F + D = g_A/g_V\,, \tag{2.120a}$$

$$A_8 = 3F - D\,. \tag{2.120b}$$

The value of A_3 may be obtained by direct fits to neutron β-decay (Groom *et al.*, 2000) while the value of A_8 (or of F and D separately) requires a global analysis of the hyperon octet decays, which must also account for

$SU(3)_f$ breaking (for example, see Flores-Mendieta, García and Sánchez-Colón, 1996; Ratcliffe, 1996, 1999a):

$$A_3 = 1.2670 \pm 0.0035\,, \tag{2.121a}$$

$$A_8 = 0.58 \pm 0.03\,. \tag{2.121b}$$

Use of (2.119c) and (2.120a, b) leads to

$$\begin{aligned}\Gamma_1^{p,n} &= \pm\frac{1}{12}\,A_3 + \frac{5}{36}\,A_8 + \frac{1}{3}\,(\Delta s + \Delta\bar{s}) \\ &= \pm\frac{1}{12}\,(F+D) + \frac{5}{36}\,(3F-D) + \frac{1}{3}\,(\Delta s + \Delta\bar{s})\,.\end{aligned} \tag{2.122}$$

If we make the assumption $\Delta s = \Delta\bar{s} = 0$, then $A_0 = A_8$ and we obtain

$$\begin{aligned}\Gamma_1^{p,n} &= \pm\frac{1}{12}\,A_3 + \frac{5}{36}\,A_8 \\ &= \pm\frac{1}{12}\,(F+D) + \frac{5}{36}\,(3F-D)\,,\end{aligned} \tag{2.123}$$

which is the *Ellis–Jaffe sum rule* (Ellis and Jaffe, 1974). Numerically, using (2.121a, b), one obtains

$$\Gamma_1^p \simeq 0.186\,, \qquad \Gamma_1^n \simeq -0.025\,. \tag{2.124}$$

It turns out that the experimental result for Γ_1^p (which is the better measured quantity) is much smaller than the value (2.124), see Sec. 2.9.

In next-to-leading order (NLO) QCD the first moment of g_1 takes the form

$$\Gamma_1^{p,n} = \left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)\left[\pm\frac{1}{12}\,A_3 + \frac{1}{36}\,A_8 + \frac{1}{9}\,A_0(Q^2)\right]. \tag{2.125}$$

Note that this result is actually scheme independent; the corresponding NS anomalous dimensions vanish to all orders in perturbation theory. As anticipated, $A_0(Q^2)$ is now Q^2-dependent and is related to the quark spin $\Delta\Sigma$ and to the gluon spin Δg by

$$A_0(Q^2) = \Delta\Sigma - N_f\,\frac{\alpha_s(Q^2)}{2\pi}\,\Delta g(Q^2)\,. \tag{2.126}$$

Here $\Delta\Sigma$ is the Q^2-independent quantity that enters the nucleon spin sum rule

$$\tfrac{1}{2} = \Delta\Sigma + \Delta g + L_z\,, \tag{2.127}$$

where L_z is the orbital angular-momentum contribution.

The presence of Δg in the expression (2.125) of the singlet matrix element is a consequence of the axial anomaly (Adler, 1969; Bell and Jackiw, 1969), which relates the divergence of the axial-vector current to the gluon field. The possible rôle of the ABJ anomaly was first pointed out by Jaffe (1987) and full discussion of the gluonic contribution to A_0 was presented by Altarelli and Ross (1988) and by Efremov and Teryaev (1988). Such a contribution can explain why the measured A_0, which we note does *not* coincide with $\Delta\Sigma$, may be small while not conflicting with quark-model expectations.

2.8.2 *The Bjorken sum rule*

An important sum rule, first derived by Bjorken (1966) in the context of current algebra, relates the first moments of g_1^p and g_1^n to the vector (g_V) and axial-vector (g_A) couplings measured in nuclear β-decay. Taking the proton–neutron difference in (2.125) and using (2.120a) gives, to leading order in QCD,

$$\Gamma_1^p - \Gamma_1^n = \frac{1}{6}\frac{g_A}{g_V}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right). \tag{2.128}$$

Theoretically, the value is known up to $\mathcal{O}(\alpha_s^3)$: the result is (Larin and Vermaseren, 1991; Larin, Tkachov and Vermaseren, 1991)

$$\Gamma_1^p - \Gamma_1^n = \frac{1}{6}\frac{g_A}{g_V}\left[1 - \frac{\alpha_s(Q^2)}{\pi} - \frac{43}{12}\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 - \frac{2911}{144}\left(\frac{\alpha_s(Q^2)}{\pi}\right)^3\right]. \tag{2.129}$$

At $Q^2 = 5\,\mathrm{GeV}^2$, taking a nominal $\alpha_s/\pi = 0.061 \pm 0.004$, one obtains numerically

$$\Gamma_1^p - \Gamma_1^n = 0.181 \pm 0.005\,, \tag{2.130}$$

which is in good agreement with the data (for a more complete discussion, see Altarelli *et al.*, 1997).

2.8.3 *The Wandzura–Wilczek relation*

Let us go back to (2.115a, b). By combining these two relations we can extract d_n

$$d_n(Q^2) = 2\int_0^1 \mathrm{d}x\, x^{n-1}\left[g_1(x,Q^2) + \frac{n}{n-1}\, g_2(x,Q^2)\right], \quad n = 3,5,7,\ldots, \tag{2.131}$$

which measures the genuine twist-three effects in polarised DIS (arising from quark–quark–gluon correlations). If we assume that these effects are negligible, then, setting d_n to zero in (2.131), we obtain

$$\int_0^1 \mathrm{d}x\, x^{n-1}\left[g_1(x,Q^2) + g_2(x,Q^2)\right] = \frac{1}{n}\int_0^1 \mathrm{d}x\, x^{n-1} g_1(x,Q^2)\,. \tag{2.132}$$

Using the convolution property of Mellin transforms and the fact that $1/n$ is the Mellin transform of unity, the inverse Mellin transform of the r.h.s. of (2.132) is immediately found to be

$$\int_x^1 \frac{\mathrm{d}y}{y} g_1(x,Q^2)\,.$$

Inverting (2.132) then gives[c]

$$g_1(x,Q^2) + g_2(x,Q^2) = \int_x^1 \frac{\mathrm{d}y}{y} g_1(x,Q^2)\,, \tag{2.133}$$

that is,

$$g_2(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{\mathrm{d}y}{y} g_1(x,Q^2)\,. \tag{2.134}$$

This is the *Wandzura–Wilczek* (WW) *relation* (Wandzura and Wilczek, 1977). Its validity relies on the assumption made above, namely that the higher-twist contribution to g_2 may be neglected. In Sec. 2.9 we shall see that the present data are in rough agreement with (2.134).

Splitting g_2 into a twist-two part g_2^{WW} (the "Wandzura–Wilczek part") and a twist-three part $\bar{g}_2$,

$$g_2(x,Q^2) = g_2^{WW}(x,Q^2) + \bar{g}_2(x,Q^2)\,, \tag{2.135}$$

[c]Taking the inverse Mellin transform of an expression that is not defined for *all* n is rather unsafe from a mathematical point of view. This *caveat* behind the derivation of (2.133) from (2.132) should not be forgotten.

Eq. (2.134) becomes an exact relation for g_2^{WW}, that is,

$$g_2^{WW}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{\mathrm{d}y}{y} g_1(x,Q^2)\,. \tag{2.136}$$

At first sight, it may appear surprising that the Wandzura–Wilczek relation (2.134), based on the twist-two approximation, does not hold for a free quark, for which we have $g_2 = 0$. This is due to the fact (in the language of the OPE) that there is a cancellation between the twist-two operators and the mass-dependent twist-three operators contributing to g_2 (Jaffe and Ji, 1991a; Roberts and Ross, 1996). This cancellation, which persists in the massless limit, leads to the vanishing of g_2.

2.8.4 *The Burkhardt–Cottingham sum rule*

Another important relation involving g_2 was derived by Burkhardt and Cottingham (1970) by means of dispersive methods and Regge theory (for a general introduction to these techniques see, *e.g.*, Barone and Predazzi, 2002). The *Burkhardt–Cottingham* (BC) *sum rule* states that the first moment of g_2 vanishes, *i.e.*,

$$\int_0^1 \mathrm{d}x\, g_2(x,Q^2) = 0\,. \tag{2.137}$$

The absence of a twist-three operator for $n = 1$ (see the discussion above), is a *hint* of the BC relation, but at the same time shows that the OPE is unable to *prove* it.

Let us sketch the derivation of (2.137) within the framework of Regge theory (Burkhardt and Cottingham, 1970; for a critical discussion see Jaffe, 1990). If we decompose the antisymmetric part of the virtual Compton scattering amplitude $T_{\mu\nu}(\nu,Q^2)$ introduced in Sec. 2.5, in analogy to what was done for $W_{\mu\nu}^{(A)}$ in (2.32), we get the following relation between g_2 and the corresponding virtual Compton amplitude (which we call T_2)

$$g_2(x,Q^2) \sim \nu^2\, \mathrm{Im}\, T_2(\nu,Q^2)\,, \tag{2.138}$$

where an irrelevant constant factor has been omitted. The first moment of g_2 then reads

$$\int_0^1 \mathrm{d}x\, g_2(x,Q^2) \sim \int_{Q^2/2M}^{\infty} \mathrm{d}\nu'\, \mathrm{Im}\, T_2(\nu',Q^2)\,. \tag{2.139}$$

The amplitude T_2 obeys the dispersion relation

$$T_2(\nu, Q^2) = \frac{2}{\pi}\nu \int_{Q^2/2M}^{\infty} \frac{\mathrm{d}\nu'}{\nu'^2 - \nu^2} \operatorname{Im} T_2(\nu', Q^2)\,. \tag{2.140}$$

which follows from general properties of the S-matrix. In Regge theory, if only poles contribute to T_2, its behaviour at large ν and fixed Q^2 is

$$T_2(\nu, Q^2) \sim \nu^{\alpha_0 - 1} \quad \text{as} \quad \nu \to \infty\,, \tag{2.141}$$

where α_0 is the intercept of the relevant Regge pole. Burkhardt and Cottingham argued that all contributing poles have an intercept below zero. This implies that T_2 vanishes asymptotically faster than $1/\nu$. Therefore, we can take the limit $\nu \to \infty$ under the integral (2.140) and get

$$T_2(\nu, Q^2) \sim -\frac{2}{\pi\nu} \int_{Q^2/2M}^{\infty} \mathrm{d}\nu' \operatorname{Im} T_2(\nu', Q^2)\,. \tag{2.142}$$

This is compatible with (2.141), in the case $\alpha_0 < 1$, only if the integral on the r.h.s. vanishes, that is, if

$$\int_{Q^2/2M}^{\infty} \mathrm{d}\nu' \operatorname{Im} T_2(\nu', Q^2) = 0\,. \tag{2.143}$$

In view of (2.139), this is equivalent to the BC sum rule (2.137).

Possible contributions from Regge-cut singularities spoil the above argument, as pointed out by Heimann (1973). Multi-pomeron cuts, in fact, lead to a highly singular behaviour $g_2(x) \sim x^{-2}$ as $x \to 0$, so that the first moment of g_2 might even diverge. However, one can argue (for a discussion, see Jaffe, 1990) that, since Regge cuts arise from non-planar graphs (*e.g.*, see Barone and Predazzi, 2002), which correspond to higher twist and hence are suppressed at large Q^2, at least asymptotically, the BC sum rule should be reliable. This conclusion has recently been challenged by Ivanov *et al.* (1999a,b), who found a steeply rising unitarity correction to g_2, which would invalidate the BC sum rule (see Sec. 5.6).

In perturbative QCD, it has been shown (Altarelli *et al.*, 1994; Kodaira *et al.*, 1995) that the BC sum rule does not receive any correction at order α_s and is free from target-mass effects (we shall discuss this point further in Sec. 5.5.1).

2.8.5 *The Efremov–Leader–Teryaev sum rule*

Using the field-theoretical definitions of the parton distributions, it is possible to derive expressions for the *even* moments of the valence parts of g_1 and g_2 (Efremov, Teryaev and Leader, 1997). In particular, for the case $n = 2$, one can prove an exact relation, namely (the superscript V stands for "valence")

$$\int_0^1 \mathrm{d}x\, x \left[g_1^V(x) + 2\, g_2^V(x) \right] = 0\,. \tag{2.144}$$

This is known as the *Efremov–Leader–Teryaev* (ELT) *sum rule* (Efremov, Teryaev and Leader, 1997). Since only the valence components (that is, quark densities *minus* antiquark densities) appear in (2.144), the ELT sum rule does not suffer from effects of high singularities as $x \to 0$. The analogue of (2.144) for the full structure functions g_1 and g_2 is a more complicated, and less useful, relation containing on the r.h.s. a combination of quark–quark–gluon correlators.

2.9 Experimental results on polarised structure functions

In about a quarter of century, polarised DIS experiments have probed the structure functions g_1 and g_2 of the proton, neutron and deuteron in a kinematic range that is rather wide in x ($0.003 < x < 0.8$), but quite limited in Q^2 ($2\,\mathrm{GeV}^2 < \langle Q^2 \rangle < 10\,\mathrm{GeV}^2$). A summary of all spin measurements is given in Table 2.1. P_B and P_T denote the *beam* and *target polarisation*, respectively. Their range reflects the different experimental techniques adopted by the various collaborations. The quantity called f is the *dilution factor*, *i.e.*, the fraction of scattered events contributing to polarised DIS. For example, in the H-butanol target only the hydrogen atoms contribute to polarised scattering because carbon and oxygen are spin-zero nuclei. Thus, the effective target polarisation is reduced by a factor $f = 10/74$, which is the ratio of the number of hydrogen nucleons over the total number of nucleons. The last column of Table 2.1 reports the total nucleon luminosity in units of 10^{32} nucleons/cm^2/s.

Assuming no time variation of the beam current, of the target density nor of the beam and target polarisations, the longitudinal and transverse

Table 2.1 A summary of spin structure-function measurements (see the text for an explanation of the symbols); adapted from Filippone and Ji (2001).

Lab.	Expt.	Year	Beam		$\langle Q^2 \rangle$	x	P_B	Target	P_T	f	$\mathcal{L} \times 10^{-32}$
			GeV	Type	GeV^2		%		%		cm^{-2}·s
SLAC	E80	75	10–16	e^-	2	0.1 – 0.5	85	H-butanol	50	0.13	400
	E130	80	16–23	e^-	5	0.1 – 0.6	81	H-butanol	58	0.15	400
	E142	92	19–26	e^-	2	0.03 – 0.6	39	^{3}He	35	0.35	2000
	E143	93	10–29	e^-	3	0.03 – 0.8	85	NH_3	70	0.15	1000
								ND_3	25	0.24	1000
	E154	95	48	e^-	5	0.01 – 0.7	82	^{3}He	38	0.55	3000
	E155	97	48	e^-	5	0.01 – 0.9	81	NH_3	90	0.15	1000
								LiD	22	0.36	1000
	E155x	99	30	e^-	3	0.02 – 0.9	83	NH_3	75	0.16	1000
								LiD	22	0.36	1000
CERN	EMC	85	100–200	μ^+	11	0.01 – 0.7	79	NH_3	78	0.16	0.3
	SMC	92	100	μ^+	4.6	0.006 – 0.6	82	D-butanol	35	0.19	0.3
		93	190	μ^+	10	0.003 – 0.7	80	H-butanol	86	0.12	0.6
		94–95					81	D-butanol	50	0.20	0.6
		96					77	NH_3	89	0.16	0.6
DESY	HERMES	95	28	e^+	2.5	0.02 – 0.6	55	^{3}He	46	1.0	1
		96–97					55	H	88	1.0	0.1
		98	28	e^-			55	D	85	1.0	0.2
		99–00	28	e^+			55	D	85	1.0	0.2
CERN	COMPASS	02	190	μ^+	10	0.005 – 0.6	80	NH_3	90	0.16	3
								LiD	40	0.50	3
JLAB	E97-103	01	4–6	e^+	0.5 – 1.4	0.14 – 0.22	75	^{3}He	41	0.93	10^4

asymmetries are determined via (we ignore radiative corrections here)

$$A_{\text{long}} \text{ (or } A_{\text{transv}}) = \frac{A_{\text{exp}}}{P_B P_T f}, \tag{2.145}$$

where A_{exp} is the experimentally measured asymmetry, which is expressed in terms of the number $N^{\pm}$ of scattered leptons per incident lepton for positive (+) and negative helicity (−) as

$$A_{\text{exp}} = \frac{N^- - N^+}{N^- + N^+}. \tag{2.146}$$

In the case when there is time dependence in the beam and target parameters, the extraction of the asymmetry from the measured quantities is slightly more complicated (*e.g.*, see Filippone and Ji, 2001).

Once A_{long} has been determined, the g_1 structure function is obtained by means of Eq. (2.56) or (2.57). The ratio g_1^p/F_1^p, which coincides with the virtual photoabsorption asymmetry A_1^p modulo an $\mathcal{O}(x^2M^2/Q^2)$ contribution – see Eq. (2.61a), is shown in Fig. 2.8. As can be seen, the

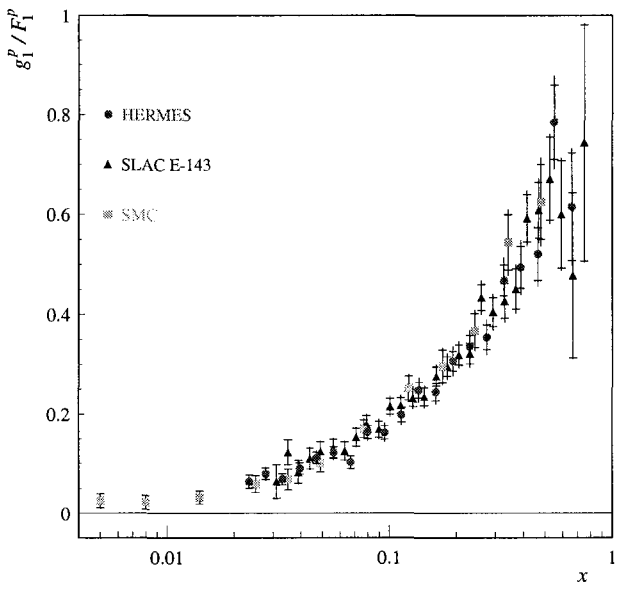

Fig. 2.8 The ratio of polarised to unpolarised proton structure function from the SMC, E143 and HERMES experiments; from Filippone and Ji (2001).

measurements have achieved a high degree of accuracy and, moreover, the agreement between the experiments is very good. Within the presently explored kinematical range, the ratio g_1^p/F_1^p turns out to be very nearly Q^2-independent.

The existing data on the g_1 structure function for proton, deuteron and neutron are collected in Fig. 2.9. The first moment of g_1^p is found to be (Anthony *et al.*, 2000)

$$\Gamma_1^p = 0.118 \pm 0.004 \pm 0.007 \quad \text{at} \quad Q^2 = 5\,\text{GeV}^2\,. \tag{2.147}$$

This value is the result of the evolution of the world data to $Q^2 = 5\,\text{GeV}^2$

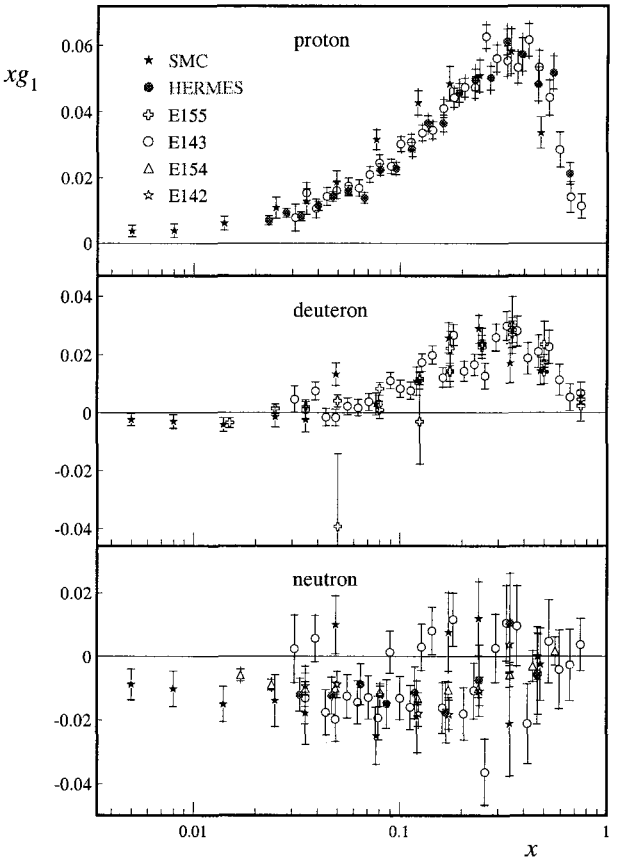

Fig. 2.9 A compilation of the data on g_1^p, g_1^d and g_1^n; from Filippone and Ji (2001).

and of an extrapolation of g_1^p to low and high x using a NLO fit. The same analysis leads for the Bjorken sum to

$$\Gamma_1^p - \Gamma_1^n = 0.176 \pm 0.003 \pm 0.007 \quad \text{at} \quad Q^2 = 5\,\text{GeV}^2\,. \tag{2.148}$$

The agreement with the theoretical expectation (2.130) is excellent.

Coming now to g_2, we recall that the determination of this structure functions is more arduous, as discussed in Sec. 2.6. One can extract g_2 either from a combination of A_{long} and A_{transv} or from A_{transv} and a fit to g_1. The former method is used by the SMC (Adams *et al.*, 1994) and the E143 (Abe *et al.*, 1996) and E155 (Anthony *et al.*, 1999) collaborations, the latter is used in the latest E155 analysis (Anthony *et al.*, 2002).

The virtual photoabsorption asymmetries A_2 for proton and deuteron are shown in Fig. 2.10. No evidence for a Q^2 dependence of A_2 is found.

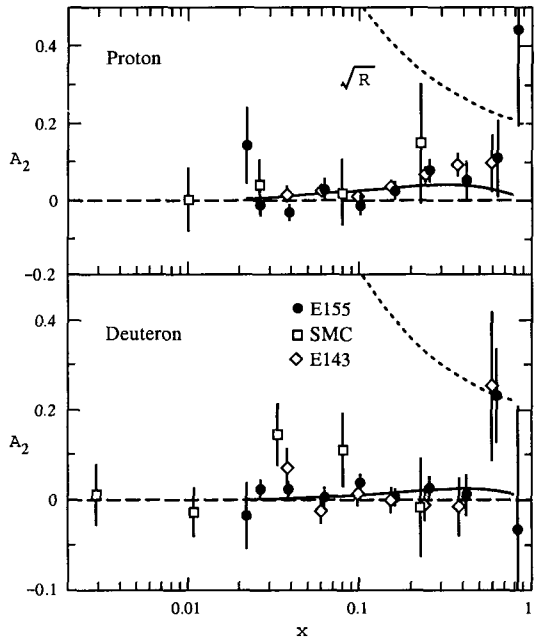

Fig. 2.10 The asymmetries A_2 for proton and deuteron. Data from SMC (Adams *et al.*, 1994), E143 (Abe *et al.*, 1996) and E155 (Anthony *et al.*, 1999) are shown. The errors are statistical; the systematic errors are negligible. The short-dashed line is the positivity bound $\sqrt{R}$. The long-dashed line is the result of a Wandzura–Wilczek calculation of A_2. From Anthony *et al.* (1999).

Note that A_2^p is significantly larger than zero around $x \sim 0.2-0.3$ and both A_2^p and A_2^d are far below the positivity limit $\sqrt{R}$ for $x \lesssim 0.5$.

The transverse-spin structure function xg_2 for proton and deuteron is displayed in Fig. 2.11. It is interesting to observe that the measured g_2 is

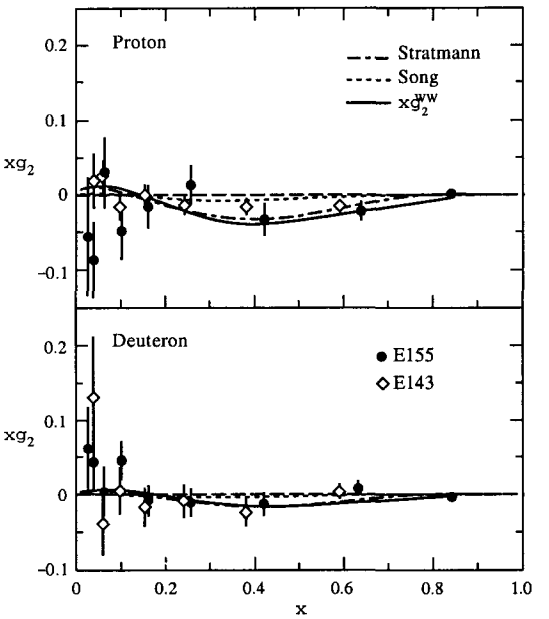

Fig. 2.11 The spin structure function xg_2 from E143 (Abe *et al.*, 1996) and E155 (Anthony *et al.*, 1999). The curves represent various model calculations (see Sec. 3.11.3). From Anthony *et al.* (1999).

consistent with the twist-two Wandzura–Wilczek calculation, that is, with g_2^{WW} computed from g_1 by means of (2.136). The twist-three reduced matrix element d_3, see Eq. (2.115b), is found to be very small, namely (Anthony *et al.*, 1999)

$$d_3^p = 0.005 \pm 0.008\,, \quad d_3^d = 0.008 \pm 0.005\,, \quad \langle Q^2 \rangle = 5\,\mathrm{GeV}^2. \tag{2.149}$$

A combined analysis of the E142 (Anthony *et al.*, 1993) and E154 (Abe *et al.*, 1997) data on neutron, and of the E143 (Abe *et al.*, 1995a,b, 1998) and E155 (Anthony *et al.*, 1999) data on proton and deuteron, yields

$$d_3^p = 0.007 \pm 0.004\,, \quad d_3^d = 0.004 \pm 0.010\,. \tag{2.150}$$

These values are compatible with the results of most of the calculations based on models or other non-perturbative tools, with the exception of the lattice result (for the comparison between the experimental findings on d_3 and the theoretical predictions see Sec. 3.11.3).

As for the first moment of g_2^p, the combined E143 and E155 data give

$$\int_{0.02}^{0.8} \mathrm{d}x\, g_2^p = -0.015 \pm 0.026 \quad \text{at} \quad Q^2 = 5\,\mathrm{GeV}^2\,, \tag{2.151}$$

a value consistent, within errors, with the Burkhardt–Cottingham sum rule (2.137).

The E155 collaboration (Anthony *et al.*, 2002) has recently reported new results on g_2 (Fig. 2.12). Some statistically significant differences between the measured g_2^p and the Wandzura–Wilczek prediction are found, possibly indicating a non-zero twist-three contribution. Moreover, the data appear to be inconsistent with the Burkhardt–Cottingham sum rule, provided that g_2 has no pathological behaviour as $x \to 0$.

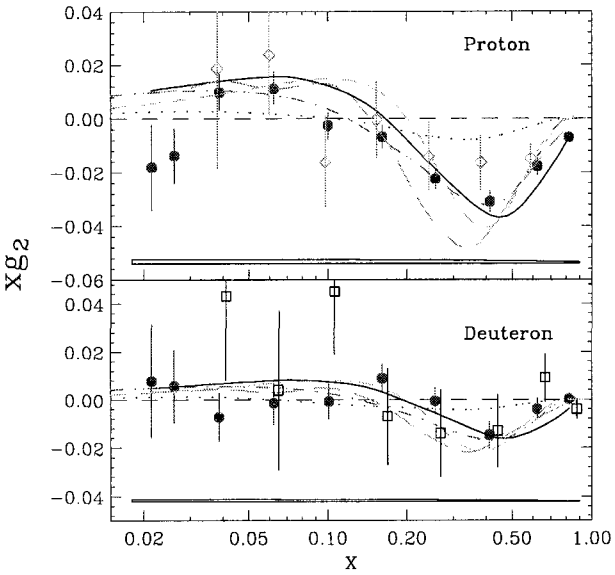

Fig. 2.12 The spin structure function xg_2 from the latest E155 analysis (Anthony *et al.*, 2002)—solid circles. The errors are statistical; the systematic errors are indicated at the foot of the figure. Also shown are the E143 (Abe *et al.*, 1996) and E155 data (Anthony *et al.*, 1999)—diamonds and squares, respectively. The curves represent various model calculations. From Anthony *et al.* (2002).

2.10 Transverse spin in electroweak DIS

We conclude this chapter with a brief discussion on the rôle of transverse spin in electroweak DIS. When the momentum transfer Q becomes comparable to the Z mass, neutral current DIS receives sizable contributions not only from photon exchange but also from Z exchange and γ–Z interference. Consequently a number of new structure functions (called g_3, g_4, g_5) appear[d]. The cross-section for the process $\ell^{\mp} N \to \ell^{\mp} X$ is the sum of three contributions,

$$\frac{\mathrm{d}^2\sigma_{NC}}{\mathrm{d}x\,\mathrm{d}y} = \frac{\pi\alpha^2 y}{Q^4} \sum_{i=\gamma,\gamma Z,Z} \eta^i L^i_{\mu\nu} W^{i,\mu\nu} \,. \tag{2.152}$$

The factors η^i in (2.152) are

$$\eta^{\gamma} = 1 \,, \tag{2.153a}$$

$$\eta^{\gamma Z} = \frac{G_F m_Z^2}{2\sqrt{2}\pi\alpha} \frac{1}{(1+Q^2/m_Z^2)^2} \,, \tag{2.153b}$$

$$\eta^{Z} = (\eta^{\gamma Z})^2 = \left[\frac{G_F m_Z^2}{2\sqrt{2}\pi\alpha} \frac{1}{(1+Q^2/m_Z^2)^2}\right]^2 \,, \tag{2.153c}$$

where G_F is the Fermi coupling constant, m_Z is the Z^0 mass and

$$\frac{G_F m_Z^2}{2\sqrt{2}\pi\alpha} = \frac{1}{4\sin^2\theta_W \cos^2\theta_W} \simeq 1.4 \,. \tag{2.154}$$

For Q^2 values up to $\sim 10^3\,\mathrm{GeV}^2$, one has

$$\eta^Z \ll \eta^{\gamma Z} \ll \eta^{\gamma} \,, \tag{2.155}$$

and therefore weak effects become relevant only at extremely high Q^2.

Assuming, as usual, that the leptons are longitudinally polarised, the

[d]The reader is warned that the nomenclature concerning these structure functions is not yet well established and may be author dependent. For instance, our conventions differ from those of Anselmino, Efremov and Leader (1995) in that our g_3 and g_5 corresponds to their $-g_5$ and $-g_3$ respectively.

leptonic tensors are

$$L^{\gamma}_{\mu\nu} = \sum_{s_{l'}} \left[\bar{u}_{l'}(\ell', s_{l'})\gamma_\mu u_l(\ell, s_l)\right]^* \left[\bar{u}_{l'}(\ell', s_{l'})\gamma_\nu u_l(\ell, s_l)\right]$$
$$= 2\left(\ell_\mu \ell'_\nu + \ell_\nu \ell'_\mu - g_{\mu\nu}\ell\cdot\ell' - \mathrm{i}\lambda_l \varepsilon_{\mu\nu\rho\sigma}\ell^\rho \ell'^\sigma\right), \tag{2.156a}$$

$$L^{\gamma Z}_{\mu\nu} = \sum_{s_{l'}} \left[\bar{u}_{l'}(\ell', s_{l'})\gamma_\mu (v_l - a_l\gamma_5) u_l(\ell, s_l)\right]^* \left[\bar{u}_{l'}(\ell', s_{l'})\gamma_\nu u_l(\ell, s_l)\right]$$
$$= (v_l - \lambda_l a_l) L^{\gamma}_{\mu\nu}, \tag{2.156b}$$

$$L^{Z}_{\mu\nu} = \sum_{s_{l'}} \left[\bar{u}_{l'}(\ell', s_{l'})\gamma_\mu (v_l - a_l\gamma_5) u_l(\ell, s_l)\right]^* \times \left[\bar{u}_{l'}(\ell', s_{l'})\gamma_\nu (v_l - a_l\gamma_5) u_l(\ell, s_l)\right]$$
$$= (v_l - \lambda_l a_l)^2 L^{\gamma}_{\mu\nu}, \tag{2.156c}$$

with

$$v_l = -\frac{1}{2} + 2\sin^2\theta_W, \qquad a_l = \mp\frac{1}{2}, \tag{2.157}$$

for $Z - \ell^{\mp}$ couplings.

The parametrisation of the hadronic tensor is

$$\begin{aligned} W^i_{\mu\nu} = {} & 2\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F^i_1 \\ & + \frac{2}{P\cdot q}\left(P_\mu - \frac{P\cdot q}{q^2} q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2} q_\nu\right) F^i_2 \\ & + \mathrm{i}\varepsilon_{\mu\nu\rho\sigma}\frac{P^\rho q^\sigma}{P\cdot q} F^i_3 \\ & + \frac{2M}{P\cdot q}\left\{\mathrm{i}\varepsilon_{\mu\nu\rho\sigma} q^\rho S^\sigma g^i_1 + \mathrm{i}\varepsilon_{\mu\nu\rho\sigma} q^\rho \left(S^\sigma - \frac{S\cdot q}{P\cdot q} P^\sigma\right) g^i_2 \right. \\ & \quad + S\cdot q\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) g^i_3 \\ & \quad + \frac{S\cdot q}{P\cdot q}\left(P_\mu - \frac{P\cdot q}{q^2} q^\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2} q^\nu\right) g^i_4 \\ & \quad + \frac{1}{2}\left[\left(P_\mu - \frac{P\cdot q}{q^2} q^\mu\right)\left(S_\nu - \frac{S\cdot q}{q^2} q^\nu\right)\right. \\ & \quad \left.\left. + \left(P_\nu - \frac{P\cdot q}{q^2} q^\nu\right)\left(S_\mu - \frac{S\cdot q}{q^2} q^\mu\right)\right] g^i_5 \right\}. \end{aligned} \tag{2.158}$$

Here we recognise the structure (2.21) in the first two terms and (2.32) in

the fourth and fifth terms. The parity violating structure functions are F_3 (unpolarised) and g_3, g_4 and g_5 (polarised). For $i = \gamma$ we obviously have $F_3^\gamma = g_3^\gamma = g_4^\gamma = g_5^\gamma = 0$. There are a total of 12 polarised structure functions (two of them, g_1^γ and g_2^γ, coincide with the electromagnetic structure functions g_1 and g_2 discussed so far in this chapter).

Let us now see which structure functions are related to the transverse spin of the target. We first simplify $W_{\mu\nu}^i$ by omitting terms that give contributions proportional to the lepton mass when contracted with the leptonic tensor,

$$W_{\mu\nu}^i = -2g_{\mu\nu}F_1^i + \frac{2P_\mu P_\nu}{P\cdot q}F_2^i + \mathrm{i}\varepsilon_{\mu\nu\rho\sigma}\frac{P^\rho q^\sigma}{P\cdot q}F_3^i + \frac{2M}{P\cdot q}\left\{\mathrm{i}\varepsilon_{\mu\nu\rho\sigma}q^\rho S^\sigma g_1^i + \mathrm{i}\varepsilon_{\mu\nu\rho\sigma}q^\rho\left(S^\sigma - \frac{S\cdot q}{P\cdot q}P^\sigma\right)g_2^i - g_{\mu\nu}S\cdot q g_3^i + \frac{S\cdot q}{P\cdot q}P_\mu P_\nu g_4^i + \frac{1}{2}\left(P_\mu S_\nu + P_\nu S_\mu\right)g_5^i\right\}. \quad (2.159)$$

With $S^\mu = (\lambda_N/M)P^\mu + S_\perp^\mu$ (where 'transverse' refers to the γ^*N axis) the hadronic tensor $W_{\mu\nu}^i$ can be rearranged as

$$W_{\mu\nu}^i = W_{\mu\nu}^i(S=0) + \frac{2M}{P\cdot q}\left\{\frac{\lambda_N}{M}\left[\mathrm{i}\varepsilon_{\mu\nu\rho\sigma}q^\rho P^\sigma g_1^i - g_{\mu\nu}P\cdot q g_3^i + P_\mu P_\nu\left(g_4^i + g_5^i\right)\right] + \mathrm{i}\varepsilon_{\mu\nu\rho\sigma}q^\rho S_\perp^\sigma\left(g_1^i + g_2^i\right) + \frac{1}{2}\left(P_\mu S_{\perp\nu} + P_\nu S_{\perp\mu}\right)g_5^i\right\}. \quad (2.160)$$

As one can see, the transverse-spin structure functions are $g_1^i + g_2^i$ and g_5^i.

The case of charged-current DIS, $\ell^\mp N \to \nu(\bar{\nu})X$, which is mediated by W bosons, is treated similarly and a hadronic tensor analogous to (2.160) is introduced, with polarised structure functions g_1^W, g_2^W, g_3^W, g_4^W and g_5^W. One can extract information on the electroweak structure functions by looking at appropriate combinations of cross-sections (Anselmino, Gambino and Kalinowski, 1994).

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Chapter 3

The transverse-spin structure of the proton

In this chapter we shall discuss in detail the leading-twist and twist-three quark and antiquark distribution functions of the proton, focusing on transverse spin. The distributions we have already encountered in Chap. 1.5 will be revisited in a more systematic fashion, and new $\boldsymbol{k}_\perp$-dependent distributions will be introduced. We shall also study the sum rules involving these functions. Finally, a brief review of the model calculations of $\Delta_T f$ and g_2 will be presented.

3.1 The quark–quark correlation matrix

Let us consider the quark–quark correlation matrix introduced in Sec. 2.7 and represented in Fig. 3.1,

$$\Phi_{ij}(k,P,S) = \int \mathrm{d}^4\xi \; \mathrm{e}^{\mathrm{i}k\cdot\xi} \, \langle PS|\bar{\psi}_j(0)\psi_i(\xi)|PS\rangle \,. \tag{3.1}$$

Here, we recall, i and j are Dirac indices and a summation over colour is implicit. The quark distribution functions are essentially integrals over k of traces of the form

$$\mathrm{Tr}(\Gamma\Phi) = \int \mathrm{d}^4\xi \; \mathrm{e}^{\mathrm{i}k\cdot\xi} \, \langle PS|\bar{\psi}(0)\,\Gamma\,\psi(\xi)|PS\rangle \,, \tag{3.2}$$

where Γ is a generic Dirac matrix structure.

In Sec. 2.7 Φ was defined within the naïve parton model. In QCD, in order to make Φ gauge invariant, a path-dependent link operator (a "Wilson

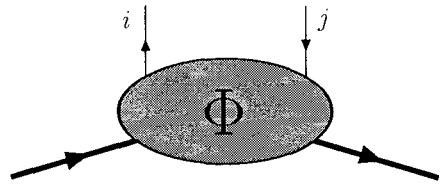


Fig. 3.1 The quark–quark correlation matrix Φ.

line")

$$\mathcal{L}(0,\xi) = \mathcal{P}\,\exp\left(-\mathrm{i}g\int_0^\xi \mathrm{d}s_\mu\, A^\mu(s)\right), \tag{3.3}$$

where $\mathcal{P}$ denotes path-ordering, must be inserted between the quark fields. It turns out that the distribution functions involve separations ξ of the form $(0,\xi^-,\mathbf{0}_\perp)$, or $(0,\xi^-,\boldsymbol{\xi}_\perp)$. Thus, by working in the axial gauge $A^+ = 0$ and choosing an appropriate path, $\mathcal{L}$ can be reduced to unity. Hereafter, we shall simply assume that the link operator is unity, and just omit it.[a]

The Φ matrix satisfies certain relations arising from hermiticity, parity invariance and time-reversal invariance (Mulders and Tangerman, 1996):

$$\Phi^\dagger(k,P,S) = \gamma^0\,\Phi(k,P,S)\,\gamma^0 \qquad \text{(hermiticity)}, \tag{3.4a}$$

$$\Phi(k,P,S) = \gamma^0\,\Phi(\widetilde{k},\widetilde{P},-\widetilde{S})\,\gamma^0 \qquad \text{(parity)}, \tag{3.4b}$$

$$\Phi^*(k,P,S) = \gamma_5 C\,\Phi(\widetilde{k},\widetilde{P},\widetilde{S})\,C^\dagger\gamma_5 \qquad \text{(time-reversal)}, \tag{3.4c}$$

where $C = \mathrm{i}\gamma^2\gamma^0$ and the tilde four-vectors are defined as $\widetilde{k}^\mu = (k^0,-\boldsymbol{k})$. As we shall see, the time-reversal condition (3.4c) plays an important rôle in the phenomenology of transverse-polarisation distributions. It is derived in a straight-forward manner by using $T\,\psi(\xi)\,T^\dagger = -\mathrm{i}\gamma_5 C\,\psi(-\widetilde{\xi})$ and $T\,|PS\rangle = (-1)^{S-S_z}\,|\widetilde{P}\widetilde{S}\rangle$, where T is the time-reversal operator. The crucial point, to be kept in mind, is the transformation of the nucleon state, which is a free particle state and therefore, under T, goes into the same

[a]Note however that, as recently stressed by Collins (2002), there are some situations in which a careful treatment of the link operator is required (see Sec. 3.10).

state with $\boldsymbol{P}$ and $\boldsymbol{S}$ reversed. Another assumption implicit in (3.4c) is that Wilson lines may be ignored.

The most general decomposition of Φ in a basis of Dirac matrices,

$$\Gamma = \{\mathbb{1},\ \gamma^\mu,\ \gamma^\mu\gamma_5,\ \mathrm{i}\gamma_5,\ \mathrm{i}\sigma^{\mu\nu}\gamma_5\}\,, \tag{3.5}$$

is (we introduce a factor $\frac{1}{2}$ for later convenience)

$$\Phi(k,P,S) = \tfrac{1}{2}\{\mathcal{S}\,\mathbb{1} + \mathcal{V}_\mu\,\gamma^\mu + \mathcal{A}_\mu\gamma_5\gamma^\mu + \mathrm{i}\mathcal{P}_5\gamma_5 + \mathrm{i}\,\mathcal{T}_{\mu\nu}\,\sigma^{\mu\nu}\gamma_5\}\,. \tag{3.6}$$

The quantities $\mathcal{S}$, $\mathcal{V}^\mu$, $\mathcal{A}^\mu$, $\mathcal{P}_5$ and $\mathcal{T}^{\mu\nu}$ are constructed with the vectors k^μ, P^μ and the pseudovector S^μ. Imposing the constraints (3.4a–c) we have, in general,

$$\mathcal{S} = \tfrac{1}{2}\,\mathrm{Tr}(\Phi) = C_1\,, \tag{3.7a}$$
$$\mathcal{V}^\mu = \tfrac{1}{2}\,\mathrm{Tr}(\gamma^\mu\Phi) = C_2\,P^\mu + C_3\,k^\mu\,, \tag{3.7b}$$
$$\mathcal{A}^\mu = \tfrac{1}{2}\,\mathrm{Tr}(\gamma^\mu\gamma_5\Phi) = C_4\,S^\mu + C_5\,k{\cdot}S\,P^\mu + C_6\,k{\cdot}S\,k^\mu\,, \tag{3.7c}$$
$$\mathcal{P}_5 = \tfrac{1}{2\mathrm{i}}\,\mathrm{Tr}(\gamma_5\Phi) = 0\,, \tag{3.7d}$$
$$\mathcal{T}^{\mu\nu} = \tfrac{1}{2\mathrm{i}}\,\mathrm{Tr}(\sigma^{\mu\nu}\gamma_5\Phi) = C_7\,P^{[\mu}S^{\nu]} + C_8\,k^{[\mu}S^{\nu]} + C_9\,k{\cdot}S\,P^{[\mu}k^{\nu]}\,, \tag{3.7e}$$

where the coefficients $C_i = C_i(k^2, k{\cdot}P)$ must be real functions, owing to hermiticity.

3.2 Leading-twist distribution functions

We are mainly interested in the leading-twist contributions, that is, the terms in Eqs. (3.7a–e) that are $\mathcal{O}(P^+)$ in the infinite-momentum frame.

The vectors at our disposal are P^μ, $k^\mu \simeq xP^\mu$, and $S^\mu \simeq \lambda_N P^\mu/M + S_\perp^\mu$, where the approximate equality signs indicate that we are neglecting terms suppressed by $(P^+)^{-2}$. Remember that the transverse-spin vector $S_\perp^\mu$ is of order $(P^+)^0$. For the time being we ignore quark transverse momentum $k_\perp^\mu$ (which in DIS is integrated over). We shall see later on how the situation becomes more complicated when $k_\perp^\mu$ enters the game.

At leading order in P^+ only the vector, axial, and tensor terms in (3.6)

survive and Eqs. (3.7b, c, e) become

$$\mathcal{V}^\mu = \tfrac{1}{2}\int \mathrm{d}^4\xi\ \mathrm{e}^{\mathrm{i}k\cdot\xi}\,\langle PS|\bar\psi(0)\gamma^\mu\psi(\xi)|PS\rangle = A_1\,P^\mu\,, \tag{3.8a}$$

$$\mathcal{A}^\mu = \tfrac{1}{2}\int \mathrm{d}^4\xi\ \mathrm{e}^{\mathrm{i}k\cdot\xi}\,\langle PS|\bar\psi(0)\gamma^\mu\gamma_5\psi(\xi)|PS\rangle = \lambda_N\,A_2\,P^\mu\,, \tag{3.8b}$$

$$\mathcal{T}^{\mu\nu} = \tfrac{1}{2\mathrm{i}}\int \mathrm{d}^4\xi\ \mathrm{e}^{\mathrm{i}k\cdot\xi}\,\langle PS|\bar\psi(0)\sigma^{\mu\nu}\gamma_5\psi(\xi)|PS\rangle = A_3\,P^{[\mu}S_\perp^{\nu]}\,, \tag{3.8c}$$

where we have introduced new real functions $A_i(k^2, k{\cdot}P)$. The leading-twist quark correlation matrix (3.6) is then (we use $P^{[\mu}S_\perp^{\nu]}\sigma_{\mu\nu} = \mathrm{i}\,\not{P}\,\not{S}_\perp$)

$$\Phi(k,P,S) = \tfrac{1}{2}\left\{A_1\,\not{P} + A_2\,\lambda_N\,\gamma_5\,\not{P} + A_3\,\not{P}\gamma_5\,\not{S}_\perp\right\}. \tag{3.9}$$

From (3.8a–c) we obtain

$$A_1 = \frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\Phi)\,, \tag{3.10a}$$

$$\lambda_N\,A_2 = \frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\gamma_5\Phi)\,, \tag{3.10b}$$

$$S_\perp^i\,A_3 = \frac{1}{2P^+}\,\mathrm{Tr}(\mathrm{i}\sigma^{i+}\gamma_5\Phi) = \frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\gamma^i\gamma_5\Phi)\,. \tag{3.10c}$$

The leading-twist distribution functions $f(x)$, $\Delta f(x)$ and $\Delta_T f(x)$ are obtained by integrating A_1, A_2 and A_3, respectively, over k, with the constraint $x = k^+/P^+$, that is,

$$\begin{Bmatrix} f(x) \\ \Delta f(x) \\ \Delta_T f(x) \end{Bmatrix} = \int \frac{\mathrm{d}^4k}{(2\pi)^4} \begin{Bmatrix} A_1(k^2, k{\cdot}P) \\ A_2(k^2, k{\cdot}P) \\ A_3(k^2, k{\cdot}P) \end{Bmatrix} \delta\!\left(x - \frac{k^+}{P^+}\right), \tag{3.11}$$

that is, using (3.10a–c) and setting $\lambda_N = +1$ and $S_\perp^i = (1,0)$ for the sake of definiteness,

$$f(x) = \frac{1}{2}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{Tr}(\gamma^+\Phi)\,\delta(k^+ - xP^+)\,, \tag{3.12a}$$

$$\Delta f(x) = \frac{1}{2}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{Tr}(\gamma^+\gamma_5\Phi)\,\delta(k^+ - xP^+)\,, \tag{3.12b}$$

$$\Delta_T f(x) = \frac{1}{2}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{Tr}(\gamma^+\gamma^1\gamma_5\Phi)\,\delta(k^+ - xP^+)\,. \tag{3.12c}$$

Finally, inserting the definition (3.1) of Φ into (3.12a–c), we obtain the three leading-twist distribution functions as light-cone Fourier transforms

of expectation values of quark-field bilinears (Collins and Soper, 1982):

$$f(x) = \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-} \langle PS|\bar\psi(0)\gamma^+\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle\,, \tag{3.13a}$$

$$\Delta f(x) = \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-} \langle PS|\bar\psi(0)\gamma^+\gamma_5\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle\,, \tag{3.13b}$$

$$\Delta_T f(x) = \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-} \langle PS|\bar\psi(0)\gamma^+\gamma^1\gamma_5\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle\,. \tag{3.13c}$$

The quark–quark correlation matrix Φ integrated over k with the constraint $x = k^+/P^+$

$$\begin{aligned}\Phi_{ij}(x) &= \int \frac{\mathrm{d}^4k}{(2\pi)^4}\, \Phi_{ij}(k,P,S)\,\delta(x-k^+/P^+) \\ &= \int \frac{\mathrm{d}\tau}{2\pi}\, \mathrm{e}^{\mathrm{i}\tau x} \langle PS|\bar\psi_j(0)\psi_i(\tau n)|PS\rangle\,,\end{aligned} \tag{3.14}$$

in terms of the three leading-twist distribution functions, reads

$$\Phi(x) = \tfrac{1}{2}\left\{f(x)\,\not{P} + \lambda_N\,\Delta f(x)\,\gamma_5\,\not{P} + \Delta_T f(x)\,\not{P}\gamma_5\,\not{S}_\perp\right\}. \tag{3.15}$$

Let us now complete the discussion by introducing the antiquarks. Their distribution functions are obtained from the correlation matrix

$$\bar\Phi_{ij}(k,P,S) = \int \mathrm{d}^4\xi\, \mathrm{e}^{\mathrm{i}k\cdot\xi} \langle PS|\psi_i(0)\bar\psi_j(\xi)|PS\rangle\,. \tag{3.16}$$

Tracing $\bar\Phi$ with the Dirac matrices Γ gives

$$\mathrm{Tr}(\Gamma\Phi) = \int \mathrm{d}^4\xi\, \mathrm{e}^{\mathrm{i}k\cdot\xi} \langle PS|\,\mathrm{Tr}\left[\Gamma\,\psi(0)\bar\psi(\xi)\right]|PS\rangle\,. \tag{3.17}$$

In deriving the expressions for $\bar f$, $\Delta\bar f$, $\Delta_T\bar f$ care is needed with the signs. By charge conjugation, the field bilinears in (3.4a) transform as

$$\bar\psi(0)\,\Gamma\,\psi(\xi) \to \pm\,\mathrm{Tr}\left[\Gamma\,\psi(0)\bar\psi(\xi)\right], \tag{3.18}$$

where the plus sign is for $\Gamma = \gamma^\mu$, $\mathrm{i}\sigma^{\mu\nu}\gamma_5$ and the minus sign for $\Gamma = \gamma^\mu\gamma_5$. We thus obtain the antiquark density number:

$$\begin{aligned}\bar f(x) &= \frac{1}{2}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{Tr}(\gamma^+\bar\Phi)\,\delta(k^+ - xP^+) \\ &= \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-} \langle PS|\,\mathrm{Tr}\left[\gamma^+\psi(0)\bar\psi(0,\xi^-,\mathbf{0}_\perp)\right]|PS\rangle\,,\end{aligned} \tag{3.19a}$$

the antiquark helicity distribution

$$\Delta\bar{f}(x) = -\frac{1}{2}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{Tr}\left(\gamma^+\gamma_5\overline{\Phi}\right)\delta(k^+ - xP^+)$$
$$= \int \frac{\mathrm{d}\xi^-}{4\pi}\,\mathrm{e}^{\mathrm{i}xP^+\xi^-}\,\langle PS|\,\mathrm{Tr}\left[\gamma^+\gamma_5\psi(0)\bar{\psi}(0,\xi^-,\mathbf{0}_\perp)\right]|PS\rangle\,, \quad (3.19\mathrm{b})$$

and the antiquark transversity distribution

$$\Delta_T\bar{f}(x) = \frac{1}{2}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{Tr}\left(\gamma^+\gamma^1\gamma_5\overline{\Phi}\right)\delta(k^+ - xP^+)$$
$$= \int \frac{\mathrm{d}\xi^-}{4\pi}\,\mathrm{e}^{\mathrm{i}xP^+\xi^-}\,\langle PS|\,\mathrm{Tr}\left[\gamma^+\gamma^1\gamma_5\psi(0)\bar{\psi}(0,\xi^-,\mathbf{0}_\perp)\right]|PS\rangle. \quad (3.19\mathrm{c})$$

Note the minus sign in the definition of the antiquark helicity distribution.

If we adhere to the definitions of quark and antiquark distributions, Eqs. (3.13a–c) and (3.19a–c), the variable $x \equiv k^+/P^+$ is not constrained *a priori* to be positive or to range from zero to unity (we shall see in Sec. 3.3 how the correct support for x comes out, hence justifying its identification with the Bjorken variable). It turns out that there is a set of symmetry relations connecting quark and antiquark distribution functions, which are obtained by continuing x to negative values. Using anticommutation relations for the fermion fields in the connected matrix elements

$$\langle PS|\bar{\psi}(\xi)\psi(0)|PS\rangle_c = -\langle PS|\psi(0)\bar{\psi}(\xi)|PS\rangle_c\,, \quad (3.20)$$

one easily obtains the following relations for the three distribution functions

$$\bar{f}(x) = -f(-x)\,, \quad (3.21\mathrm{a})$$
$$\Delta\bar{f}(x) = \Delta f(-x)\,, \quad (3.21\mathrm{b})$$
$$\Delta_T\bar{f}(x) = -\Delta_T f(-x)\,. \quad (3.21\mathrm{c})$$

Therefore, antiquark distributions are given by the continuation of the corresponding quark distributions into the negative x region.

3.3 Probabilistic interpretation of distribution functions

Distribution functions are essentially the probability densities for finding partons with a given momentum fraction and a given polarisation inside a hadron. We shall now see how this interpretation comes about from the

field-theoretical definitions of quark (and antiquark) distribution functions presented above.

Let us first of all decompose the quark fields into "good" and "bad" components:

$$\psi = \psi_{(+)} + \psi_{(-)}\,, \tag{3.22}$$

where

$$\psi_{(\pm)} = \tfrac{1}{2}\,\gamma^{\mp}\,\gamma^{\pm}\,\psi\,. \tag{3.23}$$

The usefulness of this procedure lies in the fact that "bad" components are not dynamically independent: using the equations of motion, they can be eliminated in favour of "good" components and terms containing quark masses and gluon fields. Since in the $P^+ \to \infty$ limit $\psi_{(+)}$ dominates over $\psi_{(-)}$, the presence of "bad" components in a parton distribution function signals higher twists. Using the relations

$$\bar{\psi}\,\gamma^+\,\psi = \sqrt{2}\,\psi^\dagger_{(+)}\,\psi_{(+)}\,, \tag{3.24a}$$

$$\bar{\psi}\,\gamma^+\gamma_5\,\psi = \sqrt{2}\,\psi^\dagger_{(+)}\,\gamma_5\,\psi_{(+)}\,, \tag{3.24b}$$

$$\bar{\psi}\,\mathrm{i}\sigma^{i+}\gamma_5\,\psi = \sqrt{2}\,\psi^\dagger_{(+)}\,\gamma^i\gamma_5\,\psi_{(+)}\,, \tag{3.24c}$$

the leading-twist distributions (3.13a–c) can be re-expressed as (Collins and Soper, 1982)

$$f(x) = \int \frac{\mathrm{d}\xi^-}{2\sqrt{2}\,\pi}\,\mathrm{e}^{\mathrm{i}xP^+\xi^-}\,\langle PS|\psi^\dagger_{(+)}(0)\gamma^+\psi_{(+)}(0,\xi^-,\mathbf{0}_\perp)|PS\rangle\,, \tag{3.25a}$$

$$\Delta f(x) = \int \frac{\mathrm{d}\xi^-}{2\sqrt{2}\,\pi}\,\mathrm{e}^{\mathrm{i}xP^+\xi^-}\,\langle PS|\psi^\dagger_{(+)}(0)\gamma_5\psi_{(+)}(0,\xi^-,\mathbf{0}_\perp)|PS\rangle\,, \tag{3.25b}$$

$$\Delta_T f(x) = \int \frac{\mathrm{d}\xi^-}{2\sqrt{2}\,\pi}\,\mathrm{e}^{\mathrm{i}xP^+\xi^-}\,\langle PS|\psi^\dagger_{(+)}(0)\gamma^1\gamma_5\psi_{(+)}(0,\xi^-,\mathbf{0}_\perp)|PS\rangle\,. \tag{3.25c}$$

Note that, as anticipated, only "good" components appear. It is the peculiar structure of the quark-field bilinears in Eqs. (3.25a–c) that allows us to put the distributions in a form that renders their probabilistic nature transparent.

A remark on the support of the distribution functions is now in order. We have already mentioned that, according to the definitions of the quark distributions, nothing constrains the ratio $x \equiv k^+/P^+$ to take on values between 0 and 1. The correct support of the distributions emerges, along with their probabilistic content, if one inserts (Jaffe, 1983) a complete set

of intermediate states $\{|n\rangle\}$ into (3.25a–c), see Fig. 3.2. Considering, for instance, the unpolarised distribution we obtain from (3.25a)

$$f(x) = \tfrac{1}{\sqrt{2}} \sum_n \delta\big((1-x)P^+ - P_n^+\big) \left|\langle PS|\psi_{(+)}(0)|n\rangle\right|^2 , \tag{3.26}$$

where $\sum_n$ incorporates integration over the phase space of the intermediate

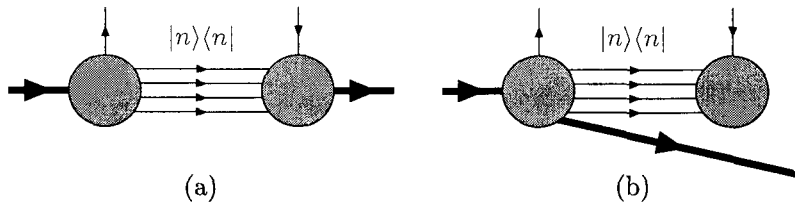


Fig. 3.2 (a) A connected matrix element with the insertion of a complete set of intermediate states and (b) a semi-connected matrix element.

states. Equation (3.26) clearly gives the probability of finding inside the nucleon a quark with longitudinal momentum k^+/P^+, irrespective of its polarisation. Since the states $|n\rangle$ are physical we must have $P_n^+ \geq 0$, that is, $E_n \geq |\boldsymbol{P}_n|$, and therefore $x \leq 1$. Moreover, if we exclude semi-connected diagrams such as that in Fig. 3.2b, which correspond to $x < 0$, we are left with the connected diagram of Fig. 3.2a and with the correct support $0 \leq x \leq 1$. A similar reasoning applies to antiquarks.

Let us turn now to the polarised distributions: using the Pauli–Lubanski projectors $\mathcal{P}_\pm = \frac{1}{2}(1 \pm \gamma_5)$ (for helicity) and $\mathcal{P}_{\uparrow\downarrow} = \frac{1}{2}(1 \pm \gamma^1\gamma_5)$ (for transverse polarisation), we obtain

$$\begin{aligned}\Delta f(x) = \tfrac{1}{\sqrt{2}} \sum_n \delta\big((1-x)P^+ - P_n^+\big) \\ \times \left\{\left|\langle PS|\mathcal{P}_+\psi_{(+)}(0)|n\rangle\right|^2 - \left|\langle PS|\mathcal{P}_-\psi_{(+)}(0)|n\rangle\right|^2\right\}, \end{aligned} \tag{3.27a}$$

$$\begin{aligned}\Delta_T f(x) = \tfrac{1}{\sqrt{2}} \sum_n \delta\left((1-x)P^+ - P_n^+\right) \\ \times \left\{\left|\langle PS|\mathcal{P}_\uparrow\psi_{(+)}(0)|n\rangle\right|^2 - \left|\langle PS|\mathcal{P}_\downarrow\psi_{(+)}(0)|n\rangle\right|^2\right\}. \end{aligned} \tag{3.27b}$$

These expressions exhibit the probabilistic content of the leading-twist polarised distributions $\Delta f(x)$ and $\Delta_T f(x)$: $\Delta f(x)$ is the number density of quarks with positive helicity minus the number density of quarks with negative helicity (assuming the parent nucleon to have positive helicity); $\Delta_T f(x)$ is the number density of quarks with transverse polarisation $\uparrow$ minus the number density of quarks with transverse polarisation $\downarrow$ (assuming the parent nucleon to have transverse polarisation $\uparrow$). It is important to notice that $\Delta_T f$ admits an interpretation in terms of probability densities only in the transverse-polarisation basis.

The three leading-twist quark distribution functions are contained in the entries of the spin density matrix for quarks in the nucleon ($\lambda(x)$ is the quark helicity density, $\boldsymbol{s}_\perp(x)$ is the quark transverse-spin density):

$$\begin{aligned}\rho_{\lambda\lambda'} &= \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 1+\lambda(x) & s_x(x) - \mathrm{i} s_y(x) \\ s_x(x) + \mathrm{i} s_y(x) & 1-\lambda(x) \end{pmatrix} . \end{aligned} \tag{3.28}$$

Recalling the probabilistic interpretation of the spin density matrix elements discussed in Sec. A.4, one finds that the spin components $(\boldsymbol{s}_\perp, \lambda)$ of the quark appearing in (3.28) are related to the spin components $(\boldsymbol{S}_\perp, \lambda_N)$ of the parent nucleon by

$$\boldsymbol{s}_\perp(x)\, f(x) = \boldsymbol{S}_\perp\, \Delta_T f(x) . \tag{3.29a}$$

$$\lambda_q(x)\, f(x) = \lambda_N\, \Delta f(x) , \tag{3.29b}$$

3.4 Vector, axial and tensor charges

If we integrate the correlation matrix $\Phi(k, P, S)$ over k, or equivalently $\Phi(x)$ over x, we obtain a local matrix element (which we call Φ, with no arguments)

$$\Phi_{ij} = \int \mathrm{d}^4 k\, \Phi_{ij}(k, P, S) = \int \mathrm{d}x\, \Phi_{ij}(x) = \langle PS | \bar{\psi}_j(0) \psi_i(0) | PS \rangle , \tag{3.30}$$

which, in view of (3.9), can be parametrised as

$$\Phi = \tfrac{1}{2} \left[g_V \not{P} + g_A\, \lambda_N\, \gamma_5 \not{P} + g_T\, \gamma_5 \not{\boldsymbol{S}}_\perp \not{P} \right] . \tag{3.31}$$

Here g_V, g_A and g_T are the vector, axial and tensor charge, respectively. They are given by the following matrix elements, recall (3.8a–c):

$$\langle PS|\bar{\psi}(0)\gamma^\mu\psi_i(0)|PS\rangle = 2g_V\,P^\mu\,, \tag{3.32a}$$

$$\langle PS|\bar{\psi}(0)\gamma^\mu\gamma_5\psi_i(0)|PS\rangle = 2g_A\,MS^\mu\,, \tag{3.32b}$$

$$\langle PS|\bar{\psi}(0)\mathrm{i}\sigma^{\mu\nu}\gamma_5\psi_i(0)|PS\rangle = 2g_T\,(S^\mu P^\nu - S^\nu P^\mu)\,. \tag{3.32c}$$

Warning: the tensor charge g_T should not be confused with the twist-three distribution function $g_T(x)$ encountered in Sec. 2.7.3.

Integrating Eqs. (3.13a–c) and using the symmetry relations (3.21a–c) yields

$$\int_{-1}^{+1} \mathrm{d}x\, f(x) = \int_0^1 \mathrm{d}x\,\big[f(x) - \bar{f}(x)\big] = g_V\,, \tag{3.33a}$$

$$\int_{-1}^{+1} \mathrm{d}x\, \Delta f(x) = \int_0^1 \mathrm{d}x\,\big[\Delta f(x) + \Delta\bar{f}(x)\big] = g_A\,, \tag{3.33b}$$

$$\int_{-1}^{+1} \mathrm{d}x\, \Delta_T f(x) = \int_0^1 \mathrm{d}x\,\big[\Delta_T f(x) - \Delta_T\bar{f}(x)\big] = g_T\,. \tag{3.33c}$$

Note that g_V is simply the valence number. As a consequence of the charge conjugation properties of the field bilinears $\bar{\psi}\gamma^\mu\psi$, $\bar{\psi}\gamma^\mu\gamma_5\psi$ and $\bar{\psi}\mathrm{i}\sigma^{\mu\nu}\gamma_5\psi$, the vector and tensor charges are the first moments of flavour non-singlet combinations (quarks minus antiquarks) whereas the axial charge is the first moment of a flavour singlet combination (quarks plus antiquarks).

3.5 Quark–nucleon helicity amplitudes

The DIS hadronic tensor is related to forward virtual Compton scattering amplitudes. Thus, leading-twist quark distribution functions can be expressed in terms of quark–nucleon forward amplitudes. In the helicity basis these amplitudes have the form $\mathcal{A}_{\Lambda\lambda,\Lambda'\lambda'}$, where λ, λ' (Λ, Λ') are quark (nucleon) helicities. There are in general 16 amplitudes. Imposing helicity conservation,

$$\Lambda - \lambda' = \Lambda' - \lambda\,, \quad \textit{i.e.,} \quad \Lambda + \lambda = \Lambda' + \lambda'\,, \tag{3.34}$$

only 6 amplitudes survive:

$$\mathcal{A}_{++,++}\,,\ \mathcal{A}_{--,--}\,,\ \mathcal{A}_{+-,+-}\,,\ \mathcal{A}_{-+,-+}\,,\ \mathcal{A}_{+-,-+}\,,\ \mathcal{A}_{-+,+-}\,. \tag{3.35}$$

Parity invariance implies

$$\mathcal{A}_{\Lambda\lambda,\Lambda'\lambda'} = \mathcal{A}_{-\Lambda-\lambda,-\Lambda'-\lambda'}\,, \tag{3.36}$$

and gives the following 3 constraints on the amplitudes:

$$\begin{aligned} \mathcal{A}_{++,++} &= \mathcal{A}_{--,--}\,, \\ \mathcal{A}_{++,--} &= \mathcal{A}_{--,++}\,, \\ \mathcal{A}_{+-,-+} &= \mathcal{A}_{-+,+-}\,. \end{aligned} \tag{3.37}$$

Time-reversal invariance,

$$\mathcal{A}_{\Lambda\lambda,\Lambda'\lambda'} = \mathcal{A}_{\Lambda'\lambda',\Lambda\lambda}\,, \tag{3.38}$$

adds no further constraints. Hence, we are left with three independent amplitudes (see Fig. 3.3):

$$\mathcal{A}_{++,++}\,, \quad \mathcal{A}_{+-,+-}\,, \quad \mathcal{A}_{+-,-+}\,. \tag{3.39}$$

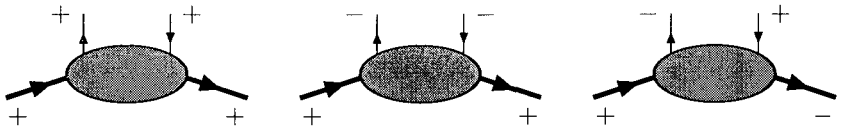


Fig. 3.3 The three quark–nucleon helicity amplitudes.

Two of the amplitudes in (3.39), $\mathcal{A}_{++,++}$ and $\mathcal{A}_{+-,+-}$, are diagonal in the helicity basis (the quark does not flip its helicity: $\lambda = \lambda'$), the third, $\mathcal{A}_{+-,-+}$, is off-diagonal (helicity flip: $\lambda = -\lambda'$). Using the optical theorem we can relate these quark–nucleon helicity amplitudes to the three leading-twist quark distribution functions, according to the scheme

$$f(x) = f_+(x) + f_-(x) \sim \mathrm{Im}(\mathcal{A}_{++,++} + \mathcal{A}_{+-,+-})\,, \tag{3.40a}$$

$$\Delta f(x) = f_+(x) - f_-(x) \sim \mathrm{Im}(\mathcal{A}_{++,++} - \mathcal{A}_{+-,+-})\,, \tag{3.40b}$$

$$\Delta_T f(x) = f_\uparrow(x) - f_\downarrow(x) \sim \mathrm{Im}\,\mathcal{A}_{+-,-+}\,. \tag{3.40c}$$

In a transversity basis (with $\uparrow$ directed along y)

$$\begin{aligned} |\uparrow\rangle &= \tfrac{1}{\sqrt{2}}\left[|+\rangle + \mathrm{i}|-\rangle\right], \\ |\downarrow\rangle &= \tfrac{1}{\sqrt{2}}\left[|+\rangle - \mathrm{i}|-\rangle\right], \end{aligned} \tag{3.41}$$

the transverse-polarisation distributions $\Delta_T f$ is related to a diagonal amplitude

$$\Delta_T f(x) = f_\uparrow(x) - f_\downarrow(x) \sim \mathrm{Im}(\mathcal{A}_{\uparrow\uparrow,\uparrow\uparrow} - \mathcal{A}_{\uparrow\downarrow,\uparrow\downarrow}). \tag{3.42}$$

Reasoning in terms of parton–nucleon forward helicity amplitudes, it is easy to understand why there is no such thing as leading-twist transverse polarisation of gluons. A hypothetical $\Delta_T g$ would imply an helicity-flip gluon–nucleon amplitude, which cannot exist owing to helicity conservation. In fact, gluons have helicity ± 1 but the nucleon cannot undergo an helicity change $\Delta\Lambda = \pm 2$. Targets with higher spin may have an helicity-flip gluon distribution.

If the transverse momenta of quarks are not neglected, the situation becomes more complicated and the number of independent helicity amplitudes increases. These amplitudes combine to form six $\boldsymbol{k}_\perp$-dependent distribution functions (three of which reduce to $f(x)$, $\Delta f(x)$ and $\Delta_T f(x)$ when integrated over $\boldsymbol{k}_\perp$).

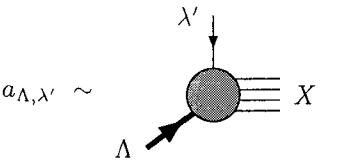
3.6 The Soffer inequality

From the definitions of f, Δf and $\Delta_T f$, that is, $\Delta f(x) = f_+(x) - f_-(x)$, $\Delta_T f(x) = f_\uparrow(x) - f_\downarrow(x)$ and $f(x) = f_+(x) + f_-(x) = f_\uparrow(x) + f_\downarrow(x)$, two bounds on Δf and $\Delta_T f$ immediately follow:

$$|\Delta f(x)| \leq f(x), \tag{3.43a}$$

$$|\Delta_T f(x)| \leq f(x). \tag{3.43b}$$

Similar inequalities are satisfied by the antiquark distributions. A further, more subtle, bound, simultaneously involving f, Δf and $\Delta_T f$, was discovered by Soffer (1995). It can be derived from the expressions (3.40a–c) of the distribution functions in terms of quark–nucleon forward amplitudes. Let us introduce the following quark–nucleon vertices $a_{\Lambda\lambda'}$:



and rewrite Eqs. (3.40a–c) in the form

$$\begin{aligned} f(x) &\sim \mathrm{Im}(\mathcal{A}_{++,++} + \mathcal{A}_{+-,+-}) \\ &\sim \sum_X (a^*_{++}a_{++} + a^*_{+-}a_{+-})\,, \end{aligned} \tag{3.44a}$$

$$\begin{aligned} \Delta f(x) &\sim \mathrm{Im}(\mathcal{A}_{++,++} - \mathcal{A}_{+-,+-}) \\ &\sim \sum_X (a^*_{++}a_{++} - a^*_{+-}a_{+-})\,, \end{aligned} \tag{3.44b}$$

$$\Delta_T f(x) \sim \mathrm{Im}\,\mathcal{A}_{+-,-+} \sim \sum_X a^*_{--}a_{++}\,. \tag{3.44c}$$

From

$$\sum_X |a_{++} \pm a_{--}|^2 \geq 0\,, \tag{3.45}$$

using parity invariance, we obtain

$$\sum_X a^*_{++}a_{++} \pm \sum_X a^*_{--}a_{++} \geq 0\,, \tag{3.46}$$

that is,

$$f_+(x) \geq |\Delta_T f(x)|\,, \tag{3.47}$$

which is equivalent to

$$f(x) + \Delta f(x) \geq 2|\Delta_T f(x)|\,. \tag{3.48}$$

An analogous relation holds for the antiquark distributions. Equation (3.48) is known as the Soffer inequality. It is an important bound, which must be satisfied by the leading-twist distribution functions. The reason it escaped attention until a relatively late discovery by Soffer (1995) is that it involves three quantities that are not diagonal in the same basis. Thus, to be derived, Soffer's inequality requires consideration of probability amplitudes, not of probabilities themselves. The constraint (3.48) is represented in Fig. 3.4.

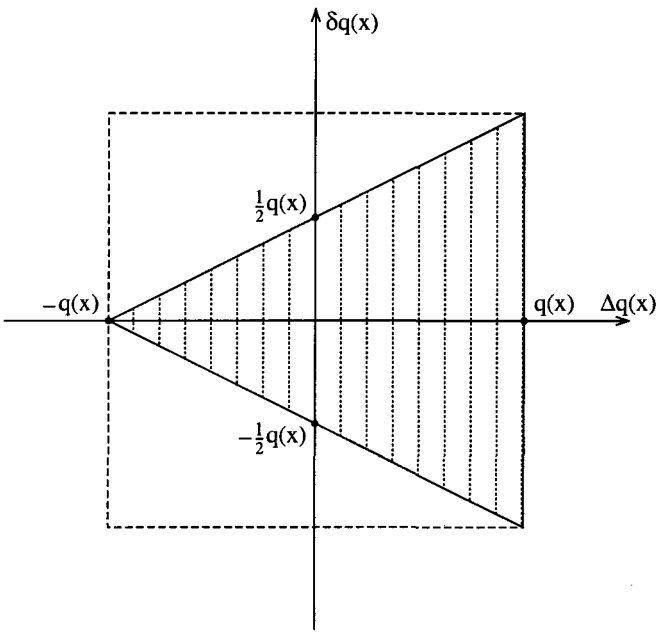


Fig. 3.4 The Soffer bound on the leading-twist distributions (Soffer, 1995). Note that there $\Delta_T f(x)$ is called $\delta q(x)$.

We shall see in Sec. 4.6 that the Soffer bound, like the other two – more obvious – inequalities (3.43a, b), is preserved by QCD evolution, as it should be.

3.7 Transverse motion of quarks

Let us now take into account quark transverse motion. This is necessary in semi-inclusive DIS, when one wants to study the $\boldsymbol{P}_{h\perp}$ distribution of the produced hadron. Thus, in this section we shall prepare the groundwork for later applications.

In this case the quark momentum is given by

$$k^\mu \simeq xP^\mu + k_\perp^\mu \,, \tag{3.49}$$

where we have retained $k_\perp^\mu$, which is zeroth order in P^+ and thus suppressed by one power of P^+ with respect to the longitudinal momentum.

At leading twist, again, only the vector, axial-vector and tensor terms in (3.6) appear and Eqs. (3.7b, c, e) become

$$\mathcal{V}^\mu = A_1\, P^\mu\,, \tag{3.50a}$$

$$\mathcal{A}^\mu = \lambda_N\, A_2\, P^\mu + \frac{1}{M}\,\widetilde{A}_1\, \boldsymbol{k}_\perp{\cdot}\boldsymbol{S}_\perp\, P^\mu\,, \tag{3.50b}$$

$$\mathcal{T}^{\mu\nu} = A_3\, P^{[\mu} S_\perp^{\nu]} + \frac{\lambda_N}{M}\,\widetilde{A}_2\, P^{[\mu} k_\perp^{\nu]} + \frac{1}{M^2}\,\widetilde{A}_3\, \boldsymbol{k}_\perp{\cdot}\boldsymbol{S}_\perp\, P^{[\mu} k_\perp^{\nu]}\,, \tag{3.50c}$$

where we have defined new real functions $\widetilde{A}_i(k^2, k{\cdot}P)$ (the tilde signals the presence of $\boldsymbol{k}_\perp$) and introduced powers of M, so that all coefficients have the same dimensionality. The quark–quark correlation matrix (3.6) then reads

$$\Phi(k,P,S) = \frac{1}{2}\Big\{ A_1\, \not{P} + A_2\, \lambda_N\, \gamma_5\, \not{P} + A_3\, \not{P}\gamma_5\, \not{S}_\perp + \frac{1}{M}\,\widetilde{A}_1\, \boldsymbol{k}_\perp{\cdot}\boldsymbol{S}_\perp\, \gamma_5 \not{P} + \widetilde{A}_2\, \frac{\lambda_N}{M}\, \not{P}\gamma_5 \not{k}_\perp + \frac{1}{M^2}\,\widetilde{A}_3\, \boldsymbol{k}_\perp{\cdot}\boldsymbol{S}_\perp\, \not{P}\gamma_5 \not{k}_\perp \Big\}\,. \tag{3.51}$$

We can project out the A_i's and $\widetilde{A}_i$'s, as we did in Sec. 3.2

$$\frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\Phi) = A_1\,, \tag{3.52a}$$

$$\frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\gamma_5\Phi) = \lambda_N\, A_2 + \frac{1}{M}\,\boldsymbol{k}_\perp{\cdot}\boldsymbol{S}_\perp\,\widetilde{A}_1\,, \tag{3.52b}$$

$$\frac{1}{2P^+}\,\mathrm{Tr}(\mathrm{i}\sigma^{i+}\gamma_5\Phi) = S_\perp^i\, A_3 + \frac{\lambda_N}{M}\, k_\perp^i\, \widetilde{A}_2 + \frac{1}{M^2}\,\boldsymbol{k}_\perp{\cdot}\boldsymbol{S}_\perp\, k_\perp^i\, \widetilde{A}_3\,. \tag{3.52c}$$

Let us rearrange the r.h.s. of the last expression in the following manner ($g_\perp^{\mu\nu}$ is defined in App. D.1)

$$S_\perp^i\, A_3 + \frac{1}{M^2}\,\boldsymbol{k}_\perp{\cdot}\boldsymbol{S}_\perp\, k_\perp^i\, \widetilde{A}_3 = S_\perp^i \left(A_3 + \frac{\boldsymbol{k}_\perp^2}{2M^2}\,\widetilde{A}_3\right) - \frac{1}{M^2}\left(k_\perp^i k_\perp^j + \frac{1}{2}\,\boldsymbol{k}_\perp^2\, g_\perp^{ij}\right) S_{\perp j}\,\widetilde{A}_3\,. \tag{3.53}$$

If we integrate Eqs. (3.52a–c) over k with the constraint $x = k^+/P^+$, the terms proportional to $\widetilde{A}_1$ and $\widetilde{A}_2$ in (3.52b, c) and to $\widetilde{A}_3$ in (3.53) vanish. We

are left with the three terms proportional to A_1, A_2 and to the combination $A_3 + (\boldsymbol{k}_\perp^2/2M^2)\,\widetilde{A}_3$, which give, upon integration, the three distribution functions $f(x)$, $\Delta f(x)$ and $\Delta_T f(x)$, respectively. The only difference from the previous case of no quark transverse momentum is that $\Delta_T f(x)$ is now related to $A_3 + (\boldsymbol{k}_\perp^2/2M^2)\,\widetilde{A}_3$ and not to A_3 alone:

$$\Delta_T f(x) \equiv \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \left(A_3 + \frac{\boldsymbol{k}_\perp^2}{2M^2}\,\widetilde{A}_3 \right) \delta(x - k^+/P^+)\,. \tag{3.54}$$

If we do not integrate over $\boldsymbol{k}_\perp$, we obtain six $\boldsymbol{k}_\perp$-dependent distribution functions. Three of them, which we call $f(x, \boldsymbol{k}_\perp^2)$, $\Delta f(x, \boldsymbol{k}_\perp^2)$ and $\Delta_T' f(x, \boldsymbol{k}_\perp^2)$, are such that $f(x) = \int \mathrm{d}^2\boldsymbol{k}_\perp f(x, \boldsymbol{k}_\perp^2)$, *etc.* The other three are completely new and are related to the terms of the correlation matrix containing the $\widetilde{A}_i$'s. We shall adopt Mulders' terminology for them (Mulders and Tangerman, 1996; Boer and Mulders, 1998).

Let us introduce the integrated correlation matrix,

$$\begin{aligned} \Phi_{ij}(x, \boldsymbol{k}_\perp) &\equiv \frac{1}{2} \int \frac{\mathrm{d}k^+\, \mathrm{d}k^-}{(2\pi)^4}\, \Phi(k, P, S)\, \delta(k^+ - xP^+) \\ &= \frac{1}{2} \int \frac{\mathrm{d}\xi^-\, \mathrm{d}^2\boldsymbol{\xi}_\perp}{(2\pi)^3}\, \mathrm{e}^{\mathrm{i}(xP^+\xi^- - \boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp)} \\ &\qquad \times \langle PS|\bar{\psi}_j(0)\,\psi_i(0, \xi^-, \boldsymbol{\xi}_\perp)|PS\rangle, \end{aligned} \tag{3.55}$$

and its projections (Γ is a generic Dirac matrix)

$$\begin{aligned} \Phi^{[\Gamma]} &\equiv \frac{1}{2} \int \frac{\mathrm{d}k^+\, \mathrm{d}k^-}{(2\pi)^4}\, \mathrm{Tr}(\Gamma\,\Phi)\, \delta(k^+ - xP^+) \\ &= \frac{1}{2} \int \frac{\mathrm{d}\xi^-\, \mathrm{d}^2\boldsymbol{\xi}_\perp}{(2\pi)^3}\, \mathrm{e}^{\mathrm{i}(xP^+\xi^- - \boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp)} \\ &\qquad \times \langle PS|\bar{\psi}(0)\Gamma\psi(0, \xi^-, \boldsymbol{\xi}_\perp)|PS\rangle. \end{aligned} \tag{3.56}$$

$\Phi(x, \boldsymbol{k}_\perp)$ has the following structure (Mulders and Tangerman, 1996)

$$\begin{aligned} \Phi(x, \boldsymbol{k}_\perp) = \frac{1}{2} \Bigg\{ & f(x, \boldsymbol{k}_\perp^2)\, \not{P} + \left[\lambda_N \Delta f(x, \boldsymbol{k}_\perp^2) \right. \\ & \left. + \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M}\, g_{1T}(x, \boldsymbol{k}_\perp^2) \right] \gamma_5 \not{P} + h_{1T}(x, \boldsymbol{k}_\perp^2)\, \not{S}_\perp \gamma_5 \not{P} \\ & + \frac{1}{M} \left[\lambda_N\, h_{1L}^\perp(x, \boldsymbol{k}_\perp^2) + \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M}\, h_{1T}^\perp(x, \boldsymbol{k}_\perp^2) \right] \not{k}_\perp \gamma_5 \not{P} \Bigg\}. \end{aligned} \tag{3.57}$$

From this we get the Dirac projections:

$$\Phi^{[\gamma^+]} = \mathcal{P}_{q/N}(x, \boldsymbol{k}_\perp) = f(x, \boldsymbol{k}_\perp^2)\,, \tag{3.58a}$$

$$\begin{aligned}\Phi^{[\gamma^+\gamma_5]} &= \mathcal{P}_{q/N}(x, \boldsymbol{k}_\perp)\,\lambda(x, \boldsymbol{k}_\perp) \\ &= \lambda_N\,\Delta f(x, \boldsymbol{k}_\perp^2) + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{M}\, g_{1T}(x, \boldsymbol{k}_\perp^2)\,,\end{aligned} \tag{3.58b}$$

$$\begin{aligned}\Phi^{[\mathrm{i}\sigma^{i+}\gamma_5]} &= \mathcal{P}_{q/N}(x, \boldsymbol{k}_\perp)\, s_\perp^i(x, \boldsymbol{k}_\perp) \\ &= S_\perp^i\, h_{1T}(x, \boldsymbol{k}_\perp^2) \\ &\quad + \frac{k_\perp^i}{M}\left[\lambda_N\, h_{1L}^\perp(x, \boldsymbol{k}_\perp^2) + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{M}\, h_{1T}^\perp(x, \boldsymbol{k}_\perp^2)\right] \\ &= S_\perp^i\, \Delta_T' f(x, \boldsymbol{k}_\perp^2) + \frac{\lambda_N}{M}\, k_\perp^i\, h_{1L}^\perp(x, \boldsymbol{k}_\perp^2) \\ &\quad - \frac{1}{M^2}\left(k_\perp^i k_\perp^j + \frac{1}{2}\,\boldsymbol{k}_\perp^2\, g_\perp^{ij}\right) S_{\perp j}\, h_{1T}^\perp(x, \boldsymbol{k}_\perp^2)\,,\end{aligned} \tag{3.58c}$$

with $\Delta_T' f = h_{1T} + (\boldsymbol{k}_\perp^2/2M)\, h_{1T}^\perp$. In Eqs. (3.58a, b) $\mathcal{P}_{q/N}(x, \boldsymbol{k}_\perp)$ is the probability of finding a quark with longitudinal momentum fraction x and transverse momentum $\boldsymbol{k}_\perp$, and $\lambda(x, \boldsymbol{k}_\perp)$, $\boldsymbol{s}_\perp(x, \boldsymbol{k}_\perp)$ are the quark helicity and transverse-spin densities, respectively. The spin density matrix of quarks now reads

$$\rho_{\lambda\lambda'} = \frac{1}{2}\begin{pmatrix} 1 + \lambda(x, \boldsymbol{k}_\perp) & s_x(x, \boldsymbol{k}_\perp) - \mathrm{i}s_y(x, \boldsymbol{k}_\perp) \\ s_x(x, \boldsymbol{k}_\perp) + \mathrm{i}s_y(x, \boldsymbol{k}_\perp) & 1 - \lambda(x, \boldsymbol{k}_\perp) \end{pmatrix}, \tag{3.59}$$

and its entries incorporate the six distributions listed above, according to Eqs. (3.58a, b).

Let us now try to understand the partonic content of the $\boldsymbol{k}_\perp$-dependent distributions. If the target nucleon is unpolarised, the only measurable quantity is $f(x, \boldsymbol{k}_\perp^2)$, which coincides with $\mathcal{P}_q(x, \boldsymbol{k}_\perp)$, the number density of quarks with longitudinal momentum fraction x and transverse momentum squared $\boldsymbol{k}_\perp^2$.

If the target nucleon is transversely polarised, there is some probability of finding the quarks transversely polarised along the same direction as the nucleon, along a different direction, or longitudinally polarised. This variety of situations is allowed by the presence of $\boldsymbol{k}_\perp$. Integrating over $\boldsymbol{k}_\perp$, the transverse-polarisation asymmetry of quarks along a different direction with respect to the nucleon polarisation, and the longitudinal polarisation

asymmetry of quarks in a transversely polarised nucleon disappear: only the case $s_\perp \parallel S_\perp$ survives.

Referring to Fig. 3.5 for the geometry in the azimuthal plane and us-

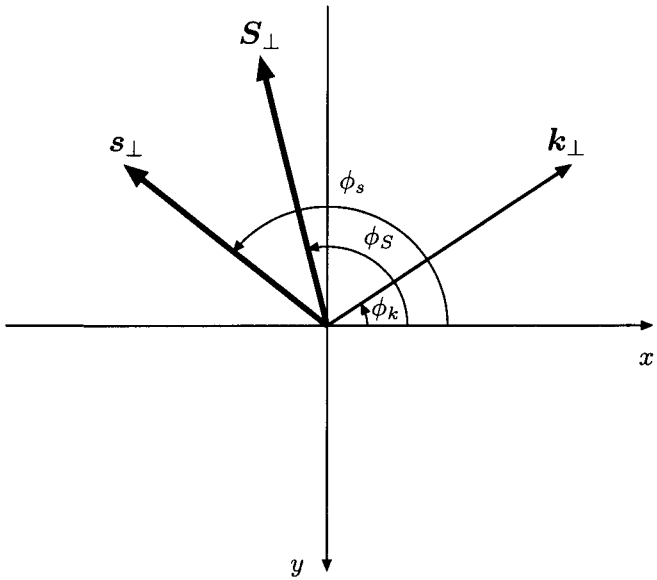


Fig. 3.5 Our definition of the azimuthal angles in the plane orthogonal to the $\gamma^* N$ axis. The photon momentum, which is directed along the positive z axis, points inwards. For our choice of axes, see Fig. 2.6.

ing the following parametrisations for the vectors at hand (we assume full polarisation of the nucleon):

$$\boldsymbol{k}_\perp = \left(|\boldsymbol{k}_\perp| \cos\phi_k, -|\boldsymbol{k}_\perp| \sin\phi_k\right), \tag{3.60a}$$

$$\boldsymbol{S}_\perp = \left(\cos\phi_S, -\sin\phi_S\right), \tag{3.60b}$$

$$\boldsymbol{s}_\perp = \left(|\boldsymbol{s}_\perp| \cos\phi_s, -|\boldsymbol{s}_\perp| \sin\phi_s\right), \tag{3.60c}$$

we find for the $\boldsymbol{k}_\perp$-dependent transverse-polarisation distributions of quarks in a transversely polarised nucleon ($\pm$ denote, as usual, longitudinal polarisation whereas $\uparrow\downarrow$ denote transverse polarisation)

$$\begin{aligned}\mathcal{P}_{q\uparrow/N\uparrow}(x,\boldsymbol{k}_\perp) - \mathcal{P}_{q\downarrow/N\uparrow}(x,\boldsymbol{k}_\perp) \\ = \cos(\phi_S - \phi_s)\, \Delta'_T f(x,\boldsymbol{k}_\perp^2)\end{aligned}$$

$$+\frac{\boldsymbol{k}_\perp^2}{2M^2}\cos(2\phi_k-\phi_S-\phi_s)\,h_{1T}^\perp(x,\boldsymbol{k}_\perp^2)\,, \qquad (3.61\text{a})$$

and for the longitudinal polarisation distribution of quarks in a transversely polarised nucleon

$$\mathcal{P}_{q+/N\uparrow}(x,\boldsymbol{k}_\perp)-\mathcal{P}_{q-/N\uparrow}(x,\boldsymbol{k}_\perp)=\frac{|\boldsymbol{k}_\perp|}{M}\cos(\phi_S-\phi_k)\,g_{1T}(x,\boldsymbol{k}_\perp^2)\,. \quad (3.61\text{b})$$

Owing to transverse motion, quarks may also be transversely polarised in a longitudinally polarised nucleon. The polarisation asymmetry in this case is

$$\mathcal{P}_{q\uparrow/N+}(x,\boldsymbol{k}_\perp)-\mathcal{P}_{q\downarrow/N+}(x,\boldsymbol{k}_\perp)=\frac{|\boldsymbol{k}_\perp|}{M}\cos(\phi_k-\phi_s)\,h_{1L}^\perp(x,\boldsymbol{k}_\perp^2)\,. \quad (3.61\text{c})$$

As we shall see in Sec. 7.7, the $\boldsymbol{k}_\perp$-dependent distribution function $h_{1L}^\perp$ plays a rôle in the azimuthal asymmetries of semi-inclusive leptoproduction.

3.8 Twist-three distributions

At twist three the quark–quark correlation matrix, integrated over k, has the structure (Jaffe and Ji, 1992)

$$\Phi(x)=\cdots+\frac{M}{2}\left\{e(x)+g_T(x)\,\gamma_5\,\not{S}_\perp+\frac{\lambda_N}{2}\,h_L(x)\,\gamma_5\,[\not{p},\not{n}]\right\}, \qquad (3.62)$$

where the ellipsis represents the twist-two contribution, Eq. (3.15), and p, n are the Sudakov vectors (see App. B). Three more distributions appear in (3.62): $e(x)$, $g_T(x)$ and $h_L(x)$. We have already encountered $g_T(x)$, which is the twist-three partner of $\Delta f(x)$:

$$\int\frac{\mathrm{d}\tau}{2\pi}\,\mathrm{e}^{\mathrm{i}\tau x}\,\langle PS|\bar\psi(0)\gamma^\mu\gamma_5\psi(\tau n)|PS\rangle$$
$$=2\lambda_N\,\Delta f(x)\,p^\mu+2M\,g_T(x)\,S_\perp^\mu+\text{twist-four terms.} \quad (3.63\text{a})$$

Analogously, $h_L(x)$ is the twist-three partner of $\Delta_T f(x)$ and appears in the tensor term of the quark–quark correlation matrix:

$$\int\frac{\mathrm{d}\tau}{2\pi}\,\mathrm{e}^{\mathrm{i}\tau x}\,\langle PS|\bar\psi(0)\mathrm{i}\sigma^{\mu\nu}\gamma_5\psi(\tau n)|PS\rangle$$
$$=2\Delta_T f(x)\,p^{[\mu}S_\perp^{\nu]}+2M\,h_L(x)\,p^{[\mu}n^{\nu]}+\text{twist-four terms.} \quad (3.63\text{b})$$

The third distribution, $e(x)$, has no counterpart at leading twist. It appears in the expansion of the scalar field bilinear:

$$\int \frac{\mathrm{d}\tau}{2\pi}\, \mathrm{e}^{\mathrm{i}\tau x}\, \langle PS|\bar{\psi}(0)\psi(\tau n)|PS\rangle = 2M\, e(x) + \text{ twist-four terms.} \quad (3.63\mathrm{c})$$

The higher-twist distributions do not admit any probabilistic interpretation. To see this, consider for instance $g_T(x)$. Upon separation of ψ into good and bad components, it turns out to be

$$g_T(x) = \frac{P^+}{M} \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-}\, \langle PS|\psi^\dagger_{(+)}(0)\gamma^0\gamma^1\gamma_5\psi_{(-)}(0,\xi^-,\mathbf{0}_\perp) - \psi^\dagger_{(-)}(0)\gamma^0\gamma^1\gamma_5\psi_{(+)}(0,\xi^-,\mathbf{0}_\perp)|PS\rangle\,. \quad (3.64)$$

This distribution cannot be put into a form such as (3.27a, b). Thus, g_T cannot be regarded as a probability density. Just as all higher-twist distributions, it involves quark–quark–gluon correlations and hence has no simple partonic meaning. It is precisely this fact that renders $g_T(x)$ and the structure function that contains $g_T(x)$, *i.e.*, $g_2(x, Q^2)$, quite subtle and difficult to handle within the framework of parton model, as discussed in Secs. 2.7 and 2.7.3.

A polarised gluon distribution in a transversely polarised proton, analogous to $g_T(x)$, has been studied by Soffer and Teryaev (1997). This distribution, called $\Delta G_T(x)$ by Soffer and Teryaev, arises owing either to the non-zero virtuality or to gluon the transverse momentum and contributes to double transverse-spin asymmetries in two-jet production.

It should be borne in mind that the twist-three distributions in (3.62) are, in a sense, "effective" quantities, which incorporate various kinematical and dynamical effects that contribute to higher twist: quark masses, intrinsic transverse motion and gluon interactions. As shown by Tangerman and Mulders (1994), Mulders and Tangerman (1996), and Henneman, Boer and Mulders (2002), $e(x)$, $h_L(x)$ and $g_T(x)$ admit the decomposition

$$e(x) = \frac{m_q}{M}\frac{f(x)}{x} + \widetilde{e}(x)\,, \quad (3.65\mathrm{a})$$

$$h_L(x) = \frac{m_q}{M}\frac{\Delta f(x)}{x} - \frac{2}{x}\, h_{1L}^{\perp(1)}(x) + \widetilde{h}_L(x)\,, \quad (3.65\mathrm{b})$$

$$g_T(x) = \frac{m_q}{M}\frac{\Delta_T f(x)}{x} + \frac{1}{x}\, g_{1T}^{(1)}(x) + \widetilde{g}_T(x)\,, \quad (3.65\mathrm{c})$$

where we have introduced the weighted distributions

$$h_{1L}^{\perp(1)}(x) \equiv \int \mathrm{d}^2\boldsymbol{k}_\perp \, \frac{\boldsymbol{k}_\perp^2}{2M^2} \, h_{1L}^\perp(x, \boldsymbol{k}_\perp^2) \,, \tag{3.66a}$$

$$g_{1T}^{(1)}(x) \equiv \int \mathrm{d}^2\boldsymbol{k}_\perp \, \frac{\boldsymbol{k}_\perp^2}{2M^2} \, g_{1T}(x, \boldsymbol{k}_\perp^2) \,. \tag{3.66b}$$

The three tilde functions $\widetilde{e}(x)$, $\widetilde{h}_L(x)$ and $\widetilde{g}_T(x)$ are the genuine interaction-dependent twist-three parts of the subleading distributions, arising from non-handbag diagrams such as that of Fig. 2.7.

To understand the origin of the various terms in (3.65a–c), let us define the quark–quark–gluon correlation matrix (see Fig. 3.6)

$$\Phi_{A,ij}^\mu(k, \widetilde{k}, P, S) = \int \mathrm{d}^4\xi \int \mathrm{d}^4 z \; \mathrm{e}^{\mathrm{i}\widetilde{k}\cdot\xi} \; \mathrm{e}^{\mathrm{i}(k-\widetilde{k})\cdot z} \times \langle PS|\bar{\psi}_j(0)\, g A^\mu(z)\, \psi_i(\xi)|PS\rangle \,. \tag{3.67}$$

Note that in the diagram of Fig. 3.6 the momenta of the quarks on the left

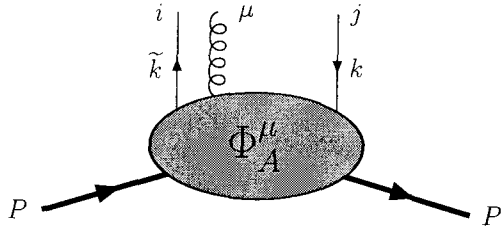


Fig. 3.6 The quark–quark–gluon correlation matrix Φ_A^μ.

and on the right are different. We call the two momentum fractions x and y, *i.e.*,

$$k \simeq xP \,, \qquad \widetilde{k} \simeq yP \,, \tag{3.68}$$

and integrate (3.67) over k and $\widetilde{k}$ with the constraints (3.68)

$$\begin{aligned}\Phi^{\mu}_{A,ij}(x,y) &= \int\frac{\mathrm{d}^4k}{(2\pi)^4}\int\frac{\mathrm{d}^4\widetilde{k}}{(2\pi)^4}\,\Phi^{\mu}_{A}(k,\widetilde{k},P,S)\\ &\qquad\times\delta(x-k^+/P^+)\,\delta(y-\widetilde{k}^+/P^+)\\ &= \int\frac{\mathrm{d}\tau}{2\pi}\int\frac{\mathrm{d}\eta}{2\pi}\,\mathrm{e}^{\mathrm{i}\tau y}\,\mathrm{e}^{\mathrm{i}\eta(x-y)}\\ &\qquad\times\langle PS|\bar{\psi}_j(0)\,gA^{\mu}(\eta n)\,\psi_i(\tau n)|PS\rangle\,, \qquad (3.69)\end{aligned}$$

where in the latter equality we set $\tau = P^+\xi^-$ and $\eta = P^+z^-$, and n is the usual Sudakov vector. If a further integration over y is performed, one obtains a quark–quark–gluon correlation matrix where one of the quark fields and the gluon field are evaluated at the same space–time point:

$$\Phi^{\mu}_{A,ij}(x) = \int\frac{\mathrm{d}\tau}{2\pi}\,\mathrm{e}^{\mathrm{i}\tau x}\langle PS|\bar{\psi}_j(0)\,gA^{\mu}(\tau n)\,\psi_i(\tau n)|PS\rangle\,. \qquad (3.70)$$

The matrix $\Phi^{\mu}_{A}(x,y)$ contains four multiparton distributions G_A, $\widetilde{G}_A$, H_A and E_A, and has the following structure ($\varepsilon^{\mu\nu}_{\perp}$ is defined in App. D.1)

$$\begin{aligned}\Phi^{\mu}_{A}(x,y) = \frac{M}{2}\Big\{&\mathrm{i}\,G_A(x,y)\,\varepsilon^{\mu\nu}_{\perp}S_{\perp\nu}\not{P} + \widetilde{G}_A(x,y)\,S^{\mu}_{\perp}\gamma_5\not{P}\\ &+ H_A(x,y)\,\lambda_N\gamma_5\gamma^{\mu}_{\perp}\not{P} + E_A(x,y)\,\gamma^{\mu}_{\perp}\not{P}\Big\}\,. \qquad (3.71)\end{aligned}$$

Time-reversal invariance implies that G_A, $\widetilde{G}_A$, H_A and E_A are real functions. By hermiticity, $\widetilde{G}_A$ and H_A are symmetric whereas G_A and E_A are antisymmetric under interchange of x and y. As we shall see, it turns out that $\widetilde{g}_T(x)$, $\widetilde{h}_L(x)$ and $\widetilde{e}(x)$ are indeed related to G_A, $\widetilde{G}_A$, H_A and E_A, in particular to the integrals over y of these functions.

In analogy with (3.67) and (3.69), we introduce the correlation matrix Φ^{μ}_{D} containing the covariant derivative $D_\mu = \partial_\mu - \mathrm{i}gA_\mu$,

$$\begin{aligned}\Phi^{\mu}_{D,ij}(k,\widetilde{k},P,S) &= \int\mathrm{d}^4\xi\int\mathrm{d}^4z\,\mathrm{e}^{\mathrm{i}\widetilde{k}\cdot\xi}\,\mathrm{e}^{\mathrm{i}(k-\widetilde{k})\cdot z}\\ &\qquad\times\langle PS|\bar{\psi}_j(0)\,\mathrm{i}D^{\mu}(z)\,\psi_i(\xi)|PS\rangle\\ &= \int\frac{\mathrm{d}\tau}{2\pi}\int\frac{\mathrm{d}\eta}{2\pi}\,\mathrm{e}^{\mathrm{i}\tau y}\,\mathrm{e}^{\mathrm{i}\eta(x-y)}\\ &\qquad\times\langle PS|\bar{\psi}_j(0)\,\mathrm{i}D^{\mu}(\eta n)\,\psi_i(\tau n)|PS\rangle\,. \qquad (3.72)\end{aligned}$$

The decomposition of Φ_D^μ is

$$\Phi_D^\mu(x,y) = \frac{M}{2}\Big\{\mathrm{i}\,G_D(x,y)\,\varepsilon_\perp^{\mu\nu}S_{\perp\nu}\not{P} + \widetilde{G}_D(x,y)\,S_\perp^\mu\gamma_5\not{P} + H_D(x,y)\,\lambda_N\gamma_5\gamma_\perp^\mu\not{P} + E_D(x,y)\,\gamma_\perp^\mu\not{P}\Big\}. \quad (3.73)$$

The multiparton distributions G_D, $\widetilde{G}_D$, H_D and E_D have similar time-reversal and hermiticity properties as G_A, $\widetilde{G}_A$, H_A and E_A.

Another useful correlation matrix is Φ_∂^μ, defined in terms of $\Phi(x,\boldsymbol{k}_\perp)$, Eq. (3.55), as

$$\Phi_\partial^\mu(x) \equiv \int \mathrm{d}^2\boldsymbol{k}_\perp\,\frac{k_\perp^\mu}{M}\,\Phi(x,\boldsymbol{k}_\perp)\,, \quad (3.74)$$

and related to $\Phi_A^\mu(x,y)$ and $\Phi_D^\mu(x,y)$ by

$$\begin{aligned}\Phi_\partial^\mu(x) &= \int \mathrm{d}y\,[\Phi_D^\mu(x,y) - \Phi_A^\mu(x,y)]\\ &= \Phi_D^\mu(x) - \Phi_A^\mu(x)\,. \end{aligned} \quad (3.75)$$

Using (3.57), the structure of Φ_∂^μ is found to be

$$\Phi_\partial^\mu(x) = \frac{1}{2}\Big\{g_{1T}^{(1)}(x)\,S_\perp^\mu\gamma_5\not{P} + \mathrm{i}\,\lambda_N\,h_{1L}^{(1)}(x)\,\sigma^{\mu\nu}\gamma_5 P_\nu\Big\}, \quad (3.76)$$

where $g_{1T}^{(1)}$ and $h_{1L}^{(1)}$ are the weighted distributions defined in (3.66a, b).

Now, the identities $\gamma^\mu\gamma^\nu = g^{\mu\nu} - \mathrm{i}\,\sigma^{\mu\nu}$ and (2.105) allow us to re-express the quark equation of motion $(\mathrm{i}\not{D} - m_q)\psi = 0$ in the two forms

$$\mathrm{i}\,D^\mu\psi + \sigma^{\mu\nu}D_\nu\psi - m_q\gamma^\mu\psi = 0\,, \quad (3.77a)$$

$$\mathrm{i}\gamma^\mu D^\nu\psi - \mathrm{i}\gamma^\nu D^\mu\psi - \mathrm{i}\,\varepsilon^{\mu\nu\rho\sigma}\gamma_\sigma\gamma_5\,\mathrm{i}D_\rho\psi + \mathrm{i}m_q\sigma^{\mu\nu}\psi = 0\,, \quad (3.77b)$$

whence

$$\mathrm{i}\,D^+\psi + \sigma^{+-}D^+\psi - \sigma^i D_i\psi - m_q\gamma^+\psi = 0\,, \quad (3.78a)$$

$$\mathrm{i}\gamma^+ D^\nu\psi - \mathrm{i}\gamma^\nu D^+\psi + \mathrm{i}\,\varepsilon_\perp^{\nu\rho}\gamma_\rho\gamma_5\,\mathrm{i}D^+\psi - \mathrm{i}\varepsilon_\perp^{\nu\rho}\gamma^+\gamma_5\,\mathrm{i}D_\rho\psi - \mathrm{i}m_q\sigma^{\nu+}\psi = 0\,. \quad (3.78b)$$

It is possible (see Tangerman and Mulders, 1994; Mulders and Tangerman, 1996; Henneman, Boer and Mulders, 2002) to translate (3.78a, b) into a set

of relations for the traces $\Phi^{[\Gamma]}$, $\Phi_A^{\mu\,[\Gamma]}$, $\Phi_D^{\mu\,[\Gamma]}$ and $\Phi_\partial^{\mu\,[\Gamma]}$; and to derive from these, using (3.57, 3.71, 3.73 and 3.76), the relations (3.65a–c), with

$$x\,\widetilde{e}(x) = 2\int \mathrm{d}y\, E_A(x,y)\,, \tag{3.79a}$$

$$x\,\widetilde{g}_T(x) = \int \mathrm{d}y \left[G_A(x,y) + \widetilde{G}_A(x,y)\right], \tag{3.79b}$$

$$x\,\widetilde{h}_L(x) = 2\int \mathrm{d}y\, H_A(x,y)\,. \tag{3.79c}$$

Thus, the tilde distributions $\widetilde{e}$, $\widetilde{g}_T$ and $\widetilde{h}_L$ incorporate quark–quark–gluon correlations, just as we anticipated.

Let us now derive the tensor structures relevant to the gluon contribution to g_2 (Ji, 1992b). Note first of all that, unlike the transversity case, gluonic (singlet) contributions are present here since there is no requirement of spin flip. We shall follow the development according to Ji. The starting point is the definition of the gluon field in the light-cone gauge, $A{\cdot}n \propto A^+ = 0$, with n^μ defined as in Eq. (B.1b). Thus, the gluon has three components: two transverse and A^-. It may thus be defined by $\partial^+ A^\mu = F^{+\mu}$, with the solution

$$A^\mu(\xi) = \tfrac{1}{2}\int \mathrm{d}\eta\, \varepsilon(\xi^- - \eta) F^{+\mu}(\xi^+, \xi^1, \xi^2, \eta) + \text{constant}\,, \tag{3.80}$$

where the constant is determined by the boundary condition, but is only important for zero modes.

The gluon density matrix for a polarised nucleon is

$$\begin{aligned}\rho^{\mu\nu} &= \int \frac{\mathrm{d}\lambda}{2\pi}\, \mathrm{e}^{\mathrm{i}\lambda x} \langle PS|A^\mu(0)A^\nu(\lambda n)|PS\rangle \\ &= \frac{(n^-)^2}{x^2}\int \frac{\mathrm{d}\lambda}{2\pi}\, \mathrm{e}^{\mathrm{i}\lambda x} \langle PS|F^{+\mu}(0)F^{+\nu}(\lambda n)|PS\rangle\,,\end{aligned} \tag{3.81}$$

where the second line is derived by applying Eq. (3.80). It is then easy to project out the structure relevant to a transversely polarised nucleon (we retain Ji's original notation):

$$G_3 = \frac{(n^-)^2}{2xM^2}\mathrm{i}\varepsilon_{\mu\nu\rho\sigma} S^\rho P^\sigma \int \frac{\mathrm{d}\lambda}{2\pi}\, \mathrm{e}^{\mathrm{i}\lambda x} \langle PS|F^{+\mu}(0)F^{+\nu}(\lambda n)|PS\rangle\,. \tag{3.82}$$

To render the three-gluon correlator piece explicit, consider first the equations of motion for the gluon field: $D_\mu F^{\mu\nu} = gJ^\nu$, with $J^\nu = \bar{\psi}\gamma^\nu \frac{\lambda}{2}\psi$.

Solving for ∂A^- one obtains

$$\partial^+ A^-(\lambda n) = \mathrm{i}n^- \int \frac{\mathrm{d}\lambda'}{2\pi} \frac{\mathrm{d}y}{y} \, \mathrm{e}^{\mathrm{i}(\lambda'-\lambda)y} \left[gJ^+(\lambda' n) - D_\perp F^{+\perp}(\lambda' n) \right], \quad (3.83)$$

and substitution into Eq. (3.82) leads to

$$G_3 = \frac{(n^-)^2}{Mx^2} \int \frac{\mathrm{d}\lambda}{2\pi} \cos\lambda x \times \langle PS | F^{+\perp}(0) F^{+\mu}(\lambda n) \left[gJ^+(\lambda n) - D_\perp F^{+\perp}(\lambda n) \right] | PS \rangle . \quad (3.84)$$

The first term is related to the qqg correlators introduced in Eq. (3.71) and the second term contains an analogous ggg correlator. The more general version, involving two different momentum fractions, is simply

$$M^{\mu\nu\rho}(x,y) = (\mathrm{i}n^-)^2 \int \frac{\mathrm{d}\lambda}{2\pi} \frac{\mathrm{d}\tau}{2\pi} \, \mathrm{e}^{\mathrm{i}\lambda x + \mathrm{i}\tau(y-x)} \times \langle PS | F^{+\mu}(0) \, \mathrm{i}D^\nu(\tau n) F^{+\rho}(\lambda n) | PS \rangle , \quad (3.85\mathrm{a})$$

where the indices μ, ν and ρ are taken to be transverse. This, however, is not the only three-gluon operator; two others exist, which also mix under renormalisation:

$$N^{\mu\nu\rho}(x,y) = -g(\mathrm{i}n^-)^3 \int \frac{\mathrm{d}\lambda}{2\pi} \frac{\mathrm{d}\tau}{2\pi} \, \mathrm{e}^{\mathrm{i}\lambda x + \mathrm{i}\tau(y-x)} \times \langle PS | \mathrm{i} f_{abc} F_a^{+\mu}(0) F_b^{+\nu}(\tau n) F_c^{+\rho}(\lambda n) | PS \rangle , \quad (3.85\mathrm{b})$$

$$O^{\mu\nu\rho}(x,y) = -g(\mathrm{i}n^-)^3 \int \frac{\mathrm{d}\lambda}{2\pi} \frac{\mathrm{d}\tau}{2\pi} \, \mathrm{e}^{\mathrm{i}\lambda x + \mathrm{i}\tau(y-x)} \times \langle PS | d_{abc} F_a^{+\mu}(0) F_b^{+\nu}(\tau n) F_c^{+\rho}(\lambda n) | PS \rangle , \quad (3.85\mathrm{c})$$

where f_{abc} and d_{abc} are the usual structure constants for the colour SU(3) group. At this point we simply note that to parametrise these tensors one needs eight two-variable functions (for details, see Ji, 1992b). This implies that experimental extraction of the separate quantities involved will be a very arduous task indeed and a number of different contributing processes will have to be identified.

3.9 Sum rules for $\Delta_T f$ and g_T

Going back to the Lorentz decomposition (3.6) of Φ, and expressing the distribution functions in terms of the coefficients C_i, one can obtain two

noteworthy relations for h_L and g_T, namely (Tangerman and Mulders, 1994; Mulders and Tangerman, 1996; Boglione and Mulders, 2000; Henneman, Boer and Mulders, 2002)

$$g_T(x) = \Delta f(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_{1T}^{(1)}\,, \tag{3.86}$$

$$h_L(x) = \Delta_T f(x) - \frac{\mathrm{d}}{\mathrm{d}x} h_{1L}^{\perp(1)}(x)\,, \tag{3.87}$$

where $g_{1T}^{(1)}$ and $h_{1L}^{\perp(1)}$ were defined in Eqs. (3.66a, b).

Let us focus for the moment on (3.86). If we combine this relation with (3.65c), and solve for $g_{1T}^{(1)}$, we obtain (Jaffe and Ji, 1992)

$$g_T(x) = x\int_x^1 \frac{\mathrm{d}y}{y}\,\Delta f(y) + \frac{m_q}{M}\left[\frac{\Delta_T f(x)}{x} - x\int_x^1 \frac{\mathrm{d}y}{y^2}\,\Delta_T f(y)\right] + \left[\widetilde{g}_T(x) - \int_x^1 \frac{\mathrm{d}y}{y}\,\widetilde{g}_T(y)\right]. \tag{3.88}$$

On the other hand, solving for g_T leads to

$$\frac{g_{1T}^{(1)}(x)}{x} = \int_x^1 \frac{\mathrm{d}y}{y}\,\Delta f(y) - \frac{m_q}{M}\int_x^1 \frac{\mathrm{d}y}{y^2}\,\Delta_T f(y) - \int_x^1 \frac{\mathrm{d}y}{y}\,\widetilde{g}_T(y)\,. \tag{3.89}$$

Equation (3.88) displays the three contributions to g_T (and therefore to g_2): the leading-twist term, the quark mass term and the twist-three interaction-dependent term.

If the mass and higher-twist components are ignored in (3.88), then one obtains

$$g_T(x) = \int_x^1 \frac{\mathrm{d}y}{y}\,\Delta f(y)\,, \tag{3.90}$$

and from this, recalling (2.103),

$$g_2(x) = -g_1(x) + \int_x^1 \frac{\mathrm{d}y}{y}\,g_1(y)\,, \tag{3.91}$$

which is just the parton-model version of the Wandzura–Wilczek relation (2.134). Retaining only the leading-twist part, Eqs. (3.86, 3.89) lead to

$$\frac{g_{1T}^{(1)}(x)}{x} = \int_x^1 \frac{\mathrm{d}y}{y}\,\Delta f(y) = g_T(x)\,. \tag{3.92}$$

Let us now come to (3.87). Proceeding as above, we find

$$h_L(x) = 2x\int_x^1 \frac{\mathrm{d}y}{y^2}\,\Delta_T f(y) + \frac{m_q}{M}\left[\frac{\Delta f(x)}{x} - 2x\int_x^1 \frac{\mathrm{d}y}{y^3}\,\Delta f(y)\right] + \left[\widetilde{h}_L(x) - 2x\int_x^1 \frac{\mathrm{d}y}{y^2}\,\widetilde{h}_L(y)\right] \tag{3.93}$$

and

$$\frac{h_{1L}^{\perp(1)}(x)}{x^2} = -\int_x^1 \frac{\mathrm{d}y}{y^2}\,\Delta_T f(y) + \frac{m_q}{M}\int_x^1 \frac{\mathrm{d}y}{y^3}\,\Delta f(y) + \int_x^1 \frac{\mathrm{d}y}{y^2}\,\widetilde{h}_L(y)\,. \tag{3.94}$$

The transversity analogue of the WW relation is

$$h_L(x) = 2x\int_x^1 \frac{\mathrm{d}y}{y^2}\,\Delta_T f(y)\,, \tag{3.95}$$

and another leading-twist relation is

$$h_{1L}^{\perp(1)}(x) = -x^2\int_x^1 \frac{\mathrm{d}y}{y^2}\,\Delta_T f(y) = -\frac{1}{2}\,x h_L(x)\,. \tag{3.96}$$

Integrating $h_L(x)$, as given by Eq. (3.93), yields an integral relation analogous to the Burkhardt–Cottingham sum rule, which was first derived by Burkardt (1993, 1995) and Tangerman and Mulders (1994). For a recent study, see Burkardt and Koike (2001).

3.10 *T*-odd distributions

Let us suppose we relax the constraint (3.4c) of time-reversal invariance (we postpone the discussion of the physical relevance of this until the end of this section), then three more terms appear in (3.7b, d, e),

$$\mathcal{V}^\mu = \cdots + C_{10}\,\varepsilon^{\mu\nu\rho\sigma} S_\nu P_\rho k_\sigma\,, \tag{3.7b$'$}$$

$$\mathcal{P}_5 = C_{11}\,k{\cdot}S\,, \tag{3.7d$'$}$$

$$\mathcal{T}^{\mu\nu} = \cdots + C_{12}\,\varepsilon^{\mu\nu\rho\sigma} P_\rho k_\sigma\,. \tag{3.7e$'$}$$

At leading twist the relevant terms are

$$\mathcal{V}^\mu = \cdots + \frac{1}{M} A_1' \, \varepsilon^{\mu\nu\rho\sigma} P_\nu k_{\perp\rho} S_{\perp\sigma} \,, \tag{3.8a'}$$

$$\mathcal{T}^{\mu\nu} = \cdots + \frac{1}{M} A_2' \, \varepsilon^{\mu\nu\rho\sigma} P_\rho k_{\perp\sigma} \,, \tag{3.8c'}$$

which give rise to two $\boldsymbol{k}_\perp$-dependent T-odd distribution functions, $f_{1T}^\perp$ and $h_1^\perp$ (Boer and Mulders, 1998)

$$\Phi^{[\gamma^+]} = \cdots - \frac{\varepsilon_\perp^{ij} k_{\perp i} S_{\perp j}}{M} \, f_{1T}^\perp(x, \boldsymbol{k}_\perp^2) \,, \tag{3.97a}$$

$$\Phi^{[i\sigma^{i+}\gamma_5]} = \cdots - \frac{\varepsilon_\perp^{ij} k_{\perp j}}{M} \, h_1^\perp(x, \boldsymbol{k}_\perp^2) \,. \tag{3.97b}$$

Let us see the partonic interpretation of the new distributions. The first of them, $f_{1T}^\perp$, is related to the number density of unpolarised quarks in a transversely polarised nucleon. More precisely, it is given by

$$\begin{aligned} \mathcal{P}_{q/N\uparrow}(x, \boldsymbol{k}_\perp) - \mathcal{P}_{q/N\downarrow}(x, \boldsymbol{k}_\perp) &= \mathcal{P}_{q/N\uparrow}(x, \boldsymbol{k}_\perp) - \mathcal{P}_{q/N\uparrow}(x, -\boldsymbol{k}_\perp) \\ &= -2 \frac{|\boldsymbol{k}_\perp|}{M} \, \sin(\phi_k - \phi_S) \, f_{1T}^\perp(x, \boldsymbol{k}_\perp^2) \,. \end{aligned} \tag{3.98a}$$

The other T-odd distribution, $h_1^\perp$, measures quark transverse polarisation in an unpolarised hadron and is defined via

$$\mathcal{P}_{q\uparrow/N}(x, \boldsymbol{k}_\perp) - \mathcal{P}_{q\downarrow/N}(x, \boldsymbol{k}_\perp) = -\frac{|\boldsymbol{k}_\perp|}{M} \, \sin(\phi_k - \phi_s) \, h_1^\perp(x, \boldsymbol{k}_\perp^2) \,. \tag{3.98b}$$

We shall encounter again these distributions in the analysis of hadron production (Sec. 8.1). For later convenience we define two quantities $\Delta_0^T f$ and $\Delta_T^0 f$, related respectively to $f_{1T}^\perp$ and $h_1^\perp$ by (see Sec. 1.4 for the notation)

$$\Delta_0^T f(x, \boldsymbol{k}_\perp^2) \equiv -2 \frac{|\boldsymbol{k}_\perp|}{M} \, f_{1T}^\perp(x, \boldsymbol{k}_\perp^2) \,, \tag{3.99a}$$

$$\Delta_T^0 f(x, \boldsymbol{k}_\perp^2) \equiv -\frac{|\boldsymbol{k}_\perp|}{M} \, h_1^\perp(x, \boldsymbol{k}_\perp^2) \,. \tag{3.99b}$$

In field-theoretical terms we have

$$\Delta_0^T f_a(x, \boldsymbol{k}_\perp^2) = \mathrm{Im} \int \frac{\mathrm{d}\xi^- \, \mathrm{d}^2\boldsymbol{\xi}_\perp}{2(2\pi)^3} \, \mathrm{e}^{\mathrm{i}(xP^+\xi^- - \boldsymbol{k}_\perp \cdot \boldsymbol{\xi}_\perp)} \times \langle P, - | \bar{\psi}_a(0) \gamma^+ \psi_a(0, \xi^-, \boldsymbol{\xi}_\perp) | P, + \rangle \,, \quad (3.100\mathrm{a})$$

$$\Delta_T^0 f_a(x, \boldsymbol{k}_\perp^2) = \int \frac{\mathrm{d}\xi^- \, \mathrm{d}^2\boldsymbol{\xi}_\perp}{2(2\pi)^3} \, \mathrm{e}^{\mathrm{i}(xP^+\xi^- - \boldsymbol{k}_\perp \cdot \boldsymbol{\xi}_\perp)} \times \langle P | \bar{\psi}_a(0) \mathrm{i}\sigma^{2+} \psi_a(0, \xi^-, \boldsymbol{\xi}_\perp) | P \rangle \,, \quad (3.100\mathrm{b})$$

where $|P, \pm\rangle$ are the momentum–helicity eigenstates of the proton, a is a flavour index and for definiteness we have taken the transverse polarisation of the quarks, or of the proton, to be directed along the y-axis. One can check the time-reversal properties of the two distributions directly from expressions (3.100a, b). Using the standard action of the T operator on the quark fields, that is,

$$T \, \psi_a(x) \, T^\dagger = -\mathrm{i}\gamma_5 C \, \psi_a(-\widetilde{x}) \,, \quad (3.101)$$

it is easy to see that the matrix elements in (3.100a, b) change sign under T and hence the corresponding distributions must vanish owing to time-reversal invariance (Collins, 1993b).

One may legitimately wonder whether T-odd quark distributions, such as $f_{1T}^\perp$ and $h_1^\perp$, which violate the time-reversal condition (3.4c), make any sense at all. In order to justify the existence of T-odd distribution functions, some of their proponents (Anselmino and Murgia, 1998) advocate initial-state effects, which prevent implementation of time-reversal invariance by naïvely imposing the condition (3.4c). The idea, similar to that which leads to admitting T-odd fragmentation functions as a result of final-state effects (see Sec. 7.5.1), is that the colliding particles interact strongly with non-trivial relative phases. As a consequence, time reversal no longer implies the constraint (3.4c)[b]. If hadronic interactions in the initial state are crucial to explain the existence of $f_{1T}^\perp$ and $h_1^\perp$, these distributions should only be observable in reactions involving two initial hadrons (Drell–Yan processes, hadron production in proton–proton collisions *etc.*). This mechanism is known as the Sivers effect (1990; 1991). Clearly, it should be absent in leptoproduction.

[b]Thus, "T-odd" really means that condition (3.4c) is not satisfied, not that time-reversal invariance itself is violated.

An alternative way of viewing T-odd distributions has been proposed by Anselmino *et al.* (2002a). Applying a general argument on time reversal for particle multiplets suggested by Weinberg (1995, p. 100), Anselmino *et al.* argue that, if the internal structure of hadrons is described at some low momentum scale by a chiral lagrangian, time reversal might be realised in a "non-standard" manner that would mix the multiplet components. Equation (3.101) would then be replaced by

$$T\,\psi_a(x)\,T^\dagger = -\mathrm{i}(\tau_2)_{ab}\,\gamma_5 C\,\psi_b(-\widetilde{x})\,, \tag{3.102}$$

where τ_2 is a Pauli matrix. If time reversal acts as in (3.102), $f_{1T}^\perp$ and $h_1^\perp$ are not forced to vanish. In fact, when (3.102) is used, the u (d) distribution transforms into the d (u) distribution, and time-reversal invariance simply establishes a relation between the u and d sectors.

Notice that, if the conjecture of Anselmino *et al.* (2002a) is correct, since no initial-state effects are required, the T-odd distributions should also be observable in semi-inclusive leptoproduction (where they would give rise to single transverse-spin asymmetries).

Very recently, Collins (2002) has reconsidered his proof of the vanishing of $f_{1T}^\perp$ and $h_1^\perp$, based on the field-theoretical expressions of the two distributions. He has noticed that, if one reinstates the link operators into (3.100a, b), the distributions do not simply change sign under T, because a future-pointing Wilson line is transformed into a past-pointing Wilson line. As a consequence, time-reversal invariance, rather than constraining $f_{1T}^\perp$ and $h_1^\perp$ to be zero, gives a relation between processes that probe Wilson lines pointing in opposite directions. In particular, Collins predicts that the Sivers asymmetry should have opposite signs in DIS and in Drell–Yan (DY) processes.

At higher twist more T-odd distribution functions emerge. For future reference we give the T-odd twist-three quark–quark correlation matrix, which contains three T-odd distribution functions (Boer and Mulders, 1998; Boer, 1999)

$$\Phi(x)|_{T-\mathrm{odd}} = \frac{M}{2}\left\{ f_T(x)\,\varepsilon_\perp^{\mu\nu} S_{\perp\mu}\gamma_\nu - \mathrm{i}\lambda_N\, e_L(x)\,\gamma_5 + \frac{\mathrm{i}}{2}\,h(x)\,[\not{p},\not{n}] \right\}. \tag{3.103}$$

We shall find these distributions again in Sec. 6.4, where it will be shown that they can effectively arise from quark–quark–gluon correlations.

The quark (and antiquark) distribution functions at leading twist and twist three are collected in Table 3.1.

Table 3.1 The quark distributions at twist two and three. S denotes the polarisation state of the parent hadron (0 indicates unpolarised). The asterisk indicates T-odd quantities.

Quark distributions			
S	0	L	T
twist 2	$f(x)$	$\Delta f(x)$	$\Delta_T f(x)$
twist 3	$e(x)$	$h_L(x)$	$g_T(x)$
(*)	$h(x)$	$e_L(x)$	$f_T(x)$

3.11 Model calculations

Since we have no experimental information on transversity and rather scarce data on g_2, model calculations are presently an important avenue to acquire knowledge of these quantities.

What models can provide is the nucleon state appearing in the field-theoretical expressions for quark distributions. The matrix elements of quark operators in this state turn out to be just numbers, with no scale dependence. The problem therefore arises as to how to reconcile quark models with QCD perturbation theory. Since the early days of QCD various authors (Parisi and Petronzio, 1976; Glück and Reya, 1977; Jaffe and Ross, 1980) have proposed a possible answer to this question: the twist-two matrix elements computed in quark models should be interpreted as representing the nucleon at some *fixed*, low scale Q_0^2 (we shall call it the *model scale*). In other terms, quark models provide the *initial conditions* for QCD evolution. The experience accumulated with radiative generation models (Glück, Godbole and Reya, 1989; Barone *et al.*, 1993a,b) has taught us that, in order to obtain a picture of the nucleon at large Q^2 in agreement with experiment (at least in the unpolarised case), the nucleon must actually contain a relatively large fraction of sea and glue, even at very low momentum scales. Purely valence models are usually unable to fit the data at all well.

Although the model scale has the same order of magnitude in all models

($Q_0 \sim 0.3 - 0.8\,\mathrm{GeV}$), its precise value depends on the details of the model and on the procedure adopted to determine it. The smallness of Q_0^2 clearly raises another problem: namely, to what extent one can apply perturbative evolution to extrapolate the quark distributions from such low scales to large Q^2. This problem is still unresolved (for an attempt to model a non-perturbative evolution mechanism in an effective theory see Ball and Forte, 1994; Ball *et al.*, 1994), although the success of fits based on radiatively generated parton distributions (Glück, Godbole and Reya, 1989; Glück, Reya and Vogelsang, 1990; Glück, Reya and Vogt, 1992, 1998) inspires some confidence that the realm of perturbative QCD may extend to fairly small scales.

Another problem arising in model calculations is the fact that solutions of classical equations of motion (*i.e.*, the mean-field approximation) break translational invariance and thus do not provide momentum eigenstates. Suitable projection procedures, such as the Peierls–Yoccoz projection (Ring and Schuck, 1980), need then to be applied.

3.11.1 *Models for the transversity distributions*

The transversity distributions have been computed using a large variety of models (for a review, see Barone, Drago and Ratcliffe, 2002). In the following we shall present some general considerations based mostly on relativistic confinement models, such as the MIT bag model (Chodos *et al.*, 1974a,b; DeGrand *et al.*, 1975) and the chromodielectric model (Birse, 1990; Pirner, 1992; Banerjee, 1993). In these models confinement is implemented by situating the quarks in a region characterised by a value of the colour-dielectric constant of order unity. The value of this constant is zero outside the nucleon, that is, in the true vacuum, from which the coloured degrees of freedom are expelled.

In the (projected) mean-field approximation, the matrix elements defining the leading-twist distribution functions can be rewritten in terms of single-particle (quark or antiquark) wave functions, after inserting a complete set of states between the two fermionic fields ψ and $\bar{\psi}$ (Signal and Thomas, 1988; Schreiber and Thomas, 1988). The intermediate states that contribute are $2q$ and $3q1\bar{q}$ states for the quark distributions, and $4q$ states for the antiquark distributions (see Fig. 3.7).

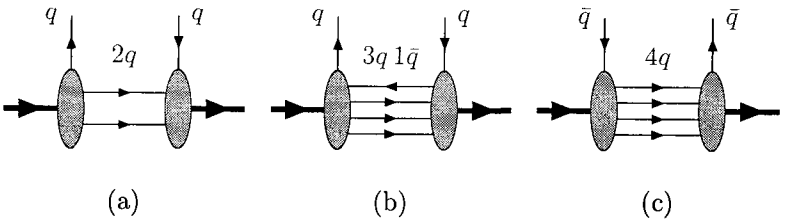


Fig. 3.7 Intermediate states in the parton model: (a) $2q$, (b) $3q1\bar{q}$ and (c) $4q$.

The leading-twist quark distribution functions read (f is the flavour)

$$f(x) = \sum_{\alpha} \sum_{m} P(f,\alpha,m)\, F_\alpha(x)\,, \tag{3.104a}$$

$$\Delta f(x) = \sum_{\alpha} \sum_{m} P(f,\alpha,m)\, (-1)^{(m+\frac{3}{2}+i_\alpha)}\, G_\alpha(x)\,, \tag{3.104b}$$

$$\Delta_T f(x) = \sum_{\alpha} \sum_{m} P(f,\alpha,m)\, (-1)^{(m+\frac{3}{2}+i_\alpha)}\, H_\alpha(x)\,, \tag{3.104c}$$

where

$$\left.\begin{matrix} F_\alpha(x) \\ G_\alpha(x) \\ H_\alpha(x) \end{matrix}\right\} = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3(2p^0)}\, A_\alpha(p)\, \delta\big((1-x)P^+ - p^+\big)$$

$$\times \frac{1}{2} \begin{cases} u^2(p) + 2u(p)\, v(p)\, \frac{p_z}{|\boldsymbol{p}|} + v^2(p)\,, \\ u^2(p) + 2u(p)\, v(p)\, \frac{p_z}{|\boldsymbol{p}|} + v^2(p) \left(2\, \frac{p_z^2}{|\boldsymbol{p}|^2} - 1\right), \\ u^2(p) + 2u(p)\, v(p)\, \frac{p_z}{|\boldsymbol{p}|} + v^2(p) \frac{p_z^2}{|\boldsymbol{p}|^2}\,. \end{cases} \tag{3.105}$$

In (3.105) u and v are respectively the upper and lower components of the single-quark wave functions, m is the projection of the quark spin along the direction of the nucleon spin, and $P(f,\alpha,m)$ is the probability of extracting a quark (or inserting an antiquark) of flavour f and spin m, leaving a state generically labelled by the quantum number α. The index i_α takes the value 0 when a quark is extracted and 1 when an antiquark is inserted. The overlap function $A_\alpha(p)$ contains the details of the intermediate states and of the projection procedures needed to construct momentum eigenstates. The integration in (3.105) is over the intermediate-state momentum (notice the delta function of energy–momentum conservation). The anti-

quark distributions are obtained in a similar manner (the index i_α is 1 for the $4q$ states).

In the non-relativistic limit, where the lower components of the quark wave functions are neglected, the three currents in (3.105) coincide. This implies, in the light of (3.104b, c), that ignoring relativistic effects the helicity distributions are equal to the transversity distributions,

$$\text{NR limit:} \qquad \Delta f(x, Q_0^2) = \Delta_T f(x, Q_0^2)\,. \tag{3.106}$$

This is obviously only valid at the model scale Q_0^2 (*i.e.*, at very low Q^2) since, as we shall see in Sec. 4.5, QCD evolution discriminates between the two distributions, in particular at small x.

It is also interesting to observe that the three quantities F_α, G_α and H_α defined in (3.105) satisfy

$$F_\alpha(x) + G_\alpha(x) = 2H_\alpha(x)\,. \tag{3.107}$$

This has sometimes led to the erroneous conclusion that the Soffer inequality is saturated for a relativistic quark model, such as the MIT bag model. It is clear from (3.104a–c) that the spin–flavour structure of the proton, which results in the appearance of the probabilities $P(f, \alpha, m)$, spoils this argument and in general prevents saturation of the inequality. Indeed, Soffer's inequality is only saturated in very specific (and quite unrealistic) cases. For instance, it is saturated when $P(f, \alpha, -\frac{1}{2}) = 0$, which happens if the proton is modelled as a bound state of a quark and scalar diquark. Another instance of saturation occurs when $F_\alpha = G_\alpha = H_\alpha$ and $P(f, \alpha, -\frac{1}{2}) = 2\,P(f, \alpha, \frac{1}{2})$. It is easy to verify that this happens for the d quark distribution in a non-relativistic model of the proton with an SU(6) wavefunction. Apart from these special cases, Soffer's inequality should not generally be expected to be saturated.

If an extremely simplified approach is adopted and no projections are performed, $A_\alpha(p)$ does not appear in (3.105), and the functions $F_\alpha(x)$, $G_\alpha(x)$ and $H_\alpha(x)$ are actually independent of the quantum number α. In the MIT bag model these functions can be computed analytically (at least in part) and one gets (Jaffe and Ji, 1992)

$$F(x) = \frac{\omega_n M R}{2\pi(\omega_n - 1) j_0^2(\omega_n)} \times \int_{|y_{\min}|}^{\infty} \mathrm{d}y\, y \left[t_0^2 + 2 t_0 t_1 \frac{y_{\min}}{y} + t_1^2 \right], \tag{3.108a}$$

$$G(x) = \frac{\omega_n MR}{2\pi(\omega_n - 1)j_0^2(\omega_n)} \times \int_{|y_{\min}|}^{\infty} \mathrm{d}y\, y \left[t_0^2 + 2t_0 t_1 \frac{y_{\min}}{y} + t_1^2 \left(2\left(\frac{y_{\min}}{y}\right)^2 - 1 \right) \right], \tag{3.108b}$$

$$H(x) = \frac{\omega_n MR}{2\pi(\omega_n - 1)j_0^2(\omega_n)} \times \int_{|y_{\min}|}^{\infty} \mathrm{d}y\, y \left[t_0^2 + 2t_0 t_1 \frac{y_{\min}}{y} + t_1^2 \left(\frac{y_{\min}}{y}\right)^2 \right], \tag{3.108c}$$

where ω_n is the n-th root of the equation

$$\tan\omega_n = -\frac{\omega_n}{(\omega_n - 1)}, \tag{3.109}$$

and $y_{\min} = xRM - \omega_n$. R and M are the bag radius and nucleon mass, respectively. The functions t_0 and t_1 are defined as

$$t_l(\omega_n, y) = \int_0^1 \mathrm{d}u\, u^2 j_l(u\,\omega_n)\, j_l(uy), \tag{3.110}$$

where j_l is the l-th order spherical Bessel function.

To obtain the quark distributions one must insert $F(x)$, $G(x)$ and $H(x)$ into (3.104a–c), along with the probabilities $P(f, \alpha, n)$. If only valence quarks are assumed to contribute to the distributions, the intermediate states $|\alpha\rangle$ reduce to the diquark state alone. The quark distributions are then proportional to $F(x)$, $G(x)$ and $H(x)$. In particular, with an SU(6) spin–flavour wave function one simply has

$$\Delta u(x) = \tfrac{4}{3}\, G(x), \qquad \Delta d(x) = -\tfrac{1}{3}\, G(x), \tag{3.111a}$$

$$\Delta_T u(x) = \tfrac{4}{3}\, H(x), \qquad \Delta_T d(x) = -\tfrac{1}{3}\, H(x). \tag{3.111b}$$

The quantities $F(x)$, $G(x)$ and $H(x)$ are plotted in Fig. 3.8. Since translational invariance is lost, the distributions do not have the correct support. Thus, one has to renormalise them so that the integral of $F(x)$ over x between zero and one is unity, as it should be.

A more sophisticated calculation of the transversity distributions within the MIT bag model, incorporating a Peierls–Yoccoz projection to partially restore translational invariance, was reported by Stratmann (1993). His

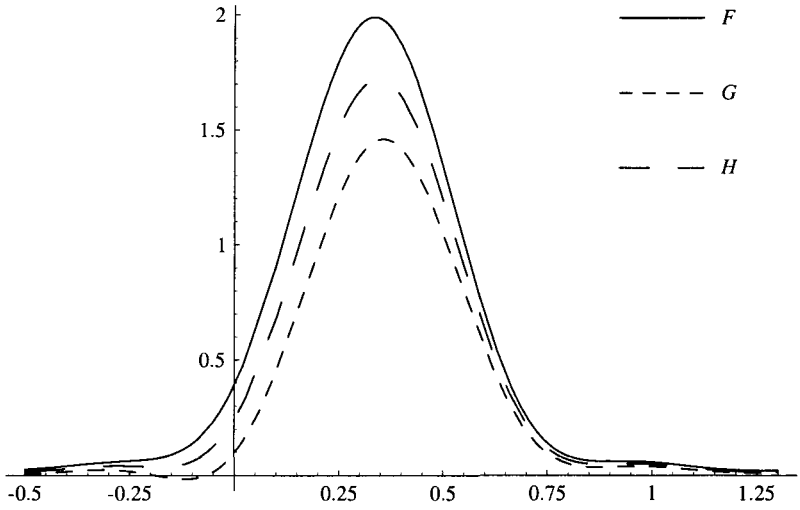


Fig. 3.8 Single-quark contributions to the distribution functions in the MIT bag model of Jaffe and Ji (1992); from Barone, Drago and Ratcliffe (2002).

results are shown in Fig. 3.9. The MIT bag model was also used to compute $\Delta_T f$ by Scopetta and Vento (1998).

Another relativistic confinement model that has been used (Barone, Calarco and Drago, 1997b) to calculate the transversity distributions is the chiral colour-dielectric model (CDM—Birse, 1990; Pirner, 1992; Banerjee, 1993). Its main technical advantage with respect to the MIT bag model is that it allows a full "variation after projection" procedure to be performed, since confinement is implemented by a dynamical field, not by a static bag surface. It turns out that the valence-number sum rules are fulfilled (to within a few percent) if the masses of the intermediate states are consistently computed within the model. The Soffer inequalities are also satisfied, for both quarks and antiquarks. The solution of the field equations proceeds through the introduction of the so-called hedgehog ansatz, which corresponds to a mean-field approximation (Fiolhais, Urbano and Goeke, 1985). The technique used to compute the physical nucleon state is based on a double projection of the mean-field solution onto linear- and angular-momentum eigenstates. In Fig. 3.10 we show the results of the calculation of Barone, Calarco and Drago (1997b). One of the features of the distributions computed in the CDM is their rapid falloff and vanishing

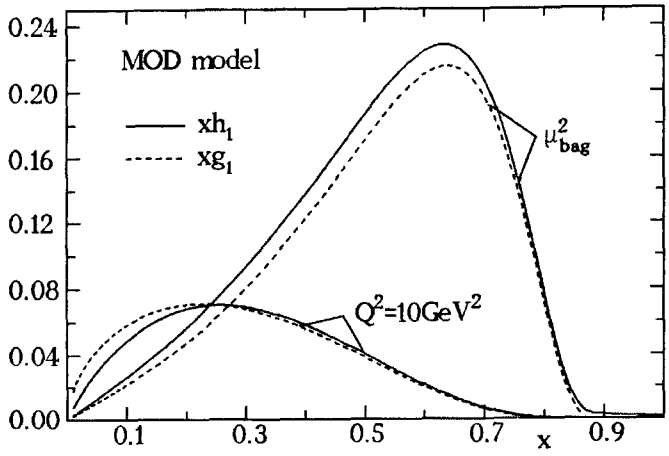


Fig. 3.9 The transversity "structure function" $h_1 = \frac{1}{2}\sum_a e_a^2 \Delta_T f_a$ and the spin structure function g_1 for the MIT bag model at the initial scale $\mu^2_{\rm bag} = 0.08\,{\rm GeV}^2$ and at $Q^2 = 10\,{\rm GeV}^2$; from Stratmann (1993).

for $x > 0.6$. This is due to the soft confinement of quarks, which do not carry large momenta.

It should be stressed that the Peierls–Yoccoz procedure, which is based on a non-relativistic approximation, becomes unreliable at large x. Note also that the sea contribution is rather small. As for the model scale Q_0^2, it was determined by matching the value of the momentum fraction carried by the valence, as computed in the model, with that obtained by evolving backwards the value experimentally determined at large Q^2 (Barone and Drago, 1993; Barone, Drago and Fiolhais, 1994; Barone, Calarco and Drago, 1997b). The result is $Q_0^2 = 0.16\,{\rm GeV}^2$. Proceeding in a similar manner, Stratmann (1993) found $Q_0^2 = 0.08\,{\rm GeV}^2$ in the MIT bag model. Needless to say, perturbative evolution from such low Q_0^2 values should be taken with some caution.

An important class of models extensively used to calculate quark distribution functions is that of chiral models (Manohar and Georgi, 1984; Diakonov and Petrov, 1986; Diakonov, Petrov and Pobylitsa, 1988), in which the $q\bar{q}$ excitations are described in terms of effective degrees of freedom given by chiral fields. As for the spin structure, a crucial feature of these models is the depolarisation of the valence quarks, due to the transfer of total angular momentum into the orbital angular momentum of the sea

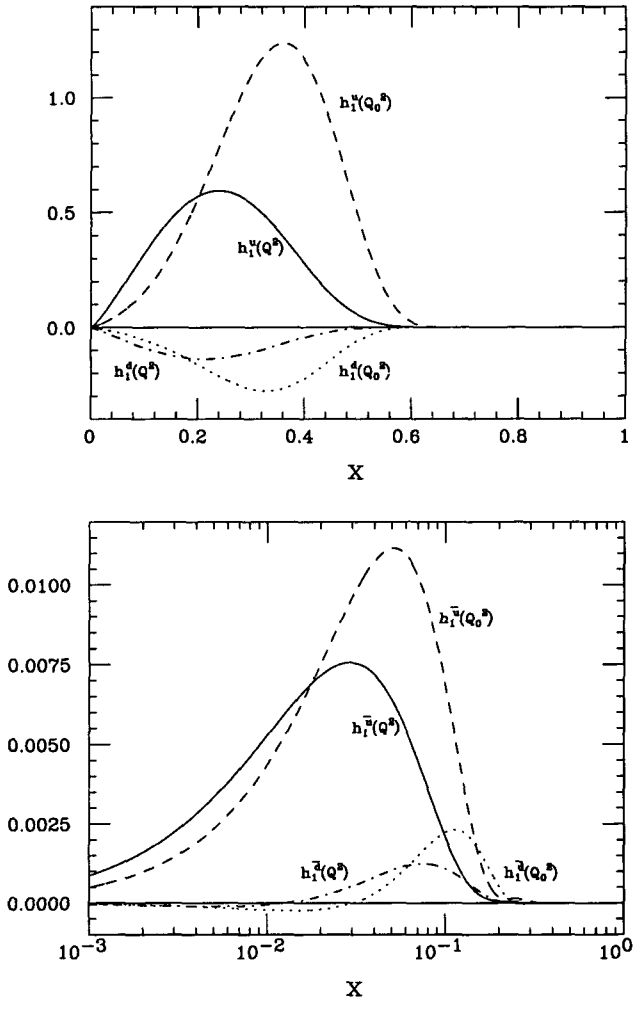


Fig. 3.10 The transversity distributions $x\Delta_T q(x)$ in the colour-dielectric model (from Barone, Calarco and Drago, 1997b). Top: the quark distributions $x\Delta_T u$ and $x\Delta_T d$. Bottom: the antiquark distributions $x\Delta_T \bar{u}$ and $x\Delta_T \bar{d}$. The distributions are shown at the model scale $Q_0^2 = 0.16\,\mathrm{GeV}^2$ and evolved to $Q^2 = 25\,\mathrm{GeV}^2$.

(*i.e.*, of the chiral fields). This has made chiral models quite popular for the study of the nucleon spin.

In the chiral quark–soliton model (CQSM—Diakonov and Petrov, 1986; Diakonov, Petrov and Pobylitsa, 1988) chiral symmetry is dynamically bro-

ken and a non-trivial topology is introduced in order to stabilise the soliton. The nucleon is described as a state of N_c valence quarks bound by a self-consistent hedgehog-like pion field whose energy coincides with the aggregate energy of quarks from the negative-energy Dirac continuum. At zeroth order in $1/N_c$, the CQSM essentially corresponds to a mean-field picture. Some observables, however, vanish at this order of approximation (this is the case of unpolarised isovector and polarised isoscalar distribution functions) and for these quantities a calculation at first order in $1/N_c$ is clearly needed. Without $1/N_c$ corrections the tensor (and also the axial) charge of the u quark would be equal and opposite to that of the d quark. It is also important to notice that, while the axial singlet charge is substantially reduced owing to the presence of the chiral fields, the tensor charges are close to those obtained in other models where the chiral fields are absent, or only play a minor rôle.

Calculations of the transversity distributions within the CQSM have been reported by many authors (Kim, Polyakov and Goeke, 1996; Pobylitsa and Polyakov, 1996; Gamberg, Reinhardt and Weigel, 1998; Wakamatsu and Kubota, 1999; Schweitzer *et al.*, 2001; Wakamatsu, 2001). The calculations differ in the order of $1/N_c$ expansion considered, in the projections and in other details (for a critical discussion, see Barone, Drago and Ratcliffe, 2002). In Fig. 3.11 we show the findings of Wakamatsu (2001). An

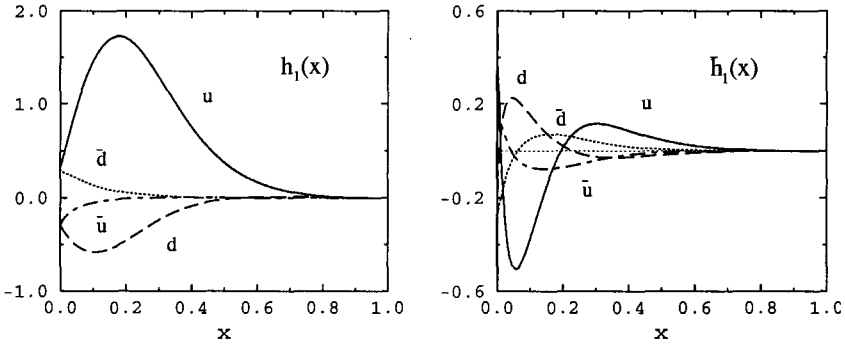


Fig. 3.11 Transversity distributions of quarks and antiquarks as computed in the chiral quark–soliton model by Wakamatsu (2001).

intriguing feature of the CQSM results is that the antiquark transversity

distributions have the opposite sign as compared to the antiquark helicity distributions. Note, however, that the *antiquark* distributions are obtained by extending the *quark* distributions to *negative* x. That is, see (3.21b, c), by using

$$\Delta\bar{f}(x) = \Delta\bar{f}(-x)\,, \tag{3.112a}$$

$$\Delta_T\bar{f}(x) = -\Delta_T\bar{f}(-x)\,. \tag{3.112b}$$

Such a procedure is, in principle, not correct (although it may occasionally work in practice). The reason, as was explained in Sec. 3.3, is that there are *semi-connected* diagrams that contribute to the distributions for $x < 0$, whereas in computing the quark distribution functions in the physical region only *connected* diagrams should be considered (indeed as stressed by Jaffe, 1983, this *defines* the parton model).

Other models that have been used to compute transversity distributions include: the Manohar–Georgi chiral quark model (Suzuki and Weise, 1998), light-cone models with Melosh rotation (Ma, Schmidt and Soffer, 1998; Ma and Schmidt, 1998), the hypercentral potential model (Cano, Faccioli and Traini, 2000), and spectator diquark models (Jakob, Mulders and Rodrigues, 1997; Suzuki and Shigetani, 1997).

3.11.2 *Calculations of the tensor charges*

Tensor charges have been evaluated in the models mentioned in the previous section and also by means of various non-perturbative tools, such as QCD sum rules (He and Ji, 1995, 1996; Jin and Tang, 1997) and lattice simulations (Aoki *et al.*, 1997; Brower *et al.*, 1997; Göckeler *et al.*, 1997; Capitani *et al.*, 1999; Dolgov *et al.*, 2001). An estimate of δq based on axial-vector dominance has also been recently provided by Gamberg and Goldstein (2001).

Some of the results for the axial (Δq) and tensor charges (δq) are collected in Table 3.2. To allow a homogeneous comparison, the tensor charges have been evolved from the model scales Q_0^2 to $Q^2 = 10\,\mathrm{GeV}^2$ in LO QCD. Given the very low input scales, the result of such an evolution should be taken with caution (indeed, it only serves to provide a qualitative idea of the trend). Unfortunately, Q_0 has not been evaluated in the same manner in all models. As stated above, a possible method for estimating the model scale is to choose it in such a manner that, by starting from the computed

Table 3.2 The axial and tensor charges as calculated in various models. Tensor charges evolved in LO QCD from the intrinsic scale of the model (Q_0^2) up to $Q^2 = 10\,\mathrm{GeV}^2$ are also shown. The references are: $\diamond$ (Jaffe and Ji, 1992); $\oplus$ (Barone, Calarco and Drago, 1997b); $\times$ (Kim, Polyakov and Goeke, 1996); + (Wakamatsu and Kubota, 1999); $\otimes$ (Suzuki and Weise, 1998); $\circ$ (Schmidt and Soffer, 1997); $*$ (Jakob, Mulders and Rodrigues, 1997); and $\triangleright$ (Fukugita *et al.*, 1995; Aoki *et al.*, 1997). From Barone, Drago and Ratcliffe (2002).

Model	Δu	Δd	δu	δd	Q_0[GeV]	$\delta u(Q^2)$	$\delta d(Q^2)$
NRQM $\star$	1.33	-0.33	1.33	-0.33	0.28	0.97	-0.24
MIT $\diamond$	0.87	-0.22	1.09	-0.27	0.87	0.99	-0.25
CDM $\oplus$	1.08	-0.29	1.22	-0.31	0.40	0.99	-0.25
CQSM1 $\times$	0.90	-0.48	1.12	-0.42	0.60	0.97	-0.37
CQSM2 +	0.88	-0.53	0.89	-0.33	0.60	0.77	-0.29
CQM $\otimes$	0.65	-0.22	0.80	-0.15	0.80	0.72	-0.13
LC $\circ$	1.00	-0.25	1.17	-0.29	0.28	0.85	-0.21
Spect. $*$	1.10	-0.18	1.22	-0.25	0.25	0.83	-0.17
Latt. $\triangleright$	0.64	-0.35	0.84	-0.23	1.40	0.80	-0.22

value of the momentum fraction carried by the valence, and evolving it to larger Q^2, the experimentally observed value is reproduced. This procedure, albeit with slight differences, has been adopted to find the intrinsic scales in the non-relativistic quark model (NRQM) (Scopetta and Vento, 1998), the MIT bag model (Jaffe and Ross, 1980; Stratmann, 1993), the CDM (Barone and Drago, 1993; Barone, Drago and Fiolhais, 1994; Barone, Calarco and Drago, 1997b), and the spectator model (Meyer and Mulders, 1991). For the light-cone (LC) model of Schmidt and Soffer (1997) it is reasonable to take the same scale as for the NRQM, since the starting point of that calculation is just given by the rest-frame spin distributions of quarks. In other calculations, particularly in chiral models, the authors have chosen Q_0 as the scale up to which the model is expected to incorporate the relevant degrees of freedom.

Such a variety of procedures adopted to determine Q_0 introduces a further element of uncertainty into the results for $\delta q(Q^2)$ presented in Table 3.2. The QCD-evolved tensor charges are also displayed in Fig. 3.12 for a more direct comparison. As can be seen, they approximately span the

ranges

$$\delta u \sim 0.7 - 1.0\,, \quad \delta d \sim -(0.1 - 0.4) \quad \text{at} \quad Q^2 = 10\,\text{GeV}^2\,. \tag{3.113}$$

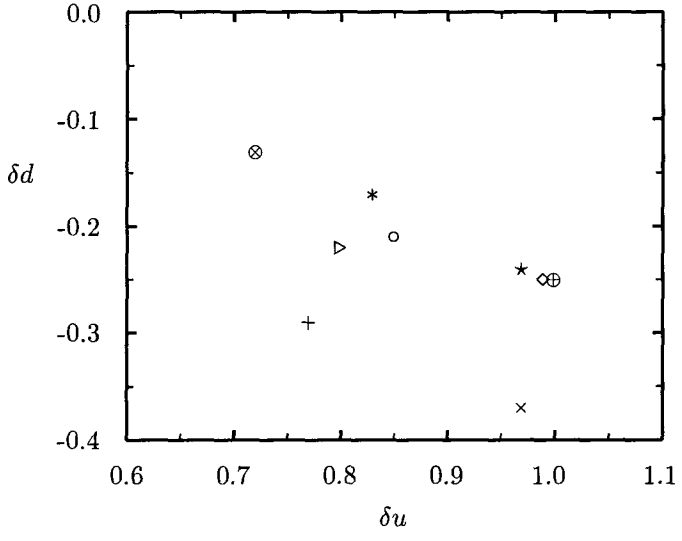


Fig. 3.12 The tensor charges, as computed in various models and evolved to a common scale, $Q^2 = 10\,\text{GeV}^2$. The symbols are as in Table 3.2. For clarity, the MIT and CDM points have been displaced slightly.

3.11.3 *Models for g_2*

An important feature of g_2 on which models may help to shed some light (or at least provide useful indications) is the magnitude of its twist-three component. As already discussed in Sec. 2.9, the available data are not conclusive in this respect, especially in the region of low-x.

In the unprojected MIT bag model the g_T distribution, which builds up g_2, is given by (Jaffe and Ji, 1991a)

$$g_T^u(x) = \frac{4}{3} G_T(x)\,, \qquad g_T^d(x) = -\frac{1}{3} G_T(x)\,, \tag{3.114}$$

with

$$G_T(x) = \frac{\omega_n M R}{2\pi(\omega_n - 1) j_0^2(\omega_n)} \int_{|y_{\min}|}^{\infty} \mathrm{d}y\, y \left[t_0^2 - t_1^2 \left(\frac{y_{\min}}{y} \right)^2 \right]. \qquad (3.115)$$

The transverse-spin structure function g_2 then reads

$$\begin{aligned} g_2(x) &= \tfrac{1}{2} \sum_a e_a^2 g_T^a(x) - \tfrac{1}{2} \sum_a e_a^2 \Delta f_a(x) \\ &= \tfrac{5}{18} \left[G_T(x) - G(x) \right]. \end{aligned} \qquad (3.116)$$

The twist-two and twist-three components of g_2 can be separated by means of (2.135, 2.136). The result is shown in Fig. 3.13 (the dot–dashed lines labelled MIT).

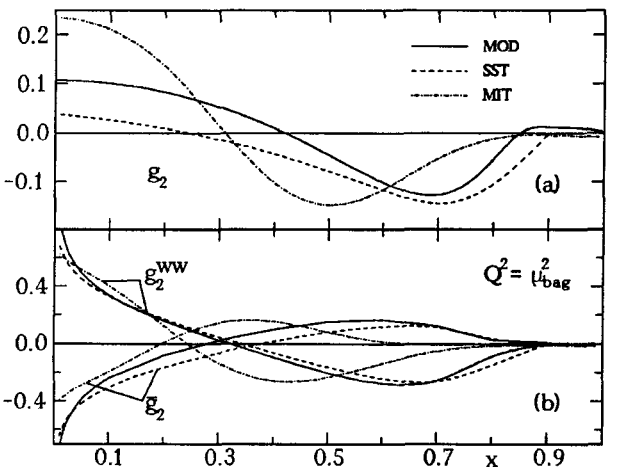


Fig. 3.13 The g_2 structure function in the MIT bag model. The curves labelled MIT are the unprojected results of Jaffe and Ji (1991a). The curves labelled SST and MOD are the results of Stratmann (1993), corresponding to two different methods of repairing the valence-number sum-rule violation.

In the more sophisticated approach, where translational invariance is restored via a Peierls–Yoccoz projection, eqs (3.114, 3.115) then become,

cf. eqs. (3.104b, 3.105),

$$g_T(x) = \sum_\alpha \sum_m P(f,\alpha,m)\,(-1)^{(m+\frac{3}{2}+i_\alpha)}\,G_\alpha^T(x)\,, \tag{3.117}$$

where

$$G_\alpha^T(x) = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3(2p^0)}\,A_\alpha(p)\,\delta\big((1-x)P^+ - p^+\big) \times \frac{1}{2}\left[u^2(p) - v^2(p)\left(\frac{p_z}{|\boldsymbol{p}|}\right)^2\right]. \tag{3.118}$$

A calculation of (3.118) within the MIT bag model has been performed by Stratmann (1993). These results are shown in Fig. 3.13. As one can see, g_2 starts out positive at low x, crosses zero at intermediate x and then finally becomes negative for large values of x. Note that the twist-two and twist-three contributions are comparable in size and specular to each other. The evolution of g_2 (computed using the large-N_c simplified approach of Ali, Braun and Hiller (1991) for $\bar{g}_2$ – see Sec. 5.7) is illustrated in Fig. 3.14.

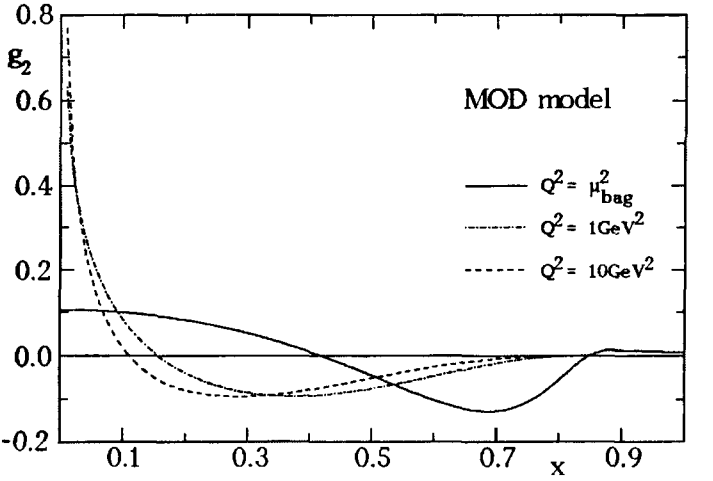


Fig. 3.14 Evolution of g_2, taken from Stratmann (1993).

Another version of the bag model (known as the modified centre-of-mass bag model) has been used by Song and McCarthy (1994) and Song

(1996) to investigate the spin structure of the nucleon. In this approach the g_2 structure function is found to be sensibly smaller, compared to the results of the original MIT bag model. On the other hand, the twist-three component turns out to be larger than in other models (see below for a comparison of the values of d_3).

Predictions for g_2 within the chiral quark–soliton model have been offered by Weigel, Gamberg and Reinhardt (1997) and by Wakamatsu (2000). In Fig. 3.15 we show the results of the Tübingen group, based on the valence approximation.

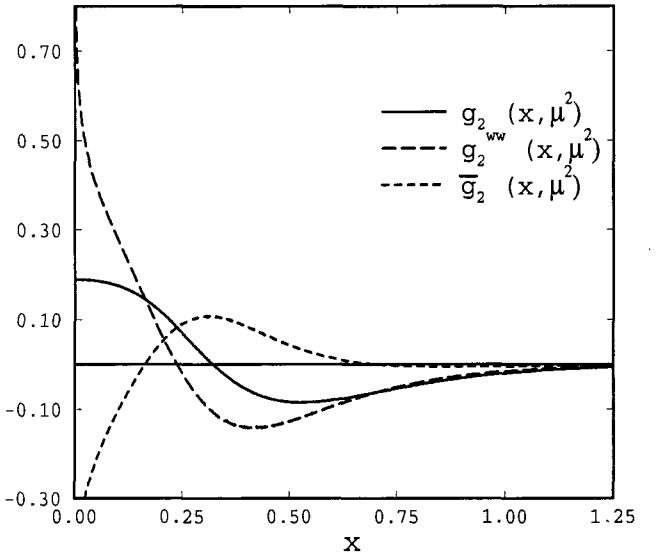


Fig. 3.15 The valence-quark approximation for g_2 in the chiral quark–soliton model of Weigel, Gamberg and Reinhardt (1997).

As can be seen from Figs. 2.11 and 2.12 in Sec. 2.9, all models roughly reproduce the available data on g_2 (which are not, however, very accurate). Should the more precise, though still preliminary, E155x data (Fig. 2.12) be confirmed, only the modified centre-of-mass bag model of Song (1996), which predicts a very small g_2 (the dotted curves in Fig. 2.12), would seem to be ruled out.

The parameter quantifying the size of the twist-three part of g_2 is the reduced matrix element d_3, introduced in Eq. (2.115b). In Fig. 3.16 the

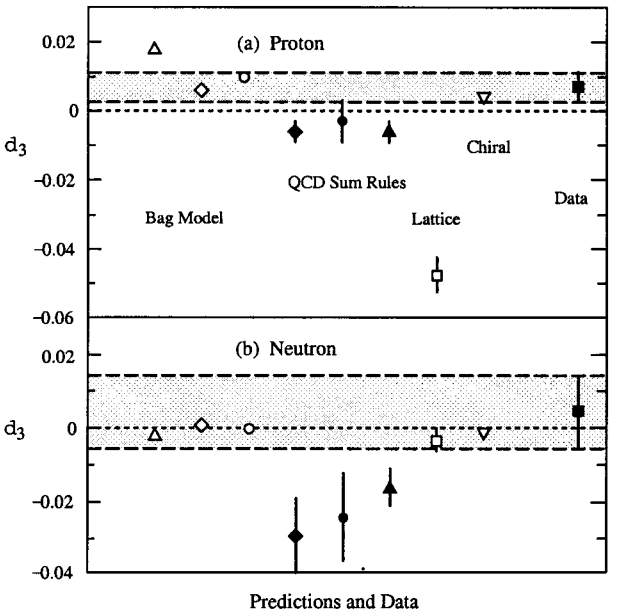


Fig. 3.16 The reduced matrix element d_3. The shaded areas represent the data with their uncertainties. A number of non-perturbative and model predictions are also shown: from left to right, bag models (Song, 1996; Stratmann, 1993; Ji and Unrau, 1994), QCD sum rules (Stein *et al.*, 1995; Balitsky, Braun and Kolesnichenko, 1990; Ehrnsperger and Schäfer, 1995), lattice QCD (Göckeler *et al.*, 1996), chiral quark–soliton model (Weigel, Gamberg and Reinhardt, 1997). Figure from Anthony *et al.* (1999).

model predictions for d_3 together with the results of other non-perturbative approaches are collected and compared to the data. Except for the lattice point[c], all predictions for proton lie quite close to the data and would thus suggest that the twist-three component is very small.

[c]Note that Göckeler *et al.* (2001) have recently improved their previous lattice calculation, obtaining a value for d_3 consistent with the data.

Chapter 4

The QCD evolution of transversity

As well-known, the principal effect of QCD on the naïve parton model is to induce, via renormalisation, a (logarithmic) dependence on Q^2 (Gross and Wilczek, 1973, 1974; Georgi and Politzer, 1974), the energy scale at which the distributions are defined or, in other words, the resolution with which they are measured. The two techniques with which we shall exemplify the following discussions of this dependence and of the general calculational framework are the renormalisation-group equation (RGE—Stueckelberg and Petermann, 1953; Gell-Mann and Low, 1954) applied to the operator-product expansion (OPE) – providing a solid formal basis – and the ladder-diagram summation approach (Craigie and Jones, 1980; Dokshitzer, Diakonov and Troian, 1980) – providing a physically more intuitive picture. The variation of the distributions as functions of energy scale may be expressed in the form of the standard Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP—Gribov and Lipatov, 1972a; Lipatov, 1974; Altarelli and Parisi, 1977; Dokshitzer, 1977) so-called evolution equations.

Further consequences of higher-order QCD are mixing and, beyond the leading-logarithmic approximation (LLA), eventual scheme ambiguity in the definitions of the various quark and gluon distributions; *i.e.*, the densities lose their precise meaning in terms of real physical probability and require further conventional definition. In this and the following chapter we shall discuss the present status with respect to QCD evolution of the two aspects of transverse polarisation presented in this book: the transversity dependent parton densities $\Delta_T q$ and the DIS structure function g_2. As should already be quite clear these are two distinct aspects of transverse

polarisation (although QCD does bring about some connection) and thus require separate treatments. Transversity, although a relative newcomer to the panorama of parton phenomenology, being a twist-two effect is far simpler to treat in perturbative QCD and it is with this that we open the discussion of higher-order corrections.

In this chapter then we shall examine the Q^2 evolution of the transversity distribution at LO and NLO. In particular, we shall compare its evolution with that of both the unpolarised and helicity-dependent distributions. Such a comparison is especially relevant to the Soffer inequality (Soffer, 1995), which involves all three types of distribution, see Sec. 3.6. The chapter closes with a detailed examination of the question of parton-density positivity and the so-called generalised positivity bounds (of which Soffer's is then just one example).

4.1 The renormalisation-group equation

In order to establish our conventions for the definition of operators and their renormalisation *etc.*, it will be useful to briefly recall (with a somewhat pedagogical eye) the RGE as applied to the OPE in QCD. Before doing so let us make two remarks related to the problem of scheme dependence. Firstly, in order to lighten the notation, where applicable and unless otherwise stated, all expressions will be understood to refer to the so-called modified minimal subtraction ($\overline{\text{MS}}$) scheme, see App. E. A further complication that arises in the case of polarisation at NLO is the extension of γ_5 to $D \neq 4$ dimensions (Akyeampong and Delbourgo, 1974; Breitenlohner and Maison, 1977; Chanowitz, Furman and Hinchliffe, 1979). An in-depth discussion of this problem is beyond the scope of the present review and the interested reader is referred to the paper by Kamal (1996), where it is also considered in the context of transverse polarisation.

For a generic Green function G, the scale independent so-called bare, G_{bare}, and renormalised, $G_{\text{ren}}(\mu^2)$, Green functions are related via a renormalisation constant $Z(\mu^2)$, where μ is the scale at which the renormalisation procedure is carried out:

$$G_{\text{ren}}(\mu^2) = Z^{-1}(\mu^2)\, G_{\text{bare}}\,. \tag{4.1}$$

The crucial observation is that G^B, while depending on the regulator (*e.g.*, Λ or ϵ), does not depend on μ, which is only introduced when Z, $G_{\text{ren}}(\mu^2)$,

general vertices and hence α_s are defined according to the chosen renormalisation scheme.

The scale dependence of $G_{\rm ren}(\mu^2)$ is then obtained by solving the RGE, which expressed in terms of the QCD coupling constant $\alpha_s = g^2/4\pi$ is

$$\frac{\mathrm{d}G_{\rm bare}}{\mathrm{d}\ln\mu^2} = \frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}[ZG_{\rm ren}] = 0\,, \tag{4.2}$$

or

$$Z\left[\frac{\partial}{\partial\ln\mu^2} + \frac{\partial\alpha_s}{\partial\ln\mu^2}\frac{\partial}{\partial\alpha_s} + \frac{1}{Z}\frac{\partial Z}{\partial\ln\mu^2}\right]G_{\rm ren} = 0\,. \tag{4.3}$$

Introducing the RGE β-function, which governs the renormalisation scale dependence of the effective QCD coupling constant $\alpha_s(\mu^2)$,

$$\beta(\alpha_s) = \frac{\partial\alpha_s(\mu^2)}{\partial\ln\mu^2}\,, \tag{4.4}$$

and the anomalous dimension for the Green function $G_{\rm ren}(\mu^2)$ defined by

$$\gamma(\alpha_s) = \frac{\partial\ln Z}{\partial\ln\mu^2}\,, \tag{4.5}$$

this can be rewritten as

$$\left[\frac{\partial}{\partial\ln\mu^2} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s} + \gamma(\alpha_s)\right]G_{\rm ren} = 0\,. \tag{4.6}$$

Note that $\beta(\alpha_s)$ does not depend on which Green function G is under consideration, but it is a property of the theory and the renormalisation scheme adopted, while $\gamma(\alpha_s)$ does depend on G. Indeed, the so-called *running* coupling constant $\alpha_s(t)$ is thus *defined* by the integral relation

$$t \equiv \ln\frac{Q^2}{\mu^2} = \int_{\alpha_s(\mu^2)}^{\alpha_s(Q^2)} \frac{\mathrm{d}\alpha_s'}{\beta(\alpha_s')}\,, \tag{4.7}$$

where, since the dependence on μ^2 and/or Q^2 is logarithmic, a new variable t has been introduced. The trick, via this change of variable, is then simply to transform dependence on the renormalisation scale μ^2 into dependence on the process scale Q^2.

Let us then apply the RGE to some hard process at a large scale[a] Q, related to a Green function G. Without loss of generality, G may always

[a]By large scale one does, of course, mean a scale at which α_s is sufficiently small that perturbative QCD should be applicable.

be taken as dimensionless since any dimensionality can always be absorbed into a suitable power of Q. Thus, one may write $G_{\rm ren} \equiv F(t, \alpha_s)$, where possible trivial dependence on (dimensionless) scaling variables has been suppressed. In the naïve scaling limit F would be independent of t. To find the scale-breaking dependence, one must solve the RGE, which now becomes

$$\left[-\frac{\partial}{\partial t} + \beta(\alpha_s)\frac{\partial}{\partial \alpha_s} + \gamma(\alpha_s)\right] G_{\rm ren} = 0\,, \tag{4.8}$$

with initial condition at $t = 0$ (or equivalently at $Q^2 = \mu^2$): $G_{\rm ren}(0) = F(0, \alpha_s)$. One can easily confirm by substitution that the solution to Eq. (4.8) is given by

$$F[t, \alpha_s(0)] = F[0, \alpha_s(t)] \exp \int_{\alpha_s(0)}^{\alpha_s(t)} \frac{\gamma(\alpha_s')}{\beta(\alpha_s')}\, \mathrm{d}\alpha_s'\,. \tag{4.9}$$

Two points should be observed in this so-called renormalisation-group improved solution: firstly there is the simple but fundamental correction applied by evaluating α_s at Q^2, the scale of the process, and then a Green function dependent piece involving the anomalous dimension γ, which is indeed a sort of anomalous scale dependence, induced by radiative corrections.

The anomalous dimension γ and the QCD β-function both have perturbative expansions, at NLO accuracy the two series are

$$\gamma(\alpha_s) = \frac{\alpha_s}{2\pi}\,\gamma^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \gamma^{(1)} + \mathcal{O}(\alpha_s^3)\,, \tag{4.10}$$

$$\beta(\alpha_s) = -\alpha_s\left[\frac{\alpha_s}{4\pi}\,\beta_0 + \left(\frac{\alpha_s}{4\pi}\right)^2 \beta_1 + \mathcal{O}(\alpha_s^3)\right], \tag{4.11}$$

note that while γ starts at $\mathcal{O}(\alpha_s)$ (*i.e.*, its effects are naturally absent in free field theory), the β-function actually starts at $\mathcal{O}(\alpha_s^2)$. This is reflected in the fact that the first and first two coefficients of γ and β respectively are renormalisation-scheme independent. Explicitly, the first two coefficients of the β-function are:

$$\beta_0 = \frac{11}{3}C_{\rm G} - \frac{4}{3}T_{\rm F} = 11 - \frac{2}{3}N_{\rm f} \tag{4.12a}$$

and

$$\beta_1 = \frac{2}{3}(17C_{\rm G}^2 - 10C_{\rm G}T_{\rm F} - 6C_{\rm F}T_{\rm F}) = 102 - \frac{38}{3}N_{\rm f}\,, \tag{4.12b}$$

where $C_G = N_c$ and $C_F = (N_c^2 - 1)/2N_c$ are the usual Casimirs related respectively to the gluon and the fermion representations of the colour symmetry group, SU(N_c), and $T_F = \frac{1}{2}N_f$, for active quark flavour number N_f. This leads to the following NLO expression for the QCD coupling constant:

$$\frac{\alpha_s(Q^2)}{4\pi} \simeq \frac{1}{\beta_0 \ln Q^2/\Lambda^2}\left[1 - \frac{\beta_1}{\beta_0{}^2}\frac{\ln\ln Q^2/\Lambda^2}{\ln Q^2/\Lambda^2}\right], \tag{4.13}$$

where Λ is the so-called QCD scale parameter.

A generic observable derived from some operator O may be defined by $f(Q^2) \sim \langle PS\,|O(Q^2)|\,PS\rangle$. Inserting the above expansions into (4.9), the NLO evolution equation for $f(Q^2)$ is then obtained (note that the equations apply directly to O if it already represents a physical observable):

$$\frac{f(Q^2)}{f(\mu^2)} = \left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right)^{-\frac{2\gamma^{(0)}}{\beta_0}}\left[\frac{\beta_0 + \beta_1\alpha_s(Q^2)/4\pi}{\beta_0 + \beta_1\alpha_s(\mu^2)/4\pi}\right]^{-\left(\frac{4\gamma^{(1)}}{\beta_1} - \frac{2\gamma^{(0)}}{\beta_0}\right)}, \tag{4.14}$$

which, to NLO accuracy, may be expanded thus

$$\simeq \left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right)^{-\frac{2\gamma^{(0)}}{\beta_0}}\left[1 + \frac{\alpha_s(\mu^2) - \alpha_s(Q^2)}{\pi\beta_0}\left(\gamma^{(1)} - \frac{\beta_1}{2\beta_0}\gamma^{(0)}\right)\right]. \tag{4.14'}$$

In order to obtain physical hadronic cross-sections at NLO level, the NLO contribution to $f(Q^2)$ has then to be combined with the NLO contribution to the relevant hard partonic cross-section; indeed, it is only this combination that is fully scheme independent.

It turns out that the quantities typically measured experimentally (*i.e.*, cross-sections, DIS structure functions or, more simply, quark distributions) are related via Mellin-moment transforms to expectation values of composite quark and gluon field operators. The definition we adopt for the Mellin transform of structure functions, anomalous dimensions *etc.* is as follows:[b,c]

$$f(n) = \int_0^1 \mathrm{d}x\, x^{n-1} f(x)\,. \tag{4.15}$$

[b]The definition with $n-1$ replaced by n is also found in the literature.

[c]We choose to write n as an argument to avoid confusion with the label indicating perturbation order.

We may also define the Altarelli–Parisi (AP—Altarelli and Parisi, 1977) splitting function, $P(x)$, as precisely the function of which the Mellin moments, Eq. (4.15), are just the anomalous dimensions,

$$\gamma(n) = \int_0^1 \mathrm{d}x\, x^{n-1} P(x)\,. \tag{4.16}$$

Note that $P(x)$ may be expanded in powers of the QCD coupling constant in a manner analogous to γ and therefore also depends on Q^2. In x space the evolution equations may be written in the following schematic form:

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2} f(x,Q^2) = P(x,Q^2) \otimes f(x,Q^2)\,, \tag{4.17}$$

where the symbol $\otimes$ stands for a convolution in x,

$$g(x) \otimes f(x) = \int_x^1 \frac{\mathrm{d}y}{y}\, g\left(\frac{x}{y}\right) f(y)\,, \tag{4.18}$$

which becomes a simple product in Mellin-moment space. With these expressions it is then possible to perform numerical evolution either via direct integration of (4.17) using suitable parametric forms to fit data, or in the form of (4.14′) via inversion of the Mellin moments.

The operators governing the twist-two[d] evolution of moments of the f, Δf and $\Delta_T f$ distributions are[e]

$$O(n) = \mathcal{S}\,\bar{\psi}\gamma^{\mu_1}\mathrm{i}D^{\mu_2}\cdots\,\mathrm{i}D^{\mu_n}\psi\,, \tag{4.19a}$$

$$O^L(n) = \mathcal{S}\,\bar{\psi}\gamma_5\gamma^{\mu_1}\mathrm{i}D^{\mu_2}\cdots\,\mathrm{i}D^{\mu_n}\psi\,, \tag{4.19b}$$

$$O^T(n) = \mathcal{S}'\,\bar{\psi}\gamma_5\gamma^{\sigma}\gamma^{\mu_1}\mathrm{i}D^{\mu_2}\cdots\,\mathrm{i}D^{\mu_n}\psi\,, \tag{4.19c}$$

where the symbol $\mathcal{S}$ conventionally indicates symmetrisation over the indices $\mu_1, \mu_2, \ldots \mu_n$ while the symbol $\mathcal{S}'$ indicates simultaneous *anti*symmetrisation over the indices σ and μ_1 and symmetrisation over the indices $\mu_1, \mu_2, \ldots \mu_n$.

[d]There are, of course, possible higher-twist contributions too, but we shall ignore these here.

[e]All composite operators appearing herein are to be considered implicitly traceless.

4.2 QCD evolution at leading order

The Q^2 evolution coefficients for f_1 and g_1 at leading order have long been known while the LO Q^2 evolution for h_1 was first specifically presented by Baldracchini *et al.* (1981) in a ground-breaking paper on spin properties in QCD, which unfortunately went virtually unnoticed for many years. The reason for the neglect was, for course, the experimental inaccessibility of transversity in the standard hard scattering processes then available. Nearly ten years later Artru and Mekhfi (1990) independently repeated the calculation. However, the first calculations of the one-loop anomalous dimensions related to the operators governing the evolution of $\Delta_T q(x, Q^2)$ date back, in fact, to early (though incomplete) work on the evolution of the DIS transverse-spin structure function g_2 (Kodaira *et al.*, 1979), which, albeit in an indirect manner, involves the operators of interest here. Mention (though again incomplete) is also made by Antoniadis and Kounnas (1981). Following this, and with various approaches, the complete derivation of the complex system of evolution equations for the twist-three operators governing g_2 was presented by Bukhvostov, Kuraev and Lipatov (1983b), Bukhvostov *et al.* (1985), and Ratcliffe (1986). We shall, of course, present a detailed discussion of g_2 and its QCD evolution in the following chapter; suffice it to say here that among the operators mixing with the leading contributions one finds the following:

$$O^m(n) = \mathcal{S}' m_q \bar{\psi} \gamma_5 \gamma^{\mu_1} \gamma^{\mu_2} \mathrm{i}D^{\mu_3} \cdots \mathrm{i}D^{\mu_n} \psi \,, \tag{4.20}$$

where m_q is the (current) quark mass. It is immediately apparent that this is none other than the twist-two operator responsible for $\Delta_T q(x)$, multiplied here however by a quark mass and thereby rendered twist three—its evolution is, of course, identical to the twist-two version.

For reasons already mentioned, see for example Eq. (3.42) and the discussion following it, calculation of the anomalous dimensions governing $\Delta_T q(x)$ turns out to be surprisingly simpler than for the other twist-two structure functions, owing to its peculiar chirally-odd properties. Indeed, as we have seen, the gluon field cannot contribute at LO in the case of spin-half hadrons as it would require helicity flip of two units in the corresponding hadron–parton amplitude. Thus, in the case of baryons the evolution is of a purely non-singlet (NS) type. Note that this is no longer the case for targets of spin greater than one half and, as pointed out by Artru and Mekhfi (1990), a separate contribution due to linear gluon polar-

isation is possible; we shall also consider the situation for spin-one mesons and/or indeed photons in what follows.

Using dimensional regularisation (DR), see App. E, calculation of the anomalous dimensions requires evaluation of the $1/\epsilon$ poles in the diagrams depicted in Fig. 4.1 (recall that at this order there is no scheme dependence). Although not present in baryon scattering, the linearly polarised gluon

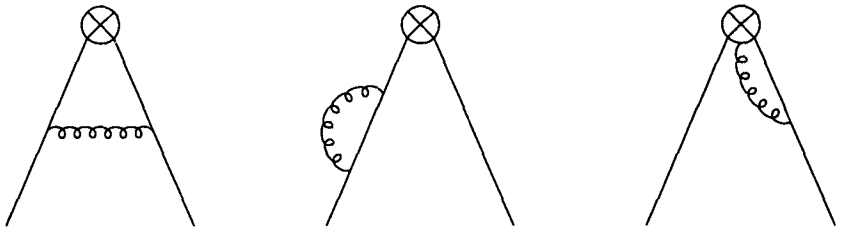


Fig. 4.1 Example one-loop diagrams contributing to the $\mathcal{O}(\alpha_s)$ anomalous dimensions of $\Delta_T q$.

distribution, $\Delta_T g$, can contribute to scattering involving polarised spin-one mesons. Thus, to complete the discussion of LO evolution, we include here too the anomalous dimensions for this density. For the three cases that are diagonal in parton type in the fermionic sector (spin-averaged and helicity-weighted, Altarelli and Parisi, 1977; and transversity, Artru and Mekhfi, 1990) one finds in Mellin-moment space:

$$\gamma_{qq}(n) = C_{\mathrm{F}} \left[\frac{3}{2} + \frac{1}{n(n+1)} - 2 \sum_{j=1}^{n} \frac{1}{j} \right], \tag{4.21a}$$

$$\Delta\gamma_{qq}(n) = \gamma_{qq}(n), \tag{4.21b}$$

$$\begin{aligned} \Delta_T\gamma_{qq}(n) &= C_{\mathrm{F}} \left[\frac{3}{2} - 2 \sum_{j=1}^{n} \frac{1}{j} \right] \\ &= \gamma_{qq}(n) - C_{\mathrm{F}} \frac{1}{n(n+1)}. \end{aligned} \tag{4.21c}$$

The equality expressed in Eq. (4.21b) is a direct consequence of fermion-helicity conservation by purely vector interactions in the limit of negligible fermion mass. The three cases diagonal in the gluonic sector (spin-averaged

and helicity-weighted, Altarelli and Parisi, 1977; and linear gluon polarisation, Artru and Mekhfi, 1990) are

$$\gamma_{gg}(n) = C_{\mathrm{G}}\left[\frac{1}{n-1}+\frac{1}{n}-\frac{2}{n+1}-2\sum_{j=1}^{n}\frac{1}{j}\right]+\frac{\beta_0}{2}, \tag{4.22a}$$

$$\Delta\gamma_{gg}(n) = C_{\mathrm{G}}\left[\frac{4}{n}-\frac{4}{n+1}-2\sum_{j=1}^{n}\frac{1}{j}\right]+\frac{\beta_0}{2}, \tag{4.22b}$$

$$\Delta_T\gamma_{gg}(n) = C_{\mathrm{G}}\left[-2\sum_{j=1}^{n}\frac{1}{j}\right]+\frac{\beta_0}{2}. \tag{4.22c}$$

As well known, the first moment of $\Delta\gamma_{gg}$ is just $\beta_0/2$. Thus, the first moment of Δg grows as $1/\alpha_s$; that of $\Delta_T g$ grows more slowly, while that of g grows faster (although it does, of course, usually diverge) essentially owing to the singularity at $n = 1$. However, all distributions evolve similarly in the region $x \to 1$.

The first point to stress is the commonly growing negative value, for increasing n, indicative of the tendency of all the x-space distributions to migrate towards $x = 0$ with increasing Q^2. In other words, evolution has a degrading effect on the densities. Secondly, in contrast to the behaviour of both q and Δq, the anomalous dimensions governing $\Delta_T q$ do *not* vanish for $n = 0$ and hence there is no sum rule associated with the tensor charge (Jaffe and Ji, 1992). Moreover, comparison to Δq reveals that $\Delta_T\gamma_{qq}(n) < \Delta\gamma_{qq}(n)$ for all n. This implies that for (hypothetically) identical starting distributions (*i.e.*, $\Delta_T q(x,Q_0^2) = \Delta q(x,Q_0^2)$), $\Delta_T q(x,Q^2)$ will fall more rapidly than $\Delta q(x,Q^2)$ everywhere in x with increasing Q^2. We shall return in more detail to this point in Sec. 4.6.

For completeness, we also present the AP splitting functions (*i.e.*, the inverse Mellin transforms of the anomalous dimensions). For the purely fermionic sector one has

$$P_{qq}^{(0)} = C_{\mathrm{F}}\left(\frac{1+x^2}{1-x}\right)_+, \tag{4.23a}$$

$$\Delta P_{qq}^{(0)} = P_{qq}^{(0)}, \tag{4.23b}$$

$$\Delta_T P_{qq}^{(0)} = C_{\mathrm{F}} \left[\left(\frac{1+x^2}{1-x} \right)_+ - 1 + x \right]$$
$$= P_{qq}^{(0)}(x) - C_{\mathrm{F}}(1-x)\,, \tag{4.23c}$$

where the "plus" regularisation prescription is defined in App. F in Eq. (F.1) and we also have made use of the identity given in Eq. (F.2). Naturally, the plus prescription is to be ignored when multiplying functions that vanish at $x = 1$. Once again, the inequality $\Delta_T P_{qq}^{(0)} < \Delta P_{qq}^{(0)}$ is manifest for all $x < 1$, indeed, one has

$$\tfrac{1}{2}\left[P_{qq}^{(0)}(x) + \Delta P_{qq}^{(0)}(x) \right] - \Delta_T P_{qq}^{(0)}(x) = C_{\mathrm{F}}(1-x) \geq 0\,. \tag{4.24}$$

The purely gluonic kernels are

$$P_{gg}^{(0)} = C_{\mathrm{G}} \left[\frac{2x}{(1-x)_+} + 2x(1-x) + \frac{(1-x)}{x} \right] + \frac{\beta_0}{2}\,\delta(x-1)\,, \tag{4.25a}$$

$$\Delta P_{gg}^{(0)} = C_{\mathrm{G}} \left[\frac{2x}{(1-x)_+} + 4(1-x) \right] + \frac{\beta_0}{2}\,\delta(x-1)\,, \tag{4.25b}$$

$$\Delta_T P_{gg}^{(0)} = C_{\mathrm{G}} \left[\frac{2x}{(1-x)_+} \right] + \frac{\beta_0}{2}\,\delta(x-1)\,. \tag{4.25c}$$

Note that the essential difference between the polarised and unpolarised kernels lies in the presence of the term $1/x$.

The non-mixing of the transversity distributions for quarks, $\Delta_T q$, and gluons, $\Delta_T g$, is afforded a physical demonstration via the ladder-diagram summation technique. This approach rests on the observation that in certain physical gauges (axial or light-like gauges) the diagrammatic structure order-by-order is greatly simplified. It can be shown (Craigie and Jones, 1980; Dokshitzer, Diakonov and Troian, 1980) that by using a gauge of the type $A{\cdot}n = 0$ with n a light-like vector, usually chosen as in Eq. (B.1b), at leading twist all non-planar diagrams are suppressed and only simple ladder-type diagrams contribute in the LLA. This then permits the construction of a simple Bethe–Salpeter type equation governing the Q^2 evolution of the distributions, see Fig. 4.2.

In Fig. 4.3 the general LO one-particle irreducible (1PI) kernels are displayed. If the four external lines are all quarks (*i.e.*, a gluon rung, see Fig. 4.3a), the kernel is clearly diagonal (in parton type) and therefore con-

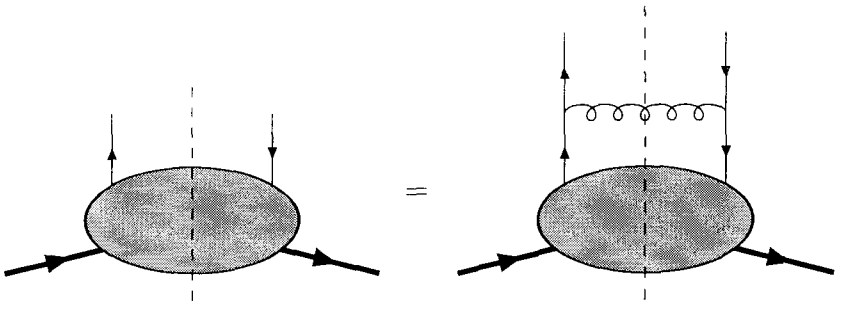


Fig. 4.2 The diagrammatic representation of the Bethe–Salpeter equation for the evolution of general twist-two quark distributions.

tributes to the evolution of $\Delta_T q$. For the case in which one pair of external lines are quarks and the other gluons (*i.e.*, a quark rung, see Figs. 4.3b and c), helicity conservation along the quark line in the chiral limit implies a vanishing contribution to transversity evolution. Likewise, the known properties of four-body amplitudes, namely t-channel helicity conservation, preclude any contribution that might mix the evolution of $\Delta_T q$ and $\Delta_T g$.

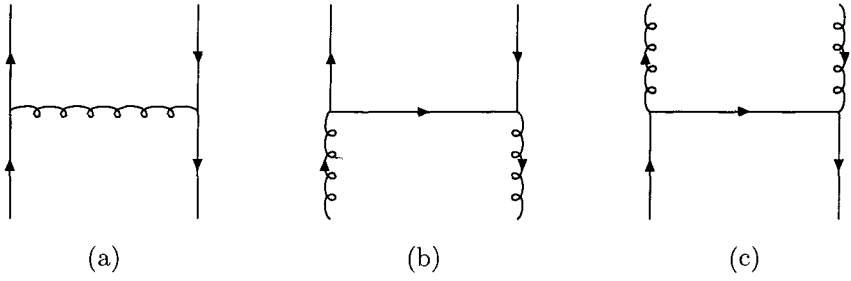


Fig. 4.3 The 1PI kernels contributing to the $\mathcal{O}(\alpha_s)$ evolution of $\Delta_T q$ in the axial gauge.

The same reasoning clearly holds at higher orders since the only manner for gluon and quark ladders to mix is via diagrams in which an incoming quark line connects to its Hermitian-conjugate partner. Thus, quark-helicity conservation in the chiral limit will always protect against such contributions.

Before continuing to NLO, a comment is in order here on a recent debate in the literature (Meyer-Hermann, Kuhn and Schützhold, 2001; Mukherjee and Chakrabarti, 2001; Blümlein, 2001) regarding the calculation of

the anomalous dimensions for h_1 and the validity of a certain approach. Meyer-Hermann, Kuhn and Schützhold attempted to calculate the anomalous dimensions relevant to h_1 exploiting a method by Ioffe and Khodjamirian (1995). The motivation was that use of so-called time-ordered or old-fashioned perturbation theory in the Weizsäcker–Williams approximation (von Weizsäcker, 1934; Williams, 1934), as adopted by, *e.g.*, Artru and Mekhfi (1990), encounters a serious difficulty: it is only applicable to the region $x < 1$ while the end-point ($x = 1$) contributions cannot be evaluated directly. Where there is a conservation law (*e.g.*, quark number), then appeal to the resulting sum rule allows indirect extraction at this point (the common δ-function contribution). In the case of transversity no such conserved quantum number exists and one might doubt the validity of such calculations. Indeed, the claim by Meyer-Hermann, Kuhn and Schützhold was that direct calculation, based on a dispersion relation approach, yielded a different result to that reported in Eq. (4.23c) above.

A priori, from a purely theoretical point of view, such an apparent discrepancy is hard to credit: were it real, it would imply precisely the type of ambiguity to which the singularity structure of the theory on the light cone is supposedly immune. Meyer-Hermann, Kuhn and Schützhold calculated the anomalous dimensions via the one-loop corrections to the Compton amplitude or classic handbag diagram (*e.g.*, Fig. 2.5 with on-shell external quark states and no lower hadronic blob). In order to mimic the required chiral structure, one of the upper vertices is taken to be $\gamma_5\gamma^\mu$ and the other γ^μ or $\mathbb{1}$ for g_1 or h_1 respectively. The results for g_1 are in agreement with other approaches while the anomalous dimensions for h_1 differ in the coefficient of the δ-function contribution. What immediately casts doubt on such findings is the fact that in a physical gauge, as used for example in the ladder-diagram summation approach (Dokshitzer, Diakonov and Troian, 1980; Craigie and Jones, 1980) mentioned earlier, precisely all such vertex corrections are in fact *absent* (to this order in QCD). Moreover, it is just this property that gives rise, in such an approach, to the universal short-distance behaviour, *independent* of the particular nature of the vertices involved.

Various cross checks of these potentially disturbing findings have been performed (Mukherjee and Chakrabarti, 2001; Blümlein, 2001) with the conclusion that the original calculations are after all correct. In particular, Blümlein (2001) provided a very thorough appraisal of the situation, in which, moreover, he uncovered a fatal conceptual oversight: the scalar

current is *not* conserved[f]. To appreciate the relevance of this observation let us briefly recall the salient points of the RGE approach (for more detail the reader is referred to Blümlein, 2001).

A product of two currents (as in the peculiar Compton amplitude under consideration) may be expanded as

$$j_1(z)j_2(0) = \sum_n C(n;z)\, O(n;0)\,, \tag{4.26}$$

where typically then $j_i = j_{V,A}$ (*i.e.*, vector or axial-vector currents) but here a scalar current j_S must be introduced. The RGE for the Wilson coefficients $C(n;z)$ is

$$\left[\mathcal{D} + \gamma_{j_1}(g) + \gamma_{j_2}(g) - \gamma_O(n;g)\right] C(n;z) = 0\,, \tag{4.27}$$

where $\gamma_{j_i}(g)$ and $\gamma_O(n;g)$ are the anomalous dimensions of the currents j_i and the composite operators $O(n)$ respectively, and (neglecting quark masses) the renormalisation group (RG) operator is defined as

$$\mathcal{D} = \mu^2 \frac{\partial}{\partial \mu^2} + \beta(g)\frac{\partial}{\partial g}\,. \tag{4.28}$$

Thus, the corrections to the Compton amplitude in the LLA have coefficients

$$\gamma_C(n;g) = \gamma_{j_1}(g) + \gamma_{j_2}(g) - \gamma_O(n;g)\,. \tag{4.29}$$

The point then is that while in the better-known spin-averaged and helicity-weighted cases $\gamma_{j_i} = 0$ for both currents (axial and/or vector), the scalar current necessary for the transversity case is *not* conserved and $\gamma_{j_S} \neq 0$. Thus, in contrast to the former, γ_C and $-\gamma_O$ do *not* coincide in the calculation for h_1. The apparent discrepancy signalled by Meyer-Hermann, Kuhn and Schützhold is due precisely then to the neglect of γ_{j_S}.

4.3 QCD evolution at next-to-leading order

Reliable QCD analysis of the sort of data samples we have come to expect in modern experiments requires full NLO accuracy. For this it is necessary to calculate both the anomalous dimensions to two-loop level and the

[f]We are particularly grateful to Johannes Blümlein for illuminating discussion on this point.

constant terms (*i.e.*, the part independent of $\ln Q^2$) of the so-called coefficient function (or hard-scattering process) at the one-loop level, together, of course, with the two-loop β-function.

The calculation of the two-loop anomalous dimensions for h_1 has been presented in three papers: Hayashigaki, Kanazawa and Koike (1997) and Kumano and Miyama (1997) used the minimal subtraction (MS) scheme in the Feynman gauge while Vogelsang (1998b) adopted the $\overline{\text{MS}}$ scheme in the light-cone gauge. These complement the earlier two-loop calculations for the two other better-known twist-two structure functions: f_1 (Floratos, Ross and Sachrajda, 1977, 1979; González-Arroyo, López and Ynduráin, 1979; Curci, Furmanski and Petronzio, 1980; Furmanski and Petronzio, 1980; Floratos, Lacaze and Kounnas, 1981a,b) and g_1 (Mertig and van Neerven, 1996; Vogelsang, 1996a,b). Such knowledge has been exploited in the past for the phenomenological parametrisation of f_1 (Glück, Reya and Vogt, 1995; Martin, Roberts and Stirling, 1995; Lai *et al.*, 1997) and g_1 (Glück *et al.*, 1996; Gehrmann and Stirling, 1996; Altarelli *et al.*, 1997) in order to perform global analyses of the experimental data; and will certainly be of value when the time comes to analyse data on transversity.

The situation at NLO is still relatively simple, as compared to the unpolarised or helicity-weighted cases. Examples of the relevant two-loop diagrams are shown in Fig. 4.4. It remains impossible for the gluon to contribute, for the reasons already given. The only complication is the usual mixing, possible at this level, between quark and antiquark distributions, for which quark helicity conservation poses no restriction since the quark and antiquark lines do not connect directly to one another, see Fig. 4.4d.

It is convenient to introduce the following combinations of quark transversity distributions (the $\pm$ subscript is not to be confused with helicity):

$$\Delta_T q_\pm(n) = \Delta_T q(n) \pm \Delta_T \bar{q}(n)\,, \tag{4.30a}$$

$$\Delta_T \widetilde{q}_+(n) = \Delta_T q_+(n) - \Delta_T q'_+(n)\,, \tag{4.30b}$$

$$\Delta_T \Sigma(n) = \sum_q \Delta_T q_+(n)\,, \tag{4.30c}$$

where q and q' represent quarks of differing flavours. The specific evolution equations may then be written as (*e.g.*, see Ellis and Vogelsang, 1996)

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2}\Delta_T q_-(n,Q^2) = \Delta_T \gamma_{qq,-}(n,\alpha_s(Q^2))\,\Delta_T q_-(n,Q^2)\,, \tag{4.31a}$$

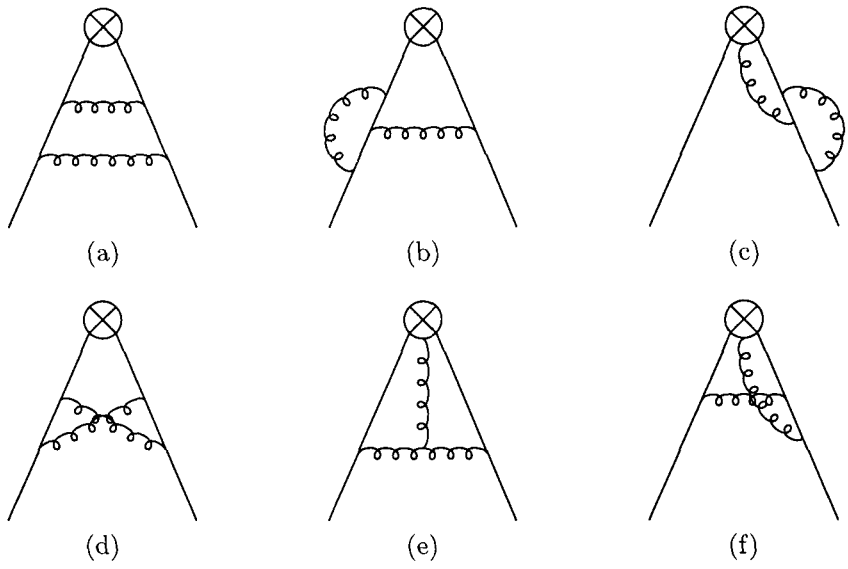


Fig. 4.4 Examples of two-loop diagrams contributing at $\mathcal{O}(\alpha_s^2)$ to the anomalous dimensions of $\Delta_T q$.

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2}\Delta_T\widetilde{q}_+(n,Q^2) = \Delta_T\gamma_{qq,+}(n,\alpha_s(Q^2))\,\Delta_T\widetilde{q}_+(n,Q^2)\,, \qquad (4.31\mathrm{b})$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2}\Delta_T\Sigma(n,Q^2) = \Delta_T\gamma_{\Sigma\Sigma}(n,\alpha_s(Q^2))\,\Delta_T\Sigma(n,Q^2)\,. \qquad (4.31\mathrm{c})$$

Note that the first moment ($n = 1$) in Eq. (4.31a) corresponds to evolution of the nucleon's tensor charge (Jaffe and Ji, 1991b, 1992; Schmidt and Soffer, 1997). The anomalous dimensions $\Delta_T\gamma_{qq,\pm}$ and $\Delta_T\gamma_{\Sigma\Sigma}$ have expansions in powers of the coupling constant, which take the following form:

$$\Delta_T\gamma_{ii}(n,\alpha_s) = \left(\frac{\alpha_s}{2\pi}\right)\Delta_T\gamma_{qq}^{(0)}(n) + \left(\frac{\alpha_s}{2\pi}\right)^2\Delta_T\gamma_{ii}^{(1)}(n) + \ldots\,, \qquad (4.32)$$

where $\{ii\} = \{qq,\pm\}$, $\{\Sigma\Sigma\}$ and we have taken into account the fact that $\Delta_T\gamma_{qq,+}$, $\Delta_T\gamma_{qq,-}$ and $\Delta_T\gamma_{\Sigma\Sigma}$ are all equal at LO. It is convenient then to introduce (Ellis and Vogelsang, 1996)

$$\Delta_T\gamma_{qq,\pm}^{(1)}(n) = \Delta_T\gamma_{qq}^{(1)}(n) \pm \Delta_T\gamma_{q\bar{q}}^{(1)}(n)\,, \qquad (4.33\mathrm{a})$$

$$\Delta_T\gamma_{\Sigma\Sigma}^{(1)}(n) = \Delta_T\gamma_{qq,+}^{(1)}(n) + \Delta_T\gamma_{qq,PS}^{(1)}(n)\,. \qquad (4.33\mathrm{b})$$

Since it turns out that $\Delta_T\gamma^{(1)}_{qq,PS}(n) = 0$, the two evolution Eqs. (4.31b, c) may be replaced by a single equation:

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2}\,\Delta_T q_+(n,Q^2) = \Delta_T\gamma_{qq,+}(n,\alpha_s(Q^2))\,\Delta_T q_+(n,Q^2)\,. \qquad (4.31\mathrm{b}')$$

The formal solution to Eqs. (4.31a) and (4.31b′) is well-known (*e.g.*, see Glück and Reya, 1982) and reads

$$\Delta_T q_\pm(n,Q^2) = \left\{1 + \frac{\alpha_s(Q_0^2)-\alpha_s(Q^2)}{\pi\beta_0}\left[\Delta_T\gamma^{(1)}_{qq,\pm}(n) - \frac{\beta_1}{2\beta_0}\Delta_T\gamma^{(0)}_{qq}(n)\right]\right\} \times \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\frac{-2\Delta_T\gamma^{(0)}_{qq}(n)}{\beta_0}} \Delta_T q_\pm(n,Q_0^2)\,, \qquad (4.34)$$

with the input distributions $\Delta_T q_\pm(n,Q_0^2)$ given at the input scale Q_0. Of course, the corresponding LO expressions may be recovered from the above expressions by setting the NLO quantities, $\Delta_T\gamma^{(1)}_{ij,\pm}$ and β_1, to zero.

In the $\overline{\mathrm{MS}}$ scheme the $\gamma^{(1)}(n)$ relevant to h_1 are as follows:[g]

$$\begin{aligned}\Delta_T\gamma^{(1)}_{qq,\eta}(n) = C_\mathrm{F}^2\Big\{&\tfrac{3}{8} + \tfrac{2}{n(n+1)}\delta_{\eta-} - 3S_2(n) - 4S_1(n)\left[S_2(n) - S_2'\left(\tfrac{n}{2}\right)\right] \\ &- 8\widetilde{S}(n) + S_3'\left(\tfrac{n}{2}\right)\Big\} \\ + \tfrac{1}{2}C_\mathrm{F}N_\mathrm{c}\Big\{&\tfrac{17}{12} - \tfrac{2}{n(n+1)}\delta_{\eta-} - \tfrac{134}{9}S_1(n) + \tfrac{22}{3}S_2(n) \\ &+ 4S_1(n)\left[2S_2(n) - S_2'\left(\tfrac{n}{2}\right)\right] + 8\widetilde{S}(n) - S_3'\left(\tfrac{n}{2}\right)\Big\} \\ + \tfrac{2}{3}C_\mathrm{F}T_\mathrm{F}\Big\{&-\tfrac{1}{4} + \tfrac{10}{3}S_1(n) - 2S_2(n)\Big\}\,, \end{aligned} \qquad (4.35)$$

where $\eta = \pm$ and the S sum functions are defined by

$$S_k(n) = \sum_{j=1}^{n} j^{-k}, \qquad (4.36\mathrm{a})$$

$$S_k'\left(\tfrac{n}{2}\right) = 2^k \sum_{j=2,\mathrm{even}}^{n} j^{-k}, \qquad (4.36\mathrm{b})$$

$$\widetilde{S}(n) = \sum_{j=1}^{n}(-1)^j S_1(j) j^{-2}\,. \qquad (4.36\mathrm{c})$$

[g]Note that $\frac{2}{3}S_1(n)$ in the second line of (A.8) of González-Arroyo, López and Ynduráin (1979) should read $\frac{2}{3}S_3(n)$.

In Fig. 4.5 we show the n dependence of the two-loop anomalous dimensions, as presented by Hayashigaki, Kanazawa and Koike (1997)[h]. From the

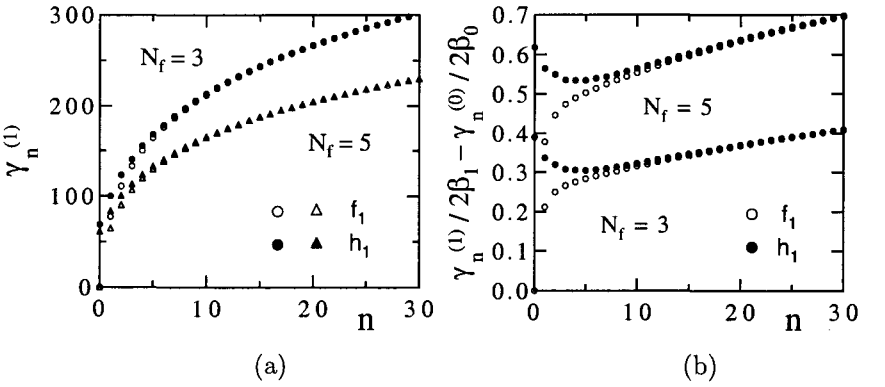


Fig. 4.5 The comparison between f_1 and h_1 of the variation with n of (a) the two-loop anomalous dimensions $\gamma_n^{(1)}$ for $N_f = 3$ (circles) and 5 (triangles), and (b) the combination $\gamma^{(1)}(n)/2\beta_1 - \gamma^{(0)}(n)/2\beta_0$ for $N_f = 3$ and 5; from Hayashigaki, Kanazawa and Koike (1997).

figure, one clearly sees that for n small, $\Delta_T\gamma^{(1)}(n)$ is significantly larger than $\gamma^{(1)}(n)$ but, with growing n, very quickly approaches $\gamma^{(1)}(n)$ while maintaining the inequality $\Delta_T\gamma^{(1)}(n) > \gamma^{(1)}(n)$. For the specific moments $n = 1$ (corresponding to the tensor charge) and $n = 2$, we display the Q^2 variation in Fig. 4.6.

To express the corresponding results in x space it is convenient to introduce the following definitions:[i]

$$\Delta_T R^{(0)}(x) = \frac{2x}{(1-x)_+}, \tag{4.37}$$

$$\begin{aligned} S_2(x) &= \int_{\frac{x}{1+x}}^{\frac{1}{1+x}} \frac{\mathrm{d}z}{z} \ln\left(\frac{1-z}{z}\right) \\ &= -2\,\mathrm{Li}_2(-x) - 2\ln x \ln(1+x) + \tfrac{1}{2}\ln^2 x - \tfrac{1}{6}\pi^2, \end{aligned} \tag{4.38}$$

[h]According to the convention adopted for the moments by Hayashigaki, Kanazawa and Koike, $n = 0$ there corresponds to $n = 1$ in the present work

[i]In order to avoid confusion with the tensor-charge anomalous dimensions, the notation adopted here corresponding to $\Delta_T R^{(0)}(x)$ is different from that commonly adopted.

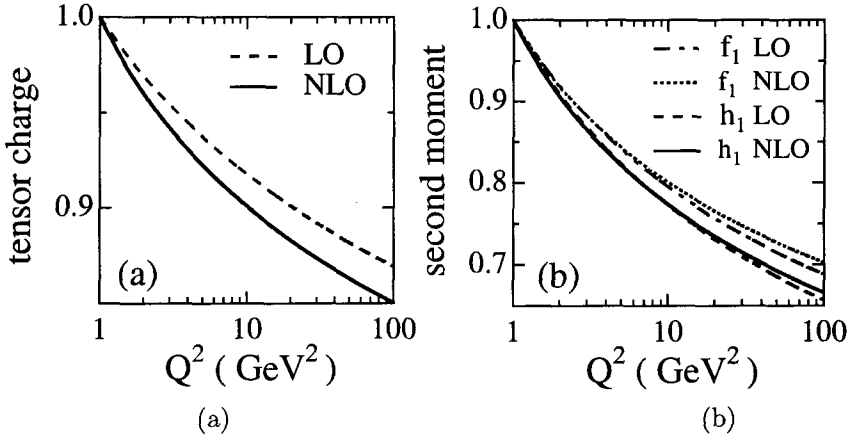


Fig. 4.6 The LO and NLO Q^2-evolution of (a) the tensor charge and (b) the second moments of $h_1(x,Q^2)$ and $f_1(x,Q^2)$ (both are normalised at $Q^2 = 1\,\mathrm{GeV}^2$); from Hayashigaki, Kanazawa and Koike (1997).

where $\mathrm{Li}_2(x)$ is the usual dilogarithm function. In the $\overline{\mathrm{MS}}$ scheme, defining

$$\Delta_T P^{(1)}_{qq,\pm}(x) = \Delta_T P^{(1)}_{qq}(x) \pm \Delta_T P^{(1)}_{q\bar{q}}(x)\,, \tag{4.39}$$

cf. Eq. (4.33a), one then has

$$\begin{aligned}\Delta_T P^{(1)}_{qq}(x) = C_{\mathrm{F}}^2 \Big\{ & 1 - x - \left[\tfrac{3}{2} + 2\ln(1-x)\right] \ln x\, \Delta_T R^{(0)}(x) \\ & + \left[\tfrac{3}{8} - \tfrac{1}{2}\pi^2 + 6\zeta(3)\right] \delta(1-x) \Big\} \\ + \tfrac{1}{2} C_{\mathrm{F}} C_{\mathrm{G}} \Big\{ & -(1-x) + \left[\tfrac{67}{9} + \tfrac{11}{3}\ln x + \ln^2 x - \tfrac{1}{3}\pi^2\right] \Delta_T R^{(0)}(x) \\ & + \left[\tfrac{17}{12} + \tfrac{11}{9}\pi^2 - 6\zeta(3)\right] \delta(1-x) \Big\} \\ + \tfrac{2}{3} C_{\mathrm{F}} T_{\mathrm{F}} \Big\{ & \left[-\ln x - \tfrac{5}{3}\right] \Delta_T R^{(0)}(x) - \left[\tfrac{1}{4} + \tfrac{1}{3}\pi^2\right] \delta(1-x) \Big\}\,, \end{aligned} \tag{4.40a}$$

$$\Delta_T P^{(1)}_{q\bar{q}}(x) = C_{\mathrm{F}} \left[C_{\mathrm{F}} - \tfrac{1}{2} C_{\mathrm{G}}\right] \Big\{ -(1-x) + 2S_2(x)\, \Delta_T R^{(0)}(-x) \Big\}\,, \tag{4.40b}$$

where $\zeta(3) \approx 1.202057$ is the usual Riemann Zeta function. Note that the plus prescription is to be ignored in $\Delta_T R^{(0)}(-x)$.

To complete this section we also report on the corresponding NLO calculation for linear (transverse) gluon polarisation (Vogelsang, 1998a). As already noted, $\Delta_T g$ is precluded in the case of spin-half hadrons – it may, however, be present in objects of spin one, such as the deuteron (Jaffe and Manohar, 1989; Sather and Schmidt, 1990) or indeed even the photon (Delduc, Gourdin and Oudrhiri-Safiani, 1980).

$$\begin{aligned}\Delta_T P_{gg}^{(1)}(x) = C_{\mathrm{G}}^2 \Bigg[& \left(\tfrac{67}{18} + \tfrac{1}{2}\ln^2 x - 2\ln x \ln(1-x) - \tfrac{1}{6}\pi^2\right) \Delta_T R^{(0)}(x) \\ & + \frac{1-x^3}{6x} + S_2(x)\Delta_T R^{(0)}(-x) + \left(\tfrac{8}{3} + 3\zeta(3)\right)\delta(1-x)\Bigg] \\ & + C_{\mathrm{G}} T_{\mathrm{F}} \left[-\tfrac{10}{9}\Delta_T R^{(0)}(x) + \frac{1-x^3}{3x} - \tfrac{4}{3}\delta(1-x)\right] \\ & - C_{\mathrm{F}} T_{\mathrm{F}} \left[\frac{2(1-x^3)}{3x} + \delta(1-x)\right]. \end{aligned} \tag{4.41}$$

The corresponding expression in Mellin-moment space for the anomalous dimensions is[j]

$$\begin{aligned}\Delta_T \gamma_{gg}^{(1)}(n) = C_{\mathrm{G}}^2 \Bigg[& \tfrac{8}{3} + \tfrac{1}{2(n-1)(n+2)} \\ & + S_1(n)\left(2S_2'(\tfrac{n}{2}) - \tfrac{67}{9}\right) + \tfrac{1}{2}S_3'(\tfrac{n}{2}) - 4\widetilde{S}(n)\Bigg] \\ & + C_{\mathrm{G}} T_{\mathrm{F}} \left[-\tfrac{4}{3} + \tfrac{20}{9}S_1(n) + \tfrac{1}{(n-1)(n+2)}\right] \\ & - C_{\mathrm{F}} T_{\mathrm{F}} \tfrac{n(n+1)}{(n-1)(n+2)}\,. \end{aligned} \tag{4.42}$$

As noted by Vogelsang (1998a), the result for the part proportional to $C_{\mathrm{F}} T_{\mathrm{F}}$ in (4.41) was presented by Delduc, Gourdin and Oudrhiri-Safiani for the region $x < 1$ (corresponding to the two-loop splitting function for linearly polarised gluons into *photons*). However, the two calculations appear to be at variance: the results of Delduc, Gourdin and Oudrhiri-Safiani imply a small-x behaviour of $\mathcal{O}(1/x^2)$ for the relevant splitting function, which would then be more singular than the unpolarised case.

There are two aspects of the splitting function (4.41) that warrant particular comment. Firstly, the small-x behaviour changes significantly on going from LO to NLO. At LO, the splitting function is $\mathcal{O}(x)$ for $x \to 0$ whereas at NLO there are $\mathcal{O}(1/x)$ terms (as in the unpolarised case): we

[j]We are very grateful to Werner Vogelsang for providing us with the exact expression.

have

$$\Delta_T P_{gg}^{(1)}(x) \approx \frac{1}{6x}\left(N_c^2 + 2N_c T_F - 4C_F T_F\right) + \mathcal{O}(x) \qquad (x \to 0)\,. \quad (4.43)$$

Notice that all logarithmic terms $\sim x \ln^2 x$ cancel in this limiting region.

The second comment regards the so-called supersymmetric limit: *i.e.*, $C_F = N_c = 2T_F$ (Bukhvostov *et al.*, 1985), which was investigated for the unpolarised and longitudinally polarised NLO splitting functions by Antoniadis and Floratos (1981), Mertig and van Neerven (1996), and Vogelsang (1996b,a), for the time-like case by Stratmann and Vogelsang (1997) and for the case of transversity by Vogelsang (1998a). In the supersymmetric limit the LO splitting functions for quark transversity and for linearly polarised gluons are equal (Delduc, Gourdin and Oudrhiri-Safiani, 1980; Artru and Mekhfi, 1990):

$$\Delta_T P_{qq}^{(0)}(x) = N_c\left[\frac{2x}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right] = \Delta P_{gg}^{(0)}(x)\,. \quad (4.44)$$

Hence, we may consider linear polarisation of the gluon as the supersymmetric partner to transversity (see also Bukhvostov *et al.*, 1985). Indeed, as was natural, we have already applied the terminology to spin-half and spin-one, without distinction.

Vogelsang (1998a) checked that the supersymmetric relation still holds at NLO. To do so it is necessary to transform to a regularisation scheme that respects supersymmetry, namely dimensional reduction. As noted by Vogelsang (1998a) the transformation is rendered essentially trivial owing to the absence of $\mathcal{O}(\epsilon)$ terms in the D-dimensional LO splitting functions for transversity or linearly polarised gluons at $x < 1$; such terms are always absent in dimensional *reduction* but may be present in dimensional *regularisation*. Thus, at NLO the results for the splitting functions for quark transversity – see Eqs. (4.40a, b) – and for linearly polarised gluons, Eq. (4.41), automatically coincide for $x < 1$ with their respective $\overline{\text{MS}}$ expressions in dimensional *reduction*. These expressions may therefore be immediately compared in the supersymmetric limit and indeed for $C_F = N_c = 2T_F$

$$\Delta_T P_{qq,+}^{(1)}(x) \equiv \Delta_T P_{gg}^{(1)}(x) \qquad (x < 1)\,. \quad (4.45)$$

Note, in addition, that the supersymmetric relation is trivially satisfied for $x = 1$; see Vogelsang (1996b,a), where the appropriate factorisation-scheme

transformation to dimensional reduction for $x = 1$ is given.

4.4 Fragmentation functions at next-to-leading order

Before moving on to discuss the phenomenological implications of the results just described, we should remark that an analogous theoretical framework exists for fragmentation functions. The evolution of fragmentation functions was calculated (also beyond LO) in the early days of QCD (for example, see Owens, 1978; Uematsu, 1978; Altarelli *et al.*, 1979; Berger, 1980; Wu, 1980).

The form of the evolution equations for fragmentation functions is identical to that for parton distribution functions, moreover, the same evolution kernels drive the evolution of the so-called interference fragmentation functions introduced by Jaffe, Jin and Tang (1998b,a). Indeed, in the LLA the kernels are also identical to those of the distribution function evolution. However, at higher order differences do arise. We shall now examine these differences and their origins. We note before so doing that at present almost all partonic cross-sections involving transverse polarisation in the final state are known only to LO. The exceptions are $e^+e^- \to q^\uparrow \bar{q}^\uparrow X$ and $eq^\uparrow \to eq^\uparrow X$, for which first-order corrections are available (Contogouris *et al.*, 1995; Contogouris, Merebashvili and Korakianitis, 1996).

While not in general identical, the AP splitting functions for densities and fragmentation are closely related, as has already been discussed in the literature for the unpolarised and longitudinally polarised cases (Curci, Furmanski and Petronzio, 1980; Furmanski and Petronzio, 1980; Stratmann and Vogelsang, 1997; Blümlein, Ravindran and van Neerven, 2000). Stratmann and Vogelsang (2002) continuing their previous work, in which they had already calculated first-order corrections to fragmentation for unpolarised and longitudinally polarised quarks and hadronic products, have also calculated the corresponding quantities for transverse polarisations.

The obvious difference between distribution and fragmentation evolution is that the splitting kernels are space-like for the former and time-like for the latter. At LO this has no effect and the kernels are just those presented in Eqs. (4.21c) and (4.23c). This identity is a manifestation of the Gribov–Lipatov relation (GLR—Gribov and Lipatov, 1972a,b), which relates space-like and time-like splitting functions for physical values of their arguments ($x \leq 1$). The two sets of LO splitting functions are also related

via analytic continuation across the point $x = 1$:

$$\Delta_T P_{qq,\pm}^{(T,0)}(z) = -z\,\Delta_T P_{qq,\pm}^{(S,0)}\left(\frac{1}{z}\right) \equiv \mathcal{AC}\left[\Delta_T P_{qq,\pm}^{(S,0)}(x)\right], \tag{4.46}$$

where the variables appropriate for each region are $x = 1/z$ (with $z \leq 1$) and we recall that the $\pm$ definitions correspond to the same NS combinations used earlier, see Eq. (4.30a); the indices T and S stand for the time-like and space-like cases respectively. In the second line (following Stratmann and Vogelsang, 2002) we have introduced the operator $\mathcal{AC}$, an analytical continuation of a space-like function to $x \to 1/z > 1$ and correctly adjusts the normalisation. Eq. (4.46) represents the Drell–Levy–Yan relation (DLYR—Drell, Levy and Yan, 1969, 1970), sometimes also known as the analytic continuation relation. Note, of course, that the endpoint contributions, $\propto \delta(1-z)$, are not subject to the $\mathcal{AC}$ operation; they are, however, necessarily identical in the space-like and time-like cases.

Both the GLR and the DLYR are known to be violated beyond the LO (Curci, Furmanski and Petronzio, 1980; Stratmann and Vogelsang, 1997; Blümlein, Ravindran and van Neerven, 2000). However, the DLYR is based on symmetries of tree diagrams under crossing (the interchange of s and t), and therefore the origin of its failure at NLO is clearly kinematical, as was shown by Stratmann and Vogelsang (1997). The main source of the difference is an extra factor of $z^{-2\epsilon}$ arising in the phase space for the time-like case (in $\overline{\text{MS}}$ with $D = 4 - 2\epsilon$ space–time dimensions). This, when combined with poles in ϵ, generates extra terms $\propto \ln z$ in the time-like NLO splitting functions. It is thus straight-forward to examine the calculation of the space-like NLO transversity splitting functions by Vogelsang (1998b) and identify, diagram by diagram, the DLYR breaking terms arising in the analytic continuation to $x > 1$.

In $\overline{\text{MS}}$, combining all extra terms, one has (for $z < 1$)

$$\Delta_T P_{qq,\pm}^{(T,1)}(z) = \mathcal{AC}\left[\Delta_T P_{qq,\pm}^{(S,1)}(x)\right] + \beta_0\,\Delta_T P_{qq,\pm}^{(S,0)}(z)\ln z\,. \tag{4.47}$$

The last term in Eq. (4.47) is that responsible for breaking the DLYR. Notice that the structure of this term, being proportional to both β_0 and the LO splitting function, is typically that of a factorisation-scheme transformation. In other words, it should be possible to define a factorisation scheme (non-$\overline{\text{MS}}$) in which the time-like transversity splitting functions is

given by the analytic continuation of its space-like counterpart, with no extra term; *i.e.*, such that the DLYR is not broken. Such a possibility was first realised by Baulieu, Floratos and Kounnas (1980) for the unpolarised case and by Stratmann and Vogelsang (1997) for that of longitudinal polarisation. The case of transversity, with the absence of singlet mixing, is rather simpler and one may in fact remain within the confines of the more conventional $\overline{\rm MS}$ scheme.

Exploiting the results of Vogelsang (1998b) for $\Delta_T P^{(S,1)}_{qq,\pm}(x)$ at NLO, performing the analytic continuation, and finally adding the endpoint contributions, one obtains the result for $\Delta_T P^{(T,1)}_{qq,\pm}(z)$. Thus, defining

$$\Delta_T P^{(T,1)}_{qq,\pm}(z) \equiv \Delta_T P^{(T,1)}_{qq}(z) \pm \Delta_T P^{(T,1)}_{q\bar{q}}(z)\,, \tag{4.48}$$

one has

$$\begin{aligned}
\Delta_T P^{(T,1)}_{qq}(z) = C_{\rm F}^2 \Bigg\{ & 1 - z + \left[\frac{3}{2} + 2\ln(1-z) - 2\ln z\right] \ln z\, \Delta_T R^{(0)}(x) \\
& + \left[\frac{3}{8} - \frac{\pi^2}{2} + 6\zeta(3)\right] \delta(1-z) \Bigg\} \\
+ \frac{1}{2} C_{\rm F} C_{\rm G} \Bigg\{ & -(1-z) + \left[\frac{67}{9} + \frac{11}{3}\ln z + \ln^2 z - \frac{\pi^2}{3}\right] \Delta_T R^{(0)}(x) \\
& + \left[\frac{17}{12} + \frac{11\pi^2}{9} - 6\zeta(3)\right] \delta(1-z) \Bigg\} \\
+ \frac{2}{3} C_{\rm F} T_{\rm F} N_{\rm f} \Bigg\{ & \left[-\ln z - \frac{5}{3}\right] \Delta_T R^{(0)}(x) - \left[\frac{1}{4} + \frac{\pi^2}{3}\right] \delta(1-z) \Bigg\},
\end{aligned} \tag{4.49}$$

$$\Delta_T P^{(T,1)}_{q\bar{q}}(z) = C_{\rm F}\left(C_{\rm F} - \tfrac{1}{2} C_{\rm G}\right) \left[-(1-z) + 2\mathcal{S}_2(z) \Delta_T p^{(0)}_{qq}(-z)\right], \tag{4.50}$$

where $\Delta_T R^{(0)}(z)$ and $\mathcal{S}_2(z)$ are as defined in Eqs. (4.37) and (4.38) respectively.

The difference $\Delta_T P^{(T,1)}_{qq,\pm}(z) - \Delta_T P^{(S,1)}_{qq,\pm}(z)$ then is non-zero if and only if the GLR is violated. Using Eqs. (4.49) and (4.50) along with the results of Vogelsang (1998b), we thus obtain

$$\begin{aligned}
& \Delta_T P^{(T,1)}_{qq,\pm}(z) - \Delta_T P^{(S,1)}_{qq,\pm}(z) \\
& \qquad = C_{\rm F}^2 \ln z\, \Delta_T R^{(0)}(x) \left[3 + 4\ln(1-z) - 2\ln z\right].
\end{aligned} \tag{4.51}$$

Note that as in the unpolarised and longitudinally polarised cases (Curci, Furmanski and Petronzio, 1980; Stratmann and Vogelsang, 1997) the violation of the GLR only occurs in the C_F^2 part of the splitting function.

Defining Mellin moments of the NLO splitting functions $\Delta_T P_{qq,\pm}^{(T,1)}(z)$ as usual,

$$\Delta_T \gamma_{qq,\eta}^{(T,1)}(n) \equiv \int_0^1 \mathrm{d}z\, z^{n-1} \Delta_T P_{qq,\pm}^{(T,1)}(z)\,, \tag{4.52}$$

one obtains

$$\begin{aligned}
\Delta_T \gamma_{qq,\eta}^{(T,1)}(n) &= \Delta_T \gamma_{qq,\eta}^{(S,1)}(n) + 6\,C_F^2 \left[\frac{4}{3}S_1(n) - 1\right]\left[\frac{\pi^2}{6} - S_2(n)\right] \\
&= C_F^2 \left\{ \frac{3}{8} + \frac{1-\eta}{n(n+1)} + 3S_2(n) - 4S_1(n)\left[3S_2(n) - S_2'\left(\frac{n}{2}\right)\right] \right. \\
&\qquad \left. - 8\tilde{S}(n) + S_3'\left(\frac{n}{2}\right) + \pi^2\left[\frac{4}{3}S_1(n) - 1\right]\right\} \\
&\quad + \frac{1}{2} C_F C_G \left\{ \frac{17}{12} - \frac{1-\eta}{n(n+1)} - \frac{134}{9}S_1(n) + \frac{22}{3}S_2(n) \right. \\
&\qquad \left. + 4S_1(n)\left[2S_2(n) - S_2'\left(\frac{n}{2}\right)\right] + 8\tilde{S}(n) - S_3'\left(\frac{n}{2}\right)\right\} \\
&\quad + \frac{2}{3} C_F T_F N_f \left\{ -\frac{1}{4} + \frac{10}{3}S_1(n) - 2S_2(n)\right\},
\end{aligned} \tag{4.53}$$

where, as before, $\eta = \pm$ and the sums (S_1, S_2 *etc.*) appearing above are those defined in Eqs. (4.36a–c). In the first line the result has been expressed in terms of the moments of the space-like NLO transversity splitting functions (Kumano and Miyama, 1997; Hayashigaki, Kanazawa and Koike, 1997; Vogelsang, 1998b), see Eq. (4.35).

4.5 Evolution of the transversity distributions

The interest in the effects of evolution in the case of transversity is twofold: first, there is the obvious question of the relative magnitude of the distributions at high energies given some low-energy starting point (*e.g.*, a non-perturbative model calculation, for a detailed discussion and examples see Sec. 3.11) and second is the problem raised by the Soffer inequality. It

is to the first that we now address our attention while we shall deal with latter shortly.

Let us for the moment simply pose the question of the effect of QCD evolution (Vogelsang and Weber, 1993; Barone, 1997; Barone, Calarco and Drago, 1997a,b; Hayashigaki, Kanazawa and Koike, 1997; Bourrely, Soffer and Teryaev, 1998; Contogouris *et al.*, 1998, 1999; Hirai, Kumano and Miyama, 1998; Koike, 1998; Kumano and Miyama, 1998; Martin *et al.*, 1998, 1999; Scopetta and Vento, 1998; Vogelsang, 1998a,b) on the overall magnitude of the transversity densities that might be constructed at some low-energy scale. As already noted above, there is no conservation rule associated with the tensor charge of the nucleon (*cf.* the vector and axial-vector charges) and, indeed, the sign of the anomalous dimensions at both LO and NLO is such that the first moment of h_1 falls with increasing Q^2. Thus, one immediately deduces that the tensor charge will eventually disappear in comparison to the vector and axial charges. Such behaviour could have a dramatic impact on the feasibility of high-energy measurement of h_1 and thus requires carefully study.

An analytic functional form for the LO anomalous dimensions governing the evolution of h_1 is

$$\Delta_T\gamma^{(0)}(n) = \tfrac{4}{3}\left\{\tfrac{3}{2} - 2\left[\psi(n+1) + \gamma_{\mathrm{E}}\right]\right\}, \tag{4.54}$$

where $\psi(z) = \frac{\mathrm{d}\ln\Gamma(z)}{\mathrm{d}z}$ is the digamma function and $\gamma_{\mathrm{E}} \approx 0.577216$ is the Euler–Mascheroni constant. Since $\Delta_T\gamma^{(0)}(1) = -\frac{2}{3}$, the first moment of h_1 and the tensor charges, $\delta q = \int_0^1 \mathrm{d}x\,(\Delta_T q - \Delta_T\bar{q})$, decrease with Q^2 as

$$\begin{aligned}\delta q(Q^2) &= \left[\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right]^{-2\Delta_T\gamma_{qq}^{(0)}(1)/\beta_0} \delta q(Q_0^2) \\ &= \left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right]^{-4/27} \delta q(Q_0^2)\,,\end{aligned} \tag{4.55}$$

where, to obtain the second equality, we have set $N_{\mathrm{f}} = 3$. Despite the smallness of the exponent, $-4/27$, we shall see that the evolution of $\Delta_T q(x, Q^2)$ is rather different from that of the helicity distributions $\Delta q(x, Q^2)$, especially

for small x. At NLO this becomes

$$\begin{aligned}\delta q(Q^2) &= \left[\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right]^{-2\Delta_T\gamma_{qq}^{(0)}(1)/\beta_0} \\ &\quad\times\left[1+\frac{\alpha_s(Q_0^2)-\alpha_s(Q^2)}{\pi\beta_0}\left(\Delta_T\gamma_{qq,-}^{(1)}(1)-\frac{\beta_1}{2\beta_0}\,\Delta_T\gamma_{qq}^{(0)}(1)\right)\right]\delta q(Q_0^2) \\ &= \left[\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right]^{\frac{4}{27}}\left[1-\frac{337}{486\pi}\left(\alpha_s(Q_0^2)-\alpha_s(Q^2)\right)\right], \qquad (4.56)\end{aligned}$$

where in the second equality we have used

$$\Delta_T\gamma_{qq,-}^{(1)}(1)=\frac{19}{8}\,C_{\rm F}^2-\frac{257}{72}\,C_{\rm F}N_c+\frac{13}{18}\,C_{\rm F}T_{\rm F}=-\frac{181}{18}+\frac{13}{27}\,N_{\rm f}\,, \qquad (4.57)$$

and once again we have set $N_{\rm f}=3$.

Recall that the first moments of the $q\to qg$ polarised and unpolarised splitting functions vanish to all orders in perturbation theory and that the $g\to q\bar{q}$ polarised anomalous dimension $\Delta\gamma_{qg}(1)$ is zero at LO; thus, $\Delta q(Q^2)$ is constant. This can be seen analytically from the following argument by Barone (1997) based on the double-logarithmic approximation. The leading behaviour of the parton distributions for small x is governed by the rightmost singularity of their anomalous dimensions in Mellin-moment space. From Eq. (4.54) we see that this singularity is located at $n=-1$ for $\Delta_T q$ at LO. Expanding $\Delta_T\gamma(n)$ around this point gives

$$\Delta_T\gamma(n)\sim\frac{8}{3(n+1)}+\mathcal{O}(1)\,. \qquad (4.58)$$

Equivalently, in x space, expanding the splitting function $\Delta_T P$ in powers of x yields

$$\Delta_T P(x)\sim\frac{8}{3}x+\mathcal{O}(x^2)\,. \qquad (4.59)$$

In contrast, the rightmost singularity for Δq in moment space is located at $n=0$ and the splitting functions ΔP_{qq} and ΔP_{qg} behave as constants as $x\to 0$. Therefore, owing to QCD evolution, $\Delta_T q$ acquires an extra suppression factor of one power of x with respect to Δq at small x. We note too that at NLO the rightmost singularity for $\Delta_T q$ is located at $n=0$, so that NLO evolution renders the DGLAP asymptotics for $x\to 0$ in the case of transversity compatible with Regge theory (Kirschner *et al.*, 1997), see Sec. 4.7 for further discussion of the small-x asymptotics.

As mentioned earlier, this problem may be investigated numerically by integrating the DGLAP equations (4.17) with suitable starting input for h_1 and g_1. As a reasonable trial model one may assume the various $\Delta_T q$ and Δq to be equal at some small scale Q_0^2 and then allow the two types of distributions to evolve separately, each according to its own evolution equations. The input hypothesis $\Delta_T q(x, Q_0^2) = \Delta q(x, Q_0^2)$ is suggested by various quark-model calculations of $\Delta_T q$ and Δq (Jaffe and Ji, 1992; Barone, Calarco and Drago, 1997b, see also Sec. 3.11 here), in which these two distributions are found to be very similar at a scale $Q_0^2 \lesssim 0.5\,\mathrm{GeV}^2$. For $\Delta q(x, Q_0^2)$, we then use the LO Glück–Reya–Vogelsang (GRV—Glück, Reya and Vogelsang, 1995) parametrisation (Glück, Reya and Vogelsang, 1995), whose input scale is $Q_0^2 = 0.23\,\mathrm{GeV}^2$. The result for the u-quark distributions is shown in Fig. 4.7 (the situation is similar for the other flavours). The dashed line is the input, the solid line and the dotted line are

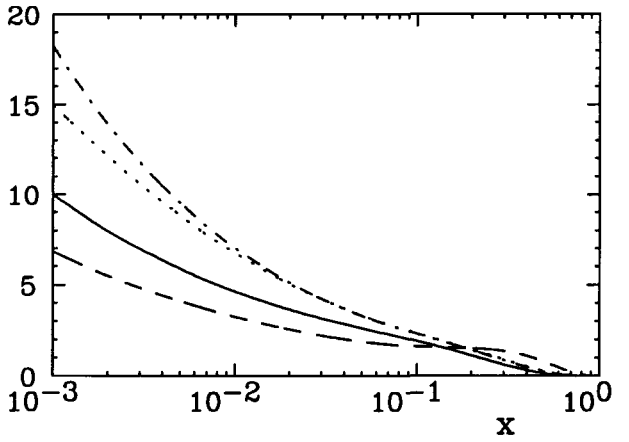


Fig. 4.7 Evolution of the helicity and transversity distributions for the u flavour (Barone, 1997). The dashed curve is the input $\Delta_T u = \Delta u$ at $Q_0^2 = 0.23\,\mathrm{GeV}^2$ taken from the GRV (Glück, Reya and Vogelsang, 1995) parametrisation. The solid (dotted) curve is $\Delta_T u$ (Δu) at $Q^2 = 25\,\mathrm{GeV}^2$. The dot–dashed curve is the result of the evolution of $\Delta_T u$ to $Q^2 = 25\,\mathrm{GeV}^2$ driven by P_{qq}, *i.e.*, with the term $\Delta_T P$ in P_h turned off.

the results of the evolution of $\Delta_T u$ and Δu, respectively, to $Q^2 = 25\,\mathrm{GeV}^2$. For completeness, the evolution of $\Delta_T u$ when driven only by P_{qq} – *i.e.*, with the $\Delta_T P$ term turned off, see Eq. (4.23c) – is also shown (dot–dashed line). The large difference in the evolution of $\Delta_T u$ (solid curve) and Δu (dotted curve) at small x is evident. Note also the difference between the

correct evolution of $\Delta_T u$ and the evolution driven purely by P_{qq} (dot–dashed curve).

As a further comparison of the behaviour of h_1 and g_1, in Figs. 4.8 (a) and (b) we display the LO and NLO Q^2-evolution of $\Delta_T u$ and Δu, starting respectively from the LO and NLO input function for Δu given by Glück *et al.* (1996) for $Q^2 = 0.23$ and $0.34\,\mathrm{GeV}^2$. The LO evolution leads to a significant divergence between $\Delta_T u$ and Δu at $Q^2 = 20\,\mathrm{GeV}^2$ and this tendency is strengthened by the NLO evolution, in particular, in the small-x region. Although the evolution of Δu shown in Fig. 4.8 is affected by mixing with the gluon distribution, the NS quark distributions also show the same trend.

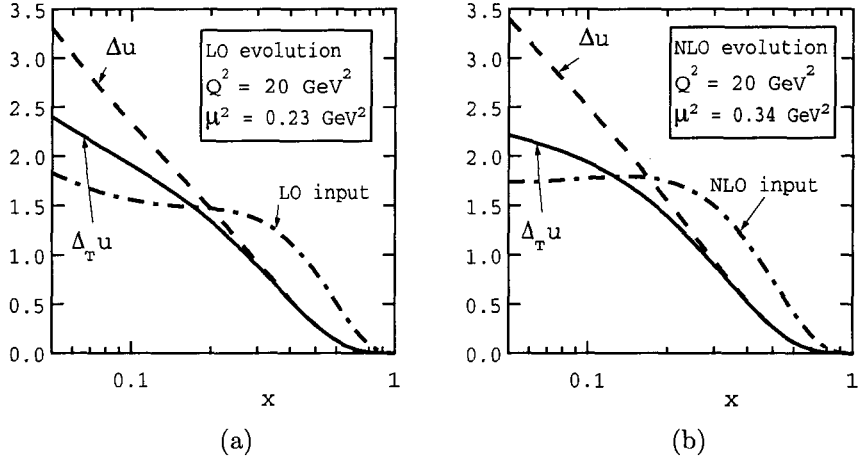


Fig. 4.8 A comparison of the Q^2-evolution of $\Delta_T u(x, Q^2)$ and $\Delta u(x, Q^2)$ at (a) LO and (b) NLO; from Hayashigaki, Kanazawa and Koike (1997).

In Fig. 4.9, we compare the NLO Q^2-evolution of $\Delta_T \bar{u}$, $\Delta_T \bar{d}$ and $\Delta \bar{u}$, starting from the same input distribution function (the NLO input function for the sea-quark distribution to g_1 given by Glück *et al.*, 1996). The difference between $\Delta_T \bar{u}$ and $\Delta \bar{u}$ is again significant. Although the input sea-quark distribution is taken to be flavour symmetric ($\Delta_T \bar{u} = \Delta_T \bar{d}$ at $Q^2 = 0.34\,\mathrm{GeV}^2$), NLO evolution violates this symmetry owing to the appearance of $\Delta_T P_{q\bar{q}}^{(1)}$ – see (4.40b). However, this effect is very small, as is evident from Fig. 4.9 and discussed by Martin *et al.* (1998).

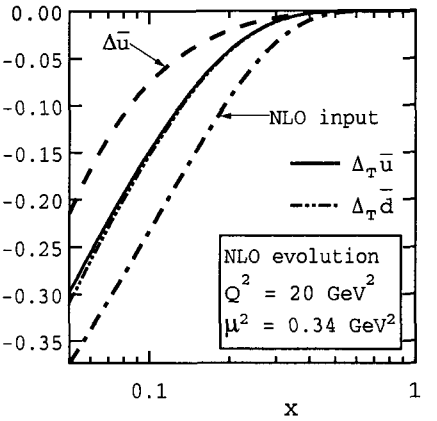


Fig. 4.9 A comparison of the NLO Q^2-evolution of $\Delta_T\bar{u}(x,Q^2)$, $\Delta_T\bar{d}(x,Q^2)$ and $\Delta\bar{u}(x,Q^2)$; from Hayashigaki, Kanazawa and Koike (1997).

4.6 Evolution of the Soffer inequality and positivity constraints

Particular interest in the effects of evolution arises in connection with the inequality derived by Soffer (1995), Eq. (3.48). It has been argued by Goldstein, Jaffe and Ji (1995) that this inequality, which was derived within a parton-model framework, may be spoilt by radiative corrections, much as the Callan–Gross relation. Such an analogy, however, is somewhat misleading, since the Soffer inequality is actually very similar and rather closely related to the more familiar (and relatively trivial) positivity bound $|\Delta q(x)| \leq q(x)$. The LO QCD evolution of the inequality is governed by Eq. (4.24) and hence it is not endangered, as pointed out by Barone (1997). At NLO the situation is complicated by the well-known problems of scheme dependence *etc.*

Indeed, it is perhaps worth stressing at this point that the entire question of positivity is ill-defined beyond LO, inasmuch as the parton distributions themselves as physical quantities become ill-defined: *a priori* there is no guarantee in a given scheme that any form of positivity will survive higher-order corrections. This observation may, of course, be turned on its head and used to impose conditions on the scheme choice, such that positivity might be guaranteed (Bourrely, Leader and Teryaev, 1997). At any rate, if this is possible, then at the hadronic level any natural positiv-

ity bounds should always be respected, independently of the regularisation scheme applied.

An instructive and rather general manner in which to examine the problem, developed by Bourrely, Soffer and Teryaev (1998), is to recast the system of evolution equations into a form analogous to the Boltzmann equation. First of all, let us rewrite Eq. (4.17) in a slightly more suggestive form for the NS case:

$$\frac{\mathrm{d}q(x,t)}{\mathrm{d}t} = \int_x^1 \frac{\mathrm{d}y}{y}\, P\left(\frac{x}{y},t\right) q(y,t)\,, \tag{4.60}$$

where $t = \ln Q^2$. One may thus interpret the equation as describing the time, t, evolution of densities, $f(x,t)$, in a one-dimensional x space. The flow is constrained to run from large to small x owing to the ordering $x < y$ under the integral. Such an interpretation facilitates dealing with the infrared (IR) singularities present in the expressions for $P(x)$. Indeed, a key element is provided precisely by consideration of the IR singularities (Durand and Putikka, 1987; Collins and Qiu, 1989).

Let us now rewrite the plus regularisation in the following form:

$$P_+(x,t) = P(x,t) - \delta(1-x)\int_0^1 \frac{\mathrm{d}y}{y}\, P(y,t)\,, \tag{4.61}$$

which then permits the evolution equations to be rewritten as

$$\frac{\mathrm{d}q(x,t)}{\mathrm{d}t} = \int_x^1 \frac{\mathrm{d}y}{y}\, q(y,t)\, P\left(\frac{x}{y},t\right) - q(x,t)\int_0^1 \mathrm{d}y\, P(y,t)\,. \tag{4.62}$$

Reading the second term as describing the flow of partons at the point x (Collins and Qiu, 1989), the kinetic interpretation is immediate. It is useful to render the analogy more direct by the change of variables $y \to y/x$ in the second term, leading to the following more symmetric form:

$$\frac{\mathrm{d}q(x,t)}{\mathrm{d}t} = \int_x^1 \frac{\mathrm{d}y}{y}\, q(y,t)\, P\left(\frac{x}{y},t\right) - \int_0^x \frac{\mathrm{d}y}{x}\, q(x,t)\, P\left(\frac{y}{x},t\right). \tag{4.63}$$

In this fashion the equation has been translated into a form analogous to the Boltzmann equation: namely,

$$\frac{\mathrm{d}q(x,t)}{\mathrm{d}t} = \int_0^1 \mathrm{d}y\, \left[\sigma(y\to x;t)\, q(y,t) - \sigma(x\to y;t)\, q(x,t)\right], \tag{4.64}$$

where the one-dimensional analogue of the Boltzmann "scattering probability" may be defined as

$$\sigma(y \to x; t) = \theta(y - x) \frac{1}{y} P\left(\frac{x}{y}, t\right). \tag{4.65}$$

Cancellation of the IR divergencies between contributions involving real and virtual gluons is therefore seen to occur as a consequence of the continuity condition on "particle number"; *i.e.*, the equality of flow in and out in the neighbourhood of $y = x$ in both terms of Eq. (4.64).

In the spin-averaged case the particle density (at some initialisation point) is positive by definition. Now, the negative second term in Eq. (4.64) cannot change the sign of the distribution because it is "diagonal" in x, *i.e.*, it is proportional to the function at the same point x. When the distribution is sufficiently close to zero, it stops decreasing. This is true for both "plus" and $\delta(1 - x)$ terms, for any value of their coefficients (if positive, it only reinforces positivity of the distribution).

Turning next to the spin-dependent case, for simplicity we consider first the flavour NS and allow the spin-dependent and spin-independent kernels to be different, as they indeed are at NLO. Rather than the usual helicity sums and differences, it turns out to be convenient to cast the equations in terms of definite parton helicities. Although such a form mixes contributions of different helicities, the positivity properties emerge more clearly. We thus have

$$\frac{\mathrm{d}q_+(x,t)}{\mathrm{d}t} = P_{++}(x,t) \otimes q_+(x,t) + P_{+-}(x,t) \otimes q_-(x,t)\,, \tag{4.66a}$$

$$\frac{\mathrm{d}q_-(x,t)}{\mathrm{d}t} = P_{+-}(x,t) \otimes q_+(x,t) + P_{++}(x,t) \otimes q_-(x,t)\,, \tag{4.66b}$$

where $P_{+\pm}(x,t) = \frac{1}{2}[P(x,t) \pm \Delta P(x,t)]$ are the evolution kernels for helicity non-flip and flip respectively. For $x < y$, positivity of the initial distributions, $q_\pm(x,t_0) \geq 0$ or $|\Delta q(x,t_0)| \leq q(x,t_0)$, is preserved if both kernels $P_{+\pm}$ are positive, which is true if

$$|\Delta P(x,t)| \leq P(x,t) \qquad (x < 1)\,. \tag{4.67}$$

Terms that are singular at $x = 1$ cannot alter positivity as they only appear in the diagonal (in helicity) kernel, P_{++}; non-forward scattering is completely IR safe. Once again in the kinetic interpretation, the distributions q_+ and q_- stop decreasing on approaching zero.

To extend the proof to include the case in which there is quark–gluon mixing is trivial—we need the full expressions for the evolutions of quark and gluon distributions of each helicity:

$$\frac{\mathrm{d}q_+(x,t)}{\mathrm{d}t} = P_{++}^{qq}(x,t)\otimes q_+(x,t) + P_{+-}^{qq}(x,t)\otimes q_-(x,t) + P_{++}^{qg}(x,t)\otimes g_+(x,t) + P_{+-}^{qg}(x,t)\otimes g_-(x,t)\,, \quad (4.68a)$$

$$\frac{\mathrm{d}q_-(x,t)}{\mathrm{d}t} = P_{+-}^{qg}(x,t)\otimes q_+(x,t) + P_{++}^{qg}(x,t)\otimes q_-(x,t) + P_{+-}^{qg}(x,t)\otimes g_+(x,t) + P_{++}^{qg}(x,t)\otimes g_-(x,t)\,, \quad (4.68b)$$

$$\frac{\mathrm{d}g_+(x,t)}{\mathrm{d}t} = P_{++}^{gq}(x,t)\otimes q_+(x,t) + P_{+-}^{gq}(x,t)\otimes q_-(x,t) + P_{++}^{gg}(x,t)\otimes g_+(x,t) + P_{+-}^{gg}(x,t)\otimes g_-(x,t)\,, \quad (4.68c)$$

$$\frac{\mathrm{d}g_-(x,t)}{\mathrm{d}t} = P_{+-}^{gq}(x,t)\otimes q_+(x,t) + P_{++}^{gq}(x,t)\otimes q_-(x,t) + P_{+-}^{gg}(x,t)\otimes g_+(x,t) + P_{++}^{gg}(x,t)\otimes g_-(x,t)\,. \quad (4.68d)$$

Since inequality (4.67) is clearly valid separately for each type of parton (Bourrely, Leader and Teryaev, 1997),

$$|\Delta P_{ij}(x,t)| \le P_{ij}(x,t) \qquad (x<1, \quad i,j=q,g)\,, \quad (4.69)$$

all the kernels appearing on the r.h.s. of this system, are positive. With regard to the singular terms, they are again diagonal (in parton type here) and hence cannot affect positivity. The validity of the equations at LO is guaranteed via their derivation, just as the (positive) helicity-dependent kernels were in fact first calculated by Altarelli and Parisi (1977). At NLO, the situation is more complex (see Bourrely, Leader and Teryaev, 1997).

To recapitulate, the maintenance of positivity under Q^2 evolution has two sources: (a) inequalities (4.69), leading to the increase of distributions and (b) the kinetic interpretation of the decreasing terms. For the latter, it is crucial that they are diagonal in x, helicity and also parton type, which is a prerequisite for their IR nature.

We now finally return to the Soffer inequality: in analogy with the previous analysis, it is convenient to define the following "super" distributions (henceforth the argument t will be suppressed)

$$Q_+(x) = q_+(x) + \Delta_T q(x)\,, \quad (4.70a)$$

$$Q_-(x) = q_+(x) - \Delta_T q(x)\,. \quad (4.70b)$$

According to the Soffer inequality, both distributions must be positive at some scale (say Q_0^2) and the evolution equations for the NS case take the form

$$\frac{\mathrm{d}Q_+(x)}{\mathrm{d}t} = P_{++}^Q(x) \otimes Q_+(x) + P_{+-}^Q(x) \otimes Q_-(x)\,, \tag{4.71a}$$

$$\frac{\mathrm{d}Q_-(x)}{\mathrm{d}t} = P_{+-}^Q(x) \otimes Q_+(x) + P_{++}^Q(x) \otimes Q_-(x)\,, \tag{4.71b}$$

where the "super" kernels at LO are just

$$\begin{aligned} P_{++}^Q(x) &= \tfrac{1}{2}[P_{qq}^{(0)}(x) + P_h^{(0)}(x)] \\ &= \tfrac{1}{2}C_\mathrm{F}\left[\frac{(1+x)^2}{(1-x)_+} + 3\delta(1-x)\right], \end{aligned} \tag{4.72a}$$

$$\begin{aligned} P_{+-}^Q(x) &= \tfrac{1}{2}[P_{qq}^{(0)}(x) - P_h^{(0)}(x)] \\ &= \tfrac{1}{2}C_\mathrm{F}(1-x)\,. \end{aligned} \tag{4.72b}$$

One can plainly see that the inequalities analogous to (4.69) are satisfied, so that both $P_{++}^Q(x)$ and $P_{+-}^Q(x)$ are positive for $x < 1$, while the singular term appears only in the diagonal kernel. Thus, both requirements are fulfilled and the Soffer inequality is maintained under LO evolution. The extension to the singlet case is trivial owing to the exclusion of gluon mixing. Therefore, only evolution of quarks is affected, leading to the presence of the same extra terms on the r.h.s., as in Eqs. (4.68a):

$$\begin{aligned} \frac{\mathrm{d}Q_+(x)}{\mathrm{d}t} &= P_{++}^Q(x) \otimes Q_+(x) + P_{+-}^Q(x) \otimes Q_-(x) \\ &\quad + P_{+-}^{qG}(x) \otimes G_+(x) + P_{++}^{qG}(x) \otimes G_-(x)\,, \end{aligned} \tag{4.73a}$$

$$\begin{aligned} \frac{\mathrm{d}Q_-(x)}{\mathrm{d}t} &= P_{+-}^Q(x) \otimes Q_+(x) + P_{++}^Q(x) \otimes Q_-(x) \\ &\quad + P_{+-}^{qG}(x) \otimes G_+(x) + P_{++}^{qG}(x) \otimes G_-(x)\,, \end{aligned} \tag{4.73b}$$

which are all positive and singularity free; this concludes the demonstration that positivity is indeed preserved.

4.7 The low-x behaviour of h_1

In recent years the behaviour of structure functions (or equivalently the parton distributions) for $x \to 0$ has come under close scrutiny, from both

experimental and theoretical points of view. This region corresponds to the standard Regge limit and it is therefore of interest to examine, for example, the extent to which the leading reggeons are reproduced in perturbative QCD. We recall the pioneering work of Fadin, Kuraev and Lipatov (1975), Kuraev, Lipatov and Fadin (1977), and Balitsky and Lipatov (1978) leading to the so-called BFKL pomeron. The main result of this analysis is that the leading behaviour for the singlet unpolarised structure functions should be of the form

$$\frac{1}{x}F_2^{\mathrm{S}}(x) \propto g(x) \propto \left(\frac{1}{x}\right)^{1+4\ln 2\, N_c \alpha_s/\pi} . \tag{4.74}$$

Such a steep rise with respect to naïve Regge theory expectations (which traditionally is associated with a power equal to unity), while being a little more than required by the data, is welcome and with NLO corrections actually comes more-or-less into line with the experimental picture emerging from the HERA experiments.

Apart from the obvious theoretical importance of such questions, which are beyond the scope of this volume, the small-x behaviour is very relevant to the problem of sum-rule fulfilment and experimental analysis. Similar calculations have been performed in the case of g_1 (Bartels, Ermolaev and Ryskin, 1996a,b), but our concern here is with h_1, which has been examined by Kirschner *et al.* (1997).

The results of Kirschner *et al.* show a rather different behaviour in the case of h_1 when compared to the asymptotics of F_1 or g_1. Whereas the latter two both appear to rise rapidly in the small-x region, $h_1(x)$ tends to a roughly constant behaviour. This should also be compared with the behaviour of the behaviour of the g_2 DIS structure function discussed in the following chapter. The experimental consequence is that the necessary extrapolation $x \to 0$ becomes much more reliable than in other cases and thus too the evaluation of any associated sum rules.

Chapter 5

The g_2 structure function in QCD

While the transversity distribution is not accessible in fully inclusive DIS, transverse-polarisation degrees of freedom are also linked to the DIS structure function known as g_2. This structure function occupies a special position in the general hierarchy: although not leading twist (it enters at the twist-three level) it has a leading rôle in the effects it governs (*i.e.*, no lower-twist structure contributes to the same behaviour). Moreover, it is, so to speak, only marginally higher-twist: in unpolarised DIS higher twist begins at level four. Thus, to complete the discussion of spin effects in *inclusive* DIS and the rôle of QCD, we now turn to the evolution of g_2.

Summarising the richer and, indeed, more complex behaviour one finds that:

- g_2 has no simple partonic/probabilistic interpretation;
- the evolution of g_2 is governed by an infinite tower of operators that mix under renormalisation;
- operators proportional to the equations of motion also contribute via mixing;
- perhaps most important of all, the involvement of three-parton correlators depending on *two* light-cone momentum fractions (from which g_2 is obtained only after integration in one such variable) means that g_2 itself does not evolve autonomously.

In Sec. 2.2 we presented the definition of the structure function g_2, Eq. (2.32), here we shall examine the operators governing its QCD evolution. Unlike the cases of the unpolarised or longitudinal-spin dependent structure functions, the case of g_2 is complicated by the fact that there are a

number of twist-three operators of any given spin that are involved and the number actually grows proportionally to the spin. The existence of various operators of the same twist and with same Lorentz structure already indicates that we shall have to deal with the problem of operator mixing under renormalisation. Moreover, as one might expect (for example, see Politzer, 1980), higher twist inevitably involves multiparton densities—in the case at hand we shall find that new three-parton (qqg) correlators or quasi-parton operators enter the picture. It is precisely the presence of such structures that spoils the usual probabilistic interpretation of the partonic densities. As has also been discussed more generally in the literature (Collins, 1984, and references therein), in the case of higher-twist, operators proportional to the equations of motion also contribute via mixing under renormalisation (Kodaira, Yasui and Uematsu, 1995; Kodaira *et al.*, 1996).

In the early eighties various groups derived the equations governing the evolution of the g_2 structure function: Bukhvostov, Kuraev and Lipatov (1983b) and Bukhvostov *et al.* (1985) adopted the OPE, while Ratcliffe (1986) worked with ladder diagrams in a light-like gauge. Later work was also presented by Balitsky and Braun (1989) and Ji and Chou (1990), who also evaluated the next-to-leading order (NLO) coefficient functions (Ji *et al.*, 2000; Ji and Osborne, 2001). The NLO singlet coefficient functions were evaluated by Belitsky, Efremov and Teryaev (1995) and Belitsky *et al.* (2001); note, however, that in the former paper the effects of the twist-three two-gluon operators were not taken into account. An important simplification of the system was found by Ali, Braun and Hiller (1991)—see also Sasaki (1998); Derkachov, Korchemsky and Manashov (2000); Braun, Korchemsky and Manashov (2000, 2001a,b): in the large-N_c limit only one combination of the mixing operators survives and thus diagonalises the evolution equations, permitting a DGLAP-like formulation and consequent solution. We shall discuss this more in detail in Sec. 5.7.

A useful property common to many structure functions (and partonic densities) is their satisfying of sum rules (usually corresponding to some global symmetry or conserved current); the g_2 structure function presents a particular case in the form of a sum rule derived by Burkhardt and Cottingham (1970), long before the advent of QCD. Sum rules provide experimentally useful indications of sign and magnitude; they can, moreover, provide clear-cut tests of the theoretical framework. It is therefore very important to understand the effects of QCD evolution (for example, see Mertig and van Neerven, 1993; Altarelli *et al.*, 1994).

Thus, in the following section we shall present first the NS evolution using the OPE approach, a short discussion in terms of ladder-diagram summation will also be given—this latter technique affords a more intuitive physical picture and clarifies the rôle of non-planar diagrams. We then briefly review the gluon (singlet) contributions—these will naturally involve *ggg* correlators. With the expressions to hand and a clear picture of the multiparton correlators and non-planar contributions, it will then be possible to examine the large-N_c limit. In the final section the calculation of the coefficient functions will be outlined.

5.1 The operator-product expansion—non-singlet

Let us start then with an examination of the operators responsible for spin dependence in DIS. Recall first that the spin-dependent part of the hadronic polarisation tensor has the following decomposition, see Eq. (2.32):

$$W_{\mu\nu}^{(\mathrm{A})} = \frac{2M\,\varepsilon_{\mu\nu\rho\sigma}\,q^{\rho}}{P{\cdot}q}\left\{S^{\sigma}\,g_1(x,Q^2) + \left[S^{\sigma} - \frac{S{\cdot}q}{P{\cdot}q}\,P^{\sigma}\right] g_2(x,Q^2)\right\}.$$

Applying the OPE to the product of currents contributing to $W_{\mu\nu}^{(\mathrm{A})}$ one has the following expansion:

$$\mathrm{i}\int \mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}q\cdot x}\,T\left(J_\mu(x)J_\nu(0)\right)^{(\mathrm{A})} = -\mathrm{i}\varepsilon_{\mu\nu\lambda\sigma}q^{\lambda}\sum_{n\ \mathrm{odd}}\left(\frac{2}{Q^2}\right)^{n} q_{\mu_1}\cdots\, q_{\mu_{n-1}} \times \sum_i E_i^n(Q^2,\alpha_s)R_i^{\sigma\mu_1\cdots\,\mu_{n-1}}, \quad (5.1)$$

where the index i runs over operator types (of both twist two and three), the R_i^n are the composite operators (which we shall now list) and the E_i^n are the corresponding Wilson coefficients. We shall initially consider the flavour NS case and for simplicity suppress the explicit flavour structure of the operators.

The following twist-two operator alone governs the helicity structure of parton of distributions, *i.e.*, that contributing to g_1:

$$R_q^{\sigma\mu_1\cdots\,\mu_{n-1}} = \mathrm{i}^{n-1}\mathcal{S}\bar{\psi}\gamma_5\gamma^{\sigma}D^{\mu_1}\cdots\, D^{\mu_{n-1}}\psi\,, \quad (5.2)$$

where, we recall, the symbol $\mathcal{S}$ indicates symmetrisation over the indices $\sigma,\mu_1,\ldots\mu_{n-1}$.

The case of g_2 requires the inclusion of twist-three operators; together with an antisymmetrised version of the preceding operator and the transversity operator multiplied by the quark mass, there appears moreover a tower of operators that are naïvely proportional to the QCD coupling constant, g. Indeed, it is probably this apparent higher order in g that led to initial misformulation of the problem through their neglect (among others, Hey and Mandula, 1972; Heimann, 1973; Ahmed and Ross, 1975, 1976; Sasaki, 1975; Kodaira *et al.*, 1979; Antoniadis and Kounnas, 1981; Ahmed, 1983). We shall demonstrate that the constraints due to current conservation and to the equations of motion lead to a relation between the expectation values of these new operators and those already mentioned, hence they effectively enter at leading order in QCD. The full set of twist-three NS operators is then

$$R_F^{\sigma\mu_1\cdots\mu_{n-1}} = \frac{\mathrm{i}^{n-1}}{n}\left[(n-1)\bar\psi\gamma_5\gamma^\sigma D^{\{\mu_1}\cdots D^{\mu_{n-1}\}}\psi - \sum_{l=1}^{n-1}\bar\psi\gamma_5\gamma^{\mu_l}D^{\{\sigma}D^{\mu_1}\cdots D^{\mu_{l-1}}D^{\mu_{l+1}}\cdots D^{\mu_{n-1}\}}\psi\right], \tag{5.3a}$$

$$R_m^{\sigma\mu_1\cdots\mu_{n-1}} = \mathrm{i}^{n-2}m_q\mathcal{S}'\bar\psi\gamma_5\gamma^\sigma D^{\mu_1}\cdots D^{\mu_{n-2}}\gamma^{\mu_{n-1}}\psi\,, \tag{5.3b}$$

$$R_k^{\sigma\mu_1\cdots\mu_{n-1}} = \frac{1}{2n}\left(V_k - V_{n-1-k} + U_k + U_{n-1-k}\right), \tag{5.3c}$$

where the symbol $\mathcal{S}'$ indicates simultaneous *anti*symmetrisation over the indices σ and μ_1 and symmetrisation over the indices $\mu_1, \mu_2, \ldots \mu_{n-1}$; the factor m_q in (5.3b) represents the (current) quark mass. Note that the operators in (5.3c) are explicitly proportional to either the gluon field strength tensor $G^{\mu\nu}$ or its dual $\widetilde{G}^{\mu\nu} \equiv \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}G_{\alpha\beta}$ and are given by

$$V_k = \mathrm{i}^n\, g\, S\,\bar\psi\gamma_5 D^{\mu_1}\cdots G^{\sigma\mu_k}\cdots D^{\mu_{n-2}}\gamma^{\mu_{n-1}}\psi\,, \tag{5.4a}$$

$$U_k = \mathrm{i}^{n-3}\, g\, S\,\bar\psi D^{\mu_1}\cdots \widetilde{G}^{\sigma\mu_k}\cdots D^{\mu_{n-2}}\gamma^{\mu_{n-1}}\psi\,, \tag{5.4b}$$

where g is the QCD coupling constant. Such operators clearly arise from the action of antisymmetrising products of D^μ's:

$$[D_\mu, D_\nu] = -\mathrm{i}\, g\, T^a G^a_{\mu\nu}\,, \tag{5.5}$$

where $G_{\mu\nu}$ is the usual gluon field-strength tensor given by

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc}A^b_\mu A^c_\nu\,. \tag{5.6}$$

The twist-three operators in above set, Eqs. (5.3a–c) are related via operators proportional to the equations of motion (EoM—Shuryak and Vainshtein, 1982; Efremov and Teryaev, 1982, 1984; Bukhvostov, Kuraev and Lipatov, 1983a; Ratcliffe, 1986; Balitsky and Braun, 1989; Jaffe, 1990; Ji and Chou, 1990; Ali, Braun and Hiller, 1991):

$$R_{\mathrm{EoM}}^{\sigma\mu_1\cdots\,\mu_{n-1}} = \mathrm{i}^{n-2}\frac{(n-1)}{2n}\,\mathcal{S}\left[\bar{\psi}\gamma_5\gamma^\sigma D^{\mu_1}\cdots\,D^{\mu_{n-2}}\gamma^{\mu_{n-1}}(\mathrm{i}\not{D}-m_q)\psi\right.$$
$$\left.+\,\bar{\psi}(\mathrm{i}\not{D}-m_q)\gamma_5\gamma^\sigma D^{\mu_1}\cdots\,D^{\mu_{n-2}}\gamma^{\mu_{n-1}}\psi\right]. \qquad (5.7)$$

The equations of motion lead directly to the following linear relation among the coefficients:

$$R_F^{\sigma\mu_1\cdots\,\mu_{n-1}} = \frac{(n-1)}{n}R_m^{\sigma\mu_1\cdots\,\mu_{n-1}} + \sum_{k=1}^{n-2}(n-1-k)R_k^{\sigma\mu_1\cdots\,\mu_{n-1}}$$
$$+\,R_{\mathrm{EoM}}^{\sigma\mu_1\cdots\,\mu_{n-1}}. \qquad (5.8)$$

Thus, the full set of operators, Eqs. (5.3a–c) and (5.7), are not linearly independent. Although the physical matrix elements of the EoM operators must clearly vanish (Politzer, 1980), their off-shell Green functions do not. Hence, they should be retained for a complete study of the operator mixing via renormalisation.

The matrix elements of these operators between nucleon states with momentum P and spin S may be expressed as

$$\langle PS|R_q^{\sigma\mu_1\cdots\mu_{n-1}}|PS\rangle = -e_q^n S^{\{\sigma}P^{\mu_1}\cdots P^{\mu_{n-1}\}}, \qquad (5.9a)$$

$$\langle PS|R_F^{\sigma\mu_1\cdots\mu_{n-1}}|PS\rangle = -\frac{(n-1)}{n}\,e_F^n\,(S^\sigma P^{\mu_1} - S^{\mu_1}P^\sigma)$$
$$\times\,P^{\mu_2}\cdots P^{\mu_{n-1}}, \qquad (5.9b)$$

$$\langle PS|R_m^{\sigma\mu_1\cdots\mu_{n-1}}|PS\rangle = -e_m^n\,(S^\sigma P^{\mu_1} - S^{\mu_1}P^\sigma)\,P^{\mu_2}\cdots P^{\mu_{n-1}}, \qquad (5.9c)$$

$$\langle PS|R_k^{\sigma\mu_1\cdots\mu_{n-1}}|PS\rangle = -e_k^n\,(S^\sigma P^{\mu_1} - S^{\mu_1}P^\sigma)\,P^{\mu_2}\cdots P^{\mu_{n-1}}, \qquad (5.9d)$$

$$\langle PS|R_{EoM}^{\sigma\mu_1\cdots\mu_{n-1}}|PS\rangle = 0. \qquad (5.9e)$$

The operator normalisation is chosen such that for a free quark target $e_q^n = e_F^n = e_m^n = 1$, while $e_k^n = \mathcal{O}(g^2)$. The moment sum rules for g_1 and

g_2 are then, using Eq.(5.9a–e),

$$\int_0^1 \mathrm{d}x\, x^{n-1} g_1(x,Q^2) = \frac{1}{2} e_q^n E_q^n(Q^2). \tag{5.10a}$$

$$\int_0^1 \mathrm{d}x\, x^{n-1} g_2(x,Q^2) = -\frac{(n-1)}{2n}\, e_q^n E_q^n(Q^2) + \frac{1}{2}\left[\frac{(n-1)}{n}\, e_F^n E_F^n(Q^2) + e_m^n E_m^n(Q^2) + \sum_k e_k^n E_k^n(Q^2)\right]. \tag{5.10b}$$

In terms of the coefficients e_F^n, e_m^n and e_k^n, Eq.(5.8) becomes

$$\frac{(n-1)}{n}\, e_F^n = \frac{(n-1)}{n}\, e_m^n + \sum_{k=1}^{n-2} (n-1-k) e_k^n\,. \tag{5.11}$$

Renaming the product $e_q^n E_q^n(Q^2)$ as simply $a_n(Q^2)$ and incorporating all twist-three terms (which indeed multiply the same Lorentz structure) into the coefficients $d_n(Q^2)$, Eqs. (5.10a, b) become Eqs. (2.115a, b), which we rewrite here for convenience:

$$\int_0^1 \mathrm{d}x\, x^{n-1}\, g_1(x,Q^2) = \frac{1}{2} a_n(Q^2)\,, \qquad n = 1,3,5,\ldots\,,$$

$$\int_0^1 \mathrm{d}x\, x^{n-1}\, g_2(x,Q^2) = \frac{1}{2}\frac{n-1}{n}\left[d_n(Q^2) - a_n(Q^2)\right], \quad n = 3,5,7,\ldots\,,$$

As already mentioned, one sees from Eqs.(5.3a–c) that *no* twist-three operators are defined for $n = 1$. That is, e_F^n, e_m^n, and e_k^n are identically zero for $n = 1$ in (5.10b). Furthermore, there is a "kinematical" factor $(n-1)$ multiplying the contribution of the twist-two operators in the sum rule for g_2, Eq.(5.10b). Thus, an OPE analysis implies that the BC sum rule does *not* receive radiative corrections.

The problem of renormalisation and operator mixing in the presence of EoM operators has been thoroughly dealt with by Kodaira, Yasui and Uematsu (1995). Suffice it to note here that the coefficient functions for the EoM operators do not actually mix with those for the other operators. That this is so may be seen from the triangular nature (with respect to the EoM operators) of the anomalous-dimension matrix in the full set of RGE's. Thus, one can legitimately choose any convenient basis in which to perform the calculations; the standard choice made in the literature corresponds to excluding the operator $R_F^{\sigma\mu_1\cdots\mu_{n-1}}$.

Further discussion may be found in Kodaira and Tanaka (1999): in particular, the rôle of BRST invariance is clearly explained and the relation with approaches using non-local operators. Moreover, the operators relevant for the singlet case are presented.

The calculation of the various twist-three splitting kernels is rather long and tedious as it involves a number of operators and, in particular, some of these carry two indices; the k in Eq. (5.4) is effectively a second index in addition to n. All of the twist-three operators involved mix and thus the final form is considerably more complicated than the twist-two case. The interested reader is referred to the original papers for the complete expressions (Bukhvostov, Kuraev and Lipatov, 1983b; Bukhvostov *et al.*, 1985; Ratcliffe, 1986; Ji and Chou, 1990)[a]. However, a useful, physical approach complementary to the RG and OPE uses the summation of ladder diagrams. We shall now present some details using this technique.

5.2 Ladder-diagram summation

One of the early works (Ratcliffe, 1986) on the evolution of g_2 adopted the ladder-diagram summation technique in order to better elucidate the physical nature of the problem. In particular, the rôle of the triple-parton correlators is brought out clearly and the contribution of non-planar diagrams is explicit. This latter point is relevant to the simplification, already mentioned, induced in the large-N_c limit.

The presence of three-parton correlators considerably complicates the construction of the evolution equations, which are depicted diagrammatically in Fig. 5.1. The two- and three-legged hadronic blobs shown there represent the functions $\widetilde{g}_T(x)$ and $[G_A(x,y) + \widetilde{G}_A(x,y)]$, already encountered in Eq. (3.79b) and which then clearly mix under renormalisation.

For completeness, the precise form of the kernels is shown in Fig. 5.2, note how the triple-gluon vertex is also now explicitly present. Although the set of diagrammatical equations appears considerably more complex than the twist-two case, the relations between the various twist-three structures given in Eq. (3.79b) limits the number of new equations to be derived to just one, precisely that for the combination of three-parton correlators given there. Note that the evolution equation for the structure explicitly

[a]Note that in the last reference the result presented corresponds to the transpose of the standard definition.

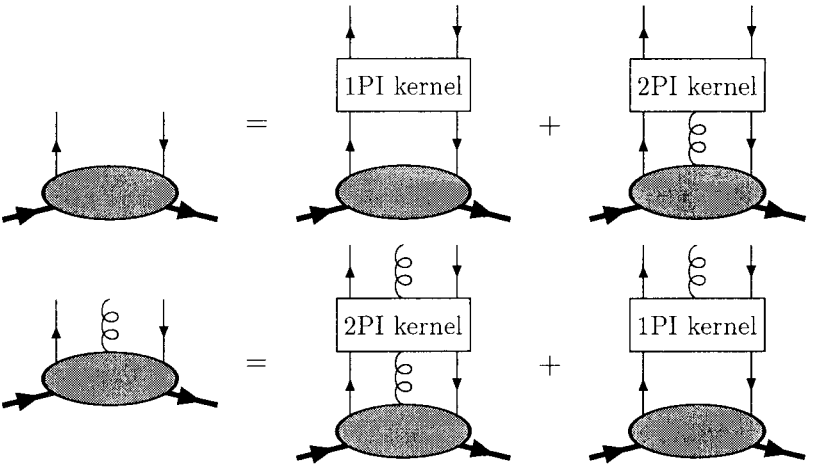


Fig. 5.1 The diagrammatic representation of the Bethe-Salpeter equations for the evolution of the twist-three quark distributions and qqg correlators.

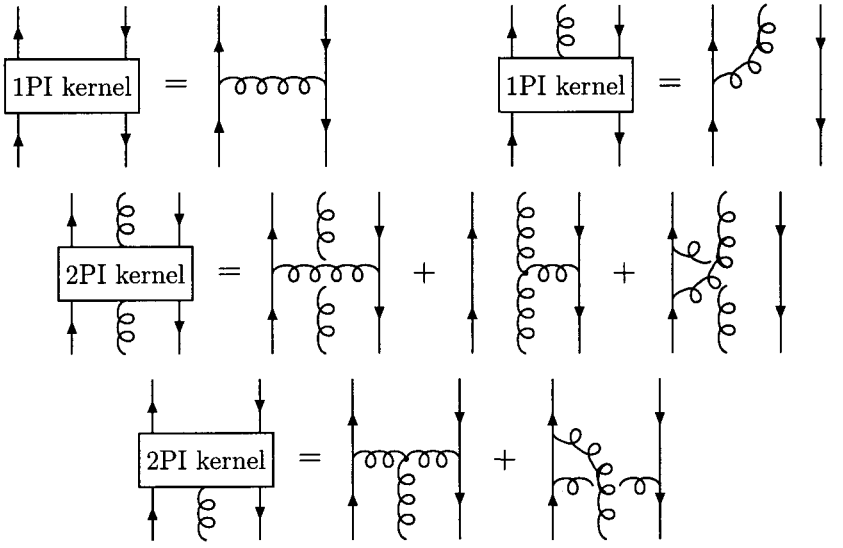


Fig. 5.2 The 1PI and 2PI evolution kernels for the twist-three transverse-spin dependent quark distributions and qqg correlators, where relevant the Hermitian-conjugate graphs should also be included.

proportional to the quark mass is precisely that of h_1, presented in the preceding chapter.

The results of Ratcliffe (1986) are given in terms of the relevant function defined there as $Y(x,y)$ and which has the following correspondence with the correlator functions defined here:

$$Y(x,y) \equiv (x-y)\left[G_A(x,y) + \widetilde{G}_A(x,y)\right]. \tag{5.12}$$

The evolution equation is then

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\ln Q^2} Y_n^{j,k} = Y_n^{j,k} \Bigg\{ & C_\mathrm{F}\left(3 - 2S_1(j) - 2S_1(k)\right) \\
& + C_\mathrm{G}\left[\frac{1}{k} - \frac{1}{k+1} - \frac{1}{k+2} - \frac{1}{j+1} - S_1(j) - S_1(k)\right] \\
& + (2C_\mathrm{F} - C_\mathrm{G})\left[\frac{2(-1)^k}{k(k+1)(k+2)} - \frac{(-1)^j}{j+1} + \frac{1}{n}\right] \Bigg\} \\
+ \sum_{m=1}^{j-1} Y_n^{j+m,k+m} \Bigg\{ & C_\mathrm{G} \frac{(j-m)(j-m+1)}{k(k+1)m} \\
& + (2C_\mathrm{F} - C_\mathrm{G})(-1)^m \left[\frac{C_{n-1}^{k+m}}{C_{n-1}^k}\left(\frac{1}{n} + \frac{1}{m}\right) - (-1)^k \frac{C_{j-1}^m}{j+1}\right] \Bigg\} \\
+ \sum_{m=1}^{k-1} Y_n^{j+m,k-m} \Bigg\{ & C_\mathrm{G} \frac{(k-m+1)(k-m+2)}{(k+1)(k+2)m} \\
& + (2C_\mathrm{F} - C_\mathrm{G})(-1)^m \left[\frac{C_{n-1}^{j+m}}{C_{n-1}^j}\left(\frac{1}{n} + \frac{1}{m}\right)\right. \\
& \qquad \left. - 2(-1)^j \frac{C_k^m}{k(k+1)(k+2)}\right] \Bigg\} \\
+ C_\mathrm{F} & \frac{4}{k(k+1)(k+2)} \frac{m_q}{M} \Delta_T f^n ,
\end{aligned} \tag{5.13}$$

where C_n^j are the binomial coefficients and the sums $S_1(j)$ are as given in Eq. (4.36a). The last (inhomogeneous) term is just that appearing in Eq. (3.65c) and is due to the operator related to transversity distribution. One immediately sees at least part of the simplification arising in the large-N_c limit: the colour factor $(2C_\mathrm{F} - C_\mathrm{G})$, common to many terms, is $\mathrm{O}(1/N_\mathrm{c}^2)$

with respect C_G in this limit.

5.3 Singlet g_2 in LO

Extending the evolution to the full singlet sector clearly involves evermore operators: in particular, there now arise three-gluon operators. All these operators will, of course, mix under renormalisation. Various authors have considered the singlet sector and mixing for the case of g_2 (for example, see Bukhvostov, Kuraev and Lipatov, 1984; Balitsky and Braun, 1989; Muller, 1997; Kodaira and Tanaka, 1999; Braun, Korchemsky and Manashov, 2001a); we refer the interested reader to the original papers for explicit expressions of the evolution kernels.

5.4 Non-singlet and singlet coefficients g_2 in NLO

The first step towards higher-order corrections to QCD hard-scattering processes is always the calculation of the $\mathcal{O}(\alpha_s)$ corrections to the coefficient functions (*i.e.*, the first non-logarithmic corrections to the hard-scattering cross-section). Such calculations are not, in fact, scheme independent and one also needs the NLO anomalous dimensions or splitting functions for a complete and unambiguous NLO calculation. However, they do, in general, contain the larger part of the corrections (see, *e.g.*, Altarelli, Ellis and Martinelli, 1979; Ratcliffe, 1983).

In the NS sector, the first approaches to this problem were performed in order to check the validity under renormalisation of the BC sum rule (Mertig and van Neerven, 1993; Altarelli *et al.*, 1994; Kodaira *et al.*, 1995), who considered DIS off a quark target. Note, however, that these calculations do not include twist-three contributions; the full NS coefficient functions at NLO were calculated by Ji *et al.* (2000) and Ji and Osborne (2001).

In the singlet sector, the first approaches to this problem were also performed as a check of the stability of the BC sum rule (Mathews, Ravindran and Sridhar, 1996; Gabrieli and Ridolfi, 1998), who considered DIS off a gluon target. Again the nature of the approach adopted does not include twist three. The full singlet coefficient functions at NLO were calculated by Belitsky, Efremov and Teryaev (1995) and Belitsky *et al.* (2001). In the approach of the first paper, non-local operators also play a rôle and thus the usual statement that the BC sum rule is protected (owing to the absence

of $n = 1$ operators) is not *a priori* available. However, the second paper presents a local formulation and, in any case, the results of all coefficient function calculations corroborate the continued validity of the sum rule.

5.5 Sum rules for g_2 in QCD

In this section we examine the possible effects of QCD evolution on the various sum rules that have been found related to the g_2 structure function.

5.5.1 *The Burkhardt–Cottingham sum rule in QCD*

Let us first examine in some detail the effect of QCD evolution on the Burkhardt–Cottingham (BC) sum rule (Burkhardt and Cottingham, 1970), which simply corresponds to the first moment of g_2, see Eq. (2.137). As already noted and as may be seen from Eqs. (5.3a–c), the relevant twist-three operators do *not* exist for $n = 1$. Therefore, an OPE analysis suggests that the BC sum rule is not affected (for details of the calculational and discussion see Burkhardt and Cottingham, 1970; Mertig and van Neerven, 1993[b]; Altarelli *et al.*, 1994; and Kodaira *et al.*, 1995).

Possible δ-function singularities in g_2 (and at twist three in general) have been discussed by Burkardt (1993, 1995), Burkardt and Koike (2001), and also by Soffer and Teryaev (1995). Moreover, Harindranath and Zhang (1997) have examined g_T in a light-front time-ordered formalism and, by explicit calculation, have found that there is no δ-function contribution at $x = 0$ for g_T; these results have been confirmed by Burkardt and Koike (2001), again by explicit calculation for a massive quark target

The final expression to $\mathcal{O}(\alpha_s)$ is

$$g_T(x, Q^2) = \delta(1-x) + \frac{\alpha_s}{2\pi} C_F \ln \frac{Q^2}{m_q^2} \left[2x + \frac{1+x^2}{[1-x]_+} + \frac{1}{2}\delta(1-x) \right] . \quad (5.14)$$

First of all, note that this result was actually first derived by Altarelli and Muzzetto (1979) in a very early discussion of the rôle of quark masses in the evaluation in perturbative QCD of radiative corrections to $g_1 + g_2$. Note also that, in contrast, the analogous calculations for $e(x)$ and $h_L(x)$ do lead to a δ-function contribution at $x = 0$ and thus the corresponding sum rules

[b]Note that this work contains an error, corrected in the published *erratum*, initially leading the authors to conclude that the BC is actually violated by QCD evolution.

for the first moments of those structure functions should be violated in any experimental analysis, owing to the inaccessibility of the region $x \to 0$. While, of course, such perturbative results give no guarantee of continuation to the realistic case of a nucleon target (where, of course, non-perturbative physics dominates), the sum-rule violation for $e(x)$ and $h_L(x)$ is strongly indicative of the expected experimental situation.

The singlet calculations (Belitsky *et al.*, 2001) also demonstrate a similar protection of the sum rule, even in the presence of three-gluon operators. Let us briefly present the results: the general form for the OPE of g_T may be written as

$$g_T(x,Q^2) = \sum_i \int_{-1}^{1} \frac{\mathrm{d}z}{z}\frac{\mathrm{d}y}{y}\left\{C_i\left(\frac{x}{z},\frac{x}{y};\alpha_s\right)K_i(z,y) + (x \to -x)\right\}, \tag{5.15}$$

where C_i are the coefficient functions of the expectation values, K_i, of the various operators associated with the twist-three singlet component.

The correlation function of the relevant generic operator is

$$\begin{aligned}\Gamma^{\mu}_{3g}(x,y) = \int \frac{\mathrm{d}\lambda}{2\pi}\frac{\mathrm{d}\tau}{2\pi}\,\mathrm{e}^{\mathrm{i}\lambda x}\,\mathrm{e}^{\mathrm{i}\tau(y-x)} \\ \times \langle PS| - \mathrm{i}g f_{abc} A^{\mu}_a(0) A_{b\alpha}(\tau n) A^{\alpha}_c(\lambda n)|PS\rangle\,. \end{aligned} \tag{5.16}$$

In fact, gauge invariance demands that the final result should depend only on a certain combination of such distributions:

$$\begin{aligned}\Gamma^{\mu}_{g}(x,y) &= \Gamma^{\mu}_{2g}(x,y) + \Gamma^{\mu}_{3g}(x,y) \\ &= \int \frac{\mathrm{d}\lambda}{2\pi}\frac{\mathrm{d}\tau}{2\pi}\,\mathrm{e}^{\mathrm{i}\lambda x}\,\mathrm{e}^{\mathrm{i}\tau(y-x)} \\ &\qquad \times \langle PS|A^{\mu}_a(0)\mathrm{i}D_{\alpha a b}(\tau n) A^{\alpha}_b(\lambda n)|PS\rangle\,, \end{aligned} \tag{5.17}$$

where $D^{\mu}_{\alpha a b} \equiv \delta_{ab}\partial^{\mu} - g f_{acb} A_c$ is the covariant derivative associated with the gauge field. By applying the usual insertions of light-cone gauge links, generated by summing over states with additional longitudinally-polarised gluons, K_g may also be rendered gauge invariant.

The coefficient function for this combination is obtained from a lengthy calculation, in which care must be taken to correctly subtract the two-gluon

twist-three contributions. The final answer is (Belitsky *et al.*, 2001)

$$
\begin{aligned}
&C_g^{(1)}\left(\frac{x}{z},\frac{x}{y}\right)\\
&\quad=\frac{\alpha_s(\mu^2)T_F}{8\pi}\frac{1}{2zy}\left\{3\left[y+2\frac{(y-z)^2}{z}\right](x-z)\,\theta\Big(\frac{z}{x}-1\Big)\right.\\
&\qquad-\left[3(4z-3y)(x-y)+14z(y-z)-20\,\frac{xz}{y}\,(y-z)\right]\theta\Big(\frac{y}{x}-1\Big)\\
&\qquad-\frac{6z^2+zy-9y^2}{z-y}\left[x-(z-y)\right]\theta\Big(\frac{z-y}{x}-1\Big)\\
&\qquad+\left[(y-2z)(x-z)+3y(y-z)-6\,\frac{xy}{z}\,(y-z)\right]\\
&\qquad\qquad\times\ln\Big(\frac{z}{x}-1\Big)\,\theta\Big(\frac{z}{x}-1\Big)\\
&\qquad+\left[(4z-3y)(x-y)+4z(y-z)-8\,\frac{xz}{y}\,(y-z)\right]\\
&\qquad\qquad\times\ln\Big(\frac{y}{x}-1\Big)\,\theta\Big(\frac{y}{x}-1\Big)\\
&\qquad+\left[\frac{(z+y)(2z-3y)}{z-y}(x-(z-y))-zy\right]\\
&\qquad\qquad\left.\times\ln\left(\frac{z-y}{x}-1\right)\theta\left(\frac{z-y}{x}-1\right)\right\},
\end{aligned}
\tag{5.18}
$$

where $\theta(z)$ is the usual Heaviside function.

It is now possible to check the BC sum rule: all that is necessary is an integration over x. Assuming the order of integration over z and y can be interchanged with that of x, one readily obtains that the sum rule is also respected at one loop in the purely gluonic sector.

5.5.2 *The Wandzura–Wilczek relation in QCD*

Formally, the WW relation must fail since while g_1 contains only odd-signature amplitudes, g_2 receives contributions from both odd and even signatures (see, for example, Heimann, 1973; Ioffe, Khoze and Lipatov, 1984). Indeed, using light-front techniques, Harindranath and Zhang (1997) have shown that even in free-field theory the relation is badly broken and already so at order α_s. More importantly, the asymptotic small-x behaviour found, for example, by Bartels and Ryskin (2001), see Sec. 5.6, shows that

in this region the twist-three contribution is likely to be important and thus violate the sum rule.

5.5.3 *The Efremov–Leader–Teryaev sum rule in QCD*

The case of the Efremov–Leader–Teryaev sum rule is somewhat unusual since it is not derivable from a local OPE. As the sum rule appears in Eq. (2.144), *i.e.*, referring only to valence contributions, there is explicitly no Q^2 dependence. This is trivial since the very existence of the sum rule depends on the absence of operators of corresponding spin. At any rate its examination experimentally, should flavour separation in this structure function turn out to be viable, would be an interesting window onto the workings of QCD evolution.

5.6 Low-x behaviour of g_2

The low-x behaviour of g_2 for DIS off a quark target has been examined in the double-logarithmic approximation in QCD by Ermolaev and Troian (1997) and by Bartels and Ryskin (2001). Although some of the details of the calculations differ, the main final result is essentially the same:

$$g_2 \sim \left(\frac{1}{x}\right)^{\omega_0} \left(\frac{Q^2}{\mu^2}\right)^{\omega_0/2}, \tag{5.19}$$

where $\omega_0 \simeq \sqrt{2C_F\alpha_s/\pi}$. Thus, if we take $\alpha_s = 0.2$, the power of $1/x$ is approximately 0.4.

A different viewpoint is taken by Ivanov *et al.* (1999a,b), who promote the idea of a diffraction-driven steep rise of the spin structure function[c] $g_1 + g_2$ at small x. By relating this structure function to the longitudinal–transverse interference cross-section of diffractive DIS, they arrive at the following form:

$$g_1 + g_2 \sim \left[g(x, Q^2)\right]^2 \sim \left(\frac{1}{x}\right)^{2(1+\delta_g)}, \tag{5.20}$$

where δ_g is the exponent of the small-x rise of the *un*polarised gluon density in the nucleon $g(x, Q^2)$.

[c]Note that Ivanov *et al.* indicate the combination $g_{LT} = g_1 + g_2$ with g_{LT} instead of g_T, as in this volume.

A model calculation of the possible magnitude of such a strongly rising diffractive contribution compared to the standard prediction obtained via the WW relation would suggest that this contribution should make its appearance for x roughly below 10^{-3}. Noting also that such a rise is steeper than anything seen in g_1 and that indeed the diffractive mechanism they suggest does not contribute to the asymmetry A_1, Ivanov *et al.* point out that this contribution is then concentrated in g_2 and thus invalidates the assumption of superconvergence necessary for the BC sum rule.

5.7 Twist-three evolution equations in the large-N_c limit

In DIS off a transversely polarised target, only a particular projection of the two-variable correlators may be measured. Thus, since the evolution equations for the generating qqg correlation function require as input the entire function (Bukhvostov, Kuraev and Lipatov, 1983a, 1984; Ratcliffe, 1986), knowledge of just the particular projection $g_2(x, Q_0^2)$ for some starting value of Q_0^2 does not allow prediction of $g_2(x, Q^2)$ at a different scale. In other words, a standard DGLAP-type evolution equation for $g_2(x, Q^2)$ in QCD cannot be constructed.

From the phenomenological point of view such a situation is highly disturbing since it means that measurements of $g_2(x, Q^2)$ at different values of Q^2 cannot be related to each other without model assumptions. Thus, in particular, there is no real predictive power gained from the evolution equations for experimental comparison. In order to avoid the use of models or *ad hoc* assumptions, it is clearly necessary to reduce the effective degrees of freedom of the qqg correlators involved from two to one. In such a case an approximation to the scale dependence introducing a minimum of non-perturbative uncertainty may be found.

It is thus natural to seek a simplification and attempt to find some approximation in which the equations effectively diagonalise. As already noted in the discussion on the summation of ladder-diagrams, the non-diagonal structures arise from the inclusion of non-planar diagrams and a well-known property of QCD in the large-N_c limit is the suppression of such contributions. Indeed, Ali, Braun and Hiller (1991) showed how the equations simplify in this approximation and thus produced a self-contained evolution equation for g_2 itself.

Let us examine the case of the flavour NS contribution to the structure

function. In the OPE formulation the odd moments $n = 3, 5, \ldots$ of the twist-three part $\bar{g}_2(x, Q^2)$ of the structure function g_2, see Eq. (2.135), can be expanded as a sum of contributions of multiplicatively renormalised local qqg operators, schematically we may write

$$\int_0^1 dx\, x^{n-1} \bar{g}_2^{\rm NS}(x, Q^2) = \sum_{k=0}^{n-3} C_n^k \left(\frac{\alpha_s(Q)}{\alpha_s(\mu)} \right)^{\gamma_n^k/\beta_0} \langle PS | O_n^k(\mu) | PS \rangle \,, \quad (5.21)$$

where C_n^k are coefficient functions, $\langle PS|O_n^k(\mu)|PS\rangle$ are reduced matrix elements normalised at a scale μ and γ_n^k are the corresponding anomalous dimensions taken as ordered in k such that: $\gamma_n^0 < \gamma_n^1 < \cdots < \gamma_{n-3}^n$ for each n. Note that, in general, the *number* of contributing operators increases linearly with n in the r.h.s. of (5.21). Compare this to the case of twist-two distributions, where only a single (flavour NS) operator exists for each moment. The contributions of different operators cannot be separated in measurements of the moments of $g_2(x)$ and, hence, the scale dependence cannot be predicted.

A dramatic simplification occurs, however, in the large-$N_{\rm c}$ limit. On inspection, it turns out (Ali, Braun and Hiller, 1991) that all tree-level coefficient functions, C_n^k, apart from those of operators with the lowest anomalous dimension, for each n are suppressed by powers of $1/N_{\rm c}^2$. Thus, to this accuracy, the sum in (5.21) may be approximated with just the first term, $k = 0$. The corresponding anomalous dimension $\gamma_n^{k=0}$ can be calculated analytically and the result can then be reformulated as a DGLAP-type evolution equation:

$$Q^2 \frac{\rm d}{{\rm d}Q^2} \bar{g}_2^{\rm NS}(x, Q^2) = \frac{\alpha_s}{4\pi} \int_x^1 \frac{{\rm d}z}{z} P^{\rm NS}(x/z)\, \bar{g}_2^{\rm NS}(z, Q^2) \,, \quad (5.22)$$

with

$$P^{\rm NS}(z) = \left[\frac{4C_{\rm F}}{1-z} \right]_+ + \left[C_{\rm F} + \frac{1}{N_{\rm c}} \left(2 - \frac{\pi^2}{3} \right) \right] \delta(1-z) - 2C_{\rm F} \,. \quad (5.23)$$

Here, the $1/N_{\rm c}^2$ corrections calculated by Braun, Korchemsky and Manashov (2000) have also been included.

In the formalism adopted by Ali, Braun and Hiller of working directly with the non-local operator contributing to the twist-three part of g_2, it was shown that local operators involving gluons effectively decouple from evolution equation for large $N_{\rm c}$. Note that the analysis was performed using

massless quarks. Various authors have since discussed the approximation (for example, see Sasaki, 1998; Braun, Korchemsky and Manashov, 2000; Derkachov, Korchemsky and Manashov, 2000).

The paper by Braun, Korchemsky and Manashov (2000) examines the QCD corrections beyond LO in the large-N_c expansion, also in the case of the other (chirally-odd) twist-three distributions, see Eq. (3.62). The results obtained there, using a Hamiltonian formulation, show that the first non-leading $\mathcal{O}(1/N_c^2)$ corrections are rather small and concentrated in the region of soft-gluon emission. Thus, the accuracy of the approximation is guaranteed by the favourable conjunction of small mixing coefficients and the low probability for finding multiparton components with a large gluon momentum.

The analysis of Sasaki (1998) is also of particular interest since he attacks the problems of the choice of operator basis and that (not unrelated) of quark masses. Thus, Sasaki reanalysed the Q^2 evolution of the flavour NS twist-three part of g_2 in the framework of the standard OPE and the RG using massive quarks. Including both the operators proportional to the current quark mass and those proportional to the equation of motion, there are then the four different types of twist-three operators contributing to g_2 listed in Eq. (5.3). Taking a new basis of the independent operators, it can be shown that the Q^2 evolution of the twist-three part of g_2 obeys a simple DGLAP equation in the $N_c \to \infty$ limit and thus the results of Ali, Braun and Hiller (1991) are also reproduced for the case of massive quarks.

Indeed, one finds that if instead of the standard basis, R_m, R_k of Eqs. (5.3b, c) and R_{EoM} of Eq. (5.7), one adopts the combination R_F, Eq. (5.3a), together with R_k and R_{EoM}, then the simplification of the large-N_c limit is almost immediate. However, one can, of course, equally choose any other basis and the same result follows, albeit after careful analysis of the ensuing operator mixing.

As a concluding remark to this section, we note a legitimate question that now arises is whether or not this simplification might continue to higher orders. A general examination by Ji and Osborne (1999) arrives at the conclusion that such a simplification is limited to the three twist-three distribution functions $g_T(x)$, $h_L(x)$ and $e(x)$ and there to only the LLA, *i.e.*, to one-loop order. The observation is that once a further gluon is inserted into a one-loop diagram, any gluon already present propagating between the two quarks may now change wavelength at different points and thus the divergence subtraction operators will no longer have a unique

coefficient. Explicit calculation of a few test diagrams shows this to be the case and since cancellation among different Feynman diagrams is always unlikely (unless there exists some hitherto undiscovered symmetry), the indication is then that the simplification stops at the LLA.

5.8 Evolution of the g_2 fragmentation function

We close this chapter with a brief remark on the evolution of the fragmentation equivalent of g_2. Belitsky (1997) has performed the calculation of the splitting kernels for the various twist-three fragmentation functions, including that corresponding to g_2, in the large-N_c limit. The interest here is first of all to check on the possibility of simplified DGLAP-like evolution also in the time-like region and secondly to examine the Gribov–Lipatov relation (GLR—Gribov and Lipatov, 1972a,b). This latter is only proven to hold at twist-two level and thus a test at the twist-three level is an important indicator of its possible extension.

While the GLR is actually found to be strongly violated at twist three already in the LLA, the above-mentioned large-N_c simplification of diagonalising the evolution equations still holds in the space-like case. This is an important result in that, together with the standard DGLAP evolution for the twist-two distribution and fragmentation functions and the approximate ladder form in the large-N_c limit valid at twist three, it will allow full scaling violation analyses of future data on semi-inclusive processes involving transverse spin.

For completeness, we present here the results of Belitsky for what he calls $\mathcal{G}_T^{\text{tw-3}}$: namely, the genuine twist-three contributions to the corresponding fragmentation functions,

$$\mathcal{G}_T(\zeta) = \frac{1}{4}\int \frac{\mathrm{d}\lambda}{2\pi}\, e^{i\lambda\zeta}\langle 0|\gamma_\perp\gamma_5\psi(\lambda n)|h,X\rangle\langle h,X|\bar{\psi}(0)|0\rangle\,. \tag{5.24}$$

The anomalous dimensions (in the large-N_c limit) are as follows:

$$\Delta_T\gamma_n^{(T,0)} = N_c\left[-2\sum_{j=1}^{n-2}\frac{1}{j} - \frac{1}{n-1} + \frac{1}{2}\right]. \tag{5.25}$$

Inverting the moments one thus obtains the AP splitting kernel itself:

$$\Delta_T P_{qq}^{(T,0)} = N_c \left[\frac{2}{(1-z)_+} + \frac{1}{z} + \frac{1}{2}\delta(1-z) \right]. \tag{5.26}$$

Comparing Eq. (5.26) with the corresponding space-like AP splitting function in Eq. (5.23) one immediately sees that the GLR is indeed badly violated.

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Chapter 6

Transversity in Drell–Yan production

As noted repeatedly, the transversity distributions do not appear in fully inclusive DIS. Transversity is probed in reactions involving no less than two *bona fide* hadrons, of which at least one must be transversely polarised. The two hadrons may be either both in the initial state, or one in the initial state and the other in the final state.

We shall consider three classes of reactions:

- scattering of two transversely polarised hadrons (in particular, the transversely polarised Drell–Yan process);
- leptoproduction on a transversely polarised target;
- hadroproduction with one transversely polarised initial hadron.

This chapter focuses on Drell–Yan production with two transversely polarised hadrons. This turns out to be the cleanest reaction for studying the transversity distributions (the case of Drell–Yan processes with *one* transversely polarised hadron will be briefly discussed in Sec. 6.4). Indeed, the pioneering work of Ralston and Soper (1979) and Pire and Ralston (1983) concentrated precisely on this process. We shall see, however, that the expected transverse asymmetries are rather small, and therefore their measurement represents a serious challenge for experimentalists.

6.1 Double transverse-spin asymmetries

In general, when both colliding hadrons are transversely polarised, the typical experimental quantities to be measured are double transverse-spin asym-

metries, which take the following form:

$$A_{TT} = \frac{\mathrm{d}\sigma(\boldsymbol{S}_T, \boldsymbol{S}_T) - \mathrm{d}\sigma(\boldsymbol{S}_T, -\boldsymbol{S}_T)}{\mathrm{d}\sigma(\boldsymbol{S}_T, \boldsymbol{S}_T) + \mathrm{d}\sigma(\boldsymbol{S}_T, -\boldsymbol{S}_T)} \,. \tag{6.1}$$

Since there is no gluon transversity distribution for spin-half hadrons, those transversely polarised pp reactions that are dominated at the partonic level by qg or gg scattering are expected to yield a very small A_{TT} (Ji, 1992a; Jaffe and Saito, 1996). Thus, direct-photon production (with lowest-order subprocesses $gq \to q\gamma$ and $q\bar{q} \to \gamma g$), heavy-quark production ($q\bar{q} \to Q\bar{Q}$ and $gg \to Q\bar{Q}$), and two-gluon-jet production ($gg \to gg$ and $q\bar{q} \to gg$) do not seem to be promising reactions to detect quark transverse polarisation.

The only good candidate process for measuring transversity in doubly polarised pp (or $p\bar{p}$) collisions is Drell–Yan lepton-pair production (Ralston and Soper, 1979; Cortes, Pire and Ralston, 1991; Jaffe and Ji, 1992). We shall see that at lowest order A_{TT}^{DY} contains combinations of the products

$$\Delta_T f(x_A)\, \Delta_T \bar{f}(x_B) \,.$$

The advantage of studying quark transverse polarisation via Drell–Yan is twofold: *i*) transversity distributions appear at leading-twist level; *ii*) the cross-section contains no unknown quantities, besides the transversity distributions themselves. This renders theoretical predictions relatively easier, with respect to other reactions.

6.2 The Drell–Yan process

Drell–Yan lepton-pair production is the process (see Fig. 6.1)

$$A\,(P_1) + B\,(P_2) \to l^+\,(\ell) + l^-\,(\ell') + X \,, \tag{6.2}$$

where A and B are protons or antiprotons and X is an undetected hadronic system. The centre-of-mass energy squared of this reaction is $s = (P_1 + P_2)^2 \simeq 2\,P_1 \cdot P_2$ (in the following, the hadron masses M_1 and M_2 will be systematically neglected, unless otherwise stated). The lepton pair originates from a virtual photon (or from a Z^0) with four-momentum $q = \ell + \ell'$. Note that, in contrast to DIS, q is a time-like vector: $Q^2 = q^2 > 0$. This is also the invariant mass of the lepton pair. We shall consider the deeply inelastic limit where $Q^2, s \to \infty$ while the ratio $\tau = Q^2/s$ is fixed and finite.

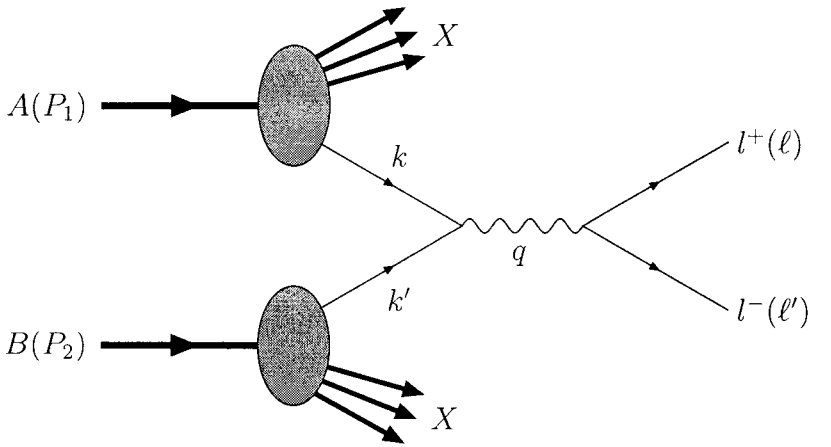

Fig. 6.1 Drell–Yan dilepton production.

The DY cross-section is

$$\mathrm{d}^6\sigma = \frac{8\pi^2\alpha_{\mathrm{em}}^2}{s\,Q^4}\,2L_{\mu\nu}W^{\mu\nu}\,(2\pi)^4\,\frac{\mathrm{d}^3\boldsymbol{\ell}}{(2\pi)^3\,2E}\,\frac{\mathrm{d}^3\boldsymbol{\ell}'}{(2\pi)^3\,2E'}\,, \tag{6.3}$$

where the leptonic tensor, neglecting lepton masses and ignoring their polarisation, is given by

$$L_{\mu\nu} = 2\left(\ell_\mu\ell'_\nu + \ell_\nu\ell'_\mu - \frac{Q^2}{2}\,g_{\mu\nu}\right), \tag{6.4}$$

and the hadronic tensor is defined as

$$\begin{aligned} W^{\mu\nu} &= \frac{1}{(2\pi)^4}\sum_X \int \frac{\mathrm{d}^3\boldsymbol{P}_X}{(2\pi)^3\,2E_X}\,(2\pi)^4\delta^4(P_1+P_2-q-P_X) \\ &\qquad \times \langle P_1S_1, P_2S_2|J^\mu(0)|X\rangle\langle X|J^\nu(0)|P_1S_1, P_2S_2\rangle \\ &= \frac{1}{(2\pi)^4}\int \mathrm{d}^4\xi\ \mathrm{e}^{\mathrm{i}q\cdot\xi}\,\langle P_1S_1, P_2S_2|J^\mu(z)J^\nu(0)|P_1S_1, P_2S_2\rangle. \end{aligned} \tag{6.5}$$

The phase space in (6.3) can be rewritten as

$$\frac{\mathrm{d}^3\boldsymbol{\ell}}{(2\pi)^3\,2E}\,\frac{\mathrm{d}^3\boldsymbol{\ell}'}{(2\pi)^3\,2E'} = \frac{\mathrm{d}^4q\,\mathrm{d}\Omega}{8(2\pi)^6}\,, \tag{6.6}$$

where Ω is the solid angle identifying the direction of the leptons in their rest frame. Using (6.6) the DY cross-section takes the form

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}^4q\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{2sQ^4}\,L_{\mu\nu}W^{\mu\nu}\,. \tag{6.7}$$

We define now the two invariants

$$x_1 = \frac{Q^2}{2P_1{\cdot}q}\,, \qquad x_2 = \frac{Q^2}{2P_2{\cdot}q}\,. \tag{6.8}$$

In the parton model x_1 and x_2 will be interpreted as the fractions of the longitudinal momenta of the hadrons A and B carried by the quark and the antiquark that annihilate into the virtual photon.

In a frame where the two colliding hadrons are collinear (hadron A being taken to move in the positive z direction), the photon momentum can be parametrised as

$$q^\mu = \frac{Q^2}{x_2 s}\,P_1^\mu + \frac{Q^2}{x_1 s}\,P_2^\mu + q_T^\mu\,. \tag{6.9}$$

Neglecting $\mathcal{O}(1/Q^2)$ terms, one finds $Q^2/x_1x_2s \simeq 1$, that is, $\tau = Q^2/s = x_1x_2$, and therefore

$$q^\mu = x_1\,P_1^\mu + x_2\,P_2^\mu + q_T^\mu = \left[x_1\,P_1^+, x_2\,P_2^-, \boldsymbol{q}_T\right]. \tag{6.10}$$

Note that

$$x_1 \simeq \frac{P_2{\cdot}q}{P_1{\cdot}P_2}\,, \qquad x_2 \simeq \frac{P_1{\cdot}q}{P_1{\cdot}P_2}\,. \tag{6.11}$$

In terms of x_1, x_2 and $\boldsymbol{q}_T$, the DY cross-section reads

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}^2\boldsymbol{q}_T\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{4Q^4}\,L_{\mu\nu}W^{\mu\nu}\,. \tag{6.12}$$

It is customary (Ralston and Soper, 1979; Tangerman and Mulders, 1995a) to introduce three vectors Z^μ, X^μ and Y^μ defined as

$$Z^\mu = \frac{P_2{\cdot}q}{P_1{\cdot}P_2}\,P_1^\mu - \frac{P_1{\cdot}q}{P_1{\cdot}P_2}\,P_2^\mu\,, \tag{6.13a}$$

$$X^\mu = -\frac{1}{P_1{\cdot}P_2}\left[P_2{\cdot}Z\,\tilde{P}_1^\mu - P_1{\cdot}Z\,\tilde{P}_2^\mu\right], \tag{6.13b}$$

$$Y^\mu = \frac{1}{P_1{\cdot}P_2}\,\varepsilon^{\mu\nu\rho\sigma}\,P_{1\nu}P_{2\rho}q_\sigma\,. \tag{6.13c}$$

where $\widetilde{P}^{\mu}_{1,2} = P^{\mu}_{1,2} - (P_{1,2}{\cdot}q/q^2)\, q^{\mu}$. These vectors are mutually orthogonal and orthogonal to q^{μ}, and satisfy

$$Z^2 \simeq -Q^2\,, \qquad X^2 \simeq Y^2 \simeq -\boldsymbol{q}_T^2\,. \tag{6.14}$$

Thus, they form a set of space-like axes and have only spatial components in the dilepton rest frame. Using (6.10), Z^{μ}, X^{μ} and Y^{μ} can be expressed as

$$Z^{\mu} = x_1\, P_1^{\mu} - x_2\, P_2^{\mu}\,, \qquad X^{\mu} = q_T^{\mu}\,, \qquad Y^{\mu} = \varepsilon_T^{\sigma\mu}\, q_{T\sigma}\,, \tag{6.15}$$

where

$$\varepsilon_T^{\sigma\mu} \equiv \frac{1}{P_1{\cdot}P_2}\, \varepsilon^{\sigma\mu\nu\rho} P_{1\nu} P_{2\rho}\,. \tag{6.16}$$

In terms of the unit vectors

$$\hat{x}^{\mu} = \frac{X^{\mu}}{\sqrt{-X^2}}\,, \quad \hat{y}^{\mu} = \frac{Y^{\mu}}{\sqrt{-Y^2}}\,, \quad \hat{z}^{\mu} = \frac{Z^{\mu}}{\sqrt{-Z^2}}\,, \quad \hat{q}^{\mu} = \frac{q^{\mu}}{\sqrt{Q^2}}\,, \tag{6.17}$$

the lepton momenta can be expanded as

$$\ell^{\mu} = \tfrac{1}{2}\, q^{\mu} + \tfrac{1}{2}\, Q\, (\sin\theta\, \cos\phi\, \hat{x}^{\mu} + \sin\theta\, \sin\phi\, \hat{y}^{\mu} + \cos\theta\, \hat{z}^{\mu}), \tag{6.18a}$$

$$\ell'^{\mu} = \tfrac{1}{2}\, q^{\mu} - \tfrac{1}{2}\, Q\, (\sin\theta\, \cos\phi\, \hat{x}^{\mu} + \sin\theta\, \sin\phi\, \hat{y}^{\mu} + \cos\theta\, \hat{z}^{\mu}). \tag{6.18b}$$

The geometry of the process in the dilepton rest frame is shown in Fig. 6.2.

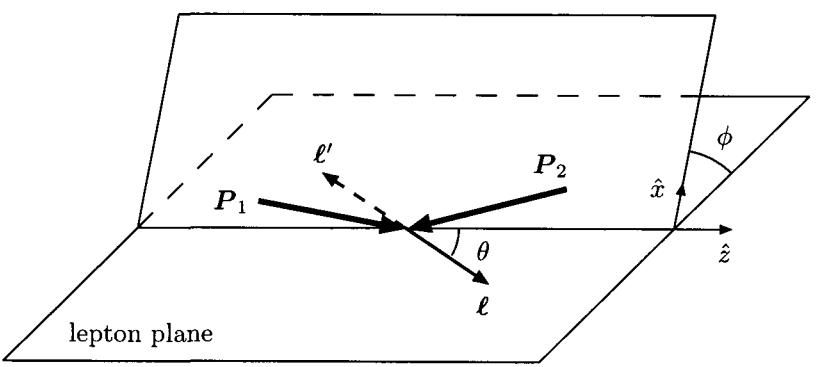

Fig. 6.2 The geometry of Drell–Yan production in the rest frame of the lepton pair.

The leptonic tensor then reads

$$L^{\mu\nu} = -\tfrac{1}{2}Q^2 \Big[(1+\cos^2\theta)\, g_\perp^{\mu\nu} - 2\sin^2\theta\, \hat{z}^\mu \hat{\zeta}^\nu + 2\sin^2\theta\, \cos 2\phi\, (\hat{x}^\mu \hat{x}^\nu + \tfrac{1}{2} g_\perp^{\mu\nu}) + \sin^2\theta\, \sin 2\phi\, \hat{x}^{\{\mu} \hat{y}^{\nu\}} + \sin 2\theta\, \cos\phi\, \hat{z}^{\{\mu} \hat{x}^{\nu\}} + \sin 2\theta\, \sin\phi\, \hat{z}^{\{\mu} \hat{y}^{\nu\}} \Big], \tag{6.19}$$

where

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \hat{q}^\mu \hat{q}^\nu + \hat{z}^\mu \hat{z}^\nu. \tag{6.20}$$

In the parton model, calling k and k' the momenta of the quark (or antiquark) coming from hadron A and B respectively, the hadronic tensor is (see Fig. 6.3)

$$W^{\mu\nu} = \frac{1}{3} \sum_a e_a^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \int \frac{\mathrm{d}^4 k'}{(2\pi)^4}\, \delta^4(k + k' - q)\, \mathrm{Tr}[\Phi_1\, \gamma^\mu\, \overline{\Phi}_2 \gamma^\nu]. \tag{6.21}$$

Here Φ_1 is the quark correlation matrix for hadron A, Eq. (3.1), $\overline{\Phi}_2$ is the

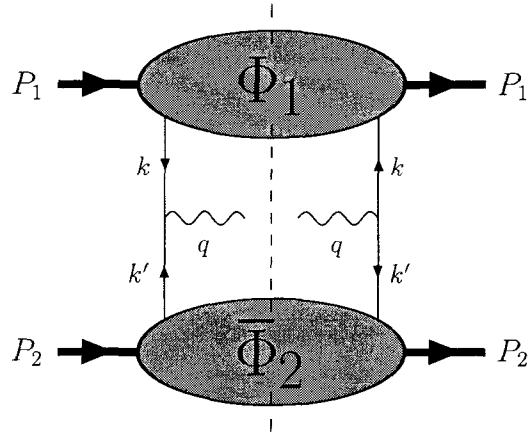

Fig. 6.3 The parton-model diagram for the Drell–Yan hadronic tensor.

antiquark correlation matrix for hadron B, Eq. (3.16), and the factor 1/3 has been added since in Φ_1 and $\overline{\Phi}_2$ summations over colours are implicit. It is understood that, in order to obtain the complete expression of the hadronic tensor, one must add to (6.21) a term with Φ_1 replaced by Φ_2 and $\overline{\Phi}_2$ replaced by $\overline{\Phi}_1$, which accounts for the case where a quark is extracted

from B and an antiquark is extracted from A. In the following formulæ we shall denote this term symbolically by $[1 \leftrightarrow 2]$.

Hereafter, quark transverse motion (which is discussed at length by Tangerman and Mulders, 1995a) will be ignored and only the ordinary collinear configuration will be considered.

We now evaluate the hadronic tensor in a frame where A and B move collinearly, with large longitudinal momentum. Setting $k \simeq \xi_1 P_1$, $k' \simeq \xi_2 P_2$ and using (6.10), the delta function in (6.21) gives

$$\begin{aligned}\delta^4(k + k' - \kappa) &= \delta(k^+ + k'^+ - q^+)\,\delta(k^- + k'^- - q^-)\,\delta^2(\boldsymbol{q}_T) \\ &\simeq \delta(\xi_1 P_1^+ - x_1 P_1^+)\,\delta(\xi_2 P_2^- - x_2 P_2^-)\,\delta^2(\boldsymbol{q}_T) \\ &= \frac{1}{P_1^+ P_2^-}\,\delta(\xi_1 - x_1)\,\delta(\xi_2 - x_2)\,\delta^2(\boldsymbol{q}_T)\,. \qquad (6.22)\end{aligned}$$

The hadronic tensor then becomes

$$\begin{aligned}W^{\mu\nu} = \frac{1}{3}\sum_a e_a^2 \int \frac{\mathrm{d}k^-\,\mathrm{d}^2\boldsymbol{k}_T}{(2\pi)^4} \int \frac{\mathrm{d}k'^+\,\mathrm{d}^2\boldsymbol{k}'_T}{(2\pi)^4}\,\delta^2(\boldsymbol{q}_T) \\ \times \mathrm{Tr}[\Phi_1\,\gamma^\mu\,\overline{\Phi}_2\gamma^\nu]_{k^+ = x_1 P_1^+,\, k'^- = x_2 P_2^-} + [1 \leftrightarrow 2]\,. \qquad (6.23)\end{aligned}$$

Since the leptonic tensor is symmetric, then too only the symmetric part of $W^{\mu\nu}$ can contribute to the cross-section. The Fierz decomposition of $\mathrm{Tr}[\Phi_1\,\gamma^{\{\mu}\,\overline{\Phi}_2\,\gamma^{\nu\}}]$ is

$$\begin{aligned}\mathrm{Tr}\left[\Phi_1\,\gamma^{\{\mu}\,\overline{\Phi}_2\,\gamma^{\nu\}}\right] = \tfrac{1}{2}\Big\{ &\mathrm{Tr}\left[\Phi_1\right]\mathrm{Tr}\left[\Xi\right] + \mathrm{Tr}\left[\mathrm{i}\,\Phi_1\,\gamma_5\right]\mathrm{Tr}\left[\mathrm{i}\,\overline{\Phi}_2\,\gamma_5\right] \\ &- \mathrm{Tr}\left[\Phi_1\,\gamma^\alpha\right]\mathrm{Tr}\left[\overline{\Phi}_2\,\gamma_\alpha\right] \\ &- \mathrm{Tr}\left[\Phi_1\,\gamma^\alpha\,\gamma_5\right]\mathrm{Tr}\left[\overline{\Phi}_2\,\gamma_\alpha\,\gamma_5\right] \\ &+ \tfrac{1}{2}\,\mathrm{Tr}\left[\mathrm{i}\,\Phi_1\,\sigma^{\alpha\beta}\,\gamma_5\right]\mathrm{Tr}\left[\mathrm{i}\,\overline{\Phi}_2\,\sigma_{\alpha\beta}\,\gamma_5\right]\Big\}\,g^{\mu\nu} \\ &+ \tfrac{1}{2}\,\mathrm{Tr}\left[\Phi_1\,\gamma^{\{\mu}\right]\mathrm{Tr}\left[\overline{\Phi}_2\,\gamma^{\nu\}}\right] \\ &+ \tfrac{1}{2}\,\mathrm{Tr}\left[\Phi_1\,\gamma^{\{\mu}\,\gamma_5\right]\mathrm{Tr}\left[\overline{\Phi}_2\,\gamma^{\nu\}}\,\gamma_5\right] \\ &+ \tfrac{1}{2}\,\mathrm{Tr}\left[\mathrm{i}\,\Phi_1\,\sigma^{\alpha\{\mu}\,\gamma_5\right]\mathrm{Tr}\left[\mathrm{i}\,\overline{\Phi}_2\,\sigma_\alpha^{\nu\}}\,\gamma_5\right]. \qquad (6.24)\end{aligned}$$

Using (3.12a–c) and (3.19a–c), we then obtain, the spins of the two hadrons

are $\boldsymbol{S}_1 = (\boldsymbol{S}_{1T}, \lambda_1)$ and $\boldsymbol{S}_2 = (\boldsymbol{S}_{2T}, \lambda_2)$,

$$\int \mathrm{d}^2\boldsymbol{q}_T\, W^{\mu\nu} = \frac{1}{3}\sum_a e_a^2 \Big\{ -g_T^{\mu\nu} \Big[f_a(x_1)\, \bar{f}_a(x_2) + \lambda_1\lambda_2\, \Delta f_a(x_1)\, \Delta\bar{f}_a(x_2) \Big] - \Big[S_{1T}^{\{\mu} S_{2T}^{\nu\}} + \boldsymbol{S}_{1T}\cdot\boldsymbol{S}_{2T}\, g_T^{\mu\nu} \Big]\, \Delta_T f_a(x_1)\, \Delta_T \bar{f}_a(x_2) \Big\} + [1 \leftrightarrow 2]\,. \tag{6.25}$$

In contracting the leptonic and the hadronic tensors, it is convenient to pass from the AB collinear frame to the $\gamma^* A$ collinear frame. We recall that, at leading twist, the transverse (T) vectors approximately coincide with the vectors perpendicular to the photon direction (denoted by a subscript $\perp$): $\boldsymbol{S}_{1T} \simeq \boldsymbol{S}_{1\perp}$, $\boldsymbol{S}_{2T} \simeq \boldsymbol{S}_{2\perp}$, $g_T^{\mu\nu} \simeq g_\perp^{\mu\nu}$. Therefore, the contraction $L_{\mu\nu}\, W^{\mu\nu}$ can be performed by means of the following identities

$$-g_\perp^{\mu\nu} L_{\mu\nu} = Q^2(1+\cos^2\theta)\,, \tag{6.26a}$$

$$\Big[S_{1\perp}^{\{\mu} S_{2\perp}^{\nu\}} + \boldsymbol{S}_{1\perp}\cdot\boldsymbol{S}_{2\perp}\, g_\perp^{\mu\nu} \Big]\, L_{\mu\nu} = -Q^2 \sin^2\theta\, |\boldsymbol{S}_{1\perp}|\, |\boldsymbol{S}_{2\perp}| \cos(2\phi - \phi_{S_1} - \phi_{S_2})\,, \tag{6.26b}$$

where θ is the polar angle of the lepton pair in the dilepton rest frame and ϕ_{S_1} (ϕ_{S_2}) is the azimuthal angle of $\boldsymbol{S}_{1\perp}$ ($\boldsymbol{S}_{2\perp}$), measured with respect to the lepton plane. For the Drell–Yan cross-section we finally obtain

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{4Q^2} \sum_a \frac{e_a^2}{3} \Big\{ \big[f_a(x_1)\, \bar{f}_a(x_2) + \lambda_1\,\lambda_2\, \Delta f_a(x_1)\, \Delta\bar{f}_a(x_2) \big]\, (1+\cos^2\theta) + |\boldsymbol{S}_{1\perp}|\, |\boldsymbol{S}_{2\perp}|\, \cos(2\phi - \phi_{S_1} - \phi_{S_2}) \times \Delta_T f_a(x_1)\, \Delta_T \bar{f}_a(x_2)\, \sin^2\theta \Big\} + [1 \leftrightarrow 2]\,. \tag{6.27}$$

From this we derive the parton-model expression for the double transverse-spin asymmetry:

$$A_{TT}^{\mathrm{DY}} = \frac{\mathrm{d}\sigma(\boldsymbol{S}_{1\perp}, \boldsymbol{S}_{2\perp}) - \mathrm{d}\sigma(\boldsymbol{S}_{1\perp}, -\boldsymbol{S}_{2\perp})}{\mathrm{d}\sigma(\boldsymbol{S}_{1\perp}, \boldsymbol{S}_{2\perp}) + \mathrm{d}\sigma(\boldsymbol{S}_{1\perp}, -\boldsymbol{S}_{2\perp})} = |\boldsymbol{S}_{1\perp}||\boldsymbol{S}_{2\perp}|\, \frac{\sin^2\theta\, \cos(2\phi - \phi_{S_1} - \phi_{S_2})}{1+\cos^2\theta} \times \frac{\sum_a e_a^2\, \Delta_T f_a(x_1) \Delta_T \bar{f}_a(x_2) + [1 \leftrightarrow 2]}{\sum_a e_a^2\, f_a(x_1)\bar{f}_a(x_2) + [1 \leftrightarrow 2]}\,, \tag{6.28}$$

and we see that a measurement of A_{TT}^{DY} directly provides a product of quark and antiquark transverse-polarisation distributions $\Delta_T f(x_1)\,\Delta_T f(x_2)$, not mixed with other unknown quantities. Thus, the Drell–Yan process seems to be, at least in principle, a very good reaction to probe transversity. Note that in LO QCD Eq. (6.28) is still valid, with Q^2 dependent distribution functions, namely

$$A_{TT}^{\mathrm{DY}} = |\boldsymbol{S}_{1\perp}||\boldsymbol{S}_{2\perp}|\,\frac{\sin^2\theta\,\cos(2\phi-\phi_{S_1}-\phi_{S_2})}{1+\cos^2\theta} \times \frac{\sum_a e_a^2\,\Delta_T f_a(x_1,Q^2)\Delta_T \bar{f}_a(x_2,Q^2)+[1\leftrightarrow 2]}{\sum_a e_a^2\,f_a(x_1,Q^2)\bar{f}_a(x_2,Q^2)+[1\leftrightarrow 2]}\,. \tag{6.29}$$

Here $\Delta_T f(x,Q^2)$ are then the transversity distributions evolved at LO. In Sec. 6.5 we shall see some predictions for A_{TT}^{DY}.

6.2.1 Z^0-mediated Drell–Yan processes

If Drell–Yan dilepton production is mediated by the exchange of a Z^0 boson, the vertex $e_i\,\gamma^\mu$, where e_i is the electric charge of particle i (quark or lepton), is replaced by $(V_i + A_i\,\gamma_5)\,\gamma^\mu$, where the vector and axial-vector couplings are

$$\begin{aligned} V_i &= T_3^i - 2e_i\,\sin^2\vartheta_W\,, \\ A_i &= T_3^i\,. \end{aligned} \tag{6.30}$$

The weak isospin T_3^i is $+\frac{1}{2}$ for $i=u$ and $-\frac{1}{2}$ for $i=l^-,d,s$.

The resulting double transverse-spin asymmetry has a form similar to (6.28), with the necessary changes in the couplings. Omitting the interference contributions, it reads

$$A_{TT}^{\mathrm{DY},Z} = |\boldsymbol{S}_{1\perp}||\boldsymbol{S}_{2\perp}|\,\frac{\sin^2\theta\,\cos(2\phi-\phi_{S_1}-\phi_{S_2})}{1+\cos^2\theta} \times \frac{\sum_a (V_a^2-A_a^2)\,\Delta_T f_a(x_1)\Delta_T \bar{f}_a(x_2)+[1\leftrightarrow 2]}{\sum_a (V_a^2+A_a^2)\,f_a(x_1)\bar{f}_a(x_2)+[1\leftrightarrow 2]}\,. \tag{6.31}$$

6.3 Factorisation in Drell–Yan processes

With a view to extending the results previously obtained to NLO in QCD, we now rederive them in the framework of QCD factorisation (Collins,

1993a). The Drell–Yan cross-section may be written in a factorised form as (hereafter, we omit the interchanged term $[1 \leftrightarrow 2]$)

$$\mathrm{d}\sigma = \sum_a \sum_{\alpha\alpha'\beta\beta'} \int \mathrm{d}\xi_1 \int \mathrm{d}\xi_2 \, \rho^{(1)}_{\alpha'\alpha} \, f_a(\xi_1, \mu^2) \, \rho^{(2)}_{\beta'\beta} \, \bar{f}_a(\xi_2, \mu^2) \times \left[\mathrm{d}\hat{\sigma}(Q^2/\mu^2, \alpha_s(\mu^2)) \right]_{\alpha\alpha'\beta\beta'} , \qquad (6.32)$$

where ξ_1 and ξ_2 are the momentum fractions of the quark (from hadron A) and antiquark (from B), $\rho^{(1)}$ and $\rho^{(2)}$ are the quark and antiquark spin density matrices, and $(\mathrm{d}\hat{\sigma})_{\alpha\alpha'\beta\beta'}$ is the cross-section matrix (in the quark and antiquark spin space) of the elementary subprocesses. As usual, μ denotes the factorisation scale.

At LO $\mathrm{d}\hat{\sigma}$ incorporates an overall energy–momentum conservation delta function, namely $\delta^4(k + k' - q)$, which sets $\xi_1 = x_1$, $\xi_2 = x_2$ and $\boldsymbol{q}_T = 0$. Thus, Eq. (6.32) becomes (omitting the momentum scales)

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}x_1 \, \mathrm{d}x_2 \, \mathrm{d}\Omega} = \sum_a \sum_{\alpha\alpha'\beta\beta'} \rho^{(1)}_{\alpha'\alpha} \, \rho^{(2)}_{\beta'\beta} \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega} \right)_{\alpha\alpha'\beta\beta'} f_a(\xi_1) \, \bar{f}_a(\xi_2) , \qquad (6.33)$$

where the only contributing subprocess is $q\bar{q} \to l^+l^-$ (see Fig. 6.4), with

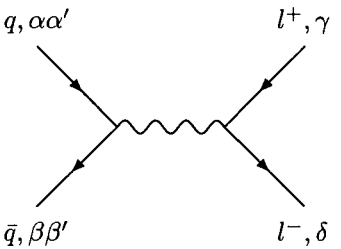

Fig. 6.4 The $q\bar{q} \to l^+l^-$ process contributing to Drell–Yan production at LO.

cross-section

$$\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega} \right)_{\alpha\alpha'\beta\beta'} = \frac{1}{64\pi^2 \hat{s}} \sum_{\gamma\delta} \mathcal{M}^*_{\alpha\beta\gamma\delta} \, \mathcal{M}_{\alpha'\beta'\gamma\delta} . \qquad (6.34)$$

The contributing scattering amplitudes are

$$\mathcal{M}_{++++} = \mathcal{M}_{----} , \qquad \mathcal{M}_{++--} = \mathcal{M}_{--++} , \qquad (6.35)$$

and cross-section (6.33) reduces to

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}\Omega} = \sum_a \left\{ \left(\rho^{(1)}_{++}\rho^{(2)}_{++} + \rho^{(1)}_{--}\rho^{(2)}_{--}\right)\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega}\right)_{++++} + \left(\rho^{(1)}_{+-}\rho^{(2)}_{+-} + \rho^{(1)}_{-+}\rho^{(2)}_{-+}\right)\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega}\right)_{+-+-}\right\} \times f_a(x_1)\,\bar{f}_a(x_2)\,. \tag{6.36}$$

Here the spin density matrix elements are, for the quark

$$\begin{aligned} \rho^{(1)}_{++} &= \tfrac{1}{2}(1+\lambda)\,, & \rho^{(1)}_{--} &= \tfrac{1}{2}(1-\lambda)\,, \\ \rho^{(1)}_{+-} &= \tfrac{1}{2}(s_x - \mathrm{i}\,s_y)\,, & \rho^{(1)}_{-+} &= \tfrac{1}{2}(s_x + \mathrm{i}\,s_y)\,, \end{aligned} \tag{6.37a}$$

and for the antiquark

$$\begin{aligned} \rho^{(2)}_{++} &= \tfrac{1}{2}(1+\bar{\lambda})\,, & \rho^{(2)}_{--} &= \tfrac{1}{2}(1-\bar{\lambda})\,, \\ \rho^{(2)}_{+-} &= \tfrac{1}{2}(\bar{s}_x - \mathrm{i}\,\bar{s}_y)\,, & \rho^{(2)}_{-+} &= \tfrac{1}{2}(\bar{s}_x + \mathrm{i}\,\bar{s}_y)\,, \end{aligned} \tag{6.37b}$$

so that (6.36) becomes

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}\Omega} = \frac{1}{2}\sum_a \left\{ (1+\lambda\bar{\lambda})\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega}\right)_{++++} + (s_x\,\bar{s}_x - s_y\,\bar{s}_y)\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega}\right)_{+-+-}\right\} f_a(x_1)\,\bar{f}_a(x_2)\,. \tag{6.38}$$

At LO, the scattering amplitudes for the $q\bar{q}$ annihilation process are

$$\mathcal{M}_{++++} = \mathcal{M}_{----} = e^2\,e_a\,(1-\cos\theta)\,, \tag{6.39a}$$

$$\mathcal{M}_{++--} = \mathcal{M}_{--++} = e^2\,e_a\,(1+\cos\theta)\,, \tag{6.39b}$$

and the elementary cross-sections then read

$$\begin{aligned} \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega}\right)_{++++} = \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega}\right)_{----} &= \frac{1}{64\pi^2\hat{s}}\left(|\mathcal{M}_{++++}|^2 + |\mathcal{M}_{++--}|^2\right) \\ &= \frac{\alpha^2_{\mathrm{em}}\,e_a^2}{2\hat{s}}\,\frac{1}{3}\left(1+\cos^2\theta\right), \end{aligned} \tag{6.40a}$$

$$\begin{aligned} \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega}\right)_{+-+-} = \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega}\right)_{-+-+} &= \frac{1}{64\pi^2\hat{s}}\,2\,\mathrm{Re}\left(\mathcal{M}^*_{++++}\,\mathcal{M}_{++--}\right) \\ &= \frac{\alpha^2_{\mathrm{em}}\,e_a^2}{2\hat{s}}\,\frac{1}{3}\,\sin^2\theta\,. \end{aligned} \tag{6.40b}$$

Inserting (6.40a, b) into (6.38) we obtain

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{4Q^2}\sum_a \frac{e_a^2}{3}\left\{(1+\lambda\bar{\lambda})\,(1+\cos^2\theta) + (s_x\,\bar{s}_x - s_y\,\bar{s}_y)\,\sin^2\theta\right\} f_a(x_1)\,\bar{f}_a(x_2)\,. \quad (6.41)$$

Using

$$\lambda\, f_a(x_1) = \lambda_1\,\Delta f_a(x_1)\,, \qquad \boldsymbol{s}_\perp\, f_a(x_1) = \boldsymbol{S}_{1\perp}\,\Delta_T f_a(x_1)\,, \quad (6.42a)$$

$$\bar{\lambda}\,\bar{f}_a(x_2) = \lambda_2\,\Delta\bar{f}_a(x_2)\,, \qquad \bar{\boldsymbol{s}}_\perp\,\bar{f}_a(x_2) = \boldsymbol{S}_{2\perp}\,\Delta_T\bar{f}_a(x_2)\,, \quad (6.42b)$$

we finally arrive at

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{4Q^2}\sum_a \frac{e_a^2}{3}\left\{\left[f_a(x_1)\,\bar{f}_a(x_2) + \lambda_1\,\lambda_2\,\Delta f_a(x_1)\,\Delta\bar{f}_a(x_2)\right](1+\cos^2\theta) + |\boldsymbol{S}_{1\perp}|\,|\boldsymbol{S}_{2\perp}|\,\cos(2\phi-\phi_{S_1}-\phi_{S_2}) \times \Delta_T f_a(x_1)\,\Delta_T\bar{f}_a(x_2)\,\sin^2\theta\right\}, \quad (6.43)$$

which is what we obtained in Sec. 6.2 in a different manner, see Eq. (6.27). Note that the angular factor appearing in A_{TT}^{DY} – Eq. (6.28) – is the elementary double transverse-spin asymmetry of the $q\bar{q}$ scattering process, namely

$$\hat{a}_{TT} \equiv \frac{\mathrm{d}\hat{\sigma}(\boldsymbol{s}_\perp,\bar{\boldsymbol{s}}_\perp) - \mathrm{d}\hat{\sigma}(\boldsymbol{s}_\perp,-\bar{\boldsymbol{s}}_\perp)}{\mathrm{d}\hat{\sigma}(\boldsymbol{s}_\perp,\bar{\boldsymbol{s}}_\perp) + \mathrm{d}\hat{\sigma}(\boldsymbol{s}_\perp,-\bar{\boldsymbol{s}}_\perp)} = \frac{\mathrm{d}\hat{\sigma}_{+-+-}}{\mathrm{d}\hat{\sigma}_{++++}}\,(s_x\bar{s}_x - s_y\bar{s}_y) = \frac{\sin^2\theta}{1+\cos^2\theta}\,\cos(2\phi-\phi_s-\phi_{\bar{s}})\,. \quad (6.44)$$

The Drell–Yan cross-section is most often expressed as a function of the rapidity of the virtual photon y defined as

$$y \equiv \frac{1}{2}\,\ln\frac{q^+}{q^-} = \frac{1}{2}\,\ln\frac{x_1}{x_2}\,. \quad (6.45)$$

In the lepton c.m. frame $y = \frac{1}{2}(1+\cos\theta)$. From $x_1x_2 \equiv \tau = Q^2/s$, we obtain

$$x_1 = \sqrt{\tau}\,\mathrm{e}^{y}\,, \qquad x_2 = \sqrt{\tau}\,\mathrm{e}^{-y}\,, \quad (6.46)$$

and (6.43) thus becomes (recalling $\mathrm{d}y\,\mathrm{d}Q^2 = s\,\mathrm{d}x_1\,\mathrm{d}x_2$)

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}y\,\mathrm{d}Q^2\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{4Q^2 s}\sum_a \frac{e_a^2}{3}\Big\{\Big[f_a(x_1)\,\bar{f}_a(x_2) + \lambda_1\,\lambda_2\,\Delta f_a(x_1)\,\Delta\bar{f}_a(x_2)\Big](1+\cos^2\theta) + |\boldsymbol{S}_{1\perp}|\,|\boldsymbol{S}_{2\perp}|\cos(2\phi-\phi_{S_1}-\phi_{S_2}) \times \Delta_T f_a(x_1)\,\Delta_T\bar{f}_a(x_2)\,\sin^2\theta\Big\}. \tag{6.47}$$

If we integrate over $\cos\theta$, we obtain

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}y\,\mathrm{d}Q^2\,\mathrm{d}\phi} = \frac{2\alpha_{\mathrm{em}}^2}{9Q^2 s}\sum_a e_a^2\Big\{f_a(x_1)\,\bar{f}_a(x_2) + \lambda_1\,\lambda_2\,\Delta f_a(x_1)\,\Delta\bar{f}_a(x_2) + \tfrac{1}{2}\,|\boldsymbol{S}_{1\perp}|\,|\boldsymbol{S}_{2\perp}|\cos(2\phi-\phi_{S_1}-\phi_{S_2}) \times \Delta_T f_a(x_1)\,\Delta_T\bar{f}_a(x_2)\Big\}. \tag{6.48}$$

Let us now extend the previous results to NLO. Here we are interested in the transverse-polarisation contribution to the Drell–Yan cross-section, which can be written as (reintroducing the scales)

$$\frac{\mathrm{d}^3\sigma_T}{\mathrm{d}y\,\mathrm{d}Q^2\,\mathrm{d}\phi} = \sum_a e_a^2\int\mathrm{d}\xi_1\int\mathrm{d}\xi_2\,\frac{\mathrm{d}\hat{\sigma}_T(Q^2/\mu^2,\alpha_s)}{\mathrm{d}y\,\mathrm{d}Q^2\,\mathrm{d}\phi} \times \Delta_T f_a(\xi_1,\mu^2)\,\Delta_T f_a(\xi_2,\mu^2). \tag{6.49}$$

We have seen that at LO, *i.e.*, $\mathcal{O}(\alpha_s^0)$, the elementary cross-section is

$$\text{LO}: \quad \frac{\mathrm{d}^3\hat{\sigma}_T}{\mathrm{d}y\,\mathrm{d}Q^2\,\mathrm{d}\phi} = \frac{\alpha_{\mathrm{em}}^2}{9Q^2 s}\cos(2\phi-\phi_s-\phi_{\bar{s}})\,\delta(\xi_1-x_1)\,\delta(\xi_2-x_2), \tag{6.50}$$

where $\phi_s(\phi_{\bar{s}})$ is the azimuthal angle of the quark (antiquark) spin, with respect to the lepton plane. Integrating over y we obtain

$$\text{LO}: \quad \frac{\mathrm{d}^3\hat{\sigma}_T}{\mathrm{d}Q^2\,\mathrm{d}\phi} = \frac{\alpha_{\mathrm{em}}^2}{9Q^2\hat{s}}\cos(2\phi-\phi_s-\phi_{\bar{s}})\,\delta(1-z), \tag{6.51}$$

with $z \equiv \tau/\xi_1\xi_2 = Q^2/\xi_1\xi_2 s$.

At NLO, *i.e.*, $\mathcal{O}(\alpha_s)$, the subprocesses contributing to Drell–Yan production are those shown in Fig. 6.5: virtual-gluon corrections and real-gluon emission. The NLO cross-section $\mathrm{d}\hat{\sigma}_T/\,\mathrm{d}y\,\mathrm{d}Q^2\,\mathrm{d}\phi$ exhibits ultraviolet singularities (arising from loop integrations), infrared singularities (due to soft gluons), and collinear singularities (when the gluon is emitted parallel to

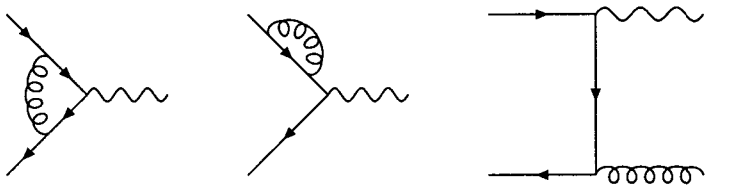

Fig. 6.5 Elementary processes contributing to the transverse Drell–Yan cross-section at NLO: (a, b) virtual-gluon corrections and (c) real-gluon emission.

the quark or antiquark). Summing virtual and real diagrams, only the collinear divergences survive. Working in $D = 4 - 2\epsilon$ dimensions, they are of the type $1/\epsilon$. These singularities are then subtracted and absorbed in the definition of the parton distributions.

The NLO elementary cross-sections have been computed by several authors with different methods (Vogelsang and Weber, 1993; Contogouris, Kamal and Merebashvili, 1994; Kamal, 1996; Vogelsang, 1998b)[a]. Vogelsang and Weber (1993) were the first to perform this calculation using massive gluons to regularise the divergences. Soon after, Contogouris, Kamal and Merebashvili (1994) presented a calculation based on dimensional reduction. The result was then translated into dimensional regularisation (DR) by Kamal (1996). As a check of the expression given by Kamal, Vogelsang (1998b) has shown how to exploit the earlier result obtained by Vogelsang and Weber (1993). From the detailed structure of the collinear singularities for both dimensional and off-shell regularisation, it is straight-forward to translate results from one scheme to another.

The expression for $\mathrm{d}^3\hat{\sigma}_T/\,\mathrm{d}y\,\mathrm{d}Q^2\,\mathrm{d}\phi$ is rather cumbersome and we do not repeat it here (instead, we refer the reader to the original papers). The y-integrated cross-section is more legible and reads, in the $\overline{\mathrm{MS}}$ scheme (Vogelsang, 1998b)

$$\mathrm{NLO}: \quad \frac{\mathrm{d}^2\hat{\sigma}_T}{\mathrm{d}Q^2\,\mathrm{d}\phi} = \frac{\alpha_{\mathrm{em}}^2}{9Q^2\hat{s}}\cos(2\phi - \phi_s - \phi_{\bar{s}})$$

[a]We recall that the NLO corrections to unpolarised Drell–Yan were presented by Altarelli, Ellis and Martinelli (1979) and Harada, Kaneko and Sakai (1979), and to longitudinally polarised Drell–Yan by Ratcliffe (1983).

$$\times C_{\mathrm{F}}\left\{8z\left[\frac{\ln(1-z)}{1-z}\right]_+ - \frac{4z\ln z}{1-z} - \frac{6z\ln^2 z}{1-z}\right.$$
$$\left. + 4(1-z) + \left(\frac{2\pi^2}{3} - 8\right)\delta(1-z)\right\}. \quad (6.52)$$

The quantity in curly brackets is the NLO Wilson coefficient $\Delta_T C_{\mathrm{DY}}^{(1)}$ for Drell–Yan. If we call $\Delta_T\widetilde{C}$ the quantity to be added to the Wilson coefficient in order to pass from the scheme of Kamal (1996) to $\overline{\mathrm{MS}}$, the result (6.52) coincides with that of Kamal for the choice $\Delta_T\widetilde{C} = -\delta(1-z)$. On the other hand, the expression for $\Delta_T\widetilde{C}$ claimed by Kamal as providing the translation to the $\overline{\mathrm{MS}}$ scheme (in DR) is $\Delta_T\widetilde{C} = 2(1-z) - \delta(1-z)$. Vogelsang (1998b) noted that the reason for this difference lies in the discrepancy between his own calculation and that of Kamal (1996) for the $(4-2\epsilon)$-dimensional LO splitting function, where extra $\mathcal{O}(\epsilon)$ terms were found. The correctness of the result of Vogelsang (1998b) for this quantity is supported by the observation that the D-dimensional $2 \to 3$ squared matrix element for the process $q\bar{q} \to \mu^+\mu^- g$ (with transversely polarised incoming (anti)quarks) given there factorises into the product of the D-dimensional $2 \to 2$ squared matrix element for $q\bar{q} \to \mu^+\mu^-$ multiplied by the splitting function $\Delta_T P_{qq}^{(0)}$ in $D = 4-2\epsilon$ dimensions, when the collinear limit of the gluon aligning parallel to one of the incoming quarks is correctly performed. It is thus claimed that the result of Kamal (1996) for the NLO transversely polarised Drell–Yan cross-section (in DR) corresponds to a different (non-$\overline{\mathrm{MS}}$) factorisation scheme.

Finally, a first step towards a next-to-next-to-leading order (NNLO) calculation of the transversely polarised Drell–Yan cross-section was taken by Chang, Corianò and Elwood (1997).

6.4 Twist-three contributions to the Drell–Yan process

At twist three, transversity distributions are also probed in Drell–Yan processes when one of the two hadrons is transversely polarised and the other is longitudinally polarised. In this case, ignoring subtleties related to quark masses and transverse motion (so that $h_L(x) = \widetilde{h}_L(x)$ and $g_T(x) = \widetilde{g}_T(x)$, see Sec. 3.8), the cross-section is (Jaffe and Ji, 1992; Tangerman and Mulders, 1995a; Boer, Mulders and Teryaev, 1997), the transversely polarised

hadron is A,

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{4Q^2}\sum_a \frac{e_a^2}{3}\left\{\ldots + |\boldsymbol{S}_{1\perp}|\,\lambda_2\,\sin 2\theta\,\cos(\phi-\phi_{S_1})\right.$$
$$\left.\times\left[\frac{2M_1}{Q}\,x_1\,g_T^a(x_1)\,\Delta\bar{f}_a(x_2) + \frac{2M_2}{Q}\,x_2\,\Delta_T f_a(x_1)\,\bar{h}_L^a(x_2)\right]\right\}, \quad (6.53)$$

where the ellipsis denotes the leading-twist contributions already shown in Eq. (6.27). The transversity distribution of quarks in hadron A is coupled to the twist-three antiquark distribution $\bar{h}_L$. The longitudinal–transverse asymmetry resulting from (6.53) is (we assume the masses of the two hadrons to be equal, *i.e.*, $M_1 = M_2 \equiv M$)

$$A_{LT}^{\mathrm{DY}} = |\boldsymbol{S}_{1\perp}|\,\lambda_2\,\frac{2\sin 2\theta\,\cos(\phi-\phi_{S_1})}{1+\cos^2\theta}\,\frac{M}{Q}$$
$$\times\,\frac{\sum_a e_a^2\left[x_1\,g_T^a(x_1)\,\Delta\bar{f}_a(x_2) + x_2\,\Delta_T f_a(x_1)\,\bar{h}_L^a(x_2)\right]}{\sum_a e_a^2\,f_a(x_1)\bar{f}_a(x_2)}. \quad (6.54)$$

Let us now consider the case where one of the two hadrons is unpolarised while the other is transversely polarised. In this situation single-spin asymmetries might arise at twist three from T-odd distribution functions. The corresponding contribution to the cross-section is

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{4Q^2}\sum_a \frac{e_a^2}{3}\left\{\cdots + |\boldsymbol{S}_{1\perp}|\,\sin 2\theta\,\sin(\phi-\phi_{S_1})\right.$$
$$\left.\times\left[\frac{2M_1}{Q}\,x_1\,\widetilde{f}_T^a(x_1)\,\bar{f}_a(x_2) + \frac{2M_2}{Q}\,x_2\,\Delta_T f_a(x_1)\,\widetilde{\bar{h}}_a(x_2)\right]\right\}. \quad (6.55)$$

Here $\widetilde{f}_T(x)$ and $\widetilde{\bar{h}}(x)$ are the twist-three T-odd distribution functions introduced in Sec. 3.10. From (6.55) we obtain the single-spin asymmetry (Boer, Mulders and Teryaev, 1997; Boer, Jakob and Mulders, 2000; Di Salvo, 2001)

$$A_T^{\mathrm{DY}} = |\boldsymbol{S}_{1\perp}|\,\frac{2\sin 2\theta\,\sin(\phi-\phi_{S_1})}{1+\cos^2\theta}\,\frac{M}{Q}$$
$$\times\,\frac{\sum_a e_a^2\left[x_1\,f_T^a(x_1)\,\bar{f}_a(x_2) + x_2\,\Delta_T f_a(x_1)\,\widetilde{\bar{h}}_a(x_2)\right]}{\sum_a e_a^2\,f_a(x_1)\bar{f}_a(x_2)}. \quad (6.56)$$

The existence of T-odd distribution functions has also been advocated by Boer (1999) to explain, at leading-twist level, an anomalously large $\cos 2\phi$ term in the unpolarised Drell–Yan cross-section (Falciano *et al.*,

1986; Guanziroli *et al.*, 1988; Conway *et al.*, 1989), which cannot be accounted for by LO or NLO QCD (Brandenburg, Nachtmann and Mirkes, 1993)—it can however be attributed to higher-twist effects (Berger and Brodsky, 1979; Berger, 1980; Brandenburg *et al.*, 1994). Boer has shown that, on introducing initial-state T-odd effects, the unpolarised Drell–Yan cross-section indeed acquires a $\cos 2\phi$ contribution involving the product $h_1^\perp(x_1, \boldsymbol{k}_\perp^2)\, \bar{h}_1^\perp(x_2, \boldsymbol{k}'^2_\perp)$. If hadron A is transversely polarised, the same mechanism generates a $\sin(\phi + \phi_{S_1})$ term, which depends on the product $\Delta_T f(x_1, \boldsymbol{k}_\perp^2)\, \bar{h}_1^\perp(x_2, \boldsymbol{k}'^2_\perp)$.

It must be stressed once again that the mechanism based on initial-state interactions is highly hypothetical, if not at all unlikely. However, it was shown by Hammon, Teryaev and Schäfer (1997), Boer, Mulders and Teryaev (1997); Boer, Jakob and Mulders (2000); and Boer and Qiu (2002) that single-spin asymmetries might arise in Drell–Yan processes owing to the so-called gluonic poles in twist-three multiparton correlation functions (Efremov and Teryaev, 1982, 1985; Qiu and Sterman, 1991a, 1992, 1999). Let us briefly address this issue (for a general discussion of higher twists in hadron scattering see Efremov and Teryaev, 1984; Ratcliffe, 1986, 1999b; Qiu and Sterman, 1991b,c).

At twist three the Drell–Yan process is governed by diagrams such as that in Fig. 6.6. The hadronic tensor is then (we drop the subscripts 1 and

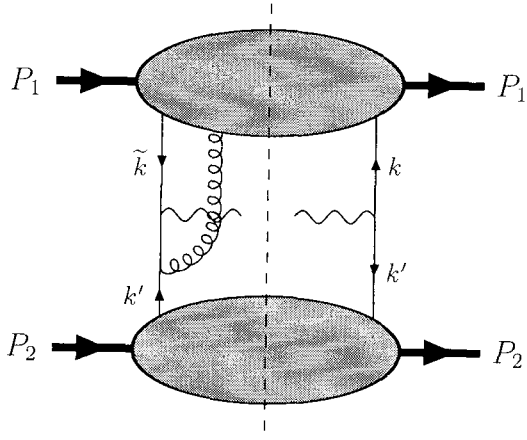

Fig. 6.6 One of the diagrams contributing to the Drell–Yan cross-section at twist three.

2 from the quark correlation matrices for simplicity)

$$W^{\mu\nu} = \frac{1}{3}\sum_a e_a^2 \left\{ \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int \frac{\mathrm{d}^4k'}{(2\pi)^4}\, \delta(k+k'-q)\ \mathrm{Tr}\left[\Phi\,\gamma^\mu\,\overline{\Phi}\,\gamma^\nu\right] \right.$$
$$- \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int \frac{\mathrm{d}^4k'}{(2\pi)^4} \int \frac{\mathrm{d}^4\widetilde{k}}{(2\pi)^4}\, \delta(k+k'-q)$$
$$\left. \times\ \mathrm{Tr}\left[\gamma_\sigma\, \frac{\widetilde{\not{k}} - \not{q}}{(\widetilde{k}-q)^2 + \mathrm{i}\epsilon}\, \gamma^\nu\, \Phi_A^\sigma\, \gamma^\mu\, \overline{\Phi}\right] + \ldots \right\}, \quad (6.57)$$

where we have retained only one of the twist-three contributions, and Φ_A^σ is the quark–quark–gluon correlation matrix defined in (3.67). Neglecting $1/Q^2$ terms, the quark propagator in (6.57) gives ($\widetilde{k} = y_1 P_1$)

$$\frac{\widetilde{\not{k}} - \not{q}}{(\widetilde{k}-q)^2 + \mathrm{i}\epsilon} \to -\frac{\gamma^-}{2x_2 P_2^-}\, \frac{x_1 - y_1}{x_1 - y_1 + \mathrm{i}\epsilon}\,. \quad (6.58)$$

Let us now introduce another quark–quark–gluon correlator ($F^{\mu\nu}$ is the gluon field-strength tensor)

$$\Phi_{Fij}^\mu(x,y) = \int \frac{\mathrm{d}\tau}{2\pi} \int \frac{\mathrm{d}\eta}{2\pi}\, \mathrm{e}^{\mathrm{i}\tau y}\, \mathrm{e}^{\mathrm{i}\eta(x-y)}$$
$$\times \langle PS|\bar{\psi}_j(0)\, F^{+\mu}(\eta n)\, \psi_i(\tau n)|PS\rangle\,, \quad (6.59)$$

which can be parametrised as, see the analogous decomposition of $\Phi_A^\mu(x,y)$ Eq. (3.71),

$$\Phi_F^\mu(x,y) = \frac{M}{2}\left\{ \mathrm{i}\, G_F(x,y)\, \varepsilon_\perp^{\mu\nu} S_{\perp\nu} \not{P} + \widetilde{G}_F(x,y)\, S_\perp^\mu \gamma_5 \not{P} \right.$$
$$\left. + H_F(x,y)\, \lambda_N \gamma_5 \gamma_\perp^\mu \not{P} + E_F(x,y)\, \gamma_\perp^\mu \not{P} \right\}. \quad (6.60)$$

In the $A^+ = 0$ gauge one has $F^{+\mu} = \partial^+ A_\perp^\mu$ and by partial integration one finds the following relation between $\Phi_A^\mu(x,y)$ and $\Phi_F^\mu(x,y)$

$$(x-y)\, \Phi_A^\mu(x,y) = -\mathrm{i}\, \Phi_F^\mu(x,y)\,. \quad (6.61)$$

Thus, if some projection of $\Phi_F^\mu(x,x)$ is non-zero, the corresponding projection of $\Phi_A^\mu(x,x)$ must have a pole (the "gluonic pole").

From (6.58) and (6.61), we see that the trace in the twist-three term of (6.57) contains the quantity (P.V. stands for principal value)

$$\begin{aligned}\frac{x_1-y_1}{x_1-y_1+\mathrm{i}\epsilon}\,\Phi_A^\sigma(x_1,y_1) &= \frac{-\mathrm{i}}{x_1-y_1+\mathrm{i}\epsilon}\,\Phi_F^\sigma(x_1,y_1)\\ &= \mathrm{P.V.}\,\frac{-\mathrm{i}}{x-y}\,\Phi_F^\mu(x,y)\\ &\quad -\pi\,\delta(x_1-y_1)\,\Phi_F^\sigma(x_1,x_1)\,. \qquad (6.62)\end{aligned}$$

Keeping the real term in (6.62), one ultimately finds that the Drell–Yan cross-section with one transversely polarised hadron and one unpolarised hadron involves, at twist three, the multiparton distributions $G_F(x_1,x_1)$ and $E_F(x_1,x_1)$; the former is proportional to the distribution $T(x_1,x_1)$ introduced by Qiu and Sterman (1992) and by Hammon, Teryaev and Schäfer (1997). The single-spin asymmetry is then expressed as

$$\begin{aligned}A_T^{\mathrm{DY}} \propto |\boldsymbol{S}_{1\perp}|\,&\frac{2\sin 2\theta\,\sin(\phi-\phi_{S_1})}{1+\cos^2\theta}\,\frac{M}{Q}\\ &\times\frac{\sum_a e_a^2\,[G_F^a(x_1,x_1)\,\bar f_a(x_2)+\Delta_T f_a(x_1)\,E_F^a(x_2,x_2)]}{\sum_a e_a^2\,f_a(x_1)\bar f_a(x_2)}\,. \qquad (6.63)\end{aligned}$$

To establish a connection between (6.63) and (6.56), let us now invert $F^{+\mu}=\partial^+ A_\perp^\mu$, hence obtaining

$$\begin{aligned}\Phi_A^\mu(x,y) &= \frac{1}{2}\,\delta(x-y)\left[\Phi_{A(\infty)}^\mu(x)+\Phi_{A(-\infty)}^\mu(x)\right]\\ &\quad +\mathrm{P.V.}\,\frac{-\mathrm{i}}{x-y}\,\Phi_F^\mu(x,y)\,. \qquad (6.64)\end{aligned}$$

If we now impose antisymmetric boundary conditions (Boer, Mulders and Teryaev, 1997), *i.e.*,

$$\Phi_{A(\infty)}^\mu(x) = -\Phi_{A(-\infty)}^\mu(x)\,, \qquad (6.65)$$

then (6.64) reduces to

$$\Phi_A^\mu(x,y) = \mathrm{P.V.}\,\frac{-\mathrm{i}}{x-y}\,\Phi_F^\mu(x,y)\,, \qquad (6.66)$$

and (6.62) becomes ("eff" stands for "effective")

$$\begin{aligned}\Phi_A^{\sigma,\mathrm{eff}} &\equiv \frac{x_1-y_1}{x_1-y_1+\mathrm{i}\epsilon}\,\Phi_A^\sigma(x_1,y_1)\\ &= \Phi_A^\sigma(x_1,y_1)-\pi\,\delta(x_1-y_1)\,\Phi_F^\sigma(x_1,x_1)\,. \qquad (6.67)\end{aligned}$$

The important observation is that Φ_A^σ and Φ_F^σ have opposite behaviour with respect to time reversal and hence $\Phi_A^{\sigma,\text{eff}}$ has no definite behaviour under this transformation. Consequently, the T-even functions of Φ_F^σ can be identified with T-odd functions in the effective correlation matrix $\Phi_A^{\sigma,\text{eff}}$. This mechanism gives rise to two effective T-odd distributions $\widetilde{f}_T^{\text{eff}}(x)$ and $\widetilde{h}^{\text{eff}}(x)$, which were related to the multiparton distribution functions by Boer, Mulders and Teryaev (1997), omitting some factors:

$$\widetilde{f}_T^{\text{eff}}(x) \sim \int \mathrm{d}y \, \mathrm{Im}\, G_A^{\text{eff}}(x,y) \sim G_F(x,x)\,, \tag{6.68a}$$

$$\widetilde{h}^{\text{eff}}(x) \sim \int \mathrm{d}y \, \mathrm{Im}\, E_A^{\text{eff}}(x,y) \sim E_F(x,x)\,. \tag{6.68b}$$

In the light of this correspondence one can see that Eq. (6.56), based on T-odd distributions, and Eq. (6.63), based on multiparton distributions, translate into each other. Thus, at least in the case in which the T-odd functions appear at twist three, there is an explanation for them in terms of quark–gluon interactions, with no need for initial-state effects.

In conclusion, let us summarise the various contributions of transversity to the Drell–Yan cross-section in Table 6.1.

Table 6.1 Contributions to the Drell–Yan cross-section involving transversity. The asterisk denotes T-odd terms.

Drell–Yan cross-section			
	A	B	observable
twist 2	T	T	$\Delta_T f(x_1)\Delta_T \bar{f}(x_2)$
twist 3	T	0	$\Delta_T f(x_1)\, \bar{h}(x_2)$ $^{(*)}$
	T	L	$\Delta_T f(x_1)\, \bar{h}_L(x_2)$

6.5 Predictions for Drell–Yan double transverse-spin asymmetries

Owing to the current lack of knowledge of $\Delta_T f$ the calculations of physical observables related to transversity are somewhat model-dependent and should therefore be taken with a pinch of salt. They essentially provide

an indication of the order of magnitude of some phenomenological quantities. The case of Drell–Yan processes is rather fortunate because no other unknown ingredients, such as transverse-polarisation fragmentation functions, occur: this gives us confidence that predictions in this context are quite reliable.

The Drell–Yan double transverse-spin asymmetries were calculated at LO by Barone, Calarco and Drago (1997a,b) and at NLO by Martin *et al.* (1998) – see also Hino *et al.* (1998). Earlier estimates by Ji (1992a); Bourrely and Soffer (1994) suffered a serious problem: they assumed the same QCD evolution for Δf and $\Delta_T f$, which (as we have explained) will always lead to over-optimistic values for A_{TT}^{DY} at higher energies.

In the calculation of Barone, Calarco and Drago (1997a) the equality $\Delta_T f(x, Q_0^2) = \Delta f(x, Q_0^2)$ was assumed to hold at some very low scale (the input scale $Q_0^2 = 0.23\,\mathrm{GeV}^2$ of the GRV distributions), as suggested by various non-perturbative and confinement model calculations (see Sec. 3.11). The transversity distributions were then evolved according to their own DGLAP equation at LO. The resulting asymmetry (divided by the partonic asymmetry) is shown in Fig. 6.7. Its value is just a few percent, which renders the planned RHIC measurement of A_{TT}^{DY} rather difficult. The asymmetry for the Z^0-mediated Drell–Yan process is plotted in Fig. 6.8 and has the same order of magnitude as in the electromagnetic case.

Martin *et al.* (1998) use a different procedure to estimate the transversity distributions. They set $|\Delta_T f| = 2\,(f + \Delta f)$ at the GRV input scale, thus imposing saturation of the Soffer inequality. Such a choice yields the maximal value for A_{TT}^{DY}. The transversity distributions are then evolved in NLO QCD. Although non-negligible, the NLO corrections are found to be relatively small. Curves for the predicted A_{TT}^{DY} are shown in Fig. 6.9.

Summarising the results of the calculations of A_{TT}^{DY}, we can say that at the typical energies of the RHIC experiments ($\sqrt{s} > 100\,\mathrm{GeV}$, see Bland, 2000; Enyo, 2000) one expects for the double-spin asymmetry, integrated over the invariant mass Q^2 of the dileptons, a value

$$A_{TT}^{\mathrm{DY}} \sim (1-2)\%, \quad \text{at most.} \tag{6.69}$$

It is interesting to note that as $\sqrt{s}$ falls the asymmetry tends to increase, as was first pointed out by Barone, Calarco and Drago (1997a). Thus, at $\sqrt{s} = 40\,\mathrm{GeV}$, which would correspond to the c.m. energy of the proposed (though later cancelled) HERA-$\vec{N}$ experiment (Anselmino *et al.*, 1996a),

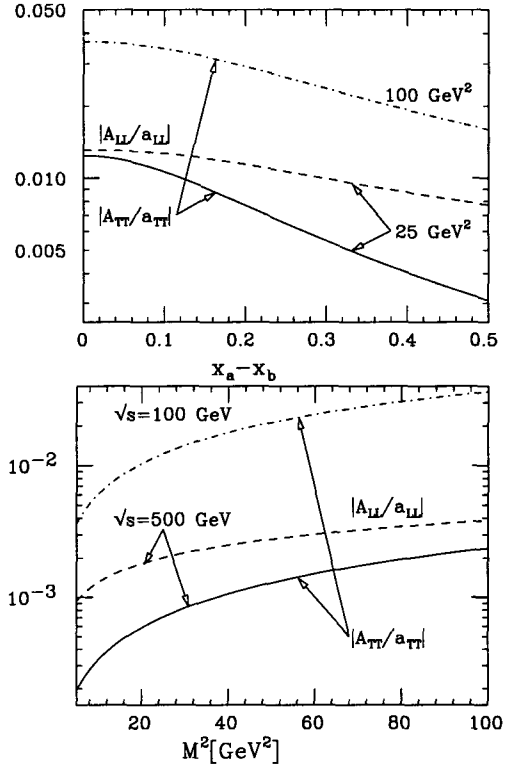

Fig. 6.7 Drell–Yan double longitudinal- and transverse-spin asymmetries normalised to the partonic asymmetry, as a function of $x_a - x_b$ (*i.e.*, $x_1 - x_2$) for two values of the dilepton invariant mass (upper), and as a function of the invariant mass of the dilepton pair M^2 for two values of the c.m. energy (lower). From Barone, Calarco and Drago (1997a).

A_{TT}^{DY} could reach a value of $\sim (3-4)\%$.

Model calculations of A_{TT}^{DY} have been presented by Barone, Calarco and Drago (1997b) and by Cano, Faccioli and Traini (2000). The longitudinal–transverse Drell–Yan asymmetry A_{LT}^{DY} (see Sec. 6.4) has been estimated by Kanazawa, Koike and Nishiyama (1998, 2000) and found to be five to ten times smaller than the double transverse-spin asymmetry. Polarised proton–deuteron Drell–Yan processes have been investigated by Hino and Kumano (1999a,b, 2000), and by Kumano and Miyama (2000).

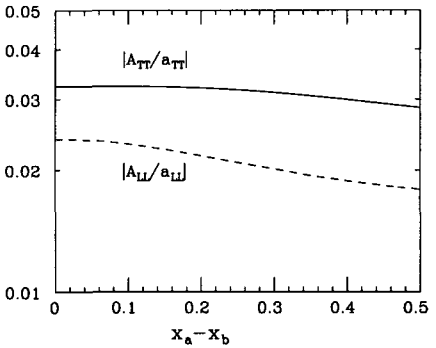

Fig. 6.8 Double longitudinal- and transverse-spin asymmetries (normalised to the partonic asymmetry) for the Z^0-mediated Drell–Yan process. From Barone, Calarco and Drago (1997a).

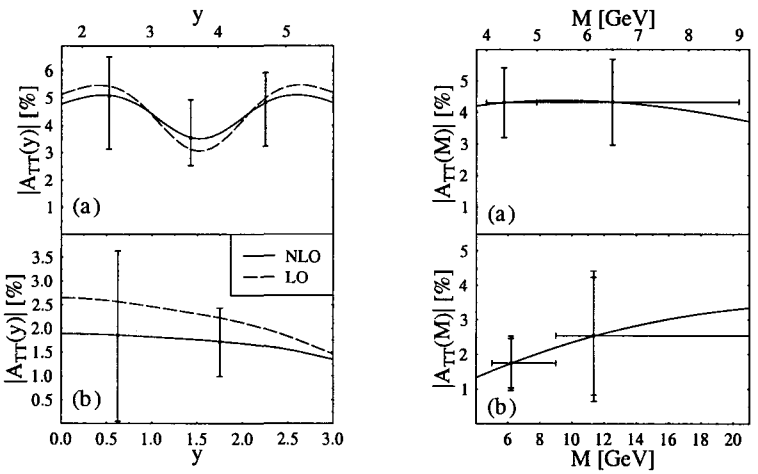

Fig. 6.9 The Drell–Yan double transverse-spin asymmetry as a function of the virtual photon rapidity y and of the dilepton invariant mass M, for two values of the c.m. energy: (a) $\sqrt{s} = 40\,\text{GeV}$ (corresponding to HERA-$\vec{N}$) and (b) $\sqrt{s} = 200\,\text{GeV}$ (corresponding to RHIC). The error bars represent the estimated statistical uncertainties of the two experiments. From Martin *et al.* (1998).

6.6 Transversity at RHIC

At the Brookhaven National Laboratory the Relativistic Heavy Ion Collider (RHIC), see Fig. 6.10, operates with gold ions and protons. With the

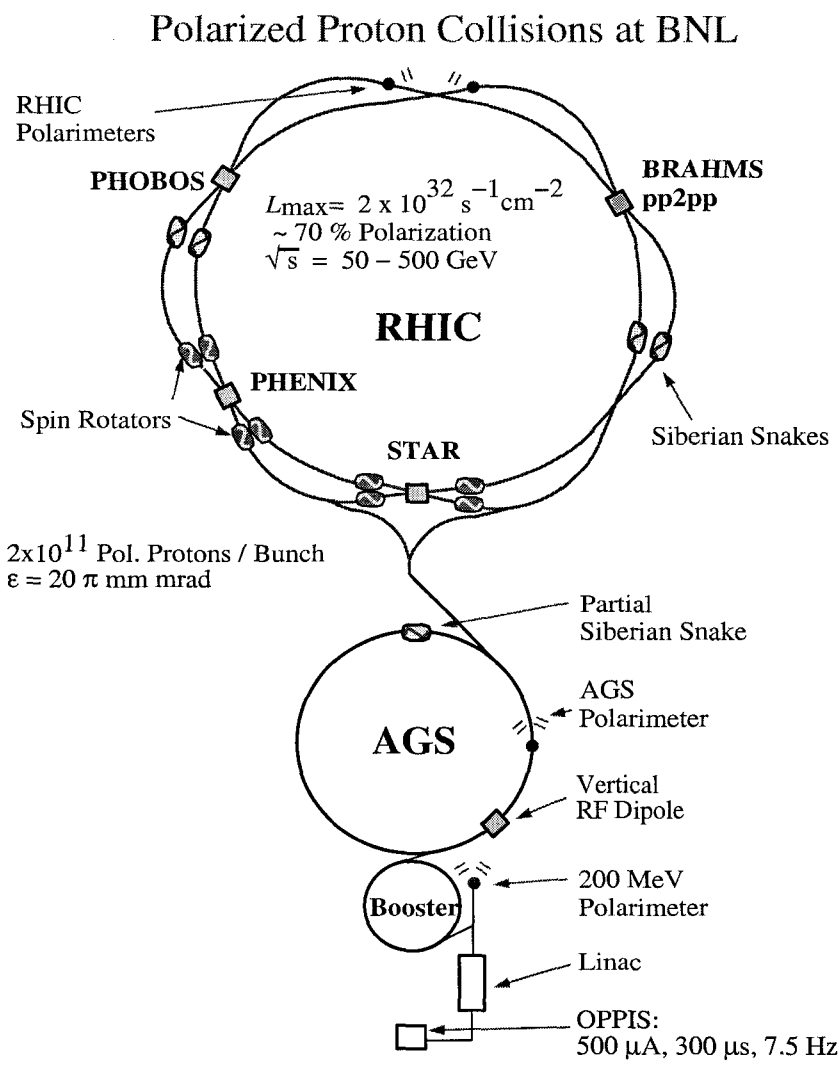

Fig. 6.10 An overview of the Relativistic Heavy Ion Collider (RHIC).

addition of Siberian snakes and spin rotators, there will be the possibility of accelerating intense polarised proton beams up to energies of 250 GeV per beam. The spin-physics program at RHIC will study reactions involving two polarised proton beams with both longitudinal and transverse-spin ori-

entations, at an average centre-of-mass energy of 500 GeV (for an overview of spin physics at RHIC, see Bunce *et al.*, 2000; Saito, 2001). The expected luminosity is up to $\sim 2{\cdot}10^{32}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-2}$, with 70% beam polarisation. Two detectors will be in operation: STAR (see, *e.g.*, Bland, 2000) and PHENIX (see, *e.g.*, Enyo, 2000). The former is a general purpose detector with a large solid angle; the latter is a dedicated detector mainly for leptons and photons. Data taking with gold ions started in 2000; runs with polarised protons are expected for the near future.

Experiments at RHIC aim to study Drell–Yan lepton-pair production mediated by either γ^* or Z^0. As seen in the previous section, the expected double transverse-spin asymmetry A_{TT}^{DY} is just few percent but may still be visible experimentally, provided the transversity distributions are not too small. Single-spin Drell–Yan measurements could prove to be a good testing ground for the existence of transversity in unpolarised hadrons arising from T-odd initial-state interaction effects (Boer, 1999).

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Chapter 7

Transversity in inclusive leptoproduction

While not contributing to fully inclusive DIS, transversity does play a rôle in inclusive leptoproduction, owing to the presence of (at least) two hadrons: one in the initial state and the other in the final state (Artru and Mekhfi, 1990; Cortes, Pire and Ralston, 1992; Artru, 1993; Collins, 1993b; Jaffe and Ji, 1993; Tangerman and Mulders, 1995b; Anselmino, Leader and Murgia, 1997). This process is the subject of the present chapter. As we shall see, the phenomenology of transverse polarisation in inclusive leptoproduction is extremely rich and complex, since in addition to the internal structure of the target one has to take into account the dynamics of the fragmentation process.

7.1 Single-particle leptoproduction: definitions and kinematics

Single-particle inclusive leptoproduction (see Fig. 7.1) is a DIS reaction in which a hadron h, produced in the current fragmentation region, is detected in the final state (for the general formalism see Meng, Olness and Soper, 1992; Mulders and Tangerman, 1996)

$$l(\ell) + N(P) \to l'(\ell') + h(P_h) + X(P_X)\,. \tag{7.1}$$

With a transversely polarised target, one can measure quark transverse polarisation at leading twist either by looking at a possible asymmetry in the $\boldsymbol{P}_{h\perp}$ distribution of the produced hadron—the so-called Collins effect (Collins, 1993b; Collins, Heppelmann and Ladinsky, 1994; Mulders and Tangerman, 1996; Boer and Mulders, 1998), or by polarimetry of a trans-

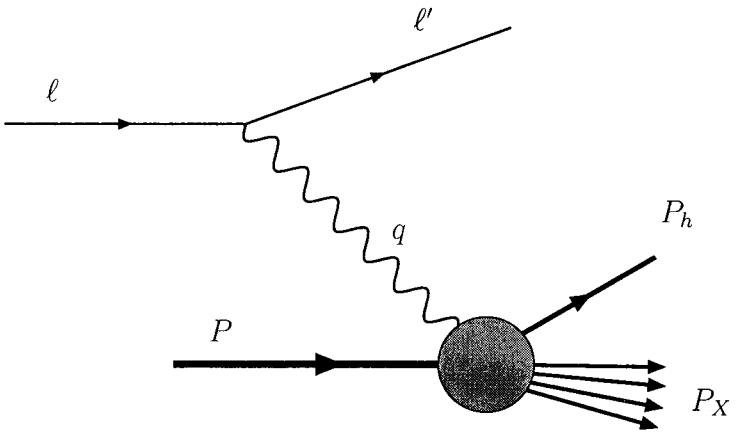

Fig. 7.1 Semi-inclusive deeply-inelastic scattering.

versely polarised final hadron, a Λ^0 hyperon for instance (Artru and Mekhfi, 1990, 1991; Jaffe, 1996; Mulders and Tangerman, 1996). Transversity distributions also appear in the $\boldsymbol{P}_{h\perp}$-integrated cross-section at higher twist (Jaffe and Ji, 1993; Mulders and Tangerman, 1996).

We define the invariants

$$x = \frac{Q^2}{2P{\cdot}q}\,, \quad y = \frac{P{\cdot}q}{P{\cdot}\ell}\,, \quad z = \frac{P{\cdot}P_h}{P{\cdot}q}\,. \tag{7.2}$$

We shall be interested in the limit where $Q^2 \equiv -q^2, P{\cdot}q, P_h{\cdot}q$ and $P_h{\cdot}P$ become large while x and z remain finite. The geometry of the process is shown in Fig. 7.2. The lepton scattering plane is identified by $\boldsymbol{\ell}$ and $\boldsymbol{\ell}'$. The virtual photon is taken to move along the z-axis. The three-momenta of the virtual photon $\boldsymbol{q}$ and of the produced hadron $\boldsymbol{P}_h$ define a second plane, which we call the hadron plane. The spin S of the nucleon and the spin S_h of the produced hadron satisfy $S^2 = S_h^2 = -1$ and $S{\cdot}P = S_h{\cdot}P_h = 0$. The cross-section for the reaction (7.1) is

$$\mathrm{d}^6\sigma = \frac{1}{4P{\cdot}\ell} \sum_{s_{l'}} \sum_X \int \frac{\mathrm{d}^3\boldsymbol{P}_X}{(2\pi)^3\, 2E_X}$$
$$\times\, (2\pi)^4 \delta^4(P + \ell - P_X - P_h - \ell')\, |\mathcal{M}|^2\, \frac{\mathrm{d}^3\boldsymbol{\ell}'}{(2\pi)^3\, 2E'}\, \frac{\mathrm{d}^3\boldsymbol{P}_h}{(2\pi)^3\, 2E_h}\,, \tag{7.3}$$

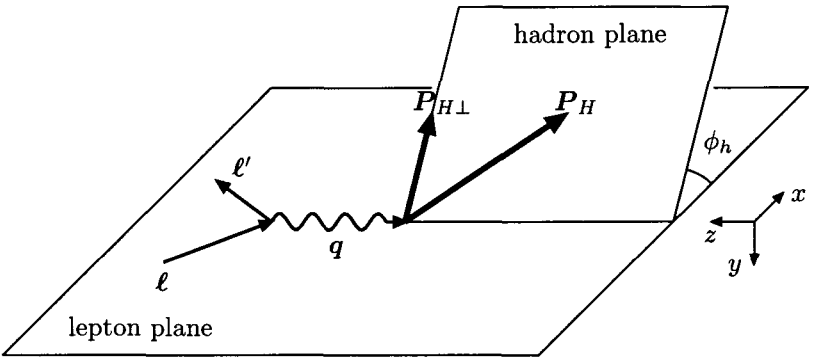

Fig. 7.2 The lepton and hadron planes in semi-inclusive leptoproduction.

where we have summed over the spin $s_{l'}$ of the outgoing lepton. The squared matrix element in (7.3) is

$$|\mathcal{M}|^2 = \frac{e^4}{q^4}\left[\bar{u}_{l'}(\ell', s_{l'})\gamma_\mu u_l(\ell, s_l)\right]^* \left[\bar{u}_{l'}(\ell', s_{l'})\gamma_\nu u_l(\ell, s_l)\right] \times \langle X, P_h S_h|J^\mu(0)|PS\rangle^* \langle X, P_h S_h|J^\nu(0)|PS\rangle\,, \tag{7.4}$$

Introducing the leptonic tensor

$$L_{\mu\nu} = \sum_{s_{l'}} \left[\bar{u}_{l'}(\ell', s_{l'})\gamma_\mu u_l(\ell, s_l)\right]^* \left[\bar{u}_{l'}(\ell', s_{l'})\gamma_\nu u_l(\ell, s_l)\right] = 2(\ell_\mu \ell'_\nu + \ell_\nu \ell'_\mu - g_{\mu\nu}\,\ell{\cdot}\ell') + 2\mathrm{i}\,\lambda_l\,\varepsilon_{\mu\nu\rho\sigma}\ell^\rho q^\sigma\,, \tag{7.5}$$

and the hadronic tensor

$$W^{\mu\nu} = \frac{1}{(2\pi)^4}\sum_X \int \frac{\mathrm{d}^3\boldsymbol{P}_X}{(2\pi)^3\,2E_X}\,(2\pi)^4\delta^4(P + q - P_X - P_h) \times \langle PS|J^\mu(0)|X, P_h S_h\rangle\langle X, P_h S_h|J^\nu(0)|PS\rangle\,, \tag{7.6}$$

the cross-section becomes

$$\mathrm{d}^6\sigma = \frac{1}{4P{\cdot}\ell}\,\frac{e^4}{Q^4}\,L_{\mu\nu}W^{\mu\nu}\,(2\pi)^4\,\frac{\mathrm{d}^3\boldsymbol{\ell}'}{(2\pi)^3\,2E'}\,\frac{\mathrm{d}^3\boldsymbol{P}_h}{(2\pi)^3\,2E_h}\,, \tag{7.7}$$

In the target rest frame ($P{\cdot}\ell = ME$) one has

$$2E_h\,\frac{\mathrm{d}^5\sigma}{\mathrm{d}E'\,\mathrm{d}\Omega\,\mathrm{d}^3\boldsymbol{P}_h} = \frac{\alpha_{\mathrm{em}}^2}{2MQ^4}\,\frac{E'}{E}\,L_{\mu\nu}W^{\mu\nu}\,, \tag{7.8}$$

In terms of the invariants x, y and z, Eq. (7.8) reads

$$2E_h \frac{\mathrm{d}^5\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}^3\boldsymbol{P}_h} = \frac{\pi\alpha_{\mathrm{em}}^2 y}{Q^4} L_{\mu\nu} W^{\mu\nu} . \tag{7.9}$$

If we decompose the momentum $\boldsymbol{P}_h$ of the produced hadron into a longitudinal ($\boldsymbol{P}_{h\parallel}$) and a transverse ($\boldsymbol{P}_{h\perp}$) component with respect to the γ^*N axis and if $|\boldsymbol{P}_{h\perp}|$ is small compared to the energy E_h, then we can write approximately

$$\frac{\mathrm{d}^3\boldsymbol{P}_h}{2E_h} = \frac{1}{2z}\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P}_{h\perp} , \tag{7.10}$$

and re-express Eq. (7.9) as

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P}_{h\perp}} = \frac{\pi\alpha_{\mathrm{em}}^2}{2Q^4}\frac{y}{z} L_{\mu\nu} W^{\mu\nu} . \tag{7.11}$$

Instead of working in a γ^*N collinear frame, it is often convenient to work in a frame where the target nucleon and the produced hadron move collinearly (the hN collinear frame, see App. D.2). In this frame the virtual photon has transverse momentum $\boldsymbol{q}_T$, which is related to $\boldsymbol{P}_{h\perp}$, up to $\mathcal{O}(1/Q^2)$ corrections, by $\boldsymbol{q}_T \simeq -\boldsymbol{P}_{h\perp}/z$. Thus, Eq. (7.11) can be written as

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{q}_T} = \frac{\pi\alpha_{\mathrm{em}}^2}{2Q^4} yz\, L_{\mu\nu} W^{\mu\nu} . \tag{7.12}$$

Let us now evaluate the leptonic tensor. In the γ^*N collinear frame the lepton momenta can be parametrised in terms of the Sudakov vectors p and n as

$$\ell^\mu = \frac{x}{y}(1-y)\,p^\mu + \frac{Q^2}{2xy}\,n^\mu + \ell_\perp^\mu , \tag{7.13a}$$

$$\ell'^\mu = \frac{x}{y}\,p^\mu + \frac{Q^2(1-y)}{2xy}\,n^\mu + \ell_\perp^\mu , \tag{7.13b}$$

with $\ell_\perp^2 = Q^2(1-y)/y^2$. The symmetric part of the leptonic tensor then

becomes

$$L^{\mu\nu(\mathrm{S})} = -\frac{Q^2}{y^2}\left[1+(1-y)^2\right] g_\perp^{\mu\nu} + \frac{4(1-y)}{y^2}\, t^\mu t^\nu + \frac{2(2-y)}{y}\left(t^\mu \ell_\perp^\nu + t^\nu \ell_\perp^\mu\right) + \frac{2Q^2\,(1-y)}{y^2}\left(\frac{2\ell_\perp^\mu \ell_\perp^\nu}{\ell_\perp^2} + g_\perp^{\mu\nu}\right), \tag{7.14a}$$

where $t^\mu = 2xp^\mu + q^\mu$; the antisymmetric part reads

$$L^{\mu\nu(A)} = \lambda_l\, \varepsilon_{\mu\nu\rho\sigma}\left[\frac{Q^2(2-y)}{y}\, p^\rho n^\sigma + \frac{Q^2}{x}\, \ell_\perp^\rho n^\sigma - 2x\, \ell_\perp^\rho p^\sigma\right]. \tag{7.14b}$$

At leading-twist level only semi-inclusive DIS processes with unpolarised lepton beams probe quark transverse-polarisation distributions (Mulders and Tangerman, 1996). Therefore, in what follows, we shall focus on this case and take only the target nucleon (and, possibly, the outgoing hadron) to be polarised. At twist three there are also semi-inclusive DIS reactions with polarised leptons, which allow the extraction of $\Delta_T f$. For these higher-twist processes we shall limit ourselves to presenting the cross-sections without derivation.

7.2 The partonic description of semi-inclusive DIS

In the parton model the virtual photon strikes a quark (or antiquark), which later fragments into a hadron h. The process is depicted in Fig. 7.3. The relevant diagram is the handbag diagram with an upper blob representing the fragmentation process. Referring to Fig. 7.3 for the notation, the hadronic tensor is given by (for simplicity we consider only the quark contribution)

$$W^{\mu\nu} = \frac{1}{(2\pi)^4}\sum_a e_a^2 \sum_X \int \frac{\mathrm{d}^3\boldsymbol{P}_X}{(2\pi)^3\, 2E_X} \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int \frac{\mathrm{d}^4\kappa}{(2\pi)^4} \times (2\pi)^4\delta^4(P-k-P_X)\,(2\pi)^4\delta^4(k+q-\kappa)\,(2\pi)^4\delta^4(\kappa-P_h-P_{X'}) \times \left[\bar{\chi}(\kappa;P_h,S_h)\gamma^\mu\phi(k;P,S)\right]^* \left[\bar{\chi}(\kappa;P_h,S_h)\gamma^\nu\phi(k;P,S)\right], \tag{7.15}$$

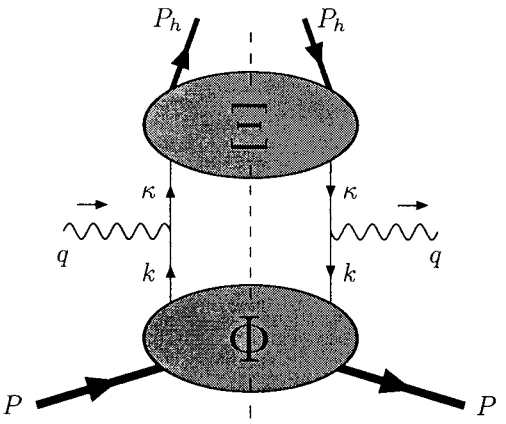

Fig. 7.3 A diagram contributing to semi-inclusive DIS at LO.

where $\phi(k;P,S)$ and $\chi(\kappa;P_h,S_h)$ are matrix elements of the quark field ψ, defined as

$$\phi(k;P,S) = \langle X|\psi(0)|PS\rangle\,, \tag{7.16a}$$

$$\chi(\kappa;P_h,S_h) = \langle 0|\psi(0)|P_hS_h,X\rangle\,. \tag{7.16b}$$

We now introduce the quark–quark correlation matrices

$$\begin{aligned}\Phi_{ij}(k;P,S) &= \sum_X \int \frac{\mathrm{d}^3\boldsymbol{P}_X}{(2\pi)^3\,2E_X}\,(2\pi)^4\delta^4(P_X+k-P)\\ &\qquad\times \phi_i(k;P,S)\,\bar{\phi}_j(k;P,S)\\ &= \int \mathrm{d}^4\xi\;\mathrm{e}^{\mathrm{i}k\cdot\xi}\,\langle PS|\bar{\psi}_j(0)\psi_i(\xi)|PS\rangle,\end{aligned} \tag{7.17a}$$

and

$$\begin{aligned}\Xi_{ij}(\kappa;P_h,S_h) &= \sum_X \int \frac{\mathrm{d}^3\boldsymbol{P}_X}{(2\pi)^3\,2E_X}\,(2\pi)^4\delta^4(P_h+P_X-\kappa)\\ &\qquad\times \chi_i(\kappa;P_h,S_h)\,\bar{\chi}_j(\kappa;P_h,S_h)\\ &= \sum_X \int \frac{\mathrm{d}^3\boldsymbol{P}_X}{(2\pi)^3\,2E_X}\int \mathrm{d}^4\xi\;\mathrm{e}^{\mathrm{i}\kappa\cdot\xi}\\ &\qquad\times \langle 0|\psi_i(\xi)|P_hS_h,X\rangle\langle P_hS_h,X|\bar{\psi}_j(0)|0\rangle\,.\end{aligned} \tag{7.17b}$$

Here Φ is the matrix already encountered in inclusive DIS, see Secs. 2.7 and 3.1, which incorporates the quark distribution functions. Ξ is a new quark–quark correlation matrix (sometimes called the decay, or fragmentation matrix), which contains the fragmentation functions of quarks into a hadron h. An average over colours is included in Ξ. Inserting Eqs. (7.17a, b) into (7.15) yields

$$W^{\mu\nu} = \sum_a e_a^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int \frac{\mathrm{d}^4\kappa}{(2\pi)^4}\, \delta^4(k+q-\kappa)\ \mathrm{Tr}[\Phi\,\gamma^\mu\,\Xi\gamma^\nu]\,. \tag{7.18}$$

It is an assumption of the parton model that k^2, $k{\cdot}P$, κ^2 and $\kappa{\cdot}P_h$ are much smaller than Q^2. Stated differently, when these quantities become large, Φ and Ξ are strongly suppressed. Let us work in the hN collinear frame (see App. D.2), the photon momentum is then

$$q^\mu \simeq -xP^\mu + \frac{1}{z}\,P_h^\mu + q_T^\mu = \left(-xP^+,\, \frac{1}{z}P_h^-,\, \boldsymbol{q}_T\right). \tag{7.19}$$

We recall that $\boldsymbol{P}_{h\perp} \simeq -z\boldsymbol{q}_T$. The quark momenta are

$$k^\mu \simeq \alpha\,P^\mu + k_T^\mu = (\xi P^+, 0, \mathbf{0}_T)\,, \tag{7.20a}$$

$$\kappa^\mu \simeq \beta\,P_h^\mu + \kappa_T^\mu = (0, P_h^-/\zeta, \mathbf{0}_T)\,. \tag{7.20b}$$

Thus, the delta function in (7.18) can be decomposed as

$$\begin{aligned}\delta^4(k+q-\kappa) &= \delta(k^+ + q^+ - \kappa^+)\,\delta(k^- + q^- - \kappa^-)\,\delta^2(\boldsymbol{k}_T + \boldsymbol{q}_T - \boldsymbol{\kappa}_T) \\ &\simeq \delta(k^+ - xP^+)\,\delta(k^- - P_h^-/z)\,\delta^2(\boldsymbol{k}_T + \boldsymbol{q}_T - \boldsymbol{\kappa}_T)\,,\end{aligned} \tag{7.21}$$

which implies $\alpha = x$ and $\beta = 1/z$, that is,

$$k^\mu \simeq xP^\mu + k_T^\mu \tag{7.22a}$$

$$\kappa^\mu \simeq \frac{1}{z}P_h^\mu + \kappa_T^\mu\,. \tag{7.22b}$$

The hadronic tensor (7.18) then becomes

$$\begin{aligned}W^{\mu\nu} = \sum_a e_a^2 \int \frac{\mathrm{d}k^+\,\mathrm{d}k^-\,\mathrm{d}^2\boldsymbol{k}_T}{(2\pi)^4} \int \frac{\mathrm{d}\kappa^+\,\mathrm{d}\kappa^-\,\mathrm{d}^2\boldsymbol{\kappa}_T}{(2\pi)^4} \\ \times\,\delta(k^+ - xP^+)\,\delta(k^- - P_h^-/z)\,\delta^2(\boldsymbol{k}_T + \boldsymbol{q}_T - \boldsymbol{\kappa}_T) \\ \times\,\mathrm{Tr}[\Phi\,\gamma^\mu\,\Xi\,\gamma^\nu]\,.\end{aligned} \tag{7.23}$$

Exploiting the delta functions in the longitudinal momenta, we obtain

$$W^{\mu\nu} = \sum_a e_a^2 \int \frac{\mathrm{d}k^-\, \mathrm{d}^2\boldsymbol{k}_T}{(2\pi)^4} \int \frac{\mathrm{d}\kappa^+\, \mathrm{d}^2\boldsymbol{\kappa}_T}{(2\pi)^4} \times \delta^2(\boldsymbol{k}_T + \boldsymbol{q}_T - \boldsymbol{\kappa}_T)\, \mathrm{Tr}[\Phi\, \gamma^\mu\, \Xi \gamma^\nu]_{k^+ = xP^+,\, \kappa^- = P_h^-/z}\,. \quad (7.24)$$

To obtain the final form of $W^{\mu\nu}$, we must insert the explicit expressions for Φ and Ξ into (7.24). The former has been already discussed in Sec. 3.1. In the following we shall concentrate on the structure of Ξ.

7.3 The fragmentation matrix

The fragmentation functions are contained in the decay matrix Ξ, which we rewrite here for convenience (from now on $\sum_X$ incorporates the integration over $\boldsymbol{P}_X$)

$$\Xi_{ij}(\kappa; P_h, S_h) = \sum_X \int \mathrm{d}^4\xi\, \mathrm{e}^{\mathrm{i}\kappa\cdot\xi} \times \langle 0|\psi_i(\xi)|P_h S_h, X\rangle\langle P_h S_h, X|\bar{\psi}_j(0)|0\rangle\,. \quad (7.25)$$

Here the path-ordered exponential $\mathcal{L} = \mathcal{P}\, \exp\left(-\mathrm{i}g \int \mathrm{d}s_\mu\, A^\mu(s)\right)$, needed to make (7.25) gauge invariant, has been omitted since in the $A^+ = 0$ gauge a proper path may be chosen such that $\mathcal{L} = 1$. Hereafter, the formalism will be similar to that developed in Sec. 3.1 for Φ and, therefore, much detail will be suppressed.

The quark fragmentation functions are related to traces of the form

$$\mathrm{Tr}[\Gamma\, \Xi] = \sum_X \int \mathrm{d}^4\xi\, \mathrm{e}^{\mathrm{i}\kappa\cdot\xi}\, \mathrm{Tr}\langle 0|\psi_i(\xi)|P_h S_h, X\rangle\langle P_h S_h, X|\bar{\psi}_j(0)\, \Gamma|0\rangle\,, \quad (7.26)$$

where Γ is a generic Dirac matrix. Ξ can be decomposed over a Dirac matrix basis as

$$\Xi(\kappa; P_h, S_h) = \frac{1}{2}\left\{\mathcal{S}\, \mathbb{1} + \mathcal{V}_\mu\, \gamma^\mu + \mathcal{A}_\mu \gamma_5 \gamma^\mu + \mathrm{i}\mathcal{P}_5 \gamma_5 + \frac{\mathrm{i}}{2}\, \mathcal{T}_{\mu\nu}\, \sigma^{\mu\nu} \gamma_5\right\}. \quad (7.27)$$

The quantities $\mathcal{S}$, $\mathcal{V}^\mu$, $\mathcal{A}^\mu$, $\mathcal{P}_5$, $\mathcal{T}^{\mu\nu}$ are constructed from the momentum of the fragmenting quark κ^μ, the momentum of the produced hadron P_h^μ

and its spin S_h^μ, and have the general form[a] (Levelt and Mulders, 1994a,b; Tangerman and Mulders, 1995b; Mulders and Tangerman, 1996; Boglione and Mulders, 1999)

$$\mathcal{S} = \frac{1}{2}\,\mathrm{Tr}(\Xi) = \mathcal{C}_1 \tag{7.28a}$$

$$\mathcal{V}^\mu = \frac{1}{2}\,\mathrm{Tr}(\gamma^\mu\,\Xi) = \mathcal{C}_2\,P_h^\mu + \mathcal{C}_3\,\kappa^\mu + \mathcal{C}_{10}\,\varepsilon^{\mu\nu\rho\sigma}S_{h\nu}P_{h\rho}\kappa_\sigma\,, \tag{7.28b}$$

$$\mathcal{A}^\mu = \frac{1}{2}\,\mathrm{Tr}(\gamma^\mu\gamma_5\,\Xi) = \mathcal{C}_4\,S_h^\mu + \mathcal{C}_5\,\kappa{\cdot}S_h\,P_h^\mu + \mathcal{C}_6\,\kappa{\cdot}S_h\,\kappa^\mu \tag{7.28c}$$

$$\mathcal{P}_5 = \frac{1}{2\mathrm{i}}\,\mathrm{Tr}(\gamma_5\,\Xi) = \mathcal{C}_{11}\,\kappa{\cdot}S_h \tag{7.28d}$$

$$\mathcal{T}^{\mu\nu} = \frac{1}{2\mathrm{i}}\,\mathrm{Tr}(\sigma^{\mu\nu}\gamma_5\,\Xi) = \mathcal{C}_7\,P_h^{[\mu}S_h^{\nu]} + \mathcal{C}_8\,\kappa^{[\mu}S_h^{\nu]} + \mathcal{C}_9\,\kappa{\cdot}S_h\,P_h^{[\mu}\kappa^{\nu]} + \mathcal{C}_{12}\,\varepsilon^{\mu\nu\rho\sigma}P_{h\rho}\kappa_\sigma\,. \tag{7.28e}$$

The quantities $\mathcal{C}_i = \mathcal{C}_i(\kappa^2, \kappa{\cdot}P_h)$ are real functions of their arguments, owing to the hermiticity property of Ξ.

The presence of the terms with coefficients $\mathcal{C}_{10}$, $\mathcal{C}_{11}$ and $\mathcal{C}_{12}$, which were forbidden in the expansion of the Φ matrix by time-reversal invariance, is justified by the fact that in the fragmentation case we cannot naïvely impose a condition similar to (3.4c), that is,

$$\Xi^*(\kappa; P_h, S_h) = \gamma_5 C\,\Xi(\widetilde{\kappa}; \widetilde{P}_h, \widetilde{S}_h)\,C^\dagger\gamma_5\,. \tag{7.29}$$

The reason is that Ξ contains the states $|P_hS_h, X\rangle$, which are out-states with possible final-state interactions between the hadron and the remnants. Under time reversal they do not simply invert their momenta and spin but transform into in-states

$$T\,|P_hS_h, X; \mathrm{out}\rangle \propto |\widetilde{P}_h\widetilde{S}_h, \widetilde{X}; \mathrm{in}\rangle\,. \tag{7.30}$$

These may differ non-trivially from $|\widetilde{P}_h\widetilde{S}_h, \widetilde{X}; \mathrm{out}\rangle$ owing to final-state interactions, which can generate relative phases between the various channels open in the $|\mathrm{in}\rangle \to |\mathrm{out}\rangle$ transition. Thus, a relation such as (7.29) cannot be obtained and consequently the terms containing $\mathcal{C}_{10}$, $\mathcal{C}_{11}$ and $\mathcal{C}_{12}$ are in principle allowed. The fragmentation functions related to these terms are called T-odd fragmentation functions (Levelt and Mulders, 1994b; Mulders and Tangerman, 1996; Boglione and Mulders, 1999). Let us now make a

[a]We consider here spin-half (or spin-zero) hadrons. For the production of spin-one hadrons, see Sec. 7.11 below.

digression to discuss the rôle that time reversal invariance plays in shaping the transverse-spin structure of hadron fragmentation (Barone, 2002).

7.4 Time reversal and transverse polarisation

The implications of time-reversal (TR) invariance on spin observables are sometimes rather subtle, as one can see from a simple example (Gasiorowicz, 1966, p. 515). Consider the decay of a particle of spin $\boldsymbol{s}$ and zero momentum into a state of spin $\boldsymbol{s}'$ and momentum $\boldsymbol{p}'$. Let $O(\boldsymbol{p}';\boldsymbol{s},\boldsymbol{s}')$ be an observable. The expectation value of O is

$$\langle O\rangle \sim \sum_{\boldsymbol{p}',\boldsymbol{s},\boldsymbol{s}'} O(\boldsymbol{p}';\boldsymbol{s},\boldsymbol{s}')\,|\langle \text{out};\boldsymbol{p}',\boldsymbol{s}'|H|\boldsymbol{s}\rangle|^2\,, \tag{7.31}$$

where H is the interaction Hamiltonian responsible for the decay. Inserting a complete set of out-states, labelled by the total angular momentum J and its third component m, the matrix element in (7.31) becomes

$$\langle \text{out};\boldsymbol{p}',\boldsymbol{s}'|H|\boldsymbol{s}\rangle = \sum_{Jm}\langle \text{out};\boldsymbol{p}',\boldsymbol{s}'|\text{out};J\,m\rangle\,\langle \text{out};J\,m|H|\boldsymbol{s}\rangle\,. \tag{7.32}$$

It is easy to check, using the TR invariance of H, that the phase of $\langle \text{out};J\,m|H|\boldsymbol{s}\rangle$ is the phase shift for the channel with angular momentum J, which we call η_J. Thus, Eq. (7.32) becomes

$$\begin{aligned}\langle \text{out};\boldsymbol{p}',\boldsymbol{s}'|H|\boldsymbol{s}\rangle &= \sum_{Jm}\langle \text{out};\boldsymbol{p}',\boldsymbol{s}'|\text{out};J\,m\rangle\;\mathrm{e}^{\mathrm{i}\eta_J}\;|\langle \text{out};J\,m|H|\boldsymbol{s}\rangle| \\ &= \sum_{J}\mathrm{e}^{\mathrm{i}\eta_J}\;M(J;\boldsymbol{p}';\boldsymbol{s},\boldsymbol{s}')\,.\end{aligned} \tag{7.33}$$

In terms of M, Eq. (7.31) reads

$$\begin{aligned}\langle O\rangle \sim \sum_{\boldsymbol{p}',\boldsymbol{s},\boldsymbol{s}'} O(\boldsymbol{p}';\boldsymbol{s},\boldsymbol{s}') \sum_{J,J'}\mathrm{e}^{\mathrm{i}(\eta_J-\eta_{J'})} \\ \times M(J;\boldsymbol{p}';\boldsymbol{s},\boldsymbol{s}')\,M^*(J';\boldsymbol{p}';\boldsymbol{s},\boldsymbol{s}')\,.\end{aligned} \tag{7.34}$$

TR invariance and the unitarity of the S-matrix $S^\dagger S = \mathbb{1}$ imply (for the derivation of this result see Gasiorowicz, 1966, p. 515)

$$M^*(J;\boldsymbol{p}';\boldsymbol{s},\boldsymbol{s}') = M(J;-\boldsymbol{p}';-\boldsymbol{s},-\boldsymbol{s}')\,. \tag{7.35}$$

Suppose now that O is *odd* under TR, that is,

$$O(\boldsymbol{p}';\boldsymbol{s},\boldsymbol{s}') = -O(-\boldsymbol{p}';-\boldsymbol{s},-\boldsymbol{s}')\,. \tag{7.36}$$

With the help of (7.35), Eq. (7.34) finally gives

$$\langle O\rangle \sim \sum_{\boldsymbol{p}',\boldsymbol{s},\boldsymbol{s}'}\sum_{J,J'}\sin(\eta_J-\eta_{J'})\,O(\boldsymbol{p}';\boldsymbol{s},\boldsymbol{s}') \times M(J;\boldsymbol{p}';\boldsymbol{s},\boldsymbol{s}')\,M(J';-\boldsymbol{p}';-\boldsymbol{s},-\boldsymbol{s}')\,. \tag{7.37}$$

This shows that, despite O being T-odd, its expectation value does not vanish if $\sin(\eta_J-\eta_{J'})\neq 0$, which may occur in the presence of final-state interactions that generate non-trivial phase differences between the various channels. Thus, the important lesson is that when non-trivial final-state (or initial-state) effects are at work, observables that are *naïvely* T-odd, according to their structure in terms of spins and momenta, may give rise to non-zero measurable quantities without violating true TR invariance. A noteworthy case is that of pion–nucleon scattering: although the correlation

$$(\boldsymbol{P}_\pi\wedge\boldsymbol{P}_N)\cdot\boldsymbol{S}_N \tag{7.38}$$

is T-odd in the sense of (7.36), its vacuum expectation value is known to be non-zero.

The T-odd correlations that are encountered when studying hadronic transverse-polarisation structure are similar to (7.38), but involve quark transverse momenta. They are

$$(\boldsymbol{k}_\perp\wedge\boldsymbol{P})\cdot\boldsymbol{S}\,,\quad (\boldsymbol{k}_\perp\wedge\boldsymbol{P})\cdot\boldsymbol{s}\,, \tag{7.39a}$$

$$(\boldsymbol{\kappa}_\perp\wedge\boldsymbol{P}_h)\cdot\boldsymbol{s}'\,. \tag{7.39b}$$

The first two correlations involve the momentum and/or the spin of the quark inside the target hadron and are related to the T-odd distribution functions $f_{1T}^\perp$ and $h_1^\perp$ introduced in Sec. 3.10. Correlation (7.39b) involves the momentum and the spin of the fragmenting quark and is related to a T-odd fragmentation function, the Collins function $H_1^\perp$ (Collins, 1993b), which we shall study in Sec. 7.6. According to the general discussion above, correlations (7.39a) may give rise to observable effects due to some hypothetical *initial-state interactions* (we have discussed this in Sec. 3.10), whereas (7.39b) may be observable owing to *final-state interactions.*

A generic mechanism giving rise to T-odd fragmentation functions is shown diagrammatically in Fig. 7.4. What is needed, in order to pro-

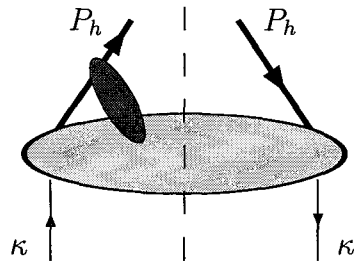

Fig. 7.4 A hypothetical mechanism giving rise to a T-odd fragmentation function.

duce such fragmentation functions, is an interference diagram in which the final-state interaction (represented in figure by the dark blob) between the produced hadron and the residual fragments cannot be reabsorbed into the quark–hadron vertex (Bianconi *et al.*, 2000a). However, as argued by Jaffe, Jin and Tang (1998b), the proliferation of channels and the sum over X might ultimately lead to an overall cancellation of the relative phases between the produced hadron and the X system, and thus to the vanishing of the T-odd fragmentation functions. If this is true (and only experiments will provide a definite answer), then, in order to observe a T-odd correlation in the fragmentation process, one should rather consider correlated variables (spins and momenta) pertaining to physical particles with known interactions. This suggests returning to a quantity *exactly* like (7.38), namely,

$$(\boldsymbol{P}_1 \wedge \boldsymbol{P}_2)\cdot\boldsymbol{S}\,, \tag{7.40}$$

where $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ are the momenta of *two final-state hadrons.* Two-hadron leptoproduction, $l\,N^\uparrow \to l'\,h_1\,h_2\,X$, has been proposed (Jaffe, Jin and Tang, 1998b), as a potential source of information about transversity and T-odd correlations. We shall consider this process in Sec. 7.10.

7.5 Leading-twist fragmentation functions

Let us now return to the fragmentation matrix and Eqs. (7.28a–e). Working in an hN collinear frame, the vectors (or pseudovectors) at our disposition

are

$$P_h^\mu\,, \quad \kappa^\mu \simeq \frac{1}{z}\,P_h^\mu + \kappa_T \quad \text{and} \quad S_h^\mu \simeq \frac{\lambda_h}{M_h}\,P_h^\mu + S_{hT}^\mu\,, \tag{7.41}$$

where one must remember that the transverse components are suppressed by a factor $1/P_h^-$ (that is, $1/Q$) compared to the longitudinal ones. To start with, consider the case of collinear kinematics. If we ignore $\boldsymbol{\kappa}_T$, the terms contributing to (7.27) at leading twist (that is, at $\mathcal{O}(P_h^-)$) are

$$\mathcal{V}^\mu = \frac{1}{2}\,\mathrm{Tr}(\gamma^\mu\,\Xi) = B_1\,P_h^\mu\,, \tag{7.42a}$$

$$\mathcal{A}^\mu = \frac{1}{2}\,\mathrm{Tr}(\gamma^\mu\gamma_5\,\Xi) = \lambda_h\,B_2\,P_h^\mu\,, \tag{7.42b}$$

$$\mathcal{T}^{\mu\nu} = \frac{1}{2\mathrm{i}}\,\mathrm{Tr}(\sigma^{\mu\nu}\gamma_5\,\Xi) = B_3\,P_h^{[\mu}S_{hT}^{\nu]}\,, \tag{7.42c}$$

where we have introduced the functions $B_i(\kappa^2, \kappa{\cdot}P_h)$. The fragmentation matrix then reads

$$\Xi(\kappa; P_h, S_h) = \tfrac{1}{2}\left\{B_1\,\not{P}_h + \lambda_h\,B_2\,\gamma_5\,\not{P}_h + B_3\,\not{P}_h\,\gamma_5\not{S}_{hT}\right\}. \tag{7.43}$$

Recalling that P_h only has a P_h^- component, Eqs. (7.42a–c) become

$$\frac{1}{2P_h^-}\,\mathrm{Tr}(\gamma^-\,\Xi) = B_1\,, \tag{7.44a}$$

$$\frac{1}{2P_h^-}\,\mathrm{Tr}(\gamma^-\gamma_5\,\Xi) = \lambda_h\,B_2\,, \tag{7.44b}$$

$$\frac{1}{2P_h^-}\,\mathrm{Tr}(\mathrm{i}\sigma^{i-}\gamma_5\,\Xi) = S_{hT}^i\,B_3\,. \tag{7.44c}$$

The three leading-twist fragmentation functions: the unpolarised case $D_q(x)$, the longitudinally polarised case $\Delta D_q(x)$, and the transversely polarised case $\Delta_T D_q(x)$ are obtained by integrating B_1, B_2 and B_3 respec-

tively, over κ, with the constraint $1/z = \kappa^-/P_h^-$. For instance,

$$\begin{aligned} D(z) &= \frac{z}{2}\int \frac{\mathrm{d}^4\kappa}{(2\pi)^4}\, B_1(\kappa^2, \kappa\cdot P_h)\,\delta(1/z - \kappa^-/P_h^-) \\ &= \frac{z}{4}\int \frac{\mathrm{d}^4\kappa}{(2\pi)^4}\, \mathrm{Tr}(\gamma^-\Xi)\,\delta(\kappa^- - P_h^-/z) \\ &= \frac{z}{4}\sum_X \int \frac{\mathrm{d}\xi^+}{2\pi}\, \mathrm{e}^{\mathrm{i}P_h^-\xi^+/z} \\ &\qquad \times \langle 0|\psi(\xi^+,0,\mathbf{0}_\perp)|P_hS_h,X\rangle\langle P_hS_h,X|\bar\psi(0)\gamma^-|0\rangle\,. \end{aligned} \tag{7.45}$$

The normalisation of $D(z)$ is such that

$$\sum_h \sum_{S_h} \int \mathrm{d}z\, z\, D(z) = 1\,, \tag{7.46}$$

where $\sum_h$ is a sum over all produced hadrons. Hence, $D(z)$ is the number density of hadrons of type h with longitudinal momentum fraction z in the fragmenting quark. Analogously, we have for $\Delta D(z)$, with $\lambda_h = 1$,

$$\begin{aligned} \Delta D(z) &= \frac{z}{2}\int \frac{\mathrm{d}^4\kappa}{(2\pi)^4}\, B_2(\kappa^2, \kappa\cdot P_h)\,\delta(1/z - \kappa^-/P_h^-) \\ &= \frac{z}{4}\sum_X \int \frac{\mathrm{d}\xi^+}{2\pi}\, \mathrm{e}^{\mathrm{i}P_h^-\xi^+/z} \\ &\qquad \times \langle 0|\psi(\xi^+,0,\mathbf{0}_\perp)|P_hS_h,X\rangle\langle P_hS_h,X|\bar\psi(0)\gamma^-\gamma_5|0\rangle\,, \end{aligned} \tag{7.47}$$

and for $\Delta_T D(z)$, with $S_{hT}^i = (1,0)$ for definiteness,

$$\begin{aligned} \Delta_T D(z) &= \frac{z}{2}\int \frac{\mathrm{d}^4\kappa}{(2\pi)^4}\, B_3(\kappa^2, \kappa\cdot P_h)\,\delta(1/z - \kappa^-/P_h^-) \\ &= \frac{z}{4}\sum_X \int \frac{\mathrm{d}\xi^+}{2\pi}\, \mathrm{e}^{\mathrm{i}P_h^-\xi^+/z} \\ &\qquad \times \langle 0|\psi(\xi^+,0,\mathbf{0}_\perp)|P_hS_h,X\rangle\langle P_hS_h,X|\bar\psi(0)\mathrm{i}\sigma^{1-}\gamma_5|0\rangle\,. \end{aligned} \tag{7.48}$$

Note that $\Delta_T D(z)$, which is often called $H_1(z)$ in the literature (see Mulders and Tangerman, 1996), is just the fragmentation-function analogue of the transverse-polarisation distribution function $\Delta_T f(x)$. Introducing the κ-integrated matrix,

$$\Xi(z) = \frac{z}{2}\int \frac{\mathrm{d}^4\kappa}{(2\pi)^4}\, \Xi(\kappa; P_h, S_h)\,\delta(1/z - \kappa^-/P_h^-)\,, \tag{7.49}$$

the leading-twist structure of the fragmentation process is summarised in the expression of $\Xi(z)$, which is

$$\Xi(z) = \tfrac{1}{2}\left\{D(z)\,\not{P}_h + \lambda_h\,\Delta D(z)\,\gamma_5\not{P}_h + \Delta_T D(z)\,\not{P}_h\gamma_5\not{S}_{hT}\right\}. \qquad (7.50)$$

The probabilistic interpretation of $D(z)$, $\Delta D(z)$ and $\Delta_T D(z)$ is analogous to that of the corresponding distribution functions (see Sec. 3.3). If we denote by $\mathcal{N}_{h/q}(z)$ the probability of finding a hadron with longitudinal momentum fraction z inside a quark q, then we have (using $\pm$ to label longitudinal polarisation states and $\uparrow\downarrow$ to label transverse-polarisation states)

$$D(z) = \mathcal{N}_{h/q}(z)\,, \qquad (7.51a)$$

$$\Delta D(z) = \mathcal{N}_{h/q+}(z) - \mathcal{N}_{h/q-}(z)\,, \qquad (7.51b)$$

$$\Delta_T D(z) = \mathcal{N}_{h/q\uparrow}(z) - \mathcal{N}_{h/q\downarrow}(z)\,. \qquad (7.51c)$$

7.5.1 $\boldsymbol{\kappa_T}$-dependent fragmentation functions

In the collinear case ($\boldsymbol{k}_T = \boldsymbol{\kappa}_T = 0$) the produced hadron is constrained to have zero transverse momentum ($\boldsymbol{P}_{h\perp} = -z\boldsymbol{q}_T = 0$). Therefore, in order to investigate its $\boldsymbol{P}_{h\perp}$ distribution within the parton model, one has to account for quark transverse motion (in QCD quark transverse momenta emerge at NLO owing to gluon emission). The kinematics in the γ^*N and hN frames is depicted in Fig. 7.5 (for simplicity the case of no quark transverse motion

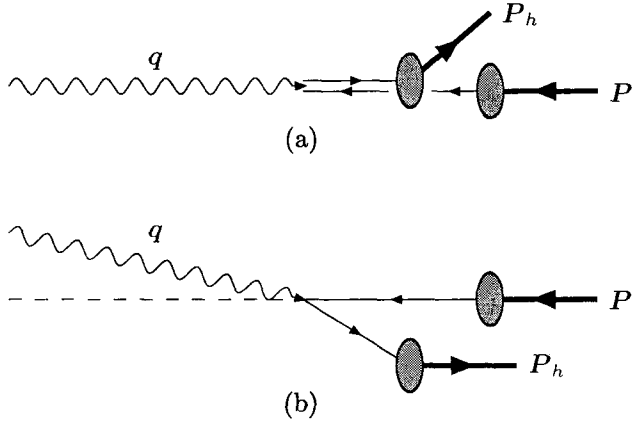

Fig. 7.5 Kinematics in (a) the γ^*N frame and (b) the hN frame.

inside the target is illustrated). Reintroducing $\boldsymbol{\kappa}_T$, we have at leading twist (Levelt and Mulders, 1994a,b; Tangerman and Mulders, 1995b; Mulders and Tangerman, 1996; Boglione and Mulders, 1999)

$$\mathcal{V}^\mu = \frac{1}{2}\,\mathrm{Tr}(\gamma^\mu\,\Xi) = B_1\,P_h^\mu + \frac{1}{M_h}\,B_1'\,\varepsilon^{\mu\nu\rho\sigma}\,P_{h\nu}\kappa_{T\rho}S_{hT\sigma}\,, \quad (7.52\mathrm{a})$$

$$\mathcal{A}^\mu = \frac{1}{2}\,\mathrm{Tr}(\gamma^\mu\gamma_5\,\Xi) = \lambda_h\,B_2\,P_h^\mu + \frac{1}{M_h}\,\widetilde{B}_1\,\boldsymbol{\kappa}_T{\cdot}\boldsymbol{S}_{hT}\,P_h^\mu\,, \quad (7.52\mathrm{b})$$

$$\begin{aligned}\mathcal{T}^{\mu\nu} = \frac{1}{2\mathrm{i}}\,\mathrm{Tr}(\sigma^{\mu\nu}\gamma_5\,\Xi) &= B_3\,P_h^{[\mu}S_{hT}^{\nu]} + \frac{\lambda_h}{M_h}\,\widetilde{B}_2\,P_h^{[\mu}\kappa_T^{\nu]} \\ &\quad + \frac{1}{M_h^2}\,\widetilde{B}_3\,\boldsymbol{\kappa}_T{\cdot}\boldsymbol{S}_{hT}\,P_h^{[\mu}\kappa_T^{\nu]} \\ &\quad + \frac{1}{M_h}\,B_2'\,\varepsilon^{\mu\nu\rho\sigma}\,P_{h\rho}\kappa_{T\sigma}\,, \end{aligned} \quad (7.52\mathrm{c})$$

where we have introduced new functions $\widetilde{B}_i(\kappa^2, \kappa{\cdot}P_h)$—the tilde signals the presence of $\boldsymbol{\kappa}_T$; $B_i'(\kappa^2, \kappa{\cdot}P_h)$—the prime labels the T-odd terms; and inserted powers of M_h so that all coefficients have the same dimensionality.

Contracting Eqs. (7.52a–c) with P_μ yields

$$\frac{1}{2P_h^-}\,\mathrm{Tr}(\gamma^-\,\Xi) = B_1 + \frac{1}{M_h}\,B_1'\,\varepsilon_T^{ij}\kappa_{Ti}S_{hTj}\,, \quad (7.53\mathrm{a})$$

$$\frac{1}{2P^-}\,\mathrm{Tr}(\gamma^-\gamma_5\Xi) = \lambda_h\,B_2 + \frac{1}{M_h}\,\boldsymbol{\kappa}_T{\cdot}\boldsymbol{S}_{hT}\,\widetilde{B}_1\,, \quad (7.53\mathrm{b})$$

$$\begin{aligned}\frac{1}{2P^-}\,\mathrm{Tr}(\mathrm{i}\sigma^{i-}\gamma_5\Xi) &= \left(B_3 + \frac{\boldsymbol{\kappa}_T^2}{2M_h^2}\,\widetilde{B}_3\right)S_{hT}^i + \frac{\lambda_h}{M_h}\,\widetilde{B}_2\,\kappa_T^i \\ &\quad - \frac{1}{M_h^2}\,\widetilde{B}_3\left(\kappa_T^i\kappa_T^j + \frac{1}{2}\,\boldsymbol{\kappa}_T^2\,g_\perp^{ij}\right)S_{hTj} \\ &\quad + \frac{1}{M_h}\,B_2'\,\varepsilon_T^{ij}\kappa_{Tj}\,. \end{aligned} \quad (7.53\mathrm{c})$$

The eight $\boldsymbol{\kappa}_T$-dependent fragmentation functions are obtained from the B coefficients as follows

$$D(z, \boldsymbol{\kappa}_T'^2) = \frac{1}{2z}\int\frac{\mathrm{d}\kappa^+\,\mathrm{d}\kappa^-}{(2\pi)^4}\,B_1(\kappa^2, \kappa{\cdot}P_h)\,\delta(1/z - \kappa^-/P_h^-)\,, \quad (7.54)$$

etc., where $\boldsymbol{\kappa}_T' \equiv -z\boldsymbol{\kappa}_T$ is the transverse momentum of the hadron h with respect to the fragmenting quark, see Eq. (7.22b). If quark transverse motion inside the target is ignored, then $\boldsymbol{\kappa}_T'$ coincides with $\boldsymbol{P}_{h\perp}$. Defining

the integrated trace

$$\begin{aligned}\Xi^{[\Gamma]} &\equiv \frac{1}{4z}\int\frac{\mathrm{d}\kappa^+\,\mathrm{d}\kappa^-}{(2\pi)^4}\,\mathrm{Tr}(\Gamma\Xi)\,\delta(\kappa^- - P_h^-/z)\\ &= \frac{1}{4z}\sum_X\int\frac{\mathrm{d}\xi^+\,\mathrm{d}^2\boldsymbol{\xi}_T}{(2\pi)^3}\,\mathrm{e}^{\mathrm{i}(P_h^-\xi^+/z-\boldsymbol{\kappa}_T\cdot\boldsymbol{\xi}_T)}\\ &\quad\times\mathrm{Tr}\langle 0|\psi(\xi^+,0,\boldsymbol{\xi}_T)|P_hS_h,X\rangle\langle P_hS_h,X|\bar\psi(0)\Gamma|0\rangle\,, \qquad (7.55)\end{aligned}$$

we obtain from (7.53a–c)

$$\begin{aligned}\Xi^{[\gamma^-]} &= \mathcal{N}_{h/q}(z,\boldsymbol{\kappa}'_T)\\ &= D(z,\boldsymbol{\kappa}'^2_T) + \frac{1}{M_h}\,\varepsilon_T^{ij}\kappa_{Ti}S_{hTj}\,D_{1T}^{\perp}(z,\boldsymbol{\kappa}'^2_T)\,, \qquad (7.56a)\\ \Xi^{[\gamma^-\gamma_5]} &= \mathcal{N}_{h/q}(z,\boldsymbol{\kappa}'_T)\,\lambda'(z,\boldsymbol{\kappa}'_T)\\ &= \lambda_h\,\Delta D(z,\boldsymbol{\kappa}'^2_T) + \frac{1}{M_h}\,\boldsymbol{\kappa}_T\cdot\boldsymbol{S}_{hT}\,G_{1T}(z,\boldsymbol{\kappa}'^2_T)\,, \qquad (7.56b)\\ \Xi^{[\mathrm{i}\sigma^{i-}\gamma_5]} &= \mathcal{N}_{h/q}(z,\boldsymbol{\kappa}'_T)\,s'^i_\perp(z,\boldsymbol{\kappa}'_T)\\ &= S_{hT}^i\,\Delta'_T D(z,\boldsymbol{\kappa}'^2_T) + \frac{\lambda_h}{M_h}\,\kappa_T^i H_{1L}^{\perp}(z,\boldsymbol{\kappa}'^2_T)\\ &\quad - \frac{1}{M_h^2}\left(\kappa_T^i\kappa_T^j + \frac{1}{2}\,\boldsymbol{\kappa}_T^2\,g_\perp^{ij}\right)S_{hTj}\,H_{1T}^{\perp}(z,\boldsymbol{\kappa}'^2_T)\\ &\quad + \frac{1}{M_h}\,\varepsilon_T^{ij}\kappa_{Tj}\,H_1^{\perp}(z,\boldsymbol{\kappa}'^2_T)\,, \qquad (7.56c)\end{aligned}$$

where $\boldsymbol{s}' = (\boldsymbol{s}'_\perp,\lambda')$ is the spin of the quark and $\mathcal{N}_{h/q}(z,\boldsymbol{\kappa}'_T)$ is the probability of finding a hadron with longitudinal momentum fraction z and transverse momentum $\boldsymbol{\kappa}'_T = -z\boldsymbol{\kappa}_T$ with respect to the quark momentum, inside a quark q. In (7.56a–c) we have adopted a more traditional notation for the three fragmentation functions, D, ΔD and $\Delta_T D$, that survive upon integration over $\boldsymbol{\kappa}_T$ whereas we have resorted to Mulders' terminology (Mulders and Tangerman, 1996) for the other, less familiar, fragmentation functions, $D_{1T}^{\perp}$, G_{1T}, $H_{1L}^{\perp}$, $H_{1T}^{\perp}$ and $H_1^{\perp}$ (note that in Mulders' scheme D, ΔD and $\Delta'_T D$ are called D_1, G_{1L} and H_{1T}, respectively, and D_1, G_1 and H_1, once integrated over $\boldsymbol{\kappa}_T$). The integrated fragmentation functions,

$D(z)$ and $\Delta D(z)$, are obtained from $D(z,{\kappa'_T}^2)$ and $\Delta D(z,{\kappa'_T}^2)$, via

$$D(z) = \int \mathrm{d}^2\boldsymbol{\kappa}'_T\, D(z,{\kappa'_T}^2)\,, \tag{7.57a}$$

$$\Delta D(z) = \int \mathrm{d}^2\boldsymbol{\kappa}'_T\, \Delta D(z,{\kappa'_T}^2)\,, \tag{7.57b}$$

whereas $\Delta_T D(z)$ is given by

$$\Delta_T D(z) = \int \mathrm{d}^2\boldsymbol{\kappa}'_T \left(\Delta'_T D(z,{\kappa'_T}^2) + \frac{{\kappa'_T}^2}{2M_h^2}\, H_{1T}^{\perp}(z,{\kappa'_T}^2) \right). \tag{7.57c}$$

The T-odd fragmentation functions are $D_{1T}^{\perp}$ and $H_1^{\perp}$. The former describes the fragmentation of unpolarised quarks into polarised hadrons, the latter describes the fragmentation of polarised quarks into unpolarised hadrons.

7.6 The Collins fragmentation function

Among the unintegrated fragmentation functions, the T-odd quantity $H_1^{\perp}$ (called the *Collins function*) plays an important rôle in the phenomenology of transversity as it is related to the *Collins effect*, *i.e.*, the observation of azimuthal asymmetries in single-inclusive production of unpolarised hadrons at leading twist (Collins, 1993b). In partonic terms, $H_1^{\perp}$ is defined – see Eq. (3.98b) for the corresponding distribution function $h_1^{\perp}$ – via

$$\mathcal{N}_{h/q\uparrow}(z,\boldsymbol{\kappa}'_T) - \mathcal{N}_{h/q\downarrow}(z,\boldsymbol{\kappa}'_T) = \frac{|\boldsymbol{\kappa}_T|}{M_h}\, \sin(\phi_\kappa - \phi_{s'})\, H_1^{\perp}(z,{\kappa'_T}^2)\,, \tag{7.58}$$

where ϕ_κ and $\phi_{s'}$ are the azimuthal angles of the quark momentum and polarisation, respectively, defined in a plane perpendicular to $\boldsymbol{P}_h$. We also introduce the function $\Delta_T^0 D(z,\boldsymbol{P}_{h\perp}^2)$ related to $H_1^{\perp}(z,\boldsymbol{P}_{h\perp}^2)$ by[b]

$$\Delta_T^0 D(z,\boldsymbol{P}_{h\perp}^2) = \frac{|\boldsymbol{P}_{h\perp}|}{zM_h}\, H_1^{\perp}(z,\boldsymbol{P}_{h\perp}^2)\,. \tag{7.59}$$

The angular factor in (7.58), that is (recall that $\boldsymbol{P}_h$ is directed along $-z$),

$$\sin(\phi_\kappa - \phi_{s'}) = \frac{(\boldsymbol{\kappa}\wedge\boldsymbol{P}_h)\cdot\boldsymbol{s}'}{|\boldsymbol{\kappa}\wedge\boldsymbol{P}_h|\,|\boldsymbol{s}'|}\,, \tag{7.60}$$

[b]Notice that our $\Delta_T^0 D$ is related to the $\Delta^N D$ of Anselmino, Boglione and Murgia (1999) by $\Delta_T^0 D = \Delta^N D/2$ (our notation is explained in Sec. 1.4).

is related to the so-called Collins angle (see Sec. 7.7), as we now show.

First of all, note that on neglecting $\mathcal{O}(1/Q)$ effects, azimuthal angles in the plane perpendicular to the hN axis coincide with the azimuthal angles defined in the plane perpendicular to the $\gamma^* N$ axis. Then, if we ignore the intrinsic motion of quarks inside the target, we have $\boldsymbol{\kappa}_T = -\boldsymbol{P}_{h\perp}/z$ and

$$\phi_\kappa = \phi_h - \pi\,. \tag{7.61}$$

The angle in (7.60) is therefore

$$\phi_\kappa - \phi_{s'} = \phi_h - \phi_{s'} - \pi = -\Phi_C - \pi\,, \tag{7.62}$$

so that

$$\sin(\phi_\kappa - \phi_{s'}) = \sin\Phi_C\,, \tag{7.63}$$

where Φ_C, the azimuthal angle between the spin vector of the fragmenting quark and the momentum of the produced hadron, is what is known as the *Collins angle* (Collins, 1993b). Just to show how the T-odd fragmentation function $H_1^\perp$ may arise from non-trivial final-state interactions, as discussed in Sec. 7.3, let us consider a toy model (Bianconi *et al.*, 2000a), see Fig. 7.6, that provides a simple example of the mechanism represented symbolically

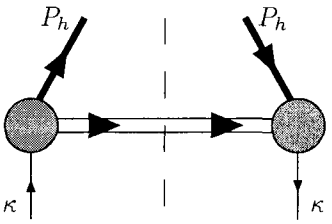

Fig. 7.6 A toy model for fragmentation.

in Fig. 7.4. Thus, we assume that a quark with momentum κ and mass m, fragments into an unpolarised hadron, leaving a remnant that is a point-like scalar diquark. The fragmentation function $H_1^\perp$ is contained in the tensor component of the matrix Ξ

$$\Xi(\kappa, P_h) = \frac{1}{2}\left\{\cdots + \frac{\mathrm{i}}{2}\mathcal{T}_{\mu\nu}\sigma^{\mu\nu}\gamma_5\right\}, \tag{7.64}$$

where – see Eq. (7.52c),

$$\mathcal{T}_{\mu\nu} = \cdots + \frac{1}{M_h}\, B_2'\, \varepsilon_{\mu\nu\rho\sigma}\, P_h^\rho \kappa^\sigma\,, \tag{7.65}$$

so that, using $\gamma_5 \sigma^{\mu\nu} = \frac{1}{2}\mathrm{i}\, \varepsilon^{\mu\nu\alpha\beta}\, \sigma_{\alpha\beta}$,

$$\Xi(\kappa, P_h) = \frac{1}{2}\left\{ \cdots + \frac{1}{M_h}\, B_2'\, \sigma^{\mu\nu} P_{h\mu} \kappa_\nu \right\}. \tag{7.66}$$

If we describe the hadron h by a plane wave

$$\psi_h(x) \sim u(P_h)\, \mathrm{e}^{\mathrm{i} P_h \cdot x}\,, \tag{7.67}$$

it is easy to show that the fragmentation matrix Ξ is

$$\begin{aligned} \Xi(\kappa, P_h) &\sim \frac{-\mathrm{i}}{\not\kappa - m}\, u(P_h)\, \bar{u}(P_h)\, \frac{\mathrm{i}}{\not\kappa - m} \\ &\sim \frac{\not\kappa + m}{\kappa^2 - m^2}\, (\not P_h + M_h)\, \frac{\not\kappa + m}{\kappa^2 - m^2}\,, \end{aligned} \tag{7.68}$$

where we have omitted some inessential factors. From (7.68) we cannot extract a term proportional to $\sigma^{\mu\nu} P_{h\mu} \kappa_\nu$ (which would produce $H_1^\perp$). Let us now suppose that a residual interaction of h with the intermediate state generates a phase in the hadron wave function. If, for instance, in (7.68) we make the replacement (assuming only two fragmentation channels)

$$u(P_h) \to u(P_h) + \mathrm{e}^{\mathrm{i}\chi} \not\kappa\, u(P_h)\,, \tag{7.69}$$

by a little algebra one can show that a term of the type (7.66) emerges in Ξ, with

$$B_2' \sim \frac{M_h}{\kappa^2 - m^2}\, \sin\chi\,. \tag{7.70}$$

Therefore, if the interference between the fragmentation channels produces a non-zero phase χ, T-odd contributions may appear. The proliferation of channels, however, might lead, as suggested by Jaffe, Jin and Tang (1998b), to the vanishing of such phases and of the resulting T-odd fragmentation functions.

A microscopic mechanism that may give rise to a T-odd fragmentation function has been recently investigated by Bacchetta *et al.* (2001, 2002). Using a simple pseudoscalar or pseudovector coupling between pions and quarks to model the fragmentation process, these authors show that the

inclusion of one-loop self-energy and vertex corrections (Fig. 7.7) generates a non-vanishing $H_1^\perp$. This is a specific example of a final-state interaction that supports the existence of the Collins fragmentation function. The question remains as to what may happen when a more realistic situation, with many open channels, is considered.

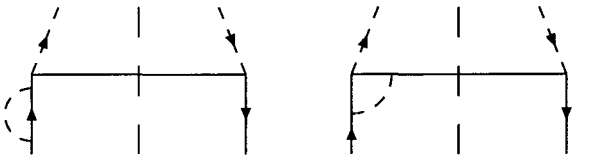

Fig. 7.7 One-loop corrections to the fragmentation of a quark (solid lines) into a pion (short-dashed line) in the model of Bacchetta *et al.* (2001, 2002). The central long-dashed line is the unitarity cut.

7.7 Cross-sections and asymmetries of inclusive leptoproduction

We shall now calculate the trace in (7.24). At leading twist, as already mentioned, transverse-polarisation distributions are probed by unpolarised lepton beams. In this case, the leptonic tensor is symmetric and couples to the symmetric part of $W^{\mu\nu}$, that is,

$$W^{\mu\nu(S)} = \frac{1}{2}\sum_a e_a^2 \int \frac{\mathrm{d}k^-\,\mathrm{d}^2\boldsymbol{k}_T}{(2\pi)^4} \int \frac{\mathrm{d}\kappa^+\,\mathrm{d}^2\boldsymbol{\kappa}_T}{(2\pi)^4} \times \delta^2(\boldsymbol{k}_T + \boldsymbol{q}_T - \boldsymbol{\kappa}_T)\,\mathrm{Tr}\left[\Phi\,\gamma^{\{\mu}\,\Xi\,\gamma^{\nu\}}\right]_{k^+ = xP^+,\,\kappa^- = P_h^-/z}. \quad (7.71)$$

The Fierz decomposition of $\mathrm{Tr}[\Phi\,\gamma^{\{\mu}\,\Xi\,\gamma^{\nu\}}]$ is similar to Eq. (6.24) with the replacements $\Phi_1 \to \Phi$ and $\bar{\Phi}_2 \to \Xi$. If we insert Eqs. (3.50a–c) and (7.52a–c) into (7.71) and integrate over k^- and κ^+ making use of Eqs. (3.58a–c) and (7.56a–c), after some algebra we obtain (Levelt and Mulders, 1994a,b; Tangerman and Mulders, 1995b; Mulders and Tanger-

man, 1996)

$$
\begin{aligned}
W^{\mu\nu(S)} = 2\sum_a e_a^2\, z \int \mathrm{d}^2\boldsymbol{\kappa}_T \int \mathrm{d}^2\boldsymbol{k}_T\, \delta^2(\boldsymbol{k}_T + \boldsymbol{q}_T - \boldsymbol{\kappa}_T) \\
\Bigg\{ -g_T^{\mu\nu} \left[f(x,\boldsymbol{k}_T^2)\, D(z,\boldsymbol{\kappa}_T'^2) + \frac{1}{M_h}\, \varepsilon_T^{\rho\sigma}\, \kappa_{T\rho} S_{hT\sigma}\, f(x,\boldsymbol{k}_T^2)\, D_{1T}^{\perp}(z,\boldsymbol{\kappa}_T'^2) \right] \\
- \left[S_T^{\{\mu} S_{hT}^{\nu\}} + \boldsymbol{S}_T{\cdot}\boldsymbol{S}_{hT}\, g_T^{\mu\nu} \right] \Delta_T' f(x,\boldsymbol{k}_T^2)\, \Delta_T' D(z,\boldsymbol{\kappa}_T'^2) \\
- \frac{\left[S_T^{\{\mu} \kappa_T^{\nu\}} + \boldsymbol{S}_T{\cdot}\boldsymbol{\kappa}_T\, g_T^{\mu\nu} \right] \boldsymbol{\kappa}_T{\cdot}\boldsymbol{S}_{hT}}{M_h^2}\, \Delta_T f(x,\boldsymbol{k}_T^2)\, H_{1T}^{\perp}(z,\boldsymbol{\kappa}_T'^2) \\
- \frac{S_T^{\{\mu} \varepsilon_T^{\nu\}\rho} \kappa_{T\rho} + \kappa_T^{\{\mu} \varepsilon_T^{\nu\}\rho} S_{T\rho}}{2M_h}\, \Delta_T' f(x,\boldsymbol{k}_T^2)\, H_1^{\perp}(z,\boldsymbol{\kappa}_T'^2) + \ldots \Bigg\}. \qquad (7.72)
\end{aligned}
$$

In (7.72) we have only considered unpolarised and transversely polarised terms, and we have omitted the $\boldsymbol{k}_T$-dependent contributions (in the following we shall assume that quark transverse motion inside the target can be neglected). Neglecting higher-twist (*i.e.*, $\mathcal{O}(1/Q)$) contributions, the transverse (T) vectors and tensors appearing in (7.72) coincide with the corresponding perpendicular ($\perp$) vectors and tensors.

The contraction of $W^{\mu\nu(S)}$ with the leptonic tensor $L_{\mu\nu}^{(S)}$ may be performed by means of the identities (Mulders and Tangerman, 1996)

$$
g_\perp^{\mu\nu}\, L_{\mu\nu}^{(S)} = -\frac{2Q^2}{y^2} \left[1 + (1-y)^2 \right], \qquad (7.73\mathrm{a})
$$

$$
\left[a_\perp^{\{\mu} b_\perp^{\nu\}} + \boldsymbol{a}_\perp{\cdot}\boldsymbol{b}_\perp\, g_\perp^{\mu\nu} \right] L_{\mu\nu}^{(S)} = \frac{4Q^2(1-y)}{y^2}\, |\boldsymbol{a}_\perp||\boldsymbol{b}_\perp| \cos(\phi_a + \phi_b), \qquad (7.73\mathrm{b})
$$

$$
\frac{1}{2} \left[a_\perp^{\{\mu} \varepsilon_\perp^{\nu\}\rho} b_{\perp\rho} + b_\perp^{\{\mu} \varepsilon_\perp^{\nu\}\rho} a_{\perp\rho} \right] L_{\mu\nu}^{(S)} = \frac{4Q^2(1-y)}{y^2}\, |\boldsymbol{a}_\perp||\boldsymbol{b}_\perp| \sin(\phi_a + \phi_b), \qquad (7.73\mathrm{c})
$$

where ϕ_a and ϕ_b are the azimuthal angles in the plane perpendicular to the photon–nucleon axis. Combining Eq. (7.72) with Eqs. (7.73a–c) leads quite straight-forwardly to the parton-model formulæ for the cross-sections. To obtain the LO QCD expressions, one must simply insert the Q^2 dependence into the distribution and fragmentation functions.

7.7.1 *Integrated cross-sections*

Consider, first of all, the cross-sections integrated over $\boldsymbol{P}_{h\perp}$. In this case, the $\boldsymbol{k}_T$ and $\boldsymbol{\kappa}_T$ integrals decouple and can be performed, yielding the integrated distribution and fragmentation functions. Hence, we obtain

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z} = \frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4} \sum_a e_a^2\, x \left\{ \frac{1}{2}\left[1+(1-y)^2\right] f_a(x)\, D_a(z) \right.$$
$$\left. - (1-y)\,|\boldsymbol{S}_\perp|\,|\boldsymbol{S}_{h\perp}|\,\cos(\phi_S+\phi_{S_h})\;\Delta_T f_a(x)\,\Delta_T D_a(z) \right\}. \quad (7.74)$$

As one can see, at leading twist, the transversity distributions are probed only when both the target and the produced hadron are transversely polarised. From (7.74) we can extract the transverse polarisation $\boldsymbol{\mathcal{P}}_h$ of the detected hadron, defined so that ('unp' = unpolarised)

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{unp}}\,(1+\boldsymbol{\mathcal{P}}_h\cdot\boldsymbol{S}_h)\,. \quad (7.75)$$

If we denote the transverse polarisation of h along y by $\mathcal{P}_{hy}^{\uparrow}$, when the target nucleon is polarised along y ($\uparrow$), and the transverse polarisation of h along x by $\mathcal{P}_{hx}^{\rightarrow}$, when the target nucleon is polarised along x ($\rightarrow$), we find

$$\mathcal{P}_{hy}^{\uparrow} = -\mathcal{P}_{hx}^{\rightarrow} = \frac{2(1-y)}{1+(1-y)^2}\,\frac{\sum_a e_a^2\,\Delta_T f_a(x)\,\Delta_T D_a(z)}{\sum_a e_a^2\, f_a(x)\, D_a(z)}\,. \quad (7.76)$$

If the hadron h is not transversely polarised, or – *a fortiori* – is spinless, the leading-twist $\boldsymbol{P}_{h\perp}$-integrated cross-section does not contain $\Delta_T f$. In this case, in order to probe the transversity distributions, one has to observe the $\boldsymbol{P}_{h\perp}$ distributions, or consider higher-twist contributions (Sec. 7.9). In the next section we shall discuss the former possibility.

7.7.2 *Single-spin azimuthal asymmetries*

We now study the (leading-twist) $\boldsymbol{P}_{h\perp}$ distributions in semi-inclusive DIS and the resulting azimuthal asymmetries. We shall assume that the detected hadron is spinless, or that its polarisation is not observed. For simplicity, we also neglect (at the beginning, at least) quark transverse motion inside the target. Thus, (7.72) simplifies as follows (recall that only the

unpolarised and the transversely polarised terms are considered):

$$W^{\mu\nu(S)} = 2\sum_a e_a^2\, z \int \mathrm{d}^2\boldsymbol{\kappa}_T\, \delta^2(\boldsymbol{\kappa}_T + \boldsymbol{P}_{h\perp}/z)$$
$$\times \Bigg\{ -g_\perp^{\mu\nu}\, f(x)\, D(z, {\boldsymbol{\kappa}'_T}^2)$$
$$- \frac{\left(S_T^{\{\mu}\varepsilon_T^{\nu\}\rho}\kappa_{T\rho} + \kappa_T^{\{\mu}\varepsilon_T^{\nu\}\rho}S_{T\rho}\right)}{2M_h}\, \Delta_T f(x)\, H_1^\perp(z, {\boldsymbol{\kappa}'_T}^2) + \ \ldots \Bigg\}. \quad (7.77)$$

Contracting $W^{\mu\nu(S)}$ with the leptonic tensor (7.14a) and inserting the result into (7.11) gives the cross-section

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P}_{h\perp}} = \frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4} \sum_a e_a^2\, x \Bigg\{ \frac{1}{2}\left[1 + (1-y)^2\right] f_a(x)\, D_a(z, \boldsymbol{P}_{h\perp}^2)$$
$$+ (1-y)\, \frac{|\boldsymbol{P}_{h\perp}|}{zM_h}\, |\boldsymbol{S}_\perp|\, \sin(\phi_S + \phi_h)$$
$$\times\, \Delta_T f_a(x)\, H_{1a}^\perp(z, \boldsymbol{P}_{h\perp}^2) \Bigg\}. \quad (7.78)$$

From this we obtain the *single transverse-spin asymmetry*

$$A_T^h \equiv \frac{\mathrm{d}\sigma(\boldsymbol{S}_\perp) - \mathrm{d}\sigma(-\boldsymbol{S}_\perp)}{\mathrm{d}\sigma(\boldsymbol{S}_\perp) + \mathrm{d}\sigma(-\boldsymbol{S}_\perp)}$$
$$= \frac{2(1-y)}{1+(1-y)^2}\, \frac{\sum_a e_a^2\, \Delta_T f_a(x)\, \Delta_T^0 D_a(z, \boldsymbol{P}_{h\perp}^2)}{\sum_a e_a^2\, f_a(x)\, D_a(z, \boldsymbol{P}_{h\perp}^2)}\, |\boldsymbol{S}_\perp|\, \sin(\phi_S + \phi_h), \quad (7.79)$$

where $\Delta_T^0 D(z, \boldsymbol{P}_{h\perp}^2)$ is related to $H_1^\perp(z, \boldsymbol{P}_{h\perp}^2)$ by (7.59). The existence of an azimuthal asymmetry in transversely polarised leptoproduction of spinless hadrons at leading twist, which depends on the T-odd fragmentation function $H_1^\perp$ and arises from final-state interaction effects, was predicted by Collins (1993b). The Collins angle Φ_C was originally defined by Collins as the angle between the transverse-spin vector of the fragmenting quark and the transverse momentum of the outgoing hadron, *i.e.*,

$$\Phi_C = \phi_{s'} - \phi_h\,. \quad (7.80)$$

One thus has

$$\sin\Phi_C = \frac{(\boldsymbol{q}\wedge\boldsymbol{P}_h)\cdot\boldsymbol{s}'}{|\boldsymbol{q}\wedge\boldsymbol{P}_h|\,|\boldsymbol{s}'|}\,. \quad (7.81)$$

Since, as dictated by QED (see Sec. 7.8), the directions of the final and initial quark spins are related to each other by (see Fig. 7.8)

$$\phi_{s'} = \pi - \phi_s \,. \tag{7.82}$$

Eq. (7.81) then becomes $\Phi_C = \pi - \phi_s - \phi_h$. Ignoring quark transverse

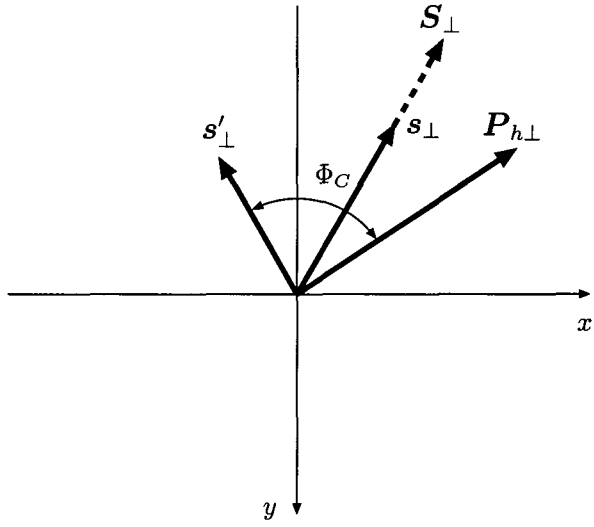

Fig. 7.8 The transverse-spin vectors and the transverse momentum of the outgoing hadron in the plane perpendicular to the $\gamma^* N$ axis. Φ_C is the Collins angle.

motion in the target, the initial quark spin is parallel to the target spin (*i.e.*, $\phi_s = \phi_S$) and Φ_C can finally be expressed in terms of measurable angles as

$$\Phi_C = \pi - \phi_S - \phi_h \,. \tag{7.83}$$

If quark transverse motion inside the target is taken into account the cross-sections become more complicated. We limit ourselves to a brief overview of such cases. Let us start from the unpolarised cross-section, which reads

$$\frac{\mathrm{d}^5\sigma_{\mathrm{unp}}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P}_{h\perp}} = \frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4} \sum_a \frac{e_a^2}{2}\, x\left[1+(1-y)^2\right] I[f_a\, D_a] \,, \tag{7.84}$$

where we have introduced the integral I, defined as (Mulders and Tangerman, 1996)

$$I[f\,D](x,z) \equiv \int \mathrm{d}^2\boldsymbol{k}_T\;\mathrm{d}^2\boldsymbol{\kappa}_T\,\delta^2(\boldsymbol{k}_T+\boldsymbol{q}_T-\boldsymbol{\kappa}_T)\,f(x,\boldsymbol{k}_T^2)\,D(z,\boldsymbol{\kappa}_T'^2)$$
$$= \int \mathrm{d}^2\boldsymbol{k}_T\;f(x,\boldsymbol{k}_T^2)\,D(z,|\boldsymbol{P}_{h\perp}-z\boldsymbol{k}_T|^2)\,. \qquad (7.85)$$

The cross-section for a transversely polarised target takes the form

$$\frac{\mathrm{d}^5\sigma(\boldsymbol{S}_\perp)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P}_{h\perp}} = \frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4}\,|\boldsymbol{S}_\perp|\sum_a e_a^2\,x(1-y)$$
$$\times\, I\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_\perp}{M_h}\,\Delta_T f_a\,H_{1a}^\perp\right]\sin(\phi_S+\phi_h)+\ldots\,, \qquad (7.86)$$

where $\hat{\boldsymbol{h}}\equiv\boldsymbol{P}_{h\perp}/|\boldsymbol{P}_{h\perp}|$ and a term giving rise to a $\sin(3\phi_h-\phi_S)$ asymmetry, but not involving $\Delta_T f$, has been omitted.

As we shall see in Sec. 7.13.2 there are presently some data on semi-inclusive DIS off nucleons polarised along the scattering axis that are of a certain interest for the study of transversity. It is therefore convenient, in view of the phenomenological analysis of those measurements, to also give the unintegrated cross-section for a longitudinally polarised target, which, although not containing $\Delta_T f$, depends on the Collins fragmentation function $H_1^\perp$, a crucial ingredient in the phenomenology of transversity. One finds

$$\frac{\mathrm{d}^5\sigma(\lambda_N)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P}_{h\perp}} = -\frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4}\,\lambda_N\sum_a e_a^2\,x(1-y)$$
$$\times\, I\left[\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_\perp)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp)-\boldsymbol{\kappa}_\perp\cdot\boldsymbol{k}_\perp}{MM_h}\,h_{1La}^\perp\,H_{1a}^\perp\right]\sin(2\phi_h)\,. \qquad (7.87)$$

Note the characteristic $\sin(2\phi_h)$ dependence of (7.86) and the appearance of the $\boldsymbol{k}_\perp$-dependent distribution function $h_{1L}^\perp$. One can factorise the x and z dependence in the above expressions by properly weighting the cross-sections with some function that depends on the azimuthal angles (Kotzinian and Mulders, 1997; Boer and Mulders, 1998). This procedure also singles out the different contributions to the cross-section for a given spin configuration of the target (and of the incoming lepton). To see how it works let us consider the case of a transversely polarised target. We

redefine the azimuthal angles so that the orientation of the lepton plane is given by a generic angle ϕ_ℓ in the transverse space. Equation (7.86) then becomes

$$\frac{\mathrm{d}^6\sigma(\boldsymbol{S}_\perp)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\phi_\ell\,\mathrm{d}^2\boldsymbol{P}_{h\perp}} = \frac{2\alpha_{\mathrm{em}}^2 s}{Q^4}\,|\boldsymbol{S}_\perp| \sum_a e_a^2\, x(1-y) \times I\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_\perp}{M_h}\,\Delta_T f_a\, H_{1a}^\perp\right] \sin(\phi_S+\phi_h-2\phi_\ell) + \sin(3\phi_h-\phi_S-2\phi_\ell)\ \text{term} \tag{7.88}$$

The weighted cross-section that projects out the first term of (7.88) and leads to a factorised expression in x and z, is

$$\int \mathrm{d}\phi_\ell\ \mathrm{d}^2\boldsymbol{P}_{h\perp}\,\frac{|\boldsymbol{P}_{h\perp}|}{M_h z}\,\sin(\phi_S+\phi_h-2\phi_\ell)\,\frac{\mathrm{d}^6\sigma(\boldsymbol{S}_\perp)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\phi_\ell\,\mathrm{d}^2\boldsymbol{P}_{h\perp}} = \frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4}\,|\boldsymbol{S}_\perp| \sum_a e_a^2\, x(1-y)\Delta_T f_a(x)\, H_{1a}^{\perp(1)}(z)\,, \tag{7.89}$$

where the weighted fragmentation function $H_1^{\perp(1)}$ is defined as

$$H_1^{\perp(1)}(z) = z^2 \int \mathrm{d}^2\boldsymbol{\kappa}_T \left(\frac{\boldsymbol{\kappa}_T^2}{2M_h^2}\right) H_1^\perp(z, z^2\boldsymbol{\kappa}_T^2)\,. \tag{7.90}$$

If one assumes that T-odd distributions (such as $f_{1T}^\perp$) exist, then there is a further single-spin asymmetry in semi-inclusive DIS, given in weighted form by

$$\int \mathrm{d}\phi_\ell\ \mathrm{d}^2\boldsymbol{P}_{h\perp}\,\frac{|\boldsymbol{P}_{h\perp}|}{M_h z}\,\sin(\phi_h-\phi_S)\,\frac{\mathrm{d}^6\sigma(\boldsymbol{S}_\perp)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\phi_\ell\,\mathrm{d}^2\boldsymbol{P}_{h\perp}} = \frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4}\,|\boldsymbol{S}_\perp| \sum_a e_a^2\, x\left(1-y+\tfrac{1}{2}y^2\right) f_{1T,a}^{\perp(1)}(x)\, D_a(z)\,, \tag{7.91}$$

where

$$f_{1T}^{\perp(1)}(x) = \int \mathrm{d}^2\boldsymbol{k}_T \left(\frac{\boldsymbol{k}_T^2}{2M^2}\right) f_{1T}^\perp(x, \boldsymbol{k}_T^2)\,. \tag{7.92}$$

Note that in (7.91) the T-odd distribution $f_{1T}^\perp$ couples to the unpolarised fragmentation function $D(z)$. The asymmetry (7.91), which arises from the transverse momentum asymmetry of unpolarised quarks in a transversely polarised target, is called the *Sivers asymmetry* (Sivers, 1990, 1991).

A different look at azimuthal asymmetries in semi-inclusive leptoproduction of unpolarised hadrons has recently been offered by Brodsky, Hwang and Schmidt (2002). These authors show that the final-state interaction given by the exchange of one gluon between the outgoing quark and the target spectator system (Fig. 7.9) produces a non-trivial phase in the amplitude and hence a single-spin asymmetry. The presence of a quark transverse

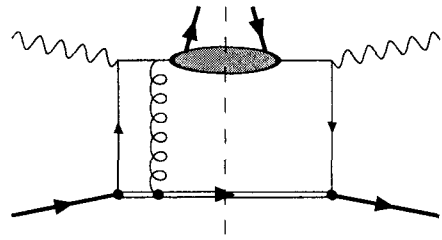

Fig. 7.9 One-gluon exchange between the outgoing quark and the spectator system (represented by the double line). The dashed line is the unitarity cut.

momentum smaller than Q ensures that this asymmetry is proportional to $M/k_\perp$, rather than to M/Q, and therefore it is a leading-twist quantity. As pointed out by Collins (2002), the final-state interaction considered by Brodsky, Hwang and Schmidt occurs over a short scale in time and distance, of the order of the scale associated with the uncertainty in the position of the vertex of the virtual photon. Therefore, the resulting single-spin asymmetry is not related to the hadronisation of the quark, but rather to an intrinsic transverse-momentum asymmetry of the quarks inside the transversely polarised target. In other terms, it is the Sivers asymmetry (7.91) associated to the T-odd distribution function $f_{1T}^{\perp}$. The proof that the Sivers asymmetry is non-zero in QCD and that the T-odd distributions are not forbidden by time-reversal invariance (see Sec. 3.10) sheds new light on the entire subject and opens the way to interesting phenomenological applications.

For a more complete discussion of transversely polarised semi-inclusive leptoproduction, with or without intrinsic quark motion, we refer the reader to the vast literature on the subject (Levelt and Mulders, 1994a; Kotzinian, 1995; Tangerman and Mulders, 1995b; Jaffe, 1996; Kotzinian and Mulders, 1996, 1997; Mulders and Tangerman, 1996; Boer and Mulders, 1998; Mulders, 1998; Boer, Jakob and Mulders, 2000; Boglione and Mulders, 2000;

Mulders and Boglione, 2000). In Sec. 7.13 we shall present some predictions and preliminary experimental results on A_T^h.

7.8 Factorisation in semi-inclusive DIS

It is instructive to use a different approach, based on QCD factorisation, to rederive the results on semi-inclusive DIS presented in the previous section. We start by considering the collinear case, that is, ignoring quark transverse motion both in the target and in the produced hadron. In such a case the factorisation theorem is known to hold. This theorem was originally demonstrated for the production of unpolarised particles (Collins, Soper and Sterman, 1984, 1985; Bodwin, 1985; Collins, Soper and Sterman, 1988, 1989) and then also shown to apply if the detected particles are polarised (Collins, 1993a). In contrast, when quark transverse motion is taken into account, factorisation is not proven and can only be regarded as a reasonable assumption.

7.8.1 *The collinear case*

The QCD factorisation theorem states that the cross-section for semi-inclusive DIS can be written, to all orders of perturbation theory, as

$$\mathrm{d}\sigma = \sum_{ab} \sum_{\lambda\lambda'\eta\eta'} \int \mathrm{d}\xi \, \mathrm{d}\zeta \, f_a(\xi,\mu) \, \rho_{\lambda'\lambda} \times \mathrm{d}\hat{\sigma}_{\lambda\lambda'\eta\eta'}(x/\xi, Q/\mu, \alpha_s(\mu)) \, \mathcal{D}_{h/b}^{\eta'\eta}(\zeta,\mu) \,, \quad (7.93)$$

where $\sum_{ab}$ is a sum over initial (a) and final (b) partons, $\rho_{\lambda\lambda'}$ is the spin density matrix of parton a in the nucleon, and $\mathrm{d}\hat{\sigma}$ is the perturbatively calculable cross-section of the hard subprocesses that contribute to the reaction. In (7.93) ξ is the fraction of the proton momentum carried by the parton a, ζ is the fraction of the momentum of parton b carried by the produced hadron, and μ is the factorisation scale. Lastly, $\mathcal{D}_{h/b}(z)$ is the fragmentation matrix of parton b into the hadron h

$$\mathcal{D}_{h/b} = \begin{pmatrix} \mathcal{D}_{h/b}^{++} & \mathcal{D}_{h/b}^{+-} \\ \mathcal{D}_{h/b}^{-+} & \mathcal{D}_{h/b}^{--} \end{pmatrix}. \quad (7.94)$$

It is defined in such a manner that

$$\tfrac{1}{2}\sum_{\eta}\mathcal{D}^{\eta\eta}_{h/b}(z)=\tfrac{1}{2}\left[\mathcal{D}^{++}_{h/b}(z)+\mathcal{D}^{--}_{h/b}(z)\right]=D_{h/b}(z)\,, \tag{7.95}$$

where $D_{h/b}(z)$ is the usual unpolarised fragmentation function, that is, the probability of finding a hadron h with longitudinal momentum fraction z inside a parton b. The difference of the diagonal elements of $\mathcal{D}_{h/b}(z)$ gives the longitudinal polarisation fragmentation function

$$\tfrac{1}{2}\left[\mathcal{D}^{++}_{h/b}(z)-\mathcal{D}^{--}_{h/b}(z)\right]=\lambda_h\,\Delta D_{h/b}(z)\,, \tag{7.96a}$$

whereas the off-diagonal elements are related to transverse polarisation

$$\tfrac{1}{2}\left[\mathcal{D}^{+-}_{h/b}(z)+\mathcal{D}^{-+}_{h/b}(z)\right]=S_{hx}\,\Delta_T D_{h/b}(z)\,, \tag{7.96b}$$

$$\tfrac{\mathrm{i}}{2}\left[\mathcal{D}^{+-}_{h/b}(z)-\mathcal{D}^{-+}_{h/b}(z)\right]=S_{hy}\,\Delta_T D_{h/b}(z)\,. \tag{7.96c}$$

Note that $\mathcal{D}_{h/b}$ is normalised such that for an unpolarised hadron it reduces to the unit matrix.

At lowest order the only elementary process that contributes to $\mathrm{d}\hat{\sigma}$ is $l\,q\,(\bar{q})\to l\,q\,(\bar{q})$, see Fig. 7.10. Thus, the sum $\sum_{ab}$ only runs over quarks

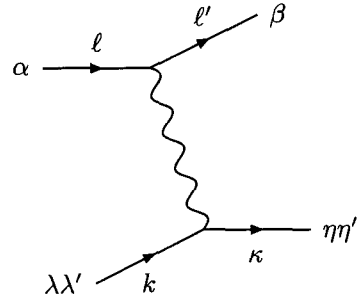

Fig. 7.10 Lepton–quark (–antiquark) scattering.

and antiquarks, and $a=b$. Equation (7.93) then becomes (omitting energy

scales)

$$E'E_h \frac{\mathrm{d}^6\sigma}{\mathrm{d}^3\boldsymbol{\ell}'\,\mathrm{d}^3\boldsymbol{P}_h} = \sum_a \sum_{\lambda\lambda'\eta\eta'} \int \mathrm{d}\xi \frac{\mathrm{d}\zeta}{\zeta^2} f_a(\xi)\,\rho_{\lambda'\lambda} \times E'E_\kappa \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}^3\boldsymbol{\ell}'\,\mathrm{d}^3\boldsymbol{\kappa}}\right)_{\lambda\lambda'\eta\eta'} \mathcal{D}_{h/a}^{\eta'\eta}(\zeta)\,. \quad (7.97)$$

The elementary cross-section in (7.97) is ($\hat{s} = xs$ is the centre-of-mass energy squared of the partonic scattering, with the hat labelling quantities defined at the subprocess level)

$$\begin{aligned} E'E_\kappa \left(\frac{\mathrm{d}^6\hat{\sigma}}{\mathrm{d}^3\boldsymbol{\ell}'\,\mathrm{d}^3\boldsymbol{\kappa}}\right)_{\lambda\lambda'\eta\eta'} &= \frac{1}{32\pi^2\hat{s}} \frac{1}{2} \sum_{\alpha\beta} \mathcal{M}_{\lambda\alpha\eta\beta}\,\mathcal{M}^*_{\lambda'\alpha\eta'\beta}\,\delta^4(\ell + k - \ell' - \kappa) \\ &= \frac{1}{2\pi} \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}\right)_{\lambda\lambda'\eta\eta'} \delta^4(\ell + k - \ell' - \kappa)\,, \end{aligned} \quad (7.98)$$

where

$$\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}\right)_{\lambda\lambda'\eta\eta'} = \frac{1}{16\pi\hat{s}} \frac{1}{2} \sum_{\alpha\beta} \mathcal{M}_{\lambda\alpha\eta\beta}\,\mathcal{M}^*_{\lambda'\alpha\eta'\beta}\,, \quad (7.99)$$

with the sum being performed over the helicities of the incoming and outgoing leptons.

Working in the hN collinear frame, where the photon momentum is given by $q^\mu \simeq -xP^\mu + \frac{1}{z} P_h^\mu + q_T^\mu$, which in light-cone components is then $q^\mu \simeq (-xP^+, \frac{1}{z} P_h^-, \boldsymbol{q}_T)$, the energy–momentum conservation delta function may be written as

$$\begin{aligned} \delta^4(\ell + k - \ell' - \kappa) &= \delta^4(q + k - \kappa) \\ &\simeq \delta(q^+ + k^+)\,\delta(q^- - \kappa^-)\,\delta^2(\boldsymbol{q}_T) \\ &= \frac{2xz}{Q^2}\,\delta(\xi - x)\,\delta(\zeta - z)\,\delta^2(\boldsymbol{q}_T)\,. \end{aligned} \quad (7.100)$$

The integrations over ξ and ζ in (7.97) can now be performed and the cross-section for semi-inclusive DIS (expressed in terms of the invariants x, y, z

and of the transverse momentum of the outgoing hadron $\boldsymbol{P}_{h\perp}$) reads

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P}_{h\perp}} = \sum_a \sum_{\lambda\lambda'\eta\eta'} f_a(\xi)\,\rho_{\lambda'\lambda} \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}\right)_{\lambda\lambda'\eta\eta'} \mathcal{D}^{\eta'\eta}_{h/a}(z)\,\delta^2(\boldsymbol{P}_{h\perp})\,. \tag{7.101}$$

Note the $\delta^2(\boldsymbol{P}_{h\perp})$ factor coming from the kinematics of the hard subprocess at lowest order. Integrating the cross-section over the hadron transverse momentum we obtain

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z} = \sum_a \sum_{\lambda\lambda'\eta\eta'} f_a(\xi)\,\rho_{\lambda\lambda'} \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}\right)_{\lambda\lambda'\eta\eta'} \mathcal{D}^{\eta\eta'}_{h/a}(z)\,. \tag{7.102}$$

Let us now look at the helicity structure of the lq scattering process. By helicity conservation, the only non-vanishing scattering amplitudes are ($y = -\hat{t}/\hat{s} = \frac{1}{2}(1-\cos\theta)$)

$$\mathcal{M}_{++++} = \mathcal{M}_{----} = 4\mathrm{i}\,e^2 e_a \frac{1}{\cos\theta} = 2\mathrm{i}\,e^2 e_a \frac{1}{y}\,, \tag{7.103a}$$

$$\mathcal{M}_{+-+-} = \mathcal{M}_{-+-+} = 2\mathrm{i}\,e^2 e_a \frac{1+\cos\theta}{1-\cos\theta} = 2\mathrm{i}\,e^2 e_a \frac{1-y}{y}, \tag{7.103b}$$

where θ is the scattering angle in the lq centre-of-mass frame. The elementary cross-sections contributing to (7.102) are

$$\begin{aligned}\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}\right)_{++++} = \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}\right)_{----} &= \frac{1}{16\pi\hat{s}}\,\frac{1}{2}\left(|\mathcal{M}_{++++}|^2 + |\mathcal{M}_{+-+-}|^2\right) \\ &= \frac{4\pi\alpha^2 xs}{Q^4}\,\frac{e_a^2}{2}\left[1+(1-y)^2\right], \end{aligned} \tag{7.104a}$$

$$\begin{aligned}\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}\right)_{+-+-} = \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}\right)_{-+-+} &= \frac{1}{16\pi\hat{s}}\,\mathrm{Re}\,\mathcal{M}_{++++}\,\mathcal{M}^*_{+-+-} \\ &= \frac{4\pi\alpha^2 xs}{Q^4}\,e_a^2\,(1-y)\,, \end{aligned} \tag{7.104b}$$

and the cross-section (7.102) then reads

$$\begin{aligned}\frac{\mathrm{d}^3\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z} = \sum_a f_a(x) \Bigg\{ &\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}\right)_{++++} \left(\rho_{++}\,\mathcal{D}^{++}_{h/a} + \rho_{--}\,\mathcal{D}^{--}_{h/a}\right) \\ &+ \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}\right)_{+-+-} \left(\rho_{+-}\,\mathcal{D}^{+-}_{h/a} + \rho_{-+}\,\mathcal{D}^{-+}_{h/a}\right)\Bigg\}. \end{aligned} \tag{7.105}$$

Inserting (7.104a, b) into (7.105) and using (3.28–3.29b) and (7.95–7.96c), we obtain

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z} = \frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4} \sum_a e_a^2\, x$$
$$\times \left\{ \frac{1}{2}\left[1+(1-y)^2\right]\left[f_a(x)\, D_{h/a}(z) + \lambda_N \lambda_h\, \Delta f_a(x)\, \Delta D_{h/a}(z)\right] \right.$$
$$\left. + (1-y)\, |\boldsymbol{S}_\perp|\, |\boldsymbol{S}_{h\perp}|\, \cos(\phi_S + \phi_{S_h})\, \Delta_T f_a(x)\, \Delta_T D_{h/a}(z) \right\}. \quad (7.106)$$

which coincides with the result already obtained in Sec. 7.7.

In the light of the present derivation of (7.106), we understand the origin of the y-dependent factor in (7.76) and (7.79). This factor,

$$\hat{a}_T \equiv \frac{\mathrm{d}\hat{\sigma}_{+-+-}}{\mathrm{d}\hat{\sigma}_{++++}} = \frac{2(1-y)}{1+(1-y)^2}\,. \quad (7.107)$$

is a spin transfer coefficient, *i.e.*, the transverse polarisation of the final quark generated by an initial transversely polarised quark in the $lq \to lq$ process. To see this, let us call $H_{\eta\eta'}$ the quantity

$$H_{\eta\eta'} \equiv \rho_{\lambda'\lambda} \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}\right)_{\lambda\lambda'\eta\eta'}, \quad (7.108)$$

and introduce the spin density matrix of the final quark, defined via

$$H_{\eta\eta'} = H_{\mathrm{unp}}\, \rho'_{\eta\eta'}\,, \quad (7.109)$$

where

$$H_{\mathrm{unp}} = H_{++} + H_{--} = (\mathrm{d}\hat{\sigma})_{++++}\,. \quad (7.110)$$

We find explicitly

$$\rho' = \begin{pmatrix} \rho_{++} & \hat{a}_T\, \rho_{-+} \\ \hat{a}_T\, \rho_{+-} & \rho_{--} \end{pmatrix}, \quad (7.111)$$

and, recalling that the final quark travels along $-z$, we finally obtain for its spin vector $\boldsymbol{s}'$

$$s'_x = -\hat{a}_T\, s_x\,, \qquad s'_y = \hat{a}_T\, s_y\,. \quad (7.112)$$

Thus, the initial and final quark spin directions are specular with respect to the y axis. The factor $\hat{a}_T$ is also known as the *depolarisation factor*. It decreases with y, being unity at $y = 0$ and zero at $y = 1$.

7.8.2 *The non-collinear case*

If quarks are allowed to have transverse momenta, QCD factorisation is no longer a proven property, but only an assumption. In this case we write, in analogy with (7.97),

$$E'E_h \frac{\mathrm{d}^6\sigma}{\mathrm{d}^3\boldsymbol{\ell}'\,\mathrm{d}^3\boldsymbol{P}_h} = \sum_a \sum_{\lambda\lambda'\eta\eta'} \int \mathrm{d}\xi \int \frac{\mathrm{d}\zeta}{\zeta^2} \int \mathrm{d}^2\boldsymbol{k}_T \int \mathrm{d}^2\boldsymbol{\kappa}'_T\, \mathcal{P}_a(\xi,\boldsymbol{k}_T)\,\rho_{\lambda'\lambda} \times E'E_\kappa \left(\frac{\mathrm{d}^6\hat{\sigma}}{\mathrm{d}^3\boldsymbol{\ell}'\,\mathrm{d}^3\boldsymbol{\kappa}}\right)_{\lambda\lambda'\eta\eta'} \mathcal{D}^{\eta'\eta}_{h/a}(\zeta,\boldsymbol{\kappa}'_T)\,, \qquad (7.113)$$

where $\mathcal{P}_a(\xi,\boldsymbol{k}_T)$ is the probability of finding a quark a with momentum fraction x and transverse momentum $\boldsymbol{k}_T$ inside the target nucleon, and $\mathcal{D}_{h/a}(\zeta,\boldsymbol{\kappa}'_T)$ is the fragmentation matrix of quark a into a hadron h, having transverse momentum $\boldsymbol{\kappa}'_T \equiv -z\boldsymbol{\kappa}_T$ with respect to the quark momentum. Evaluating Eq. (7.113), as we did with Eq. (7.97), we obtain the cross-section in terms of the invariants x, y, z and of $\boldsymbol{P}^2_{h\perp}$

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P}_{h\perp}} = \sum_a \sum_{\lambda\lambda'\eta\eta'} \int \mathrm{d}^2\boldsymbol{k}_T \int \mathrm{d}^2\boldsymbol{\kappa}'_T\, \mathcal{P}_a(\xi,\boldsymbol{k}_T)\,\rho_{\lambda'\lambda} \times \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}\right)_{\lambda\lambda'\eta\eta'} \mathcal{D}^{\eta'\eta}_{h/a}(z,\boldsymbol{\kappa}'_T)\,\delta^2(z\boldsymbol{k}_T - \boldsymbol{\kappa}'_T - \boldsymbol{P}_{h\perp})\,. \qquad (7.114)$$

Inserting the elementary cross-sections (7.104a, b) in (7.114) and writing the sum over the helicities explicitly, we obtain

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P}_{h\perp}} = \frac{4\pi\alpha^2_{\mathrm{em}}s}{Q^4} \sum_a e_a^2\, x \int \mathrm{d}^2\boldsymbol{k}_T \int \mathrm{d}^2\boldsymbol{\kappa}'_T\, \mathcal{P}_a(x,\boldsymbol{k}_T) \times \left\{\frac{1}{2}\left[1+(1-y)^2\right]\left[\rho_{++}\,\mathcal{D}^{++}_{h/a} + \rho_{--}\,\mathcal{D}^{--}_{h/a}\right] + (1-y)\left[\rho_{+-}\,\mathcal{D}^{+-}_{h/a} + \rho_{-+}\,\mathcal{D}^{-+}_{h/a}\right]\right\} \times \delta^2(z\boldsymbol{k}_T - \boldsymbol{\kappa}'_T - \boldsymbol{P}_{h\perp})\,. \qquad (7.115)$$

Let us now suppose that the hadron h is unpolarised. Using the correspondence (3.28) between the spin density matrix elements and the spin of the initial quark, and the analogous relations for the fragmentation matrix

obtained from (7.56a–c), that is,

$$\frac{1}{2}\left(\mathcal{D}^{++}_{h/a}+\mathcal{D}^{--}_{h/a}\right) = D(z,\boldsymbol{\kappa}_T'^{\,2}) + \frac{1}{M_h}\,\varepsilon_T^{ij}\kappa_{Ti}S_{hTj}\,D_{1T}^{\perp}(z,\boldsymbol{\kappa}_T'^{\,2})\,, \tag{7.116a}$$

$$\frac{1}{2}\left(\mathcal{D}^{++}_{h/a}-\mathcal{D}^{--}_{h/a}\right) = \lambda_h\,\Delta D(z,\boldsymbol{\kappa}_T'^{\,2}) + \frac{1}{M_h}\,\boldsymbol{\kappa}_T\cdot\boldsymbol{S}_{hT}\,G_{1T}(z,\boldsymbol{\kappa}_T'^{\,2})\,, \tag{7.116b}$$

$$\frac{1}{2}\left(\mathcal{D}^{+-}_{h/a}+\mathcal{D}^{-+}_{h/a}\right) = S_{hT}^1\,\Delta_T' D(z,\boldsymbol{\kappa}_T'^{\,2}) + \frac{\lambda_h}{M_h}\,\kappa_T^1 H_{1L}^{\perp}(z,\boldsymbol{\kappa}_T'^{\,2}) - \frac{1}{M_h^2}\left(\kappa_T^1\kappa_T^j+\frac{1}{2}\,\boldsymbol{\kappa}_T^2\,g_\perp^{1j}\right)S_{hTj}\,H_{1T}^{\perp}(z,\boldsymbol{\kappa}_T'^{\,2}) + \frac{1}{M_h}\,\varepsilon_T^{1j}\kappa_{Tj}\,H_1^{\perp}(z,\boldsymbol{\kappa}_T'^{\,2})\,, \tag{7.116c}$$

$$-\frac{1}{2\mathrm{i}}\left(\mathcal{D}^{+-}_{h/a}-\mathcal{D}^{-+}_{h/a}\right) = S_{hT}^2\,\Delta_T' D(z,\boldsymbol{\kappa}_T'^{\,2}) + \frac{\lambda_h}{M_h}\,\kappa_T^2 H_{1L}^{\perp}(z,\boldsymbol{\kappa}_T'^{\,2}) - \frac{1}{M_h^2}\left(\kappa_T^1\kappa_T^j+\frac{1}{2}\,\boldsymbol{\kappa}_T^2\,g_\perp^{2j}\right)S_{hTj}\,H_{1T}^{\perp}(z,\boldsymbol{\kappa}_T'^{\,2}) + \frac{1}{M_h}\,\varepsilon_T^{2j}\kappa_{Tj}\,H_1^{\perp}(z,\boldsymbol{\kappa}_T'^{\,2})\,, \tag{7.116d}$$

the transverse-polarisation contribution to the cross-section turns out to be

$$\frac{\mathrm{d}^5\sigma(\boldsymbol{S}_\perp)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P}_{h\perp}} = -\frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4}\sum_a e_a^2\,x(1-y)\int\mathrm{d}^2\boldsymbol{k}_T\int\mathrm{d}^2\boldsymbol{\kappa}_T'\,\mathcal{P}_a(x,\boldsymbol{k}_T) \times\frac{1}{M_h}\,(s_x\kappa_{Ty}+s_y\kappa_{Tx})\,H_{1a}^{\perp}(z,\boldsymbol{\kappa}_T'^{\,2}) \times\delta^2(z\boldsymbol{k}_T-\boldsymbol{\kappa}_T'-\boldsymbol{P}_{h\perp})\,. \tag{7.117}$$

If, for simplicity, we neglect the transverse momentum of the quarks inside the target, then $\boldsymbol{s}_\perp\,\mathcal{P}_a(x)=\boldsymbol{S}_\perp\,\Delta_T f_a(x)$. The integration over $\boldsymbol{\kappa}_T'$ can be performed, giving the constraint $\boldsymbol{\kappa}_T'\equiv-z\boldsymbol{\kappa}_T=\boldsymbol{P}_{h\perp}$, and Eq. (7.117) then becomes, with our convention for the axes and azimuthal angles,

$$\frac{\mathrm{d}^5\sigma(\boldsymbol{S}_\perp)}{\mathrm{d}x\;\mathrm{d}y\;\mathrm{d}z\;\mathrm{d}^2\boldsymbol{P}_{h\perp}} = \frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4}\,|\boldsymbol{S}_\perp|\sum_a e_a^2\,x(1-y)\,\Delta_T f_a(x) \times\frac{|\boldsymbol{P}_{h\perp}|}{zM_h}\,H_{1a}^{\perp}(z,\boldsymbol{P}_{h\perp}^2)\,\sin(\phi_\kappa+\phi_S)\,. \tag{7.118}$$

Since $\boldsymbol{\kappa}_T = -\boldsymbol{P}_{h\perp}/z$, we have

$$\phi_\kappa + \phi_S = \phi_h - \pi + \phi_S = \Phi_C - \pi\,, \tag{7.119}$$

and (7.118) reduces to the transverse-polarisation term of (7.78).

7.8.3 *Sudakov form factors*

The non-collinear factorisation formula (7.114) does not take into account an important dynamical effect: soft-gluon radiation. Exponentiation of soft-gluon contributions gives rise to the so-called *Sudakov form factors* (see, *e.g.*, Collins, 1989). Thus, in Eq. (7.114) the delta function of transverse momenta is replaced by

$$\int \frac{\mathrm{d}^2\boldsymbol{b}}{(2\pi)^2}\, \mathrm{e}^{-\mathrm{i}(z\boldsymbol{k}_T - \boldsymbol{\kappa}'_T - \boldsymbol{P}_{h\perp})}\, \mathrm{e}^{-S(b,Q)}\,, \tag{7.120}$$

where the Sudakov form factor $\mathrm{e}^{-S(b,Q)}$ quantifies the smearing of the transverse momentum distribution due to the emission of soft gluons. For values $b^2 \ll 1/\Lambda^2_{\mathrm{QCD}}$, the exponent $S(b,Q)$ can be calculated perturbatively and has the form

$$S(b,Q) = \int_{b_0^2/b^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2} \left[A\left(\alpha_s(\mu)\right) \ln\frac{Q^2}{\mu^2} + B\left(\alpha_s(\mu)\right) \right], \tag{7.121}$$

where $b_0 = 2\,\mathrm{e}^{-\gamma_E}$, and A and B are power series in α_s, whose first few coefficients are known (Davies and Stirling, 1984; Weber, 1992, 1993; Frixione, Nason and Ridolfi, 1999). $S(b,Q)$ is made up of two parts: one perturbative, S_P, which is obtained from (7.121) by introducing a proper regulator, and the other non-perturbative, S_{NP}, which must be modelled or parametrised from experiment. In semi-inclusive DIS, the effect of Sudakov form factors is a suppression of the azimuthal asymmetries that increases with energy. In particular, Eq. (7.79) is multiplied by the factor (Boer, 2001)

$$\mathcal{A}(Q_T) = M_h \frac{\int_0^\infty \mathrm{d}b\, b^2\, J_1(bQ_T)\, \exp[-S_P(b,Q) - S_{NP}(b,Q)]}{\int_0^\infty \mathrm{d}b\, b^2\, J_0(bQ_T)\, \exp[-S_P(b,Q) - S_{NP}(b,Q)]}\,, \tag{7.122}$$

with $Q_T \equiv |\boldsymbol{q}_T| = |\boldsymbol{P}_{h\perp}|/z$. The practical evaluation of (7.122) is made difficult by our ignorance of S_{NP}. However, as shown by Boer (2001), the Q^2 dependence of $\mathcal{A}(Q_T)$ is rather stable against variations of S_{NP} and turns out to be approximately $1/\sqrt{Q}$.

7.9 Inclusive leptoproduction at twist three

Let us now see how transversity distributions appear in semi-inclusive DIS at the higher-twist level. We shall consider only twist-three contributions and limit ourselves to quoting the main results without derivation (which may be found in Mulders and Tangerman, 1996). If the lepton beam is unpolarised, the cross-section for leptoproduction of unpolarised (or spinless) hadrons with a transversely polarised target is

$$\frac{\mathrm{d}^3\sigma(\boldsymbol{S}_\perp)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z} = \frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4}\,|\boldsymbol{S}_\perp|\,\frac{M}{Q}\sum_a e_a^2\,2(2-y)\sqrt{1-y}$$
$$\times\left\{\frac{M_h}{M}\,\sin\phi_S\,x\,\Delta_T f_a(x)\,\frac{\widetilde{H}_a(z)}{z}\right.$$
$$\left. -\,\lambda_h\,\cos\phi_S\left[x^2\,g_T^a(x)\,\Delta D_a(z)+\frac{M_h}{M}\,x\,\Delta_T f_a(x)\,\frac{\widetilde{H}_L^a(z)}{z}\right]\right\}, \quad (7.123)$$

where the factor M/Q signals that (7.123) is a twist-three quantity. Adding (7.123) to the transverse component of (7.74) gives the complete $\boldsymbol{P}_{h\perp}$-integrated cross-section of semi-inclusive DIS off a transversely polarised target up to twist three. Note that in (7.123) the leading-twist transversity distributions $\Delta_T f(x)$ are coupled to the twist-three fragmentation functions $\widetilde{H}(z)$ and $\widetilde{H}_L(z)$, while the leading-twist helicity fragmentation function $\Delta D(z)$ is coupled to the twist-three distribution $g_T(x)$. $\widetilde{H}(z)$ is a T-odd fragmentation function.

At twist three, the transversity distributions also contribute to the scattering of a longitudinally polarised lepton beam. The corresponding cross-section is

$$\frac{\mathrm{d}^3\sigma(\lambda_l,\boldsymbol{S}_\perp)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z} = -\frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4}\,\lambda_l\,|\boldsymbol{S}_\perp|\,\frac{M}{Q}\sum_a e_a^2\,2y\sqrt{1-y}$$
$$\times\left\{\cos\phi_S\left[x^2\,g_T^a(x)\,D_a(z)+\frac{M_h}{M}\,x\,\Delta_T f_a(x)\,\frac{\widetilde{E}_a(z)}{z}\right]\right.$$
$$\left. +\,\lambda_h\,\sin\phi_S\left[\frac{M_h}{M}\,x\,\Delta_T f(x)\,\frac{\widetilde{E}_L^a(z)}{z}\right]\right\}. \quad (7.124)$$

Here, again, the leading-twist transversity distributions $\Delta_T f(x)$ are coupled to the twist-three fragmentation functions $\widetilde{E}(z)$ and $\widetilde{E}_L(z)$, and the

leading-twist unpolarised fragmentation function $D(z)$ is coupled to the twist-three distribution $g_T(x)$. $\widetilde{E}_L(z)$ is a T-odd fragmentation function. Up to $\mathcal{O}(1/Q)$, there are no other observables in semi-inclusive leptoproduction involving transversity distributions. The twist-two and twist-three contributions to semi-inclusive leptoproduction involving the transversity distributions $\Delta_T f$ are collected in Tables 7.1 and 7.2.

Table 7.1 Contributions to the $\boldsymbol{P}_{h\perp}$-integrated cross-section of semi-inclusive DIS involving the transversity distributions. T, L and 0 denote transverse, longitudinal and no polarisation, respectively. The asterisk indicates T-odd observables.

Cross-section integrated over $\boldsymbol{P}_{h\perp}$				
	ℓ	N	h	observable
twist 2	0	T	T	$\Delta_T f(x)\, \Delta_T D(z)$
twist 3	0	T	0	$\Delta_T f(x)\, \widetilde{H}(z)$ (*)
	0	T	L	$\Delta_T f(x)\, \widetilde{H}_L(z)$
	L	T	0	$\Delta_T f(x)\, \widetilde{E}(z)$
	L	T	L	$\Delta_T f(x)\, \widetilde{E}_L(z)$ (*)

Table 7.2 Contributions to the $\boldsymbol{P}_{h\perp}$ distribution of semi-inclusive DIS involving transversity. The produced hadron h is taken to be unpolarised. The notation is as in Table 7.1.

$\boldsymbol{P}_{h\perp}$ distribution (h unpolarised)			
	ℓ	N	observable
twist 2	0	T	$\Delta_T f \otimes H_1^\perp$ (*)
twist 3	0	T	$\Delta_T f \otimes \widetilde{H}$ (*)
	0	T	$\Delta_T f \otimes H_1^\perp$ (*)
	L	T	$\Delta_T f \otimes \widetilde{E}$

7.10 Two-particle leptoproduction

Another partially inclusive DIS reaction where transversity can play a rôle is two-particle leptoproduction (see Fig. 7.11) with the target polarised

transversely:

$$l(\ell) + N(P) \to l'(\ell') + h_1(P_1) + h_2(P_2) + X(P_X)\,. \tag{7.125}$$

In this reaction two hadrons (for instance, two pions) are detected in the

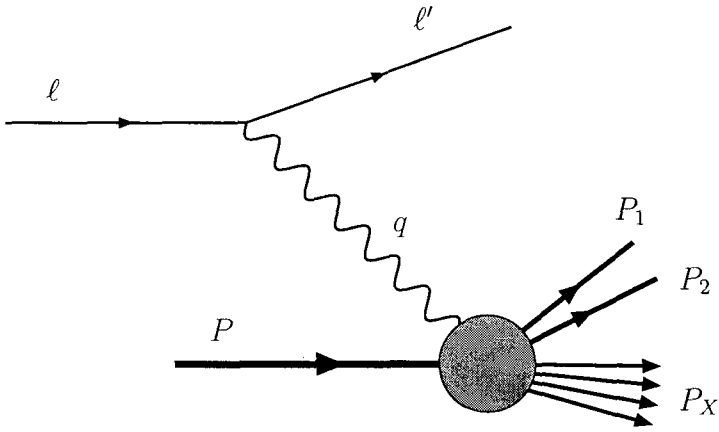

Fig. 7.11 Two-particle leptoproduction.

final state. Two-hadron leptoproduction has been proposed and studied by various authors (Collins, Heppelmann and Ladinsky, 1994; Jaffe, Jin and Tang, 1998b; Bianconi *et al.*, 2000a,b; Radici, Jakob and Bianconi, 2002) as a process that can probe the transverse-polarisation distributions of the nucleon, coupled to some interference fragmentation functions. The idea is to look at angular correlations of the form $(\boldsymbol{P}_1 \wedge \boldsymbol{P}_2)\cdot\boldsymbol{s}'$, where $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ are the momenta of the two produced hadrons and $\boldsymbol{s}'_\perp$ is the transverse-spin vector of the fragmenting quark. These correlations are not forbidden by time-reversal invariance, owing to final-state interactions between the two hadrons. To our knowledge, the first authors to suggest resonance interference as a way of producing non-diagonal quark fragmentation matrices were Cea, Nardulli and Chiappetta (1988)[c] in their attempt to explain the observed transverse polarisation of Λ^0 hyperons produced in pN interactions (Bunce *et al.*, 1976). Hereafter, we shall consider an unpolarised lepton beam and unpolarised hadrons in the final state.

[c]However, the basic idea may be traced back to a suggestion by Preparata (1985).

The cross-section for the reaction (7.125) reads, *cf.* Eq. (7.7)

$$\mathrm{d}^9\sigma = \frac{1}{4P{\cdot}\ell}\,\frac{e^4}{Q^4}\,L_{\mu\nu}W^{\mu\nu}\,(2\pi)^4\,\frac{\mathrm{d}^3\boldsymbol{\ell}'}{(2\pi)^3\,2E'}\,\frac{\mathrm{d}^3\boldsymbol{P}_1}{(2\pi)^3\,2E_1}\,\frac{\mathrm{d}^3\boldsymbol{P}_2}{(2\pi)^3\,2E_2}\,, \tag{7.126}$$

where $L_{\mu\nu}$ is the usual leptonic tensor, Eq. (7.5), and $W^{\mu\nu}$ is the hadronic tensor

$$W^{\mu\nu} = \frac{1}{(2\pi)^4}\sum_X \int \frac{\mathrm{d}^3P_X}{(2\pi)^3\,2E_X}\,(2\pi)^4\delta^4(P+q-P_X-P_1-P_2) \\ \times \langle PS|J^\mu(0)|X,P_1P_2\rangle\langle X,P_1P_2|J^\nu(0)|PS\rangle\,. \tag{7.127}$$

Following Bianconi *et al.* (2000a), we introduce the combinations

$$P_h \equiv P_1+P_2, \quad R \equiv \tfrac{1}{2}\,(P_1-P_2)\,, \tag{7.128}$$

and the invariants

$$z_1 = \frac{P{\cdot}P_1}{P{\cdot}q}\,, \qquad\qquad z_2 = \frac{P{\cdot}P_2}{P{\cdot}q}\,, \tag{7.129}$$

$$z = z_1+z_2 = \frac{P{\cdot}P_h}{P{\cdot}q}\,, \qquad \xi = \frac{z_1}{z} = \frac{P{\cdot}P_1}{P{\cdot}P_h} = 1-\frac{z_2}{z}\,, \tag{7.130}$$

in terms of which the cross-section becomes

$$\frac{\mathrm{d}^8\sigma}{\mathrm{d}x\;\mathrm{d}y\;\mathrm{d}z\;\mathrm{d}\xi\;\mathrm{d}^2\boldsymbol{P}_{h\perp}\;\mathrm{d}^2\boldsymbol{R}_\perp} = \frac{\pi\alpha_{\mathrm{em}}^2}{4(2\pi)^3\,Q^4}\,\frac{y}{\xi(1-\xi)\,z}\,L_{\mu\nu}W^{\mu\nu}\,. \tag{7.131}$$

Using

$$\mathrm{d}^2\boldsymbol{R}_\perp = \tfrac{1}{2}\;\mathrm{d}\boldsymbol{R}_\perp^2\;\mathrm{d}\phi_R = \tfrac{1}{2}\,\xi(1-\xi)\;\mathrm{d}M_h^2\;\mathrm{d}\phi_R\,, \tag{7.132}$$

where $M_h^2 = P_h^2 = (P_1+P_2)^2$ is the invariant-mass squared of the two hadrons and ϕ_R is the azimuthal angle of $\boldsymbol{R}$ in the plane perpendicular to the γ^*N axis, the cross-section can then be re-expressed as

$$\frac{\mathrm{d}^8\sigma}{\mathrm{d}x\;\mathrm{d}y\;\mathrm{d}z\;\mathrm{d}\xi\;\mathrm{d}^2\boldsymbol{P}_{h\perp}\;\mathrm{d}M_h^2\;\mathrm{d}\phi_R} = \frac{\pi\alpha_{\mathrm{em}}^2 y}{2(2\pi)^3Q^4z}\,L_{\mu\nu}W^{\mu\nu}\,. \tag{7.133}$$

In the parton model (see Fig. 7.12) the hadronic tensor has a form similar to that of the single-particle case

$$W^{\mu\nu} = \sum_q e_q^2 \int \frac{\mathrm{d}k^-\;\mathrm{d}^2\boldsymbol{k}_T}{(2\pi)^4} \int \frac{\mathrm{d}\kappa^+\;\mathrm{d}^2\boldsymbol{\kappa}_T}{(2\pi)^4}\,\delta^2(\boldsymbol{k}_T+\boldsymbol{q}_T-\boldsymbol{\kappa}_T) \\ \times \mathrm{Tr}[\Phi\,\gamma^\mu\,\Theta\,\gamma^\nu]_{k^+=xP^+,\,\kappa^-=P_h^-/z}\,, \tag{7.134}$$

except that there now appears a decay matrix for the production of a pair

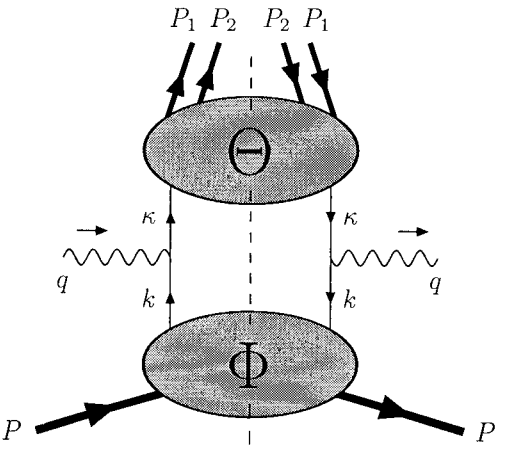

Fig. 7.12 A diagram contributing to two-hadron leptoproduction at lowest order.

of hadrons

$$\Theta_{ij}(\kappa; P_1, P_2) = \sum_X \int \mathrm{d}^4\zeta \; \mathrm{e}^{\mathrm{i}\kappa\cdot\zeta} \langle 0|\psi_i(\zeta)|P_1P_2, X\rangle\langle P_1P_2, X|\bar{\psi}_j(0)|0\rangle \,. \tag{7.135}$$

Working in a frame where P and P_h are collinear (transverse vectors in this frame are denoted, as usual, by a T subscript), the matrix (7.135) can be decomposed as was (7.27). At leading twist the contributing terms are (remember that hadrons h_1 and h_2 are unpolarised)

$$\mathcal{V}^\mu = \tfrac{1}{2}\,\mathrm{Tr}(\gamma^\mu\,\Theta) = \mathcal{B}_1\,P_h^\mu\,, \tag{7.136a}$$

$$\mathcal{A}^\mu = \frac{1}{2}\,\mathrm{Tr}(\gamma^\mu\gamma_5\,\Theta) = \frac{1}{M_1\,M_2}\,\mathcal{B}_1'\,\varepsilon^{\mu\nu\rho\sigma}\,P_{h\nu}R_\rho\kappa_{T\sigma}\,, \tag{7.136b}$$

$$\mathcal{T}^{\mu\nu} = \frac{1}{2\mathrm{i}}\,\mathrm{Tr}(\sigma^{\mu\nu}\,\gamma_5\,\Theta) = \frac{1}{M_1+M_2}\,\big[\mathcal{B}_2'\,\varepsilon^{\mu\nu\rho\sigma}P_{h\rho}\kappa_{T\sigma} + \mathcal{B}_3'\,\varepsilon^{\mu\nu\rho\sigma}P_{h\rho}R_\sigma\big]\,, \tag{7.136c}$$

where M_1 and M_2 are the masses of h_1 and h_2, respectively. In (7.136c) $\mathcal{B}_i$ and $\mathcal{B}_i'$ are functions of the invariants constructed with κ, P, P_h and R. The prime labels the so-called T-odd terms (but one should always bear in mind

that T-invariance is *not* actually broken). Contracting Eqs. (7.136a–c) with P_μ results in

$$\frac{1}{2P_h^-}\,\mathrm{Tr}(\gamma^-\,\Theta) = \mathcal{B}_1\,, \tag{7.137a}$$

$$\frac{1}{2P_h^-}\,\mathrm{Tr}(\gamma^-\gamma_5\,\Theta) = \frac{1}{M_1 M_2}\,\mathcal{B}_1'\,\varepsilon_T^{ij} R_{Ti}\kappa_{Tj}\,, \tag{7.137b}$$

$$\frac{1}{2P_h^-}\,\mathrm{Tr}(\mathrm{i}\sigma^{i-}\gamma_5\,\Theta) = \frac{1}{M_1+M_2}\left[\mathcal{B}_2'\,\varepsilon_T^{ij}\kappa_{Tj} + \mathcal{B}_3'\,\varepsilon_T^{ij} R_{Tj}\right]. \tag{7.137c}$$

Introducing the integrated trace

$$\begin{aligned}\Theta^{[\Gamma]} &= \frac{1}{4z}\int\frac{\mathrm{d}\kappa^+\,\mathrm{d}\kappa^-}{(2\pi)^4}\,\mathrm{Tr}(\Gamma\,\Theta)\,\delta\!\left(\kappa^- - \frac{1}{z}\,P_h^-\right)\\ &= \frac{1}{4z}\sum_X\int\frac{\mathrm{d}\zeta^+\,\mathrm{d}^2\boldsymbol{\zeta}_T}{(2\pi)^3}\,\mathrm{e}^{\mathrm{i}(P_h^-\zeta^+/z-\boldsymbol{\kappa}_T\cdot\boldsymbol{\zeta}_T)}\\ &\qquad\times\mathrm{Tr}\langle 0|\psi(\zeta^+,0,\mathbf{0}_\perp)|P_1P_2,X\rangle\langle P_1P_2,X|\bar\psi(0)\Gamma|0\rangle\,,\end{aligned} \tag{7.138}$$

we can rewrite Eqs. (7.137a–c) as (Bianconi *et al.*, 2000a)

$$\begin{aligned}\Theta^{[\gamma^-]} &= \mathcal{N}_{h_1h_2/q}(z,\xi,\boldsymbol{\kappa}_T',\boldsymbol{R}_T)\\ &= D(z,\xi,\boldsymbol{\kappa}_T'^2,\boldsymbol{R}_T^2,\boldsymbol{\kappa}_T'\cdot\boldsymbol{R}_T)\,,\end{aligned} \tag{7.139a}$$

$$\begin{aligned}\Theta^{[\gamma^-\gamma_5]} &= \mathcal{N}_{h_1h_2/q}(z,\xi,\boldsymbol{\kappa}_T',\boldsymbol{R}_T)\,\lambda_q'\\ &= \frac{1}{M_1M_2}\,\varepsilon_T^{ij}R_{Ti}\kappa_{Tj}G_1^\perp(z,\xi,\boldsymbol{\kappa}_T'^2,\boldsymbol{R}_T^2,\boldsymbol{\kappa}_T'\cdot\boldsymbol{R}_T)\,,\end{aligned} \tag{7.139b}$$

$$\begin{aligned}\Theta^{[\mathrm{i}\sigma^{i-}\gamma_5]} &= \mathcal{N}_{h_1h_2/q}(z,\xi,\boldsymbol{\kappa}_T',\boldsymbol{R}_T)\,s_\perp'^{\,i}\\ &= \frac{1}{M_1+M_2}\Big[\varepsilon_T^{ij}\kappa_{Tj}\,H_1^\perp(z,\xi,\boldsymbol{\kappa}_T'^2,\boldsymbol{R}_T^2,\boldsymbol{\kappa}'_T\cdot\boldsymbol{R}_T)\\ &\qquad+\varepsilon_T^{ij}R_{Tj}\,\widetilde{H}_1^\perp(z,\xi,\boldsymbol{\kappa}_T'^2,\boldsymbol{R}_T^2,\boldsymbol{\kappa}'_T\cdot\boldsymbol{R}_T)\Big]\,,\end{aligned} \tag{7.139c}$$

where recall $\boldsymbol{\kappa}_T' \equiv -z\boldsymbol{\kappa}_T$ and $\mathcal{N}_{h_1h_2/q}(z,\xi,\boldsymbol{\kappa}_T',\boldsymbol{R}_T)$ is the probability for a quark q to produce two hadrons h_1,h_2. In Eq. (7.139c) D, $G_1^\perp$, $H_1^\perp$ and $\widetilde{H}_1^\perp$ are interference fragmentation functions of quarks into a pair of unpolarised hadrons. In particular, $H_1^\perp$ and $\widetilde{H}_1^\perp$ are related to quark transverse polarisation in the target. $H_1^\perp$ has an analogue in the case of single-hadron production (where it has been denoted by the same symbol), while $\widetilde{H}_1^\perp$ is a genuinely new function. It is important to notice that $\widetilde{H}_1^\perp$ is the only

fragmentation function, besides D, that survives when the quark transverse momentum is integrated over.

The symmetric part $W^{\mu\nu(S)}$ of the hadronic tensor, the component contributing to the cross-section when the lepton beam is unpolarised (as in our case), is given by (with the same notation as in Sec. 7.7 and retaining only the unpolarised and the transverse-polarisation terms)

$$\begin{aligned} W^{\mu\nu(S)} = 2 \sum_a e_a^2 \, z \int \mathrm{d}^2\boldsymbol{\kappa}_T \int \mathrm{d}^2\boldsymbol{k}_T \, \delta^2(\boldsymbol{k}_T + \boldsymbol{q}_T - \boldsymbol{\kappa}_T) \\ \times \Big\{ - g_T^{\mu\nu} \, f(x, \boldsymbol{k}_T^2) \, D(z, \boldsymbol{\kappa}_T'^2) \\ - \frac{S_T^{\{\mu} \varepsilon_T^{\nu\}\rho} \kappa_{T\rho} + \kappa_T^{\{\mu} \varepsilon_T^{\nu\}\rho} S_{T\rho}}{2(M_1 + M_2)} \\ \times \Delta_T' f(x, \boldsymbol{k}_T^2) \, H_1^\perp(z, \xi, \boldsymbol{\kappa}_T'^2, \boldsymbol{R}_T^2, \boldsymbol{\kappa}_T' \cdot \boldsymbol{R}_T) \\ - \frac{S_T^{\{\mu} \varepsilon_T^{\nu\}\rho} R_{T\rho} + R_T^{\{\mu} \varepsilon_T^{\nu\}\rho} S_{T\rho}}{2(M_1 + M_2)} \\ \times \Delta_T' f(x, \boldsymbol{k}_T^2) \, \widetilde{H}_1^\perp(z, \xi, \boldsymbol{\kappa}_T'^2, \boldsymbol{R}_T^2, \boldsymbol{\kappa}_T' \cdot \boldsymbol{R}_T) \\ + \ldots \Big\} . \end{aligned} \tag{7.140}$$

Let us now neglect the intrinsic motion of quarks inside the target. This implies that $\boldsymbol{\kappa}_T = -\boldsymbol{P}_{h\perp}/z$. Contracting $W^{\mu\nu(S)}$ with the leptonic tensor $L_{\mu\nu}^{(S)}$ by means of the relations (7.73a–c) and integrating over $\boldsymbol{P}_{h\perp}$, we obtain the cross-section (limited to the unpolarised and transverse-polarisation contributions)

$$\begin{aligned} \frac{\mathrm{d}^6\sigma}{\mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z \, \mathrm{d}\xi \, \mathrm{d}M_h^2 \, \mathrm{d}\phi_R} = \frac{4\pi\alpha_{\mathrm{em}}^2 s}{(2\pi)^3 \, Q^4} \sum_a e_a^2 \, x \\ \times \Big\{ \frac{1}{2} [1 + (1-y)^2] \, f_a(x) \, D_a(z, \xi, M_h^2) \\ + (1-y) \, \frac{|\boldsymbol{S}_\perp| \, |\boldsymbol{R}_\perp|}{M_1 + M_2} \, \sin(\phi_S + \phi_R) \\ \times \Delta_T f_a(x) \, \widetilde{H}_{1a}^\perp(z, \xi, \boldsymbol{R}_\perp^2) \Big\} . \end{aligned} \tag{7.141}$$

The fragmentation functions appearing here are integrated over $\boldsymbol{P}_{h\perp}^2$. We

define now the interference fragmentation function $\Delta_T I(z,\xi,M_h^2)$ as

$$\Delta_T I(z,\xi,M_h^2) = \frac{|\boldsymbol{R}_\perp|}{M_1+M_2}\,\widetilde{H}_1^\perp(z,\xi,M_h^2) \propto \mathcal{N}_{h_1h_2/q\uparrow}(z,\xi,\boldsymbol{R}_\perp) - \mathcal{N}_{h_1h_2/q\downarrow}(z,\xi,\boldsymbol{R}_\perp), \quad (7.142)$$

where, we recall,

$$\boldsymbol{R}_\perp^2 = \xi(1-\xi)\,M_h^2 - (1-\xi)\,M_1^2 - \xi\,M_2^2\,. \quad (7.143)$$

Integrating (7.141) over ξ, we finally obtain

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}M_h^2\,\mathrm{d}\phi_R} = \frac{4\pi\alpha_{\rm em}^2 s}{(2\pi)^3\,Q^4}\sum_a e_a^2\,x \times\Big\{\tfrac{1}{2}\left[1+(1-y)^2\right] f_a(x)\,D_a(z,M_h^2) + (1-y)\,|\boldsymbol{S}_\perp|\,\sin(\phi_S+\phi_R)\Delta_T f_a(x)\,\Delta_T I_a(z,M_h^2)\Big\}\,. \quad (7.144)$$

From (7.144) we obtain the single transverse-spin asymmetry

$$A_T^{h_1h_2} \equiv \frac{\mathrm{d}\sigma(\boldsymbol{S}_\perp) - \mathrm{d}\sigma(-\boldsymbol{S}_\perp)}{\mathrm{d}\sigma(\boldsymbol{S}_\perp) + \mathrm{d}\sigma(-\boldsymbol{S}_\perp)} = \frac{2(1-y)}{1+(1-y)^2}\,\frac{\sum_a e_a^2\,\Delta_T f_a(x)\,\Delta_T I_a(z,M_h^2)}{\sum_a e_a^2\,f_a(x)\,D_a(z,M_h^2)}\,|\boldsymbol{S}_\perp|\,\sin(\phi_S+\phi_R)\,, \quad (7.145)$$

which probes the transversity distributions along with the interference fragmentation function $\Delta_T I$. We can introduce, into two-hadron leptoproduction, the analogue of the Collins angle Φ_C of single-hadron leptoproduction, which we call Φ_C'. We define Φ_C' as the angle between the final-quark transverse spin $\boldsymbol{s}_\perp'$ and $\boldsymbol{R}_\perp$, *i.e.*,

$$\Phi_C' \equiv \phi_{s'} - \phi_R\,. \quad (7.146)$$

We have

$$\sin\Phi_C' \equiv \frac{(\boldsymbol{P}_h\wedge\boldsymbol{R})\cdot\boldsymbol{s}'}{|\boldsymbol{P}_h\wedge\boldsymbol{R}|\,|\boldsymbol{s}'|} = \frac{(\boldsymbol{P}_2\wedge\boldsymbol{P}_1)\cdot\boldsymbol{s}'}{|\boldsymbol{P}_2\wedge\boldsymbol{P}_1|\,|\boldsymbol{s}'|}\,. \quad (7.147)$$

Since $\phi_{s'} = \pi - \phi_s$, where ϕ_s is the azimuthal angle of the initial-quark transverse spin, we can also write

$$\Phi_C' = \pi - \phi_s - \phi_R\,. \quad (7.148)$$

If the initial quark has no transverse momentum with respect to the nucleon, then $\phi_s = \phi_S$ and Φ'_C is given, in terms of measurable angles, by

$$\Phi'_C = \pi - \phi_S - \phi_R \,. \tag{7.148'}$$

In the language of QCD factorisation the cross-section for two-hadron leptoproduction is written as

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x\ \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}M_h^2\ \mathrm{d}\phi_R} = \sum_a \sum_{\lambda\lambda'\eta\eta'} f_a(x)\, \rho_{\lambda'\lambda} \left(\frac{\mathrm{d}\sigma}{\mathrm{d}y}\right)_{\lambda\lambda'\eta\eta'} \times \left[\frac{\mathrm{d}^2\mathcal{D}(z, M_h^2, \phi_R)}{\mathrm{d}M_h^2\ \mathrm{d}\phi_R}\right]_{\eta'\eta} . \tag{7.149}$$

We have found above that the fragmentation matrix $\mathrm{d}^2\mathcal{D}/\,\mathrm{d}M_h^2\,\mathrm{d}\phi_R$ factorises into z- and M_h^2-dependent fragmentation functions and certain angular coefficients. For the case at hand, the angular dependence is given by the factor $\sin(\phi_S + \phi_R)$ in (7.144). An explicit mechanism giving rise to an interference fragmentation function like $\Delta_T I$ has been suggested by Jaffe, Jin and Tang (1998a,b) (a similar mechanism was considered earlier in a different but related context by Cea, Nardulli and Chiappetta, 1988). The process considered by Jaffe, Jin and Tang and shown diagrammatically in Fig. (7.13) is the production of a $\pi^+\pi^-$ pair, via formation of a σ ($I = 0$,

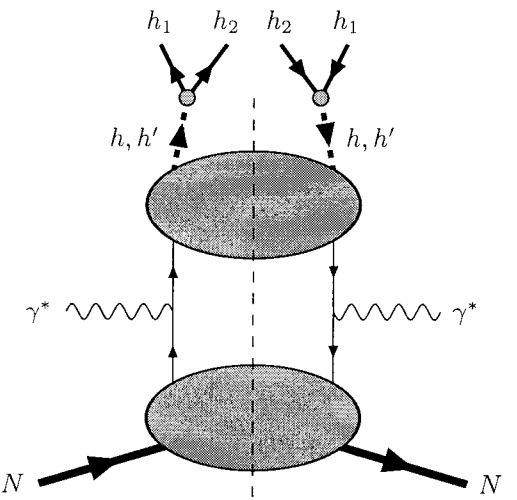

Fig. 7.13 Leptoproduction of two hadrons h_1 and h_2 via resonance (h, h') formation.

$L = 0$) and ρ ($I = 1$, $L = 1$) resonance. The single-spin asymmetry then arises from interference between the s- and p-wave of the pion system. Similar processes are πK production near to the K^* resonance, and $K\overline{K}$ production near to the ϕ. In all these cases two mesons, h_1 and h_2, are generated from the decay of two resonances h ($L = 0$) and h' ($L = 1$). The final state can be written as a superposition of two resonant states with different relative phases

$$|h_1\, h_2,\, X\rangle = \mathrm{e}^{\mathrm{i}\delta_0}\, |h,\, X\rangle + \mathrm{e}^{\mathrm{i}\delta_1}\, |h',\, X\rangle\,. \tag{7.150}$$

The interference between the two resonances is proportional to $\sin(\delta_0 - \delta_1)$. The values of δ_0 and δ_1 depend on the invariant mass M_h of the two-meson system. It turns out that the interference fragmentation function $\Delta_T I$ has the following structure

$$\Delta_T I(z, M_h^2) \sim \sin\delta_0\, \sin\delta_1\, \sin(\delta_0 - \delta_1)\, \Delta_T \hat{I}(z, M_h^2)\,, \tag{7.151}$$

where the phase factor $\sin\delta_0\, \sin\delta_1\, \sin(\delta_0 - \delta_1)$ depends on M_h^2. The maximum value that this factor can attain is $3\sqrt{3}/8$. The two-hadron spin-averaged fragmentation function $D_{h_1h_2}(z, M_h^2)$ is the superposition of the unpolarised fragmentation functions of the two resonances weighted by their phases

$$D_{h_1h_2}(z, M_h^2) = \sin^2\delta_0\; D_h(z) + \sin^2\delta_1\; D_{h'}(z)\,. \tag{7.152}$$

The resulting single-spin asymmetry is then

$$\begin{aligned} A_T^{h_1h_2} &\equiv \frac{\mathrm{d}\sigma(\boldsymbol{S}_\perp) - \mathrm{d}\sigma(-\boldsymbol{S}_\perp)}{\mathrm{d}\sigma(\boldsymbol{S}_\perp) + \mathrm{d}\sigma(-\boldsymbol{S}_\perp)} \\ &\propto \frac{2(1-y)}{1+(1-y)^2}\, |\boldsymbol{S}_\perp|\, \sin\delta_0\, \sin\delta_1\, \sin(\delta_0 - \delta_1)\, \sin(\phi_S + \phi_R) \\ &\quad \times \frac{\sum_a e_a^2\, \Delta_T f_a(x)\, \Delta_T \hat{I}_a(z, M_h^2)}{\sum_a e_a^2\, f_a(x)\, [\sin^2\delta_0\, \hat{D}_{h/a}(z) + \sin^2\delta_1\, \hat{D}_{h'/a}(z)]}\,. \end{aligned} \tag{7.153}$$

We remark that the angle ϕ defined by Jaffe, Jin and Tang (1998b) corresponds to our $\phi_S - \phi_R - \pi/2$. In the case of two-pion production, δ_0 and δ_1 can be obtained from the data on $\pi\pi$ phase shifts (Estabrooks and Martin, 1974). The factor $\sin\delta_0\, \sin\delta_1\, \sin(\delta_0 - \delta_1)$ is shown in Fig. 7.14. It is interesting to observe that the experimental value of this quantity reaches 75% of its theoretical maximum.

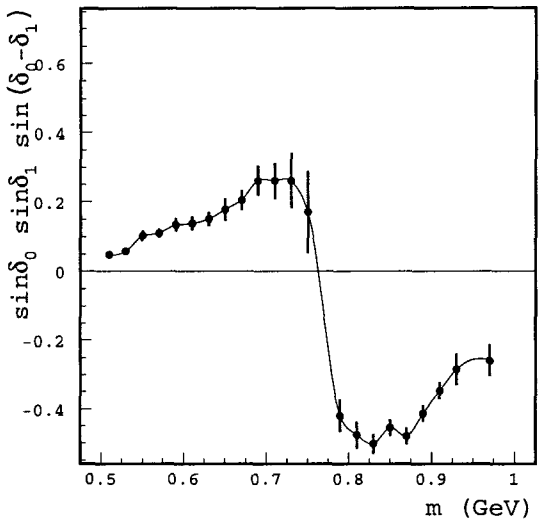

Fig. 7.14 The factor $\sin\delta_0 \sin\delta_1 \sin(\delta_0 - \delta_1)$ obtained from $\pi\pi$ phase shifts. From Jaffe, Jin and Tang (1998b).

7.11 Leptoproduction of spin-one hadrons

As first suggested by Ji (1994), see also Anselmino *et al.* (1996b), the transversity distribution can be also probed in leptoproduction of vector mesons (*e.g.*, ρ, K^*, ϕ). The fragmentation process into spin-one hadrons has been fully analysed, from a formal viewpoint, by Bacchetta and Mulders (2000, 2001). The polarisation state of a spin-one particle is described by a spin vector $\boldsymbol{S}$ and by a rank-two spin tensor T^{ij}. The latter contains five parameters, usually called S_{LL}, S^x_{LT}, S^y_{LT}, S^{xy}_{TT} and S^{xx}_{TT} (Bacchetta and Mulders, 2000). The transversity distribution $\Delta_T f$ emerges when an unpolarised beam strikes a transversely polarised target. The cross-section in this case is (Bacchetta and Mulders, 2000), we retain only the terms containing $\Delta_T f$ and we use the notation of Sec. 7.7.2,

$$\frac{\mathrm{d}^5\sigma(\boldsymbol{S}_\perp)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P}_{h\perp}} = \frac{4\pi\alpha^2_{\mathrm{em}}s}{Q^4}\,|\boldsymbol{S}_\perp| \sum_a e_a^2\, x(1-y)$$
$$\times \left\{ |S_{LT}|\, \sin(\phi_{LT} + \phi_S)\, I\left[\Delta_T f_a\, H^a_{1LT}\right] \right.$$

$$
\begin{aligned}
&+ |S_{TT}| \sin(2\phi_{TT} + \phi_S - \phi_h)\, I\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_\perp}{M_h}\, \Delta_T f_a\, H_{1TT}^a\right] \\
&+ S_{LL} \sin(\phi_S + \phi_h)\, I\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_\perp}{M_h}\, \Delta_T f_a\, H_{1LL}^{\perp a}\right] \\
&+ |S_{LT}| \sin(\phi_{LT} - \phi_S - 2\phi_h)\, I\left[\frac{[2(\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_\perp)^2 - \boldsymbol{\kappa}_\perp^2]}{2M_h^2}\, \Delta_T f_a\, H_{1LT}^{\perp a}\right] \\
&- |S_{TT}| \sin(2\phi_{TT} - \phi_S - 3\phi_h) \\
&\quad \times I\left[\frac{(\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_\perp)\,[4(\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_\perp)^2 - 3\boldsymbol{\kappa}_\perp^2]}{2M_h^3}\, \Delta_T f_a\, H_{1TT}^{\perp a}\right]\Bigg\} + \ldots \qquad (7.154)
\end{aligned}
$$

For simplicity, we have omitted the subscript h in the tensor spin parameters (which are understood to pertain to the produced hadron). The azimuthal angles ϕ_{LT} and ϕ_{TT} are defined by

$$
\tan\phi_{LT} = \frac{S_{LT}^y}{S_{LT}^x}\,, \quad \tan\phi_{TT} = \frac{S_{TT}^{xy}}{S_{TT}^{xx}}\,, \qquad (7.155)
$$

where

$$
|S_{LT}| = \sqrt{(S_{LT}^x)^2 + (S_{LT}^y)^2}\,, \quad |S_{TT}| = \sqrt{(S_{TT}^{xx})^2 + (S_{TT}^{xy})^2}\,. \qquad (7.156)
$$

Note that $\Delta_T f$ couples in (7.154) to five different fragmentation functions: H_{1LT}, H_{1TT}, $H_{1LL}^\perp$, $H_{1LT}^\perp$ and $H_{1TT}^\perp$. All these functions are T-odd. If we integrate the cross-section over $\boldsymbol{P}_{h\perp}$, only one term survives, namely

$$
\begin{aligned}
\frac{\mathrm{d}^3\sigma(\boldsymbol{S}_\perp)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z} &= \frac{4\pi\alpha_{\mathrm{em}}^2 s}{Q^4}\, |\boldsymbol{S}_\perp|\, |S_{LT}| \sin(\phi_{LT} + \phi_S) \\
&\quad \times \sum_a e_a^2\, x(1-y)\, \Delta_T f_a(x)\, H_{1LT}^a(z)\,. \qquad (7.157)
\end{aligned}
$$

The fragmentation function, H_{1LT}, appearing here was called $\hat{h}_{\mathrm{I}}$ by Ji (1994). It is a T-odd and chirally-odd function that can be measured at leading twist and without considering intrinsic transverse momenta. Probing transversity by (7.157) requires polarimetry of the produced meson. For a self-analysing particle this can be done by studying the angular distribution of its decay products (*e.g.*, $\rho^0 \to \pi^+\pi^-$). Thus, the vector-meson fragmentation function H_{1LT} represents a specific contribution to two-particle production near the vector-meson mass.

7.12 Transversity in exclusive leptoproduction processes

Let us now consider the possibility of observing the transversity distributions in exclusive leptoproduction processes. Collins, Frankfurt and Strikman (1997) remarked that the exclusive production of a transversely polarised vector meson in DIS, that is, $l\,p \to l\,V\,p$, involves the chirally-odd off-diagonal parton distributions in the proton. These distributions (also called "skewed" or "off-forward" distributions) depend on two variables x and x' since the incoming and outgoing proton states have different momenta P and P', with $(P'-P)^2 = t$ (the reader may consult Radyushkin, 1997; Ji, 1998; Martin and Ryskin, 1998, on skewed distributions). For instance, the off-forward transversity distribution (represented in Fig. 7.15a) contains a matrix element of the form $\langle PS|\bar{\psi}(0)\gamma^+\gamma_\perp\gamma_5\psi(\xi^-)|P'S\rangle$. At low x the difference between x and x' is small and the off-diagonal distributions are completely determined by the corresponding diagonal ones.

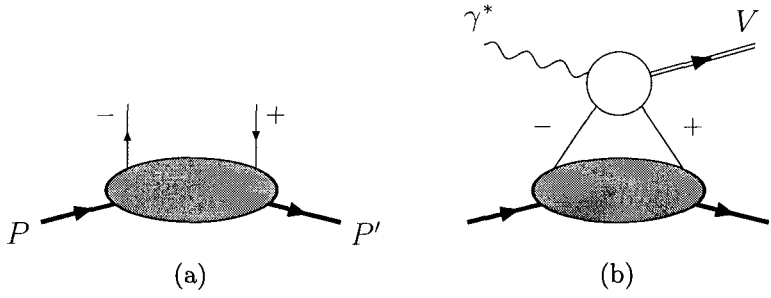

Fig. 7.15 The off-diagonal transversity distribution (a) and its contribution to exclusive vector-meson production (b).

It was shown by Mankiewicz, Piller and Weigl (1998) that the chirally-odd contribution to vector-meson production (see Fig. 7.15b) is actually zero at LO in α_s. This result was later extended by Diehl, Gousset and Pire (1999) and by Collins and Diehl (2000), where it was observed that the vanishing of the chirally-odd contribution is due to angular momentum and chirality conservation in the hard scattering and hence holds at leading twist to all orders in the strong coupling. Thus, the (off-diagonal) transversity distributions cannot be probed in exclusive vector-meson leptoproduction.

7.13 Phenomenological analyses and experimental results

Summarising the results of the previous sections, in the context of semi-inclusive DIS there are four candidate reactions for determining $\Delta_T f$ at leading twist:

(1) inclusive leptoproduction of a transversely polarised hadron from a transversely polarised target;
(2) inclusive leptoproduction of an unpolarised hadron from a transversely polarised target;
(3) inclusive leptoproduction of two hadrons from a transversely polarised target;
(4) inclusive leptoproduction of a spin-one polarised or unpolarised hadron from a transversely polarised target.

We shall review some calculations concerning the first two reactions. Two-hadron production and spin-one hadron production are more difficult to predict, as they involve interference fragmentation functions and spin-one fragmentation functions for which we have at present no independent information from other processes (a model calculation for two-hadron production has been presented by Bianconi *et al.*, 2000b).

7.13.1 Λ^0 *hyperon polarimetry*

We have seen in Sec. 7.7 that detecting a transversely polarised hadron $h^\uparrow$ in the final state of a semi-inclusive DIS process with a transversely polarised target, $l\,p^\uparrow \to l'\,h^\uparrow\,X$, probes the product $\Delta_T f_q(x)\,\Delta_T D_q(z)$ at leading twist. The relevant observable is the polarisation of $h^\uparrow$, which at lowest order reads (we take the y axis as the polarisation axis)

$$\mathcal{P}_y^{h^\uparrow} = \hat{a}_T(t)\,\frac{\sum_{q,\bar{q}} e_q^2\,\Delta_T f_q(x,Q^2)\,\Delta_T D_{h/q}(z,Q^2)}{\sum_{q,\bar{q}} e_q^2\,f_q(x,Q^2)\,D_{h/q}(z,Q^2)}\,, \tag{7.158}$$

where $\hat{a}_T(y) = 2(1-y)/[1+(1-y)^2]$ is the elementary transverse asymmetry (the QED depolarisation factor). In this class of reactions, the most promising is Λ^0 production. The Λ^0 polarisation is, in fact, easily measured by studying the angular distribution of the $\Lambda^0 \to p\,\pi$ decay. The transverse polarisation of Λ^0's produced in hard processes was studied a long time ago by Craigie *et al.* (1980), Baldracchini *et al.* (1981) and more recently by Jaffe (1996). From the phenomenological viewpoint, the main problem

is that, in order to compute the quantity (7.158), one needs to know the fragmentation functions $\Delta_T D_{h/q}(z, Q^2)$, besides the transversity distributions. Predictions for $\mathcal{P}_y^{\Lambda^\uparrow}$ have been presented by Anselmino, Boglione and Murgia (2000), Ma *et al.* (2001), and Yang (2002). In Fig. 7.16 we show the results of Anselmino, Boglione and Murgia (2000) for three different parametrisations of $\Delta_T D_{\Lambda/q}$: scenario 1 corresponds to the SU(6) non-relativistic quark model (the entire spin of the Λ^0 is carried by the strange quark); scenario 2 corresponds to the opposite sign for $\Delta_T D_{\Lambda/u,d}$ and $\Delta_T D_{\Lambda/s}$; scenario 3 corresponds to all light quarks contributing equally to the Λ^0 spin. The transverse-polarisation fragmentation functions $\Delta_T D_{\Lambda/q}$

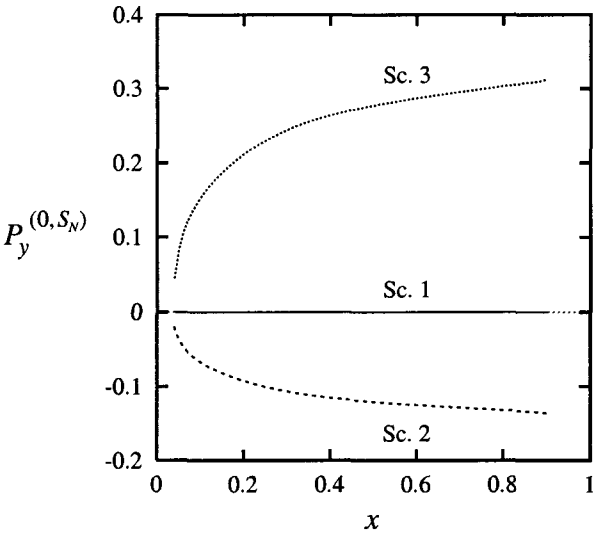

Fig. 7.16 The polarisation of Λ^0 hyperons produced in semi-inclusive DIS, as predicted by Anselmino, Boglione and Murgia (2000).

are taken to be equal to the helicity fragmentation functions $\Delta D_{\Lambda/q}$, for which the parametrisation of de Florian, Stratmann and Vogelsang (1998) is used. As for the transversity distributions, they are assumed to saturate the Soffer bound and the sea densities are neglected.

Transverse Λ^0 polarisation might also be observable in *unpolarised* semi-inclusive DIS owing to the contribution of $D_{1T}^{\perp}$, the T-odd function that describes the fragmentation of an unpolarised quark into a transversely polarised hadron (see Sec. 7.5.1). In this reaction, of course, the transversity

distributions are not probed. A similar situation occurs in Λ^0 hadroproduction, $A\,B \to \Lambda^\uparrow\,X$. A sizable transverse Λ^0 polarisation has indeed been observed in unpolarised pp scattering (Heller *et al.*, 1978, 1983; Lundberg *et al.*, 1989; Ramberg *et al.*, 1994). These data can be explained in terms of $D_{1T}^{\perp}$ (see Fig. 7.17, taken from Anselmino *et al.*, 2001) and used to extract this function, which allows a prediction of the polarisation to be measured in semi-inclusive DIS (Anselmino *et al.*, 2002b).

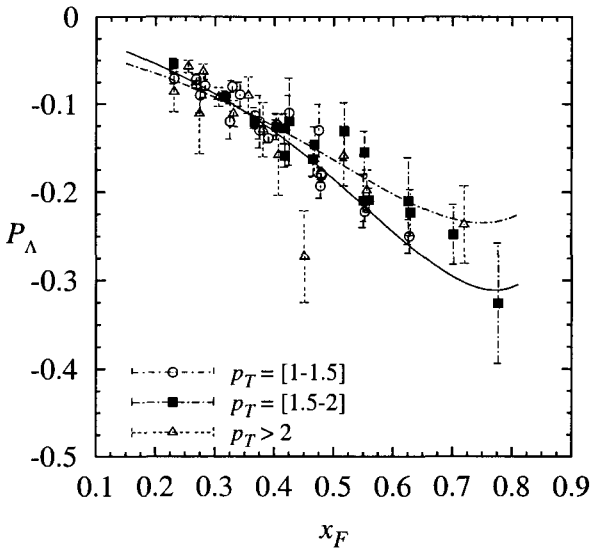

Fig. 7.17 The polarisation of Λ^0 hyperons produced in p–Be reactions, as a function of x_F and for different p_T bins. The theoretical curves are taken from Anselmino *et al.* (2001).

7.13.2 *Azimuthal asymmetries in pion leptoproduction*

A potentially relevant reaction for the study of transversity is leptoproduction of unpolarised hadrons (typically pions) with a transversely polarised target, $l\,p^\uparrow \to l'\,h\,X$. In this case, as seen in Sec. 7.7, $\Delta_T q$ may be probed as a consequence of the Collins effect (a T-odd contribution to quark fragmentation arising from final-state interactions). In this case one essentially measures the product $\Delta_T q(x)\,H_1^{\perp q}(z, \boldsymbol{P}_{h\perp}^2)$. Preliminary results on single transverse-spin asymmetries in pion leptoproduction have been recently reported by the (Bravar, 1999) and the HERMES collaboration (Airapetian

et al., 2000).

The SMC target is transversely polarised with respect to the beam direction. Neglecting $1/Q^2$ kinematical effects, the target is also transversely polarised with respect to the $\gamma^* N$ axis and the measured single transverse-spin asymmetry is given by (7.79). In contrast, the HERMES target is longitudinally polarised with respect to the beam direction. Hence, its spin has a non-zero transverse component relative to the $\gamma^* N$ axis, which is suppressed by a factor $1/Q$, according to (2.46). The single transverse-spin asymmetry measured by HERMES is therefore given by, see (7.79),

$$A_T^{\pi} \simeq \hat{a}_T(y) \frac{\sum_a e_a^2 \, \Delta_T f_a(x) \, \Delta_T^0 D_a(z, \boldsymbol{P}_{h\perp}^2)}{\sum_a e_a^2 \, f_a(x) \, D_a(z, \boldsymbol{P}_{h\perp}^2)} \times |\boldsymbol{S}| \frac{2Mx}{Q} \sqrt{1-y} \sin\phi_\pi \,. \tag{7.159}$$

We stress that the $1/Q$ factor in (7.159), which mimics a twist-three contribution, has a purely kinematical origin.

Let us turn to the data. The SMC (Bravar, 1999) presented a preliminary measurement of A_T^{π} for pion production in the DIS of unpolarised muons off a transversely polarised proton target at $s = 188.5\,\mathrm{GeV}^2$ and

$$\langle x \rangle \simeq 0.08 \,, \quad \langle y \rangle \simeq 0.33 \,, \quad \langle z \rangle \simeq 0.45 \,, \quad \langle Q^2 \rangle \simeq 5\,\mathrm{GeV}^2 \,. \tag{7.160}$$

Two data sets, with $\langle \boldsymbol{P}_{\pi\perp} \rangle = 0.5\,\mathrm{GeV}$ and $\langle \boldsymbol{P}_{\pi\perp} \rangle = 0.8\,\mathrm{GeV}$, are selected. The result for the total amount of events is (note that the SMC use a different choice of axes and, moreover, their Collins angle has the opposite sign with respect to ours)

$$A_T^{\pi^+}/\hat{a}_T = (0.11 \pm 0.06) \sin(\phi_\pi + \phi_S) \,, \tag{7.161a}$$

$$A_T^{\pi^-}/\hat{a}_T = -(0.02 \pm 0.06) \sin(\phi_\pi + \phi_S) \,. \tag{7.161b}$$

The SMC data are shown in Fig. 7.18. Evaluating the depolarisation factor $\hat{a}_T$ at $\langle y \rangle \simeq 0.33$, Eqs. (7.161a, b) imply

$$A_T^{\pi^+} = (0.10 \pm 0.06) \sin(\phi_\pi + \phi_S) \,, \tag{7.162a}$$

$$A_T^{\pi^-} = -(0.02 \pm 0.06) \sin(\phi_\pi + \phi_S) \,. \tag{7.162b}$$

The HERMES experiment at DESY (Airapetian *et al.*, 2000) has reported results on A_T^{π} for positive and negative pions produced in DIS of unpolarised positrons off a longitudinally polarised proton target at

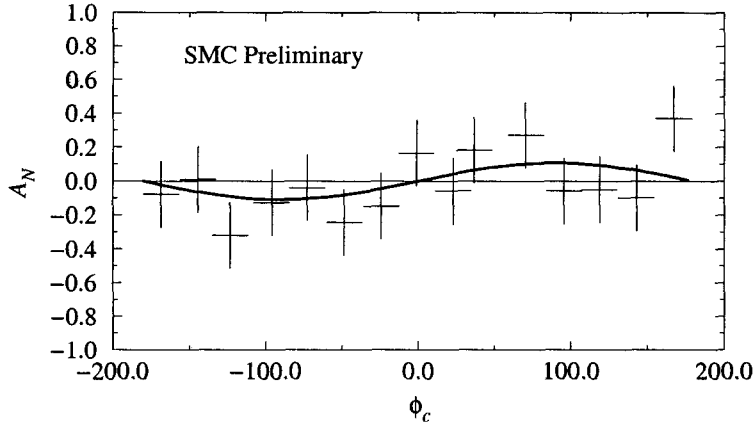

Fig. 7.18 The SMC data (Bravar, 1999) on the single transverse-spin asymmetry in pion leptoproduction, as a function of the Collins angle.

$s = 52.6\,\mathrm{GeV}^2$ and in the kinematical ranges:

$$0.023 \le x \le 0.4\,, \quad 0.1 \le y \le 0.85\,, \quad 0.2 \le z \le 0.7\,, \qquad Q^2 \ge 1\,\mathrm{GeV}^2\,. \tag{7.163}$$

The transverse momentum of the produced pions is $|\boldsymbol{P}_{\pi\perp}| \lesssim 1\,\mathrm{GeV}$. The HERMES result is (see Fig. 7.19)

$$A_T^{\pi^+} = \left[0.022 \pm 0.004\,(\mathrm{stat.}) \pm 0.004\,(\mathrm{syst.})\right] \sin\phi_\pi\,, \tag{7.164a}$$

$$A_T^{\pi^-} = -\left[0.001 \pm 0.005\,(\mathrm{stat.}) \pm 0.004\,(\mathrm{syst.})\right] \sin\phi_\pi\,. \tag{7.164b}$$

The conventions for the axes and the Collins angle used by HERMES are the same as ours. There appears to be a sign difference between the SMC and HERMES results. Unfortunately, the proliferation of conventions does not help to settle sign problems. According to the discussion above, the HERMES data, which are obtained with a longitudinally polarised target, give a single transverse-spin asymmetry suppressed by $1/Q$. Thus, higher-twist longitudinal effects might be as relevant as the leading-twist Collins effect. The result (7.164a–b) should be taken with this *caveat* in mind.

Anselmino and Murgia (2000) have analysed the SMC and HERMES data and extracted bounds on the Collins fragmentation function $\Delta_T^0 D_{\pi/q}$ (for other analyses see Efremov, Goeke and Schweitzer, 2001; Ma, Schmidt

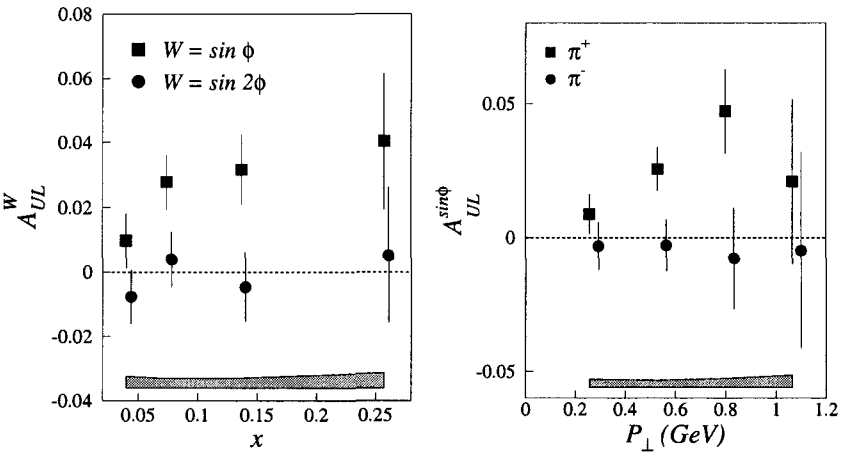

Fig. 7.19 The HERMES data (Airapetian *et al.*, 2000) on the single transverse-spin asymmetry in pion leptoproduction, as a function of x (left) and $|\boldsymbol{P}_{\pi\perp}|$ (right).

and Yang, 2001; Yang, 2002). Using isospin and charge-conjugation invariance, leads to the relations

$$\begin{aligned}\Delta_T^0 D_{\pi/q} &= \Delta_T^0 D_{\pi^+/u} = \Delta_T^0 D_{\pi^+/\bar{d}} = \Delta_T^0 D_{\pi^-/d} = \Delta_T^0 D_{\pi^-/\bar{u}} \\ &= 2\Delta_T^0 D_{\pi^0/u} = 2\Delta_T^0 D_{\pi^0/d} = 2\Delta_T^0 D_{\pi^0/\bar{d}} = 2\Delta_T^0 D_{\pi^0/\bar{u}}\,, \quad (7.165)\end{aligned}$$

and to similar constraints for $D_{\pi/q}$, and ignoring the transversity of the sea and the non-valence quark contributions in pions, the single transverse-spin asymmetries become

$$A_T^{\pi^+} \sim \frac{4\,\Delta_T u(x)}{4u(x)+\bar{d}(x)} \frac{\Delta_T^0 D_{\pi/q}(z,P_{\pi\perp})}{D_{\pi/q}(z,P_{\pi\perp})}\,, \quad (7.166a)$$

$$A_T^{\pi^-} \sim \frac{\Delta_T d(x)}{d(x)+4\bar{u}(x)} \frac{\Delta_T^0 D_{\pi/q}(z,P_{\pi\perp})}{D_{\pi/q}(z,P_{\pi\perp})}\,, \quad (7.166b)$$

$$A_T^{\pi^0} \sim \frac{4\,\Delta_T u(x)+\Delta_T d(x)}{4u(x)+d(x)+4\bar{u}(x)+\bar{d}(x)} \frac{\Delta_T^0 D_{\pi/q}(z,P_{\pi\perp})}{D_{\pi/q}(z,P_{\pi\perp})}\,. \quad (7.166c)$$

By saturating the Soffer inequality, Anselmino and Murgia (2000) derived a lower bound for the quark analysing power $\Delta_T^0 D_{\pi/q}/D_{\pi/q}$ from the data on A_T^π. From the SMC result they found (note that Anselmino and Murgia defined a function $\Delta^N D_{\pi/q^\uparrow}$, which is related to our $\Delta_T^0 D_{\pi/q}$

through $\Delta^N D_{\pi/q^\uparrow} = 2\,\Delta_T^0 D_{\pi/q}$)

$$\frac{|\Delta_T^0 D_{\pi/q}|}{D_{\pi/q}} \gtrsim 0.24 \pm 0.15\,, \quad \langle z \rangle \simeq 0.45\,, \quad \langle P_{\pi\perp} \rangle \simeq 0.65\,\mathrm{GeV}\,, \quad (7.167\mathrm{a})$$

and from the HERMES data

$$\frac{|\Delta_T^0 D_{\pi/q}|}{D_{\pi/q}} \gtrsim 0.20 \pm 0.04\,(\mathrm{stat.}) \pm 0.04\,(\mathrm{syst.})\,, \quad z \geq 0.2\,. \quad (7.167\mathrm{b})$$

These results, if confirmed, would indicate a large value of the Collins fragmentation function and would therefore also point to a relevant contribution of the Collins effect in other processes. The analysis performed by Efremov, Goeke and Schweitzer (2001), and based on the chiral quark–soliton model, yields a similar value for the quark analysing power:

$$\frac{\langle H_{1,q}^\perp \rangle}{\langle D_{\pi/q} \rangle} = 0.14 \pm 0.03\,, \quad (7.168)$$

where an average is performed over the transverse momentum and over z, with $\langle z \rangle \simeq 0.4$. For an independent determination of the Collins analysing power from a different process (e^+e^- collisions at LEP), see Sec. 7.13.3 below.

As already recalled, the interpretation of the HERMES result is made difficult by the fact that the target is longitudinally polarised with respect to the beam axis (see Sec. 2.4). Thus, focusing on the dominant $1/Q$ effects, there are in principle two types of contributions to the cross-section: 1) leading-twist contributions for a transversely polarised target; 2) twist-three contributions for a longitudinally polarised target. Type 1 is $\mathcal{O}(1/Q)$ owing to the kinematical relation (2.46); type 2 is $\mathcal{O}(1/Q)$ owing to dynamical twist-three effects. The $\sin\phi_\pi$ asymmetry measured by HERMES receives the following contributions (Tangerman and Mulders, 1995b; Mulders and Tangerman, 1996; Kotzinian and Mulders, 1997; De Sanctis, Nowak and Oganessyan, 2000; Kotzinian *et al.*, 2000; Oganessyan *et al.*, 2001a,b; Boglione and Mulders, 2000), we omit the factors in front of each

term:

$$A^{\pi}_{\sin\phi_h} \sim |\boldsymbol{S}_\perp|\,\Delta_T f \otimes H_1^{\perp(1)} + \frac{M}{Q}\,|\boldsymbol{S}_\parallel|\,h_L \otimes H_1^{\perp(1)} + \frac{M_h}{Q}\,|\boldsymbol{S}_\parallel|\,h_{1L}^{\perp(1)} \otimes \widetilde{H}$$
$$\sim \frac{M}{Q}\,|\boldsymbol{S}|\,\Delta_T f \otimes H_1^{\perp(1)} + \frac{M}{Q}\,|\boldsymbol{S}|\,h_L \otimes H_1^{\perp(1)} + \frac{M_h}{Q}\,|\boldsymbol{S}|\,h_{1L}^{\perp(1)} \otimes \widetilde{H}\,, \tag{7.169}$$

where $h_{1L}^{\perp(1)}$ and $H_1^{\perp(1)}$ are defined in (3.66a) and (7.90) respectively. The first term in (7.169) is the type 1 term described above and corresponds to the Collins effect studied by Anselmino and Murgia (2000). The other two terms (type 2) were phenomenologically investigated by Kotzinian *et al.* (2000), De Sanctis, Nowak and Oganessyan (2000), Oganessyan *et al.* (2001a,b), and Boglione and Mulders (2000).

In order to analyse the data by means of (7.169), extra input is needed given the number of unknown quantities involved. As we have seen in Sec. 7.7, when the target is longitudinally polarised there is also a $\sin 2\phi_h$ asymmetry, which appears at leading twist and has the form

$$A^{\pi}_{\sin 2\phi_\pi} \sim |\boldsymbol{S}_\parallel|\,h_{1L}^{\perp(1)} \otimes H_1^{\perp(1)}\,. \tag{7.170}$$

The smallness of $A^{\pi}_{\sin 2\phi_\pi}$, as measured by HERMES (see Fig. 7.19), seems to be an indication in favour of $h_{1L}^{\perp(1)} \simeq 0$. If we make this assumption (De Sanctis, Nowak and Oganessyan, 2000), recalling (3.86) and (3.88), we obtain $\widetilde{h}_L(x) = h_L(x) = \Delta_T f(x)$ and (7.169) reduces to a single term of the type $\Delta_T f \otimes H_1^{\perp(1)}$. Using a simple parametrisation for the Collins fragmentation function, De Sanctis, Nowak and Oganessyan fit the HERMES data fairly well (see Fig. 7.20). Boglione and Mulders (2000) pointed out that the HERMES data on the $\sin 2\phi_\pi$ asymmetry do not necessarily imply $h_{1L}^{\perp(1)} = 0$. If one assumes the interaction-dependent distribution $\widetilde{h}_L(x)$ to be vanishing, so that from (3.66a) and (3.88) one has (neglecting quark mass terms)

$$h_{1L}^{\perp(1)}(x) = -\frac{1}{2}\,x h_L(x) = -x^2 \int_x^1 \frac{\mathrm{d}y}{y^2}\,\Delta_T f(y)\,, \tag{7.171}$$

then it is still possible to obtain a $\sin\phi_\pi$ asymmetry of the order of a few

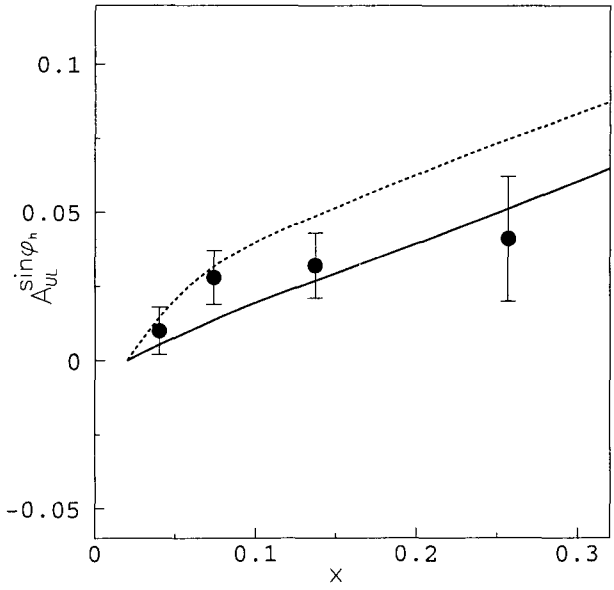

Fig. 7.20 The single-spin azimuthal asymmetry in pion leptoproduction as computed by De Sanctis, Nowak and Oganessyan (2000), compared to the HERMES data (Airapetian *et al.*, 2000). The solid line corresponds to $\Delta_T f = \Delta f$. The dashed line corresponds to saturation of the Soffer inequality.

percent (as found by HERMES), with the $\sin 2\phi_\pi$ asymmetry suppressed by a factor two. The approximation (7.171) was also adopted in the analysis of Efremov *et al.* (2000).

Two more remarks are in order: first, as stressed by Boer (2001), Sudakov form factors (Sec. 7.8.3) might affect the extraction of the Collins function from single-spin asymmetry data, although it is difficult to quantify this effect; second, the observed asymmetry might not arise from transversity and T-odd fragmentation functions, but rather from the mechanism studied by Brodsky, Hwang and Schmidt (2002)—see Sec. 7.7.2, which can be reinterpreted in terms of a T-odd distribution function (Collins, 2002). In conclusion, we can say that the interpretation of the HERMES and SMC measurements is far from clear. The experimental results seem to indicate that transversity plays some rôle but the present scarcity of data, their errors, our ignorance of most of the quantities involved in the process, and, last but not least, uncertainty in the theoretical procedures make the entire

matter still rather vague. More, and more precise, data will be of great help in settling the question (in particular, what is really called for is an accurate study of the Q^2 dependence of the process).

7.13.3 *Transverse polarisation in e^+e^- collisions*

An important source of information on the Collins fragmentation function $H_1^\perp$ is inclusive two-hadron production in electron–positron collisions (see Fig. 7.21):

$$e^+ e^- \to h_1 h_2 X . \tag{7.172}$$

This process has been studied by Chen *et al.* (1995), and by Boer, Jakob and Mulders (1997, 1998). It turns out that the cross-section has the following angular dependence (we assume that two alike hadrons are produced, omit the flavour indices and refer the reader to Fig. 7.21 for the kinematical variables):

$$\begin{aligned}\frac{\mathrm{d}^2\sigma}{\mathrm{d}\cos\theta_2\,\mathrm{d}\phi_1} &\propto (1+\cos^2\theta_2)\, D(z_1)\overline{D}(z_2) \\ &\quad + C\,\sin^2\theta_2\,\cos(2\phi_1)\, H_1^{\perp(1)}(z_1)\overline{H}_1^{\perp(1)}(z_2)\,,\end{aligned} \tag{7.173}$$

where C is a constant containing the electroweak couplings. Thus, the

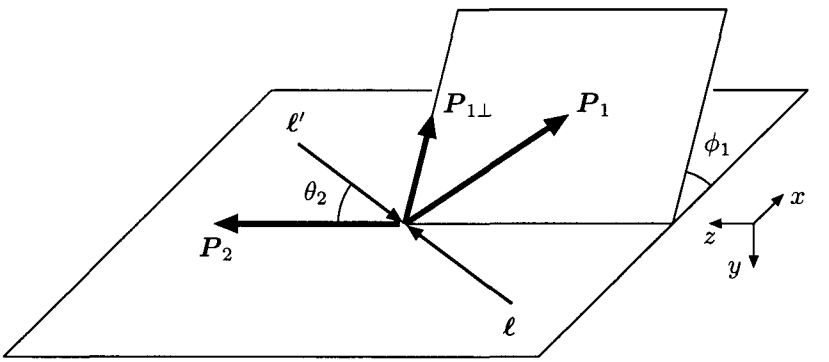

Fig. 7.21 Kinematics of two-hadron production in e^+e^- annihilation.

analysis of $\cos(2\phi_1)$ asymmetries in the process (7.172) can shed light on the ratio between unpolarised and Collins fragmentation functions. Efremov

et al. (1999); Efremov, Smirnova and Tkachev (1999a,b) have carried out such a study using the DELPHI data on Z^0 hadronic decays. Under the assumption that all produced particles are pions and that fragmentation functions have a Gaussian κ_T dependence, they find

$$\left|\frac{\langle H_1^\perp \rangle}{\langle D \rangle}\right| = (6.3 \pm 1.7)\,\%\,, \tag{7.174}$$

where the averages are over flavours and the kinematical range covered by data. The result (7.174) is an indication of a non-zero fragmentation function for transversely polarised quarks into unpolarised hadrons.

Efremov *et al.* argue that a more careful study of the θ_2 dependence of the experimentally measured cross-section could lead to an increase in the quark analysing power in (7.174) up to around 12%. This more optimistic value is very close to that extracted from an analysis of the leptoproduction data (Efremov, Goeke and Schweitzer, 2001), see (7.168). Boer (2001) has recently estimated the effect of Sudakov factors on the $\cos(2\phi_1)$ asymmetry and on the extraction of $H_1^\perp$. He concludes that the Sudakov suppression is rather strong: when one takes it into account in the analysis of the e^+e^- LEP data, the Collins function is found to be much larger, compared to the result of a tree-level extraction. Owing to the uncertainty in the non-perturbative Sudakov factor, however, this conclusion should only be taken as qualitative.

7.14 Experimental perspectives

The study of transversity distributions in leptoproduction is a more-or-less important fraction of the physics program of many ongoing and forthcoming experiments in various laboratories around the world. A useful overview of the experimental state and future prospects of the field can be found in the Proceedings of the RIKEN–BNL Workshop on "Future Transversity Measurements" (Boer and Grosse Perdekamp, 2000).

7.14.1 *HERMES*

Running since 1995, the HERMES experiment at DESY has already provided a large amount of data on polarised inclusive and semi-inclusive DIS. We have discussed their (preliminary) result of main concern in this book

(see Sec. 7.13.2): namely, the observation of a relatively large azimuthal spin asymmetry in semi-inclusive DIS off a longitudinally polarised proton target, which may involve the transversity distributions via the Collins effect.

HERMES uses the HERA 27.5 GeV positron (or electron) beam incident on a longitudinally polarised H or D gas-jet target. The hydrogen polarisation is approximately 85%. The recent upgrade of HERA has increased the average luminosity by a factor three. Two of the next five years of running of HERMES should be dedicated to a transversely polarised target with an expected statistics of $7{\cdot}10^6$ reconstructed DIS events. The foreseen target polarisation is $\sim 75\%$.

Besides the extraction of the spin structure function g_2, the transverse-polarisation program of HERMES includes (Korotkov and Nowak, 2001): *i*) a measurement of the twist-three azimuthal asymmetry in semi-inclusive pion production with a longitudinally polarised lepton beam; *ii*) the study of the Collins effect in the scattering of an unpolarised lepton beam off a transversely polarised target; and *iii*) a measurement of the transverse asymmetry in leptoproduction of two correlated mesons. According to the estimates presented by Korotkov, Nowak and Oganesian (2001) and by Korotkov and Nowak (2001), HERMES should be in a position to determine both the transversity distributions and the Collins fragmentation function (at least for the dominant u flavour) with good statistical accuracy.

7.14.2 *COMPASS*

COMPASS is a new fixed target experiment at CERN (Baum *et al.*, 1996), with two main programs: the "muon program" and the "hadron program". The former (which upgrades the SMC) aims to study the nucleon spin structure with a high-energy muon beam. COMPASS uses polarised muons of 100–200 GeV, scattering off polarised proton and deuteron targets. Expected polarisations are 90% and 50% for the proton and the deuteron, respectively. The transverse-polarisation physics program is similar to that of HERMES, but covers different kinematical regions. In particular, single-spin asymmetries in hadron leptoproduction will be measured. These will provide the transversity distributions via the Collins effect. According to an estimate presented by Baum *et al.* (1996), $\Delta_T q$ should be determined with a $\sim 10\%$ accuracy in the intermediate-x region. Data taking by COMPASS started in 2002.

7.14.3 *ELFE*

ELFE (Electron Laboratory for Europe) is a continuous electron-beam facility, which has been discussed since the early nineties. The latest proposal is for construction at CERN by exploiting the cavities and other components of LEP not required for LHC (Aulenbacher *et al.*, 1999). The maximum energy of the electron beam would be 25 GeV. The very high luminosity (about three orders of magnitude higher than HERMES and COMPASS) would allow accurate measurements of semi-inclusive asymmetries with transversely polarised targets. In particular, polarimetry in the final state should reach a good degree of precision (a month of running time with a luminosity of $10^{34}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$, allows the accumulation of about 10^6 Λ^0's with transverse momentum greater than $1\,\mathrm{GeV}/c$.

7.14.4 *TESLA-N*

The TESLA-N project (Anselmino *et al.*, 2000a) is based on the idea of using one of the arms of the e^+e^- collider TESLA to produce collisions of longitudinally polarised electrons on a fixed proton or deuteron target, which may be either longitudinally or transversely polarised. The basic parameters are: electron beam energy 250 GeV, an integrated luminosity of $100\,\mathrm{fb}^{-1}$ per year, a target polarisation $\sim 80\%$ for protons, $\sim 30\%$ for deuterium. The transversity program includes the measurement of single-spin azimuthal asymmetries and two-pion correlations (Korotkov and Nowak, 2001). The proposers of the project have estimated the statistical accuracy in the extraction of the transversity distributions via the Collins effect and found values comparable to the existing determinations of the helicity distributions. They have also shown that the expected statistical accuracy in the measurement of two-meson correlations is encouraging if the interference fragmentation function is not much smaller than its upper bound.

Chapter 8

Transversity in inclusive hadroproduction

We shall now discuss the third class of reactions that potentially probe the transversity distributions: inclusive hadron production with one transversely polarised hadron in the initial state. The possibility to explore transversity via these processes is related to the Collins effect. There already exist some data on single transverse-spin asymmetries in pion hadroproduction, which may be interpreted in terms of the coupling of transversity distributions with the Collins fragmentation function, although other explanations cannot be excluded.

8.1 Inclusive hadroproduction with a transversely polarised target

We consider the following reaction (see Fig. 8.1):

$$A^{\uparrow}(P_A) + B(P_B) \to h(P_h) + X\,, \tag{8.1}$$

where A is transversely polarised and the unpolarised (or spinless) hadron h is produced with a large transverse momentum $\boldsymbol{P}_{hT}$, so that perturbative QCD is applicable. In typical experiments A and B are protons while h is a pion. What is measured is the single-spin asymmetry

$$A_T^h = \frac{\mathrm{d}\sigma(\boldsymbol{S}_T) - \mathrm{d}\sigma(-\boldsymbol{S}_T)}{\mathrm{d}\sigma(\boldsymbol{S}_T) + \mathrm{d}\sigma(-\boldsymbol{S}_T)}\,. \tag{8.2}$$

Single-spin asymmetries are expected to vanish in perturbative QCD at leading-twist (this observation was first made by Kane, Pumplin and Repko,

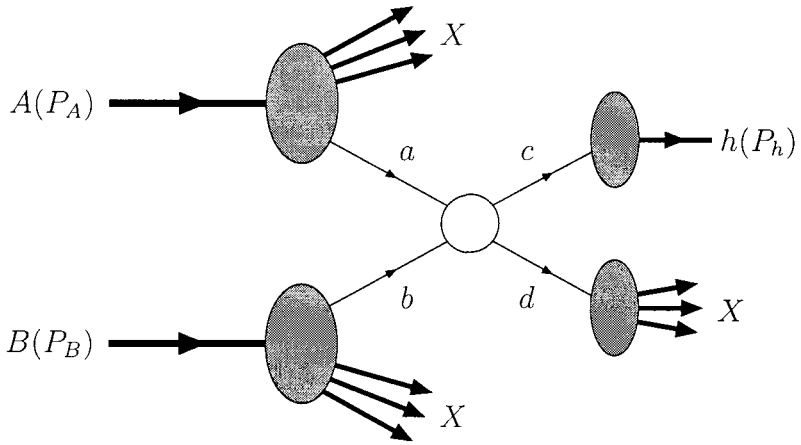

Fig. 8.1 Hadron–hadron scattering with inclusive production of a particle h.

1978). They may arise, however, as a consequence of quark transverse-motion effects (*e.g.*, see Sivers, 1990; Collins, 1993b; Collins, Heppelmann and Ladinsky, 1994) and/or higher-twist contributions (*e.g.*, see Efremov and Teryaev, 1982, 1985; Qiu and Sterman, 1991a, 1992). In the former case, one probes the following quantities related to transversity:

- Distribution functions: $\Delta_T f(x)$ (transversely polarised quarks in a transversely polarised hadron), $f_{1T}^{\perp}(x, \boldsymbol{k}_T^2)$ (unpolarised quarks in a transversely polarised hadron), $h_1^{\perp}(x, \boldsymbol{k}_T^2)$ (transversely polarised quarks in an unpolarised hadron).
- Fragmentation functions: $H_1^{\perp}(z, \boldsymbol{\kappa}_T^2)$ (transversely polarised quarks fragmenting into an unpolarised hadron), $D_{1T}^{\perp}(z, \boldsymbol{\kappa}_T^2)$ (unpolarised quarks fragmenting into a transversely polarised hadron).

The twist-three single-spin asymmetries involving transversity distributions contain, besides the familiar unpolarised quantities, the quark–quark–gluon correlation function $E_F(x, y)$ of the incoming unpolarised hadron and a twist-three fragmentation function of the outgoing hadron (see Sec. 8.3 below).

Let us now enter into some detail. The cross-section for reaction (8.1) is usually expressed as a function of $\boldsymbol{P}_{hT}^2$ and the Feynman scaling variable,

$$x_F \equiv \frac{2P_{hL}}{\sqrt{s}} = \frac{t-u}{s}\,, \tag{8.3}$$

where P_{hL} is the longitudinal momentum of h, and s, t, u are the Mandelstam variables

$$s = (P_A + P_B)^2\,, \qquad t = (P_A - P_h)^2\,, \qquad u = (P_B - P_h)^2\,. \tag{8.4}$$

The elementary processes at lowest order in QCD are two-body partonic processes

$$a(k_a) + b(k_b) \to c(k_c) + d(k_d)\,. \tag{8.5}$$

In the collinear case we set

$$k_a = x_a P_A\,, \qquad k_b = x_b P_B\,, \qquad k_c = \frac{1}{z} P_h\,, \tag{8.6}$$

and the partonic Mandelstam invariants are

$$\hat{s} = (k_a + k_b)^2 \simeq x_a x_b s\,, \tag{8.7a}$$

$$\hat{t} = (k_a - k_c)^2 \simeq \frac{x_a t}{z}\,, \tag{8.7b}$$

$$\hat{u} = (k_b - k_c)^2 \simeq \frac{x_b u}{z}\,. \tag{8.7c}$$

Thus, the condition $\hat{s} + \hat{t} + \hat{u} = 0$ implies

$$z = -\frac{x_a t - x_b u}{x_a x_b s}\,. \tag{8.8}$$

According to the factorisation theorem in perturbative QCD, the differential cross-section for reaction (8.1) can be written formally as

$$\mathrm{d}\sigma = \sum_{abc} \sum_{\alpha\alpha'\gamma\gamma'} \rho^a_{\alpha'\alpha}\, f_a(x_a) \otimes f_b(x_b) \otimes \mathrm{d}\hat{\sigma}_{\alpha\alpha'\gamma\gamma'} \otimes \mathcal{D}^{\gamma'\gamma}_{h/c}(z)\,. \tag{8.9}$$

Here f_a (f_b) is the distribution of parton a (b) inside hadron A (B), $\rho^a_{\alpha\alpha'}$ is the spin density matrix of parton a, $\mathcal{D}^{\gamma\gamma'}_{h/c}$ is the fragmentation matrix of parton c into hadron h, and $\mathrm{d}\hat{\sigma}/\mathrm{d}\hat{t}$ is the elementary cross-section:

$$\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}\right)_{\alpha\alpha'\gamma\gamma'} = \frac{1}{16\pi\hat{s}^2}\,\frac{1}{2} \sum_{\beta\delta} \mathcal{M}_{\alpha\beta\gamma\delta}\, \mathcal{M}^*_{\alpha'\beta\gamma'\delta}\,, \tag{8.10}$$

where $\mathcal{M}_{\alpha\beta\gamma\delta}$ is the scattering amplitude for the elementary partonic process (see Fig. 8.2).

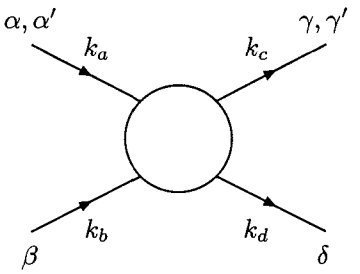

Fig. 8.2 Elementary processes contributing to hadron–hadron scattering.

If the produced hadron is unpolarised, or spinless, as will always be the case hereafter, only the diagonal elements of $\mathcal{D}_{h/c}^{\gamma\gamma'}$ are non-vanishing, *i.e.*, $\mathcal{D}_{h/c}^{\gamma\gamma'} = \delta_{\gamma\gamma'} D_{h/c}$, where $D_{h/c}$ is the unpolarised fragmentation function. Together with helicity conservation in the partonic subprocess, this implies $\alpha = \alpha'$. Therefore, in (8.10) there is no dependence on the spin of hadron A and all single-spin asymmetries are zero. To escape such a conclusion, either intrinsic quark transverse motion, or higher-twist effects must be considered.

8.2 Transverse motion of quarks and single-spin asymmetries

Let us first of all see how quark transverse motion can generate single-spin asymmetries. This can happen in three different ways:

(1) Intrinsic $\boldsymbol{\kappa}_T$ in hadron h implies that $\mathcal{D}_{h/c}^{\gamma\gamma'}$ is not necessarily diagonal (owing to T-odd effects at level of fragmentation functions).
(2) Intrinsic $\boldsymbol{k}_T$ in hadron A implies that $f_a(x_a)$ in (8.9) should be replaced by the probability density $\mathcal{P}_a(x_a, \boldsymbol{k}_T)$, which may depend on the spin of hadron A (again, owing to T-odd effects but at the level of distribution functions).
(3) Intrinsic $\boldsymbol{k}'_T$ in hadron B implies that $f_b(x_b)$ in (8.9) should be replaced by $\mathcal{P}_b(x_b, \boldsymbol{k}'_T)$. The transverse spin of parton b inside the unpolarised hadron B may then couple to the transverse spin of

parton a inside A (this too is a T-odd effect at the level of distribution functions).

The first is the Collins effect (1993b), the second is the Sivers effect (1990), and the third is the effect studied by Boer (1999) in the context of Drell–Yan processes (Sec. 6.4). We stress that all these intrinsic-$\boldsymbol{\kappa}_T$, -$\boldsymbol{k}_T$, or -$\boldsymbol{k}'_T$ effects are T-odd.

When the intrinsic quark transverse motion is taken into account, the QCD factorisation theorem is not proven. We shall assume, however, its validity and write a factorisation formula similar to (8.9), which is explicitly

$$E_h \frac{\mathrm{d}^3\sigma}{\mathrm{d}^3\boldsymbol{P}_h} = \sum_{abc} \sum_{\alpha\alpha'\beta\beta'\gamma\gamma'} \int \mathrm{d}x_a \int \mathrm{d}x_b \int \mathrm{d}^2\boldsymbol{k}_T \int \mathrm{d}^2\boldsymbol{k}'_T \int \mathrm{d}^2\boldsymbol{\kappa}_T \frac{1}{\pi z} \times \mathcal{P}_a(x_a, \boldsymbol{k}_T)\, \rho^a_{\alpha'\alpha}\, \mathcal{P}_b(x_b, \boldsymbol{k}'_T)\, \rho^b_{\beta'\beta} \times \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}\right)_{\alpha\alpha'\beta\beta'\gamma\gamma'} \mathcal{D}^{\gamma'\gamma}_{h/c}(z, \boldsymbol{\kappa}_T)\,, \tag{8.11}$$

where

$$\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}\right)_{\alpha\alpha'\beta\beta'\gamma\gamma'} = \frac{1}{16\pi\hat{s}^2} \sum_{\beta\delta} \mathcal{M}_{\alpha\beta\gamma\delta} \mathcal{M}^*_{\alpha'\beta\gamma'\delta}\,. \tag{8.12}$$

To start with, let us consider the Collins mechanism for single-spin asymmetries (Collins, 1993b; Anselmino, Boglione and Murgia, 1999). We take into account the intrinsic quark transverse motion inside the produced hadron h (which is responsible for the effect), and neglect the transverse momenta of all other quarks. Thus, Eq. (8.11) becomes

$$E_h \frac{\mathrm{d}^3\sigma}{\mathrm{d}^3\boldsymbol{P}_h} = \sum_{abc} \sum_{\alpha\alpha'\gamma\gamma'} \int \mathrm{d}x_a \int \mathrm{d}x_b \int \mathrm{d}^2\boldsymbol{\kappa}_T \frac{1}{\pi z} \times f_a(x_a)\, \rho^a_{\alpha'\alpha}\, f_b(x_b) \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}\right)_{\alpha\alpha'\gamma\gamma'} \mathcal{D}^{\gamma'\gamma}_{h/c}(z, \boldsymbol{\kappa}_T)\,, \tag{8.13}$$

and the elementary cross-sections are given by (8.10), with $\boldsymbol{\kappa}_T$ retained. We are interested in transverse-spin asymmetries $\mathrm{d}\sigma(\boldsymbol{S}_T) - \mathrm{d}\sigma(-\boldsymbol{S}_T)$. Therefore, since we are neglecting intrinsic $\boldsymbol{k}_T$ motion inside A, the spin density matrix elements of our concern are ρ^a_{+-} and ρ^a_{-+}, and the contributing elementary cross-sections are $\mathrm{d}\hat{\sigma}_{+-+-} = \mathrm{d}\hat{\sigma}_{-+-+}$ and $\mathrm{d}\hat{\sigma}_{+--+} = \mathrm{d}\hat{\sigma}_{-++-}$.

Using Eqs. (3.28) and (7.116a–d) we find, with our choice of axes,

$$
\begin{aligned}
E_h \frac{\mathrm{d}^3\sigma(\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h} - E_h \frac{\mathrm{d}^3\sigma(-\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h} \\
= -2\,|\boldsymbol{S}_T| \sum_{abc} \int \mathrm{d}x_a \int \mathrm{d}x_b \int \mathrm{d}^2\boldsymbol{\kappa}_T \, \frac{1}{\pi z}\, \Delta_T f_a(x_a)\, f_b(x_b) \\
\times \left[\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} \right)_{+-+-} \sin(\phi_\kappa + \phi_S) + \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} \right)_{+--+} \sin(\phi_\kappa - \phi_S) \right] \\
\times \Delta_T^0 D_{h/c}(z, \boldsymbol{\kappa}_T^2)\,, \qquad (8.14)
\end{aligned}
$$

where ϕ_κ and ϕ_S are the azimuthal angles of $\boldsymbol{\kappa}_T$ and $\boldsymbol{S}_T$, respectively, and $\Delta_T^0 D_{h/c}$ is the T-odd fragmentation function related to $H_1^\perp$, see (7.59).

In particular, if the spin of hadron A is directed along y, Eq. (8.14) takes the form

$$
\begin{aligned}
E_h \frac{\mathrm{d}^3\sigma(\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h} - E_h \frac{\mathrm{d}^3\sigma(-\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h} \\
= -2\,|\boldsymbol{S}_T| \sum_{abc} \int \mathrm{d}x_a \int \frac{\mathrm{d}x_b}{\pi z} \int \mathrm{d}^2\boldsymbol{\kappa}_T \Delta_T f_a(x_a)\, f_b(x_b) \\
\times \Delta_{TT}\hat{\sigma}(x_a, x_b, \boldsymbol{\kappa}_T)\, \Delta_T^0 D_{h/c}(z, \boldsymbol{\kappa}_T^2)\,, \qquad (8.15)
\end{aligned}
$$

where the elementary double-spin asymmetry $\Delta_{TT}\hat{\sigma}$ is given by

$$
\begin{aligned}
\Delta_{TT}\hat{\sigma} &= \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} \right)_{+-+-} - \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} \right)_{+--+} \\
&= \frac{\mathrm{d}\hat{\sigma}(a^\uparrow b \to c^\uparrow d)}{\mathrm{d}\hat{t}} - \frac{\mathrm{d}\hat{\sigma}(a^\uparrow b \to c^\downarrow d)}{\mathrm{d}\hat{t}}\,. \qquad (8.16)
\end{aligned}
$$

Equation (8.15) gives the single-spin asymmetry under the hypothesis that only the Collins mechanism (based on the existence of the T-odd fragmentation function $\Delta_T^0 D_{h/c}$, or $H_1^\perp$) is at work. Another source of single-spin asymmetries in hadron–hadron scattering is the Sivers effect (Sivers, 1990; Anselmino, Boglione and Murgia, 1995, 1999; Anselmino and Murgia, 1998), which relies on T-odd distribution functions. This mechanism

predicts a single-spin asymmetry of the form

$$E_h \frac{\mathrm{d}^3\sigma(\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h} - E_h \frac{\mathrm{d}^3\sigma(-\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h}$$
$$= |\boldsymbol{S}_T| \sum_{abc} \int \mathrm{d}x_a \int \frac{\mathrm{d}x_b}{\pi z} \int \mathrm{d}^2\boldsymbol{k}_T \Delta_0^T f_a(x_a, \boldsymbol{k}_T^2)\, f_b(x_b)$$
$$\times \frac{\mathrm{d}\hat{\sigma}(x_a, x_b, \boldsymbol{k}_T)}{\mathrm{d}\hat{t}}\, D_{h/c}(z)\,, \tag{8.17}$$

where $\Delta_0^T f$, (related to $f_{1T}^{\perp}$) is the T-odd distribution defined in (3.99a).

Finally, the effect studied by Boer (1999) gives rise to an asymmetry involving the other T-odd distribution, $\Delta_T^0 f$ (or $h_1^{\perp}$), defined in (3.99b). This asymmetry reads

$$E_h \frac{\mathrm{d}^3\sigma(\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h} - E_h \frac{\mathrm{d}^3\sigma(-\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h}$$
$$= -2|\boldsymbol{S}_T| \sum_{abc} \int \mathrm{d}x_a \int \frac{\mathrm{d}x_b}{\pi z} \int \mathrm{d}^2\boldsymbol{k}'_T \Delta_T f_a(x_a)\, \Delta_T^0 f_b(x_b, \boldsymbol{k}'^2_T)$$
$$\times \Delta_{TT}\hat{\sigma}'(x_a, x_b, \boldsymbol{k}'_T)\, D_{h/c}(z)\,, \tag{8.18}$$

where the elementary asymmetry is

$$\Delta_{TT}\hat{\sigma}' = \frac{\mathrm{d}\hat{\sigma}(a^{\uparrow}b^{\uparrow} \to cd)}{\mathrm{d}\hat{t}} - \frac{\mathrm{d}\hat{\sigma}(a^{\uparrow}b^{\downarrow} \to cd)}{\mathrm{d}\hat{t}}\,. \tag{8.19}$$

The *caveat* of Sec. 3.10 with regard to initial-state interaction effects, which are assumed to generate the T-odd distributions, clearly applies here and renders both the Sivers and the Boer mechanisms highly conjectural. In the next section we shall see how single-spin asymmetries emerge at higher twist.

8.3 Single-spin asymmetries at twist three

As pointed out by Efremov and Teryaev (1982, 1985), non-vanishing single-spin asymmetries can be obtained in perturbative QCD by resorting to higher twist. Such asymmetries were later evaluated in the context of QCD factorisation by Qiu and Sterman, who studied direct photon production (1991a; 1992) and, more recently, hadron production (1999). This program has been extended to cover the chirally-odd contributions by Kanazawa and Koike (2000a,b). Here we shall limit ourselves to quoting the main

general results of these works (for a phenomenological analysis, see the next section).

At twist three the cross-section for the reaction (8.1) can be written formally as

$$\begin{aligned} \mathrm{d}\sigma = \sum_{abc} \Big\{ & G_F^a(x_a, y_a) \otimes f_b(x_b) \otimes \mathrm{d}\hat{\sigma} \otimes D_{h/c}(z) \\ & + \Delta_T f_a(x_a) \otimes E_F^b(x_b, y_b) \otimes \mathrm{d}\hat{\sigma}' \otimes D_{h/c}(z) \\ & + \Delta_T f_a(x_a) \otimes f_b(x_b) \otimes \mathrm{d}\hat{\sigma}'' \otimes D_{h/c}^{(3)}(z) \Big\}, \end{aligned} \tag{8.20}$$

where $G_F(x_a, x_b)$ and $E_F(x_a, x_b)$ are the quark–quark–gluon correlation functions introduced in Sec. 6.4, $D_{h/c}^{(3)}$ is a twist-three fragmentation function (which we do not specify further), and $\mathrm{d}\hat{\sigma}$, $\mathrm{d}\hat{\sigma}'$ and $\mathrm{d}\hat{\sigma}''$ are cross-sections of hard partonic subprocesses.

The first line in (8.20), which does not contain the transversity distributions, corresponds to the chirally-even mechanism studied by Qiu and Sterman (1999). The second term in (8.20) is the chirally-odd contribution analysed by Kanazawa and Koike (2000a). The elementary cross-sections can be found in the original papers. In the next section we shall see how the predictions based on Eq. (8.20) compare with the available data on single-spin asymmetries in hadron production. In practice, it turns out that the transversity-dependent term is negligible (Kanazawa and Koike, 2000b).

8.3.1 *Experimental results and phenomenology*

In the early seventies data on single-spin asymmetries in inclusive pion hadroproduction (Dick *et al.*, 1975; Crabb *et al.*, 1977; Dragoset *et al.*, 1978) provoked a certain theoretical interest, as it was widely held that large effects could not be reproduced within the framework of perturbative QCD (Kane, Pumplin and Repko, 1978). In 1991 the E704 collaboration at Fermilab extended the results on large single-spin asymmetries in inclusive pion hadroproduction with a transversely polarised proton (Adams *et al.*, 1991; Bravar *et al.*, 1996) to higher p_T. These surprising results have prompted intense theoretical work on the subject.

In fact, one year before publication of the E704 measurements, Sivers (1990, 1991) had suggested that single-spin asymmetries could originate, at leading twist, from intrinsic quark motion in the colliding protons. This idea was pursued by Anselmino, Boglione and Murgia (1995) and by Anselmino

and Murgia (1998).

The mechanism proposed by Collins (1993b) relies on the hypothesis of *final-state* interactions, which would allow polarised quarks with non-zero transverse momentum to fragment into an unpolarised hadron (the Collins effect has been already discussed in Secs. 7.7 and 8.2).

Finally, as seen in Sec. 8.2, another way to produce single-spin asymmetries is to assume the existence of a T-odd transverse-polarisation distribution of quarks in the unpolarised initial-state hadron.

All the above effects manifest themselves at leading twist. Hereafter, we shall concentrate on the Collins mechanism.

The Collins effect in hadroproduction was investigated phenomenologically by Anselmino, Boglione and Murgia (1999), who proposed a simple parametrisation for the Collins fragmentation function $\Delta_T^0 D_{\pi/q}(z, \langle \kappa_\perp \rangle)$

$$\Delta_T^0 D_{\pi/q}(z, \langle \kappa_\perp \rangle) = N \frac{\langle \kappa_\perp(z) \rangle}{M} z^\alpha (1-z)^\beta , \tag{8.21}$$

where $M = 1\,\mathrm{GeV}$ and it is assumed that $\Delta_T^0 D_{\pi/q}$ is peaked around the average value $\langle \kappa_\perp \rangle \equiv \langle \boldsymbol{\kappa}_\perp^2 \rangle^{1/2}$. The z dependence of $\langle \kappa_\perp(z) \rangle$ is obtained from an analysis of LEP measurements of the transverse momentum of charged pions inside jets (Abreu *et al.*, 1996), remember that $\boldsymbol{\kappa}_\perp \simeq -\boldsymbol{P}_{h\perp}/z$ neglecting the intrinsic motion of quarks inside the target. The result of the fit of Anselmino, Boglione and Murgia (1999) to the single-spin asymmetry data is shown in Fig. 8.3. Good agreement is obtained if the positivity constraint $|\Delta_T^0 D_{\pi/q}| \leq D_{\pi/q}$ saturates at large z, otherwise the value of the single-spin asymmetry A_T^π is too small at large x_F. It also turns out that the resulting transversity distribution of the d quark violates the Soffer bound $2\,|\Delta_T d| \leq d + \Delta d$. Boglione and Leader (2000) pointed out that, since Δd is negative in most parametrisations, the Soffer constraint for the d distributions is a rather strict one. A fit to the A_T^π data that satisfies the Soffer inequality was performed by Boglione and Leader (2000), with good results provided Δd was allowed to become positive at large x. In this case too, the positivity constraint on $\Delta_T^0 D_{\pi/q}$ must be saturated at large z.

Another calculation of the single-spin asymmetry in pion hadroproduction, based on the Collins effect, was presented by Artru, Czyżewski and Yabuki (1997). These authors generate the T-odd fragmentation function via the Lund string mechanism and obtain fair agreement with the E704

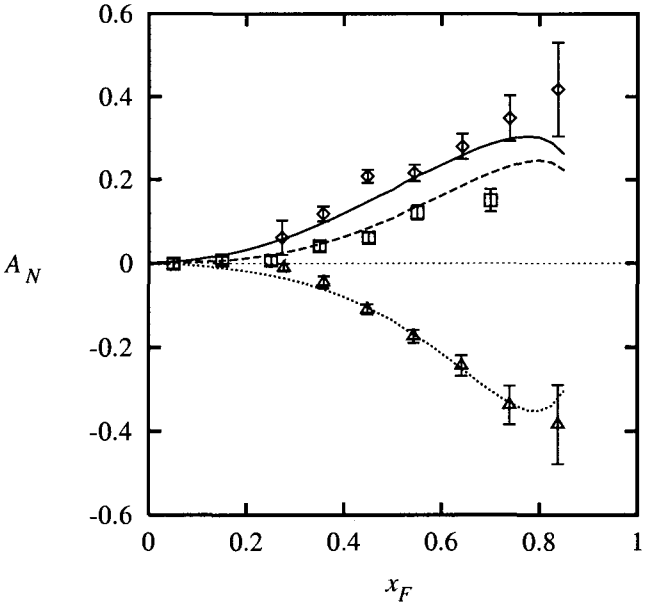

Fig. 8.3 A fit to the data on A_T^π for the process $p^\uparrow p \to \pi X$ (Adams *et al.*, 1991; Bravar *et al.*, 1996) assuming that only the Collins effect is active; the upper, middle, and lower sets of data and curves refer to π^+, π^0 and π^-, respectively. From Anselmino, Boglione and Murgia (1999).

data by assuming the following behaviour for the transversity distributions:

$$\frac{\Delta_T u(x)}{u(x)} \simeq -\frac{\Delta_T d(x)}{d(x)} \to 1\,, \quad \text{as} \quad x \to 1\,. \tag{8.22}$$

A comment on the applicability of perturbative QCD to the analysis of the E704 measurements is in order. First of all, we have already pointed out that factorisation with intrinsic quark transverse momenta is not a proven property but only a (plausible) hypothesis. Second, and more importantly, the E704 data span a range of $|\boldsymbol{P}_{\pi\perp}|$ that reaches 4 GeV for π^0 in the central region, where the asymmetry is small, and up to only 1.5 GeV for $\pi^\pm, \pi^0$ in the forward region, where the asymmetry is large. At such low values of transverse momenta perturbative QCD is not expected to be completely reliable, since cross-sections tend to rise very steeply as $|\boldsymbol{P}_{\pi\perp}| \to 0$. What allows some confidence that a perturbative QCD treatment is nevertheless meaningful is the fact that both intrinsic-$\boldsymbol{\kappa}_\perp$ effects and higher twists (see

below) regularise the cross-sections at $\boldsymbol{P}_{\pi\perp} = 0$.

A phenomenological analysis of the E704 results, based on the Sivers effect as the only source of single-spin asymmetries, was carried out by Anselmino, Boglione and Murgia (1995) and Anselmino and Murgia (1998). For other (model) calculations of A_T^π, see Suzuki *et al.* (1999) and Nakajima *et al.* (2000).

As shown in Sec. 8.3, single-spin asymmetries may also arise as a result of twist-three effects (Efremov and Teryaev, 1982, 1985; Qiu and Sterman, 1991a, 1999; Kanazawa and Koike, 2000a,b). Qiu and Sterman have used the first, chirally even, term of Eq. (8.20) to fit the E704 data on A_T^π, setting

$$G_F(x,x) = K\, f(x)\,, \tag{8.23}$$

where K is a constant parameter. Their fit is shown in Fig. 8.4.

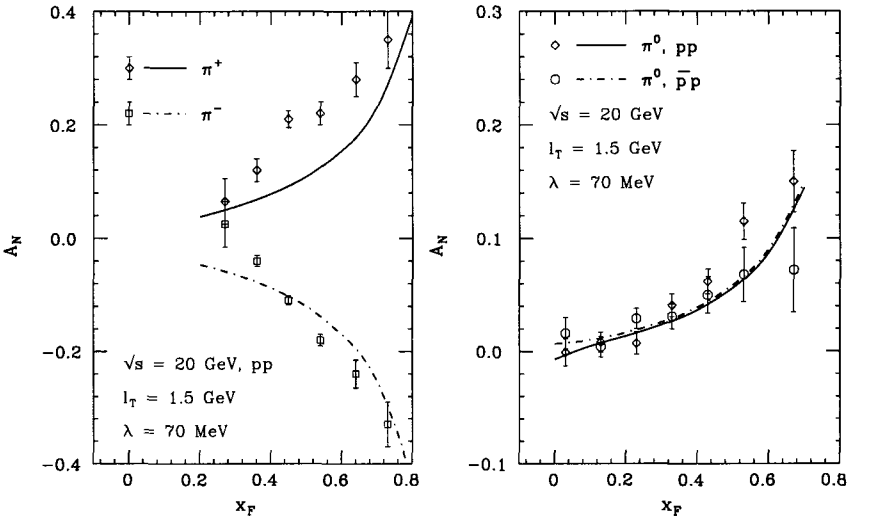

Fig. 8.4 A fit to the single-spin asymmetry data (here A_T^π is called A_N) performed by Qiu and Sterman (1999).

Another twist-three contribution, the second term in Eq. (8.20), involves the transversity distributions. This term has been evaluated by Kanazawa and Koike (2000a,b), with an assumption similar to (8.23) for the multi-parton distribution E_F, *i.e.*,

$$E_F(x,x) = K'\, \Delta_T f(x)\,. \tag{8.24}$$

They found that, owing to the smallness of the hard partonic cross-sections, this chirally-odd contribution to single-spin asymmetries turned out to be negligible.

Clearly, in order to discriminate between leading-twist intrinsic-$\boldsymbol{\kappa}_\perp$ effects and higher-twist mechanisms, a precise measurement of the $\boldsymbol{P}_{\pi\perp}$ dependence of the asymmetry is needed, in particular at large $\boldsymbol{P}_{\pi\perp}$. Given the current experimental information on A_T^π, it is just impossible to draw definite conclusions as to the dynamical source of single transverse-spin asymmetries.

Appendix A

Polarisation of a Dirac particle

A.1 The polarisation operator and spin vector

The representations of the Poincaré group are labelled by the eigenvalues of two Casimir operators, P^2 and W^2 (see, *e.g.*, Itzykson and Zuber, 1980, p. 52). P^μ is the energy–momentum operator, W^μ is the Pauli–Lubanski operator, constructed from P^μ and the angular-momentum operator $J^{\mu\nu}$

$$W^\mu = -\tfrac{1}{2}\,\varepsilon^{\mu\nu\rho\sigma} J_{\nu\rho} P_\sigma \,. \tag{A.1}$$

The eigenvalues of P^2 and W^2 are m^2 and $-s(s+1)m^2$ respectively, where m is the mass of the particle and s its spin.

The states of a Dirac particle ($s = 1/2$) are eigenvectors of P^μ and of the polarisation operator $\Pi \equiv -W{\cdot}s/m$

$$P^\mu \,|p,s\rangle = p^\mu \,|p,s\rangle \,, \tag{A.2}$$

$$-\frac{W{\cdot}s}{m}\,|p,s\rangle = \pm\frac{1}{2}\,|p,s\rangle \,, \tag{A.3}$$

where s^μ is the spin (or polarisation) vector of the particle, with the properties

$$s^2 = -1, \quad s{\cdot}p = 0 \,. \tag{A.4}$$

In general, s^μ may be written as

$$s^\mu = \left(\frac{\boldsymbol{p}{\cdot}\boldsymbol{n}}{m},\; \boldsymbol{n} + \frac{(\boldsymbol{p}{\cdot}\boldsymbol{n})\,\boldsymbol{p}}{m(m+p^0)} \right), \tag{A.5}$$

where $\boldsymbol{n}$ is a unit vector identifying a generic spatial direction. When $\boldsymbol{n}$ and $\boldsymbol{p}$ are parallel (or anti-parallel), the particle is *longitudinally polarised*; when $\boldsymbol{n}$ and $\boldsymbol{p}$ are orthogonal, the particle is *transversely polarised.*

In the high-energy limit the spin vector has the form

$$s^\mu = \lambda \frac{p^\mu}{m} + s^\mu_\perp , \tag{A.6}$$

where λ is (twice) the helicity of the particle (see Sec. A.2 below).

The polarisation operator Π can be re-expressed as

$$\Pi = \frac{1}{2m} \gamma_5 \not{s} \not{p} , \tag{A.7}$$

and if we write the plane-wave solutions of the free Dirac equation in the form

$$\psi(x) = \begin{cases} \mathrm{e}^{-\mathrm{i}p\cdot x}\, u(p) & \text{(positive energy)}, \\ \mathrm{e}^{+\mathrm{i}p\cdot x}\, v(p) & \text{(negative energy)}, \end{cases} \tag{A.8}$$

with the condition $p^0 > 0$, Π becomes

$$\Pi = +\tfrac{1}{2} \gamma_5 \not{s} \qquad \text{(positive-energy states)}, \tag{A.9a}$$

when acting on positive-energy states, $(\not{p} - m)\, u(p) = 0$, and

$$\Pi = -\tfrac{1}{2} \gamma_5 \not{s} \qquad \text{(negative-energy states)}, \tag{A.9b}$$

when acting on negative-energy states, $(\not{p} + m)\, v(p) = 0$. Thus, the eigenvalue equations for the polarisation operator read ($\alpha = 1, 2$)

$$\begin{aligned} \Pi u_{(\alpha)} &= +\tfrac{1}{2} \gamma_5 \not{s}\, u_{(\alpha)} = \pm \tfrac{1}{2}\, u_{(\alpha)} \qquad && \text{(positive energy)}, \\ \Pi v_{(\alpha)} &= -\tfrac{1}{2} \gamma_5 \not{s}\, v_{(\alpha)} = \pm \tfrac{1}{2}\, v_{(\alpha)} \qquad && \text{(negative energy)}. \end{aligned} \tag{A.10}$$

Let us now consider particles that are at rest in a given frame. The spin vector s^μ is then (set $\boldsymbol{p} = 0$ in Eq. (A.5))

$$s^\mu = (0, \boldsymbol{n}) , \tag{A.11}$$

and in the Dirac representation we have the operator

$$\tfrac{1}{2} \gamma_5 \not{s} = \begin{pmatrix} \boldsymbol{\sigma}\cdot\boldsymbol{n} & 0 \\ 0 & -\boldsymbol{\sigma}\cdot\boldsymbol{n} \end{pmatrix} , \tag{A.12}$$

acting on

$$u_{(\alpha)} = \begin{pmatrix} \varphi_{(\alpha)} \\ 0 \end{pmatrix}, \qquad v_{(\alpha)} = \begin{pmatrix} 0 \\ \chi_{(\alpha)} \end{pmatrix}. \tag{A.13}$$

Hence, the spinors $u_{(1)}$ and $v_{(1)}$ represent particles with spin $\frac{1}{2}\boldsymbol{\sigma}\cdot\boldsymbol{n} = +\frac{1}{2}$ in their rest frame whereas the spinors $u_{(2)}$ and $v_{(2)}$ represent particles with spin $\frac{1}{2}\boldsymbol{\sigma}\cdot\boldsymbol{n} = -\frac{1}{2}$ in their rest frame. Note that the polarisation operator in the form (A.9a, b) is also well defined for massless particles.

A.2 Longitudinal polarisation

For a longitudinally polarised particle ($\boldsymbol{n} = \pm\boldsymbol{p}/|\boldsymbol{p}|$), the spin vector reads

$$s^{\mu} = \pm\left(\frac{|\boldsymbol{p}|}{m}, \frac{p^0}{m}\frac{\boldsymbol{p}}{|\boldsymbol{p}|}\right). \tag{A.14}$$

In this case the polarisation operator is the *helicity* operator

$$\Pi = \frac{\boldsymbol{\Sigma}\cdot\boldsymbol{p}}{2|\boldsymbol{p}|}, \tag{A.15}$$

with $\Sigma^i = \gamma_5\gamma_0\gamma^i$. Consistently with Eq. (A.8), the *helicity states* satisfy the equations

$$\begin{aligned} \frac{\boldsymbol{\Sigma}\cdot\boldsymbol{p}}{|\boldsymbol{p}|}\, u_{\pm}(p) &= \pm\, u_{\pm}(p)\,, \\ \frac{\boldsymbol{\Sigma}\cdot\boldsymbol{p}}{|\boldsymbol{p}|}\, v_{\pm}(p) &= \mp\, v_{\pm}(p)\,. \end{aligned} \tag{A.16}$$

Here the subscript $+$ indicates positive helicity, that is, spin parallel to the momentum ($\boldsymbol{\Sigma}\cdot\boldsymbol{p} > 0$ for positive-energy states, $\boldsymbol{\Sigma}\cdot\boldsymbol{p} < 0$ for negative-energy states); the subscript $-$ indicates negative helicity, that is, spin antiparallel to the momentum ($\boldsymbol{\Sigma}\cdot\boldsymbol{p} < 0$ for positive-energy states, $\boldsymbol{\Sigma}\cdot\boldsymbol{p} > 0$ for negative-energy states). The correspondence with the spinors $u_{(\alpha)}$ and $v_{(\alpha)}$ previously introduced is: $u_+ = u_{(1)}$, $u_- = u_{(2)}$, $v_+ = v_{(2)}$, $v_- = v_{(1)}$.

In the case of massless particles one has

$$\Pi = \frac{\boldsymbol{\Sigma}\cdot\boldsymbol{p}}{2|\boldsymbol{p}|} = \tfrac{1}{2}\,\gamma_5\,. \tag{A.17}$$

Denoting again by $u_\pm, v_\pm$ the helicity eigenstates, Eqs. (A.16) become for zero-mass particles

$$\begin{aligned} \gamma_5\, u_\pm(p) &= \pm\, u_\pm(p)\,, \\ \gamma_5\, v_\pm(p) &= \mp\, v_\pm(p)\,. \end{aligned} \tag{A.18}$$

Thus, *helicity* coincides with *chirality* for positive-energy states, while it is opposite to chirality for negative-energy states. The helicity projectors for massless particles are then

$$\mathcal{P}_\pm = \begin{cases} \frac{1}{2}\,(1 \pm \gamma_5) & \text{positive-energy states,} \\ \frac{1}{2}\,(1 \mp \gamma_5) & \text{negative-energy states.} \end{cases} \tag{A.19}$$

A.3 Transverse polarisation

Let us come now to the case of transversely polarised particles. With $\boldsymbol{n}\cdot\boldsymbol{p} = 0$ and assuming that the particle moves along the z direction, the spin vector (A.5) becomes, in Cartesian components

$$s^\mu = s^\mu_\perp = (0, \boldsymbol{n}_\perp, 0)\,, \tag{A.20}$$

where $\boldsymbol{n}_\perp$ is a transverse two-vector. The polarisation operator takes the form

$$\Pi = \begin{cases} -\frac{1}{2}\,\gamma_5 \boldsymbol{\gamma}_\perp\cdot\boldsymbol{n}_\perp = \frac{1}{2}\,\gamma_0 \boldsymbol{\Sigma}_\perp\cdot\boldsymbol{n}_\perp & \text{(positive-energy states),} \\ \frac{1}{2}\,\gamma_5 \boldsymbol{\gamma}_\perp\cdot\boldsymbol{n}_\perp = -\frac{1}{2}\,\gamma_0 \boldsymbol{\Sigma}_\perp\cdot\boldsymbol{n}_\perp & \text{(negative-energy states),} \end{cases} \tag{A.21}$$

and its eigenvalue equations are

$$\begin{aligned} \tfrac{1}{2}\,\gamma_5\, \not{s}_\perp\, u_{\uparrow\downarrow} &= \pm\, \tfrac{1}{2}\, u_{\uparrow\downarrow}\,, \\ \tfrac{1}{2}\,\gamma_5\, \not{s}_\perp\, v_{\uparrow\downarrow} &= \mp\, \tfrac{1}{2}\, v_{\uparrow\downarrow}\,. \end{aligned} \tag{A.22}$$

The transverse-polarisation projectors along the directions $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ are

$$\begin{aligned} \mathcal{P}^{(x)}_{\uparrow\downarrow} &= \tfrac{1}{2}\,(1 \pm \gamma^1\gamma_5)\,, \\ \mathcal{P}^{(y)}_{\uparrow\downarrow} &= \tfrac{1}{2}\,(1 \pm \gamma^2\gamma_5)\,, \end{aligned} \tag{A.23}$$

for positive-energy states, and

$$\begin{aligned} \mathcal{P}^{(x)}_{\uparrow\downarrow} &= \tfrac{1}{2}\,(1 \mp \gamma^1\gamma_5)\,, \\ \mathcal{P}^{(y)}_{\uparrow\downarrow} &= \tfrac{1}{2}\,(1 \mp \gamma^2\gamma_5)\,, \end{aligned} \tag{A.24}$$

for negative-energy states.

The relations between transverse-polarisation states and helicity states are (for positive-energy wave functions)

$$\begin{cases} u_{\uparrow}^{(x)} = \frac{1}{\sqrt{2}}(u_+ + u_-)\,, \\ u_{\downarrow}^{(x)} = \frac{1}{\sqrt{2}}(u_+ - u_-)\,, \end{cases} \qquad \begin{cases} u_{\uparrow}^{(y)} = \frac{1}{\sqrt{2}}(u_+ + \mathrm{i}u_-)\,, \\ u_{\downarrow}^{(y)} = \frac{1}{\sqrt{2}}(u_+ - \mathrm{i}u_-)\,. \end{cases} \tag{A.25}$$

A.4 The spin density matrix

The spinor $u(p,s)$ for a particle with polarisation vector s^μ satisfies

$$u(p,s)\,\bar{u}(p,s) = (\not{p} + m)\,\tfrac{1}{2}(1 + \gamma_5 \not{s})\,. \tag{A.26}$$

If the particle is at rest, then $s^\mu = (0, \boldsymbol{s}) = (0, \boldsymbol{s}_\perp, \lambda)$ and (A.26) gives

$$\frac{1}{2m}\,u(p,s)\,\bar{u}(p,s) = \begin{pmatrix} \frac{1}{2}(1 + \boldsymbol{\sigma}\cdot\boldsymbol{s}) & 0 \\ 0 & 0 \end{pmatrix}. \tag{A.27}$$

Here one recognises the spin density matrix for a spin-half particle:

$$\rho = \tfrac{1}{2}\,(1 + \boldsymbol{\sigma}\cdot\boldsymbol{s})\,. \tag{A.28}$$

This matrix provides a general description of the spin structure of a system that is also valid when the system is not in a pure state. The polarisation three-vector $\boldsymbol{s} = (\boldsymbol{s}_\perp, \lambda)$ is, in general, such that $\boldsymbol{s}^2 \leq 1$: in particular, $\boldsymbol{s}^2 = 1$ for pure states and $\boldsymbol{s}^2 < 1$ for mixtures. Explicitly, ρ reads

$$\rho = \tfrac{1}{2}\begin{pmatrix} 1 + \lambda & s_x - \mathrm{i}s_y \\ s_x + \mathrm{i}s_y & 1 - \lambda \end{pmatrix}. \tag{A.29}$$

The entries of the spin density matrix have an obvious probabilistic interpretation. If we call $P_m(\hat{\boldsymbol{n}})$ the probability that the spin component in the $\hat{\boldsymbol{n}}$ direction is m, we can write

$$\begin{aligned} \lambda &= P_{1/2}(\hat{z}) - P_{-1/2}(\hat{z})\,, \\ s_x &= P_{1/2}(\hat{x}) - P_{-1/2}(\hat{x})\,, \\ s_y &= P_{1/2}(\hat{y}) - P_{-1/2}(\hat{y})\,. \end{aligned} \tag{A.30}$$

In the high-energy limit, using (A.6), we have

$$(1 + \gamma_5 \not{s})\,(m \pm \not{p}) = (1 \pm \lambda\gamma_5 + \gamma_5 \not{s}_\perp)\,(m \pm \not{p}), \tag{A.31}$$

and the projector (A.26) becomes (with $m \to 0$)

$$u(p,s)\,\bar{u}(p,s) = \tfrac{1}{2}\not{p}\,(1 - \lambda\gamma_5 + \gamma_5 \not{s}_\perp)\,. \tag{A.32}$$

If $u_\lambda(p)$ are helicity spinors, calling $\rho_{\lambda\lambda'}$ the elements of the spin density matrix, one has

$$\tfrac{1}{2}\not{p}\,(1 - \lambda\gamma_5 + \gamma_5 \not{s}_\perp) = \rho_{\lambda\lambda'}\,u_{\lambda'}(p)\,\bar{u}_\lambda(p), \tag{A.33}$$

where the r.h.s. is a trace in helicity space.

Appendix B

Sudakov decomposition of vectors

The Sudakov decomposition is a standard and useful tool in the high-energy regime, where it allows a simple separation between dominant and subdominant components of vectors.

The Sudakov vectors are two light-like four-vectors of the form

$$p^\mu = \tfrac{1}{\sqrt{2}}\,(\Lambda, 0, 0, \Lambda)\,, \tag{B.1a}$$

$$n^\mu = \tfrac{1}{\sqrt{2}}\,(\Lambda^{-1}, 0, 0, -\Lambda^{-1})\,, \tag{B.1b}$$

where Λ is arbitrary. These vectors satisfy

$$p^2 = 0 = n^2\,, \qquad p{\cdot}n = 1\,, \qquad n^+ = 0 = p^-\,. \tag{B.2}$$

In light-cone components they read

$$p^\mu = (\Lambda, 0, \mathbf{0}_\perp)\,, \tag{B.3a}$$

$$n^\mu = (0, \Lambda^{-1}, \mathbf{0}_\perp)\,. \tag{B.3b}$$

Any four-vector A^μ can be parametrised as

$$\begin{aligned} A^\mu &= \alpha\, p^\mu + \beta\, n^\mu + A_\perp^\mu \\ &= A{\cdot}n\, p^\mu + A{\cdot}p\, n^\mu + A_\perp^\mu\,, \end{aligned} \tag{B.4}$$

with $A_\perp^\mu = (0, \boldsymbol{A}_\perp, 0)$, defined precisely by Eq. (B.4). This is called the *Sudakov decomposition* or *parametrisation* of A^μ. The modulus squared of A^μ is then

$$A^2 = 2\alpha\beta - \boldsymbol{A}_\perp^2\,. \tag{B.5}$$

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Appendix C

Projectors for structure functions

The structure functions can be extracted from the hadronic tensor $W_{\mu\nu}$ by means of certain projectors (see, *e.g.*, Anselmino, Efremov and Leader, 1995). To start with, let us define

$$\mathcal{P}_1^{\mu\nu} = \frac{1}{4}\left(\frac{1}{a}\,P^\mu P^\nu - g^{\mu\nu}\right), \tag{C.1a}$$

$$\mathcal{P}_2^{\mu\nu} = \frac{3P{\cdot}q}{4a}\left(\frac{1}{a}\,P^\mu P^\nu - \frac{1}{3}\,g^{\mu\nu}\right), \tag{C.1b}$$

with

$$a = \frac{P{\cdot}q}{2x} + M^2\,. \tag{C.2}$$

Neglecting terms of relative order $1/Q^2$ the projectors above read

$$\mathcal{P}_1^{\mu\nu} = \frac{1}{4}\left(\frac{4x^2}{Q^2}\,P^\mu P^\nu - g^{\mu\nu}\right), \tag{C.3a}$$

$$\mathcal{P}_2^{\mu\nu} = \frac{x}{2}\left(\frac{12x^2}{Q^2}\,P^\mu P^\nu - g^{\mu\nu}\right). \tag{C.3b}$$

F_1 and F_2 are obtained by acting with $\mathcal{P}_1^{\mu\nu}$ and $\mathcal{P}_1^{\mu\nu}$, respectively, on the hadronic tensor. From (2.21) we get

$$F_1 = \mathcal{P}_1^{\mu\nu} W_{\mu\nu}\,, \tag{C.4a}$$

$$F_2 = \mathcal{P}_2^{\mu\nu} W_{\mu\nu}\,. \tag{C.4b}$$

For the polarised structure functions one has the following projectors

$$\mathcal{Q}_2^{\mu\nu} = \frac{(P{\cdot}q)^2}{bM^2(q{\cdot}S)}\,\varepsilon^{\mu\nu\rho\sigma}\left[(q{\cdot}S)\,S_\rho + q_\rho\right]P_\sigma\,, \tag{C.5a}$$

$$\mathcal{Q}_{1+2}^{\mu\nu} = \frac{1}{b}\,\varepsilon^{\mu\nu\rho\sigma}\left\{\left[\frac{(P{\cdot}q)^2}{M^2} + 2(P{\cdot}q)x\right]S_\rho + (q{\cdot}S)q_\rho\right\}P_\sigma\,, \tag{C.5b}$$

with

$$b = -4M\left[\frac{(P{\cdot}q)^2}{M^2} + 2(P{\cdot}q)x - (q{\cdot}S)^2\right]. \tag{C.6}$$

From (2.32) one obtains with some algebra

$$g_2 = \mathcal{Q}_2^{\mu\nu}W_{\mu\nu}\,, \tag{C.7a}$$

$$g_1 + g_2 = \mathcal{Q}_{1+2}^{\mu\nu}W_{\mu\nu}\,. \tag{C.7b}$$

Appendix D

Reference frames

D.1 The $\gamma^* N$ collinear frames

In DIS processes, we call the frames where the virtual photon and the target nucleon move collinearly "$\gamma^* N$ collinear frames". If the motion takes place along the z-axis, we can represent the nucleon momentum P and the photon momentum q in terms of the Sudakov vectors p and n as

$$P^\mu = p^\mu + \tfrac{1}{2} M^2 n^\mu \simeq p^\mu \,, \tag{D.1a}$$

$$q^\mu \simeq P{\cdot}q\, n^\mu - x p^\mu = M\nu\, n^\mu - x p^\mu \,, \tag{D.1b}$$

where the approximate equality sign indicates that we are neglecting M^2 with respect to large scales such as Q^2, or $(P^+)^2$ in the infinite-momentum frame. Conventionally we always take the nucleon to be directed in the positive z direction.

With the identification (D.1a) the parameter Λ appearing in the definition of the Sudakov vectors (B.1a, b) coincides with P^+ and fixes the specific frame. In particular:

- in the *target rest frame* (TRF) one has

$$P^\mu = (M, 0, 0, 0), \tag{D.2a}$$

$$q^\mu = \left(\nu, 0, 0, -\sqrt{\nu^2 + Q^2}\right), \tag{D.2b}$$

  and $\Lambda \equiv P^+ = M/\sqrt{2}$. The Bjorken limit in this frame corresponds to $q^- = \sqrt{2}\,\nu \to \infty$, with $q^+ = -Mx/\sqrt{2}$ fixed.

- in the *infinite-momentum frame* (IMF) the momenta are

$$P^\mu \simeq \frac{1}{\sqrt{2}}\,(P^+, 0, 0, P^+)\,, \tag{D.3a}$$

$$q^\mu \simeq \frac{1}{\sqrt{2}}\left(\frac{M\nu}{P^+} - xP^+, 0, 0, -\frac{M\nu}{P^+} - xP^+\right), \tag{D.3b}$$

  Here we have $P^- \to 0$ and $\Lambda \equiv P^+ \to \infty$. In this frame the vector n^μ is suppressed by a factor of $(1/P^+)^2$ with respect to p^μ.

By means of the Sudakov vectors we can construct the *perpendicular* metric tensor $g_\perp^{\mu\nu}$ that projects onto the plane perpendicular to p and n, and also to P and q (modulo M^2/Q^2 terms)

$$g_\perp^{\mu\nu} = g^{\mu\nu} - (p^\mu n^\nu + p^\nu n^\mu)\,. \tag{D.4}$$

Transverse vectors in the $\gamma^* N$ frame (or "perpendicular" vectors) will be denoted by a $\perp$ subscript. Another projector onto the transverse plane is

$$\varepsilon_\perp^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} p_\rho n_\sigma\,. \tag{D.5}$$

Consider now the spin vector of the nucleon. It may be written as

$$S^\mu = \frac{\lambda_N}{M}\left(p^\mu - \frac{M^2}{2}\,n^\mu\right) + S_\perp^\mu \simeq \frac{\lambda_N}{M}\,p^\mu + S_\perp^\mu\,, \tag{D.6}$$

with the constraint $\lambda_N^2 + \boldsymbol{S}_\perp^2 \leq 1$ (the equality sign applies to pure states). The transverse-spin vector $S_\perp^\mu$ is $\mathcal{O}(1)$, thus it is suppressed by one power of P^+ with respect to longitudinal spin $S_\parallel^\mu = \lambda_N p^\mu / M$.

Finally, in semi-inclusive DIS the momentum P_h of the produced hadron h may be parametrised in the $\gamma^* N$ collinear frame as

$$P_h^\mu \simeq z q^\mu + xz P^\mu + P_{h\perp}^\mu\,, \tag{D.7}$$

where

$$z = \frac{P{\cdot}P_h}{P{\cdot}q} = \frac{2x}{Q^2}\,P{\cdot}P_h\,. \tag{D.8}$$

D.2 The hN collinear frames

In polarised semi-inclusive DIS it is often convenient to work in a frame where the target nucleon N and the produced hadron h are collinear (an

"hN collinear frame"). In the family of such frames the momenta of N and h are parametrised, in terms of two Sudakov vectors p' and n', as

$$P^\mu = p'^\mu + \frac{M^2}{2}\, n'^\mu \simeq p'^\mu\,, \tag{D.9a}$$

$$P_h^\mu \simeq \frac{M_h^2 x}{Q^2 z}\, p'^\mu + \frac{Q^2 z}{2x}\, n'^\mu \simeq \frac{Q^2 z}{2x}\, n'^\mu\,. \tag{D.9b}$$

The projectors onto the transverse plane (vectors lying in this plane will be denoted by the subscript T) are

$$g_T^{\mu\nu} = g^{\mu\nu} - (p'^\mu n'^\nu + p'^\nu n'^\mu)\,, \tag{D.10a}$$

$$\varepsilon_T^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} p'_\rho n'_\sigma\,. \tag{D.10b}$$

In hN collinear frames the photon acquires a transverse momentum

$$q^\mu = -x p'^\mu + \frac{Q^2}{2x}\, n'^\mu + q_T^\mu\,. \tag{D.11}$$

Comparing this to the Sudakov decomposition (D.7) of the momentum of the produced hadron, we obtain

$$P_{h\perp}^\mu \simeq -z q_T^\mu\,. \tag{D.12}$$

The spin vector of h is

$$S_h^\mu = \frac{\lambda_h}{M_h}\, P_h^\mu + S_{hT}^\mu\,. \tag{D.13}$$

The relation between transverse vectors in the $\gamma^* N$ frame ($\perp$-vectors) and transverse vectors in the hN frame (T-vectors) is

$$a_\perp^\mu = a_T^\mu - \frac{2x}{Q^2}\left[a{\cdot}q_T\, P^\mu + a{\cdot}P\, q_T^\mu\right]. \tag{D.14}$$

Therefore, if we neglect order $1/Q$ corrections (that is, if we ignore higher-twist effects) we can identify transverse vectors in $\gamma^* N$ collinear frames with transverse vectors in hN collinear frames (in other terms, we have $g_\perp^{\mu\nu} \simeq g_T^{\mu\nu}$ and $\varepsilon_\perp^{\mu\nu} \simeq \varepsilon_T^{\mu\nu}$).

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Appendix E

Dimensional regularisation and minimal subtraction

The method of dimensional regularisation (DR) is both a Lorentz- and gauge-invariant technique that consists in reducing the number of space–time dimensions to $D < 4$, thus curbing the ultraviolet divergences arising in loop integrals. Although D is normally an integer, the expressions obtained in evaluating Feynman diagrams all have a natural continuation to non-integer D (apart from isolated singularities). For an integer D-dimensional space–time, the metric $g^{\mu\nu}$ simply becomes the $D\times D$ tensor: $g^{\mu\nu} = diag(1,-1,-1,\ldots,-1)$, similarly four-vectors become $k^\mu = (k^0, k^1, \ldots, k^{D-1})$. The Dirac γ matrices are $f(D)\times f(D)$ matrices; the precise form of the function $f(D)$ is not important here. It is simply sufficient to extend the usual Clifford algebra in a straight-forward manner, with rules such as

$$\gamma^\alpha\gamma_\alpha = D\mathbb{1}\,, \tag{E.1a}$$

$$\gamma^\alpha\gamma^\mu\gamma_\alpha = -(D-2)\gamma^\mu\,, \tag{E.1b}$$

$$\gamma^\alpha\gamma^\mu\gamma^\nu\gamma_\alpha = 4g^{\mu\nu} + (D-4)\gamma^\mu\gamma^\nu\,, \tag{E.1c}$$

$$\gamma^\alpha\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\alpha = -2\gamma^\rho\gamma^\nu\gamma^\mu - (D-4)\gamma^\mu\gamma^\nu\gamma^\rho\,, \tag{E.1d}$$

and

$$Tr(\gamma^\mu\gamma^\nu) = f(D)\,g_{\mu\nu}\,. \tag{E.1e}$$

As noted in the main text, the treatment of γ_5 is a very subtle problem (for example, see Akyeampong and Delbourgo, 1974; Breitenlohner and Maison, 1977; Chanowitz, Furman and Hinchliffe, 1979), being related to the so-called ABJ triangle anomaly (Adler, 1969; Bell and Jackiw, 1969). Suffice it to say here that the various techniques available generally permit

the use of DR also in the presence of γ_5, although this does in some cases lead to complications. Indeed, owing to the presence of the anomaly, the problem always arises in some form even for non-DR techniques.

The physical dimensionality of fields must change in D dimensions and, as a consequence, the gauge couplings become dimensional $g_D = \mu^\epsilon g$, where g is dimensionless, $D = 4 - 2\epsilon$ and μ is some mass scale (this is how a mass scale is introduced via the DR of massless QCD). The dimensionality of the fields is then determined by requiring that the action $S = \int \mathrm{d}^D x\, \mathcal{L}$ be dimensionless. The terms in $\mathcal{L}$ such as $m\bar{\psi}\psi$ and $g\bar{\psi}\gamma^\mu\psi A_\mu$ fix the following dimensionalities: $m \equiv [1]$, $\psi \equiv [(D-1)/2]$, $A_\mu \equiv [(D-2)/2]$ and $g \equiv [(4-D)/2]$.

Consider now a typical loop integral, the formal expression may be written for any D:

$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{1}{(k^2 - m^2)^2} = \frac{\Gamma(2 - D/2)}{(4\pi)^{D/2}} \left(-m^2\right)^{D/2-2} . \tag{E.2}$$

Setting $D = 4 - 2\epsilon$, the Euler Γ-functions may be expanded thus

$$\Gamma(\epsilon) = \frac{1}{\epsilon} - \gamma_E + \mathcal{O}(\epsilon) , \qquad \gamma_\mathrm{E} \approx 0.577216 , \tag{E.3}$$

For a given Green function G, normalised to unity at lowest order, (such as V/g, with V the three-gluon vertex function at the symmetric point $q_1^2 = q_2^2 = q_3^2$) one then typically finds at one loop:

$$G_\mathrm{bare} = 1 + \alpha_0 \left(\frac{-\mu^2}{p^2}\right)^\epsilon \left[c\left(\frac{1}{\epsilon} + \ln 4\pi - \gamma_E\right) + d + \mathcal{O}(\epsilon)\right] , \tag{E.4}$$

where c and d are constants to be calculated. In MS this may then be rewritten to one-loop accuracy (diagram by diagram, this is a virtue of the method) as

$$G_\mathrm{bare} = Z\, G_\mathrm{ren} , \tag{E.5}$$

with

$$Z = 1 + \alpha \left[c\left(\frac{1}{\epsilon} + \ln 4\pi - \gamma_E\right)\right] , \tag{E.6a}$$

$$G_\mathrm{ren} = 1 + \alpha \left[c \ln \frac{-\mu^2}{p^2} + d\right] . \tag{E.6b}$$

In the original MS prescription only $1/\epsilon$ terms were to be subtracted (in fact, ϵ effectively takes the place of a momentum cut-off). However, since the constants $\ln 4\pi$ and γ_E always appear in the expansion of D-dimensional phase-space integrals and Γ-functions, modified minimal subtraction ($\overline{\text{MS}}$) has now been introduced, in which these too are subtracted. Note that the MS or $\overline{\text{MS}}$ definitions of α are necessarily different from that, say, of the momentum-subtraction scheme because the finite terms (*i.e.*, those beyond the logarithms) are quite different. For example, in particular, here δG_{ren} does not vanish at the point $p^2 = -\mu^2$.

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Appendix F

Mellin-moment identities

We first recall here the standard definition of the so-called plus regularisation, necessary for the IR singularities present in the AP splitting kernels:

$$\int_0^1 \mathrm{d}x \, \frac{f(x)}{(1-x)_+} \equiv \int_0^1 \mathrm{d}x \, \frac{[f(x)-f(1)]}{(1-x)} \,. \tag{F.1}$$

A convenient identity regarding the above plus symbol is:

$$\int_0^1 \mathrm{d}x \, \frac{x^{n-1}}{(1-x)_+} f(x) = \int_0^1 \mathrm{d}x \left\{ \left[\frac{x^{n-1}}{(1-x)} \right]_+ - \delta(1-x) \sum_{j=1}^{n-1} \frac{1}{j} \right\} f(x) \,. \tag{F.2}$$

This allows, for example, the following particularly compact expression for the spin-averaged and helicity-dependent qq AP splitting kernels:

$$\frac{(1+x^2)}{(1-x)_+} + \frac{3}{2}\delta(1-x) = \left[\frac{1+x^2}{1-x} \right]_+ \,. \tag{F.3}$$

The plus-regularised kernels appear in the evolution equations in convolution with parton distribution functions, where the integrals naturally run from some non-zero x up to unity, for example,

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2} f(x,Q^2) = \int_x^1 \frac{\mathrm{d}y}{y} \, P_+\left(\frac{x}{y}, Q^2\right) f(y,Q^2) \,. \tag{F.4}$$

In order to see how to evaluate such integrals, let us consider the following simplified version (we suppress the Q^2 dependence as superfluous to the

present discussion):

$$\int_x^1 \mathrm{d}y\, P_+(y)\, f(y)\,, \tag{F.5}$$

where by $P_+(y)$ we intend just the plus-regularised piece, such as $1/(1-x)_+$ with constant residue at $x = 1$. In order to transform this into an integral running from zero to unity, as in the definition (F.1), take the Mellin transform:

$$M_n = \int_0^1 \mathrm{d}x\, x^{n-1} \int_x^1 \mathrm{d}y\, P_+(y)\, f(y)\,. \tag{F.6a}$$

First, change the order of integration,

$$M_n = \int_0^1 \mathrm{d}y\, P_+(y)\, f(y) \int_0^y \mathrm{d}x\, x^{n-1}\,; \tag{F.6b}$$

then apply the definition (F.1),

$$M_n = \int_0^1 \mathrm{d}y\, P(y) \left[f(y) \int_0^y \mathrm{d}x\, x^{n-1} - f(1) \int_0^1 \mathrm{d}x\, x^{n-1} \right]; \tag{F.6c}$$

finally, reverse the integration-order change,

$$M_n = \int_0^1 \mathrm{d}x\, x^{n-1} \left[\int_x^1 \mathrm{d}y\, P(y)\, f(y) - \int_0^1 \mathrm{d}y\, P(y)\, f(1) \right]. \tag{F.6d}$$

Removing the Mellin transform and comparing Eqs. (F.6a) and (F.6d) immediately reveals a more generalised definition of the plus regularisation: namely,

$$\int_x^1 \mathrm{d}y\, P_+(y)\, f(y) \equiv \int_x^1 \mathrm{d}y\, P(y)\, f(y) - \int_0^1 \mathrm{d}y\, P(y)\, f(1)\,. \tag{F.7}$$

Note that in order to apply this to, for example, Eq. (F.4), it is necessary to first change variables to $\bar{y} = x/y$ to simplify the argument of the plus-regularised function.

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This book is devoted to the theory and phenomenology of transverse-spin effects in high-energy hadronic physics. Contrary to common past belief, it is now rather clear that such effects are far from irrelevant. A decade or so of intense theoretical work has shed much light on the subject and brought to surface an entire class of new phenomena, which now await thorough experimental investigation. Over the next few years a number of experiments world-wide (at BNL, CERN, DESY and JLAB) will run with transversely polarised beams and targets, providing data that will enrich our knowledge of the transverse-spin structure of hadrons. It is therefore timely to assess the state of the art, and this is the principal aim of the volume.

An outline of the book is as follows. After a few introductory remarks (Chap. 1), attention is directed in Chap. 2 to transversely polarised deeply-inelastic scattering (DIS), which probes the transverse spin structure function g_2. The existing data are reviewed and discussed (for completeness, a brief presentation of longitudinally polarised DIS is also provided). In Chap. 3 the transverse-spin structure of the proton is illustrated in detail, with emphasis on the transversity distribution and the twist-three parton distribution contributing to g_2. Model calculations of these quantities are also presented. In Chap. 4, the QCD evolution of transversity is studied at leading and next-to-leading order. Chap. 5 illustrates the g_2 structure function and its related sum rules within the framework of perturbative QCD. The last three chapters are devoted to the phenomenology of transversity, in the context of Drell–Yan processes (Chap. 6), inclusive leptoproduction (Chap. 7) and inclusive hadroproduction (Chap. 8). The interpretation of some recent single-spin asymmetry data is discussed and the prospects for future measurements are reviewed.

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