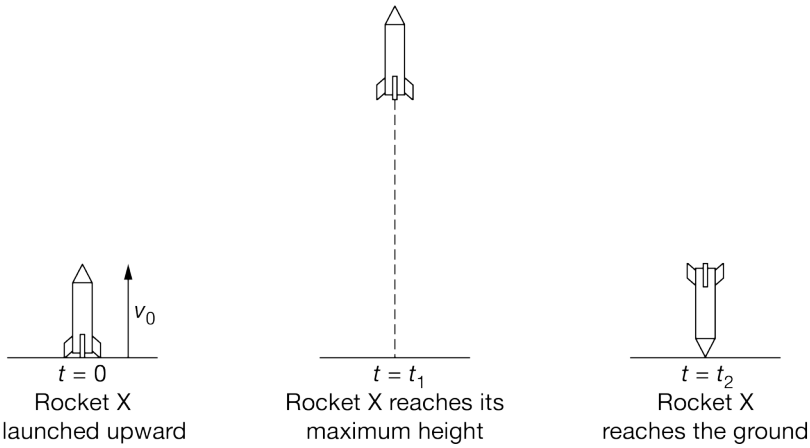
Unit 4 Progress Check: FRQ

1. This question is a long free-response question. Show your work for each part of the question.



Students conduct an experiment to study the motion of two toy rockets. In the first experiment, rocket X of mass m_R is launched vertically upward with an initial speed v_0 at time $t = 0$. The rocket continues upward until it reaches its maximum height at time t_1. As the rocket travels upward, frictional forces are considered to be negligible. The rocket then descends vertically downward until it reaches the ground at time t_2. The figure above shows the toy rocket at different times of its flight. In a second experiment, which has not yet been conducted by the students, rocket Y of mass M_R, where $M_R > m_R$, will be launched vertically upward with an initial speed v_0 at time $t = 0$ until it reaches its maximum height. Rocket Y will then descend vertically downward until it reaches the ground.

Two students in the group make predictions about the motion of rocket Y compared to that of rocket X. Their arguments are as follows.

Student 1: "Rocket Y will have a smaller maximum vertical displacement than rocket X, although it is launched upward with the same speed as rocket X and has more kinetic energy than rocket X. Because rocket Y will have a smaller maximum vertical displacement than rocket X, I predict that it will take less time for rocket Y to reach the ground compared with rocket X."

Student 2: "Rocket Y will have the same maximum vertical displacement as rocket X because both rockets have the same kinetic energy. Since both rockets will have the same maximum vertical displacement, I predict that it will take both rockets the same amount of time to reach the ground."

(a) For part (a), ignore whether the students' predictions are correct or incorrect. Do not simply repeat the students' arguments as your answers.

i. Which aspects of Student 1's reasoning, if any, are correct? Explain your answer.

ii. Which aspects of Student 1's reasoning, if any, are incorrect? Explain your answer.

iii. Which aspects of Student 2's reasoning, if any, are correct? Explain your answer.

iv. Which aspects of Student 2's reasoning, if any, are incorrect? Explain your answer.

(b) Use quantitative reasoning, including equations as needed, to derive expressions for the maximum heights

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achieved by rocket X and rocket Y. Express your answer in terms of v_0, m_R, M_R, g, and/or other fundamental constants as appropriate.

(c) Use quantitative reasoning, including equations as needed, to derive expressions for the time it takes rocket X and rocket Y to reach the ground after reaching their respective maximum heights, H_X and H_Y. Express your answer in terms of v_0, m_R, M_R, H_X, H_Y, g, and/or other fundamental constants as appropriate.

(d)

i. Explain how any correct aspects of each student's reasoning identified in part (a) are expressed by your mathematical relationships in part (b).

ii. Explain how any correct aspects of each student's reasoning identified in part (a) are expressed by your mathematical relationships in part (c).

Part (a)i

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

Student response accurately includes the following criteria.

- ☐ Student response does not accurately include the following criteria.
  1 point is earned for indicating that rocket Y is launched with the same upward speed as rocket X and for indicating that rocket Y has more kinetic energy than rocket X because rocket Y has a larger mass than rocket X.

Example Response:

Student 1 is correct that the rocket Y is launched with the same upward speed as rocket X, and since rocket Y has a larger mass than rocket X, rocket Y has more kinetic energy than rocket X.

Part (a)ii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

Student response accurately includes the following criteria.

- ☐ 1 point is earned for indicating that rocket Y will NOT travel a smaller maximum vertical displacement than rocket X, because rocket Y and rocket X have the same initial vertical velocity and the same vertical acceleration.

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Example Response:

Student 1 is incorrect in saying that rocket Y will travel a smaller maximum vertical displacement than rocket X. Both rockets have the same initial vertical velocity and the same acceleration, therefore they will reach the same maximum vertical displacement. The mass of the rocket does not affect the maximum vertical displacement.

Part (a)iii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

Student response accurately includes the following criteria.

- ☐ 1 point is earned for indicating that rocket Y will travel the same maximum vertical displacement as rocket X, because rocket and rocket X have the same initial vertical velocity and the same vertical acceleration.

Example Response:

Student 2 is correct that both rockets will have the same vertical displacement because both rockets have the same vertical velocity and the same vertical acceleration. The mass of the rocket does not affect the maximum vertical displacement.

Part (a)iv

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

Student response accurately includes the following criteria.

- ☐ 1 point is earned for indicating that rocket X and rocket Y do NOT have the same kinetic energy, because rocket X and rocket Y have different masses.

Example Response:

Student 2 is incorrect in saying that the two rockets have the same kinetic energy. Even though the two rockets have the same initial vertical velocity, they have different masses and so have different initial kinetic energies.

Part (b)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: These points can also be given for deriving an expression through using kinematic equations. In that case, the first point would be given for the use of an appropriate kinematic equation that can be used to solve for the maximum height of

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the rocket, and the second point is given for a correct equation for the maximum height of the rocket.

0	1	2 ✓

Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for indicating that the initial kinetic energy of a rocket is equal to the final gravitational potential energy of the rocket-Earth system.
- ☐ 1 point is earned for a correct equation for the maximum height of a rocket.

Example Response:

$E_0 = E_f$
$U_{g0} + U_{s0} + K_0 + W_{\text{External}} = U_{gf} + U_{sf} + K_f$
$K_0 = U_{gf}$
$\frac{1}{2}mv_0^2 = mgy_f$
$\frac{1}{2}v_0^2 = gy_f$
$y_f = \frac{1}{2}\frac{v_0^2}{g}$

Part (c)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Alternate Solution:

$v_y = v_{y0} + a_y t$
$-v_0 = 0 - gt$
$v_0 = gt$
$t = \frac{v_0}{g}$

0	1	2 ✓

Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for indicating the rocket at maximum height has an initial speed of zero and will have an acceleration toward the ground of g .
- ☐ 1 point is earned for a correct expression for the time it takes one of the rockets to reach the ground in terms of its respective height.

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Example Response:

$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$
$(0) = y_0 + (0)t + \frac{1}{2}a_y t^2$
$-y_0 = \frac{1}{2}a_y t^2$
$-y_0 = -\frac{1}{2}gt^2$
$y_0 = \frac{1}{2}gt^2$
$t = \sqrt{\frac{2y_0}{g}}$

The time for rocket X to travel from its maximum height to the ground

$t = \sqrt{\frac{2H_X}{g}}$

The time for rocket Y to travel from its maximum height to the ground

$t = \sqrt{\frac{2H_Y}{g}}$

Part (d)i

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for stating that because rocket Y has a greater mass than rocket X, both rockets are launched with the same initial speed v_0 , and , $K = \frac{1}{2}mv^2$ rocket Y has more kinetic energy than rocket X.
- ☐ 1 point is earned for stating that because $\frac{1}{2}v_0^2 = gy_f$ due to the conservation of mechanical energy, rocket X and rocket Y will reach the same vertical displacement because their masses do not affect the final height that is achieved by each rocket.

Example Response:

Because rocket Y has a greater mass than rocket X, and both rockets are launched with the same initial speed v_0 , and , $K = \frac{1}{2}mv^2$ rocket Y has more kinetic energy than rocket X. However, when conservation of energy is applied to the rocket/earth system, the masses cancel meaning that the masses of the rockets do not affect the final height that is reached by each rocket.

Part (d)ii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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Note: a response that is consistent with what was derived in part (c) will also earn full credit.

0	1	2 ✓

Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for stating that because, $-y_0 = \frac{1}{2}a_y t^2$ rocket X and rocket Y accelerate toward the ground with the same acceleration.
- ☐ 1 point is earned for stating that because , $t = \sqrt{\frac{2H_X}{g}}$ or $t = \sqrt{\frac{2H_Y}{g}}$ t both rockets will take the same amount of time to reach the ground because H_X and H_Y are the same.

Example Response:

Because both rockets will experience the same acceleration ($-y_0 = \frac{1}{2}a_y t^2$) and because both rockets reach the same maximum height, they will take the same amount of time to reach the ground. ($t = \sqrt{\frac{2H_X}{g}}$ or $t = \sqrt{\frac{2H_Y}{g}}$)

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2. This question is a short free-response question. Show your work for each part of the question.

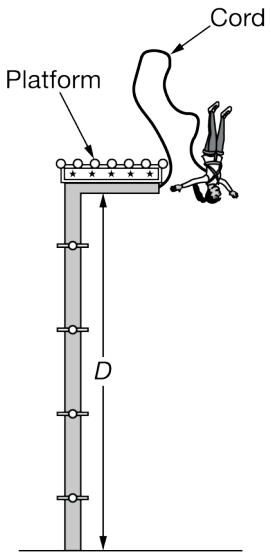

Figure 1. Initial State

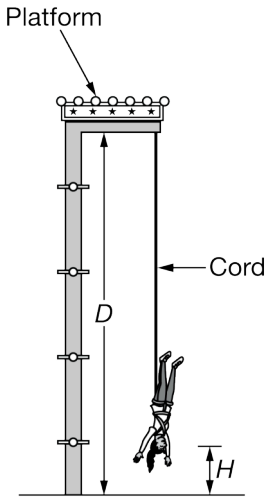

Figure 2. Final State

An amusement park ride is designed such that one end of an elastic cord is fixed to a platform that is a distance D above the ground. The other end of the cord is attached to a person of mass M. The cord has an unstretched length of L. To begin the ride, the person jumps up from the platform with an initial speed, v_0 , as shown in Figure 1. As the person falls, the elastic cord stretches and can be modeled much like a spring of negligible mass. After the cord can no longer be stretched, the person is a distance H above the ground and is no longer in motion, as shown in Figure 2. The reference position for zero gravitational potential energy of the person-cord-Earth system is a distance H above the ground. All frictional forces are considered to be negligible.

(a) The initial state of the system is shown in Figure 1, and the final state of the system is shown in Figure 2.

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i. Describe the energy transformations that take place from the initial state to the final state of the system. For each form of energy, identify the object or system that has that form of energy.

ii. Derive an expression for the spring constant of the cord, in terms of D, L, h, v_0, m, and physical constants as appropriate.

(b) Suppose that air resistance cannot be ignored. For the position at which the person has jumped from the platform and the cord reaches its maximum stretch length, how does the stretch length of the cord in the case with air resistance compare with the stretch length in the case in which air resistance is ignored? Briefly state your reasoning.

Part (a)i

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for stating that the person has kinetic energy and the person-cord-Earth system has gravitational potential energy at the initial state.
- ☐ 1 point is earned for stating that the person-cord-Earth system has elastic (spring) potential energy at the final state.

Example Response:

At the beginning, the person has kinetic energy (because the person has an initial velocity), and the person-cord-Earth system has gravitational potential and kinetic energy.

At the end, the person-cord-Earth system has spring potential energy. The person has no mechanical energy.

Part (a)ii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

Student response accurately includes all of the following criteria.

- ☐ 1 point is earned for correctly equating the forms of mechanical energy present in the initial state to the forms of mechanical energy present in the final state.
- ☐ 1 point is earned for a correct substitution for the distance that the cord stretches beyond its unstretched length: $D - H - L$

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- [ ] 1 point is earned for a correct expression for the elastic(spring) constant of the cord.

Example Response:

$\mathrm{X(stretch\,of\,the\,elastic)} = D - H - L$

$U_{g\,initial} + K_{0\,initial} = U_{s\,final}$

$Mg(D - H) + \frac{1}{2}Mv_0^2 = \frac{1}{2}k$msup

$k = \frac{2Mg(D-H)+Mv_0^2}{(D-H-L)^2}$

Part (b)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes both of the following criteria.

- [ ] 1 point is earned for stating that the cord will not stretch as far when air resistance is considered.
- [ ] 1 point is earned for indicating that as the person falls to the ground, the force of friction will do negative work on the person-cord-Earth system to remove mechanical energy from the system that would otherwise have to be stored in the cord when it reaches its maximum stretch length.

Example Response:

After the person jumps and falls to the ground, a force of friction will be exerted on the person from the air. The kinetic energy of the person will be converted into the form of non mechanical energy due to the non conservative force of friction. Therefore, the mechanical energy of the system decreases as the force of friction is exerted on the person. With less mechanical energy available in the person-cord-Earth system, there will be less mechanical energy that can be converted into elastic (spring) potential energy. Thus, the cord will not stretch as far as in the case without air resistance.