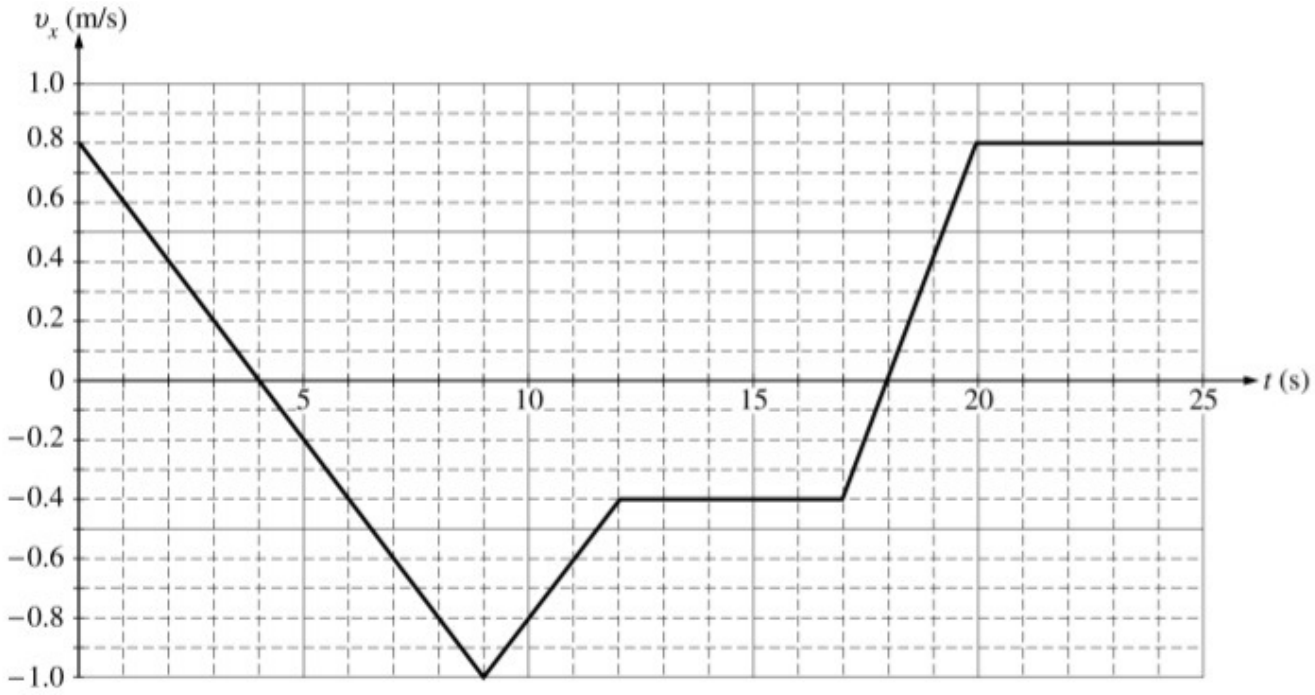
1frq

1. A 0.50 kg cart moves on a straight horizontal track. The graph of velocity v_x versus time t for the cart is given below.



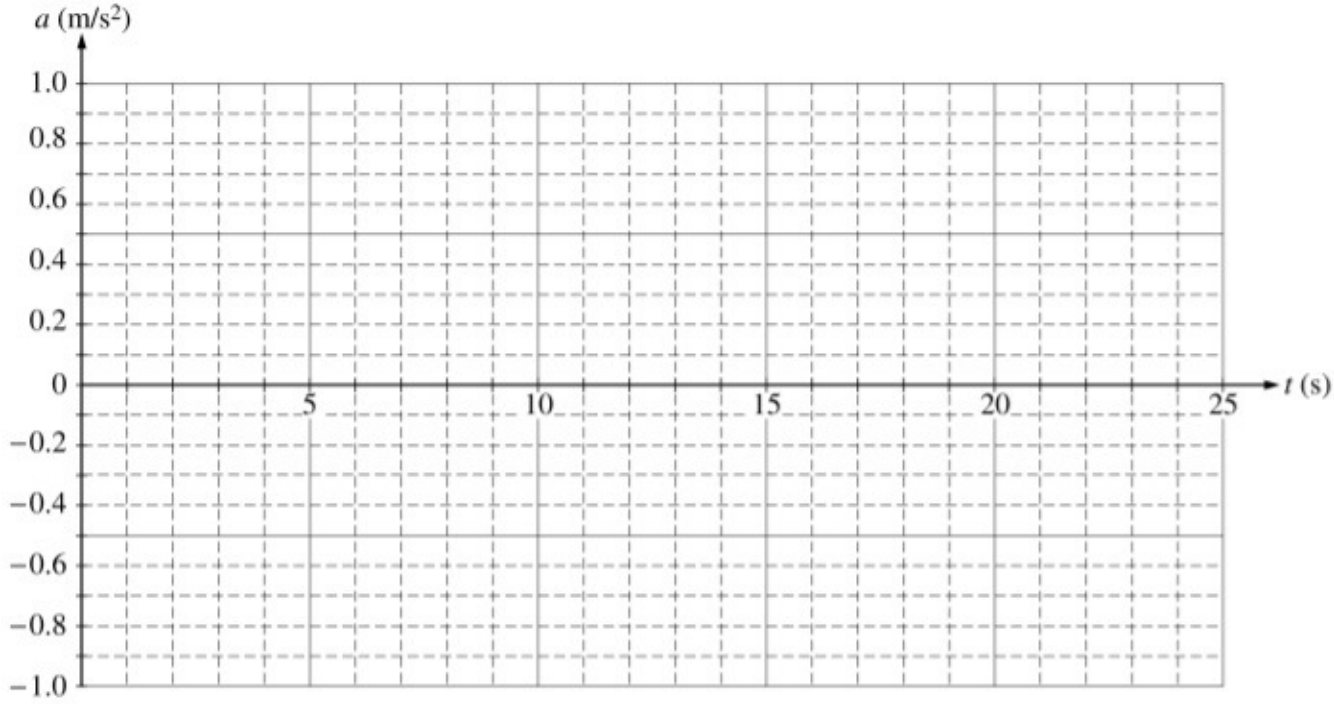
a. Indicate every time t for which the cart is at rest.

b. Indicate every time interval for which the speed (magnitude of velocity) of the cart is increasing.

c. Determine the horizontal position x of the cart at $t = 9.0$ s if the cart is located at $x = 2.0$ m at $t = 0$.

d. On the axes below, sketch the acceleration a *versus* time t graph for the motion of the cart from $t = 0$ to $t = 25$ s.

1frq



e. From $t = 25$ s until the cart reaches the end of the track, the cart continues with constant horizontal velocity. The cart leaves the end of the track and hits the floor, which is 0.40 m below the track. Neglecting air resistance, determine each of the following.

i. The time from when the cart leaves the track until it first hits the floor

ii. The horizontal distance from the end of the track to the point at which the cart first hits the floor

iii. The kinetic energy of the cart immediately before it hits the floor

Part A

The cart is at rest at $t = 4$ s and 18 s.

1 point is earned: For indicating both times that the car is at rest and not including any incorrect times

0	1 ✓

The student response earns all of the following points:

The cart is at rest at $t = 4$ s and 18 s.

1 point is earned: For indicating both times that the car is at rest and not including any incorrect times

Part B

1frq

The speed of the cart is increasing during the intervals t = 4 to 9 s and 18 to 20 s.

2 points are earned: For each correct time interval one point was awarded.

One point was deducted for each incorrect interval, for a maximum two-point deduction.

0	1	2 ✓

The student response earns all of the following points:

The speed of the cart is increasing during the intervals t = 4 to 9 s and 18 to 20 s.

2 points are earned: For each correct time interval one point was awarded.

One point was deducted for each incorrect interval, for a maximum two-point deduction.

Part C

1 point is earned: For indicating that the change in position is equal to the area under the graph, or for using an appropriate kinematic equation with non-zero initial velocity

Δx = area OR, for example, $\Delta x = v_i t + \frac{1}{2}at^2$

1 point is earned: For correct substitution of values

$\Delta x = \frac{1}{2}(4\text{ s})(0.8\text{ m/s}) + \frac{1}{2}(5\text{ s})(-1\text{ m/s})$

OR $\Delta x = (0.8\text{ m/s})(9\text{ s}) + \frac{1}{2}(-0.2\text{ m/s}^2)(9\text{ s})^2$

Δx = –0.9 m

1 point is earned: For adding the initial position to Δx

$x = x0 + \Delta x$ = 2 m + (–0.9 m)

x = 1.1 m

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For indicating that the change in position is equal to the area under the graph, or for using an appropriate kinematic equation with non-zero initial velocity

Δx = area OR, for example, $\Delta x = v_i t + \frac{1}{2}at^2$

1frq

1 point is earned: For correct substitution of values

$\Delta x = \frac{1}{2}(4\text{ s})(0.8\text{ m/s}) + \frac{1}{2}(5\text{ s})(-1\text{ m/s})$

OR $\Delta x = (0.8\text{ m/s})(9\text{ s}) + \frac{1}{2}(-0.2\text{ m/s}^2)(9\text{ s})^2$

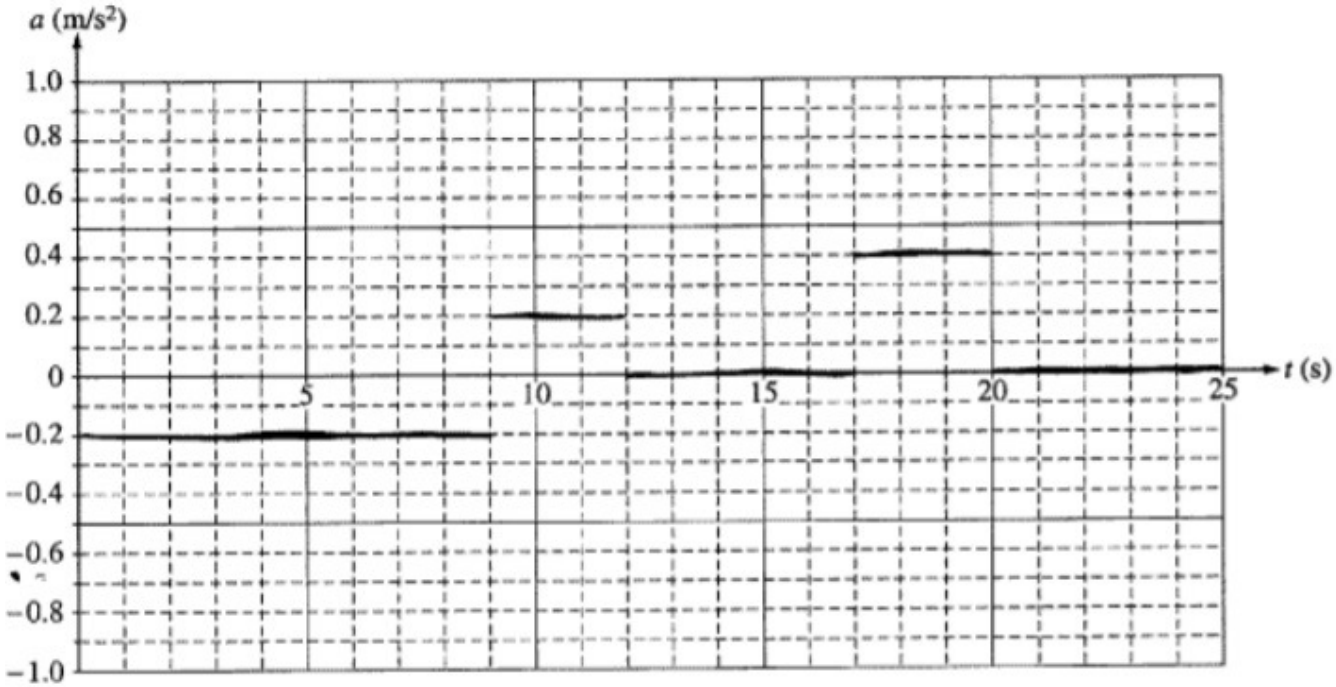
Δx = –0.9 m

1 point is earned: For adding the initial position to Δx

$x = x0 + \Delta x$ = 2 m + (–0.9 m)

x = 1.1 m

Part D



1 point is earned: For indicating that the acceleration is zero during the intervals of constant velocity, 12 to 17 s and 20 to 25 s

1 point is earned: For indicating that the acceleration is negative and constant from zero to 9 s, and positive and constant from 9 to 11 s and 17 to 20 s

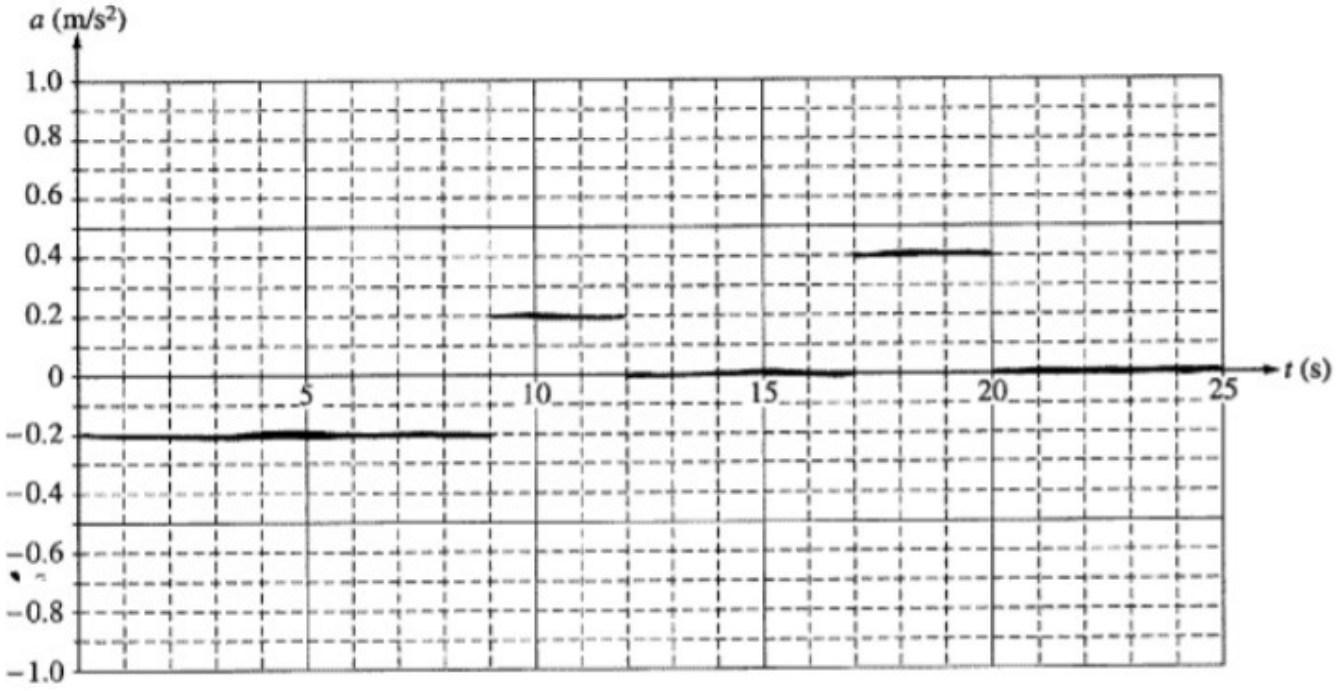
1 point is earned: For having the correct values for two of the three intervals of non-zero accelerations:

–0.20 m/s2, +0.20 m/s2, and +0.40 m/s2

0	1	2	3 ✓

1frq

The student response earns all of the following points:



1 point is earned: For indicating that the acceleration is zero during the intervals of constant velocity, 12 to 17 s and 20 to 25 s

1 point is earned: For indicating that the acceleration is negative and constant from zero to 9 s, and positive and constant from 9 to 11 s and 17 to 20 s

1 point is earned: For having the correct values for two of the three intervals of non-zero accelerations:

–0.20 m/s2, +0.20 m/s2, and +0.40 m/s2

Part E

- **1 point is earned:** For using the correct kinematic equation, with the initial vertical velocity equal to zero

  $y = \frac{1}{2}gt^2$

  Solving for *t*:

  $t = \sqrt{2y/g}$

  $t = \sqrt{2(0.40\ \text{m})/10\ \text{m/s}^2}$

  $t = 0.28\ \text{s} (\text{or } 0.29\ \text{s using } g = 9.8\ \text{m/s}^2)$

- Using the kinematic equation for the horizontal motion:

  x = *vxt*

  1 point is earned: For using the correct value for the horizontal speed

1frq

vx = 0.8 m/s

x = (0.8 m/s)(0.28 s)

1 point is earned: For the correct answer

x = 0.22 m (or 0.23 m using *g* = 9.8 m/s2)

- **2 points are earned:** For correctly applying conservation of energy

Kbottom = *Ktop* + *Utop*

$$K_{bottom} = \frac{1}{2}mv_x{}^2 + mgy$$
$$K_{bottom} = \frac{1}{2}(0.50\ \text{kg})(0.8\ \text{m/s})^2 + (0.50\ \text{kg})(10\ \text{m/s}^2)(0.40\ \text{m})$$

1 point is earned: For the correct answer

Kbottom = 2.2 J (or 2.1 J using *g* = 9.8 m/s2)

In the absence of any of the above credit, one point was awarded for any recognition that the vertical velocity changes.

(Alternate solution)

1 point is earned: For any recognition that the vertical velocity changes.

Calculating the *y*-component of velocity at the bottom:

vy = vyt – *gt*

vy = 0 + (10 m/s2)(0.28 s) = 2.8 m/s

1 point is earned: For correctly calculating the total speed from the components

*v*2 = *vx*2 + *vy*2

$$v = \sqrt{(0.8\ \text{m/s})^2 + (2.8\ \text{m/s})^2}$$

v = 2.9 m/s

1 point is earned: For correctly calculating the final kinetic energy

$$K = \frac{1}{2}mv^2$$
$$K = \frac{1}{2}(0.50\ \text{kg})(2.9\ \text{m/s})^2$$

K = 2.2 J (or 2.1 J depending on the value of *g* used and amount of rounding in intermediate calculations)

1frq

0	1	2	3	4	5	6 ✓

The student response earns all of the following points:

- **1 point is earned:** For using the correct kinematic equation, with the initial vertical velocity equal to zero

  $y = \frac{1}{2}gt^2$

  Solving for *t*:

  $t = \sqrt{2y/g}$

  $t = \sqrt{2(0.40\text{ m})/10\text{ m/s}^2}$

  $t = 0.28\text{ s} (\text{or } 0.29\text{ s using } g = 9.8\text{ m/s}^2)$

- Using the kinematic equation for the horizontal motion:

  x = *vxt*

  1 point is earned: For using the correct value for the horizontal speed

  vx = 0.8 m/s

  x = (0.8 m/s)(0.28 s)

  1 point is earned: For the correct answer

  x = 0.22 m (or 0.23 m using *g* = 9.8 m/s2)

- **2 points are earned:** For correctly applying conservation of energy

  Kbottom = *Ktop* + *Utop*

  $K_{bottom} = \frac{1}{2}mv_x{}^2 + mgy$

  $K_{bottom} = \frac{1}{2}(0.50\text{ kg})(0.8\text{ m/s})^2 + (0.50\text{ kg})(10\text{ m/s}^2)(0.40\text{ m})$

  1 point is earned: For the correct answer

  Kbottom =2.2 J (or 2.1 J using *g* = 9.8 m/s2)

  (Alternate solution)

  1 point is earned: For any recognition that the vertical velocity changes.

  Calculating the *y*-component of velocity at the bottom:

1frq

v_y = vyt – gt

v_y = 0 + (10 m/s2)(0.28 s) = 2.8 m/s

1 point is earned: For correctly calculating the total speed from the components

$v2 = vx2 + vy2$

$$v = \sqrt{(0.8\ \text{m/s})^2 + (2.8\ \text{m/s})^2}$$

v = 2.9 m/s

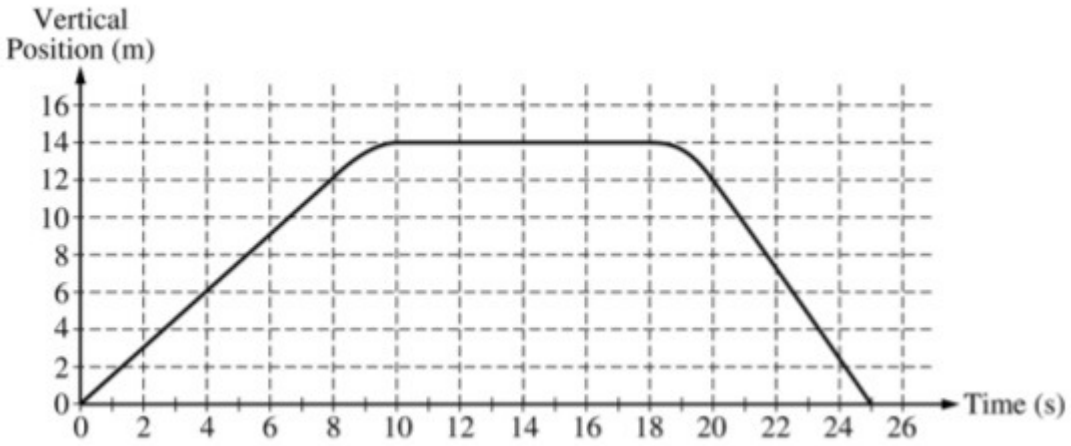
1 point is earned: For correctly calculating the final kinetic energy

$$K = \frac{1}{2}mv^2$$
$$K = \frac{1}{2}(0.50\ \text{kg})(2.9\ \text{m/s})^2$$

K = 2.2 J (or 2.1 J depending on the value of g used and amount of rounding in intermediate calculations)

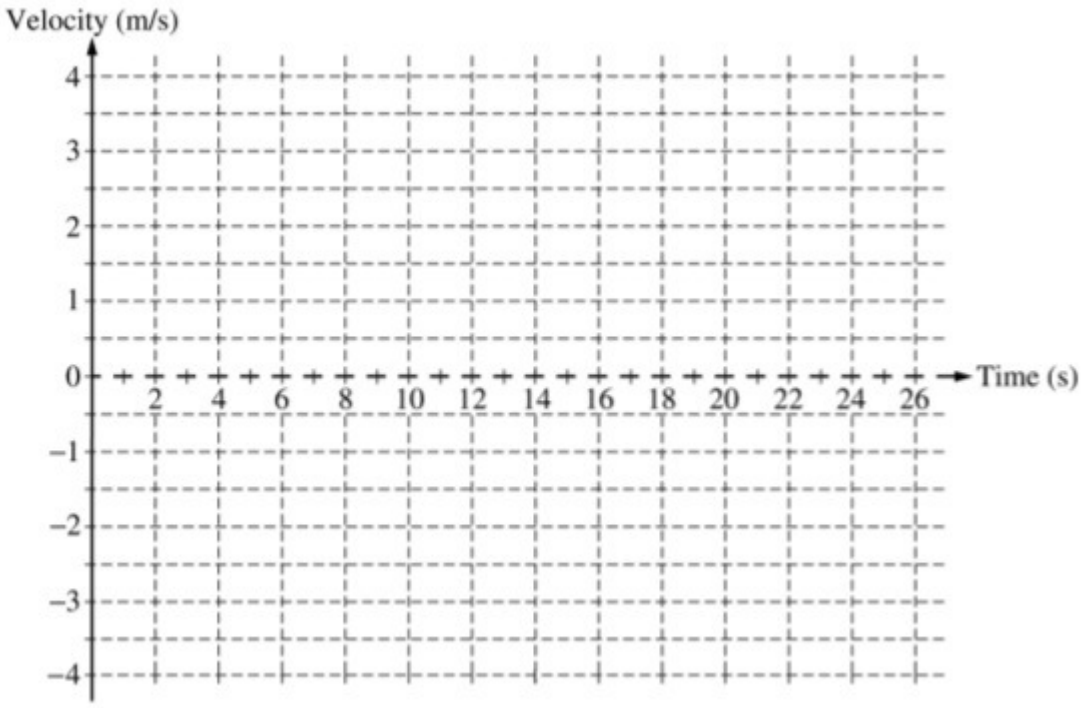
1frq

2.



The vertical position of an elevator as a function of time is shown above.

a. On the grid below, graph the velocity of the elevator as a function of time.



b. i. Calculate the average acceleration for the time period $t = 8$ s to $t = 10$ s.

ii. On the box below that represents the elevator, draw a vector to represent the direction of this average acceleration.



1frq

c. Suppose that there is a passenger of mass 70 kg in the elevator. Calculate the apparent weight of the passenger at time $t = 4$ s.

Part A



The velocities can be found from the slope of the position graph.

1 point is earned: For showing a positive velocity of magnitude 1.5 m/s (i.e., 12 m/8 s) between 0 s and 8 s inclusive

1 point is earned: For showing zero velocity between 10 s and 18 s inclusive

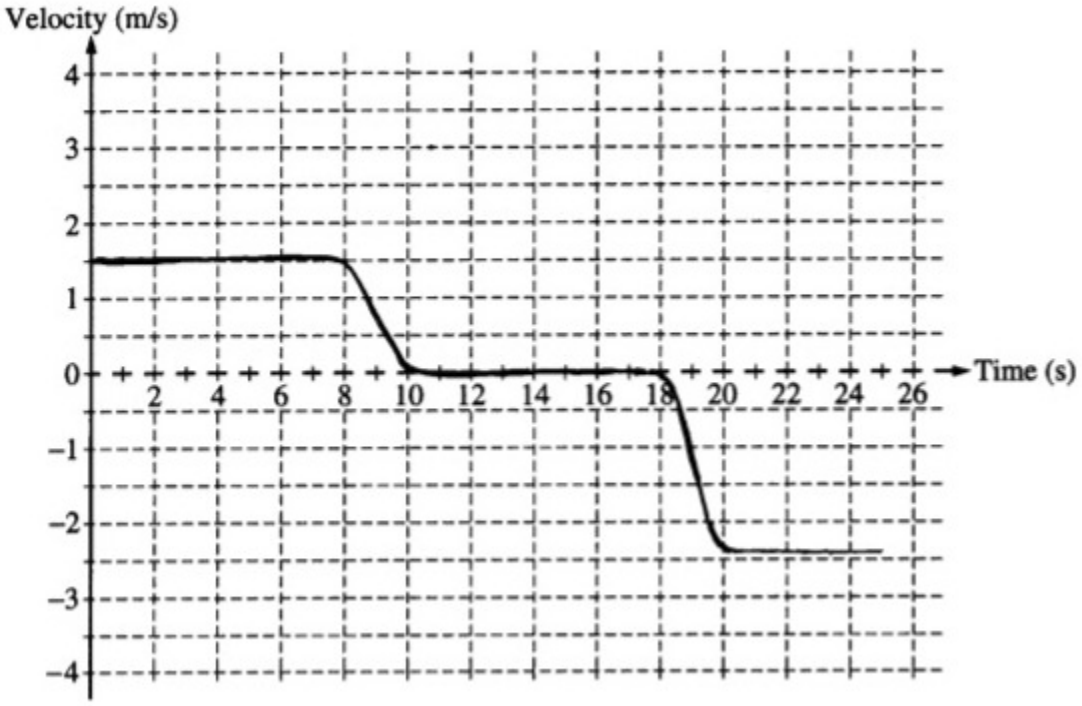
1 point is earned: For showing a negative velocity of magnitude 2.4 m/s (i.e., 12 m/5 s) between 20 s and 25 s inclusive

1 point is earned: For showing two nonvertical transition regions; between $t = 8$ s and 10 s and between $t = 18$ s and 20 s

0	1	2	3	4 ✓

The student response earns four of the following points:

1frq



The velocities can be found from the slope of the position graph.

1 point is earned: For showing a positive velocity of magnitude 1.5 m/s (i.e., 12 m/8 s) between 0 s and 8 s inclusive

1 point is earned: For showing zero velocity between 10 s and 18 s inclusive

1 point is earned: For showing a negative velocity of magnitude 2.4 m/s (i.e., 12 m/5 s) between 20 s and 25 s inclusive

1 point is earned: For showing two nonvertical transition regions; between t = 8 s and 10 s and between t = 18 s and 20 s

Part B

i. **1 point is earned:** For a definition or equation for average acceleration

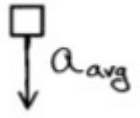
$a_{avg} = \Delta v / \Delta t$ OR $v = v_0 + at$

1 point is earned: For the correct substitution from part (a)

a_{avg} = (0 – 1.5 m/s)/2 s

1 point is earned: For the correct answer including units and sign

$a_{avg} = -0.75$ m/s^2

ii. 

1 point is earned: For a correctly drawn vector, with or without a label

1frq

0	1	2	3	4 ✓

The student response earns four of the following points:

i. **1 point is earned:** For a definition or equation for average acceleration

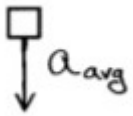
$a_{avg} = \Delta v / \Delta t$ OR $v = v_0 + at$

1 point is earned: For the correct substitution from part (a)

$a_{avg} = (0 – 1.5 \text{ m/s})/2 \text{ s}$

1 point is earned: For the correct answer including units and sign

$a_{avg} = –0.75 \text{ m/s}^2$

ii. 

1 point is earned: For a correctly drawn vector, with or without a label

Part C

The acceleration is zero, so the normal force (apparent weight) is equal to the gravitational force.

1 point is earned: For a correct relationship leading to a calculation of apparent weight

$N = W = mg$ OR $N – W = ma$

$W_{app} = (70 \text{ kg})(9.8 \text{ m/s}^2)$

1 point is earned: For the correct answer with units

$W_{app} = 686$ N (or 700 N using $g = 10 \text{ m/s}^2$)

0	1	2 ✓

The student response earns two of the following points:

The acceleration is zero, so the normal force (apparent weight) is equal to the gravitational force.

1frq

1 point is earned: For a correct relationship leading to a calculation of apparent weight

$N = W = mg$ OR $N - W = ma$

$W_{app} = (70 \text{ kg})(9.8 \text{ m/s}^2)$

1 point is earned: For the correct answer with units

$W_{app} = 686$ N (or 700 N using $g = 10 \text{ m/s}^2$)

1frq

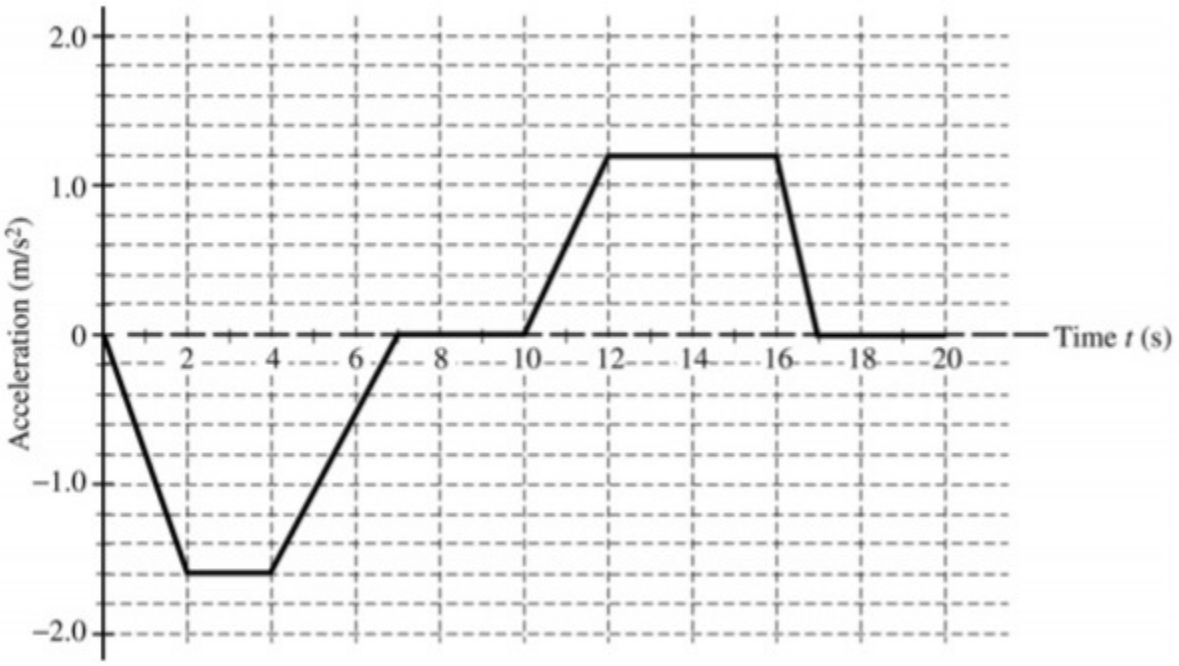
3. A student of mass m stands on a platform scale in an elevator in a tall building. The positive direction for all vector quantities is upward.

a. Draw a free-body diagram showing and labeling all the forces acting on the student, who is represented by the dot below.



b. Derive an expression for the reading on the scale in terms of the acceleration *a* of the elevator, the mass *m* of the student, and fundamental constants.

An inspector provides the student with the following graph showing the acceleration a of the elevator as a function of time *t*.



c. i. During what time interval(s) is the force exerted by the platform scale on the student a maximum value?

ii. Calculate the magnitude of that maximum force for a 45 kg student.

d. During what time interval(s) is the speed of the elevator constant?

Part A

1frq



1 point is earned: For an upward force with a label indicating a normal force or a force from the scale

1 point is earned: For a downward force with a label appropriate for a weight

One point was deducted for each extraneous force until the net score was zero.

0	1	2 ✓

The student response earns two of the following points:



1 point is earned: For an upward force with a label indicating a normal force or a force from the scale

1 point is earned: For a downward force with a label appropriate for a weight

One point was deducted for each extraneous force until the net score was zero.

Part B

The scale reading is the magnitude of the normal force on the student.

1 point is earned: For starting with Newton's second law

$F_{net} = ma$

1 point is earned: For correctly expressing the net force

$N - mg = ma$

1 point is earned: For correctly solving for the normal force

$N = m(g + a)$

1frq

0	1	2	3 ✓

The student response earns three of the following points:

The scale reading is the magnitude of the normal force on the student.

1 point is earned: For starting with Newton's second law

$F_{net} = ma$

1 point is earned: For correctly expressing the net force

$N - mg = ma$

1 point is earned: For correctly solving for the normal force

$N = m(g + a)$

Part C

i. The normal force exerted by the scale is maximum when the acceleration has its maximum positive value.

The force is maximum from 12 to 16 seconds.

1 point is earned: For an interval beginning at 12 seconds

1 point is earned: For an interval ending at 16 seconds

One point was deducted for each interval with no correct endpoints until the net score was zero.

ii. Using the expression for the normal force from part (b)

$N = m(g + a)$

$N = (45 \text{ kg})(9.8 \text{ m/s}^2 + 1.2 \text{ m/s}^2)$

$N = (45 \text{ kg})(11.0 \text{ m/s}^2)$

1 point is earned: For the correct answer with units

$N = 500 \text{ N}$

0	1	2	3 ✓

1frq

The student response earns three of the following points:

i. The normal force exerted by the scale is maximum when the acceleration has its maximum positive value.

The force is maximum from 12 to 16 seconds.

1 point is earned: For an interval beginning at 12 seconds

1 point is earned: For an interval ending at 16 seconds

One point was deducted for each interval with no correct endpoints until the net score was zero.

ii. Using the expression for the normal force from part (b)

$N = m(g + a)$

$N = (45\text{ kg})(9.8\text{ m/s}^2 + 1.2\text{ m/s}^2)$

$N = (45\text{ kg})(11.0\text{ m/s}^2)$

1 point is earned: For the correct answer with units

$N = 500\text{ N}$

Part D

The speed of the elevator is constant when the acceleration is zero.

This occurs from 7 to 10 seconds and from 17 to 20 seconds.

1 point is earned: For each correct interval

One point was deducted for each extraneous interval until the net score was zero.

One point was awarded if the scale was misread for both intervals and the response was given as both 6.5 to 10 and 16.5 to 20.

0	1	2 ✓

The student response earns two of the following points:

The speed of the elevator is constant when the acceleration is zero.

This occurs from 7 to 10 seconds and from 17 to 20 seconds.

1 point is earned: For each correct interval

One point was deducted for each extraneous interval until the net score was zero.

One point was awarded if the scale was misread for both intervals and the response was given as both 6.5 to 10 and 16.5

1frq

to 20.

1frq

4. **PHYSICS 1**

Section II

Time - 1 hour and 30 minutes

5 Questions

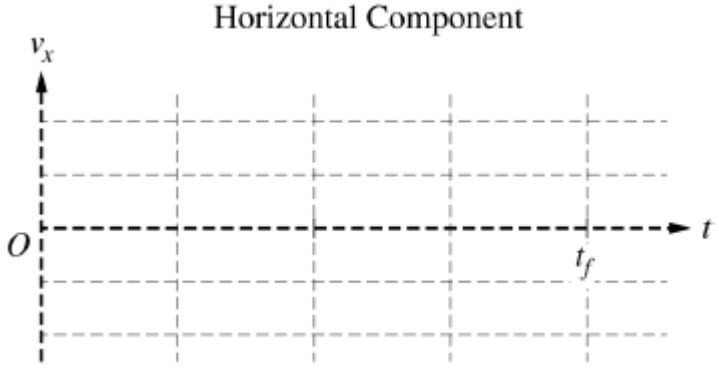
Directions: This question is a short free-response question. Show your work for each part of the question.



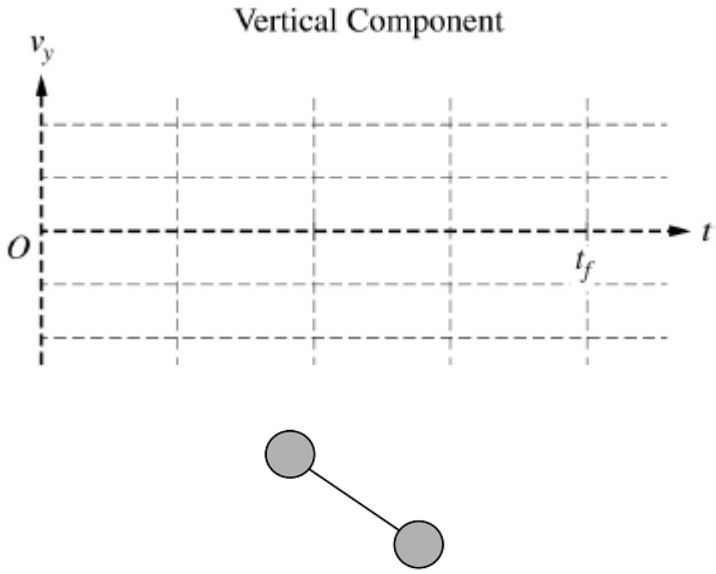
1. (7 points, suggested time 13 minutes)

A toy consists of two identical solid spheres connected by a string with negligible mass. The toy is thrown at an angle above the horizontal (not straight up) such that the string remains taut and both spheres are revolving counterclockwise in a vertical plane around the center of the string, as shown above.

(a) Sketch graphs of the horizontal and vertical components of the velocity of the center of the string as a function of time, from the instant the spheres are released at time $t = 0$ until the instant the system returns to its initial height at time t_f. Take the positive direction to be toward the right for the horizontal component and the positive direction to be upward for the vertical component.

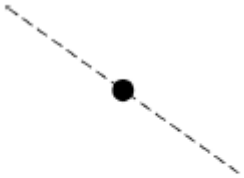


1frq



(b) The figure above shows the toy at the instant the center of the string reaches the top of its trajectory. This is a side view: the sphere on the left is higher than the sphere on the right.

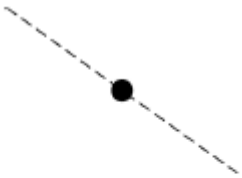
i. On the dot below, which represents the left sphere only, draw and label the forces (not components) exerted on the left sphere at this instant. Represent each force by a distinct arrow starting on, and pointing away from, the dot. The dashed line is drawn at the same angle as the string.



Left Sphere

ii. On the dot below, which represents the whole toy (the spheres-string system), draw and label the forces (not components) that act on the system at this instant. Represent each force by a distinct arrow starting on, and pointing away from, the dot. The dashed line is drawn at the same angle as the string.

1frq



Whole Toy

iii. When the toy was released, the center of the string was moving with an initial speed of $15\ \mathrm{m/s}$ at a 60° angle above the horizontal. Calculate the speed of the center of the string at the instant shown above, when the center of the string reaches the top of its trajectory.

Part A

1 point is earned: Horizontal component: For a non-zero horizontal line

1 point is earned: Vertical component: For a non-vertical line with a negative slope that crosses the horizontal axis near $t_f/2$ and reaches t_f

Note: Any part of the sketched graph beyond t_f is ignored.

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

1 point is earned: Horizontal component: For a non-zero horizontal line

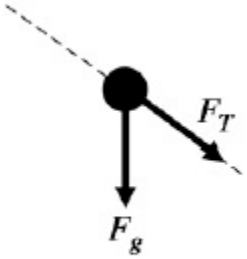
1 point is earned: Vertical component: For a non-vertical line with a negative slope that crosses the horizontal axis near $t_f/2$ and reaches t_f

Note: Any part of the sketched graph beyond t_f is ignored.

Part B

PART i

1frq



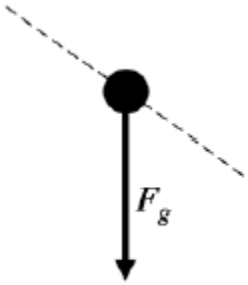
Left Sphere

1 point is earned: For a gravitational force with correct direction and label

1 point is earned: For a tension force with correct direction and label

Note: A maximum of one point may be earned if there are any extraneous forces.

PART ii



Whole Toy

1 point is earned: For a vector pointing downward representing the gravitational force with no extraneous forces

Note: A label is not required, since the use of labels is assessed in part (b)(i).

PART iii

1 point is earned: For attempting to find the horizontal component of velocity

Note: See examples below. This point is earned for the wrong use of trig functions if it is clear it is being used to find the horizontal component, but not awarded if it's not clear which component the trig function is referring to, or for stating that the vertical component is zero.

1 point is earned: For a correct numerical answer for horizontal velocity with correct units

Examples:

0 points: $v \sin\theta = 13\ \mathrm{m/s}$ (No clear indication of an attempt to find the horizontal component)

1 point: $v_x = v \sin\theta = 13\ \mathrm{m/s}$ (An attempt to find the horizontal component is implied by the v_x term)

1 point: $v \cos\theta$ (An attempt to find the horizontal component is implied by the $\cos\theta$ term)

1frq

1 point: $v\cos\theta = 7.5$ (Correct numerical answer with no units)

1 point: $7.5\ \mathrm{m/s}$ (Correct answer with units, but no work shown)

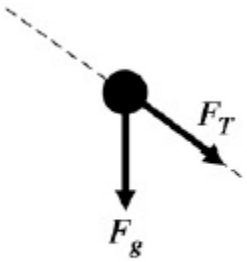
2 points: $15\cos 60 = 7.5\ \mathrm{m/s}$ (Correct answer with units)

2 points: $15\cos 60\ \mathrm{m/s}$ (Correct answer with units)

0	1	2	3	4	5 ✓

Student response earns 5 of the following 5 point(s)

PART i



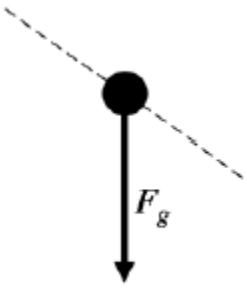
Left Sphere

1 point is earned: For a gravitational force with correct direction and label

1 point is earned: For a tension force with correct direction and label

Note: A maximum of one point may be earned if there are any extraneous forces.

PART ii



Whole Toy

1 point is earned: For a vector pointing downward representing the gravitational force with no extraneous forces

Note: A label is not required, since the use of labels is assessed in part (b)(i).

1frq

PART iii

1 point is earned: For attempting to find the horizontal component of velocity

Note: See examples below. This point is earned for the wrong use of trig functions if it is clear it is being used to find the horizontal component, but not awarded if it's not clear which component the trig function is referring to, or for stating that the vertical component is zero.

1 point is earned: For a correct numerical answer for horizontal velocity with correct units

Examples:

0 points: $v \sin\theta = 13 \text{ m/s}$ (No clear indication of an attempt to find the horizontal component)

1 point: $v_x = v \sin\theta = 13 \text{ m/s}$ (An attempt to find the horizontal component is implied by the v_x term)

1 point: $v \cos\theta$ (An attempt to find the horizontal component is implied by the $\cos\theta$ term)

1 point: $v \cos\theta = 7.5$ (Correct numerical answer with no units)

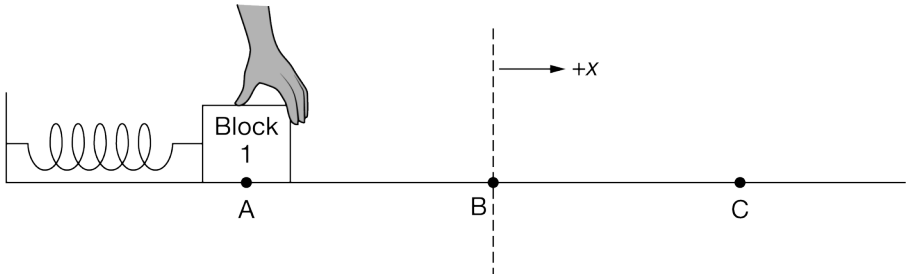
1 point: 7.5 m/s (Correct answer with units, but no work shown)

2 points: $15 \cos 60 = 7.5 \text{ m/s}$ (Correct answer with units)

2 points: $15 \cos 60 \text{ m/s}$ (Correct answer with units)

1frq

5. This question is a short free-response question. Show your work for each part of the question.



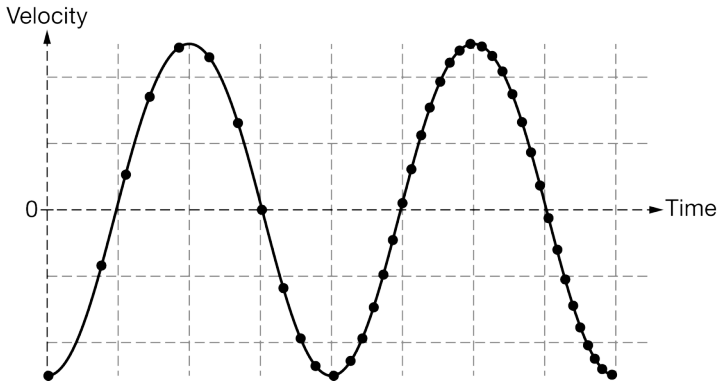
(7 points, suggested time 13 minutes)

Block 1 is at rest on a horizontal surface and is connected to a wall by an ideal spring. Friction between block 1 and the surface is negligible. Block 1 is held at rest at point A, to the left of point B which is the equilibrium position of the spring-block system, as shown in the figure. Block 1 is then released and allowed to oscillate. Some time later, block 1 is momentarily at rest at point C. Consider the positive horizontal direction to be toward the right.

(a) The dot below represents block 1 just after it is released. On the dot, draw an arrow indicating the direction of the net force exerted on the block just after it is released. The arrow should start on, and point away from, the dot.



Net Force on Block 1



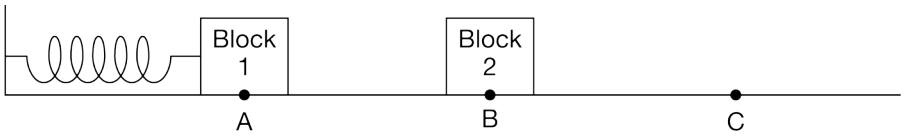
After the block has been oscillating on the table, a motion detector is turned on and the velocity of block 1 as a

1frq

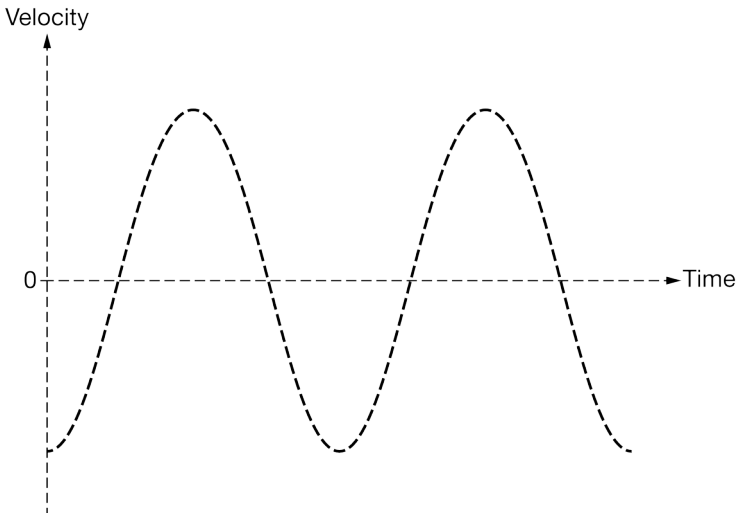
function of time is measured and plotted in a graph. The students correctly draw a best-fit curve that represents the collected data, as shown above.

(b) A student looking at the graph of velocity as a function of time makes the following claim.
"*The motion sensor was turned on when the block was at point A and was moving to the left*."

Does the data in this graph support the student's claim about the location AND the direction of the block's motion when the motion sensor was turned on? For any incorrect claims, if any, state the correct position and/or direction of motion. Justify your answer.

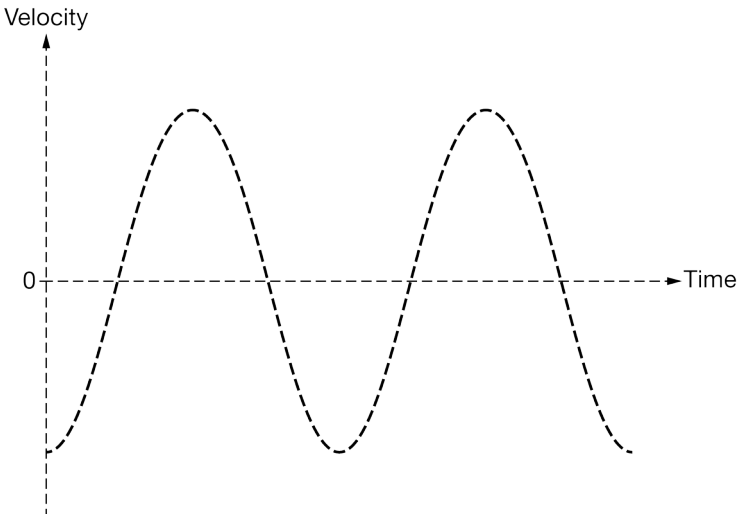


(c) Block 1 is stopped, and again block 1 is held at rest at point A. A second identical block, block 2, is placed at point B. Block 1 is released from rest, and then collides with and sticks to Block 2. After the collision, the two blocks oscillate together on the spring. On the graph below, sketch a curve to indicate the new velocity as a function of time of the two blocks after the collision. For reference, the dashed curve represents the students' best-fit curve of the velocity as a function of time described in part (b).



Note: Do any scratch (practice) work on the graph below. Sketches made below will NOT be graded.

1frq



Part A

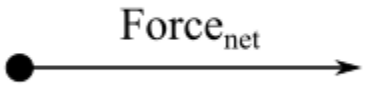
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: The amount of current that flows through each resistor must be indicated to earn the second point of part (a).

0	1 ✓

The response accurately includes **one** of the following criteria:

- ☐ For a net force arrow that is labeled and points horizontally to the right.



Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

1frq

The response accurately includes **three** of the following criteria.

- ☐ For a correct claim that the motion sensor was started when the block was at point B.
- ☐ For a correct justification of why the block is not at point B **OR** For a correct justification of why the block is at point C
- ☐ For a correct justification of why the block is moving to the left.

Example Response:

At time $t = 0$ s the block has a maximum velocity, which means that it is traveling through the equilibrium position, which is position B. Since the velocity is negative, and to the right was defined as positive, the block must be moving to the left through position B.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

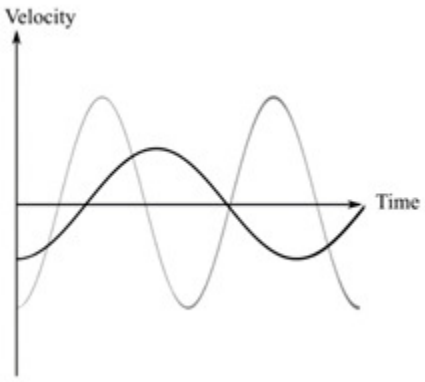
Scoring note: The starting point of this graph is not required to be correct to earn these points. Any phase shift is acceptable for this graph.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

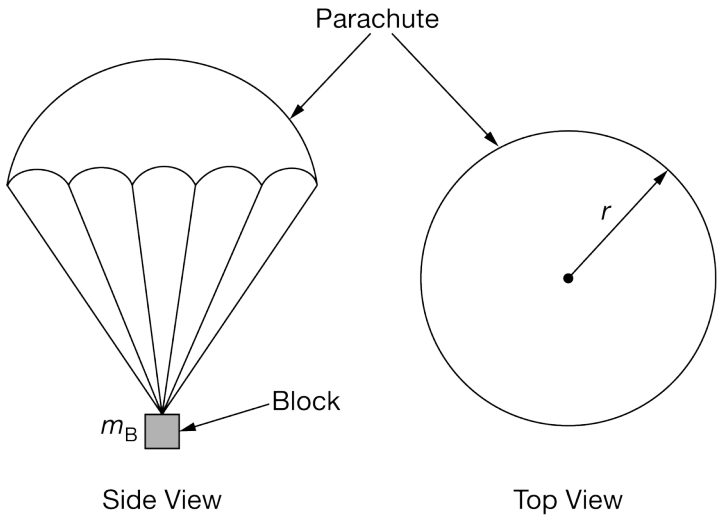
- ☐ For a sine or cosine graph
- ☐ For a graph with a smaller maximum velocity
- ☐ For a sinusoidal graph with a longer period

Example Response:



1frq

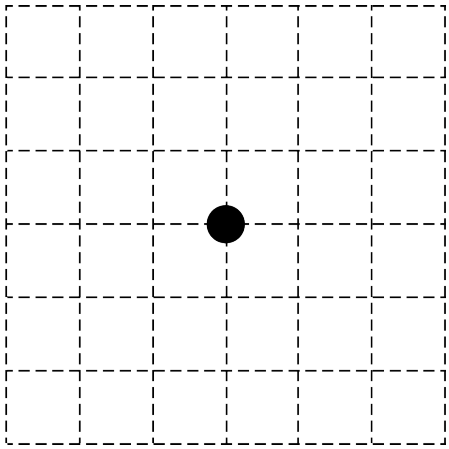
6. This question is a long free-response question. Show your work for each part of the question.



(12 points, suggested time 25 minutes)

A block of mass m_B can be attached to a parachute of negligible mass and radius r, as shown in the two views above. When falling, the parachute experiences an upward force of air resistance. This force increases as the speed of the parachute increases. The falling block and parachute speed up at first but eventually reach a constant speed called terminal speed.

(a) The dot below represents the block-parachute system when it has reached its terminal speed and is falling at this constant speed. On the dot, draw and label the forces (not components) acting on the system. Represent each force by a distinct arrow starting on, and pointing away from, the dot.
Note: Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces.



1frq

A group of students has a collection of parachutes of different radii. They are curious about how the terminal speed depends on the radius of the parachute and find the following equation on the Internet.

$$F_{\text{air}} = Crv^2,$$

where F_{air} is the force exerted by air, r is the parachute's radius, v is the parachute's speed, and C is a constant that has appropriate units.

(b) Determine the units for C.

The units for C are _____ .

(c)

i. Based on the above equation for F_{air}, derive an equation for the system's constant terminal speed v_t in terms of r, C, physical constants, and other relevant quantities that can be measured with equipment usually found in a school physics laboratory, as appropriate. Assume the mass of the parachute is negligible and that the only nonnegligible forces acting on a block-parachute system are the block's weight $m_B g$ and the force of air resistance F_{air}.

ii. Indicate below what quantities could be measured and then graphed to determine whether the equation derived in part (c)(i) correctly relates the parachute's radius and the terminal speed of the block-parachute system.

Vertical axis: _____ Horizontal axis: _____

iii. On the axes below, sketch a graph showing the relationship between the two quantities indicated in part (c)(ii) and based on the equation derived in part (c)(i).

1frq



(d) Design an experimental procedure the students could use to collect the data needed to test whether or not the relationship between the parachute's radius and the terminal speed is described by the graph drawn in part (c)(iii). Assume equipment usually found in a school physics laboratory is available.

In the table below, list the quantities that would be measured in your experiment. Define a symbol to represent each quantity, and also list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

Quantity to be Measured	Symbol for Quantity	Equipment for Measurement

Describe the overall procedure to be used, referring to the table. Provide enough detail so another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table. Include a simple diagram of the setup. As part of your description, be sure to explain how the students would know that they are measuring the constant terminal speed.

1frq

Diagram of Experimental Setup Description of Experimental Procedure

(e) In another experiment, the students collect the following data showing the height of a block above the floor at different times, where the block is dropped from rest at time $t = 0$ with the attached parachute fully open.

Time (s)	Height of Block Above Floor (m)
0.00	1.35
0.10	1.30
0.20	1.18
0.30	1.01
0.40	0.82
0.50	0.63
0.60	0.43
0.70	0.23
0.80	0.03

Use the data shown above to determine the terminal speed of the block.

Part A

1frq

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

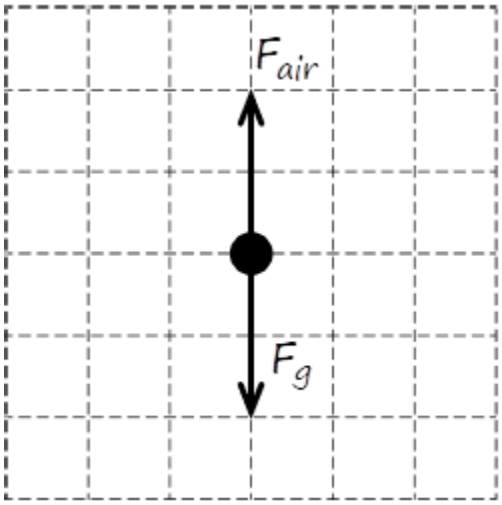
0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a downward force due to gravity, an upward force exerted by the air, and no extraneous forces.
- ☐ 1 point is earned for an upward force and a downward force that are equal in magnitude.

Scoring note: Appropriate labels for the force due to gravity include: F_{G}, F_{g}, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$, $F_{\text{Block,E}}$. The labels G or g are not appropriate labels for force due to gravity. F_{air}, "force of air", "air resistance force", "drag force", or similar labels may be used for the upward force exerted by the air.

Example Response for part (a):



Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for the correct units of kg/m^2 or equivalent.

Part C

1frq

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

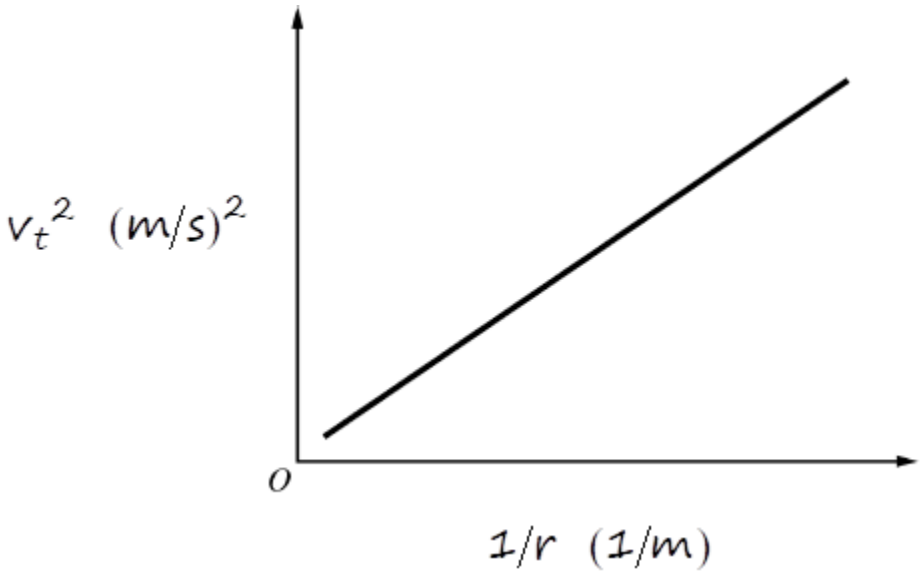
0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for recognizing that at terminal velocity, $F_{\text{air}} = m_B g$
- ☐ 1 point is earned for a correct equation for $v_t = \sqrt{\frac{m_B g}{Cr}}$
- ☐ 1 point is earned for any combination of v_t and r
- ☐ 1 point is earned for a sketch consistent with the variables listed and the equation derived above, where:
  the slope is correctly positive, negative, or zero
  AND
  the concavity is correct (concave-up, concave-down, or a straight line).

Scoring note: Graph does not have to be a straight line to earn this point.

Example Response for 4th point:



Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

1frq

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for describing how to use appropriate equipment to measure the radii of three different parachutes.
- ☐ 1 point is earned for describing how to use appropriate equipment to measure the speed of a falling parachute.
- ☐ 1 point is earned for describing a way to measure v_T that ensures measurements are taken when terminal speed has been reached.
- ☐ 1 point is earned for attempting to reduce uncertainty by averaging several terminal velocity measurements for the same parachute.

Example Response for part (d):

Quantity to be Measured	Symbol of Quantity	Equipment for Measurement
Radius of parachute	*R*	*Ruler*
Terminal velocity of parachute	v_t	*Motion sensor*

Diagram of Experimental Setup	Description of Experimental Procedure
	*Put the motion sensor on the floor pointed upwards. Measure the radius of a parachute with a ruler. Drop the parachute from a set height over the motion sensor. Record the velocity given by the motion sensor when the velocity becomes constant. Repeat the above measurements several times to reduce error and average the results. Repeat this process for parachutes of at least three different radii.*

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

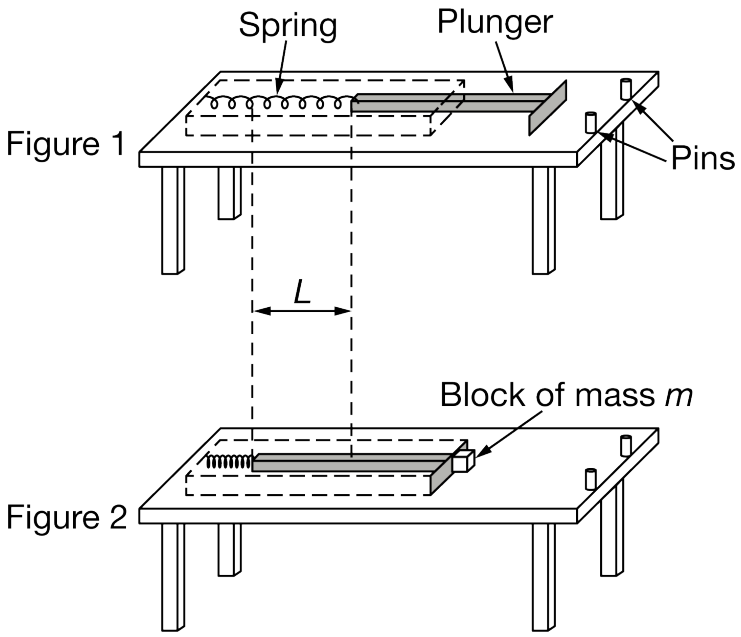
1frq

The response accurately includes:

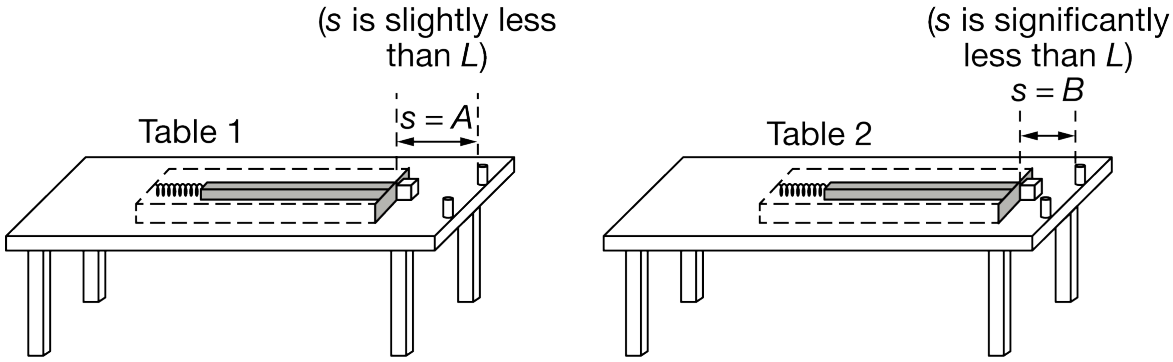
- [ ] 1 point is earned for an answer in the range of $1.9\ \mathrm{m/s}$ to $2.1\ \mathrm{m/s}$

1frq

7. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A spring-loaded launcher can be securely attached to a tabletop. When the spring is at its uncompressed length, the plunger is in the position shown above in Figure 1. A small wood block can be pressed against the end of the plunger, compressing the spring a distance L, as shown in Figure 2. When the block is released, the plunger pushes the block along the tabletop. Two pins are attached to the edge of the table to prevent the plunger from extending beyond the table. When the plunger hits the pins, the block then leaves the table. There is nonnegligible friction between the block and the tabletop.



The launcher can be moved closer to or farther from the edge of the tabletop. Students set up tables 1 and 2, as shown above, with tabletops that are made of the same material. On table 1, the launcher is positioned so that the distance $s = A$ between the launcher and the edge of the tabletop is slightly less than L. On table 2, the launcher is positioned so that the distance $s = B$ between the launcher and the edge of the tabletop is significantly less than L. On both tables, the block is released from rest and loses contact with the plunger at the moment the plunger reaches the pins. Tables 1 and 2 are the same height.

1frq

k is the spring constant of the spring.
m is the mass of the block.
L is the distance the spring is compressed.
μ (mu) is the coefficient of kinetic friction between the block and the tabletop.
s is the distance between the launcher and the edge of the tabletop.

Assume the spring is ideal, the plunger has negligible mass, the block loses contact with the plunger immediately after it reaches the end of the table, and there is non-negligible friction between the tabletops and the blocks.

(a) Without manipulating equations, explain why the block launched from table 1 could land farther from the table than the block launched from table 2 does.

(b) Does the block launched from table 1 spend more, less, or the same amount of time in the air than the block launched from table 2 does? Explain your reasoning.

(c) Consider the Earth-block system. Determine the change, if any, in the total mechanical energy of the system from the instant the block leaves the table to the instant immediately before it reaches the ground.

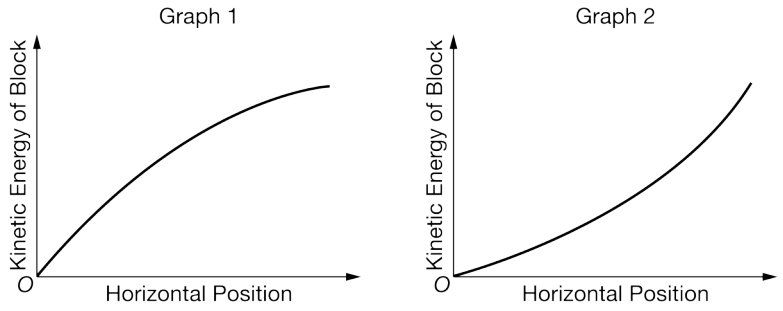
The students correctly derive an equation for the horizontal distance D from the edge of the tabletop that the block lands, in terms of s (the distance between the launcher and the edge of the tabletop):

$$D = t\sqrt{\frac{k}{m}(2Ls - s^2) - 2\mu g s},$$

where t is the time interval between the instant the block leaves the table and the instant immediately before it reaches the ground.

(d) Does this equation for D support your argument from part (a) that the block launched from table 1 could land farther from the table than the block launched from table 2? Briefly explain why or why not.

(e) The students now use the same setups to launch blocks that are identical to each other but more massive than the original blocks. The blocks are made of the same material as the original blocks. Does this change in the mass of the blocks make it more likely or less likely that the block launched from table 1 goes farther than the block launched from table 2 does? Briefly explain your reasoning.



(f) Two students each try to sketch a graph of the kinetic energy of the block as a function of its position from launch to the time the block reaches the edge of the table. Their graphs are shown above. Which graph best represents the kinetic energy of the block? Justify your selection.

1frq

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for applying work/energy or momentum/impulse to explain why launch speed could be greater for Table 1.
- ☐ 1 point is earned for indicating that a larger *s* enables the spring to exert a force on the block for a longer time or distance, or that a larger *s* corresponds to more elastic potential energy being transferred to the block.
- ☐ 1 point is earned for indicating that the block could gain more speed and/or kinetic energy and/or momentum on Table 1.

Example Response:

The distance s is greater for table 1 than table 2 which means that the force that the spring exerts on the block is exerted over a longer distance, allowing the force to do more work on the block. Since the spring force does more work on the block, the change in kinetic energy could be greater, meaning the block could be launched at a greater speed from table 1. The block from table 1 will travel a larger horizontal distance because it was launched with a greater horizontal speed.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating the same time.
- ☐ 1 point is earned for indicating that both blocks will have the same initial vertical speed or zero initial vertical speed.
- ☐ 1 point is earned for indicating that either:
  only the vertical motion determines the fall time,
  OR
  The vertical displacement is the same in both cases
  OR
  The vertical acceleration is the same in both cases.

Example Response:

1frq

The blocks launched from table 1 and table 2 will spend the same amount of time in the air because they have an initial vertical velocity of zero and start from the same height above the floor.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: just “zero” earns this point.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for indicating there is no change in the total mechanical energy.

Example Response:

After the block leaves the table the total mechanical energy does not change.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for addressing the functional dependence of *D* on *s*.
- ☐ 1 point is earned for correctly indicating how a specific term in the expression changes with increasing *s*.
- ☐ 1 point is earned for indicating that an increase in s could result in an increase in *D*, or in the right-hand-side of the equation
  Note: If a student answers “no” the third point is awarded for indicating that an increase in s results in a decrease in *D*, or in the right-hand side of the equation.

Example Response:

Yes. The closer s is to L, the greater the $(2Ls - L^\wedge 2)$ *part of the equation would be. If that term of the equation is greater, D will be greater.*

OR

Yes. Because s is always less than L, Ls *is greater than* $s^\wedge 2$*. So as s increases, the equation increases.*

Part E

1frq

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating "Less likely", with an attempt at an explanation.
- ☐ 1 point is earned for noting that the larger mass increases the force of friction.
- ☐ 1 point is earned for indicating that a larger frictional force will dissipate more energy (or result in a smaller net force on the block/acceleration of the block), which results in a lower launch speed, and therefore smaller distance.
  Note: Part (e) can be answered quantitatively. See alternate solution below.

It is less likely that the block in 1 will go farther. The larger mass means that the force of friction is larger, so the net force on the block is less, resulting in a lower launch speed. On table 1, the frictional force is exerted over a longer distance, and so has a bigger effect on reducing the speed compared to table 2.

OR

Less likely. The first term is multiplied by k/m; so this term decreases with bigger m. The second term doesn't change with m. So, the difference between the two terms is smaller, reducing the effect of increasing s on the distance traveled.

Part E – Alternate Solution

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

*The response accurately includes **all** of the following criteria.*

- ☐ *1 point is earned for indicating "less likely", with an attempt at an explanation.*
- ☐ *1 point is earned for addressing the functional dependence of D on m in the given equation.*
- ☐ *1 point is earned for explaining how the right side of the equation becomes smaller with reference to both terms in the substitution, therefore, D becomes smaller.*

Example Response:

Less likely. The first term is multiplied by k/m; so this term decreases with bigger m. The second term doesn't change with m. So, the difference between the two terms is smaller, reducing the effect of increasing s on the distance traveled."

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

1frq

0	1	2 ✓

The response accurately includes **both** of the following criteria.

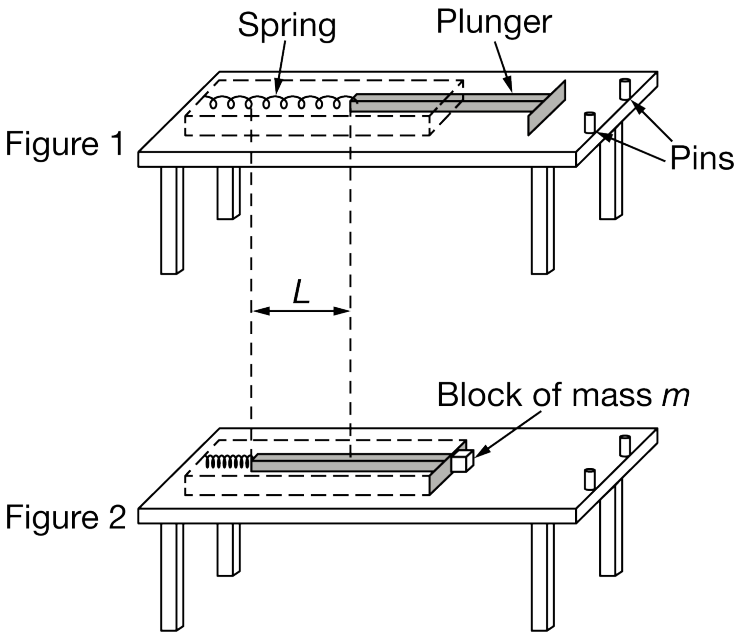
- ☐ 1 point is earned for indicating that Graph 1 is the correct graph, with an attempt at an explanation.
- ☐ 1 point is earned for correctly addressing the decreasing slope of the curve for large values of horizontal position.

Example Response:

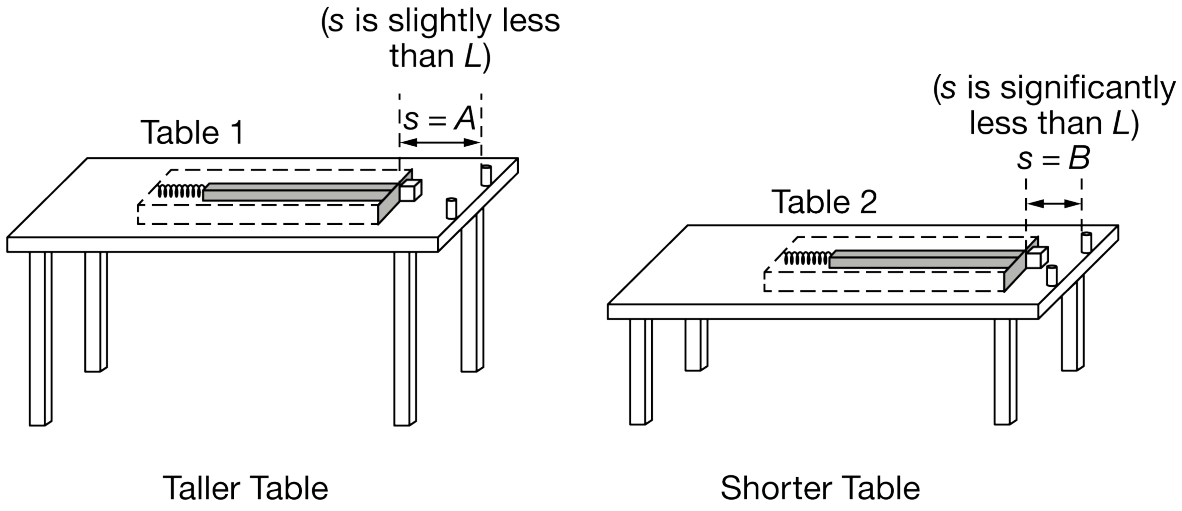
Graph 1 is the correct graph. The slope decreases with increasing position. This makes sense because, as the spring expands, it exerts a smaller force on the block. The smaller force means that the kinetic energy increases by a smaller amount for a given change in position.

1frq

8. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A spring-loaded launcher can be securely attached to a tabletop. When the spring is at its uncompressed length, the plunger is in the position shown above in Figure 1. A small wood block can be pressed against the end of the plunger, compressing the spring a distance L as shown in Figure 2. When the block is released, the plunger pushes the block along the tabletop. Two pins are attached to the edge of the table to prevent the plunger from extending beyond the table. When the plunger hits the pins, the block then leaves the table. There is nonnegligible friction between the block and the tabletop.



The launcher can be moved closer to or farther from the edge of the tabletop. Students set up tables 1 and 2, as shown above, with tabletops that are made of the same material. On table 1, which is taller than table 2, the

1frq

launcher is positioned so that the distance $s = A$ between the launcher and the edge of the tabletop is slightly less than L. On table 2, the launcher is positioned so that the distance $s = B$ between the launcher and the edge of the tabletop is significantly less than L. On both tables, the block is released from rest and loses contact with the plunger at the moment the plunger reaches the pins.

k is the spring constant of the spring.
m is the mass of the block.
L is the distance the spring is compressed.
μ (mu) is the coefficient of kinetic friction between the block and the tabletop.
s is the distance between the launcher and the edge of the tabletop.

Assume the spring is ideal, the plunger has negligible mass, the block loses contact with the plunger immediately after it reaches the end of the table, and there is non-negligible friction between the tabletops and the blocks.

(a) Without manipulating equations, explain why the block launched from table 2 could have a greater speed when it leaves the table than the block launched from table 1 does.

(b) Does the block launched from table 2 spend more, less, or the same amount of time in the air than the block launched from table 1 does? Explain your reasoning.

(c) Consider the system consisting of only the block. Determine the change, if any, in the total mechanical energy of the system from the instant the block leaves the table to the instant immediately before it reaches the ground.

The students correctly derive an equation for the speed v of the block when it is at the edge of the tabletop, in terms of s (the distance between the launcher and the edge of the tabletop):

$v = \sqrt{\frac{k}{m}(2Ls - s^2) - 2\mu g s}$.

(d) Does this equation for v support your argument from part (a) that the block launched from table 2 could have a larger speed when it leaves the table than the block on table 1 does? Briefly explain why or why not.

(e) The students now repeat the scenario with identical setups except that the coefficient of kinetic friction between the blocks and the table is larger. Does this change in the coefficient of friction make it more likely or less likely that the block launched from table 1 has a greater speed when it leaves the table than the block on table 2 does? Briefly explain your reasoning.

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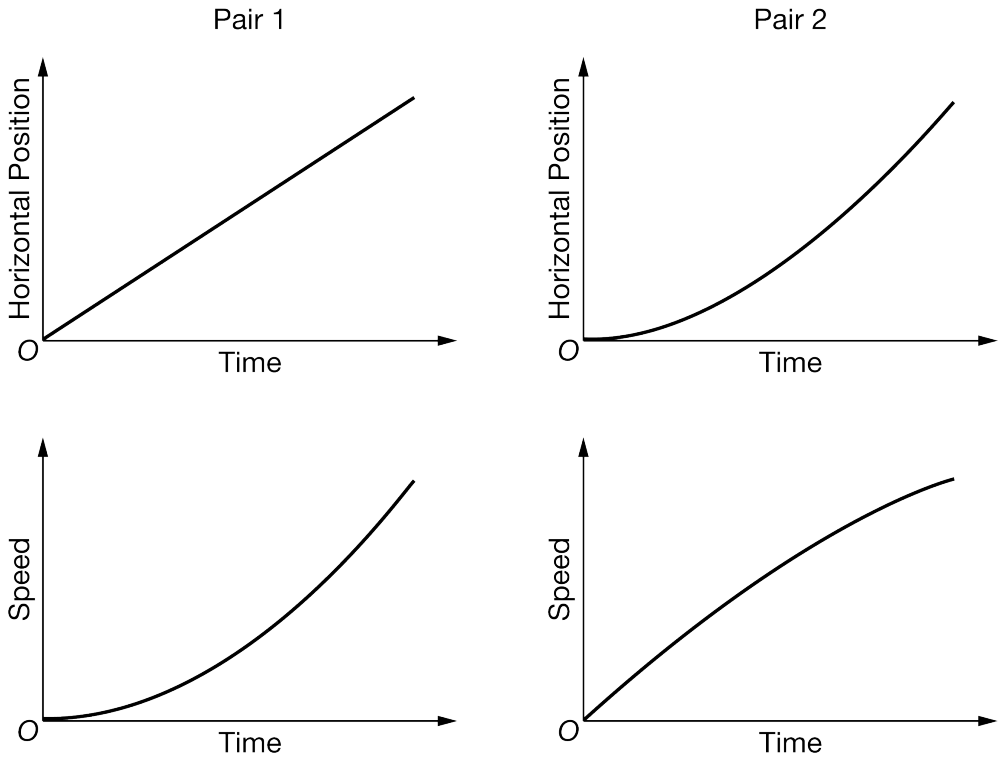


(f) Two students each try to sketch a graph of the horizontal position and speed of the block as a function of time while the block is being pushed by the plunger. Their graphs are shown above. In which pair of graphs are the two graphs consistent with each other? Justify your selection.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for attempting to apply work/energy or momentum/impulse to explain why launch speed could be greater for Table 2.
- ☐ 1 point is earned for indicating that a larger *s* for the block on Table 1 enables friction to exert a force on the block for a longer time or distance, or that a larger *s* corresponds to more energy being dissipated.
- ☐ 1 point is earned for correctly applying work/energy or momentum/impulse to explain why launch speed could be greater for Table 2.

Example Response:

The distance s is greater for table 1 than table 2 which means that the force of friction from the table can be exerted for a

1frq

longer distance, allowing the friction force to dissipate more energy. Since the friction force dissipates more energy, the reduction in kinetic energy due to friction is greater, meaning the block will be launched at a smaller speed from table 1.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the block launched from Table 2 will spend less time in the air.
- ☐ 1 point is earned for indicating that both blocks will have the same initial vertical speed or zero initial vertical speed.
- ☐ 1 point is earned for indicating that either:
  only the vertical motion determines the fall time,
  OR
  the vertical displacement is different for the two cases.

Example Response:

The block launched from table 2 will spend less time in the air because both blocks have an initial vertical velocity of zero, but the block on table 2 starts from a lower height above the floor.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for indicating that the change is mgh.

Example Response:

mgh

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

1frq

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for addressing the functional dependence of *v* on *s*.
- ☐ 1 point is earned for correctly indicating how a specific term in the expression changes with increasing *s*.
- ☐ 1 point is earned for indicating that an increase in *s* could result in a decrease in *v,* or in the right-hand-side of the equation.
  Scoring note: If a student answers "no" the third point is awarded for indicating that an increase in *s* results in an increase in *v*, or in the right-hand side of the equation.

Example Response:

Yes, because the equation indicates that v could increase as s decreases like my argument in part (a). In the equation, the expression under the radical could increase when s decreases because there are two "s" terms with negative signs. If the number under the radical increases, v will increase.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating "Less likely", with an attempt at an explanation.
- ☐ 1 point is earned for noting that the larger coefficient of friction results in a larger frictional force.
- ☐ 1 point is earned for indicating that a larger frictional force will dissipate more energy (or result in a smaller net force on the block/acceleration of the block), which results in a lower launch speed.

Example Response:

It is less likely that the block in 1 will have a greater launch speed. The larger coefficient of friction means that the force of friction is larger, so the net force on the block is less, resulting in a lower launch speed. On table 1, the frictional force is exerted over a longer distance, and so has a bigger effect on reducing the speed compared to table 2.

Part E-Alternate Solution

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

1frq

0	1	2	3 ✓

*The response accurately includes **all** of the following criteria.*

- ☐ *1 point is earned for indicating "less likely" with an attempt at an explanation.*
- ☐ *1 point is earned for addressing the functional dependence of v on the coefficient of friction (μ) in the given "correct" equation.*
- ☐ *1 point is earned for explaining how the right side of the equation becomes smaller with reference to both terms in the substitution, therefore v becomes smaller.*

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating pair 2, with an attempt at an explanation.
- ☐ 1 point is earned for either correctly ruling out the inconsistent position and time graphs in pair 1,
  OR
  For indicating that the speed graph in pair 2 is consistent with the position graph (the speed graph corresponds to the slope of the position graph).

Example Response:

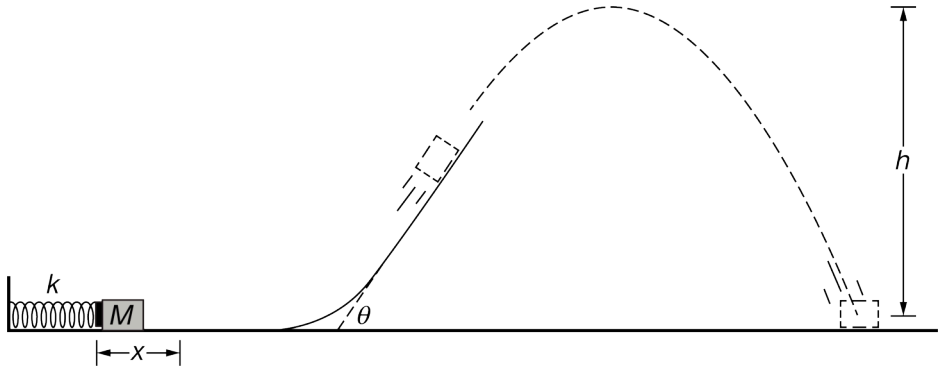
Pair 2 has the two consistent graphs. The slope of the position graph is initially zero, and then increases, which is the behavior of the speed graph.

OR

Pair 2 has the two consistent graphs. In pair 1, the position graph is a straight line, so the speed graph should be constant.

1frq

9. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A box is pressed against a horizontal spring, compressing the spring from its relaxed length. The box is then released, and the spring launches the box horizontally along a track that ends in a ramp, as shown above. The box has enough speed to leave the ramp, and the box reaches a maximum vertical height above the floor. Assume there is negligible friction between the box and the track and air resistance is negligible.

k is the spring constant of the spring.
x is the distance the spring is compressed.
M is the mass of the box.
θ (theta) is the angle of the ramp from the horizontal.
h is the maximum height reached by the box.

The scenario is repeated using a stiffer spring with a spring constant that has a greater value for k than that of the original spring. The stiffer spring is compressed the same distance x, and the same box is used.

(a) Indicate how using a stiffer spring would affect the maximum height reached by the box, and explain why without mathematically deriving a relation for h.

(b) Students derive an equation for the maximum height, $h = \frac{(\sin^2\theta)kx^2}{2Mg}$, which may or may not be correct. Is this equation consistent with your claim in part (a)? Explain why or why not.

Two students are discussing the motion of the box when it is at its maximum height and make the following statements:

Student 1: The speed of the block at its maximum height is zero because the kinetic energy is a minimum at the top of the trajectory.

Student 2: Only the vertical component of the velocity is zero at the maximum height, because the vertical component of the acceleration is zero at the top of the trajectory.

(c) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

1frq

(d) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

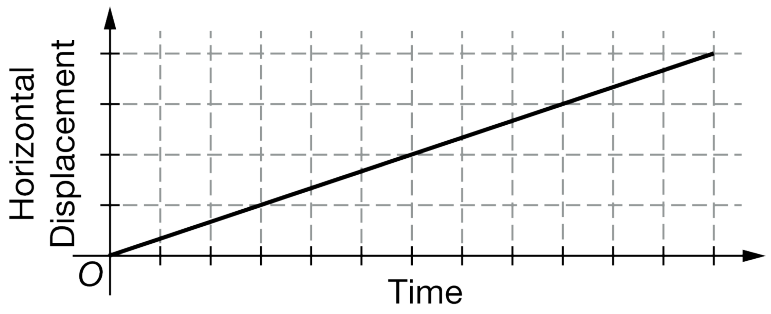
(e) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(g) Consider the system consisting of the box and the spring, but not Earth. How does the energy of the system when the spring is fully compressed compare to the energy of the system at the moment immediately before the box hits the ground? Justify your answer.

(h) If the friction between the track and the box were not negligible, state one way in which the setup could be changed such that the maximum height of the box was the same as in the original setup.

The graph below shows the horizontal component of the displacement of the box as a function of time. The graph represents the entire time the box is in the air after the box is launched from the ramp at an angle θ to the horizontal.



(i) How would the slope of the graph be different, if at all, if the ramp angle were larger than θ, but less than 90 degrees, and the height of the end of the ramp were the same? Justify your response.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that with a stiffer spring the spring constant would increase (or the average force would increase), so the speed of the box as it leaves the spring will be larger.
- ☐ 1 point is earned for a statement that because the speed of the box as it leaves the spring will be greater, and everything else remains the same, the maximum height of the block will increase.

1frq

Example Response:

A stiffer spring means that the spring constant is larger, which means that there is more initial energy stored in the spring (or that the average net force on the box is greater), which means that the launch speed is greater, so with everything else the same the box will reach a larger maximum height.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating "yes", with an attempt at a justification using functional dependence.
- ☐ 1 point is earned for indicating that, since k is in the numerator if k increases, h will increase.

Example Response:

Yes, it is consistent with my reasoning – since k is in the numerator, if k increases, h will also increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the kinetic energy is a minimum at the top of the trajectory.

OR

The second statement

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

1frq

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 1 is incorrect in saying that the speed of the block is zero at the max height

OR

The first statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that only the vertical component of the speed is zero at the max height

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

1frq

- [ ] 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 2 is incorrect in saying that the vertical component of the acceleration is zero at the top of the trajectory.

OR

The second statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- [ ] 1 point is earned for indicating that the energy of the box/spring system remains the same.
- [ ] 1 point is earned for indicating that there is mechanical energy in the spring
  OR
  For indicating that the box returns to the same height.
- [ ] 1 point is earned for indicating that just before the box lands all the energy is kinetic energy, and none is lost
  OR
  For indicating that the net work done by gravity is zero.

Example Response:

The energy of the spring-box system initially is spring potential energy, and just before the box lands, all the energy is kinetic energy, and because there is no net work done by outside forces, the energy of the spring box system is the same.

OR

The energy of the box/spring system remains the same because the box returns to the same height, and so the net work done by gravity is zero.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Valid changes also include making the ramp steeper, trading out the spring for a stiffer (larger k) spring, or using a box of less mass.

1frq

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating a valid change that could be made so that the maximum height would be the same.

Example Response:

Nonnegligible friction would cause mechanical energy to be dissipated. Therefore, the compression distance could be increased so that the spring transfers more energy to the box, offsetting the loss in energy from friction.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

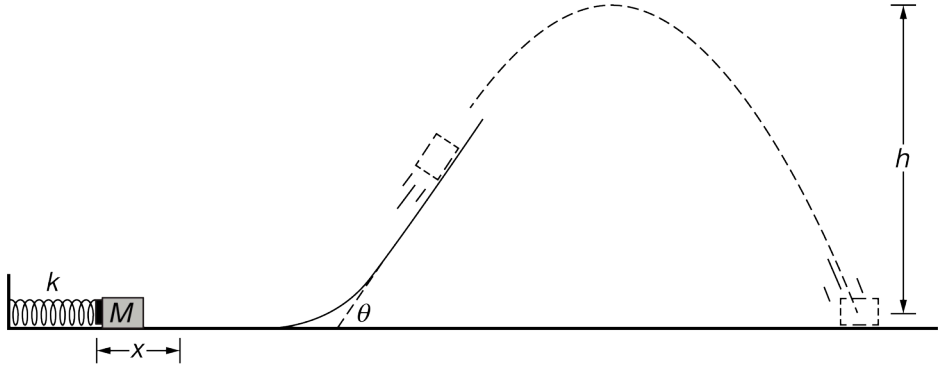
- ☐ 1 point is earned for indicating that the slope of the line would be smaller.
- ☐ 1 point is earned for indicating that with a larger angle, the horizontal component of the initial velocity is smaller.
- ☐ 1 point is earned for indicating that the slope of the horizontal position as a function of time graph is equal to the horizontal velocity of the box.

Example Response:

Since the graph is of horizontal position as a function of time, the slope of the line represents the horizontal velocity as a function of time. If the box were launched at a larger angle, the horizontal component of the velocity would be smaller and so this line would start at zero still but would have a smaller, but still positive, slope.

1frq

10. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A box is pressed against a horizontal spring, compressing the spring from its relaxed length. The box is then released and the spring launches the box horizontally along a track that ends in a ramp, as shown above. The box has enough speed to leave the ramp, and the box reaches a maximum vertical height above the floor. Assume there is negligible friction between the box and the track and air resistance is negligible.

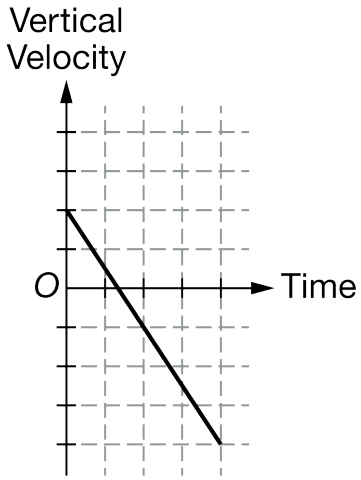
k is the spring constant of the spring.
x is the distance the spring is compressed.
M is the mass of the box.
θ (theta) is the angle of the ramp from the horizontal.
h is the maximum height reached by the box.

The scenario is repeated with the box compressing the spring a distance that is larger than the original distance x.

(a) Indicate how h in this second scenario compares to h in the original scenario, and explain why without mathematically deriving a relation for h.

(b) Students derive an equation for the maximum height h in the original scenario, $h = \frac{(\sin^2\theta)kx^2}{2Mg}$, which may or may not be correct. Is this equation consistent with your claim in part (a)? Explain why or why not.

1frq



Two students, student 1 and student 2, are discussing the graph shown above, which shows the vertical component of the velocity of the box from when it leaves the ramp to immediately before it hits the floor. The students make the following statements.

Student 1: This graph shows that the vertical acceleration of the box is constant, and that the box's speed decreases for the entire time shown.

Student 2: The area between the graph and the time axis is the box's vertical displacement, but the graph is incorrect because the positive displacement should be equal to the negative displacement.

(c) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(g) Consider the system consisting of only the box, but not the spring or Earth. How does the energy of the system when the spring is fully compressed compare to the energy of the system at the moment immediately before the box hits the ground? Justify your answer.

The angle of the ramp is changed such that it is larger than θ but less than 90 degrees, while the height of the end of the ramp remains the same.

(h) How would the maximum height of the box compare to the original scenario in which the ramp angle was equal to θ ? Justify your answer.

(i) Would the slope of the graph of vertical velocity as a function of time shown above part (c) be different for this scenario in which the box is launched from the larger angle? Justify your response.

Part A

1frq

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that with a larger compression distance the energy stored in the spring is greater (or the average force would increase or the spring acts on the box for a longer time/distance), so the speed of the box as it leaves the spring will be larger.
- ☐ 1 point is earned for a statement that because the speed of the box as it leaves the spring will be greater, and everything else remains the same, the maximum height of the block will increase.

Example Response:

A larger compression distance means that more energy is stored initially in the spring, which means that the launch speed is greater, so with everything else the same the box will reach a larger maximum height.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for the answer yes, and an attempt at a justification using functional dependence.
- ☐ 1 point is earned for indicating that, since x is in the numerator if x increases, h will increase.

Example Response:

Yes, it is consistent with my reasoning – since x is in the numerator, if x increases, h will also increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

1frq

☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the vertical acceleration of the box is constant

OR

The first statement

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 1 is incorrect in saying that the box's speed decreases for the entire time shown.

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that the area between the graph and the time axis is the box's vertical displacement

OR

1frq

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 2 is incorrect in saying that the positive displacement should be equal to the negative displacement.

OR

The second statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the energy of the box system increases
  OR
  For indicating that energy is greater from the spring doing work on box.
- ☐ 1 point is earned for indicating that there is no initial mechanical energy in the box system
  OR
  For indicating that the box returns to the same height.
- ☐ 1 point is earned for indicating that just before the box lands all the energy is kinetic energy
  OR
  For indicating that the net work done by gravity is zero.

Example Response:

The energy of the box system initially is zero because the spring is not in the system. Just before the box lands, all the mechanical energy in the system is kinetic energy, and because there is no net work done by gravity because the box returns to the same height, the energy of the box system increases.

1frq

OR

The energy of the system increases, because the system does not include the spring, and the spring does work on the box. The box eventually returns to the same height, so the net work done on the system from gravity is zero.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating that increasing the angle would increase the vertical component of the velocity which would increase the maximum height.

Example Response:

Increasing the angle of the ramp would increase the vertical component of the velocity so the box would be able to go higher.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

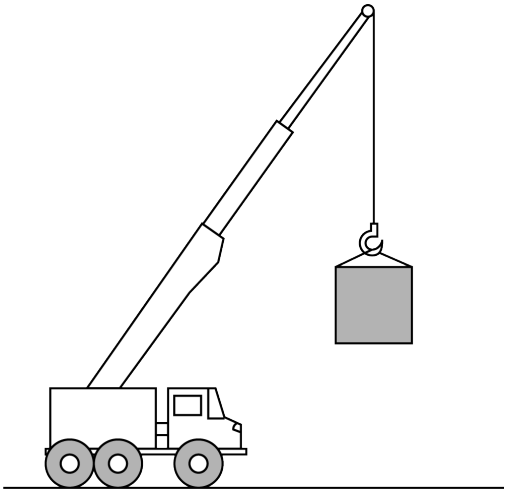
- ☐ 1 point is earned for indicating that the slope of the line would be the same.
- ☐ 1 point is earned for indicating that the slope of a velocity as a function of time graph is equal to the (vertical) acceleration.
- ☐ 1 point is earned for indicating that the vertical acceleration is constant (g).

Example Response:

Since the slope of the vertical velocity vs. time is equal to the acceleration, and the acceleration is constant throughout this problem (g), the slope of the line would remain the same even if the angle of the ramp were increased.

1frq

11. This question is a long free-response question. Show your work for each part of the question.

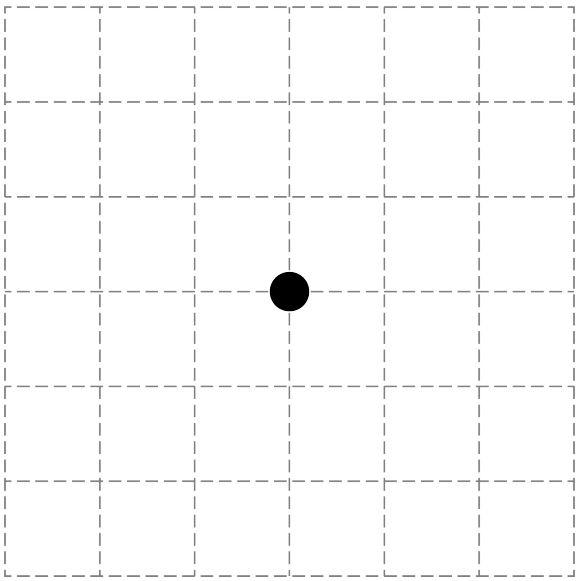


(12 points, suggested time 25 minutes)

A crane and box are both initially at rest. At time $t = 0\ \text{s}$, the crane begins to drive forward at a constant speed of $0.5\ \frac{\text{m}}{\text{s}}$, while also lifting the box with an upward acceleration of $1\ \frac{\text{m}}{\text{s}^2}$. The box does not swing while being lifted by the crane.

(a) On the dot below, which represents the box, draw and label the forces (not components) that are exerted on the box during the scenario described above. Represent each force by a distinct arrow starting on, and pointing away from, the dot.

Note: Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces.



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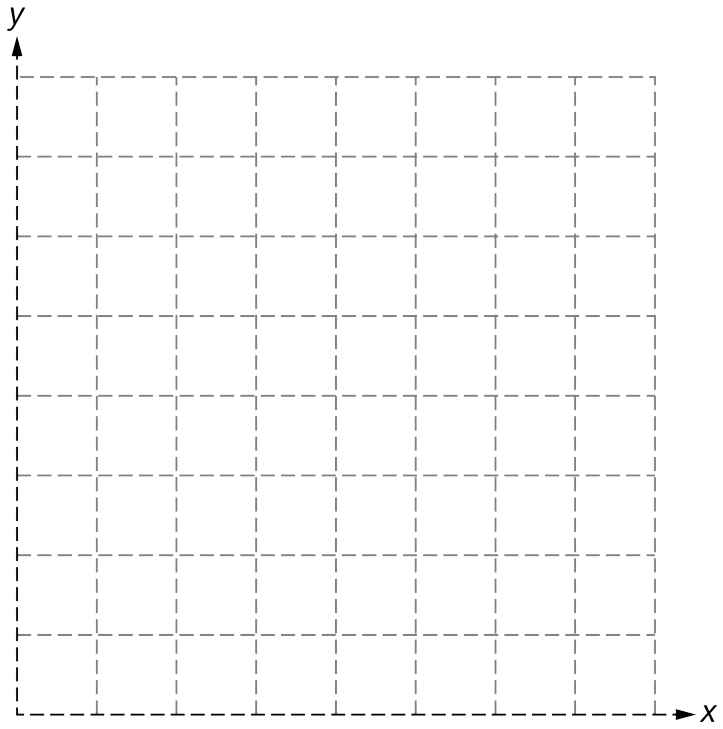
(b) On the grid below, sketch the shape of the path taken by the box as it is lifted by the crane as viewed by a stationary observer.



Justify the shape of the path you drew above.

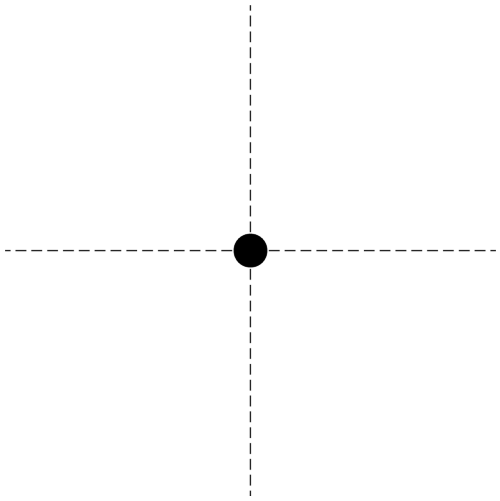
(c) Assume that, at $t = 0$ s, the horizontal position of the box is $x = 0$ and it has a vertical position of $y = 0$. Derive an equation that describes the vertical position y of the box as a function of the horizontal position x of the box.

(d) Does your equation from part (c) agree with your sketch in part (b)? Justify your response.

At time $t = t_1$, the crane begins to accelerate forward, reaching a new constant velocity at time $t = t_2$. The box accelerates upward during the entire time interval $t_1 < t < t_2$.

(e) The dot below represents the box during the time interval $t_1 < t < t_2$. On the dot, draw an arrow indicating the direction of the net force exerted on the block during this interval. The arrow should start on, and point away from, the dot.

1frq



Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Scoring notes:

· G or g are not acceptable labels for the force of gravity

· A maximum of two points can be earned if there are any extraneous forces.

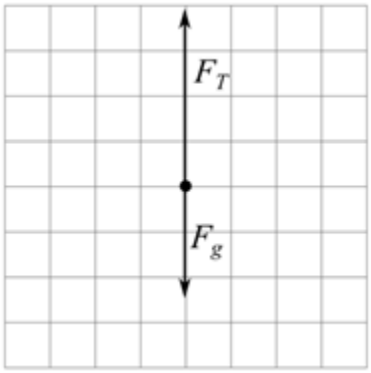
0	1	2	3 ✓

The response accurately includes **one** of the following criteria:

- ☐ For an arrow directed upwards labeled Tension (or F_T)
- ☐ For an arrow directed downwards labeled Force of Gravity (or mg, or F_g).
- ☐ For an arrow labeled tension that is larger than the arrow labeled Force of Gravity.

Example Response:

1frq



Part B

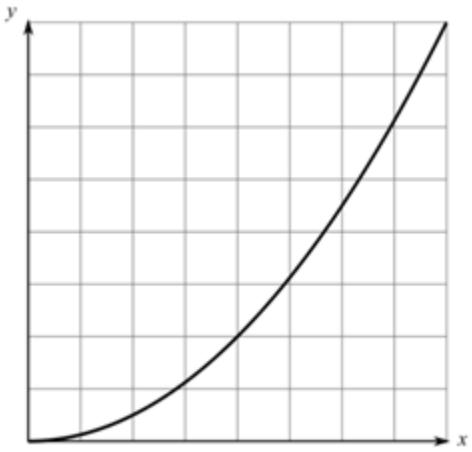
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

- ☐ For a sketch showing a non-linear path
- ☐ For a sketch that is concave up
- ☐ For a justification that addresses that the box is moving at a constant speed in the horizontal direction and accelerating in the vertical direction which leads to a concave up parabola.

Example Response:



Since the box is moving at a constant speed in the horizontal direction and accelerating in the vertical direction, the path that the box takes should look like a concave up parabola

Part C

1frq

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

- ☐ For a kinematic equation correctly relating the horizontal position of the box to the horizontal component of the speed of the box and the time
  $v_x = \frac{x}{t}$
  $0.5\ \text{m/s} = \frac{x}{t}$
  $t = \frac{x}{0.5\ \text{m/s}}$
- ☐ For a kinematic equation correctly relating the vertical position of the box as a function of the vertical acceleration of the box and the time
  $y = \frac{1}{2}a_y t^2$
  $y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)t^2$
- ☐ For correctly combining the vertical and horizontal motions of the box into a single equation.
  $y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)\left(\frac{x}{0.5\ \text{m/s}}\right)^2$
  $y = \left(2\ \text{m}^{-1}\right)x^2$

Example Response:

$v_x = \frac{x}{t}$
$0.5\ \text{m/s} = \frac{x}{t}$
$t = \frac{x}{0.5\ \text{m/s}}$

$y = \frac{1}{2}a_y t^2$
$y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)t^2$

$y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)\left(\frac{x}{0.5\ \text{m/s}}\right)^2$
$y = \left(2\ \text{m}^{-1}\right)x^2$

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

1frq

The response accurately includes **both** of the following criteria.

- ☐ For an attempt to use functional dependence to describe how the equation and the sketch in part B relate
- ☐ For correctly stating that the vertical position is proportional to the horizontal position squared and so the shape of the path traveled by the box should be a parabola.

Example Response:

Yes. My equation in part (c) describes a parabola with a minimum at the origin, which matches my sketch in part (b).

Part E

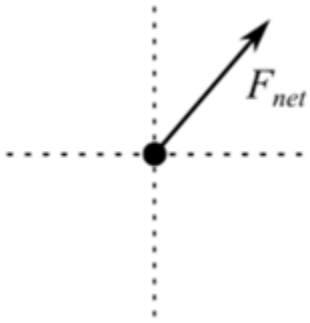
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes the following criteria:

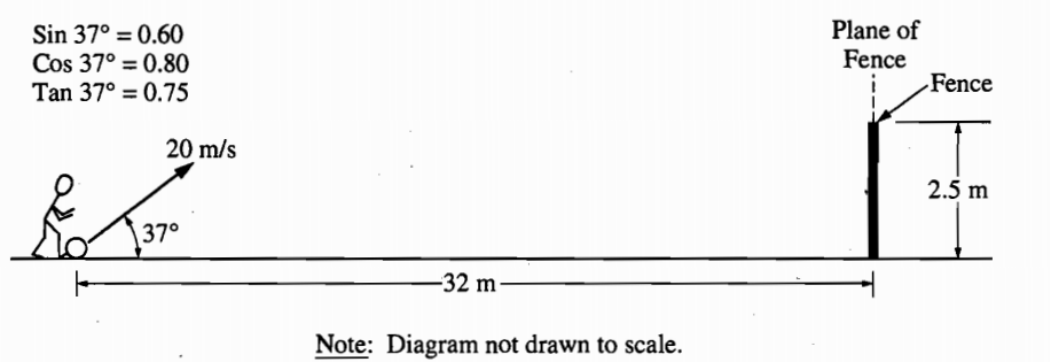
- ☐ One point for an arrow labeled ΣF or net force, pointing upwards and to the right.

Example Response:



1frq

12.



A ball of mass 0.5 kilogram, initially at rest, is kicked directly toward a fence from a point 32 meters away, as shown above. The velocity of the ball as it leaves the kicker's foot is 20 meters per second at a angle of 37° above the horizontal. The top of the fence is 2.5 meters high. The kicker's foot is in contact with the ball for 0.05 second. The ball hits nothing while in flight and air resistance is negligible.

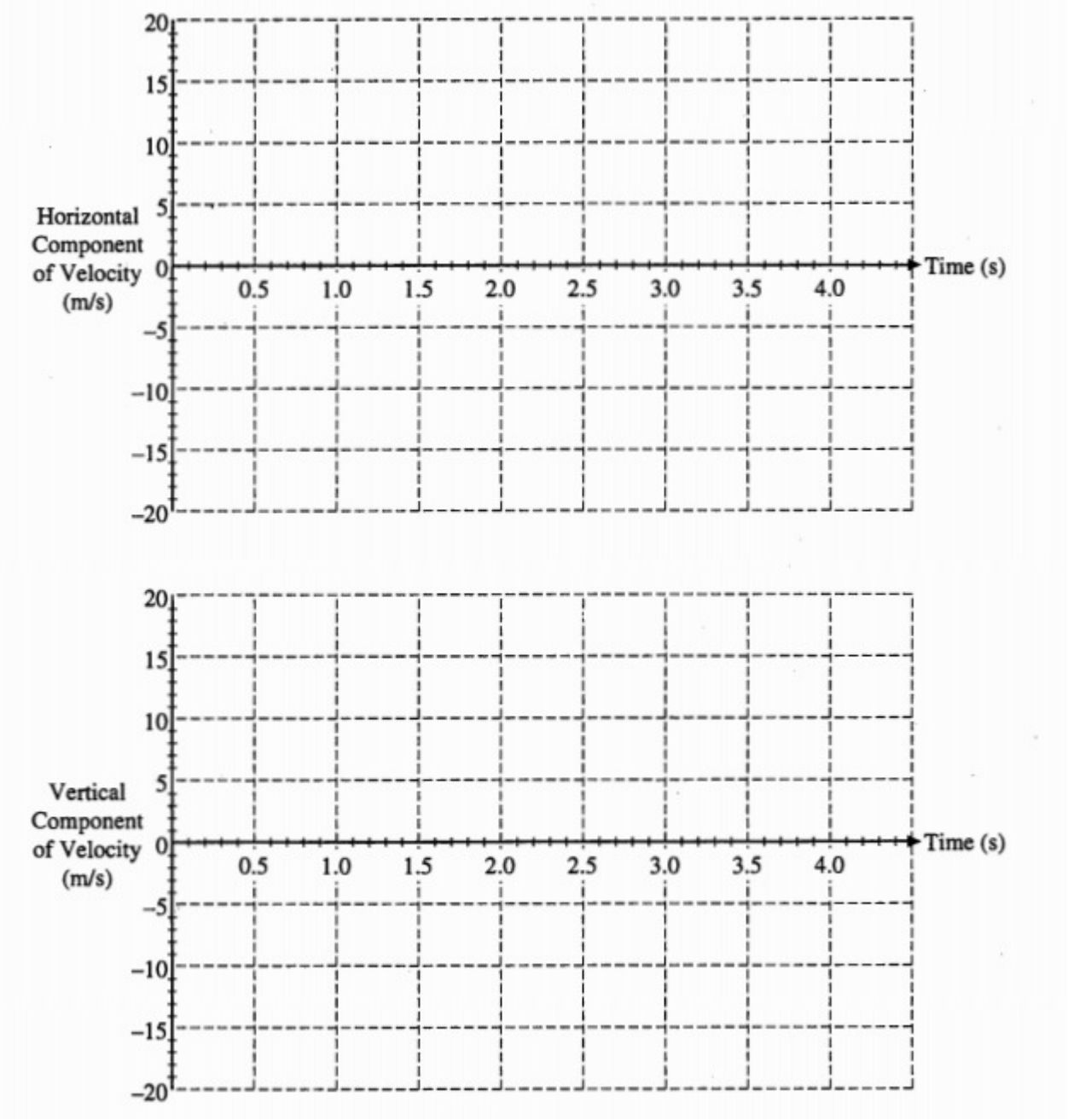
(a) Determine the magnitude of the average net force exerted on the ball during the kick.

(b) Determine the time it takes for the ball to reach the plane of the fence.

(c) Will the ball hit the fence? If so, how far below the top of the fence will it hit? If not, how far above the top of the fence will it pass?

(d) On the axes below, sketch the horizontal and vertical components of the velocity of the ball as functions of time until the ball reaches the plane of the fence.

1frq



Part A

1 point is earned : From newton's second law,

F avg = ma avg

1 point is earned : $a_{avg} = \frac{\Delta v}{\Delta t}$

$F_{avg} = m\frac{\Delta v}{\Delta t} = (0.5kg)\left(\frac{20m/s}{0.05s}\right)$

1 point is earned : F avg = 200N

1frq

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned : From newton's second law,

F avg = ma avg

1 point is earned : $a_{avg} = \frac{\Delta v}{\Delta t}$

$F_{avg} = m\frac{\Delta v}{\Delta t} = (0.5kg)\left(\frac{20m/s}{0.05s}\right)$

1 point is earned : F avg = 200N

Part B

The horizontal component of the velocity is constant

1 point is earned: Vhorizt=d

1 point is earned: Vhoriz =v0cos θ = (20m/s)(0.8)=16 m/s

t = d/ Vhoriz = 32 m / 16 m/s

1 point is earned: t = 2s

0	1	2 ✓

The student response earns two of the following points:

The horizontal component of the velocity is constant

1 point is earned: Vhorizt=d

1 point is earned: Vhoriz =v0cos θ = (20m/s)(0.8)=16 m/s

t = d/ Vhoriz = 32 m / 16 m/s

1 point is earned: t = 2s

Part C

The height y of the ball during its flight is given by

1frq

1 point is earned: y = vvertt- ½ gt2 [point also given if a issued instead of *g*]

1 point is earned: vvert = v0 sin θ = (20 m/s)(0.8) = 12 m/s

At t=2 s:

y = (12 m/s)(2s) - ½(9.8 m/s2)(2s)2

1 point is earned: y = 4.4m [Answer is 4.0 m if g = m/s2 is used]

1 point is earned: The ball is 4.4 meters above the ground at the time it reaches the fence, which is 2.5 meters high, so it passes above the fence.

1 point is earned: The distance is passes above the fence is

4.4 m - 2.5 m = 1.9 m. [Answer is 1.5 m if g = 10 m/s2 is used]

0	1	2	3	4	5 ✓

The student response earns five of the following points:

The height y of the ball during its flight is given by

1 point is earned: y = vvertt- ½ gt2 [point also given if a issued instead of *g*]

1 point is earned: vvert = v0 sin θ = (20 m/s)(0.8) = 12 m/s

At t=2 s:

y = (12 m/s)(2s) - ½(9.8 m/s2)(2s)2

1 point is earned: y = 4.4m [Answer is 4.0 m if g = m/s2 is used]

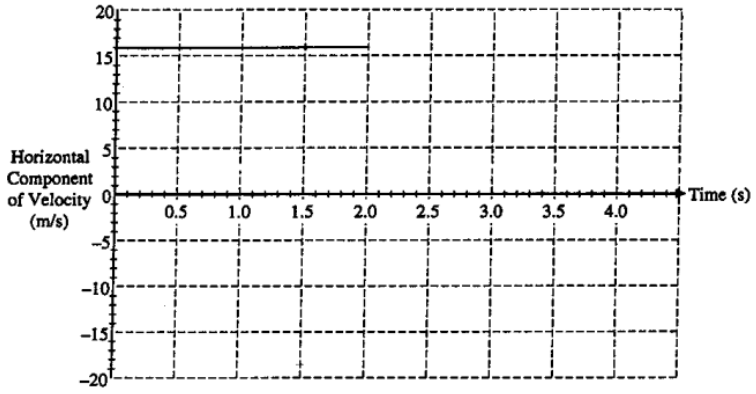
1 point is earned: The ball is 4.4 meters above the ground at the time it reaches the fence, which is 2.5 meters high, so it passes above the fence.

1 point is earned: The distance is passes above the fence is

4.4 m - 2.5 m = 1.9 m. [Answer is 1.5 m if g = 10 m/s2 is used]

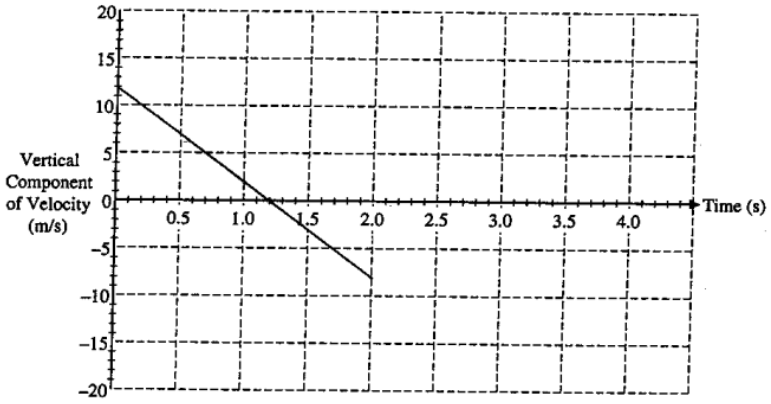
Part D

1frq



1 point is earned: For graph at a constant value (zero slope) after initial impulse period

1 point is earned: For correct value of 16 m/s



1 point is earned: For linear decrease of graph after initial impulse period

1 point is earned: For first correct point on the line

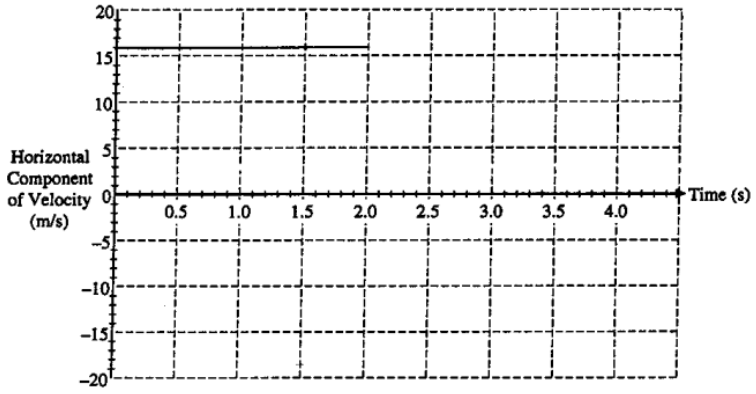
1 point is earned: For second correct point on the line or for correct slope

[Note: The graph above assume that time = 0 at the instant the ball leaves the foot.If time = 0 were assumed to be at the start of the kick, the graphs would show an initial impulse period of 0.05 second. Either assumption is OK. The graphs were graded only on the basis of the time after the initial impulse period. However, if the initial impulse period was too large by a factor of 10 on both graphs, students lost the shape point on the first part, but not on the second.]

0	1	2	3	4	5 ✓

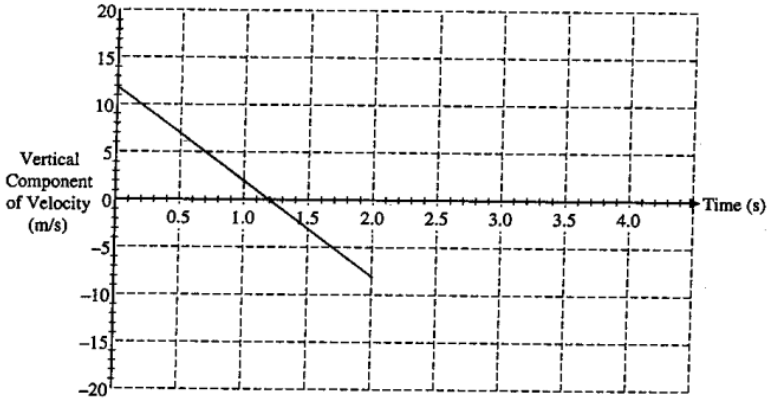
The student response earns five of the following points:

1frq



1 point is earned: For graph at a constant value (zero slope) after initial impulse period

1 point is earned: For correct value of 16 m/s



1 point is earned: For linear decrease of graph after initial impulse period

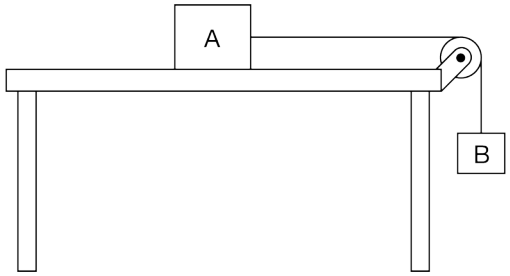
1 point is earned: For first correct point on the line

1 point is earned: For second correct point on the line or for correct slope

[Note: The graph above assume that time = 0 at the instant the ball leaves the foot.If time = 0 were assumed to be at the start of the kick, the graphs would show an initial impulse period of 0.05 second. Either assumption is OK. The graphs were graded only on the basis of the time after the initial impulse period. However, if the initial impulse period was too large by a factor of 10 on both graphs, students lost the shape point on the first part, but not on the second.]

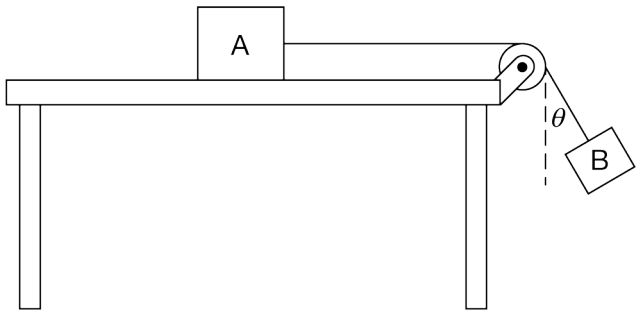
1frq

13. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)



Block A with mass m_A rests on a horizontal tabletop, while block B with mass m_B hangs from a string of negligible mass that passes over a pulley and attaches to block A, as shown above. The pulley has negligible mass and rotates with negligible friction about its axle. The blocks are at rest due to friction between block A and the tabletop. The tension in the string while the blocks are at rest has magnitude T_1, and the coefficient of static friction between block A and the tabletop is μ (mu).

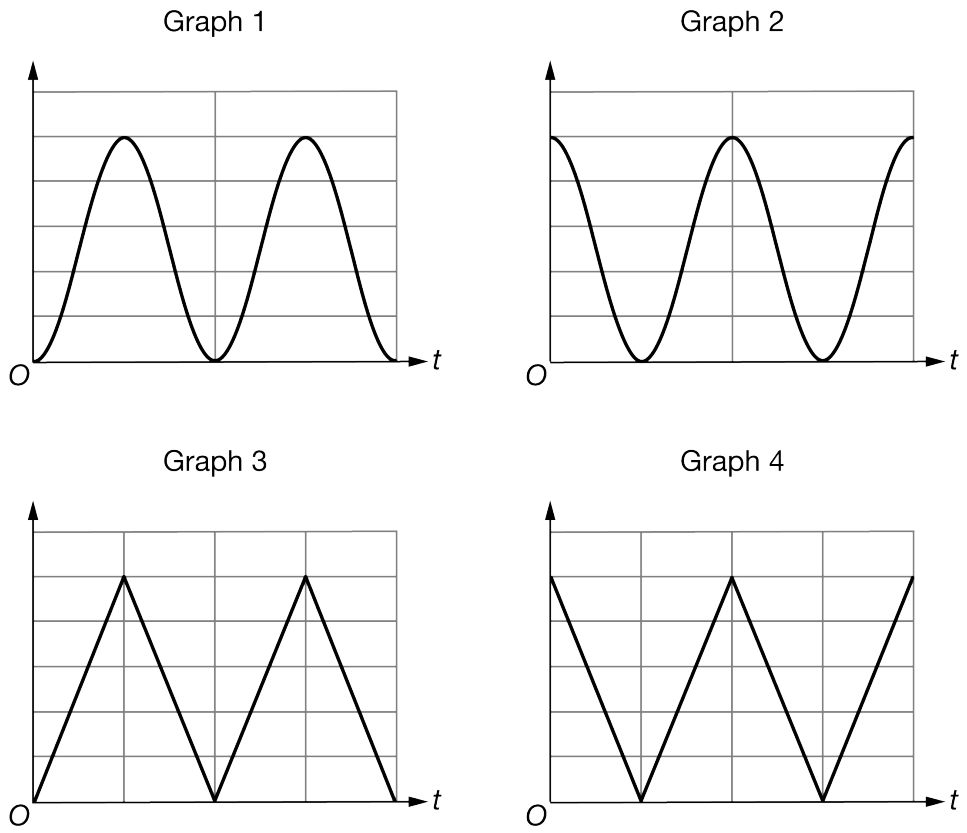
(a) What is the magnitude of the frictional force acting on block A? Justify your answer.



Note: Figure not drawn to scale.

Block B is displaced from the vertical such that the string makes a small angle θ with the vertical, as shown above. The string still has nonzero tension when block B is then released from rest.

1frq



(b) Block A is held at rest while block B swings back and forth. Which of the graphs shown above best represents the kinetic energy of block B as a function of time *t* after it is released from rest at time $t = 0$? Explain your reasoning.

Block B is again displaced from the vertical such that the string makes a small angle θ with the vertical. Block A is not held in place, and block B is released from rest.

(c) In the original scenario in which block B hung at rest, block A remained at rest. In this new scenario in which block A is not held in place, block A begins accelerating to the right across the tabletop at the instant block B passes through the bottom of its swing. In a clear, coherent paragraph-length response, explain in terms of the net force on block A why block A remained at rest in the original scenario but accelerates in this new scenario.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response does not accurately include

- ☐ 1 point is earned for a valid justification of why the friction force is either T_1 or $m_B g$.

1frq

Example Response:

The horizontal forces on block A must sum to zero because block A is not accelerating, so the friction force must be equal to T_1.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- ☐ 1 point is earned for indicating that Graph 1 is the correct graph for kinetic energy as a function of time.
- ☐ 1 point is earned for a justification that the initial speed of the block is zero and so the initial kinetic energy of the block is zero.
- ☐ 1 point is earned for a statement that includes that the kinetic energy as a function of time is not linear because it is based on a sinusoidal function.

Example Response:

Graph 1. The block is released from rest, so the kinetic energy must be zero at t = 0. This eliminates graphs 2 and 4. Kinetic energy is proportional to velocity-squared, and the velocity is a sinusoidal function. The square of a sinusoid is depicted in graph 1, not graph 3.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that object B is moving at the bottom in circular motion.
- ☐ 1 point is earned for indicating that since Block B is moving in circular motion, the acceleration or net force is toward the center of the circle or upward when at the lowest point.
- ☐ 1 point is earned for indicating that the tension is larger than when Block B is at rest
  OR
  larger than $m_B g$, or larger than T_1.
- ☐ 1 point is earned for indicating that the new string tension is greater than the maximum possible static friction on block A.
- ☐ 1 point is earned for indicating that since block A is accelerating to the right, the net force must be to the

1frq

right.

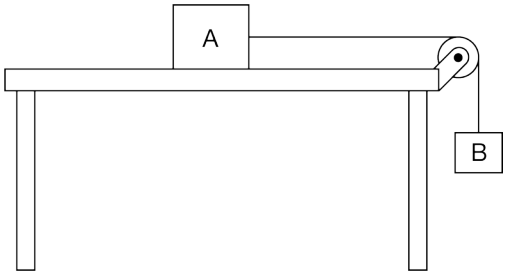
- [ ] 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

When block B is swinging and is at the bottom of its motion, it is in circular motion and has an upward acceleration and net force. Therefore, the tension in the string is now greater than $m_B g$ *or* T_1*. Because block A accelerates to the right, there is a net force to the right, which must result from the tension being greater than the maximum possible static friction force.*

1frq

14. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)



Block A with mass m_A rests on a horizontal tabletop, while block B with mass m_B hangs from a string of negligible mass that passes over a pulley and attaches to block A, as shown above. The pulley has negligible mass and rotates with negligible friction about its axle. The blocks are at rest due to friction between block A and the tabletop. The coefficient of static friction between block A and the tabletop is μ (mu), where $\mu < 1$.

(a) Determine the magnitude of the tension in the string. Briefly explain your reasoning.

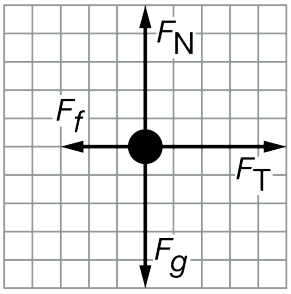


Figure 1

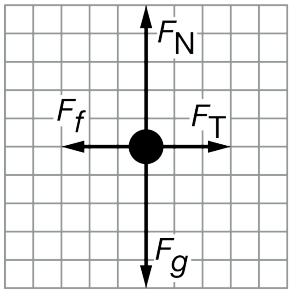


Figure 2

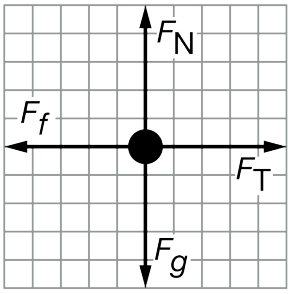


Figure 3

(b) Three students draw the free-body diagrams shown above to represent the forces acting on block A while the two-block system is at rest. Which figure correctly depicts the forces and their relative magnitudes? Explain your reasoning.

(c) A student slowly adds mass to the top of block B. When a total mass m_0 has been added to the top of block B, block A begins to accelerate across the tabletop. In a clear, coherent, paragraph-length response, explain in terms of the net force on block A why block A remained at rest even as mass was added to block B, but block A accelerates once a total mass m_0 was added to block B.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

1frq

The response accurately includes

- [ ] 1 point is earned for a valid explanation for why the tension is equal to the weight.

Example Response:

When block A is at rest, block B is at rest. So, the tension in the string must be equal to the weight of block B, $T_1 = m_B g$.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- [ ] 1 point is earned for indicating that Figure 2 is the correct free body diagram.
- [ ] 1 point is earned for an explanation which addresses that the forces must add to zero because the system is in equilibrium.
- [ ] 1 point is earned for explanation which addresses that the force of friction must be less than the normal force.

Example Response:

The system is at rest, so the forces must add to zero, and F_f *and* F_T *must be equal in magnitude. Friction also must be less than the normal force because* $\mu < 1$ *and the maximum force of friction is mu times the normal force. Therefore, Figure 2 correctly depicts the forces on Block A.*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- [ ] 1 point is earned for indicating that as the mass was added to the top of block B, the tension in the string increased.
- [ ] 1 point is earned for indicating that while block A was not moving, the static friction force was equal to the tension.
- [ ] 1 point is earned for indicating that the static friction force is not constant but can vary up to a maximum value.
- [ ] 1 point is earned for indicating that block A moved as soon as the tension was greater than the maximum

1frq

static friction force.

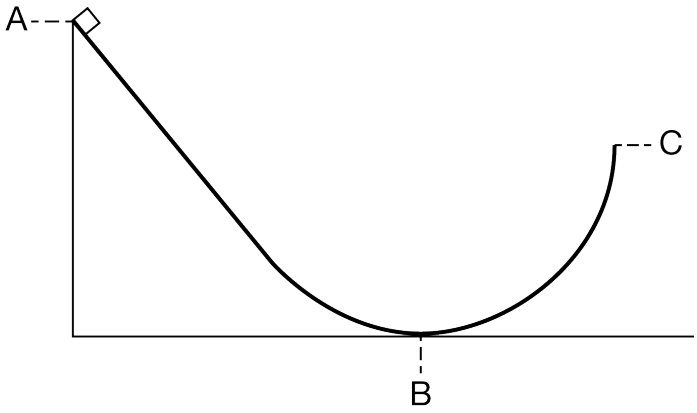
- ☐ 1 point is earned for indicating that when the tension on block A is greater than the static friction force, the block experiences a net force and accelerates to the right.
- ☐ 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

As mass was added to the top of block B, the tension in the string was increasing. As the tension was increasing, the static friction force was increasing, to keep block A at rest. Once the additional mass reached a value m_0, the tension increased past the maximum value of static friction. Since the forward tension force is greater now than the friction force, block A accelerates to the right.

1frq

15. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)



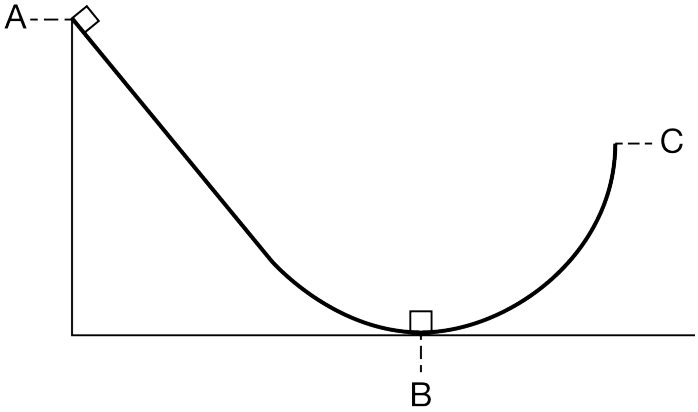
The figure above shows a track consisting of a straight section that connects to a circular section. The bottommost point of the track is at location B, and the track ends at location C, where the track is vertical. A block is released from rest at location A. There is negligible friction between the track and the box.

(a) Is the magnitude of the normal force exerted on the block by the track when the block is at location B greater than, less than, or equal to the normal force immediately before the block reaches location C? Justify your answer.

(b) In a coherent, paragraph-length response, describe the relative angle between the direction of the normal force exerted on the block by the track and the direction of the block's acceleration at the following times.

Immediately after the block is released at location A
When the block is at location B
Immediately before the block reaches location C

Specifically, indicate whether the normal force is parallel to the acceleration, perpendicular to the acceleration, or neither, at each location. Use physical principles to justify your responses.



(c) The scenario is repeated, but an identical block is at rest at location B, as shown above. When the block at

1frq

location A is released from rest, it slides down the track and collides with and sticks to the identical block. The two blocks then continue sliding up the ramp. Is the maximum height reached by the two blocks greater than, less than, or equal to the maximum height reached by the block in the original scenario? Briefly explain your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the normal force at B is greater than it is immediately before C.
- ☐ 1 point is earned for a correct explanation that correctly uses either or both of the following effects:
  · At B, the speed *v* is greater so the net force $F_{\text{net}} = mv^2/r$ is greater
  · At B, the normal force must "overcome" the downward weight force, so the normal force $F_n = F_{\text{net}} + mg$ is greater.

Example Response:

At B, the speed v is greater, so the net force mv^2/r is greater, resulting in a greater normal force than at just before location C.

OR

At B, the normal force must "overcome" the downward weight force, so the normal force $(= F_{\text{net}} + mg)$ is greater than at just before location C.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the normal force is always perpendicular to the surface of the track.
- ☐ 1 point is earned for relating the acceleration to the net force.
- ☐ 1 point is earned for indicating the acceleration at location A is along the ramp and thus perpendicular to the normal force.
- ☐ 1 point is earned for indicating that at location B the acceleration is parallel to the normal force.

1frq

- [ ] 1 point is earned for indicating that at position just before location C the acceleration is neither perpendicular nor parallel to the normal force.
- [ ] 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

The normal force is always perpendicular to the track surface. At location A, the block is accelerating from rest in a straight line down the ramp, so the acceleration is along the ramp and thus perpendicular to the normal force.

At B, the acceleration is upward, so it is in the same direction as the normal force.

Just before C, the forces of gravity (downward) and the normal force (perpendicular to the track) by vector summation result in the net force, which is in the direction of the acceleration. –The acceleration therefore is at an angle to the normal force, that is neither parallel nor perpendicular.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for indicating the height of the two blocks is less than that of the original block.
- [ ] 1 point is earned for a correct explanation of why the height is less than the original block.

Example Response:

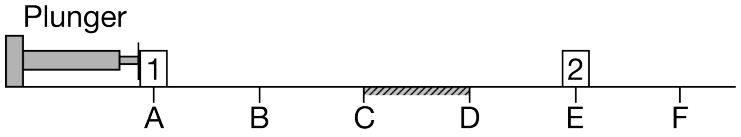
In an inelastic collision linear momentum is constant and kinetic energy is not constant. The two-block system has less energy than the first block had at the same position. All of the kinetic energy at that point can now be converted to gravitational potential energy. Which means that the two-block system will not go as high as the first block.

OR

By conservation of momentum, the speed of the two blocks after the collision is half the speed of the original block. Since the blocks are going slower, they will not go as high as the original block did.

1frq

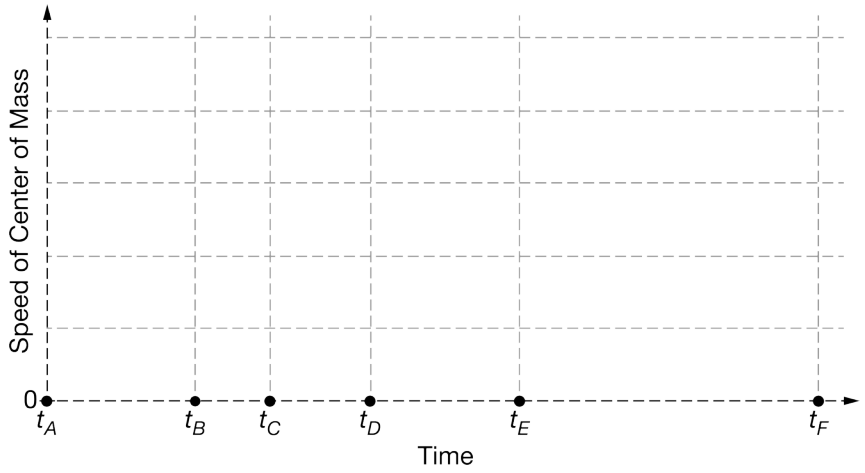
16. This question is a short free-response question. Show your work for each part of the question.

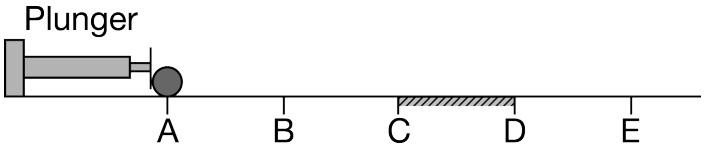


(7 points, suggested time 13 minutes)

Identical blocks 1 and 2 are placed on a horizontal surface at points A and E, respectively, as shown. The surface is frictionless except for the region between points C and D, where the surface is rough. Beginning at time t_A, block 1 is pushed with a constant horizontal force from point A to point B by a mechanical plunger. Upon reaching point B, block 1 loses contact with the plunger and continues moving to the right along the horizontal surface toward block 2. Block 1 collides with and sticks to block 2 at point E, after which the two-block system continues moving across the surface, eventually passing point F.

(a) On the axes below, sketch the speed of the center of mass of the two-block system as a function of time, from time t_A until the blocks pass point F at time t_F. The times at which block 1 reaches points A through F are indicated on the time axis.





(b) The plunger is returned to its original position, and both blocks are removed. A uniform solid sphere is placed at point A, as shown. The sphere is pushed by the plunger from point A to point B with a constant horizontal force that is directed toward the sphere's center of mass. The sphere loses contact with the plunger at point B and continues moving across the horizontal surface toward point E. In which interval(s), if any, does the sphere's angular momentum about its center of mass change? Check all that apply.

_____ A to B _____ B to C _____ C to D _____ D to E

1frq

_____ None

Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If the speed changes at t_E , the fourth point is not earned while the fifth point of part (a) may still be earned.

Note: No credit is earned for a horizontal line along the t axis.

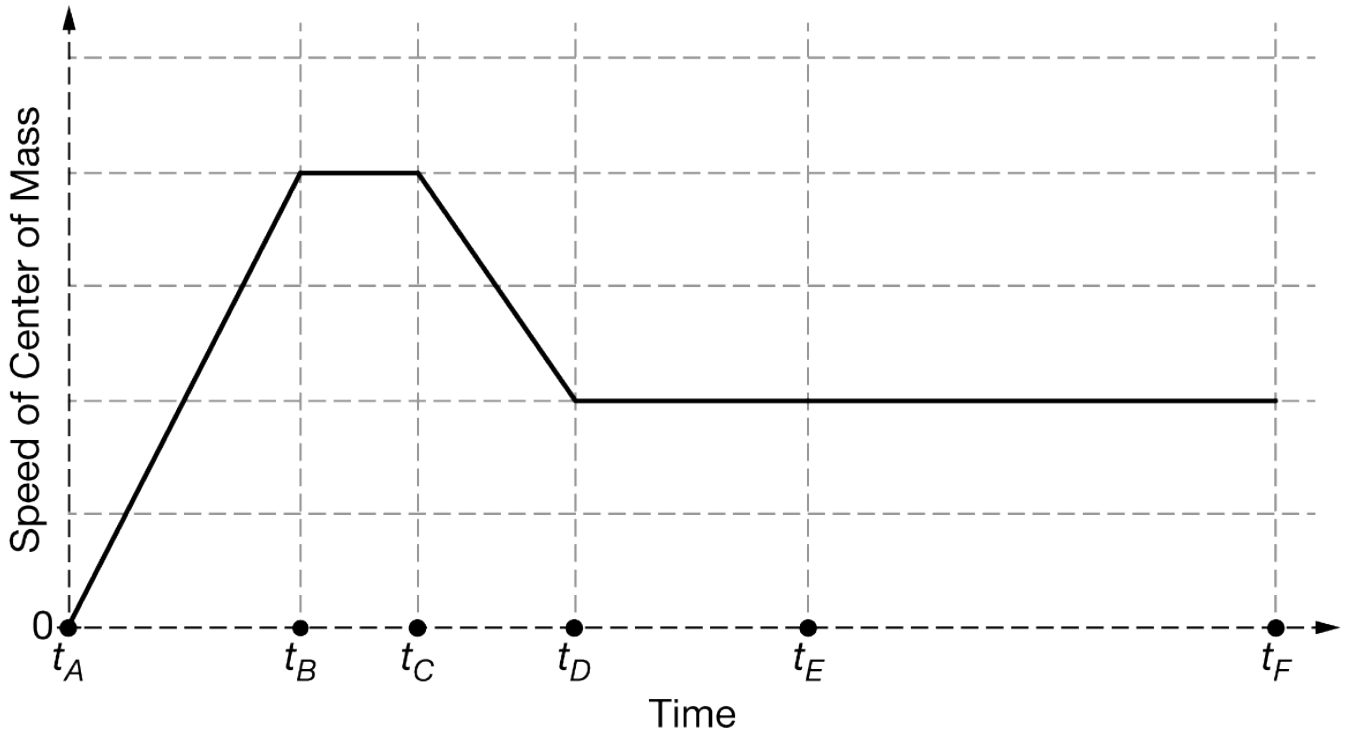
0	1	2	3	4	5 ✓

The response accurately includes **five** of the following criteria.

- ☐ For a straight line that begins at zero at t_A and increases between t_A and t_B .
- ☐ For a segment that is horizontal and nonzero between t_B and t_C .
- ☐ For a segment that decreases linearly between t_C and t_D .
- ☐ For a segment that is horizontal, nonzero, and constant (but different value than segment from t_B to t_C) from t_D through t_F (with no change at t_E).
- ☐ For a curve that is continuous from t_A through t_F , with the possible exception of t_E .

Example Response:

1frq



Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answer: “C to D ”

Note: The second point of part (b) is not earned if a statement is made that the angular momentum or angular speed decreases between C and D or that the sphere stops rotating at point D .

0	1	2 ✓

The response accurately includes **both** of the following criteria:

- ☐ For reasoning that a change in angular momentum is caused by a net external torque.
- ☐ For correctly indicating that friction from C to D is the only force producing an external torque over the entire interval from A to E .

Example Response:

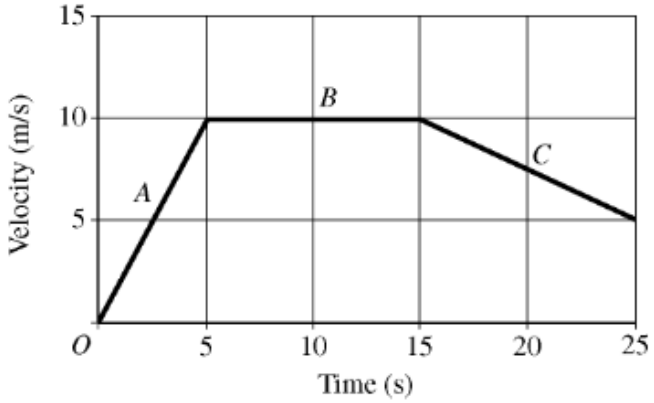
Claim: The sphere’s angular momentum about its center of mass changes in the interval C to D .

Evidence: There is friction between points C to D .

Reasoning Statements: Friction applies a torque in region C to D about the central axis of the cylinder to increase/change its angular momentum.

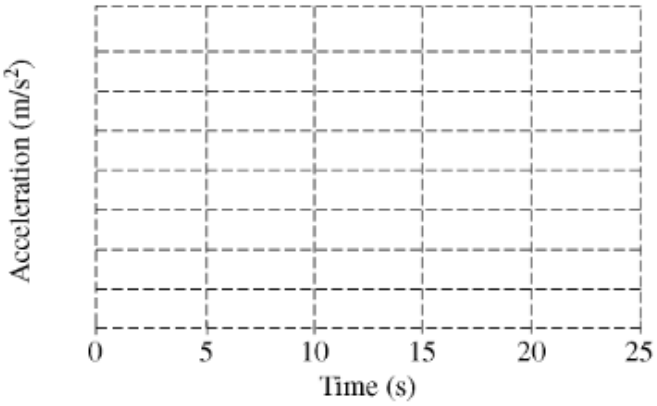
1frq

17.



A 0.40 kg object moves in a straight line under the action of a net force. The graph above shows the velocity as a function of time for the object during a 25 s interval. At time $t = 0$, the object is at the position $x = 0$.

(a) On the grid below, sketch a graph of the acceleration as a function of time for the object. Label the scale for the acceleration.



(b) Calculate the position of the object at t = 5.0 s.

(c) On which segment of the graph is the net force acting on the object zero?

____ A ____ B ____ C

Justify your answer.

(d) Calculate the net force on the object during the first 3.0 s of the motion.

(e) Calculate the amount of work done on the object by the net force during the first 15 s of the motion.

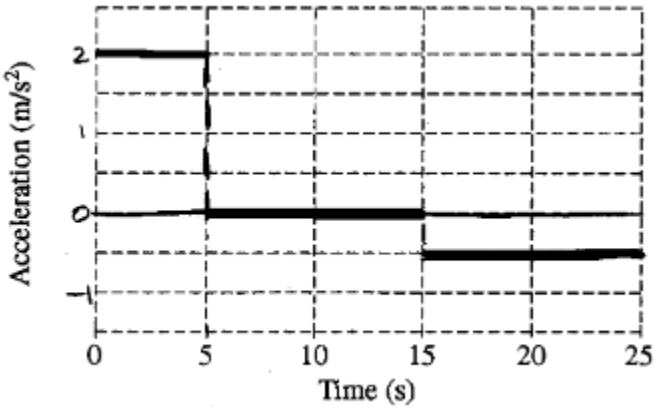
1frq

(f) For the interval t = 15 s to t = 25 s, is the work done on the object by the net force positive, negative, or zero?

____ Positive ____ Negative ____ Zero

Justify your answer.

Part A



1 point is earned: For using a linear scale on the vertical axis

1 point is earned: For showing horizontal lines in the three appropriate time frames

1 point is earned: For having the correct acceleration values for segments *B* and *C* (The correct value for segment *A* is accounted for in the answer point in part (d).)

a = $\Delta v / \Delta t$ (or the slope of the velocity versus time curve)

For segment A: a = (10m/s - 0m/s)/(5 s - 0 s) = 2 m/s2

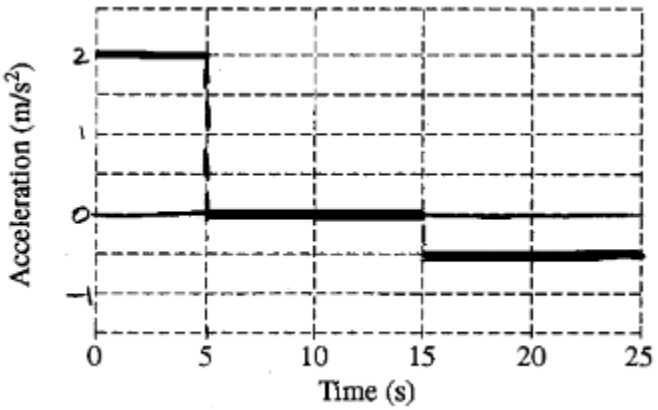
For segment B: slope is zero, so acceleration is zero

For segment C: a = (5m/s - 10m/s)/(25 s - 15 s) = -0.5 m/s2

0	1	2	3 ✓

The student response earns all of the following points:

1frq



1 point is earned: For using a linear scale on the vertical axis

1 point is earned: For showing horizontal lines in the three appropriate time frames

1 point is earned: For having the correct acceleration values for segments *B* and *C* (The correct value for segment *A* is accounted for in the answer point in part (d).)

a = $\Delta v/\Delta t$ (or the slope of the velocity versus time curve)

For segment A: *a* = (10m/s - 0m/s)/(5 s - 0 s) = 2 m/s2

For segment B: slope is zero, so acceleration is zero

For segment C: *a* = (5m/s - 10m/s)/(25 s - 15 s) = -0.5 m/s2

Part B

x = *x0* + *v0t* + (1/2)at2, where *x0* = 0

1 point is earned: For substituting v0 = 0 and *t* = 5 s and into the equation above

1 point is earned: For substituting the value on the graph of the constant acceleration between 0 and 5 s or an explicit calculation of 2m/s2

x = (1/2)(2 m/s2)(5 s)2

x = 25 m

Alternate solution

Position is the area under the velocity versus time curve.

1 point is earned: *For correctly substituting into the expression for the area under the curve*

x = (1/2)(10 m/s)(5 s)

1 point is earned: *For the correct answer*

x = 25 m

1frq

0	1	2 ✓

The student response earns all of the following points:

$x = x0 + v0t + (1/2)at2$, where $x0 = 0$

1 point is earned: For substituting v0 = 0 and $t = 5$ s and into the equation above

1 point is earned: For substituting the value on the graph of the constant acceleration between 0 and 5 s or an explicit calculation of 2m/s2

$x = (1/2)(2 \text{ m/s2})(5 \text{ s})2$

x = 25 m

Alternate solution

Position is the area under the velocity versus time curve.

1 point is earned: *For correctly substituting into the expression for the area under the curve*

x = (1/2)(10 m/s)(5 s)

1 point is earned: *For the correct answer*

x = 25 m

Part C

1 point is earned: For selecting segment *B*

1 point is earned: For an appropriate justification

Example: Fnet = ma = m ($\Delta v/\Delta t$), the velocity is constant for this segment, $\Delta v = 0$; therefore, *Fnet* = 0.

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For selecting segment *B*

1 point is earned: For an appropriate justification

Example: Fnet = ma = m ($\Delta v/\Delta t$), the velocity is constant for this segment, $\Delta v = 0$; therefore, *Fnet* = 0.

1frq

Part D

1 point is earned: For using Newton's second law

Fnet = ma

Fnet = (0.40 kg)(2 m/s2)

1 point is earned: For the correct answer

Fnet = 0.80 N

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For using Newton's second law

Fnet = ma

Fnet = (0.40 kg)(2 m/s2)

1 point is earned: For the correct answer

Fnet = 0.80 N

Part E

1 point is earned: For using the work-energy theorem

W = FΔr cos θ

To move in a straight line, the net force must be in the direction of motion, so . *W* = *Fnetd*

1 point is earned: For substituting the value of the position determined in from part (b), which is equal to the displacement, to determine the work done for segment A

For segment A, *WA* = (0.8)(25 m) = 20 J

For segment B, *WB* = (0 N)(10 m/s)(10 s) = 0 J

Wtot = *WA* + *WB* = 20 J + 0 J = 20 J

Alternate solution

$W = \Delta K$

$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

1frq

1 point is earned: *For correct substitutions into the equation above*

$W = \frac{1}{2}(0.40kg)(10m/s)^2 - \frac{1}{2}(0.40kg)(0m/s)^2$

1 point is earned: *For the correct answer*

W = 20 J

✓

0	1	2

The student response earns all of the following points:

1 point is earned: For using the work-energy theorem

W = F Δr cos θ

To move in a straight line, the net force must be in the direction of motion, so . *W = Fnetd*

1 point is earned: For substituting the value of the position determined in from part (b), which is equal to the displacement, to determine the work done for segment A

For segment A, *WA* = (0.8)(25 m) = 20 J

For segment B, *WB* = (0 N)(10 m/s)(10 s) = 0 J

Wtot = WA + WB = 20 J + 0 J = 20 J

Alternate solution

$W = \Delta K$
$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

1 point is earned: *For correct substitutions into the equation above*

$W = \frac{1}{2}(0.40kg)(10m/s)^2 - \frac{1}{2}(0.40kg)(0m/s)^2$

1 point is earned: *For the correct answer*

W = 20 J

Part F

1 point is earned: For selecting “Negative”

1 point is earned: For an appropriate explanation using kinematic or dynamic principles

Examples:

1frq

The velocity is decreasing, so the kinetic energy is also decreasing.

The acceleration is negative; therefore, the direction of the net force is opposite to the direction of the displacement.

1 point is earned: For connecting the explanation to the work done

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For selecting “Negative”

1 point is earned: For an appropriate explanation using kinematic or dynamic principles

Examples:

The velocity is decreasing, so the kinetic energy is also decreasing.

The acceleration is negative; therefore, the direction of the net force is opposite to the direction of the displacement.

1 point is earned: For connecting the explanation to the work done

Units

1 point is earned: For correct units on at least two parts with a calculated numerical answer and no incorrect units

0	1 ✓

The student response earns all of the following points:

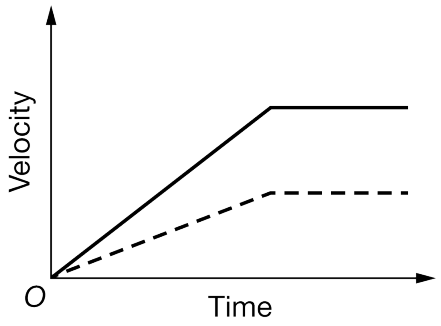
1 point is earned: For correct units on at least two parts with a calculated numerical answer and no incorrect units

1frq

18. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)



Two disks, A and B, are initially at rest at the left end of a track, as shown above. Disk A is more massive than disk B. The track has a horizontal section and then curves upward at the right end. The disks are simultaneously pushed to the right by identical, constant, horizontal forces of magnitude F for the same amount of time. The forces are removed before the disks reach the curved section of the track, and there is negligible friction between the track and the disks.



(a) The graph above shows the velocity of each disk as a function of time while the disks are moving on the horizontal section. Identify which disk's motion is represented by the solid line. Explain your reasoning.

(b) When the horizontal forces are removed, which disk, if either, has a greater momentum? If the disks have equal momentum, state this explicitly. Briefly justify your answer in terms of the impulse acting on each disk.

(c) After sliding across the horizontal section of the track, both disks slide partway up the curved section of the track. Identify which disk, if either, reaches a greater maximum height. If both disks reach the same maximum height, state this explicitly.

(d) In a clear, coherent, paragraph-length response, justify your answer to part (c).

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

1frq

- [ ] 1 point is earned for correct identification of disk B as being represented by the solid line.
- [ ] 1 point is earned for indicating that the slope of the velocity vs time graph represents the acceleration.
- [ ] 1 point is earned for explaining that the smaller mass will have a greater acceleration for the same force applied.

Example Response:

The solid line represents the velocity of disk B as a function of time. Because disk B is less massive, with the same force applied to it, it will have a greater acceleration. The slope of the velocity vs. time graph represents the acceleration, so disk B will have a line with a larger slope and will have a higher speed after the same amount of time.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for indicating the disks experience the same impulse with correct justification.

Example Response:

Both disks are pushed with the same force for the same time. Since impulse is equal to the force applied times the time, the impulses experienced by both disks is the same.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for correctly stating that disk B goes higher up the ramp.

Example Response:

Disk B goes higher up the ramp.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

1frq

0	1	2	3	4	5 ✓

The response accurately includes **all** of the following criteria.

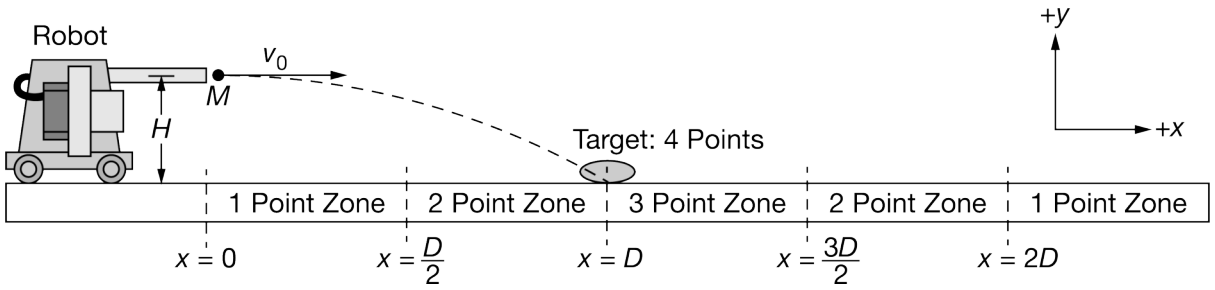
- ☐ 1 point is earned for indicating conservation of energy.
- ☐ 1 point is earned for indicating that the disk with greater initial kinetic energy corresponds to greater gravitational potential energy at the top of the slide.
- ☐ 1 point is earned for indicating that the mass terms cancel or do not affect the maximum height reached by a disk.
- ☐ 1 point is earned for indicating that the disk that corresponds to greater potential energy goes higher.
- ☐ 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

The final height of each disk only depends on its initial speed before reaching the ramp. This is because the mass term cancels when setting kinetic energy equal to final gravitational potential energy. Since block B has a larger speed after being pushed, it will have the largest kinetic energy at the bottom of the ramp. With the largest kinetic energy at the bottom of the ramp, it will also have the greatest gravitational potential energy at the top of the ramp, so it will reach the higher point on the ramp.

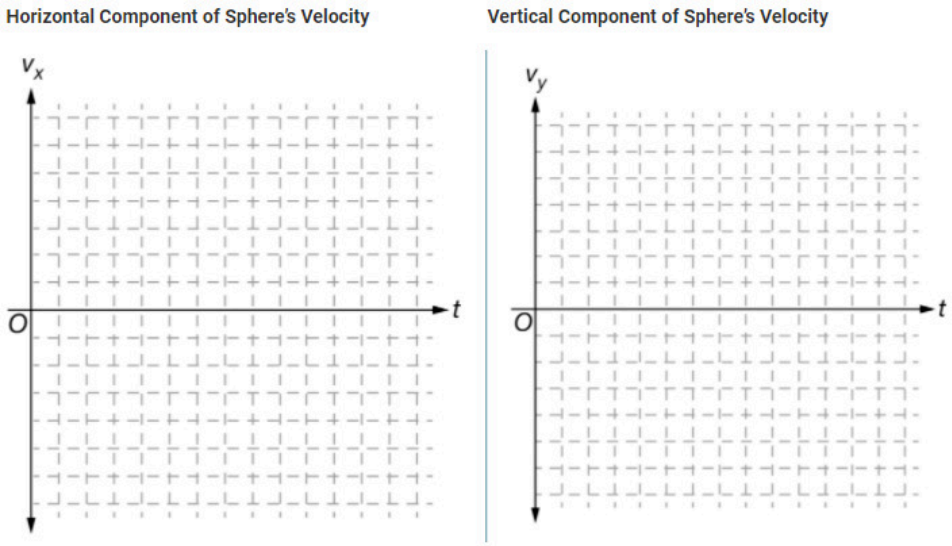
1frq

19. This question is a short free-response question. Show your work for each part of the question.



A group of students prepare for a robotic competition and build a robot that can launch large spheres of mass M in the horizontal direction with a variable speed and from a variable vertical position and a fixed horizontal position $x = 0$. The robot is calibrated by adjusting the speed at which the sphere is launched and the height of the robot's sphere launcher. Depending on where the spheres land on the ground, students earn points based on the accuracy of the robot. The robot is calibrated so that when the spheres are launched from a vertical position $y = H$ and speed v_0, they consistently land on the ground on a target that is at a position $x = D$. Positive directions for vector quantities are indicated in the figure.

(a) On the axes below, sketch the graphs of the horizontal and vertical components of the sphere's velocity v as a function of time t between $t = 0$, when the sphere is launched and $t = T$, when the sphere hits the target. Label $t = T$ for the horizontal component of the sphere's velocity and the vertical component of the sphere's velocity.



When the students arrive at the competition, it is determined that the height of the sphere launcher can no longer be adjusted due to a mechanical malfunction. Therefore, the spheres must be launched at a vertical position of $y = \frac{H}{2}$. However, the spheres may be launched at speed v_0 or $2v_0$.

(b) In a clear, coherent paragraph-length response that may also contain diagrams and/or equations, describe the speed at which a sphere should be launched so that the students earn the maximum number of points in the competition.

Part (a)

1frq

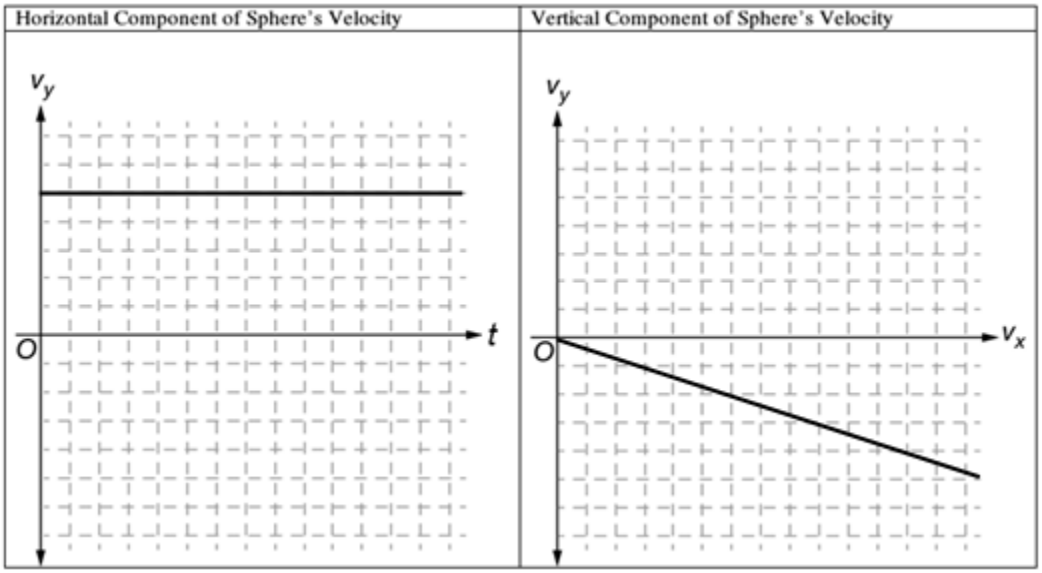
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for sketching a horizontal line above the horizontal axis from $t = 0$ to $t = T$.
- ☐ 1 point is earned for sketching a line with a constant slope that begins at the origin and becomes more negative from $t = 0$ to $t = T$.

Example Response:



Part (b)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5 ✓

Student response accurately includes all of the following criteria.

- ☐ 1 point is earned for a statement that indicates that a sphere launched from a vertical height of $y = \frac{H}{2}$ will result in a free fall time that is less than T .
- ☐ 1 point is earned for a statement that indicates that if the sphere is launched at the original speed v_0 that the distance the sphere travels before hitting the ground will be $x < D$.
- ☐ 1 point is earned for a statement that indicates that with a launch height of $y = \frac{H}{2}$, and a speed of either

1frq

v_0 or $2v_0$, the students can no longer earn 4 points.

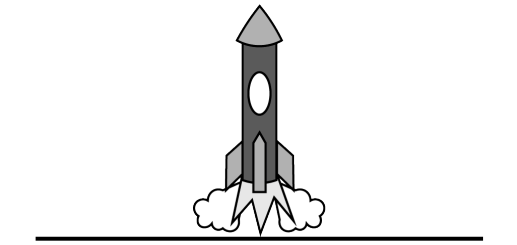
- [ ] 1 point is earned for a statement that indicates that to get the most points, since the students can no longer hit the target, they should aim for the 3-point zone by making $v = 2v_0$.
- [ ] 1 point is earned for a logical, relevant and internally consistent argument that addresses the required arguments or question asked and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

Since the sphere is to be launched from a position $y = \frac{H}{2}$, the time that the sphere takes to hit the ground will be less than when the sphere is launched from $y = H$. Since the sphere will take less time to hit the ground, if it is launched with the same initial speed v_0, it will not travel as far horizontally before hitting the ground so that $x < D$. Because students can only launch from $y = \frac{H}{2}$, with speeds of v_0 or $2v_0$, they can no longer hit the target, and should instead aim for the 3 Point zone by giving the spheres an initial speed of $2v_0$.

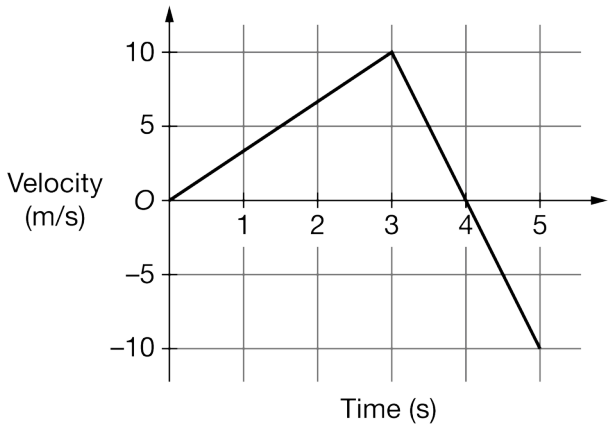
1frq

20. This question is a short free-response question. Show your work for each part of the question.



(7 points, suggested time 13 minutes)

A model rocket of constant mass 0.5 kg is launched vertically at time $t = 0$. The rocket's engine provides a constant force of magnitude F_0 for three seconds and then stops firing. The vertical velocity of the rocket as a function of time for the first 5 seconds of the flight is shown in the graph below.

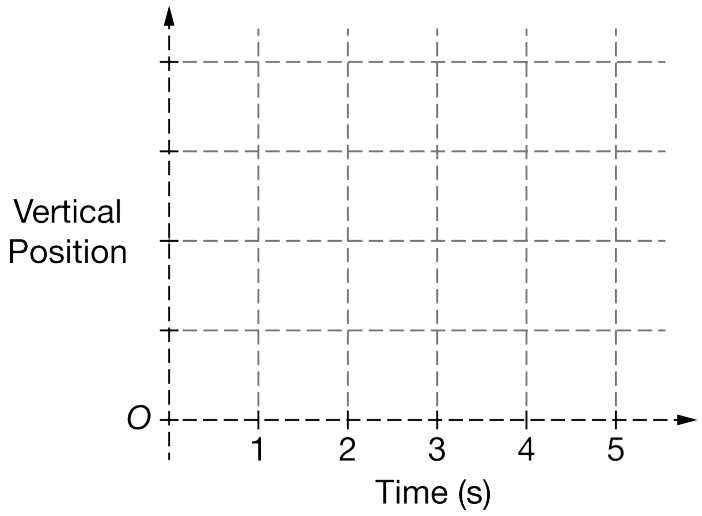


(a) Calculate F_0.

(b) Calculate the time at which the rocket hits Earth after being launched.

(c) On the axes below, sketch a graph of the vertical position of the rocket as a function of time for the first 5 seconds of the flight.

1frq



Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using the slope of the line from $t = 0$ to $t = 3$ seconds to find the acceleration of the rocket.
- ☐ 1 point is earned for using Newton's second law to correctly determine the force exerted by the engine.

Example Response for part (a):

Slope of the velocity graph from t=0 to t=3 seconds

$\text{Slope} = \frac{10\ \text{m/s}}{3\ \text{s}} = 3.3 \frac{\text{m}}{\text{s}^2}$

$\sum F_y = ma_y$

$F_{engine} - F_g = ma_y$

$F_{engine} = (0.5\ \text{kg})(3.3\ \frac{m}{s^2}) + (0.5\ \text{kg})(9.8\ \frac{\text{m}}{\text{s}^2}) = 6.6\ \text{N}$

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

1frq

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for identifying that the rocket rises for 4 seconds before reaching the maximum height.
- ☐ 1 point is earned for calculating the area under the velocity graph for the first 4 seconds to find the maximum height of the rocket:
  $\text{Max height} = (0.5)(3\text{ s})(10\text{ m/s}) + (0.5)(1\text{ s})(10\text{ m/s}) = 20\text{ meters}$
- ☐ 1 point is earned for finding the time the rocket takes to fall from its maximum height consistent with the maximum height found in the second point:
  $T_{fall} = 2\text{ seconds}$
  Scoring note: Time from launch back to ground is 6 seconds
- ☐ **Alternate Solution to part (b)**
  1 point is earned for identifying that the rocket undergoes the acceleration calculated in part (a) for the first 3 seconds.
- ☐ 1 point is earned for calculating the height reached at $t = 3\text{ seconds}$ either by using the acceleration from part (a) in a correct kinematic equation:
  $\Delta y = v_0 t + 0.5at^2$
  $\Delta y = 0 + 0.5(\frac{10}{3}\frac{\text{m}}{\text{s}^2})(3\text{ s})^2$
  $\Delta y = 15\text{ m in first 3 seconds}$
  OR by calculating the area under the velocity graph for the first 3 seconds.
- ☐ 1 point is earned for finding the time the rocket undergoes freefall acceleration until it hits the ground consistent with the distance found in the second point:
  $\Delta y = v_0 t + 0.5at^2$
  $-15\text{ m} = (10\ \frac{\text{m}}{\text{s}})t + 0.5(-9.8\frac{\text{m}}{\text{s}^2})t^2$
  $t = 3.04\text{ s}$
  Scoring note: Time from launch back to ground is 6 seconds.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

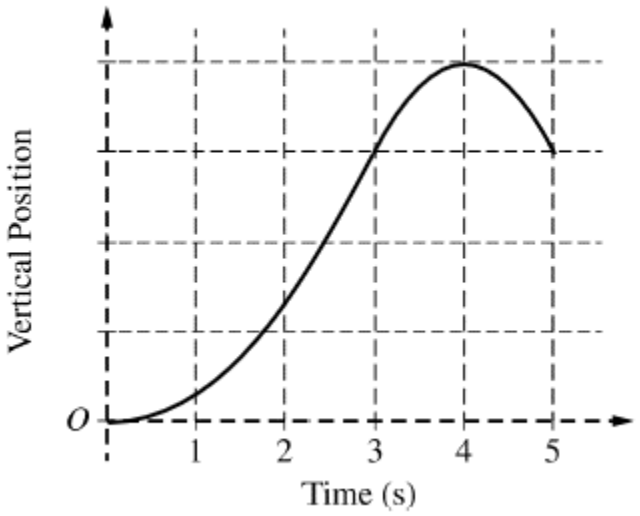
The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a graph that is concave-up from $t = 0$ to $t = 3$ seconds.

1frq

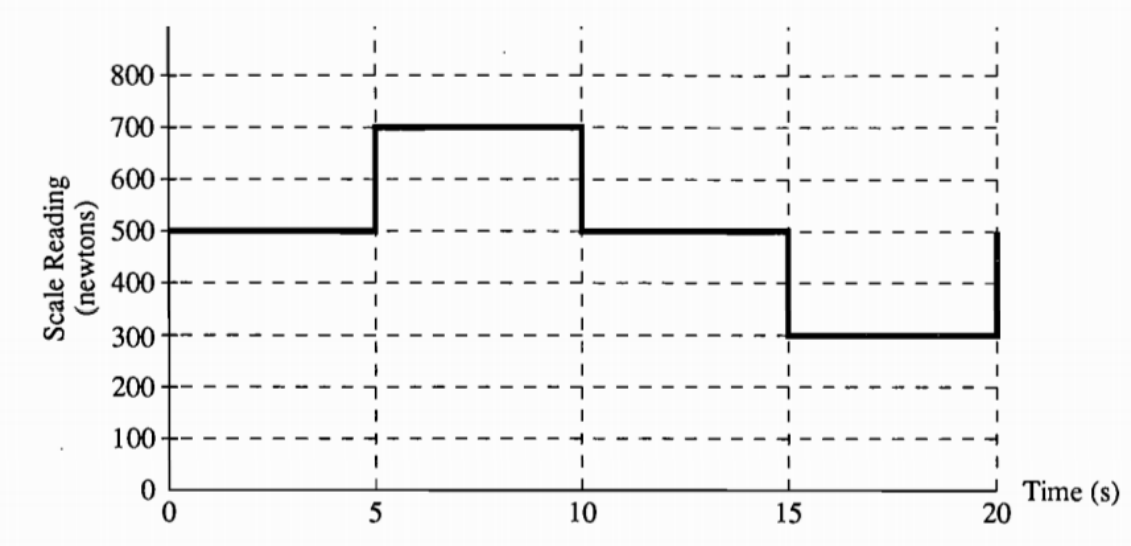
- [ ] 1 point is earned for a graph that is concave-down from $t = 3$ to $t = 5$ seconds **AND** has a maximum at 4 s.

Example Response for part (c):



1frq

21.



A student whose normal weight is 500 newtons stands on a scale in an elevator and records the scale reading as a function of time. The data are shown in the graph above. At time t = 0, the elevator is at displacement x = 0 with velocity v = 0. Assume that the positive directions for displacement, velocity, and acceleration are upward.

(a) On the diagram to the right,draw and label of the forces on the student at t = 8 seconds.



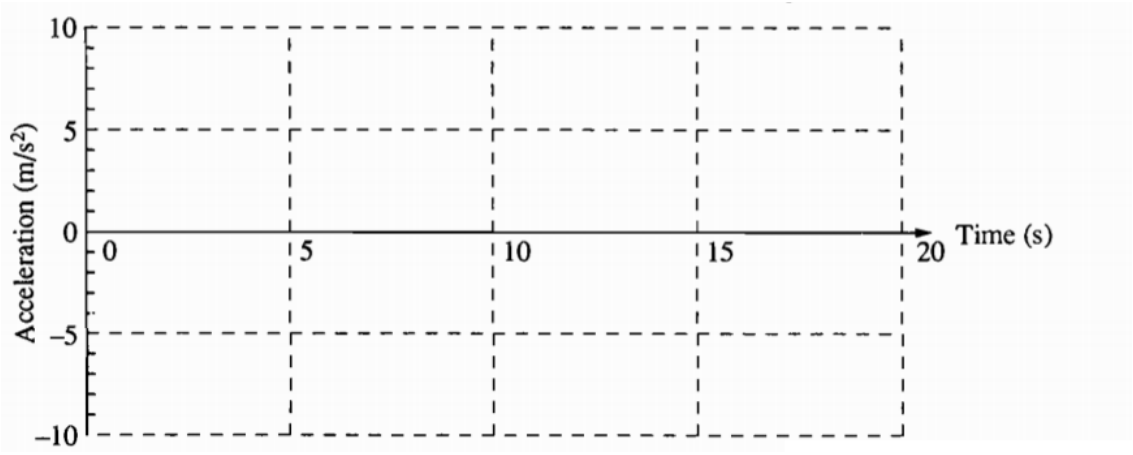
(b) Calculate the acceleration a of the elevator for each 5-second interval.

i. Indicate your results by completing the following table.

Time interval(s)	0-5	5-10	10-15	15-20
a(m/s^2)	____	____	____	____

ii. Plot the acceleration as a function of time on the following graph

1frq

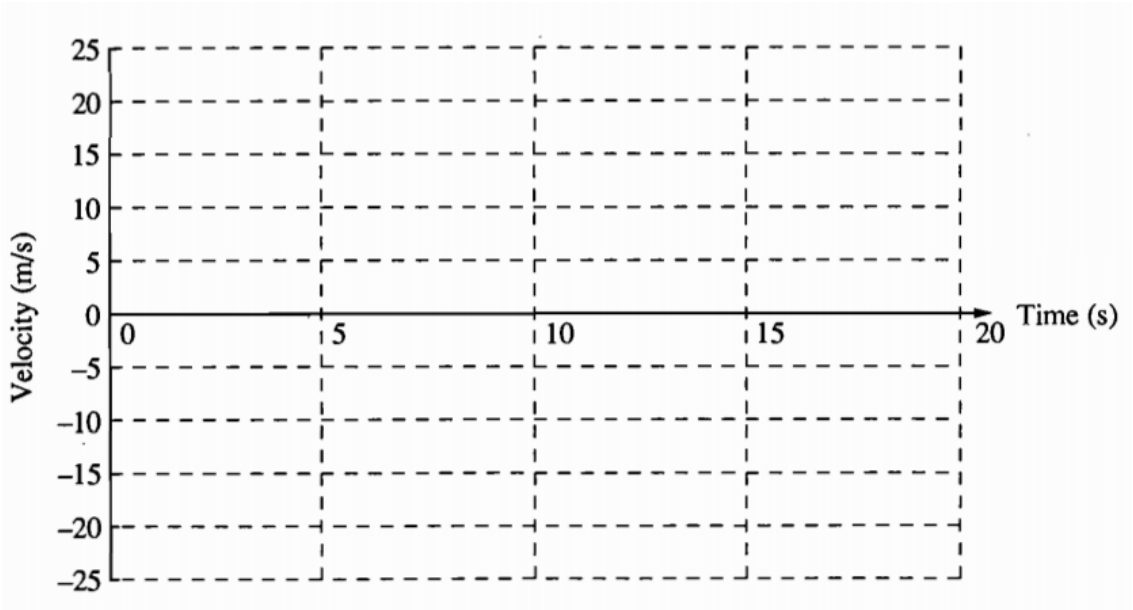


(c) Determine the velocity *v* of the elevator at the end of each 5-second interval.

i. Indicate your results by completing the following table

Time interval(s)	5	10	15	20
v(m/s)	____	____	____	____

ii. Plot the velocity as a function of time on the following graph.



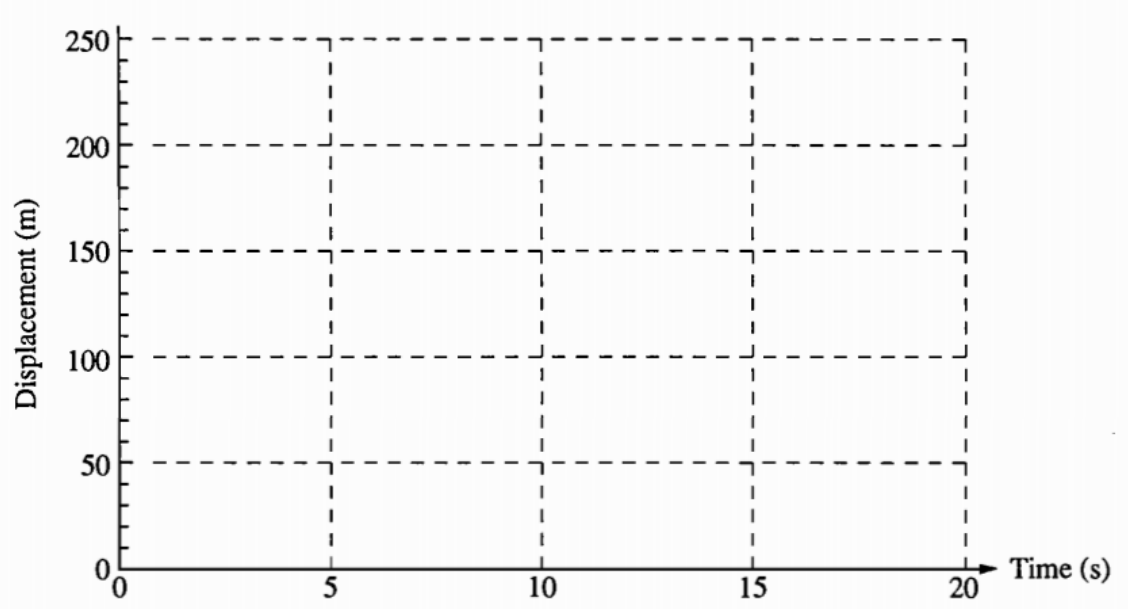
(d) Determine the displacement *x* of the elevator above the starting point at the end of each 5-second interval.

i. Indicate your results by completing the following table

Time interval(s)	5	10	15	20
x(m)	____	____	____	____

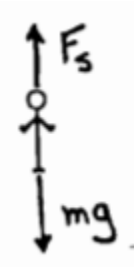
1frq

ii. Plot the displacement as a function of time on the following graph.



Part A

Note: To facilitate grading of this question, 30 points were counted and the total was divided by 2, with half points rounded up.



2 point is earned: for each vector drawn in the correct direction

1 point is earned: for correct labels on both vectors (e.g. Fs, Fn, Fe, and W, mg, gravity)

1 point is earned: for no extraneous vectors

0	1	2	3	4 ✓

The student response earns four of the following points:

Note: To facilitate grading of this question, 30 points were counted and the total was divided by 2, with half points rounded up.

1frq



2 point is earned: for each vector drawn in the correct direction

1 point is earned: for correct labels on both vectors (e.g. Fs, Fn, Fe, and W, mg, gravity)

1 point is earned: for no extraneous vectors

Part B(i,ii)

(i)

1 point is earned : For any statement of Newton's Second Law:

$\sum$ F = ma

The mass of the student is found by applying this law to the student's weight , and where is $a = g$;

$$m = \frac{F}{a} = \frac{W}{g} = \frac{500\text{N}}{10\text{m/s}^2} = 50\text{kg}$$

1 point is earned : For the correct expression for the net force on the student:

F s -mg=ma

The upward force *F s* exerted on the student by the scale is equal to the scale reading. For each time interval, applying Newton's Second Law to find the acceleration requires diving the net force by the student's mass

1 point is earned : 0-5 seconds *a* = 0N/50k = 0

1 point is earned : 5-10 seconds *a* = 200N/50k = 4 m/s 2

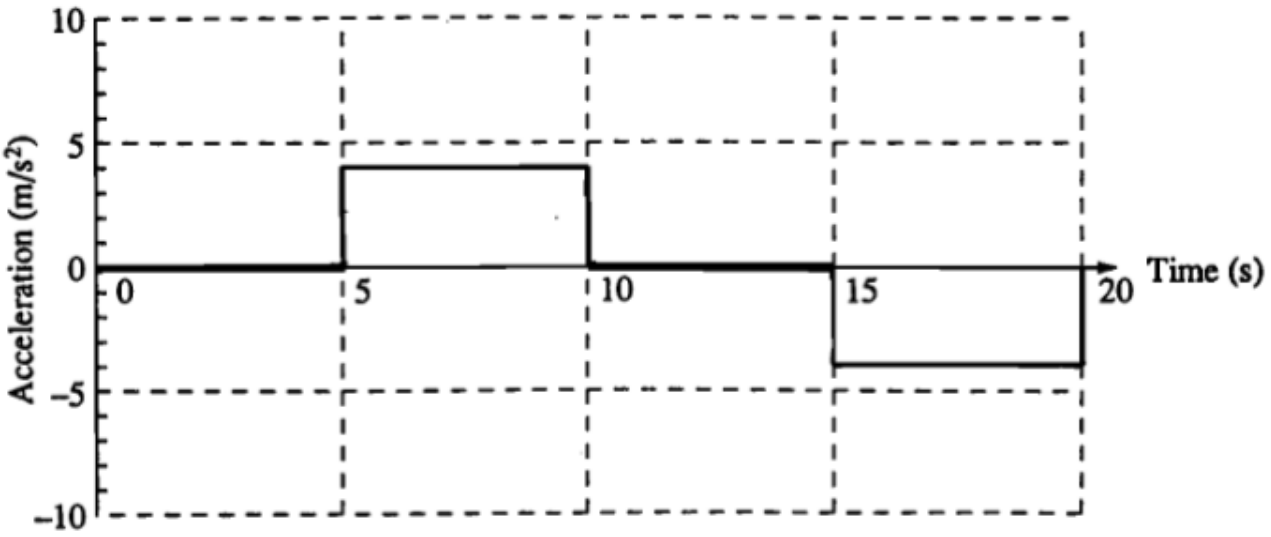
1 point is earned : 10-15 seconds *a* = 0N/50k = 0

1 point is earned : 15-20 seconds *a* = -200N/50k = -4 m/s 2

If all four values of acceleration were correct,credit was given for the first two points for the equations even if they were not written explicitly.

(ii)

1frq



1 point is earned : For the correct shape of the graph (four horizontal segments,with correct two segments above and below the axis.)

1 point is earned : For correctly graphing the magnitudes of the acceleration

0	1	2	3	4	5	6	7	8 ✓

The student response earns eight of the following points:

(i)

1 point is earned: For any statement of Newton's Second Law:

$\sum$ F = ma

The mass of the student is found by applying this law to the student's weight , and where is $a = g$;

$$m = \frac{F}{a} = \frac{W}{g} = \frac{500\text{N}}{10\text{m/s}^2} = 50\text{kg}$$

1 point is earned: For the correct expression for the net force on the student:

Fs-mg=ma

The upward force *Fs* exerted on the student by the scale is equal to the scale reading. For each time interval, applying Newton's Second Law to find the acceleration requires diving the net force by the student's mass

1 point is earned: 0-5 seconds a = 0N/50k = 0

1 point is earned: 5-10 seconds a = 200N/50k = 4 m/s2

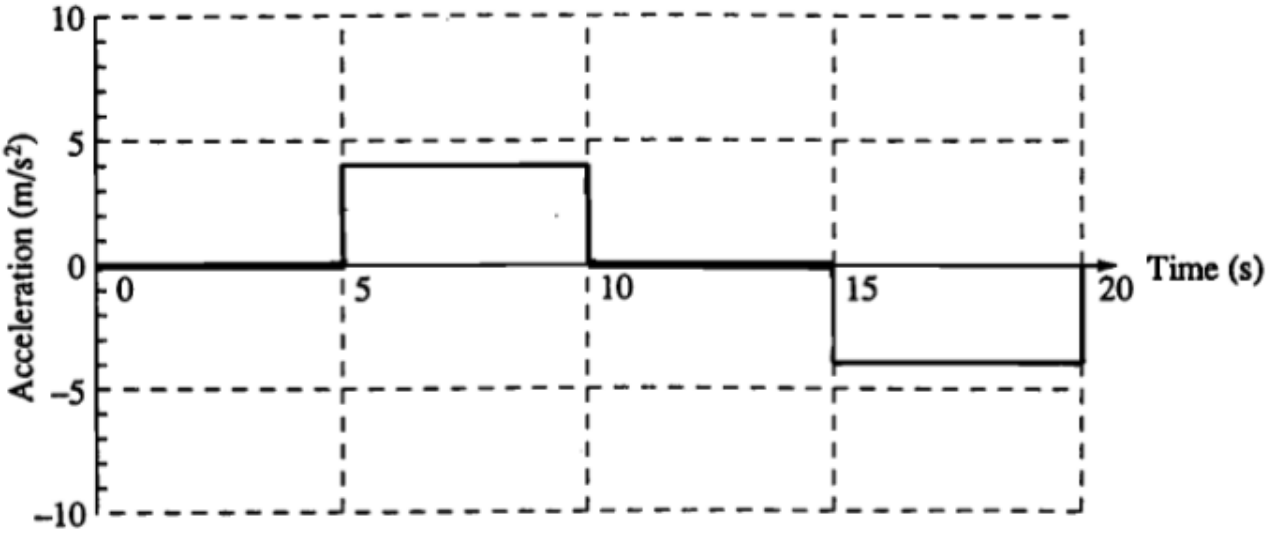
1 point is earned: 10-15 seconds a = 0N/50k = 0

1frq

1 point is earned: 15-20 seconds *a* = -200N/50k = -4 m/s2

If all four values of acceleration were correct,credit was given for the first two points for the equations even if they were not written explicitly.

(ii)



1 point is earned: For the correct shape of the graph (four horizontal segments,with correct two segments above and below the axis.)

1 point is earned: For correctly graphing the magnitudes of the acceleration

Part C

(i)

1 point is earned: For the kinematic equation applicable for constant acceleration:

v - v0 + at (addition of v0 must be indicated)

For each time interval, substitute the initial velocity for that interval, the appropriate acceleration from part (b), and a time of 5 seconds.

1 point is earned: 5 seconds: v - 0 + (0)(5 s) - 0

1 point is earned: 10 seconds: v - 0 + (4 m/s2)(5 s) - 20 m/s

1 point is earned: 15 seconds: v - 20 m/s +(0)(5 s) - 20 m/s

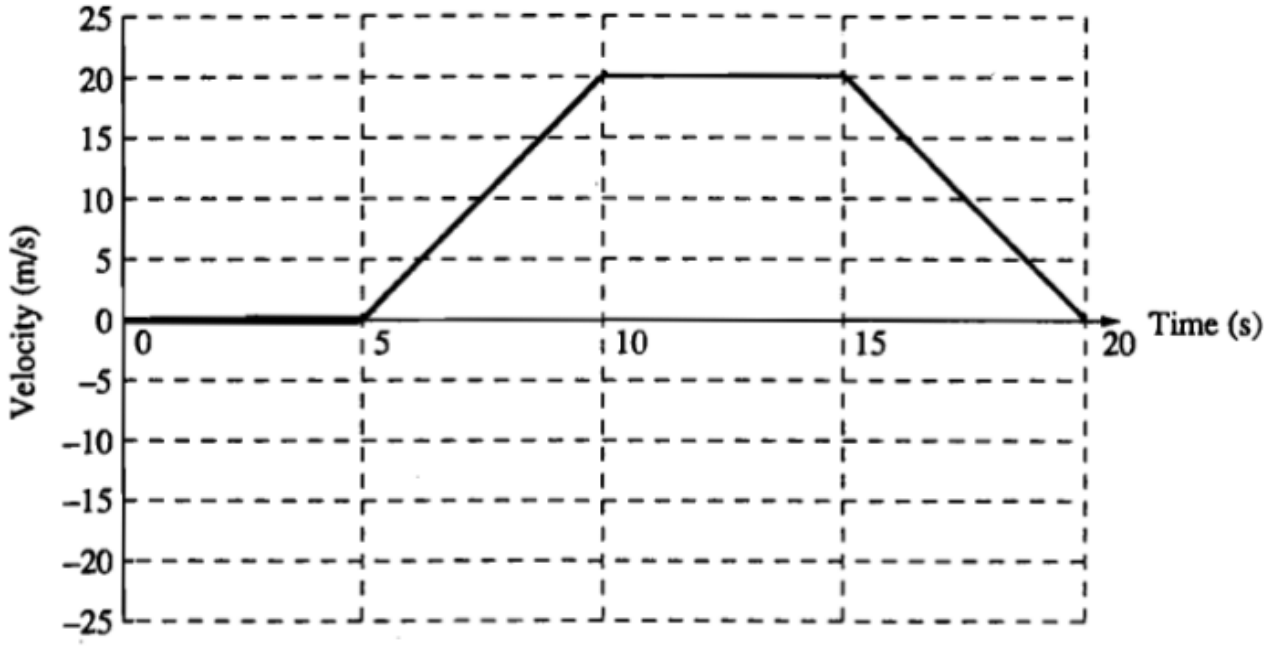
1 point is earned: 20 seconds: v - 20 m/s +(-4 m/s2)(5 s) - 0

If all four values were correct credit was given for the equation even if it was not written explicitly

Alternate solution: For indicating that at the end of each interval v = v0 + Δv

(ii)

1frq



1 point is earned(max 4): for each segment having the correct shape and slope (or which is consistent with graph in part B.

0	1	2	3	4	5	6	7	8	9 ✓

The student response earns 9 of the following points:

(i)

1 point is earned: For the kinematic equation applicable for constant acceleration:

v - *v0* + at (addition of v0 must be indicated)

For each time interval, substitute the initial velocity for that interval, the appropriate acceleration from part (b), and a time of 5 seconds.

1 point is earned: 5 seconds: v - 0 + (0)(5 s) - 0

1 point is earned: 10 seconds: v - 0 + (4 m/s2)(5 s) - 20 m/s

1 point is earned: 15 seconds: v - 20 m/s +(0)(5 s) - 20 m/s

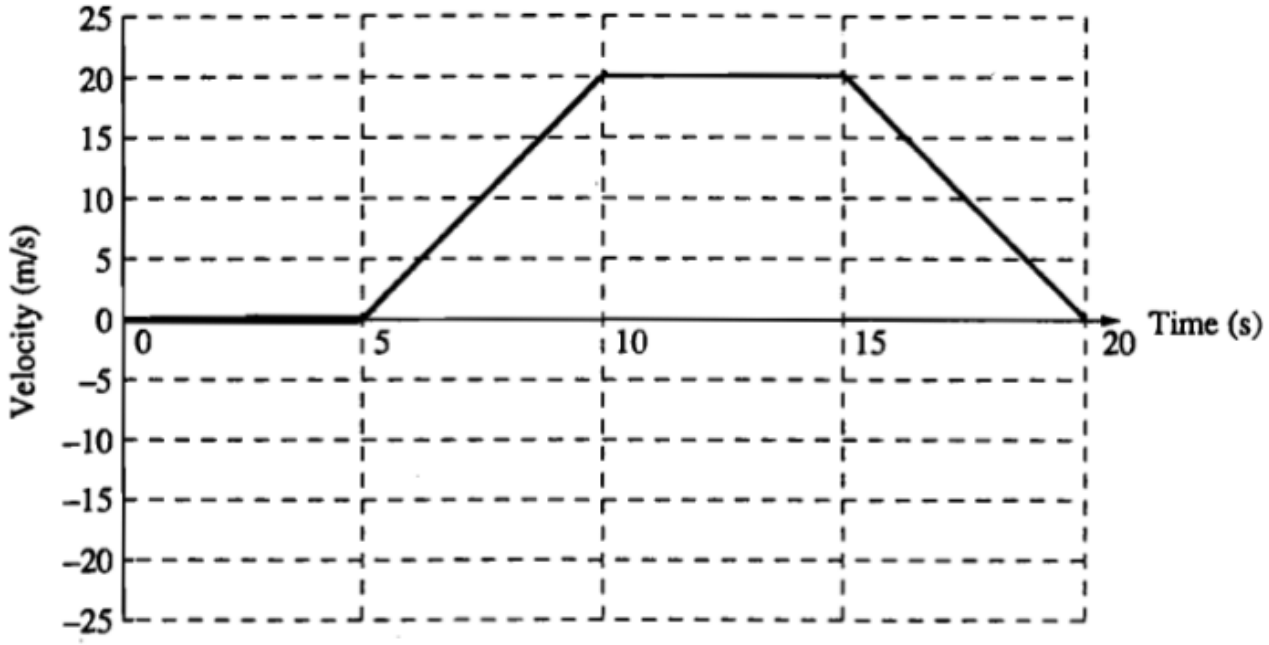
1 point is earned: 20 seconds: v - 20 m/s +(-4 m/s2)(5 s) - 0

If all four values were correct credit was given for the equation even if it was not written explicitly

Alternate solution: For indicating that at the end of each interval v = v0 + Δv

1frq

(ii)



1 point is earned(max 4): for each segment having the correct shape and slope (or which is consistent with graph in part B.

Part D

(i)

1 point is earned: For the kinematic equation applicable for constant acceleration, *x - x0 + v2t + (1/2)at*2 or equivalent

For each time interval,substitute the initial position for that interval, the initial velocity for that interval from part (c), the appropriate acceleration, and a time of 5 seconds.

1 point is earned: 5 seconds: x = 0 + (0)(5s) + (1/2)(0)(5 s)2 = 0

1 point is earned: 10 seconds: x = 0 + (0)(5s) + (1/2)(4 m/s2)(5 s)2 = 50 m

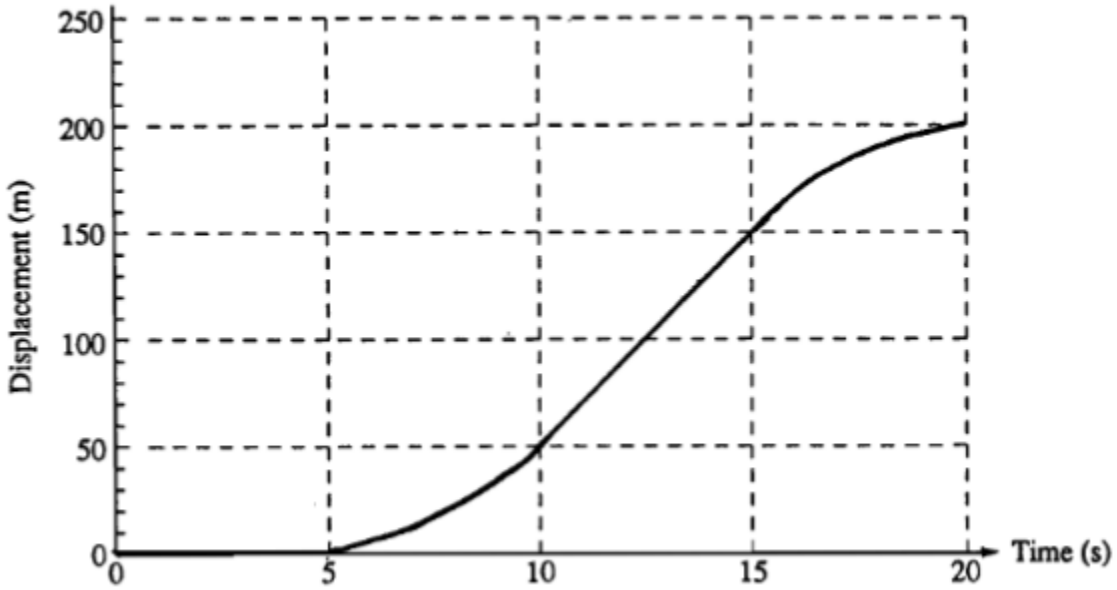
1 point is earned: 15 seconds: x = 50m + (20m/s)(5s) + (1/2)(0 m/s2)(5 s)2 = 150 m

1 point is earned: 20 seconds: x = 150 + (20 m/s)(5s) + (1/2)(-4 m/s2)(5 s)2 = 200 m

If all four values were correct, credit was given for the equation even if it was not explicitly written.)

(ii)

1frq



1 point is earned(max 4): For each segment that has the correct shape and slope (or which is consistent with graph in part (c))

0	1	2	3	4	5	6	7	8	9 ✓

The student response earns 9 of the following points:

(i)

1 point is earned: For the kinematic equation applicable for constant acceleration, *x - x0 + v2t + (1/2)at*2 or equivalent

For each time interval,substitute the initial position for that interval, the initial velocity for that interval from part (c), the appropriate acceleration, and a time of 5 seconds.

1 point is earned: 5 seconds: x = 0 + (0)(5s) + (1/2)(0)(5 s)2 = 0

1 point is earned: 10 seconds: x = 0 + (0)(5s) + (1/2)(4 m/s2)(5 s)2 = 50 m

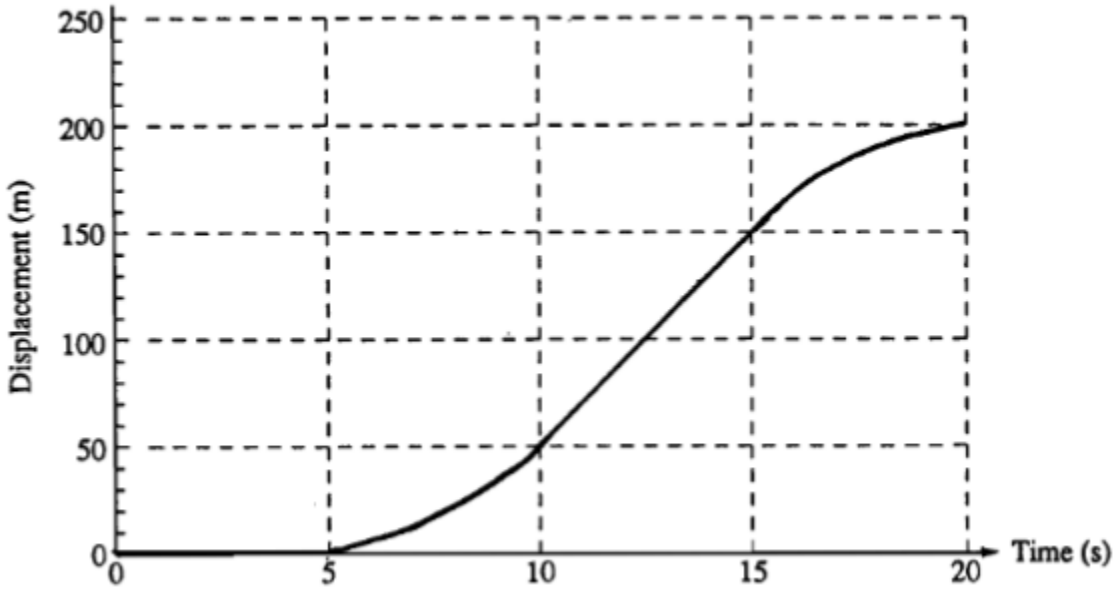
1 point is earned: 15 seconds: x = 50m + (20m/s)(5s) + (1/2)(0 m/s2)(5 s)2 = 150 m

1 point is earned: 20 seconds: x = 150 + (20 m/s)(5s) + (1/2)(-4 m/s2)(5 s)2 = 200 m

If all four values were correct, credit was given for the equation even if it was not explicitly written.)

(ii)

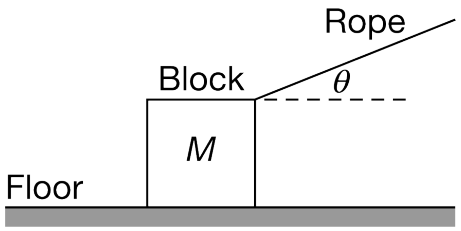
1frq



1 point is earned(max 4): For each segment that has the correct shape and slope (or which is consistent with graph in part (c))

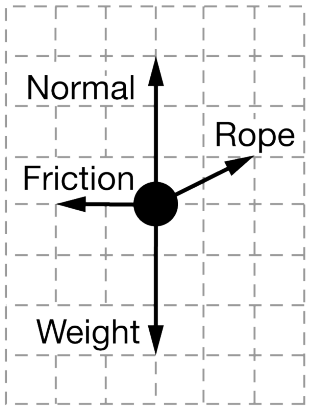
1frq

22. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

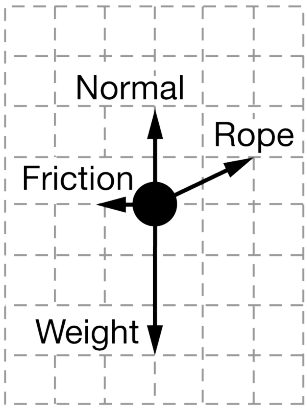


A block of mass M is pulled by a rope that makes an angle θ (theta) above the horizontal, as shown above. The coefficient of kinetic friction between the block and floor is μ (mu).

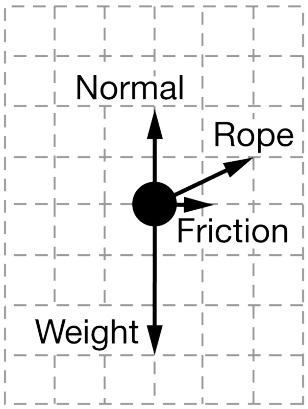
The block is initially at rest and starts accelerating to the right along the floor. Three students each draw a diagram to indicate all of the forces that are exerted on the block as it moves. In the students' diagrams, which are shown below, the lengths of the arrows indicate the relative magnitudes of the forces.



Student 1's Diagram



Student 2's Diagram



Student 3's Diagram

Only one of the students has drawn a diagram that could correctly represent the forces. The other two students' diagrams are incorrect.

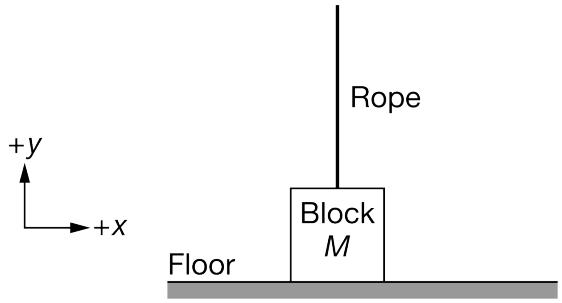
(a) Which student has drawn a diagram that could correctly represent the forces exerted on the block?

(b) For the incorrect student diagrams, explain why the diagram could not represent the forces exerted on the block. If there is more than one specific reason a diagram is incorrect, identify each of the reasons.

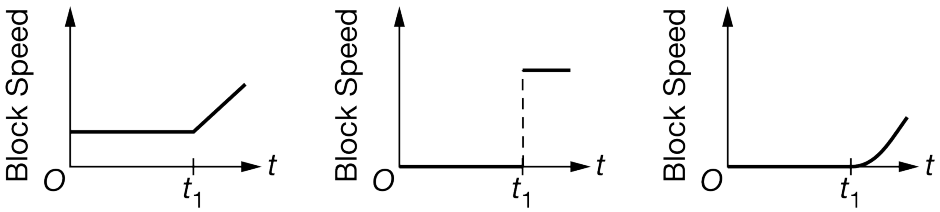
(c) Briefly explain why slightly increasing the force F exerted by the rope on the block (without changing the angle θ) would cause a decrease in the friction force.

(d) The friction force f is $f = \mu(W - F\sin\theta)$, where W is the weight of the block and F is the force exerted by the rope. Explain why this equation is consistent with the claim that an increase in F would cause a decrease in the friction force.

1frq



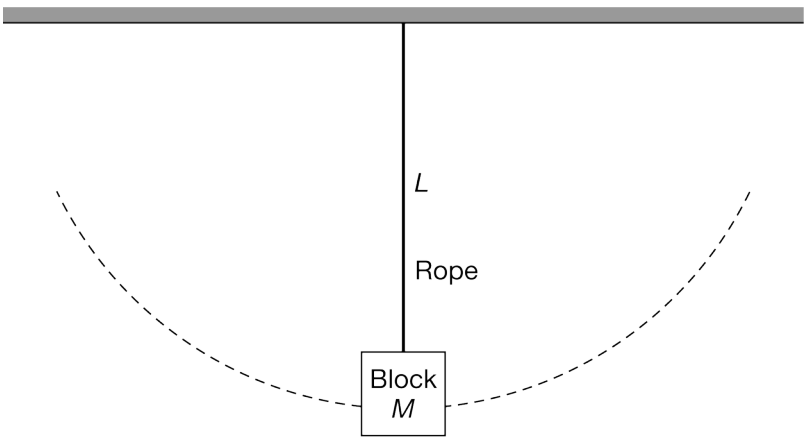
The rope is now attached to the center of the top of the block, as shown above. The force exerted by the rope is initially zero newtons at time $t = 0$. The force steadily increases, becomes equal to the block's weight W at time t_1, and continues to increase further. Three students sketch the graphs shown below to represent the speed of the block as a function of time t. One of the graphs is consistent with this scenario, while the other graphs are not consistent with this scenario.



Graph 1 Graph 2 Graph 3

(e) Which graph is consistent with the scenario in which the force exerted by the rope on the block steadily increases?

(f) Identify all features of the other two graphs that make them inconsistent with this scenario. Explain your reasoning.



Note: Figure not drawn to scale.

1frq

With one end of the rope still attached to the block, the other end of the rope is attached to a ceiling to create a pendulum with length L, where L is much greater than the dimensions of the block. A second identical pendulum is located on the Moon, where the acceleration due to gravity is less than that on Earth. Both pendulums are displaced by the same small angle and released from rest.

(g) Does the pendulum on Earth or the pendulum on the Moon have a greater period of oscillation? Briefly explain your choice without reference to equations, but by describing the restoring forces exerted on and motion of each block.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating that Student 2 drew a correct diagram.

Example Response:

The diagram for student 2 is correct.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that Student 1's diagram has equal left and right forces and thus would result in a constant horizontal velocity or no horizontal acceleration.
- ☐ 1 point is earned for indicating that Student 1's diagram would result in an upward acceleration component since the sum of the upward components is greater than the force of gravity.
- ☐ 1 point is earned for indicating that Student 3's diagram has friction in the incorrect direction.

Example Response:

Student 3 has correctly balanced the vertical forces but has the friction in the same direction as the motion. Student 1 has vertical components in the upward direction that are greater than gravity, so the block would have an upward acceleration. Also, student 1 has equal horizontal components which would not cause an acceleration to the right.

1frq

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that as the pulling force increases, its vertical component increases, decreasing the normal force on the block.
- ☐ 1 point is earned for indicating that since the friction force is proportional to the normal force, the friction force decreases.

Example Response:

As the pulling force increases, its vertical component increases, decreasing the normal force on the block. Since the friction force is proportional to the normal force, the friction force decreases.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for any attempt to use a functional dependence between the friction force and the force *F*.
- ☐ 1 point is earned for indicating that the term with F is subtracted, or that the expression $(W - F\sin\theta)$ decreases when *F* increases.

Example Response:

In the given expression for friction, there is a term that depends on the pulling force F, but that term is subtracted from the weight term. So as F increases, friction decreases.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

1frq

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for identifying Graph 3 as consistent with the scenario in which the force exerted by the rope on the block steadily increases.

Example Response:

Graph 3 is consistent with the scenario in which the force exerted by the rope on the block steadily increases.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that graph 1 shows an initial non-zero speed even though the initial speed is zero.
  Note: If it was indicated that there is ONLY a horizontal component of the velocity before the vertical force of the rope acts on the block (from the previous part of the question), this point can be earned for thinking that graph 1 is correct in the first segment but graphs 2 & 3 are not correct in the first segment.
- ☐ 1 point is earned for indicating that the speed after time in graph 1 is incorrect because the upward force is not constant, so the acceleration (slope of speed graph) should not be constant.
- ☐ 1 point is earned for indicating that t_1 graph 2 is incorrect because of the discontinuity in the graph at t_1.
- ☐ 1 point is earned for indicating that graph 2 is incorrect because the speed is not constant after time t_1, it increases since there is a net force.

Example Response:

Graph 1 is incorrect because the speed should be zero before time t_1 and after t_1 it should have an increasing slope since the net force is increasing. Graph 2 is incorrect because, although the speed is initially zero, the speed is not constant after time t_1 because there is an upward net force.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: A response *can* mention the equation for a pendulum, and still get full credit as long as the points described below are earned.

1frq

0	1	2 ✓

The response accurately includes **both** of the following criteria.

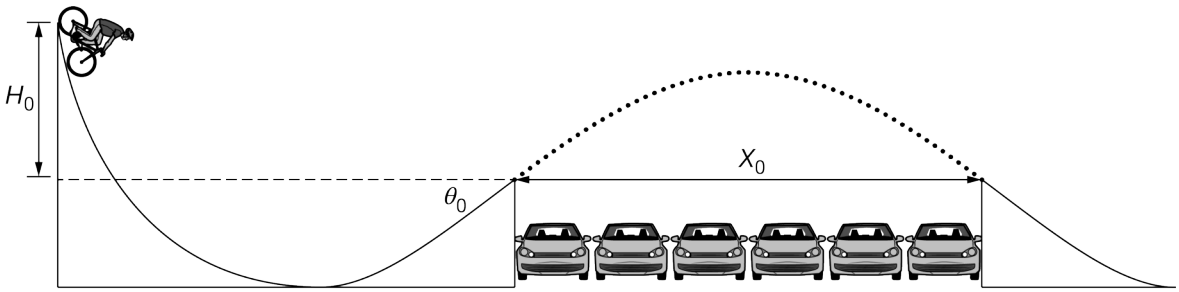
- ☐ 1 point is earned for indicating that the pendulum on the Earth has a greater restoring force at maximum displacement from equilibrium.
- ☐ 1 point is earned for indicating that since the two pendulums have the same length, the amplitude of the motion is the same, and thus the pendulum on the moon, with a smaller acceleration, will take longer to complete once cycle.

Example Response:

Since both pendulums have the same length, they will both have the same amplitude. But the pendulum on the Earth will have a greater restoring force and acceleration, so it will take less time to return to equilibrium. Thus, the pendulum on the moon will have a greater period.

1frq

23. This question is a short free-response question. Show your work for each part of the question.



Note: Figure not drawn to scale.

(7 points, suggested time 13 minutes)

A stunt cyclist builds a ramp that will allow the cyclist to coast down the ramp and jump over several parked cars, as shown above. To test the ramp, the cyclist starts from rest at the top of the ramp, then leaves the ramp, jumps over six cars, and lands on a second ramp.

H_0 is the vertical distance between the top of the first ramp and the launch point.
θ_0 is the angle of the ramp at the launch point from the horizontal.
X_0 is the horizontal distance traveled while the cyclist and bicycle are in the air.
m_0 is the combined mass of the stunt cyclist and bicycle.

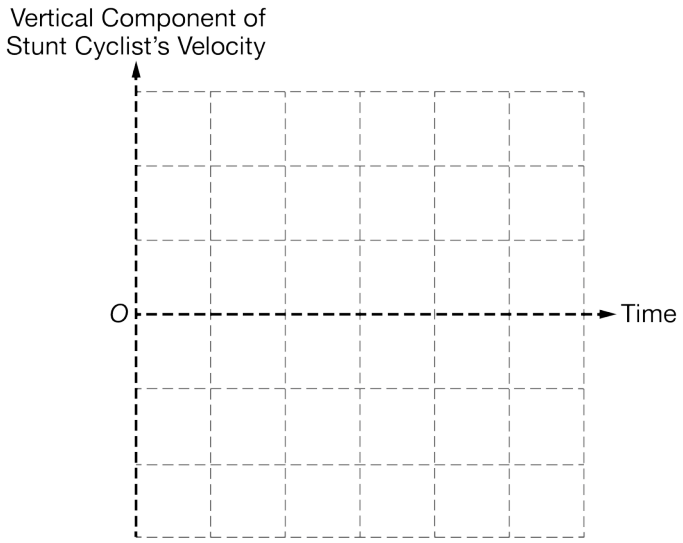
(a) Derive an expression for the distance X_0 in terms of H_0, θ_0, m_0, and physical constants, as appropriate.

(b) If the vertical distance between the top of the first ramp and the launch point were $2H_0$ instead of H_0, with no other changes to the first ramp, what is the maximum number of cars that the stunt cyclist could jump over?

Justify your answer, using the expression you derived in part (a).

(c) On the axes below, sketch a graph of the vertical component of the stunt cyclist's velocity as a function of time from immediately after the cyclist leaves the ramp to immediately before the cyclist lands on the second ramp. On the vertical axis, clearly indicate the initial and final vertical velocity components in terms of H_0, θ_0, m_0, and physical constants, as appropriate. Take the positive direction to be upward.

1frq



Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for using conservation of energy to find the speed v of the bicycle as it leaves the ramp.
- ☐ 1 point is earned for using kinematics, vertical components, attempting to find the time the bicycle is in the air.
- ☐ 1 point is earned for a correct expression for X_0 in terms of given quantities.

Example Response:

$E_{top} = E_{bottom}$
$m_0 g H_0 = \frac{1}{2} m_0 v^2$
$v = \sqrt{2gH_0}$
$v_{fy} = v_{iy} + at$
$-v \sin\theta = v \sin\theta - gt$
$-2v \sin\theta = -gt$

$2 \sin\theta_0 \sqrt{2gH_0} = gt$
$t = \frac{2 \sin\theta_0 \sqrt{2gH_0}}{g}$
$X_0 = v_x t$
$X_0 = \cos\theta_0 \sqrt{2gH_0} \frac{2 \sin\theta_0 \sqrt{2gH_0}}{g}$
$X_0 = 4H_0 \cos\theta_0 \sin\theta_0$

Scoring Note:

Using the range equation to get $X_0 = 2H_0 \sin 2\theta_0$ is sufficient to earn the second and third points.

1frq

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an answer and justification that attempts to use the functional dependence of the horizontal distance on the initial height.
- ☐ 1 point is earned for an answer consistent with the expression derived in part (a).

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

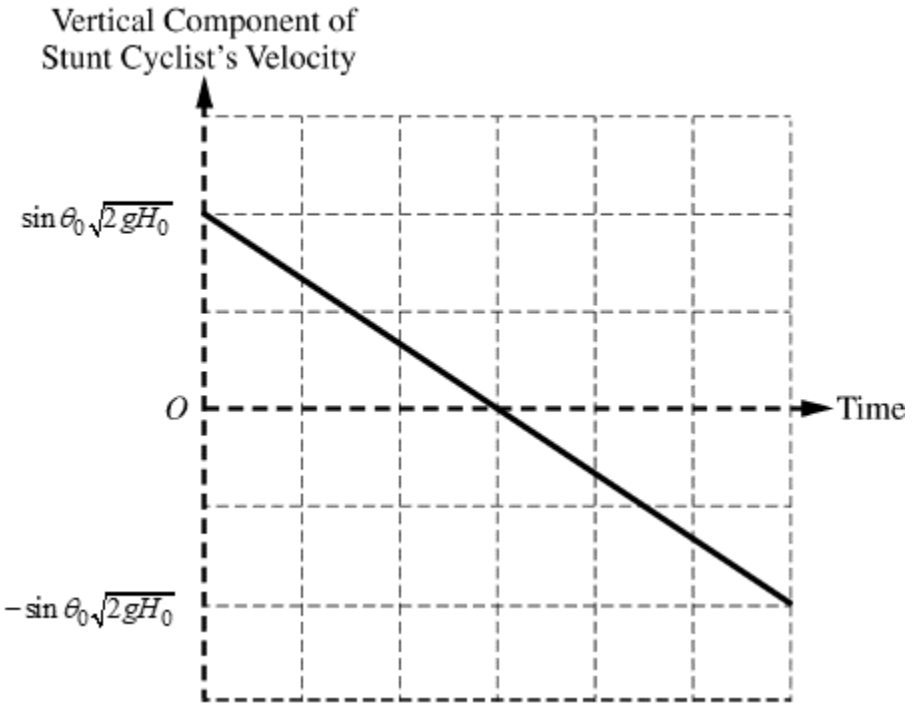
0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a linear graph with a constant negative slope.
- ☐ 1 point is earned for a graph that starts at v_y and ends at $-v_y$, using only allowed variables.

Example Response:

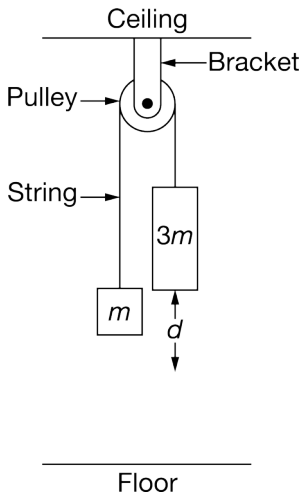
1frq


Vertical Component of
Stunt Cyclist's Velocity
$\sin\theta_0\sqrt{2gH_0}$
O
Time
$-\sin\theta_0\sqrt{2gH_0}$

1frq

24. This question is a short free-response question. Show your work for each part of the question.

(7 points, suggested time 13 minutes)



Two blocks, with masses m and $3m$, are attached to the ends of a string with negligible mass that passes over a pulley, as shown above. The pulley has negligible mass and friction and is attached to the ceiling by a bracket. The blocks are simultaneously released from rest.

(a) Derive an equation for the speed v of the block of mass $3m$ after it falls a distance d in terms of m, d, and physical constants, as appropriate.

(b) Determine the work done by the string on the two-block system as each block moves a distance d.

(c) The acceleration of the center of mass of the blocks-string-pulley system has magnitude a_{COM}. Briefly explain, in terms of any external forces acting on the system, why a_{COM} is less than g.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

This solution applies dynamics and kinematics.

0	1	2	3	4 ✓

The response accurately includes four of the following criteria.

- ☐ For using an appropriate equation for Newton's second law

1frq

- For using Newton’s second law to find the magnitude of the common acceleration of the blocks
- For using correct kinematics equation(s) to find the speed of the block
- For a correct answer, or an answer consistent with the acceleration identified earlier, with supporting work

Example Response:

$\Sigma \overrightarrow{F} = m\overrightarrow{a}$

Let F_{gA} and F_{gB} be the gravitational forces exerted on the blocks of mass m and $3m$, respectively.

Example 1:

Apply Newton’s second law to the two-block system along the path of motion.

$\Sigma \overrightarrow{F} = (3m + m)\overrightarrow{a}$
$F_{gB} - F_{gA} = (3m + m)a$
$3mg - mg = 4ma$
$2mg = 4ma$
$a = 2mg/4m = 2g/4 = g/2$

Example 2:

For block B: $\Sigma \overrightarrow{F} = 3m\overrightarrow{a}$
$F_{gB} - F_T = 3ma$
$3mg - F_T = 3ma$
$F_T = 3mg - 3ma$

For block A: $\Sigma \overrightarrow{F} = m\overrightarrow{a}$
$F_T - F_{gA} = ma$
$3mg - 3ma - mg = ma$
$4ma = 2mg$
$a = 2mg/4m = 2g/4 = g/2$

Example 1:

$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$

Example 2:

$y = y_0 + v_{y0}t + 1/2a_yt^2 \text{ and } v_y = v_{y0} + a_yt$

Example 1:

1frq

$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$
$v_y^2 = (0)^2 - 2(\frac{g}{2})(0 - d) = (-g)(-d) = gd$
$v_y = \pm\sqrt{gd}$
$|v_y| = \sqrt{gd}$

Example 2:

$\text{First apply } y = y_0 + v_{y0}t + 1/2a_yt^2$
$0 = d + (0)(t) - (1/2)(g/2)t^2$
$-d = -(g/4)t^2$
$t^2 = 4d/g, \text{ thus } t = \pm 2\sqrt{d/g}$
$|t| = 2\sqrt{d/g}$
$\text{Then apply } \ v_y = v_{y0} + a_yt$
$v_y = (0) - (g/2)(2\sqrt{d/g}) = -\sqrt{gd}$
$|v_y| = \sqrt{gd}$

Part A - Alternate Solution

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

This solution applies the conservation of energy.

0	1	2	3	4 ✓

The response accurately includes four of the following criteria.

- ☐ *For using an appropriate equation for conservation of energy*
- ☐ *For correctly relating the types of energy present (e.g., correct expressions are used for the gravitational potential energy of both mass-Earth systems and the kinetic energy of both masses)*
- ☐ *For correct qualitative application for the upward motion of the block of mass M (i.e. - appropriate signs for gravitational potential energy terms for the two blocks)*
- ☐ *For a correct answer with supporting work*

Example Response:

Example 1:

$-\Delta U = \Delta K$

Example 2:

1frq

$U_{iB} + U_{iA} + K_{iB} + K_{iA} = U_{fB} + U_{fA} + K_{fB} + K_{fA}$

$-\Delta U = \Delta K$
$-(\Delta U_B + \Delta U_A) = K_B + K_A$
$3mgd - mgd = (1/2)(3m)v^2 + (1/2)(m)v^2$
$2mgd = 2mv^2$
$gd = v^2$
$v = \pm\sqrt{gd}$
$|v| = \sqrt{gd}$

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

This point can be earned if an incorrect unit is written or if no unit is indicated.

0	1 ✓

The response accurately includes:

- ☐ A correct answer of zero OR an algebraic expression (in terms of m , d , and/or g) that is equivalent to zero.

Example Response:

Example 1:

0 J

Example 2:

$W_T = F_T d + (-F_T d) = (3/2)mgd - (3/2)mgd = 0\,\text{J}$

Example 3:

$W_S + W_g = \Delta K$

$W_S = \Delta K - W_g$
$= (1/2)(m + 3m)v^2 - (3mgd - mgd)$
$= 2mv^2 - 2mgd = 2mgd - 2mgd = 0\,\text{J}$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

1frq

Note: String tension is NOT a force on the system because the string is part of the system

0	1	2 ✓

The response accurately includes both of the following criteria:

- ☐ For indicating that there is an upward force on the system
- ☐ For relating the net force to the acceleration of the center of mass to show that $a_{\text{COM}} < g$ OR for showing that F_{net} is less than the weight of the system

Example Response:

The force of gravity acts downward on the system, but there is an upward external force exerted on the pulley by the bracket attached to the ceiling, This makes the net force on the system less than the weight of the system. Thus, the acceleration of the system is less than g $F_{\text{net}} = ma_{\text{COM}}$ and if $F_{\text{net}} < mg$ then $a_{\text{COM}} < g$.

Claim: a_{COM} is less than g .

Evidence: The system is supported at the pulley by the bracket (shown in earlier figure).

Reasoning Statements: The upward supporting force exerted by the bracket makes the net force on the system less than the weight of the system. Thus, the acceleration of the system is less than g .

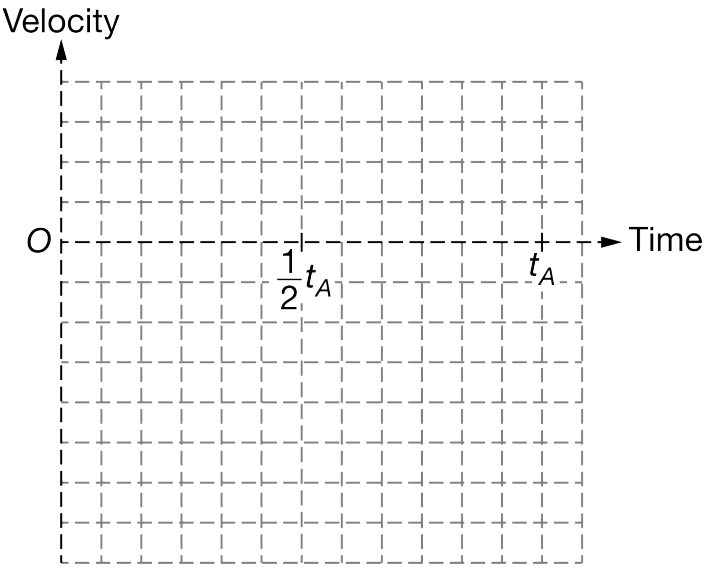
1frq

25. This question is a short free-response question. Show your work for each part of the question.

(7 points, suggested time 13 minutes)

In an experiment two identical rocks are simultaneously thrown from the edge of a cliff a distance h_0 above the ground. Rock A is thrown vertically upward with speed v_0 and rock B is thrown vertically downward with speed v_0. Rock A and rock B strike the ground at times t_A and t_B, respectively. Consider the positive vertical direction to be upward.

(a) On the axes given below, sketch and label graphs of the velocity as a function of time for rock A and rock B. Label the time t_B. Times t_A and $\frac{1}{2}t_A$ are given on the graph.



(b) Rock B hits the ground at time t_B. Derive an equation for the time t_A it takes rock A to hit the ground in terms of v_0, t_B, and physical constants, as appropriate.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: The amount of current that flows through each resistor must be indicated to earn the second point of part (a).

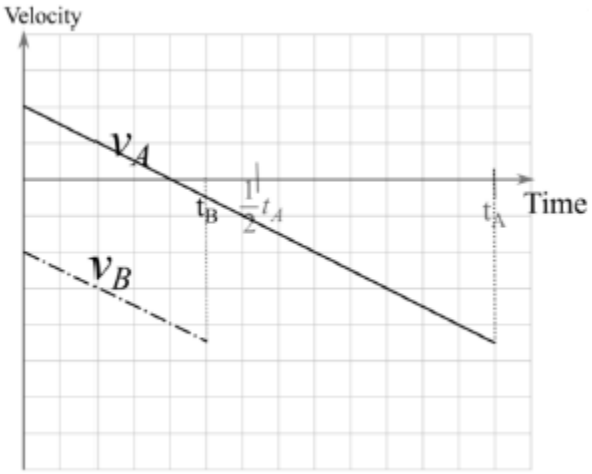
0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria:

1frq

- [ ] For a graph that shows that the initial velocity of rock A is positive, and the initial velocity of rock B is the negative value of the initial velocity of rock A.
- [ ] For a graph that contains two parallel lines with the same negative slope
- [ ] For a graph of the velocity of rock A that only crosses the horizontal axis before $\frac{1}{2}t_A$.
- [ ] For a graph of the velocity of rock B where the velocity of rock B at t_{B} is the same as the velocity of rock A at t_{A}

Example Response:



Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

- [ ] For a correctly determining the time rock A takes to move upwards and return to its original position
  $v = v_0 + at$
  $-v_0 = v_0 - gt_{\text{return}}$
  $-2v_0 = -gt_1$
  $t_{\text{return}} = \frac{2v_0}{g}$
- [ ] For implicitly or explicitly recognizing that the time for both rocks to move downwards from the top of the cliff to the ground is the same.
  $t_{\text{ground}} = t_B$
- [ ] For a correct equation for t_{A} in terms of t_{B}
  $t_A = t_{\text{return}} + t_{\text{ground}}$
  $t_A = \frac{2v_0}{g} + t_B$

1frq

Example Response:

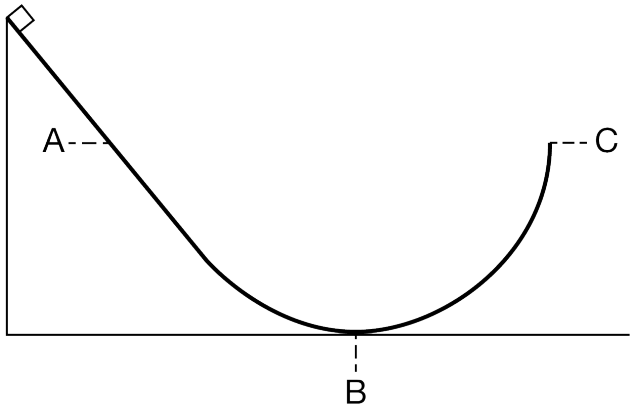
$v = v_0 + at$
$-v_0 = v_0 - gt_{\text{return}}$
$-2v_0 = -gt_1$
$t_{\text{return}} = \frac{2v_0}{g}$

$t_{\text{ground}} = t_B$

$t_A = t_{\text{return}} + t_{\text{ground}}$
$t_A = \frac{2v_0}{g} + t_B$

1frq

26. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)



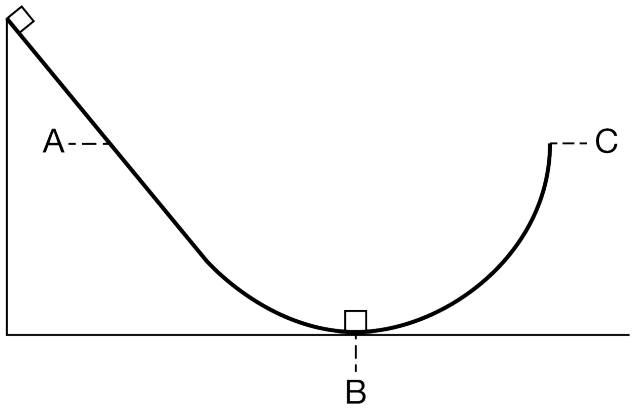
The figure above shows a track consisting of a straight section that connects to a circular section. Three locations are indicated: location A on the straight section of the track, location B at the bottommost point of the track, and location C at the point where the track is vertical. Locations A and C are at the same height. A block is released from rest at the top of the straight section of the track. There is negligible friction between the track and the block.

(a) Is the magnitude of the normal force exerted on the block by the track at location A greater than, less than, or equal to the normal force at location B? Justify your answer.

(b) In a coherent, paragraph-length response, describe the relative angle between the direction of the block's velocity and the direction of the block's acceleration at the following times.

When the block is at location A
When the block is at location B
Immediately before the block reaches location C

Specifically, indicate whether the velocity is parallel to the acceleration, perpendicular to the acceleration, or neither, at each location. Use physical principles to justify your responses.



1frq

(c) The scenario is repeated, but an identical block is at rest at location B, as shown above. When the block at the top of the straight section of the track is released from rest, it slides down the track and collides elastically with the identical block and comes to rest. The identical block then begins sliding up the ramp. Is the maximum height reached by the identical block greater than, less than, or equal to the maximum height reached by the block in the original scenario? Briefly explain your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the normal force at A is less than at B.
- ☐ 1 point is earned for a correct explanation including both locations A and B (e.g., at A the normal force is less than the weight, but at B it is greater than the weight).

Example Response:

At A, the block's acceleration is along the straight track, so the normal force must cancel the component of the gravitational force normal to the track and is therefore less in magnitude than the block's weight. At B, the block's acceleration and net force are upward, so the upward normal force must be greater than the block's weight.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the velocity is always parallel to the track.
- ☐ 1 point is earned for relating the acceleration to the net force.
- ☐ 1 point is earned for indicating the acceleration at location A is along the track and thus parallel to velocity.
- ☐ 1 point is earned for indicating that at location B the velocity is perpendicular to the acceleration.
- ☐ 1 point is earned for indicating that just before location C the acceleration is neither perpendicular nor parallel to the normal force.
- ☐ 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the

1frq

paragraph-length response.

Example Response:

The velocity of the box is along the track.

At location A, the box is accelerating down the ramp because there is a net force exerted on the box down the ramp. The net force and acceleration always have the same direction. If the net force is down the track then the acceleration is down the track. So that makes the velocity and acceleration in the same direction at location A, therefore parallel.

At location B, the bottommost point, the box is moving in a circle. Here the velocity is still along the track. Since location B is the bottommost point of a circle, the net force and acceleration would be directed toward the center of that circle. So that makes the velocity and acceleration perpendicular at location B.

Just before location C, the box is both moving in a circle and slowing down as it rises above the ground. The velocity is in the direction of motion, which is upward. Since the box is slowing down, the acceleration must have a downward component. The acceleration would also have a component toward the center of the block's circular path. So, the acceleration is neither parallel nor perpendicular to the upward velocity at just before location C.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the maximum height reached by the second block is the same as that of the original block.
- ☐ 1 point is earned for a correct explanation of why the second block reaches the same height.

Example Response:

Since the collision is elastic and the first block comes to rest, the second block has the same kinetic energy that the first block had just before the collision, which is the same as the first block's kinetic energy at the bottom of the ramp in the original scenario.