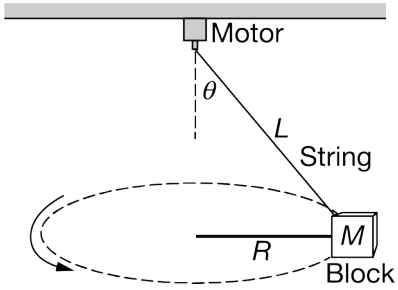
Unit 3 Progress Check: FRQ

1. This question is a long free-response question. Show your work for each part of the question.



In the apparatus shown above, one end of a string of length L is attached to a block of mass M and the other end is connected to the axle of a motor that rotates, causing the block to move in a circle of radius R at a constant speed v_T such that the string makes an angle θ with the vertical. A student wants to use the apparatus to make measurements and create a graph that can be used to estimate the acceleration g due to gravity at the surface of Earth. The student can adjust the motor to achieve different tangential speeds of the block.

(a)

i. The dot below represents the object as it travels in a horizontal circle at a constant tangential speed at the position shown in the figure above. On the dot, draw and label the forces (not components) exerted on the object. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



ii. Derive an expression for the block's centripetal acceleration a_c in terms of M, θ, and physical constants, as appropriate.

(b) Using the experimental apparatus shown above and the available equipment, design an experimental procedure the student could use to estimate the acceleration g due to gravity at the surface of Earth.

i. In the table below, list the quantities and associated symbols that would be measured in the experiment and the equipment used to measure them. Also list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

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Quantity to be Measured	Symbol for Quantity	Equipment for Measurement

ii. Describe an experimental procedure to estimate the acceleration g due to gravity at the surface of Earth using the apparatus. Give enough detail so that another student could replicate the experiment. As needed, include a diagram of the experimental setup. Assume equipment usually found in a school physics laboratory is available.

(c) The student wants to analyze data collected from the experiment by creating a linear graph that can be used to estimate the acceleration g due to gravity at the surface of Earth.

i. State what quantity should be plotted on the horizontal axis and what quantity should be plotted on the vertical axis to produce the linear graph.

Horizontal axis:_____

Vertical axis:_____

ii. Indicate which feature of the graph could be used to determine g.

Part (a)i

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for drawing a downward force due to gravity.
- ☐ 1 point is earned for drawing a tension force directed along the string (upward and to the left).

Example Response:

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Part (a)ii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for correctly applying Newton's second law to the horizontal and vertical forces.
- ☐ 1 point is earned for combining the two equations to express the centripetal acceleration in terms of the angle θ.

Example Response:

$\Sigma F_y = ma_y$
$F_{Ty} - F_g = ma_y$
$F_T \cos(\theta) - mg = m(0)$
$F_T \cos(\theta) = mg$

$\Sigma F_x = ma_x$
$F_{Tx} = ma_x$
$F_T \sin(\theta) = ma_c$

$F_T \cos(\theta) = mg$
$F_T = \frac{mg}{\cos(\theta)}$
$F_T \sin(\theta) = ma_c$
$\frac{mg}{\cos(\theta)} \sin(\theta) = ma_c$
$g \tan(\theta) = a_c$

Part (b)i

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

Student response accurately includes the following criteria.

- ☐ 1 point is earned for listing relevant/appropriate quantities and equipment

Example Response:

Quantity to be Measured	Symbol for Quantity	Equipment for Measurement
Length of string	L	Meterstick
Radius of circular path	R	Meterstick
Time for block to make one revolution	T	Stopwatch

Part (b)ii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

Student response accurately includes all of the following criteria.

- ☐ 1 point is earned for stating a plausible method to use the apparatus to estimate g.
- ☐ 1 point is earned for measuring quantities that can be used to determine the tangential speed (e.g., the radius R and the time for the block to travel around the circle once).
- ☐ 1 point is earned for measuring quantities that can be used to determine θ and R. Note that the relation $\sin(\theta) = \frac{R}{L}$ may be used.
- ☐ 1 point is earned for an attempt to reduce experimental uncertainty.

Example Response:

The speed of the motor should be set so that the object revolves around its circular path at a constant tangential speed. A stopwatch can be used to measure the time it takes for the object to make one complete revolution around its circular path, and the meterstick can be used to measure both the radius of the circular path and the length of the string. The angle can be found by relating the radius of the circular path and the length of the string with the sine of the angle: $\sin(\theta) = \frac{R}{L}$ *. Alternatively, the angle from the vertical may be measured by filming the experiment using a slow motion camera and using a protractor to measure the angle from the vertical in the camera images.The procedure can be*

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repeated for five different tangential speeds that cover as wide a range as possible using the available equipment.

Part (c)i

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	✓ 2

Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for listing quantities that were either measured, or can be determined from the measurements, in the procedure given in part (b).
- ☐ 1 point is earned for listing quantities that can be graphed in order to determine *g*.

Example Response:

Horizontal axis: $\frac{R}{T^2}$, where *T* is the time to complete one revolution

Vertical axis: $\tan\theta$

Part (c)ii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	✓ 1

Student response accurately includes the following criteria.

- ☐ 1 point is earned for giving an appropriate feature of the graph that can be used to determine *g*

Example Response:

The slope (for a graph of $\tan\theta$ as a function of $\frac{R}{T^2}$)

Derivation: The centripetal acceleration is $a_c = g\tan\theta = \frac{{v_T}^2}{R}$. Substituting the expression $\frac{2\pi R}{T}$ for v_T results in the equation $g\tan\theta = \frac{4\pi^2 R}{T^2}$, so the slope of a graph of $\tan\theta$ as a function of $\frac{R}{T^2}$ can be used to determine *g*.

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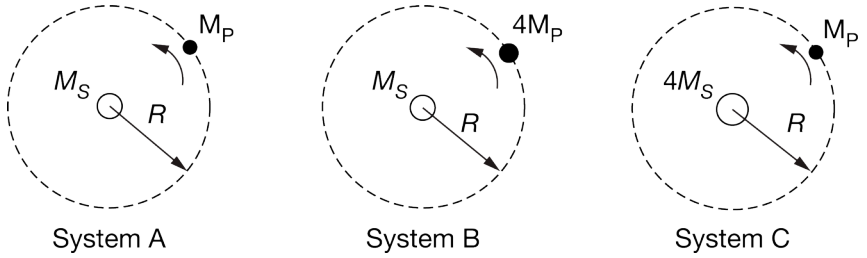
2. This question is a short free-response question. Show your work for each part of the question.



Three different planet-star systems, which are far apart from one another, are shown above. The masses of the planets are much less than the masses of the stars.

In System A, Planet A of mass M_p orbits Star A of mass M_s in a circular orbit of radius R.

In System B, Planet B of mass $4M_p$ orbits Star B of mass M_s in a circular orbit of radius R.

In System C, Planet C of mass M_p orbits Star C of mass $4M_s$ in a circular orbit of radius R.

(a) The gravitational force exerted on Planet A by Star A has a magnitude of F_0. Determine the magnitudes of the gravitational forces exerted in System B and System C.

___ Magnitude of gravitational force exerted on Planet B by Star B

___ Magnitude of gravitational force exerted on Planet C by Star C

(b) How do the tangential speeds of planets B and C compare to that of Planet A? In a clear, coherent paragraph-length response that may also contain equations and/or drawings, provide claims about

why the tangential speed of Planet B is either greater than, less than, or the same as that of Planet A, and
why the tangential speed of Planet C is either greater than, less than, or the same as that of Planet A.

Part (a)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for indicating a force of $4F_0$ for system B.

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- [ ] 1 point is earned for indicating a force of $4F_0$ for system C.

Part (b)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5 ✓

Student response accurately includes all of the following criteria.

- [ ] 1 point is earned for equating the net centripetal exerted on a planet from its star to the gravitational force exerted on the planet from its star.
- [ ] 1 point is earned for correct reasoning or a derivation relating a planet's speed to the star's mass.
- [ ] 1 point is earned for a statement that Planet B has the same tangential speed as Planet A.
- [ ] 1 point is earned for a statement that Planet C has a greater tangential speed than Planet A.
- [ ] 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response

Example Response:
For each system, the centripetal force responsible for the planet's circular motion is due to the gravitational force exerted by the star. Equating the centripetal and gravitational forces gives

$$\Sigma \vec{F} = m\vec{a}$$
$$F_c = F_g = ma_c$$
$$\frac{Gm_{planet}m_{star}}{r^2} = m_{planet}\frac{v^2}{r}$$
$$\frac{Gm_{star}}{r} = v^2$$
$$v = \sqrt{\frac{Gm_{star}}{r}}$$

where $m_{\text{Planet}}, v,$ and m_{Star} refer to a planet's mass, planet's tangential speed, and star's mass, respectively. A planet's tangential speed is, therefore, proportional to the square root of its star's mass, and independent of the planet's mass. As a result, the tangential speeds are the same for Planet A and Planet B (stars of equal mass), and greater for Planet C (star of greater mass).