第二章 数组和矩阵：编程语言的基础数据结构

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2.1 数组的内存布局

2.1.1 一维数组在内存中的连续存储

一维数组是最简单的数据结构，它在内存中占据一段连续的存储空间。假设数组的起始地址为 $base_address$，每个元素占用 $element_size$ 个字节，那么第 i 个元素的地址可以通过以下公式计算：

$$address(i) = base_address + i \times element_size$$

这个公式的时间复杂度为 $O(1)$，这也是数组支持随机访问的根本原因。

2.1.2 二维数组的行优先与列优先存储

二维数组在内存中同样是连续存储的，但需要将二维结构映射到一维空间。常见的映射方式有两种：行优先（Row-major order）和列优先（Column-major order）。

行优先存储：先存储第一行的所有元素，再存储第二行，依此类推。C、C++、Java、Python 的 NumPy 默认采用行优先。

列优先存储：先存储第一列的所有元素，再存储第二列，依此类推。Fortran、MATLAB、R 采用列优先。

对于一个 $m \times n$ 的二维数组 A，元素 $A[i][j]$ 在行优先存储下的地址为：

$$address(A[i][j]) = base_address + (i \times n + j) \times element_size$$

在列优先存储下的地址为：

$$address(A[i][j]) = base_address + (j \times m + i) \times element_size$$

2.1.3 多维数组的推广

对于 d 维数组，设第 k 维的长度为 n_k，则元素 $A[i_1][i_2]\dots[i_d]$ 在行优先存储下的地址为：

$$address = base_address + \left(\sum_{k=1}^{d} i_k \prod_{j=k+1}^{d} n_j\right) \times element_size$$

2.2 稀疏矩阵的压缩存储

2.2.1 三元组表示法

稀疏矩阵是指非零元素远少于零元素的矩阵。设矩阵 A 的规模为 $m \times n$，非零元素个数为 t，且 $t \ll m \times n$。三元组表示法将每个非零元素用 $(row, col, value)$ 三元组存储，总存储量为 $3t$。

2.2.2 十字链表法

十字链表为每个非零元素建立一个结点，结点包含 5 个域：行号、列号、值、同行下一个结点指针、同列下一个结点指针。这样可以高效地进行行遍历和列遍历。

2.2.3 压缩稀疏行（CSR）格式

CSR 格式使用三个一维数组：`values` 存储非零元素，`col_indices` 存储对应的列号，`row_ptr` 存储每行第一个非零元素在 `values` 中的位置。`row_ptr` 的长度为 $m + 1$，最后一个元素等于 t。

2.3 数组的常见操作

2.3.1 插入与删除

在数组末尾插入或删除元素的时间复杂度为 $O(1)$（不考虑扩容）。在数组中间插入或删除元素，平均需要移动 $\frac{n}{2}$ 个元素，时间复杂度为 $O(n)$。

2.3.2 查找

无序数组的查找只能采用顺序查找，时间复杂度为 $O(n)$。有序数组可以采用二分查找，时间复杂度为 $O(\log n)$。

2.3.3 排序

数组是排序算法最常作用的对象。常见的比较排序算法的平均时间复杂度为 $O(n \log n)$，而计数排序、基数排序等非比较排序在特定条件下可以达到 $O(n)$。

2.4 本章小结

本章介绍了数组的内存布局、稀疏矩阵的压缩存储以及数组的常见操作。理解数组的连续存储特性，是后续学习链表、哈希表等数据结构的基础。

2.5 习题

1. 证明：对于 $m \times n$ 的二维数组，行优先与列优先存储下的地址差为常数。
2. 设稀疏矩阵 A 的规模为 1000×1000，非零元素个数为 5000，分别计算三元组表示法和 CSR 格式所需的存储量。
3. 实现一个支持 $O(1)$ 随机访问的动态数组，并分析其均摊时间复杂度。

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