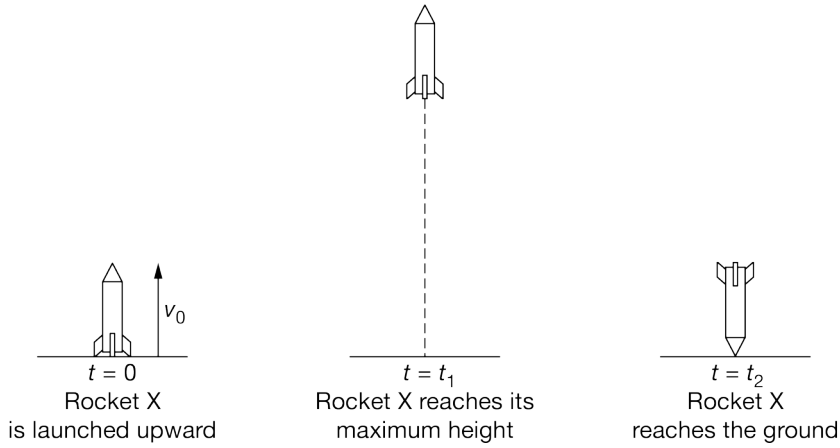
## Unit 4 Progress Check: FRQ

**1.** This question is a long free-response question. Show your work for each part of the question.




Students conduct an experiment to study the motion of two toy rockets. In the first experiment, rocket $\mathrm{X}$ of mass $m_R$ is launched vertically upward with an initial speed $v_0$ at time $t = 0$. The rocket continues upward until it reaches its maximum height at time $t_1$. As the rocket travels upward, frictional forces are considered to be negligible. The rocket then descends vertically downward until it reaches the ground at time $t_2$. The figure above shows the toy rocket at different times of its flight. In a second experiment, which has not yet been conducted by the students, rocket $\mathrm{Y}$ of mass $M_R$, where $M_R > m_R$, will be launched vertically upward with an initial speed $v_0$ at time $t = 0$ until it reaches its maximum height. Rocket $\mathrm{Y}$ will then descend vertically downward until it reaches the ground.

Two students in the group make predictions about the motion of rocket $\mathrm{Y}$ compared to that of rocket $\mathrm{X}$. Their arguments are as follows.

Student 1: "Rocket $\mathrm{Y}$ will have a smaller maximum vertical displacement than rocket $\mathrm{X}$, although it is launched upward with the same speed as rocket $\mathrm{X}$ and has more kinetic energy than rocket $\mathrm{X}$. Because rocket $\mathrm{Y}$ will have a smaller maximum vertical displacement than rocket $\mathrm{X}$, I predict that it will take less time for rocket $\mathrm{Y}$ to reach the ground compared with rocket $\mathrm{X}$."

Student 2: "Rocket $\mathrm{Y}$ will have the same maximum vertical displacement as rocket $\mathrm{X}$ because both rockets have the same kinetic energy. Since both rockets will have the same maximum vertical displacement, I predict that it will take both rockets the same amount of time to reach the ground."

(a) For part (a), ignore whether the students' predictions are correct or incorrect. Do not simply repeat the students' arguments as your answers.

i. Which aspects of Student 1's reasoning, if any, are correct? Explain your answer.

ii. Which aspects of Student 1's reasoning, if any, are incorrect? Explain your answer.

iii. Which aspects of Student 2's reasoning, if any, are correct? Explain your answer.

iv. Which aspects of Student 2's reasoning, if any, are incorrect? Explain your answer.

(b) Use quantitative reasoning, including equations as needed, to derive expressions for the maximum heights achieved by rocket $\mathrm{X}$ and rocket $\mathrm{Y}$. Express your answer in terms of $v_0$, $m_R$, $M_R$, $g$, and/or other fundamental

constants as appropriate.

(c) Use quantitative reasoning, including equations as needed, to derive expressions for the time it takes rocket X and rocket Y to reach the ground after reaching their respective maximum heights, $H_X$ and $H_Y$. Express your answer in terms of $v_0$, $m_R$, $M_R$, $H_X$, $H_Y$, $g$, and/or other fundamental constants as appropriate.

(d)

i. Explain how any correct aspects of each student's reasoning identified in part (a) are expressed by your mathematical relationships in part (b).

ii. Explain how any correct aspects of each student's reasoning identified in part (a) are expressed by your mathematical relationships in part (c).

## Unit 4 Progress Check: FRQ

**2.** This question is a short free-response question. Show your work for each part of the question.

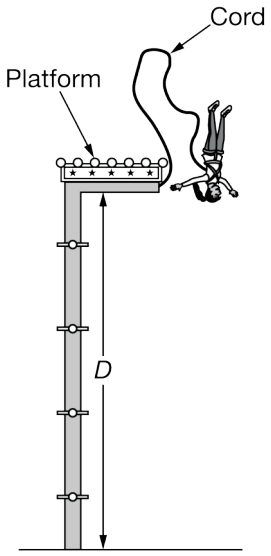



Figure 1. Initial State

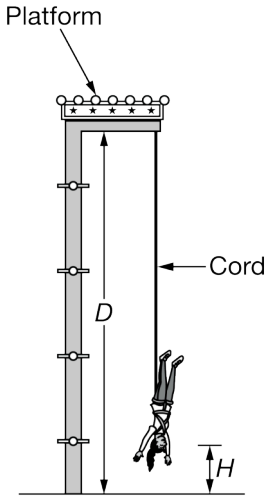



Figure 2. Final State

An amusement park ride is designed such that one end of an elastic cord is fixed to a platform that is a distance $D$ above the ground. The other end of the cord is attached to a person of mass $M$. The cord has an unstretched length of $L$. To begin the ride, the person jumps up from the platform with an initial speed, $v_0$ , as shown in Figure 1. As the person falls, the elastic cord stretches and can be modeled much like a spring of negligible mass. After the cord can no longer be stretched, the person is a distance $H$ above the ground and is no longer in motion, as shown in Figure 2. The reference position for zero gravitational potential energy of the person-cord-Earth system is a distance $H$ above the ground. All frictional forces are considered to be negligible.

(a) The initial state of the system is shown in Figure 1, and the final state of the system is shown in Figure 2.

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i. Describe the energy transformations that take place from the initial state to the final state of the system. For each form of energy, identify the object or system that has that form of energy.

ii. Derive an expression for the spring constant of the cord, in terms of $D$, $L$, $h$, $v_0$, $m$, and physical constants as appropriate.

(b) Suppose that air resistance cannot be ignored. For the position at which the person has jumped from the platform and the cord reaches its maximum stretch length, how does the stretch length of the cord in the case with air resistance compare with the stretch length in the case in which air resistance is ignored? Briefly state your reasoning.