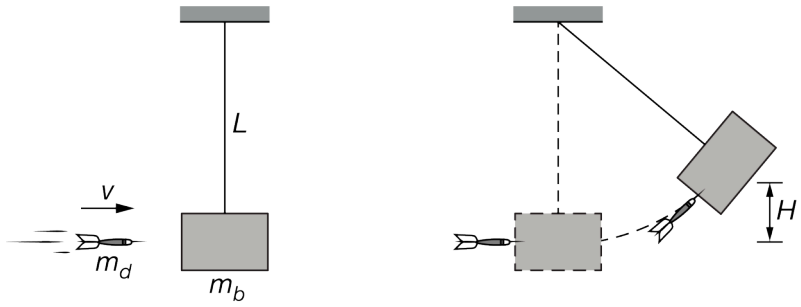
frq2

1. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown at left above. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown at right above. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, the dart or the block, if either, experiences an impulse of larger magnitude during the collision. If the impulse is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the speed of the dart as it embeds itself into the block is greater than *v*, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the motion of the dart-block system at the instant it reaches height *H*. The students make the following statements:

Student 1: The speed of the system at height *H* is zero. Since it is moving in circular motion and centripetal acceleration is given by $a_c = \frac{v^2}{r}$, the acceleration must also be zero.

Student 2: The acceleration of the system at height *H* can't be zero, or the system would not swing back down. The acceleration must be equal to *g* at height *H*.

(c) Which underlined phrase or phrases, if either, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if either, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if either, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases, if either, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{m_d v}{m_d + m_b}\right)^2$. Explain how this

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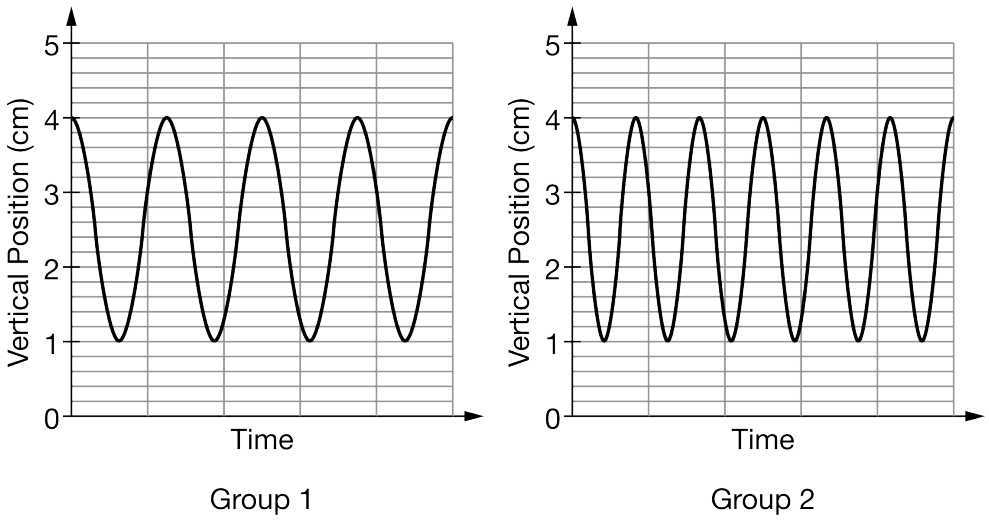
equation supports your claim in part (b).

The students perform several experimental trials. In each trial, a different speed v is used, and the resulting maximum height reached by the center of mass of the dart-block system is measured.

(h) Which quantities could the students graph to produce a straight line whose slope can be used to find the acceleration due to gravity, g ?

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart mass, initial dart speed, block mass, or string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct explanation, in terms of force & time (using Newton's third law)
  OR
  a correct explanation in terms of change in momentum (using conservation of momentum).

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Example Response:

Both the dart and the block experience the same impulse because they exert equal and opposite forces on each other because of Newton's third law, and the forces are exerted for equal times.

OR

By conservation of momentum, the decrease in the momentum of the dart must be equal and opposite to the increase in the momentum of the block, and therefore the impulse is the same in magnitude for both objects.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the final height is greater.
- ☐ 1 point is earned for a statement of conservation of momentum to indicate that the final speed of the dart-block system is greater **AND** an indication of conservation of energy leading to a greater height.

Example Response:

Because the momentum of the dart before the collision must be equal to the momentum of the dart-block system after the collision, a higher dart speed (a higher dart momentum) will lead to a higher dart-block speed after the collision (a higher dart-block momentum). Because the dart-block is traveling faster after the collision, it has more kinetic energy which means that the dart-block will swing higher.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Responses:

Student 1 is correct in saying that the speed at height H is zero

OR

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The first statement.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Responses:

Student 1 is incorrect in saying that the acceleration is zero at the height H

OR

The second statement.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Responses:

Student 2 is correct in saying the acceleration of the system at height H can't be zero or the system would not swing back down

OR

The first statement.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Responses:

Student 2 is incorrect in saying that the acceleration is g at height H

OR

The second statement.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement including functional dependence of the variables.
- ☐ 1 point is earned for correctly asserting that as the speed increases in the equation *H* also increases .

Example Response:

This equation matches my reasoning in part (b) – if the speed of the dart increases, the height of the swing increases because v is in the numerator.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for plotting a variant of *v* on one axis.

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- [ ] 1 point is earned for plotting a variant of H on the other axis.
- [ ] 1 point is earned for a correct power relationship between the two variables, i.e., H vs. v squared.

Example Response:

Students could graph H as a function of v squared.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- [ ] 1 point is earned for recognizing that the period in the second graph is shorter.
- [ ] 1 point is earned for a statement about the dependence of the period on the length of the pendulum.
- [ ] 1 point is earned for indicating that the length of the pendulum must be shorter for a shorter period.

Example Response:

Since the period is shorter for group 2, and the period is related to the length, if the period became shorter with no other changes, the length of the string must have been shortened.

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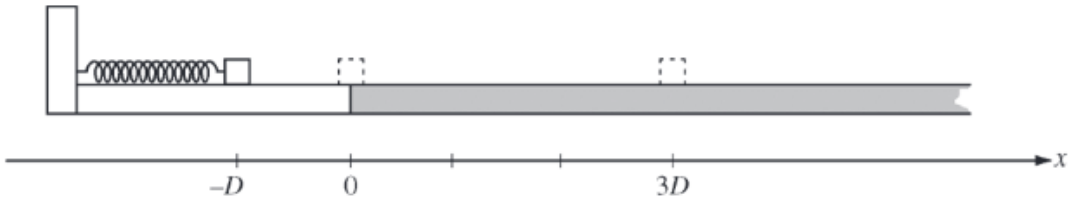
2.

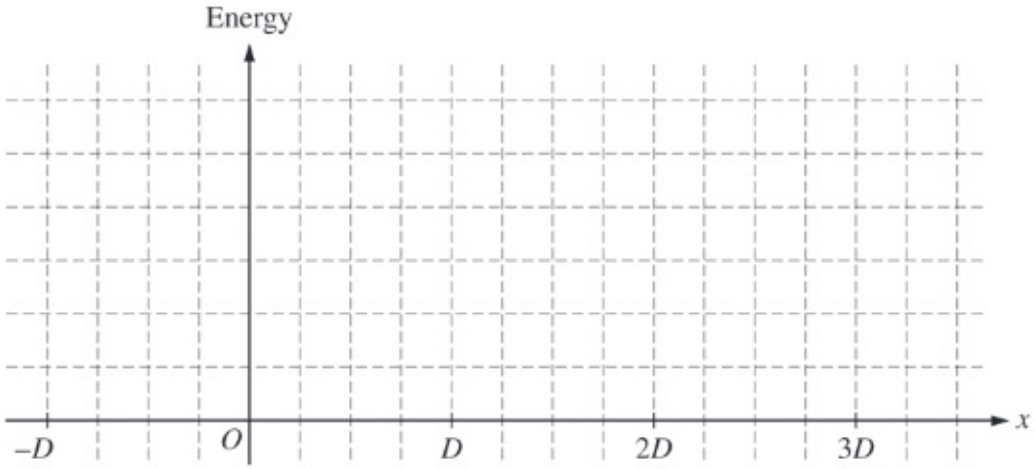


A block is initially at position x = 0 and in contact with an uncompressed spring of negligible mass. The block is pushed back along a frictionless surface from position *x = 0 to x = -D*, as shown above, compressing the spring by an amount $\Delta x = D$. The block is then released. At *x = 0* the block enters a rough part of the track and eventually comes to rest at position *x = D*. The coefficient of kinetic friction between the block and the rough track is μ.

(a) On the axes below, sketch and label graphs of the following two quantities as a function of the position of the block between x = -D and x = 3D. You do not need to calculate values for the vertical axis, but the same vertical scale should be used for both quantities.

i. The kinetic energy *K* of the block

ii. The potential energy *U* of the block-spring system



The spring is now compressed twice as much, to $\Delta x = 2D$. A student is asked to predict whether the final position of the block will be twice as far at *x = 6D*. The student reasons that since the spring will be compressed twice as much as before, the block will have more energy when it leaves the spring, so it will slide farther along the track before stopping at position *x = 6D*.

b)

i. Which aspects of the student's reasoning, if any, are correct? Explain how you arrived at your answer.

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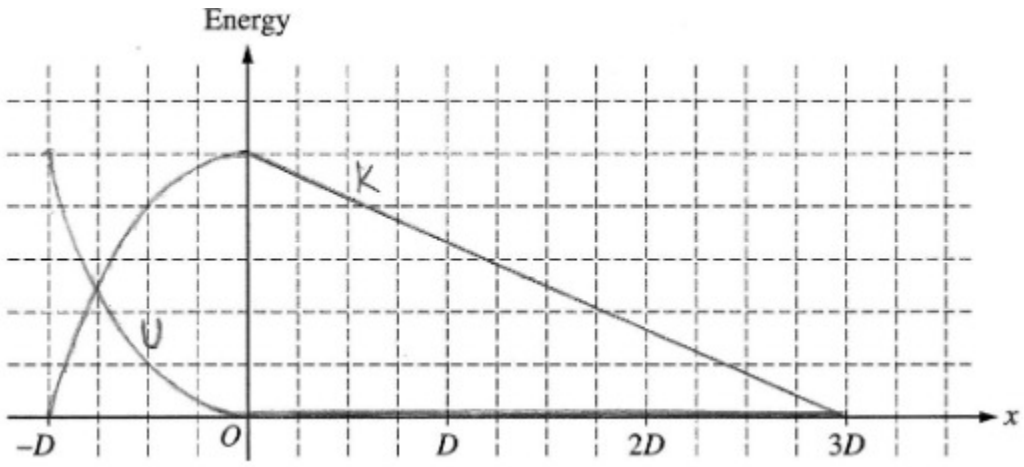
ii. Which aspects of the student's reasoning, if any, are incorrect? Explain how you arrived at your answer.

(c) Use quantitative reasoning, including equations as needed, to develop an expression for the new final position of the block. Express your answer in terms of D.

(d) Explain how any correct aspects of the student's reasoning identified in part (b) are expressed by your mathematical relationships in part (c). Explain how your relationships in part (c) correct any incorrect aspects of the student's reasoning identified in part (b). Refer to the relationships you wrote in part (c), not just the final answer you obtained by manipulating those relationships.

Part A



1 point is earned: For sketching either energy curve with a reasonably correct shape between $x = -D$ and $x = 0$, with zero and maximum values at the correct locations

1 point is earned: For sketching two curves from $x = -D$ to $x = 0$ with shapes and values such that the total energy is constant (even if the curves are incorrect)

1 point is earned: For sketching potential energy equal to zero from $x = 0$ to $x = 3D$

1 point is earned: For sketching kinetic energy as a linear function from its maximum value at $x = 0$ to zero at $x = 3D$

0	1	2	3	4 ✓

The student response earns all of the following points:

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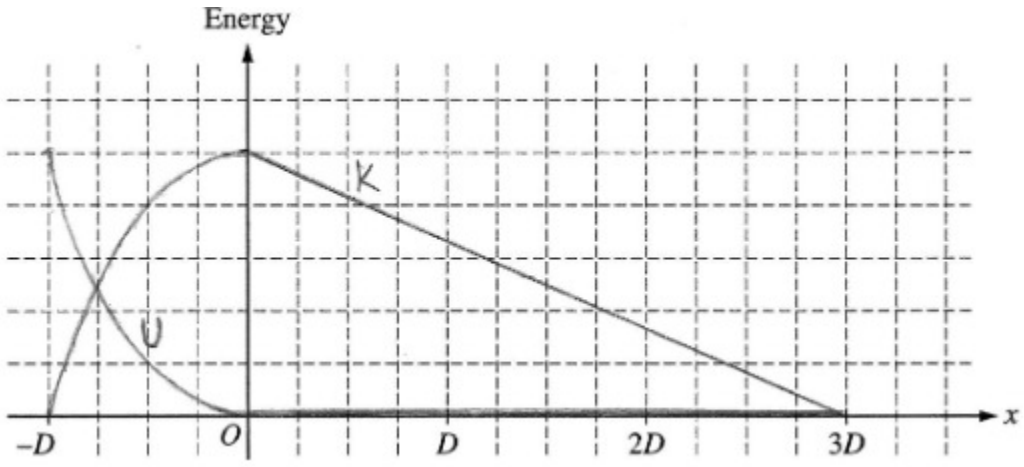

1 point is earned: For sketching either energy curve with a reasonably correct shape between $x = -D$ and $x = 0$, with zero and maximum values at the correct locations

1 point is earned: For sketching two curves from $x = -D$ to $x = 0$ with shapes and values such that the total energy is constant (even if the curves are incorrect)

1 point is earned: For sketching potential energy equal to zero from $x = 0$ to $x = 3D$

1 point is earned: For sketching kinetic energy as a linear function from its maximum value at $x = 0$ to zero at $x = 3D$

Part B

(i)

1 point is earned: For identifying that the student is correct that the block will have more energy when it leaves the spring

(ii)

1 point is earned: For identifying that the student is incorrect about the new final position of the block because the spring's energy does not scale linearly with its compression

✓

0	1	2

The student response earns all of the following points:

(i)

1 point is earned: For identifying that the student is correct that the block will have more energy when it leaves the spring

(ii)

1 point is earned: For identifying that the student is incorrect about the new final position of the block because the

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spring’s energy does not scale linearly with its compression

Part C

1 point is earned: For indicating that the final energy in the spring (which becomes the mechanical energy of the block as it reaches the rough track) is four times the original energy in the spring

1 point is earned: For indicating that the frictional force remains the same

1 point is earned: For equating the initial energy in the spring to an expression that shows that the energy dissipated by friction is proportional to the distance the block slides down the rough track

Example:

$U_1 = \frac{1}{2}kD^2 and, U_2 = \frac{1}{2}k(2D)^2, so, U_2 = 4U_1$
$W_1 = \mu mg(3D) and, W_2 = \mu mg\Delta x_2$
$W_1 = U_1 and, W_2 = U_2 = 4U_1 = 4W_1$
$\mu mg\Delta x_2 = 4(\mu mg(3D))$
$\Delta x_2 = 4(3D) = 12D$

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For indicating that the final energy in the spring (which becomes the mechanical energy of the block as it reaches the rough track) is four times the original energy in the spring

1 point is earned: For indicating that the frictional force remains the same

1 point is earned: For equating the initial energy in the spring to an expression that shows that the energy dissipated by friction is proportional to the distance the block slides down the rough track

Example:

$U_1 = \frac{1}{2}kD^2 and, U_2 = \frac{1}{2}k(2D)^2, so, U_2 = 4U_1$
$W_1 = \mu mg(3D) and, W_2 = \mu mg\Delta x_2$
$W_1 = U_1 and, W_2 = U_2 = 4U_1 = 4W_1$
$\mu mg\Delta x_2 = 4(\mu mg(3D))$
$\Delta x_2 = 4(3D) = 12D$

Part D

1 point is earned: For indicating that the student’s correct reasoning that the block has more energy in the second situation is expressed by the calculations comparing the initial energy in the spring

1 point is earned: For indicating that the student’s correct reasoning that the block will slide farther is expressed by an

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equation that indicates that the work done by friction to stop the block in the second situation is some factor greater than the work done in the first situation

1 point is earned: For indicating that the student's incorrect reasoning that energy scales linearly with the spring's compression is corrected by the expression for the initial energy of the spring

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For indicating that the student's correct reasoning that the block has more energy in the second situation is expressed by the calculations comparing the initial energy in the spring

1 point is earned: For indicating that the student's correct reasoning that the block will slide farther is expressed by an equation that indicates that the work done by friction to stop the block in the second situation is some factor greater than the work done in the first situation

1 point is earned: For indicating that the student's incorrect reasoning that energy scales linearly with the spring's compression is corrected by the expression for the initial energy of the spring

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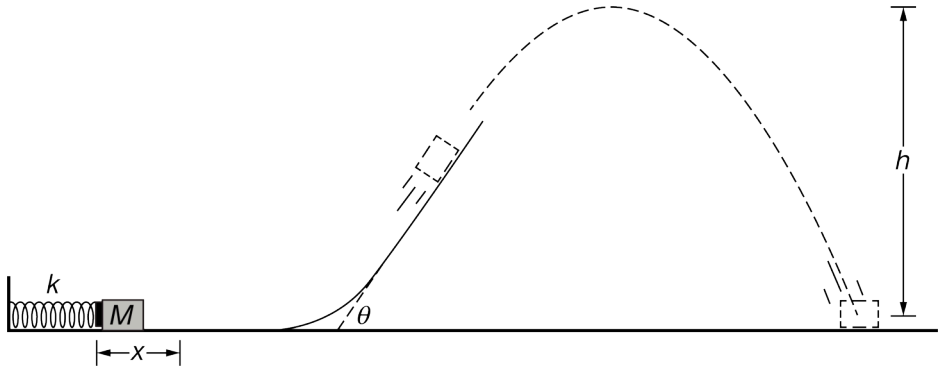
3. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A box is pressed against a horizontal spring, compressing the spring from its relaxed length. The box is then released, and the spring launches the box horizontally along a track that ends in a ramp, as shown above. The box has enough speed to leave the ramp, and the box reaches a maximum vertical height above the floor. Assume there is negligible friction between the box and the track and air resistance is negligible.

k is the spring constant of the spring.
x is the distance the spring is compressed.
M is the mass of the box.
θ (theta) is the angle of the ramp from the horizontal.
h is the maximum height reached by the box.

The scenario is repeated using a stiffer spring with a spring constant that has a greater value for k than that of the original spring. The stiffer spring is compressed the same distance x, and the same box is used.

(a) Indicate how using a stiffer spring would affect the maximum height reached by the box, and explain why without mathematically deriving a relation for h.

(b) Students derive an equation for the maximum height, $h = \frac{(\sin^2\theta)kx^2}{2Mg}$, which may or may not be correct. Is this equation consistent with your claim in part (a)? Explain why or why not.

Two students are discussing the motion of the box when it is at its maximum height and make the following statements:

Student 1: The speed of the block at its maximum height is zero because the kinetic energy is a minimum at the top of the trajectory.

Student 2: Only the vertical component of the velocity is zero at the maximum height, because the vertical component of the acceleration is zero at the top of the trajectory.

(c) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

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(d) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

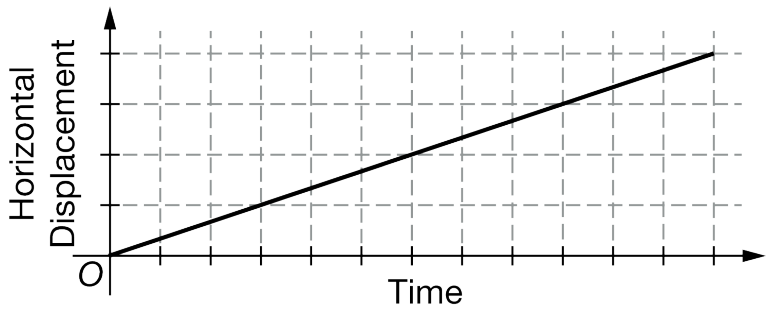
(e) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(g) Consider the system consisting of the box and the spring, but not Earth. How does the energy of the system when the spring is fully compressed compare to the energy of the system at the moment immediately before the box hits the ground? Justify your answer.

(h) If the friction between the track and the box were not negligible, state one way in which the setup could be changed such that the maximum height of the box was the same as in the original setup.

The graph below shows the horizontal component of the displacement of the box as a function of time. The graph represents the entire time the box is in the air after the box is launched from the ramp at an angle θ to the horizontal.



(i) How would the slope of the graph be different, if at all, if the ramp angle were larger than θ, but less than 90 degrees, and the height of the end of the ramp were the same? Justify your response.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that with a stiffer spring the spring constant would increase (or the average force would increase), so the speed of the box as it leaves the spring will be larger.
- ☐ 1 point is earned for a statement that because the speed of the box as it leaves the spring will be greater, and everything else remains the same, the maximum height of the block will increase.

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Example Response:

A stiffer spring means that the spring constant is larger, which means that there is more initial energy stored in the spring (or that the average net force on the box is greater), which means that the launch speed is greater, so with everything else the same the box will reach a larger maximum height.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating "yes", with an attempt at a justification using functional dependence.
- ☐ 1 point is earned for indicating that, since k is in the numerator if k increases, h will increase.

Example Response:

Yes, it is consistent with my reasoning – since k is in the numerator, if k increases, h will also increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the kinetic energy is a minimum at the top of the trajectory.

OR

The second statement

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 1 is incorrect in saying that the speed of the block is zero at the max height

OR

The first statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that only the vertical component of the speed is zero at the max height

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

frq2

- [ ] 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 2 is incorrect in saying that the vertical component of the acceleration is zero at the top of the trajectory.

OR

The second statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- [ ] 1 point is earned for indicating that the energy of the box/spring system remains the same.
- [ ] 1 point is earned for indicating that there is mechanical energy in the spring
  OR
  For indicating that the box returns to the same height.
- [ ] 1 point is earned for indicating that just before the box lands all the energy is kinetic energy, and none is lost
  OR
  For indicating that the net work done by gravity is zero.

Example Response:

The energy of the spring-box system initially is spring potential energy, and just before the box lands, all the energy is kinetic energy, and because there is no net work done by outside forces, the energy of the spring box system is the same.

OR

The energy of the box/spring system remains the same because the box returns to the same height, and so the net work done by gravity is zero.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Valid changes also include making the ramp steeper, trading out the spring for a stiffer (larger k) spring, or using a box of less mass.

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0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating a valid change that could be made so that the maximum height would be the same.

Example Response:

Nonnegligible friction would cause mechanical energy to be dissipated. Therefore, the compression distance could be increased so that the spring transfers more energy to the box, offsetting the loss in energy from friction.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the slope of the line would be smaller.
- ☐ 1 point is earned for indicating that with a larger angle, the horizontal component of the initial velocity is smaller.
- ☐ 1 point is earned for indicating that the slope of the horizontal position as a function of time graph is equal to the horizontal velocity of the box.

Example Response:

Since the graph is of horizontal position as a function of time, the slope of the line represents the horizontal velocity as a function of time. If the box were launched at a larger angle, the horizontal component of the velocity would be smaller and so this line would start at zero still but would have a smaller, but still positive, slope.

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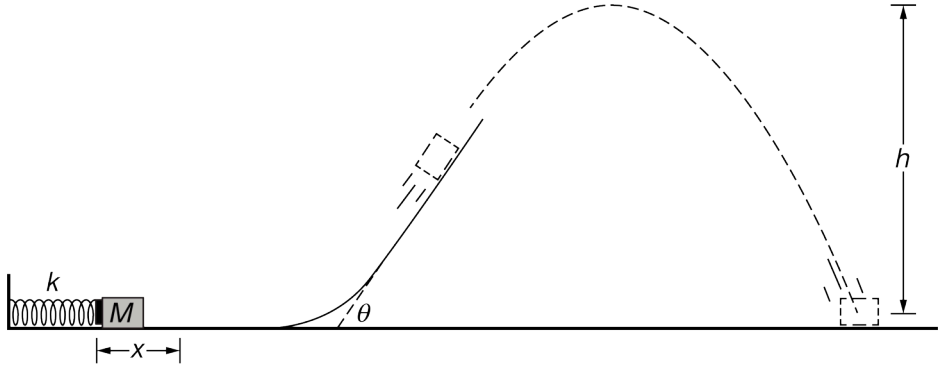
4. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A box is pressed against a horizontal spring, compressing the spring from its relaxed length. The box is then released and the spring launches the box horizontally along a track that ends in a ramp, as shown above. The box has enough speed to leave the ramp, and the box reaches a maximum vertical height above the floor. Assume there is negligible friction between the box and the track and air resistance is negligible.

k is the spring constant of the spring.
x is the distance the spring is compressed.
M is the mass of the box.
θ (theta) is the angle of the ramp from the horizontal.
h is the maximum height reached by the box.

The scenario is repeated with the box compressing the spring a distance that is larger than the original distance x.

(a) Indicate how h in this second scenario compares to h in the original scenario, and explain why without mathematically deriving a relation for h.

(b) Students derive an equation for the maximum height h in the original scenario, $h = \frac{(\sin^2\theta)kx^2}{2Mg}$, which may or may not be correct. Is this equation consistent with your claim in part (a)? Explain why or why not.

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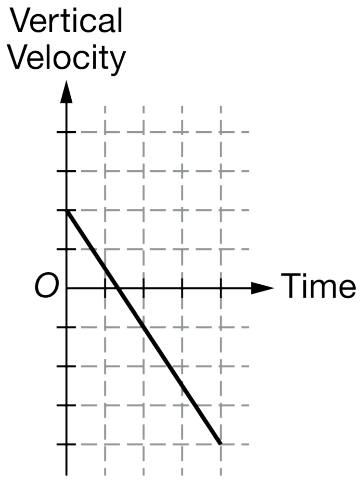


Two students, student 1 and student 2, are discussing the graph shown above, which shows the vertical component of the velocity of the box from when it leaves the ramp to immediately before it hits the floor. The students make the following statements.

Student 1: This graph shows that the vertical acceleration of the box is constant, and that the box's speed decreases for the entire time shown.

Student 2: The area between the graph and the time axis is the box's vertical displacement, but the graph is incorrect because the positive displacement should be equal to the negative displacement.

(c) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(g) Consider the system consisting of only the box, but not the spring or Earth. How does the energy of the system when the spring is fully compressed compare to the energy of the system at the moment immediately before the box hits the ground? Justify your answer.

The angle of the ramp is changed such that it is larger than θ but less than 90 degrees, while the height of the end of the ramp remains the same.

(h) How would the maximum height of the box compare to the original scenario in which the ramp angle was equal to θ ? Justify your answer.

(i) Would the slope of the graph of vertical velocity as a function of time shown above part (c) be different for this scenario in which the box is launched from the larger angle? Justify your response.

Part A

frq2

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that with a larger compression distance the energy stored in the spring is greater (or the average force would increase or the spring acts on the box for a longer time/distance), so the speed of the box as it leaves the spring will be larger.
- ☐ 1 point is earned for a statement that because the speed of the box as it leaves the spring will be greater, and everything else remains the same, the maximum height of the block will increase.

Example Response:

A larger compression distance means that more energy is stored initially in the spring, which means that the launch speed is greater, so with everything else the same the box will reach a larger maximum height.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for the answer yes, and an attempt at a justification using functional dependence.
- ☐ 1 point is earned for indicating that, since x is in the numerator if x increases, h will increase.

Example Response:

Yes, it is consistent with my reasoning – since x is in the numerator, if x increases, h will also increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

frq2

☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the vertical acceleration of the box is constant

OR

The first statement

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 1 is incorrect in saying that the box's speed decreases for the entire time shown.

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that the area between the graph and the time axis is the box's vertical displacement

OR

frq2

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 2 is incorrect in saying that the positive displacement should be equal to the negative displacement.

OR

The second statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the energy of the box system increases
  OR
  For indicating that energy is greater from the spring doing work on box.
- ☐ 1 point is earned for indicating that there is no initial mechanical energy in the box system
  OR
  For indicating that the box returns to the same height.
- ☐ 1 point is earned for indicating that just before the box lands all the energy is kinetic energy
  OR
  For indicating that the net work done by gravity is zero.

Example Response:

The energy of the box system initially is zero because the spring is not in the system. Just before the box lands, all the mechanical energy in the system is kinetic energy, and because there is no net work done by gravity because the box returns to the same height, the energy of the box system increases.

frq2

OR

The energy of the system increases, because the system does not include the spring, and the spring does work on the box. The box eventually returns to the same height, so the net work done on the system from gravity is zero.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating that increasing the angle would increase the vertical component of the velocity which would increase the maximum height.

Example Response:

Increasing the angle of the ramp would increase the vertical component of the velocity so the box would be able to go higher.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

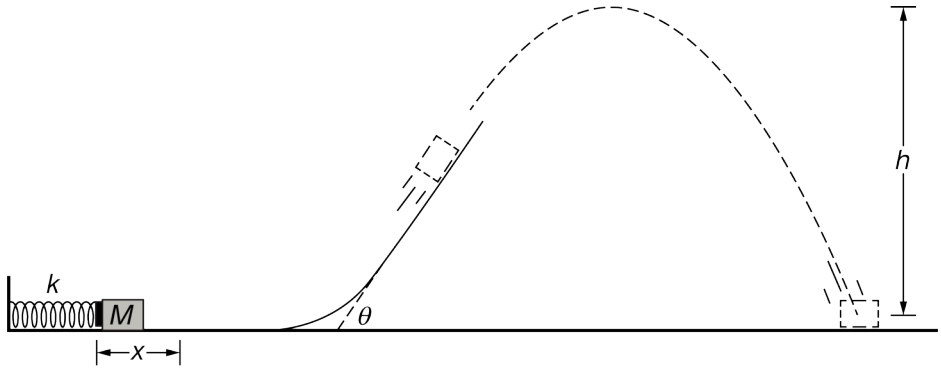
- ☐ 1 point is earned for indicating that the slope of the line would be the same.
- ☐ 1 point is earned for indicating that the slope of a velocity as a function of time graph is equal to the (vertical) acceleration.
- ☐ 1 point is earned for indicating that the vertical acceleration is constant (g).

Example Response:

Since the slope of the vertical velocity vs. time is equal to the acceleration, and the acceleration is constant throughout this problem (g), the slope of the line would remain the same even if the angle of the ramp were increased.

frq2

5. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A box is pressed against a horizontal spring, compressing the spring from its relaxed length. The box is then released and the spring launches the box horizontally along a track that ends in a ramp, as shown above. The box has enough speed to leave the ramp, and the box reaches a maximum vertical height above the floor. Assume there is negligible friction between the box and the track and air resistance is negligible.

k is the spring constant of the spring.
x is the distance the spring is compressed.
M is the mass of the box.
θ (theta) is the angle of the ramp from the horizontal.
h is the maximum height reached by the box.

The scenario is repeated using a different box with a greater mass. The spring is compressed the same distance x.

(a) Indicate how h in this second scenario compares to h in the original scenario, and explain why without mathematically deriving a relation for h.

(b) Students derive an equation for h in the original scenario, $h = \frac{(\sin^2\theta)kx^2}{2Mg}$, which may or may not be correct. Is this equation consistent with your claim in part (a)? Explain why or why not.

Two students are discussing the work done on the system consisting of only the box between the instant just before the spring begins accelerating the box and the instant the box reaches the end of the ramp. The students make the following statements.

Student 1: At first, the mechanical energy of the system increases because the spring does positive work on the system. Then, the mechanical energy of the system increases again as the box travels up the ramp because the box's potential energy increases.

Student 2: The mechanical energy of the system remains constant while the spring pushes the box because the work done on the box by the spring cancels the work done by the spring on the box. Then, when the block is on the ramp, the mechanical energy of the system decreases because the box slows down.

(c) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***,

frq2

write “none.”

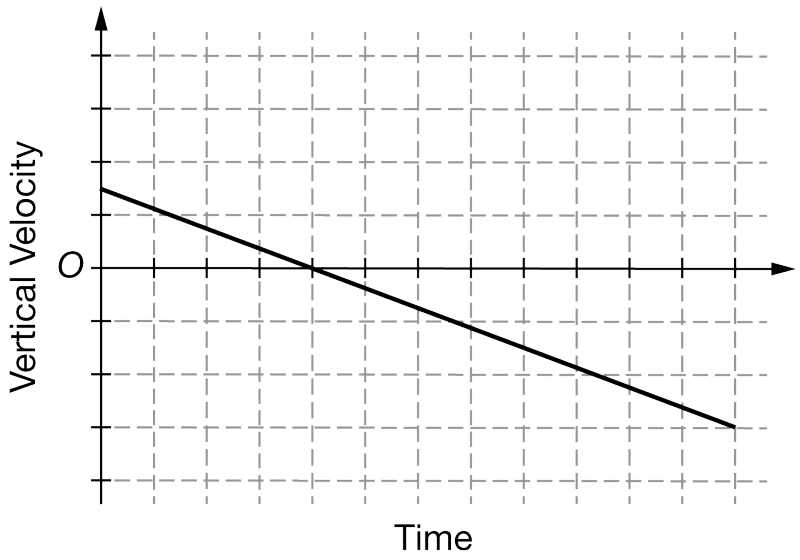
(d) Which underlined phrase or phrases in student 1’s statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write “none.”

(e) Which underlined phrase or phrases in student 2’s statement are ***correct***, if either? If neither phrase is ***correct***, write “none.”

(f) Which underlined phrase or phrases in student 2’s statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write “none.”

(g) Does the mechanical energy of the box-spring-Earth system increase, decrease, or remain the same from the moment the spring is released until just before the box hits the ground? Justify your answer.

(h) If air resistance were not negligible, how would that affect your answer to part (g), if at all? Justify your answer.



(i) The graph above shows the vertical component of the velocity of the box as a function of time. The graph represents the entire time the box is in the air when the box is launched from the ramp at an angle θ to the horizontal. How would the total area under the curve be different, if at all, if the ramp angle θ were increased but still less than 90 degrees, without changing the height of the end of the ramp?

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

frq2

- [ ] 1 point is earned for a statement that with a larger mass (in the second scenario), the energy stored in the spring is the same, but the speed of the box on the table is smaller
  OR
  For a larger mass and an equivalent average force, the average acceleration will be smaller so the speed of the box on the table will be smaller.
- [ ] 1 point is earned for a statement that because the speed of the box as it leaves the spring will be less, and everything else remains the same. The maximum height of the block will decrease.

Example Response:

A larger mass means that although there is the same spring potential energy, the box will have a smaller speed as it approaches the bottom of the ramp. This means that the launch speed is less, so with everything else the same the box will reach a smaller maximum height.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for the answer yes, and an attempt at a justification using functional dependence.
- [ ] 1 point is earned for Since M is in the denominator, if M increases, h will decrease.

Example Response:

Yes, it is consistent with my reasoning – since M is in the denominator, if M increases, h will decrease.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the mechanical energy of the system increases because the spring does positive work on the system

frq2

OR

The first statement

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that when the block is on the ramp, the mechanical energy of the system decreases because the box slows down

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 2 is incorrect in saying that the mechanical energy of the system remains constant while the spring pushes the box because the work done on the box by the spring cancels the work done by the spring on the box

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the energy of the box-spring-Earth system remains constant.
- ☐ 1 point is earned for indicating that initially there is spring potential energy.
- ☐ 1 point is earned for indicating that just before the box lands all the energy is kinetic energy.

Example Response:

The energy of the box-spring-Earth system initially is elastic potential energy because all the energy is stored in the spring. Just before the box lands, all the mechanical energy in the system is kinetic energy, and because there is no net work done by gravity because the box returns to the same height, the energy of the box-spring-Earth system is constant.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a statement that air resistance would decrease the mechanical energy of the system.

Example Response:

If there was air resistance it would remove energy from the box-spring-Earth system, decreasing the mechanical energy of the system.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that the total area under the curve would be the same.

frq2

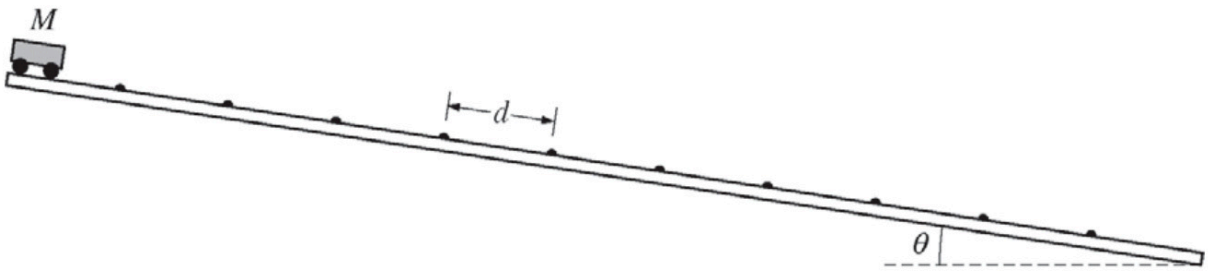
- [ ] 1 point is earned for a statement that the area under the curve of a vertical velocity vs. time graph is the vertical displacement.
- [ ] 1 point is earned for a statement that the box starts and ends at the same height as in the original scenario, so the vertical displacement is the same as in the original.

Example Response:

Since the area under the vertical velocity vs. time graph is the vertical displacement, and the box starts and ends at the same height, the vertical displacement will still be the same in both cases, and so the total area under the curve will be the same for both as well.

frq2

6.



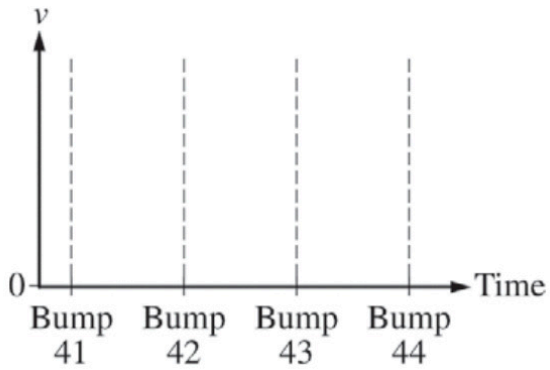
Note: Figure not drawn to scale.

A long track, inclined at an angle θ to the horizontal, has small speed bumps on it. The bumps are evenly spaced a distance d apart, as shown in the figure above. The track is actually much longer than shown, with over 100 bumps. A cart of mass M is released from rest at the top of the track. A student notices that after reaching the 40th bump the cart's average speed between successive bumps no longer increases, reaching a maximum value v_{avg}. This means the time interval taken to move from one bump to the next bump becomes constant.

a. Consider the cart's motion between bump 41 and bump 44.

   i. In the figure below, sketch a graph of the cart's velocity v as a function of time from the moment it reaches bump 41 until the moment it reaches bump 44.

   ii. Over the same time interval, draw a dashed horizontal line at $v = v_{avg}$. Label the line"v_{avg}"



b. Suppose the distance between the bumps is increased but everything else stays the same. Is the maximum speed of the cart now greater than, less than, or the same as it was with the bumps closer together?

   ____Greater than ____Less than ____The same as

   Briefly explain your reasoning.

c. With the bumps returned to the original spacing, the track is tilted to a greater ramp angle θ. Is the maximum speed of the cart greater than, less than, or the same as it was when the ramp angle was smaller?

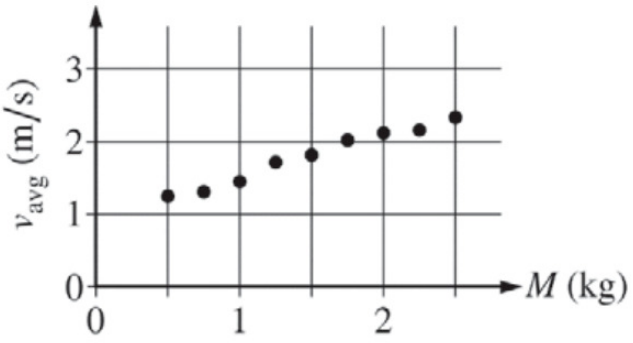
frq2

____Greater than ____Less than ____The same as

Briefly explain your reasoning.

d. Before deriving an equation for a quantity such as v_{avg} , it can be useful to come up with an equation that is intuitively expected to be true. That way, the derivation can be checked later to see if it makes sense physically. A student comes up with the following equation for the cart's maximum average speed: $v_{avg} = C\frac{Mg\sin\theta}{d}$, where C is a positive constant.

i. To test the equation, the student rolls a cart down the long track with speed bumps many times in front of a motion detector. The student varies the mass M of the cart with each trial but keeps everything else the same. The graph shown below is the student's plot of the data for v_{avg} as a function of M.



Are these data consistent with the student's equation?

____Yes ____No

Briefly explain your reasoning.

ii. Another student suggests that whether or not the data above are consistent with the equation, the equation could be incorrect for other reasons. Does the equation make physical sense?

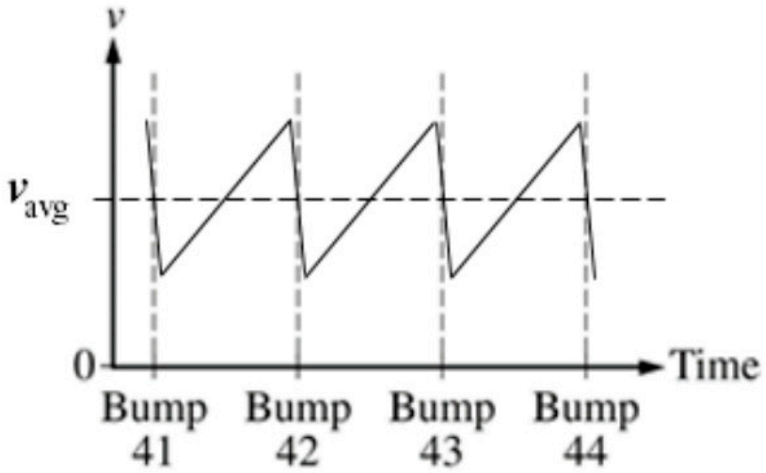
____Yes ____No

Briefly explain your reasoning.

Part A

Part(i)

frq2



1 point is earned: For having a constant upward slope in each segment between bumps

1 point is earned: For having the velocity “reset” abruptly at each bump (i.e., as a sawtooth function, not a sinusoidal curve) to a minimum positive value that is the same for each bump

1 point is earned: For having the same maximum value in each cycle that occurs near the bump times (This point can be earned for a sinusoidal curve with peaks at the bump times.)

Part(ii)

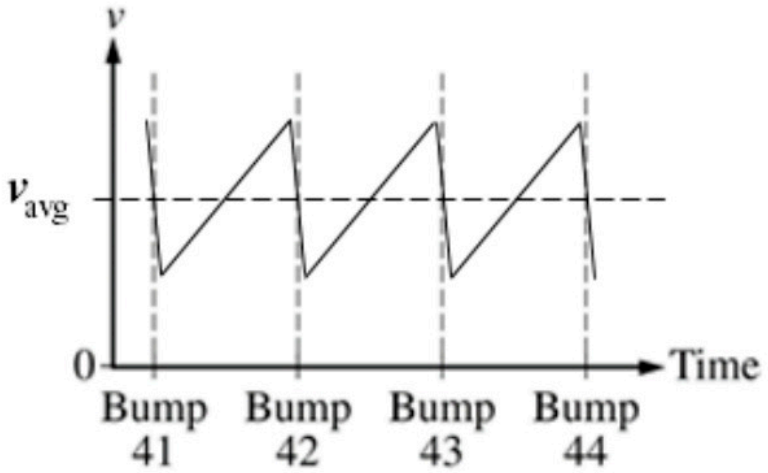
1 point is earned: For drawing a vavg line that is horizontal and consistent with the graph drawn, even if that graph is wrong

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Part(i)

frq2



1 point is earned: For having a constant upward slope in each segment between bumps

1 point is earned: For having the velocity “reset” abruptly at each bump (i.e., as a sawtooth function, not a sinusoidal curve) to a minimum positive value that is the same for each bump

1 point is earned: For having the same maximum value in each cycle that occurs near the bump times (This point can be earned for a sinusoidal curve with peaks at the bump times.)

Part(ii)

1 point is earned: For drawing a vavg line that is horizontal and consistent with the graph drawn, even if that graph is wrong

Part B

Correct answer: Greater than

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

No points are earned if the wrong answer is selected.

1 point is earned: For indicating that there is more time between bumps

1 point is earned: For connecting that the cart has more time or more distance to accelerate between bumps

Example: The maximum speed is greater because the cart has more space (or time) to accelerate (build up speed) between bumps.

For indicating that the potential energy difference increases due to increased height between successive bumps

For relating the increase in potential energy to an increase in kinetic energy

frq2

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

Correct answer: Greater than

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

No points are earned if the wrong answer is selected.

1 point is earned: For indicating that there is more time between bumps

1 point is earned: For connecting that the cart has more time or more distance to accelerate between bumps

Example: The maximum speed is greater because the cart has more space (or time) to accelerate (build up speed) between bumps.

For indicating that the potential energy difference increases due to increased height between successive bumps

For relating the increase in potential energy to an increase in kinetic energy

Part C

Correct answer: Greater than

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

No points are earned if the wrong answer is selected.

1 point is earned: For indicating that the acceleration is greater

1 point is earned: For indicating that the component of the gravitational force increases

Example: The maximum speed is greater because the cart has more space (or time) to accelerate (build up speed) between bumps.

For indicating that the potential energy difference increases due to increased height between successive bumps

For relating the increase in potential energy to an increase in kinetic energy

0	1	2 ✓

frq2

Student response earns 2 of the following 2 point(s)

Correct answer: Greater than

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

No points are earned if the wrong answer is selected.

1 point is earned: For indicating that the acceleration is greater

1 point is earned: For indicating that the component of the gravitational force increases

Example: The maximum speed is greater because the cart has more space (or time) to accelerate (build up speed) between bumps.

For indicating that the potential energy difference increases due to increased height between successive bumps

For relating the increase in potential energy to an increase in kinetic energy

Part D

Part (i)

Correct answer: No

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

1 point is earned: For indicating that vavg is not proportional to M

1 point is earned: For connecting the equation to the data

Examples:

> The y-intercept of the graph is not zero, but the equation indicates that it should be zero.
>
> Doubling the mass from the graph does not double vavg, but the equation indicates that it should double.
>
> If “yes” is selected, one point may be earned for explaining that vavg increases with M or that vavg looks (approximately) proportional to M for a limited portion of the data (e.g., the points at M = 1.0 kg and M = 2.0 kg).

Part (ii)

Correct answer: No

1 point is earned: For indicating that the distance dependency is incorrect

1 point is earned: For indicating that, according to the equation, greater d leads to a smaller vavg,

OR for stating or implying the contradiction between this inverse relation and the reasoning of part (b)

Credit is earned for any answer that is consistent with reasoning in part (b).

frq2

Example:

> According to the equation, a larger d corresponds to a smaller vavg, because they are inversely proportional. But according to the reasoning of part (b), a bigger distance d between the bumps leads to a larger maximum average speed, showing that the equation is implausible.

If “yes” is selected, one point can be earned for indicating that an increase in the angle increases vavg.

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Part (i)

Correct answer: No

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

1 point is earned: For indicating that vavg is not proportional to M

1 point is earned: For connecting the equation to the data

Examples:

> The y-intercept of the graph is not zero, but the equation indicates that it should be zero.
>
> Doubling the mass from the graph does not double vavg, but the equation indicates that it should double.
>
> If “yes” is selected, one point may be earned for explaining that vavg increases with M or that vavg looks (approximately) proportional to M for a limited portion of the data (e.g., the points at M = 1.0 kg and M = 2.0 kg).

Part (ii)

Correct answer: No

1 point is earned: For indicating that the distance dependency is incorrect

1 point is earned: For indicating that, according to the equation, greater d leads to a smaller vavg,

OR for stating or implying the contradiction between this inverse relation and the reasoning of part (b)

Credit is earned for any answer that is consistent with reasoning in part (b).

Example:

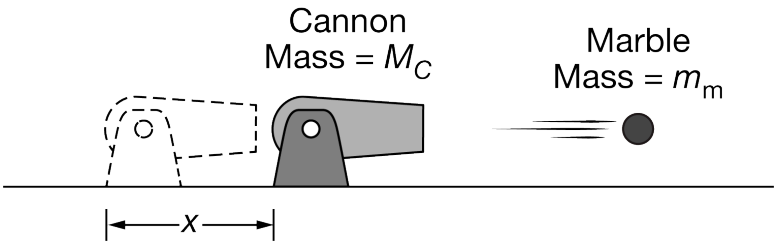
> According to the equation, a larger d corresponds to a smaller vavg, because they are inversely proportional. But according to the reasoning of part (b), a bigger distance d between the bumps leads to a larger maximum average speed, showing that the equation is implausible.

frq2

If “yes” is selected, one point can be earned for indicating that an increase in the angle increases vavg.

frq2

7. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch marbles of mass m_{m} horizontally from a toy cannon of mass M_C, where M_C is always greater than m_{m}, as shown above. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students use a new marble with a different mass launched at the same speed v relative to the ground. The students record the distance x the cannon slides backward for each launch.

(a) Without manipulating equations, briefly describe what happens to the distance x that the cannon slides backward as students increase the mass of the marble. Explain your reasoning.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\mathrm{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) Without manipulating or stating equations, briefly describe whether it would take more work to stop the marble or to stop the cannon after the marble has been launched. Justify your answer.

(d) Students correctly derive the following equation relating the work W done on an object initially at rest, the momentum p of the object, and the mass m of the object: $W = \frac{p^2}{2m}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble-cannon system during the short time interval in which the marble is accelerated inside the cannon and make the following claims:

Student 1: The cannon is pushing the marble forward, and the marble is pushing back with equal and opposite force. Since the work done by the cannon cancels out the work done by the marble, the mechanical energy of the system is constant.

Student 2: The mechanical energy of the system increases while the marble is being accelerated. The marble gains kinetic energy to the right, and the cannon gains an equal amount of kinetic energy to the left.

(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

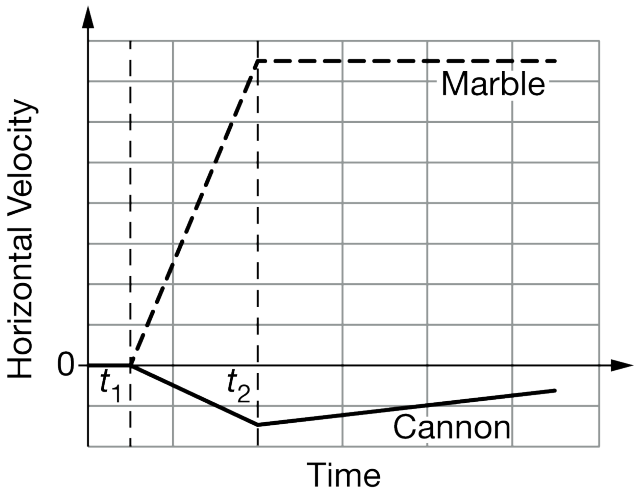
(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is

frq2

incorrect, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity of the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Provide three pieces of evidence that together explain why the magnitude of the rate of change of the cannon's velocity is less than the magnitude of the rate of change of the marble's velocity during the time interval from t_1 to t_2.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to indicate that a larger marble mass means a larger cannon speed
  OR
  For using a combination of Newton's 3rd law leading to Newton's 2nd law to indicate that a larger marble mass means a larger cannon speed.
- ☐ 1 point is earned for explaining why the larger cannon speed results in sliding a greater distance.

frq2

Example Response:

The initial momentum of the marble-cannon system is zero, so it will be zero after the marble is launched as well. A greater marble mass then means that the backward cannon speed is greater. Since the acceleration due to friction is the same regardless of cannon speed, a greater cannon speed means that the cannon will slide a greater distance.

OR

While the marble is in the cannon, the force on the marble has the same magnitude as the force on the cannon (according to Newton's 3rd law). If the final speed of the marble is the same each time but the mass of the marble increases, the net force on the marble must increase (according to Newton's 2nd law) and, therefore, the net force on the cannon must increase as well (the force of friction remains constant) giving the cannon a larger final speed resulting in a greater distance.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence between the mass m_m and x.
- ☐ 1 point is earned for a claim that as m_m in the equation increases, x in the equation also increases.

Example Response:

Yes, the equation matches that as the mass of the marble increases the slide distance of the cannon also increases. The mass of the marble is in the numerator of the equation so if the numerator increases the slide distance will also increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim that the marble requires more work to stop, with attempt at a justification.
- ☐ 1 point is earned for a valid justification.

Example Response:

The marble requires more work to stop because the marble has a greater kinetic energy thus a greater change in kinetic

frq2

energy in order to stop.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim consistent with the answer to (c).
- ☐ 1 point is earned for a claim that both the marble and cannon have equal momentum when the marble is launched and for recognizing that the marble's smaller mass in the denominator means it has more work done on it.

Example Response:

Yes. By conservation of momentum, the cannon and marble have the same magnitude of momentum immediately after the launch. Since p is the same, but m is smaller for the marble, the kinetic energy is greater for the marble since m is in the denominator, which means that the marble had more work done on it.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1's statement that is correct is the marble is pushing back with equal and opposite force

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 1's statement that is incorrect is work done by the cannon cancels out the work done by the marble and mechanical energy is constant.

OR

The second statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If a student considered the cannon to include a spring launching mechanism, then the first statement would be incorrect. A student who states "none" in part (g) and "both" in part (h) earns full credit for both parts.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2's statement that is correct is mechanical energy increases while the marble is being accelerated.

OR

The first statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student's 2 statement that is incorrect is the cannon gains an equal amount of kinetic energy to the left.

OR

The second statement

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Though the slope change in the graph occurs at t_2 (rather than "in the interval from t_1 to t_2 "), a student who indicates the effect of force of friction by discussing the slope change at t_2 is awarded this point.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

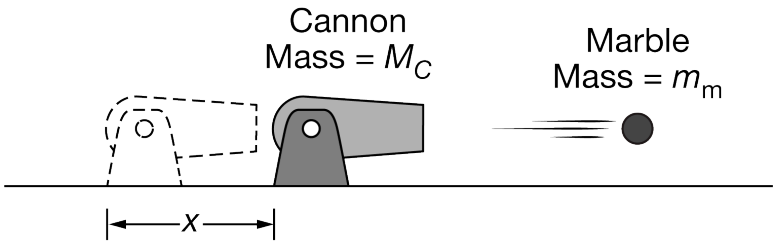
- ☐ 1 point is earned for recognizing that the same force is exerted on both the marble and cannon.
- ☐ 1 point is earned for indicating that the larger mass of the cannon results in its acceleration being less.
- ☐ 1 point is earned for indicating the force of friction acting opposite the cannon's direction of motion will further reduce the acceleration of the cannon.

Example Response:

Even the though the force on the cannon and the marble is the same due to Newton's third law, the cannon's acceleration is smaller. So, its velocity increases less than the marble because the cannon's mass is larger. The cannon also has a frictional force opposing its motion, which decreases the net force and therefore the acceleration.

frq2

8. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch identical marbles of mass m_{m} horizontally from a toy cannon of varying mass M_C, where M_C is always greater than m_{m}, as shown above. The students can attach weights to the cannon to change M_C. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students vary the mass of the cannon and launch a marble with the same speed v relative to the ground. The students record the distance x that the cannon slides backward.

(a) Without manipulating equations, explain how the distance x that the cannon slides backward changes, if at all, when marbles are launched from more massive cannons.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\text{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) If constant horizontal forces were exerted over the same distance on the cannon and on the marble after the launch, which object would require a force with a greater magnitude to completely stop it? Without manipulating or referencing equations, briefly explain your reasoning.

(d) Students correctly derive the following equation relating the force F exerted on an object initially at rest over a distance d, the mass m of the object, and the final momentum p of the object: $F = \frac{p^2}{2md}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble after it has been launched by the cannon but before it reaches the ground. Both students agree that air resistance is negligible and make the following claims.

Student 1: The mechanical energy of the marble is constant. There are no external forces exerted on the marble, so its energy does not change.

Student 2: The mechanical energy decreases as the marble falls. This happens because the gravitational force is doing work on the marble.

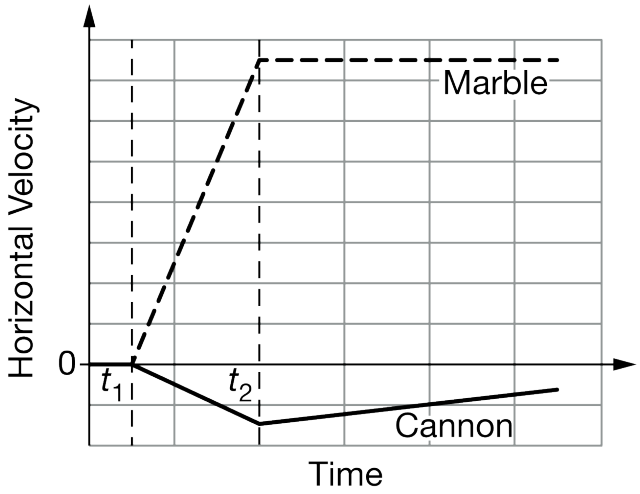
(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

frq2

(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity of the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Describe the motion of the center of mass of the marble-cannon system during the time interval from t_1 to t_2. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Students may earn the points for a correct discussion regarding the force of friction. A more massive cannon leads to a greater normal force increasing the force of friction. This would cause the net force on the cannon to the left to be smaller while the marble was in the cannon (giving it an initially smaller speed once the marble has left the cannon) resulting in a smaller distance traveled.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to indicate that a larger cannon mass means a smaller cannon speed

frq2

OR

For using a combination of Newton's 3rd law leading to Newton's 2nd law to indicate that a larger cannon mass means a smaller cannon speed.

☐ 1 point is earned for explaining why the smaller cannon speed results in sliding a shorter distance.

Example Response:

The initial momentum of the marble-cannon system is zero, so it will be zero after the marble is launched as well. A more massive cannon then means that the backward cannon speed is less. A smaller cannon speed means that the cannon will slide a shorter distance.

OR

While the marble is in the cannon, the force on the marble has the same magnitude as the force on the cannon (according to Newton's 3rd law). If the mass of the marble stays constant but the mass of the cannon increases, the cannon will have smaller acceleration and smaller speed (according to Newton's 2nd law) resulting in a smaller distance.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

☐ 1 point is earned for using functional dependence between the mass and the distance x.

☐ 1 point is earned for a claim that as M_c in the equation increases, x in the equation decreases.

Example Response:

Yes, the equation matches the prediction that as the mass of the cannon increases the distance the cannon slides backwards decreases. From the equation, the mass of the cannon and the distance it slides backwards are inversely proportional. The mass of the cannon is in the denominator of the equations so as the mass increases the distance the cannon slides backwards will decreases.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

frq2

- ☐ 1 point is earned for a claim that it would take more force to stop the marble, with an attempt at a justification.
- ☐ 1 point is earned for a valid justification using either work/energy or momentum.

Example Response:

It would take more force to stop the marble because the change in kinetic energy to stop the marble is greater so the work on the marble needs to be greater.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim consistent with the answer to (c).
- ☐ 1 point is earned for a claim that both the marble and cannon have equal momentum when the marble is launched and for recognizing that the marble's smaller mass in the denominator means it requires more force to stop.

Example Response:

Yes. By conservation of momentum, the cannon and marble have the same magnitude of momentum immediately after the launch. Since p is the same, but m is smaller for the marble, the force is greater for the marble since m is in the denominator.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a statement of "None".

Example Response:

None

Part F

frq2

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating both statements are incorrect.

Example Response:

Both parts of Student 1's statement are incorrect.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

The gravitational force is doing work on the marble

OR

The second statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

frq2

Example Response:

The mechanical energy decreases as the marble falls.

OR

The first statement

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

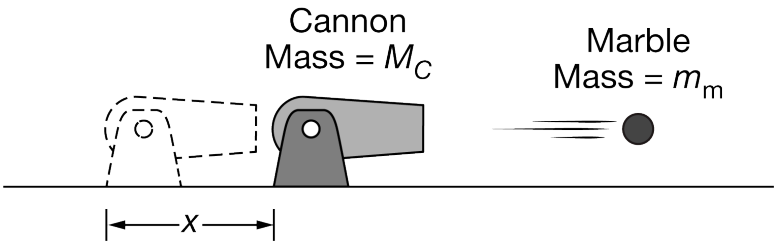
- ☐ 1 point is earned for indicating the center of mass moves to the right.
- ☐ 1 point is earned for indicating that there is an external force on the system due to friction.
- ☐ 1 point is earned for indicating that the net force on the system is needed to move the center on mass.

Example Response:

During the launch, the only horizontal external force exerted on the marble-cannon system is the frictional force exerted on the cannon. The friction force is directed to the right, so the net force on the system is to the right. This means the center of mass of the system is accelerating to the right.

frq2

9. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch identical marbles of mass m_{m} horizontally from a toy cannon of mass M_C, where $M_C > m_{\text{m}}$, as shown above. The cannon can be adjusted to change the launch speed v of the marble relative to the ground. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students vary the launch speed v and record the distance x the cannon slides backward for each launch speed.

(a) Without manipulating equations, briefly describe what happens to the distance x that the cannon slides backward as students increase the launch speed. Explain your reasoning.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\text{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) Without manipulating or stating equations, briefly describe whether the cannon or the marble has more kinetic energy immediately after the marble leaves the cannon. Justify your answer.

(d) Students correctly derive an equation relating the kinetic energy K, the momentum p, and the mass m of an object: $K = \frac{p^2}{2m}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble after it has been launched by the cannon but before it reaches the ground. Both students agree that air resistance is negligible and make the following claims.

Student 1: There are two external forces exerted on the marble while it is in the air, the gravitational force and the force exerted by the cannon. Therefore, the mechanical energy of the marble changes while it is in the air.

Student 2: The cannon does not exert a force on the marble after the launch. Therefore, the mechanical energy of the marble is constant.

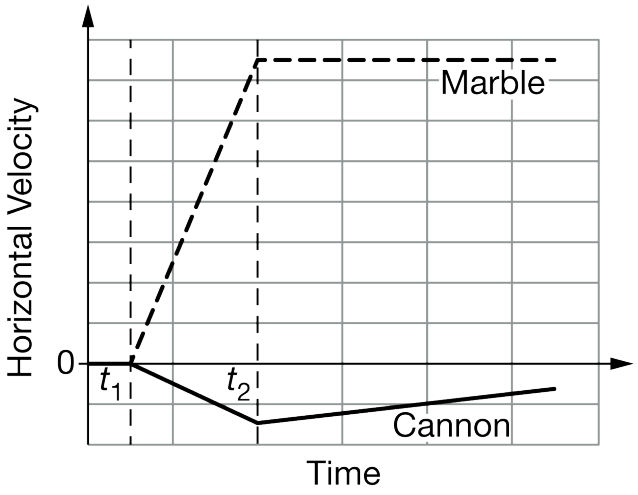
(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

frq2

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity for the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Describe how to find the change in the distance between the center of mass of the cannon and the center of mass of the marble from time t_1 to time t_2. Assume that before time t_1 the center of mass of the marble is at the same position as the center of mass of the cannon.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to indicate that a larger marble speed means a larger cannon speed
  OR
  For using a combination of Newton's 3rd law leading to Newton's 2nd law to indicate that a larger marble speed means a larger cannon speed.
- ☐ 1 point is earned for correctly explaining why the larger cannon speed results in sliding a greater distance.

frq2

Example Response:

The initial momentum of the marble-cannon system is zero, so it will be zero after the marble is launched as well. A greater marble speed then means that the backward cannon speed is greater. Since the acceleration due to friction is the same regardless of cannon speed, a greater cannon speed means that the cannon will slide a greater distance.

OR

While the marble is in the cannon, the force on the marble has the same magnitude as the force on the cannon (according to Newton's 3rd law). If the mass of the marble stays constant but the final speed of the marble increases, the net force on the marble must increase (according to Newton's 2nd law) and, therefore, the net force on the cannon must increase as well (the force of friction remains constant) giving the cannon a larger final speed resulting in a greater distance.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence.
- ☐ 1 point is earned for a claim that as v in the equation increases, x in the equation also increases.

Example Response:

Yes, the equation matches the prediction that as the velocity of the marble increases the distance the cannon moves backward also increases. In the equation as velocity increases the distance would increase. Since the velocity is in the numerator of the equation, as the numerator of a fraction increases the distance x would increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim that the marble has more kinetic energy, with an attempt at a justification.
- ☐ 1 point is earned for a valid justification.

Example Response:

By conservation of momentum, mass and velocity are inversely proportional. Since velocity is squared when determining

frq2

kinetic energy, a greater speed increases the kinetic energy by a larger factor than a smaller mass decreases the energy.

OR

The cannon and marble exert forces of equal magnitude on each other. However, the marble will move a greater distance than the cannon while these forces are exerted. So, the marble has more work done on it, and therefore a greater kinetic energy.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim consistent with the answer to (c).
- ☐ 1 point is earned for a claim that both the marble and cannon have equal momentum when the marble is launched and for recognizing that the marble's smaller mass in the denominator means it has more kinetic energy.

Example Response:

Yes. By conservation of momentum, the cannon and marble have the same magnitude of momentum immediately after the launch. Since p is the same, but m is smaller for the marble, the kinetic energy is greater for the marble since m is in the denominator.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

The correct statement from student 1 is that mechanical energy of the marble changes while it is in the air

OR

The second statement

frq2

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

The incorrect statement for student 1 is that the cannon exerts a force on the marble while it is in the air

OR

The first statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

The correct statement for student 2 is that the cannon no longer exerts a force on the marble

OR

The first statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

The incorrect statement for student 2 is that the mechanical energy of the marble is constant

OR

The second statement

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

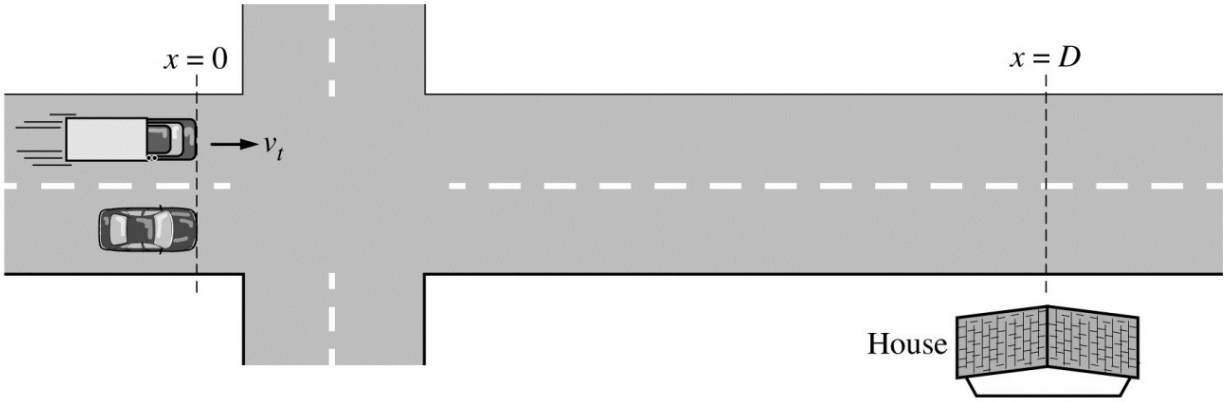
- ☐ 1 point is earned for recognizing that displacement can be calculated by finding the area under a *v* vs. *t* graph.
- ☐ 1 point is earned for recognizing that the total displacement will be represented by combining the areas under each *v* vs. *t* graph.
- ☐ 1 point is earned for indicating that, for the cannon, either the area (which is negative) is to be subtracted
  OR
  the absolute value of the area is to be added, to the area under the marble graph.

Example Response:

The area under the velocity vs. time graph represents the displacement of an object. To find the change in the distance, subtract the areas of the two graphs between t_1 *and* t_2.

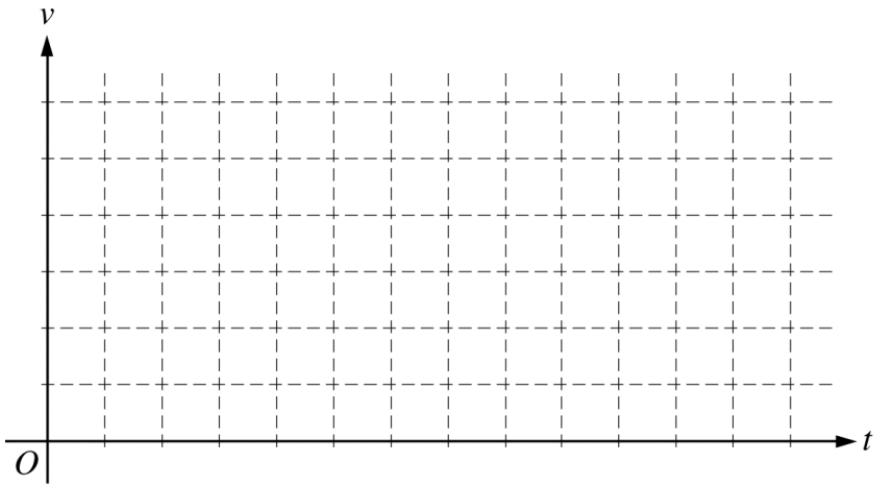
frq2

10.



A car is stopped at a traffic light. The light turns green, and at time t = 0 the car starts moving and travels with a constant acceleration. At that instant a truck traveling at constant speed v_t is alongside the car, with the front of each vehicle at position x = 0 , as shown above. The truck passes the car, but the car later catches up to the truck in front of a house, such that at time t_D the front of each vehicle is at position x = D.

a. On the axes below, sketch and label graphs of the velocity of the car and the velocity of the truck as a function of time. Indicate any important velocities or times.



b. Two students are discussing how the speed of the car compares to the speed of the truck when both vehicles are in front of the house.

Student 1 says, “The distance traveled by the car and the truck is the same, and the time is the same, so they must have the same speed.”

Student 2 says, “I don’t see how that can be. The car catches up to the truck, so the car has to be going faster.”

i. Which aspects of Student 1’s reasoning, if any, are correct? Support your answer in terms of relevant features of your graphs in part (a).

frq2

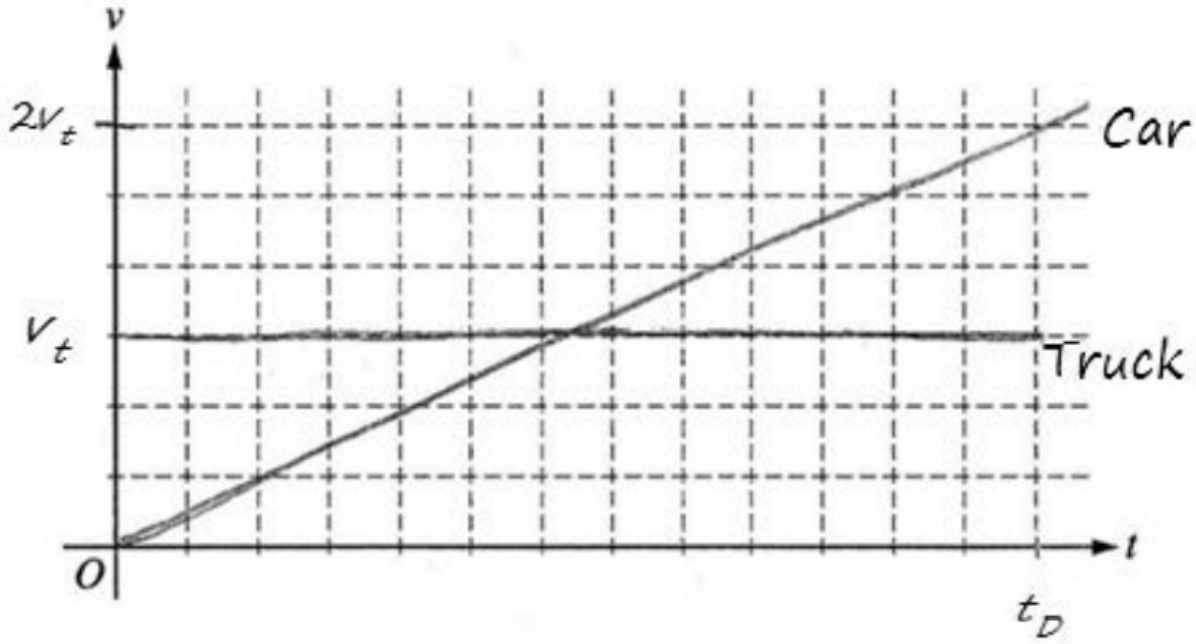
ii. Which aspects of Student 2's reasoning, if any, are correct? Support your answer in terms of relevant features of your graphs in part (a).

c. Derive an expression for the acceleration of the car. Express your answer in terms of D and v_t

d. Determine the time at which the speed of the car is equal to the speed v_t of the truck. Express your answer in terms of t_D. Justify your answer.

Part A

image.png



1 point is earned: For a clearly labeled horizontal line for the truck

1 point is earned: For a clearly labeled straight line with positive slope for the car

1 point is earned: For the speed of the car ending higher than the speed of the truck at a time labeled to indicate that the house has been reached

0	1	2	3 ✓

Student response earns 3 of the following 3 point(s)

image.png

frq2

1 point is earned: For a clearly labeled horizontal line for the truck

1 point is earned: For a clearly labeled straight line with positive slope for the car

1 point is earned: For the speed of the car ending higher than the speed of the truck at a time labeled to indicate that the house has been reached

Part B

Part(i)

1 point is earned: For indicating that student 1 is correct that the car and truck travel the same distance in the same time

1 point is earned: For identifying a feature of the graph that supports this: e.g., the area under both curves is the same, or the visual average of the speed of the car is the same as the speed of the truck

Part(ii)

1 point is earned: For indicating that student 2 is correct that the car's speed is greater when the vehicles are in front of the house

1 point is earned: For identifying a feature of the graph that supports this: e.g., the final speed of the car is twice the speed of the truck

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Part(i)

1 point is earned: For indicating that student 1 is correct that the car and truck travel the same distance in the same time

1 point is earned: For identifying a feature of the graph that supports this: e.g., the area under both curves is the same, or the visual average of the speed of the car is the same as the speed of the truck

Part(ii)

1 point is earned: For indicating that student 2 is correct that the car's speed is greater when the vehicles are in front of the house

1 point is earned: For identifying a feature of the graph that supports this: e.g., the final speed of the car is twice the speed of the truck

Part C

1 point is earned: For indicating that the final speed of the car is twice the speed of truck

1 point is earned: For applying any other appropriate kinematic relationship(s)

(2vt)2 = ve02 + 2aD

frq2

1 point is earned: For solving for the acceleration in terms of D and Vt

a = 2vt2/D

0	1	2	3 ✓

Student response earns 3 of the following 3 point(s)

1 point is earned: For indicating that the final speed of the car is twice the speed of truck

1 point is earned: For applying any other appropriate kinematic relationship(s)

(2vt)2 = ve02 + 2aD

1 point is earned: For solving for the acceleration in terms of D and Vt

a = 2vt2/D

Part D

1 point is earned: For indicating that the car and truck have the same speed at t = tD/2

1 point is earned: For a correct justification

Examples: That's where the graphs intersect.

At tD the car's speed is twice the truck's speed, so they'll be equal at tD/2

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

1 point is earned: For indicating that the car and truck have the same speed at t = tD/2

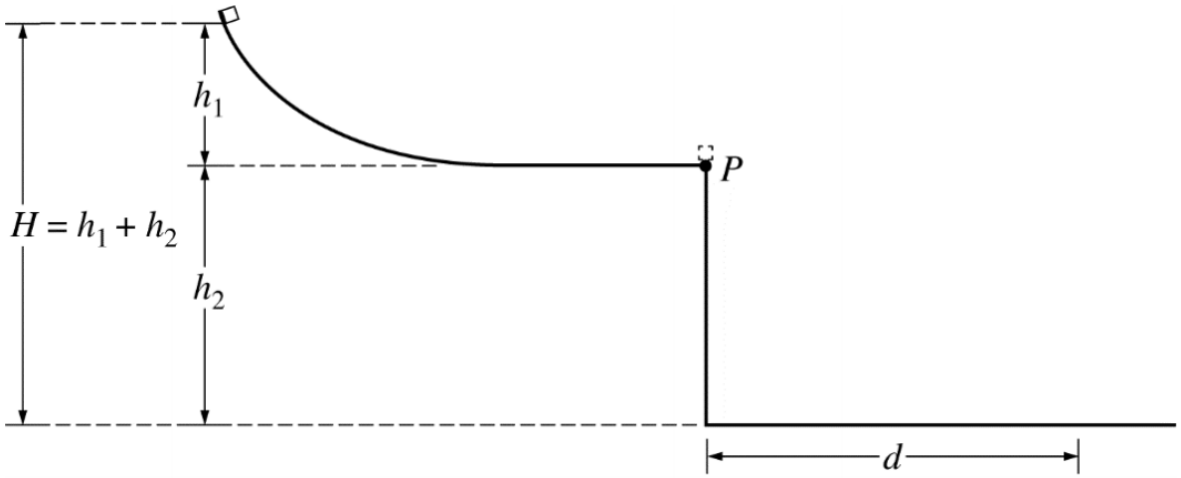
1 point is earned: For a correct justification

Examples: That's where the graphs intersect.

At tD the car's speed is twice the truck's speed, so they'll be equal at tD/2

frq2

11.



A student releases a block of mass m from rest at the top of a slide of height h_1. The block moves down the slide and off the end of a table of height h_2 , landing on the floor a horizontal distance d from the edge of the table. Friction and air resistance are negligible. The overall height H of the setup is determined by the height of the room. Therefore, if h_1 is increased, h_2 must decrease by the same amount so that the sum $h_1 + h_2$ remains equal to H. The student wants to adjust h_1 and h_2 to make d as large as possible.

a. i. Without using equations, explain why making h_1 very small would cause d to be small, even though h_2 would be large.

ii. Without using equations, explain why making h_2 very small would cause d to be small, even though h_1 would be large

b. Derive an equation for d in terms of h_1, h_2, m, and physical constants, as appropriate.

c. i. Write the equation or step in your derivation in part (b) (not your final answer) that supports your reasoning in part (a)i.

Briefly explain your choice.

ii. Write the equation or step in your derivation in part (b) (not your final answer) that supports your reasoning in part (a)ii.

Briefly explain your choice.

d. If the experiment is repeated on the Moon without changing h_1 or h_2 , will the new landing distance d be greater than, less than, or the same as the landing distance when the experiment is performed on Earth?

_____Greater than _____Less than _____The same as

Briefly explain your choice.

frq2

Part A

Part(i)

1 point is earned: For recognizing a small horizontal velocity for the block at point P

1 point is earned: For stating or implying that the block doesn't go very far even though there is a longer fall time

Example: Even though the large h2 allows the block to stay in the air for a long time, the small h1 means the system loses little gravitational potential energy and hence gains little kinetic energy so that it leaves the table with too little speed to cover much distance while aloft.

Part(ii)

1 point is earned: For recognizing a large horizontal velocity for the block at point P

1 point is earned: For mentioning a short time of flight for the falling block

Example: Now the block gains a lot of kinetic energy on the ramp so that it leaves the table with a large horizontal speed. But the small h2 means the block spends so little time in the air that it lands before covering much horizontal distance.

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Part(i)

1 point is earned: For recognizing a small horizontal velocity for the block at point P

1 point is earned: For stating or implying that the block doesn't go very far even though there is a longer fall time

Example: Even though the large h2 allows the block to stay in the air for a long time, the small h1 means the system loses little gravitational potential energy and hence gains little kinetic energy so that it leaves the table with too little speed to cover much distance while aloft.

Part(ii)

1 point is earned: For recognizing a large horizontal velocity for the block at point P

1 point is earned: For mentioning a short time of flight for the falling block

Example: Now the block gains a lot of kinetic energy on the ramp so that it leaves the table with a large horizontal speed. But the small h2 means the block spends so little time in the air that it lands before covering much horizontal distance.

Part B

1 point is earned: For using an energy conservation statement (corresponds to step 1 in example below)

1 point is earned: For setting up a time of flight calculation using vertical acceleration (corresponds to step 2a in example

frq2

below)

Note: derivation of t must be shown to earn credit.

1 point is earned: For using vt = d to determine distance traveled (corresponds to step 2b in example below)

Example:

Step 1: Use energy conservation to find the block's speed at the bottom of ramp, which equals the launch speed v1 at point P:

mgh1 = 1/2mv12 ⇒ v1 = √gh1

Step 2a: Find the time of flight, using the independence of the horizontal and vertical motion. Since the block leaves the table with zero vertical speed, the vertical kinematic equation y = y0 + vy0t + 1/2ayt2 reduces h2 = 1/2 gt2 (taking downward as positive). Solve for time to get t = √(2h2/g).

Step 2b. Find the horizontal distance d covered while in the air after point P. Since there are no horizontal forces exerted on the block, its horizontal velocity stays constant at v1. So, the block travels a horizontal distance

$$d = v_i t = \sqrt{2gh_1}\sqrt{\frac{2h_2}{g}} = 2\sqrt{h_1 h_2}$$

✓

0	1	2	3

Student response earns 3 of the following 3 point(s)

1 point is earned: For using an energy conservation statement (corresponds to step 1 in example below)

1 point is earned: For setting up a time of flight calculation using vertical acceleration (corresponds to step 2a in example below)

Note: derivation of t must be shown to earn credit.

1 point is earned: For using vt = d to determine distance traveled (corresponds to step 2b in example below)

Example:

Step 1: Use energy conservation to find the block's speed at the bottom of ramp, which equals the launch speed v1 at point P:

mgh1 = 1/2mv12 ⇒ v1 = √gh1

Step 2a: Find the time of flight, using the independence of the horizontal and vertical motion. Since the block leaves the table with zero vertical speed, the vertical kinematic equation y = y0 + vy0t + 1/2ayt2 reduces h2 = 1/2 gt2 (taking downward as positive). Solve for time to get t = √(2h2/g).

Step 2b. Find the horizontal distance d covered while in the air after point P. Since there are no horizontal forces exerted on the block, its horizontal velocity stays constant at v1. So, the block travels a horizontal distance

frq2

$$d = v_i t = \sqrt{2gh_1}\sqrt{\frac{2h_2}{g}} = 2\sqrt{h_1 h_2}$$

Part C

Part(i)

1 point is earned: For mentioning a correct equation or step of reasoning (energy conservation on ramp)

1 point is earned: For correctly explaining why this equation or step mirrors the reasoning of (a)i

Example: My energy conservation reasoning (part (b) step 1) mirrors my reasoning that

a small ramp height corresponds to a low potential energy (mgh1), which is converted into low kinetic energy and hence a low speed.

Part(ii)

1 point is earned: For mentioning a correct equation or step of reasoning (time of flight)

1 point is earned: For correctly explaining why that equation or step mirrors the reasoning of (a)ii

Example: My time-of-flight calculation (part (b) step 2a) shows that t is proportional to √(h2), so a smaller falling distance h2 corresponds to a smaller flight time.

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Part(i)

1 point is earned: For mentioning a correct equation or step of reasoning (energy conservation on ramp)

1 point is earned: For correctly explaining why this equation or step mirrors the reasoning of (a)i

Example: My energy conservation reasoning (part (b) step 1) mirrors my reasoning that

a small ramp height corresponds to a low potential energy (mgh1), which is converted into low kinetic energy and hence a low speed.

Part(ii)

1 point is earned: For mentioning a correct equation or step of reasoning (time of flight)

1 point is earned: For correctly explaining why that equation or step mirrors the reasoning of (a)ii

Example: My time-of-flight calculation (part (b) step 2a) shows that t is proportional to √(h2), so a smaller falling distance h2 corresponds to a smaller flight time.

Part D

frq2

Correct answer: “The same as”

Note: explanation is scored regardless of checkbox.

Point can be earned if response is consistent with answer in part (b)

1 point is earned: For reasoning that is consistent with the functional dependence of the part (b) answer

Example:

As found in part (b), the distance d does not depend on g. So, doing the experiment on the Moon instead of Earth makes no difference. The increased time of flight (since weaker gravity makes the block take longer to land) compensates for the small speed gained by the block on the ramp.

Checked “Greater than” then g must be in the denominator in the incorrect equation in part (b)

Checked “Less than” then g must be in the numerator in the incorrect equation in part (b)

0	1 ✓

Student response earns 1 of the following 1 point(s)

Correct answer: “The same as”

Note: explanation is scored regardless of checkbox.

Point can be earned if response is consistent with answer in part (b)

1 point is earned: For reasoning that is consistent with the functional dependence of the part (b) answer

Example:

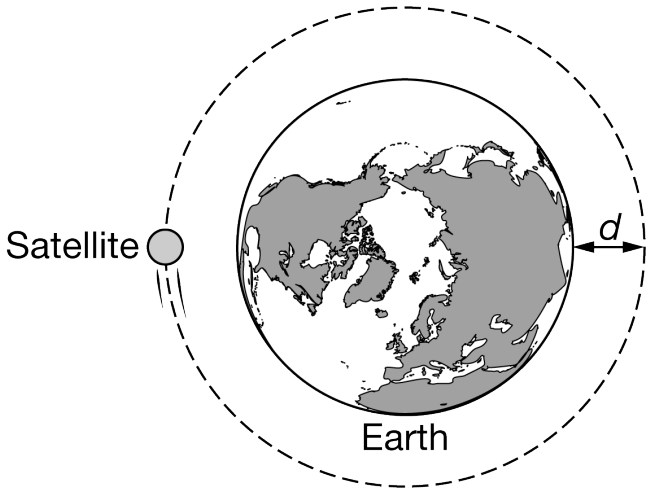
As found in part (b), the distance d does not depend on g. So, doing the experiment on the Moon instead of Earth makes no difference. The increased time of flight (since weaker gravity makes the block take longer to land) compensates for the small speed gained by the block on the ramp.

Checked “Greater than” then g must be in the denominator in the incorrect equation in part (b)

Checked “Less than” then g must be in the numerator in the incorrect equation in part (b)

frq2

12. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface, as shown above.

An identical satellite is set into orbit around a new planet. The new planet has the same radius as the radius of Earth but is significantly more massive than Earth. The satellite orbits the same distance d from the surface of the new planet.

(a) Is the speed of the satellite orbiting the new planet greater than, less than, or the same as the speed of the satellite orbiting Earth? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

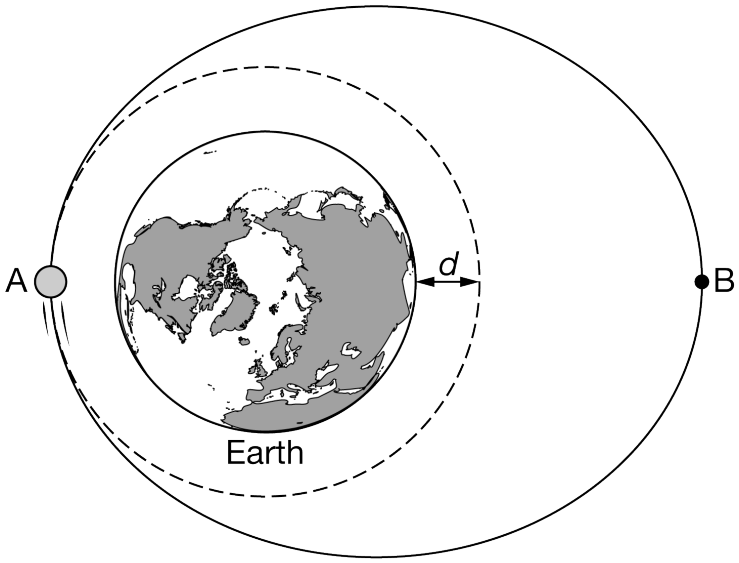
$$\frac{MT^2}{R^3} = C$$

In the equation above

M is the mass of a planet;
T is the orbital period of a satellite in a circular orbit;
R is the distance from the planet's center to the satellite; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

frq2



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point A to point B, and state the following:

Student 1: The satellite is moving faster at point A because the satellite's angular momentum is greater at point A than at point B.

Student 2: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is higher at point B than at point A.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

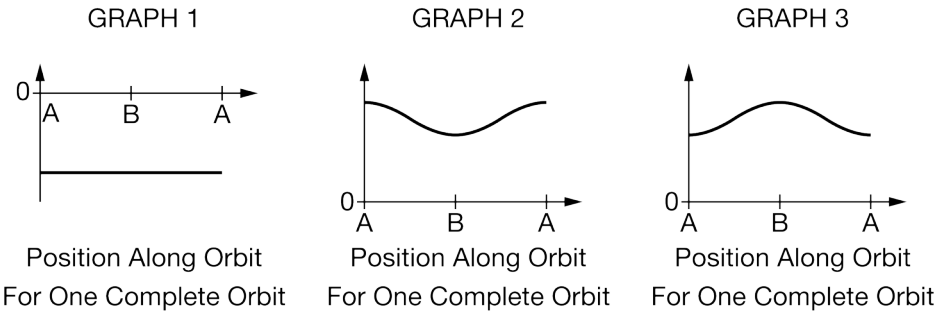
(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite

frq2

as it completes one elliptical orbit, starting and ending at point A.



(h) Which of the above graphs best represents the magnitude of the gravitational field at the location of the satellite as it orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the total mechanical energy of the satellite-Earth system as the satellite orbits? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the satellite around the new planet will orbit with a faster speed.
- ☐ 1 point is earned for a correct explanation in terms of the relationship between gravitational force and centripetal force (or acceleration).

Example Response:

There is greater gravitational force acting on the satellite around the new planet. The increased gravitational force means that the centripetal force $m\frac{v^2}{R}$ *(or acceleration* $\frac{v^2}{R}$*) is greater. Since R is the same in both cases, v must be greater for the satellite orbiting the new planet.*

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

frq2

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a consistent answer AND an attempt at reasoning using functional dependence.
- ☐ 1 point is earned for correctly relating the increased M to a decreased T.
- ☐ 1 point is earned for correctly explaining how a decreased T increases v.

Example Response:

Yes, this is consistent with my reasoning. Since the mass of the new planet is larger, the time it will take to go in an orbit must decrease because M is inversely proportional to T squared. Since the new satellite travels the same circumference orbit, if the second satellite takes less time to go around, it must be going faster.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	✓ 2

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the force was applied in the satellite's direction of motion.
- ☐ 1 point is earned for a correct explanation relating the direction of force with the change in speed.

Example Response:

A force in the direction of the satellite's motion would make the satellite move faster

OR

A force in the direction of the satellite's motion would do positive work on the satellite.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	✓ 1

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

frq2

Student 1 is correct in saying that the satellite is moving faster at A

OR

The first statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is incorrect in saying that the satellite's angular momentum is larger at A than at B

OR

The second statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that the potential energy of the satellite-Earth system is higher at B than at A

OR

The second statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is incorrect in saying that the satellite is moving faster at B

OR

The first statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 2.
- ☐ 1 point is earned for a correct explanation relating field strength to distance.

Example Response:

Graph 2. The field is larger where the satellite is closer to the planet, so it is larger at A than at B.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 1.

frq2

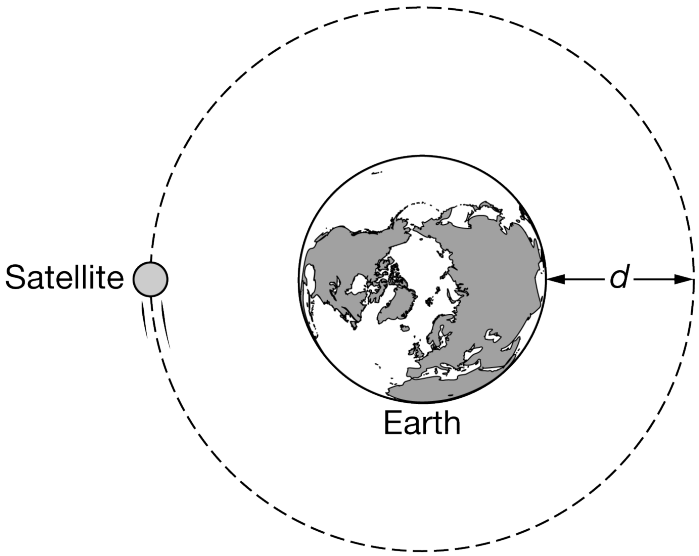
- [ ] 1 point is earned for a correct explanation of why the total mechanical energy is constant in terms of net external force.

Example Response:

Graph 1. The total mechanical energy is constant for the satellite-Earth system because there are no external forces on the system.

frq2

13. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface, as shown above.

An identical satellite is set into orbit around a new planet. The new planet's radius is the same as the radius of Earth but is significantly more massive than Earth. The satellite orbits the new planet with the same orbital period.

(a) Is the distance of the satellite from the surface of the new planet greater than, less than, or equal to d ? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

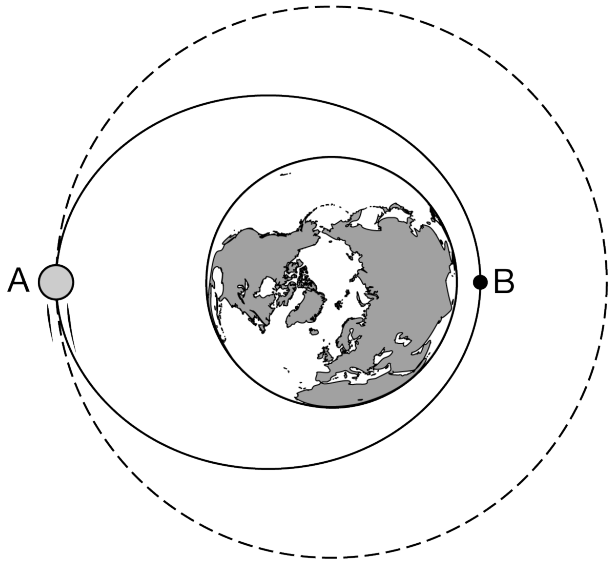
$\frac{Rv^2}{M} = C$

In the equation above

R is the distance from a planet's center to a satellite in a circular orbit;
v is the speed of the satellite;
M is the mass of the planet; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

frq2



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point A to point B, and state the following:

Student 1: The satellite is moving faster at point A because the total mechanical energy of the satellite-Earth system is constant.

Student 2: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is higher at point B than at point A.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

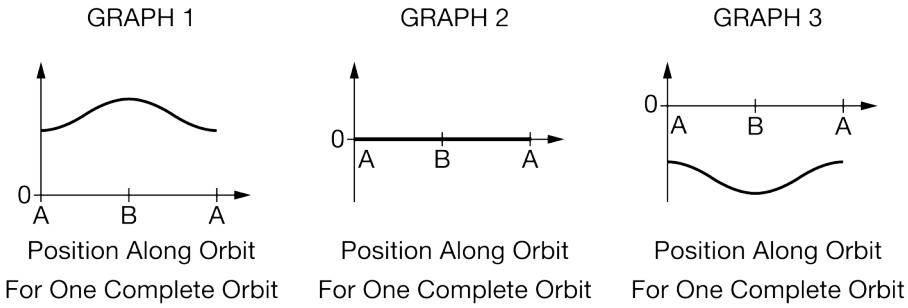
(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite

frq2

as it completes one elliptical orbit, starting and ending at point A.



(h) Which of the above graphs best represents the potential energy of the satellite-Earth system as the satellite orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the torque about the center of Earth exerted on the satellite by Earth's gravitational force as the satellite orbits? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the satellite around the new planet will orbit at a greater distance.
- ☐ 1 point is earned for correctly explaining an increasing distance from the planet to satellite is caused by an increasing speed to maintain a constant period.
  OR
  For correctly explaining an increased distance from the planet to decrease the gravitational force or acceleration on the satellite.

Example Response:

For the satellite to maintain the same period, it must orbit at a greater distance from the new planet because the satellite's orbital speed will be now be greater than before.

OR

"*The distance of the satellite must be further since increasing mass increases the force of gravity keeping the satellite in orbit, to maintain the same period, you would need to be further from the planet to decrease the gravitational force from the larger mass.*

Part B

frq2

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a consistent answer AND an attempt at reasoning using functional dependence.
- ☐ 1 point is earned for using $v = \frac{2\pi R}{T}$ or arguing that *R* must increase if *T* remains the same.
- ☐ 1 point is earned for correctly relating the increase in mass of the planet increases the distance from the planet to satellite.

Example Response:

Yes, this matches with my reasoning in part (a). The mass of the new planet is greater, which means that for C to stay constant the top of the fraction must also be greater. Since v is $\frac{2\pi R}{T}$ *and T is constant, R must be larger for the speed to be larger.*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the force was applied opposite to the satellite's direction of motion.
- ☐ 1 point is earned for a correct explanation relating the direction of force with the changing speed.

Example Response:

The force was applied opposite to the motion of the satellite's direction of motion because this would make the satellite move slower which does negative work on the satellite.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

frq2

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the total mechanical energy of the satellite-Earth system is constant

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is incorrect in saying that the satellite is moving faster at A

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that the satellite is moving faster at B

frq2

OR

The first statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is incorrect in saying that the potential energy of the satellite-Earth system is higher at B than at A

OR

The second statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 3.
- ☐ 1 point is earned for a correct explanation relating potential energy to distance.

Example Response:

Graph 3. The potential energy is higher where the satellite is farther from the planet, so it is higher at A than at B.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

0	1	2 ✓

The response accurately includes **both** of the following criteria.

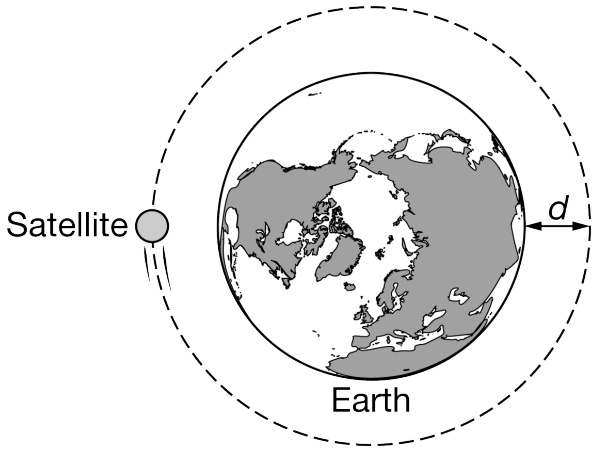
- ☐ 1 point is earned for indicating graph 2.
- ☐ 1 point is earned for a correct explanation why the torque is zero.

Example Response:

Graph 2. The torque is zero since the angle between $\vec{F}$ *and* $\vec{r}$ *is 180 degrees.*

frq2

14. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface.

An identical satellite is set into orbit around a new planet. The new planet's radius is the same as the radius of Earth but the new planet is significantly more massive than Earth. The satellite orbits the new planet with the same speed v.

(a) Is the distance of the satellite from the surface of the new planet greater than, less than, or equal to d ? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

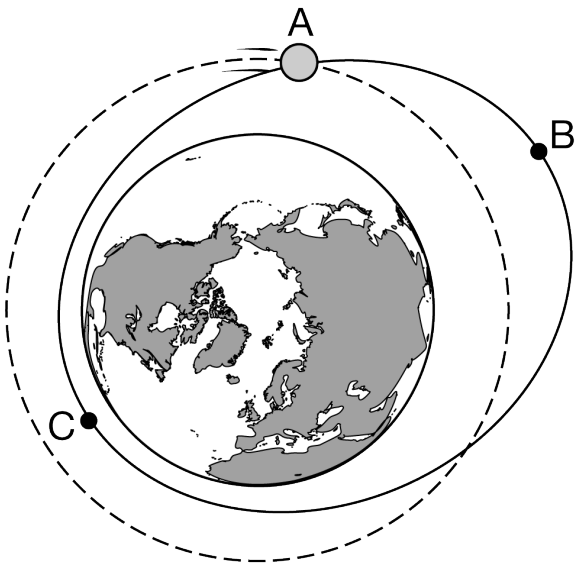
$$\frac{R^3}{MT^2} = C$$

In the equation above

R is the distance from a planet's center to a satellite in a circular orbit;
M is the mass of the planet;
T is the orbital period of the satellite; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

frq2



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit. In the modified orbit, the satellite's speed at point A, after the thrusters were fired, is the same as it was in the original orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point B to point C, and state the following:

Student 1: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is negative.

Student 2: The satellite is moving faster at point C because the total mechanical energy of the satellite-Earth system is greater at point C than at point B.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

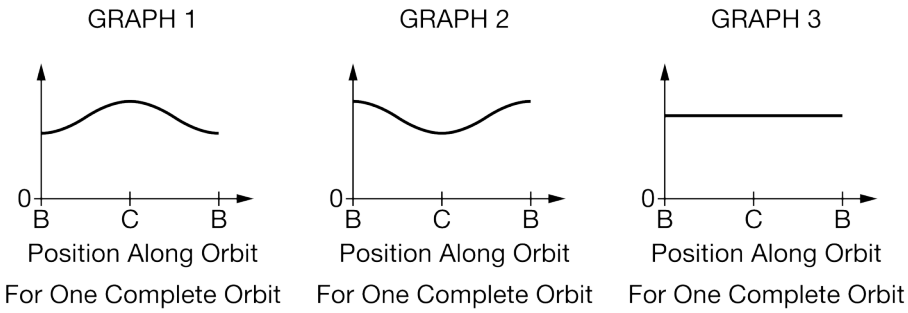
(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is

frq2

incorrect, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite as it completes one elliptical orbit, starting and ending at point B.



(h) Which of the above graphs best represents the magnitude of the acceleration of the satellite as it orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the magnitude of the angular momentum of the satellite with respect to the center of Earth as the satellite orbits? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the satellite around the new planet will orbit at a greater distance.
- ☐ 1 point is earned for a correct explanation in terms of centripetal and gravitational force or acceleration.

Example Response:

Equating the centripetal acceleration v^2/r *with the gravitational acceleration* GM/r^2, *we see that* v^2 *is proportional to* M/r. *So a bigger mass M means that the distance must be greater as well.*

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for an answer that is consistent with part (a) AND an attempt at reasoning using functional dependence.
- ☐ 1 point is earned for using $v = \frac{2\pi R}{T}$ or for arguing that *R* must increase if *v* remains the same.
- ☐ 1 point is earned for correctly relating the increase in mass of the planet increases the distance from the planet to the satellite.

Example Response:

Yes, it matches my reasoning. Since v is $\frac{2\pi R}{T}$ *and v is the same, if R increases then T must also increase. Since M and T both increase, R must increase to keep C constant.*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the force was applied perpendicular to the satellite's motion.
- ☐ 1 point is earned for a correct explanation relating the direction of force with change in speed.

Example Response:

The force was applied perpendicular to the satellite's motion because this force did not change the speed of the satellite.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

frq2

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the potential energy of the satellite-Earth system is negative

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is incorrect in saying that the satellite is moving faster at B

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that the satellite is moving faster at C

OR

frq2

The first statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is incorrect in saying that the total mechanical energy of the satellite-Earth system is greater at C than at B.

OR

The second statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 1.
- ☐ 1 point is earned for a correct explanation that the gravitational field or acceleration increases as distance decreases.

Example Response:

Graph 1. The acceleration (or gravitational field) is greater closer to Earth, so it is greater at C than at B.

OR

Graph 1. The centripetal acceleration v^2/r is greater at C because v is greater, and r is smaller than at B.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

0	1	2 ✓

The response accurately includes **both** of the following criteria.

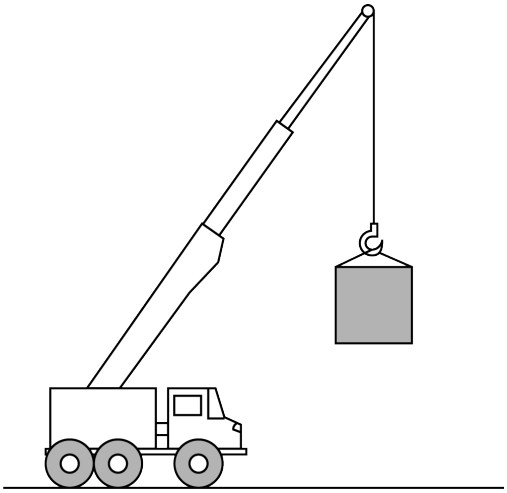
- ☐ 1 point is earned for indicating graph 3.
- ☐ 1 point is earned for a correct explanation why the angular momentum is constant throughout the motion.

Example Response:

Graph 3. The angular momentum is constant because the torque on the satellite is zero.

frq2

15. This question is a long free-response question. Show your work for each part of the question.

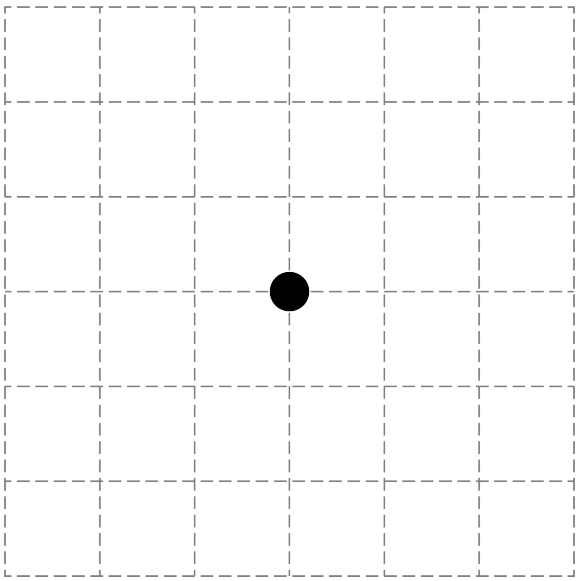


(12 points, suggested time 25 minutes)

A crane and box are both initially at rest. At time $t = 0\ \text{s}$, the crane begins to drive forward at a constant speed of $0.5\ \frac{\text{m}}{\text{s}}$, while also lifting the box with an upward acceleration of $1\ \frac{\text{m}}{\text{s}^2}$. The box does not swing while being lifted by the crane.

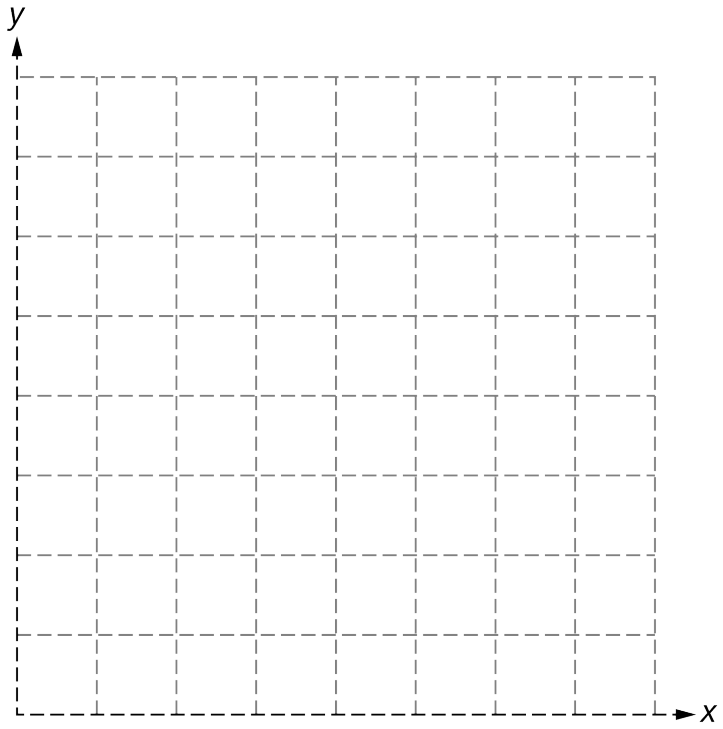
(a) On the dot below, which represents the box, draw and label the forces (not components) that are exerted on the box during the scenario described above. Represent each force by a distinct arrow starting on, and pointing away from, the dot.

Note: Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces.



frq2

(b) On the grid below, sketch the shape of the path taken by the box as it is lifted by the crane as viewed by a stationary observer.



Justify the shape of the path you drew above.

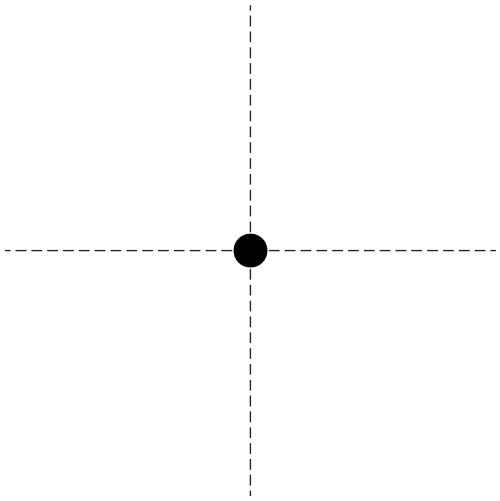
(c) Assume that, at $t = 0$ s, the horizontal position of the box is $x = 0$ and it has a vertical position of $y = 0$. Derive an equation that describes the vertical position y of the box as a function of the horizontal position x of the box.

(d) Does your equation from part (c) agree with your sketch in part (b)? Justify your response.

At time $t = t_1$, the crane begins to accelerate forward, reaching a new constant velocity at time $t = t_2$. The box accelerates upward during the entire time interval $t_1 < t < t_2$.

(e) The dot below represents the box during the time interval $t_1 < t < t_2$. On the dot, draw an arrow indicating the direction of the net force exerted on the block during this interval. The arrow should start on, and point away from, the dot.

frq2



Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Scoring notes:

· G or g are not acceptable labels for the force of gravity

· A maximum of two points can be earned if there are any extraneous forces.

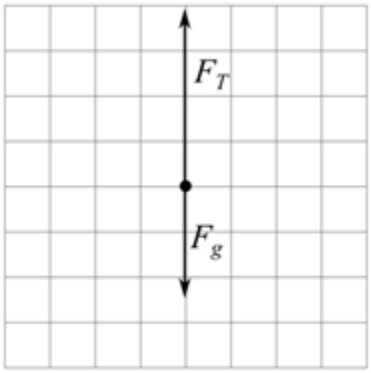
0	1	2	3 ✓

The response accurately includes **one** of the following criteria:

- ☐ For an arrow directed upwards labeled Tension (or F_T)
- ☐ For an arrow directed downwards labeled Force of Gravity (or mg, or F_g).
- ☐ For an arrow labeled tension that is larger than the arrow labeled Force of Gravity.

Example Response:

frq2



Part B

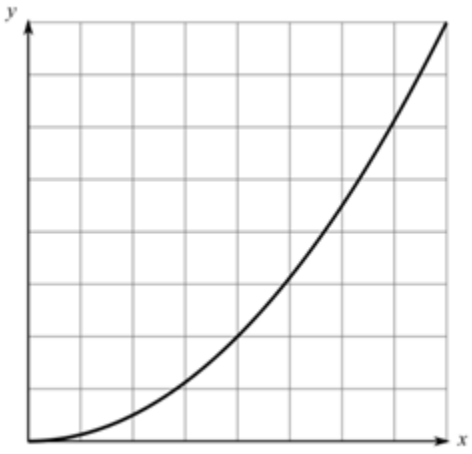
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

- ☐ For a sketch showing a non-linear path
- ☐ For a sketch that is concave up
- ☐ For a justification that addresses that the box is moving at a constant speed in the horizontal direction and accelerating in the vertical direction which leads to a concave up parabola.

Example Response:



Since the box is moving at a constant speed in the horizontal direction and accelerating in the vertical direction, the path that the box takes should look like a concave up parabola

Part C

frq2

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

- ☐ For a kinematic equation correctly relating the horizontal position of the box to the horizontal component of the speed of the box and the time
  $v_x = \frac{x}{t}$
  $0.5\ \text{m/s} = \frac{x}{t}$
  $t = \frac{x}{0.5\ \text{m/s}}$
- ☐ For a kinematic equation correctly relating the vertical position of the box as a function of the vertical acceleration of the box and the time
  $y = \frac{1}{2}a_y t^2$
  $y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)t^2$
- ☐ For correctly combining the vertical and horizontal motions of the box into a single equation.
  $y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)\left(\frac{x}{0.5\ \text{m/s}}\right)^2$
  $y = \left(2\ \text{m}^{-1}\right)x^2$

Example Response:

$v_x = \frac{x}{t}$
$0.5\ \text{m/s} = \frac{x}{t}$
$t = \frac{x}{0.5\ \text{m/s}}$

$y = \frac{1}{2}a_y t^2$
$y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)t^2$

$y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)\left(\frac{x}{0.5\ \text{m/s}}\right)^2$
$y = \left(2\ \text{m}^{-1}\right)x^2$

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

frq2

The response accurately includes **both** of the following criteria.

- [ ] For an attempt to use functional dependence to describe how the equation and the sketch in part B relate
- [ ] For correctly stating that the vertical position is proportional to the horizontal position squared and so the shape of the path traveled by the box should be a parabola.

Example Response:

Yes. My equation in part (c) describes a parabola with a minimum at the origin, which matches my sketch in part (b).

Part E

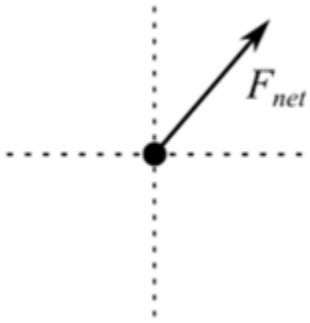
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes the following criteria:

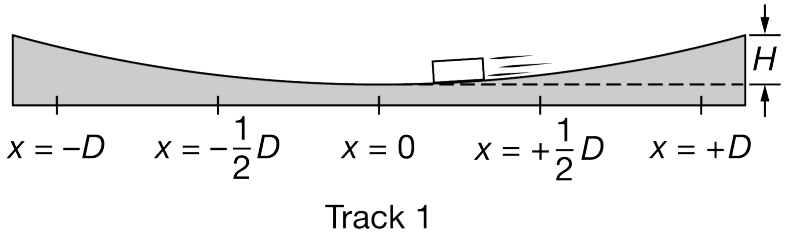
- [ ] One point for an arrow labeled ΣF or net force, pointing upwards and to the right.

Example Response:



frq2

16. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

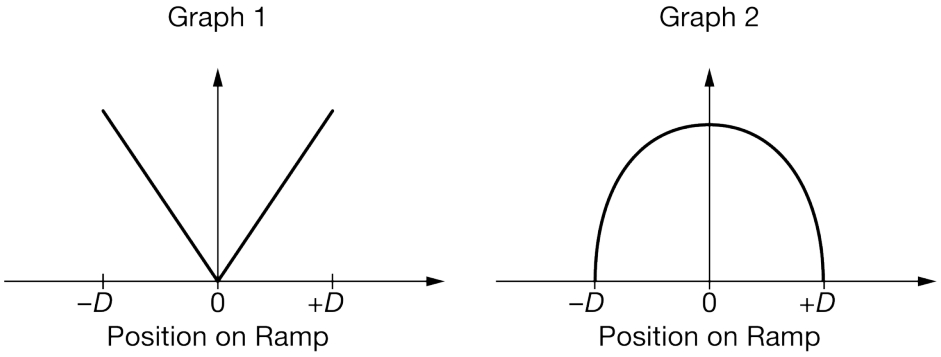


Track 1

Note: Figure not drawn to scale.

A block slides back and forth without any appreciable loss of energy on a track with a slight circular curvature, as shown above. The maximum height H of the track at each end is much smaller than D.

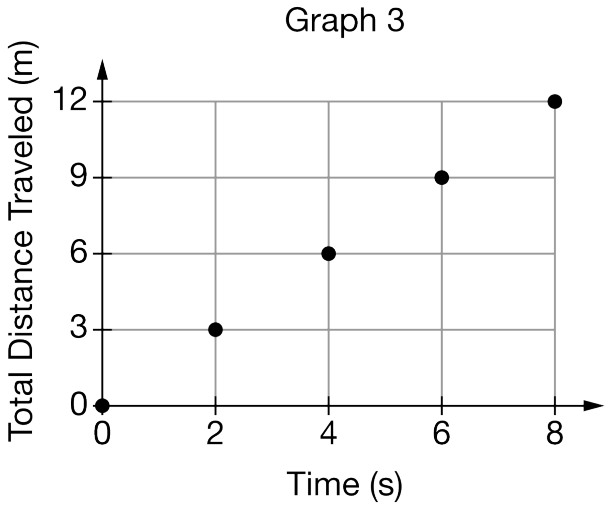
The center of mass of the block oscillates between $x = -D$ and $x = +D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. The block undergoes two full oscillations in 8.0 seconds.



(a) Which of the graphs above, Graph 1 or Graph 2, could represent the magnitude of the horizontal component of the block's acceleration as a function of its position on the ramp, and which graph could represent the magnitude of the block's momentum as a function of its position on the ramp? Justify both your choices with physics principles.

(b) For the system consisting of only the block, describe how the mechanical energy of the system changes, if at all, as the block moves from $x = 0$ to $x = -D$. Justify your answer.

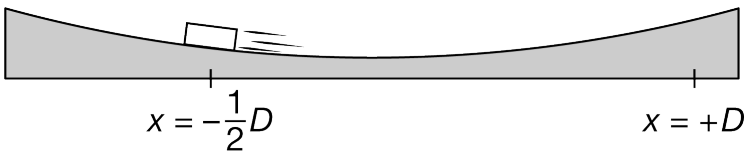
frq2



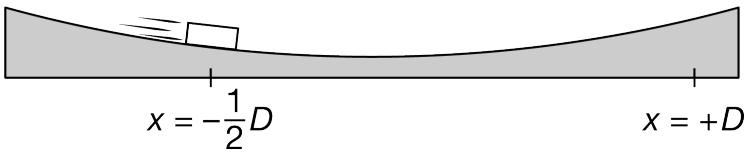
(c) Graph 3 above shows the total horizontal distance traveled by the block as a function of time as it completes two oscillations. Using the graph, determine the distance D between the midpoint of the ramp and the maximum position of the block on the ramp. Explain how you arrived at your answer.

(d) Suppose the entire setup was moved to the Moon, where the gravitational field strength is less than that on Earth. The center of mass of the block again oscillates between $x = -D$ and $x = +D$. Would the period of the block's oscillation on the Moon be greater than, less than, or equal to the period of oscillation on Earth? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{D^2}{2Hg}}$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?



Block on Track 2 at Time $t = T_1$



Block on Track 2 at Time $t = T_2$

Note: Figures not drawn to scale.

(f) Track 2 is identical to Track 1 except there is nonnegligible friction between the block and Track 2. The block

frq2

is released from rest on Track 2 at position $x = +D$. At time $t = T_1$, the block moves to the left, passing through position $x = -\frac{1}{2}D$, as shown above. After the block reverses direction near $x = -D$, the block moves to the right, passing through position $x = -\frac{1}{2}D$ at time $t = T_2$. At which time, T_1 or T_2, if either, is the magnitude of the tangential acceleration of the block greater? Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that only Graph 1 represents acceleration or that only Graph 2 represents momentum.
- ☐ 1 point is earned for a valid justification correctly relating force to acceleration and including a reference to a specific feature of the graph.
- ☐ 1 point is earned for a valid justification relating velocity to momentum and including a reference to a specific feature of the graph.

Example Response:

Graph 1 represents the magnitude of the cart's horizontal acceleration, and Graph 2 represents the block's momentum. The horizontal component of the acceleration will look like the net force graph, which should be a restoring force, and so should be proportional to D. The momentum of the cart graph will look like the velocity graph which has a maximum at $x = 0$ *and is zero at D and* $-D$.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for any statement that between $x = 0$ and $x = -D$, the kinetic or mechanical energy decreases.
- ☐ 1 point is earned for a clear indication that the mechanical energy of the system consists of only the kinetic energy of the block.

Example Response:

frq2

The mechanical energy of the block is only kinetic energy, so between $x = 0$ and $x = -D$, the mechanical energy (kinetic energy), decreases.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an attempt at relating the distance traveled to the amplitude.
- ☐ 1 point is earned for a correct answer: $D = 1.5\ \text{m}$.

Example Response:

D is equal to 1.5 meters. In two complete oscillations the block travels $8D$, in 8 seconds. From the graph we can see that the total distance traveled in 8 seconds is 12 meters, so the distance D is $(12\ \text{m})/8 = 1.5\ \text{m}$.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that the period of oscillation is longer on the moon.
- ☐ 1 point is earned for correct physical reasoning why the average speed or average acceleration is less on the moon.
- ☐ 1 point is earned for connecting smaller average speed or smaller average acceleration over the same distance to a longer time.

Example Response:

The period of oscillation on the moon would be greater. The average speed of the box on the moon would be smaller because the acceleration due to gravity is smaller, so for a smaller average speed, over the same distance, the time to oscillate would be larger.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence of *T* on *g*, with an attempt at justification.
- ☐ 1 point is earned for correctly indicating whether the dependence of *T* on *g* is consistent with the answer to part (d).
  Note: *T* should increase when *g* decreases. If part (d) was answered correctly, the equation is consistent.

Example Response:

Yes, this makes sense with my reasoning. Since the acceleration due to gravity, g, is in the denominator, when g is smaller, (like it is on the Moon), the period T is larger.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

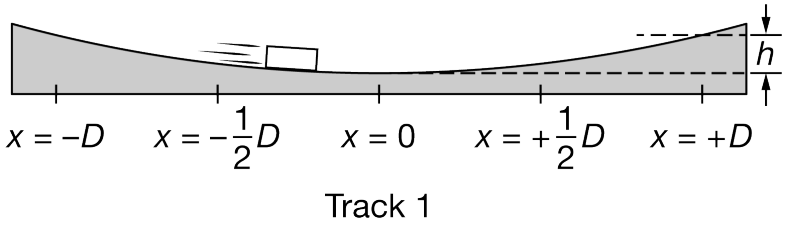
- ☐ 1 point is earned for a claim that the acceleration is greater at T_1.
- ☐ 1 point is earned for stating that both forces (component of gravity and friction) act on the block.
- ☐ 1 point is earned for correctly relating the sum of the forces on the block to the acceleration of the block at both T_1 OR T_2.

Example Response:

The tangential acceleration is greater at time T_1. At time T_1 both the component of gravity along the track and the friction force are in the same direction, while at T_2, the force of friction is in the opposite direction, so the force along the track is less at T_2, or greater at T_1. Since the force along the track is greater at T_1, the tangential acceleration of the block is greater at T_1.

frq2

17. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

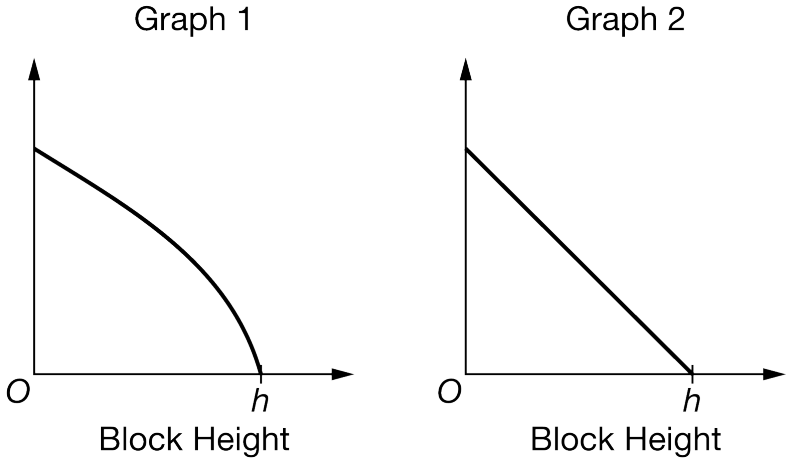


Track 1

Note: Figure not drawn to scale.

A block slides back and forth on a track with a slight circular curvature, where friction is negligible, as shown above. The maximum height h of the track at each end is much smaller than D.

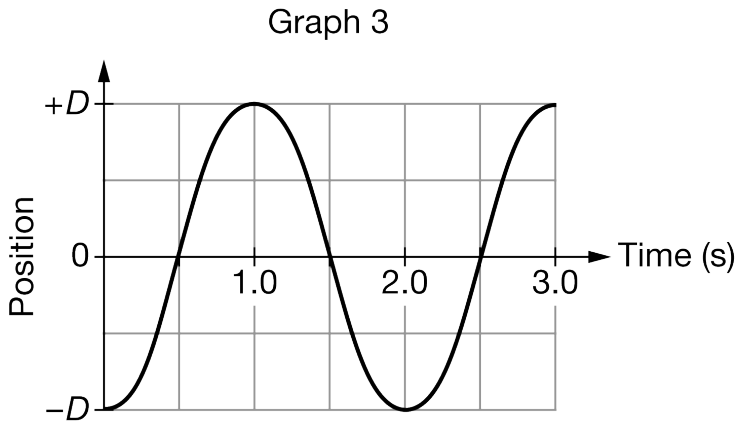
The center of mass of the block oscillates between $x = -D$ and $x = +D$, reaching a maximum height h every time the block is at $x = +D$ or $x = -D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. At time $t = 0$, the speed of the block is zero. The next time that the block's speed is zero is at time $t = 2.0$ s.



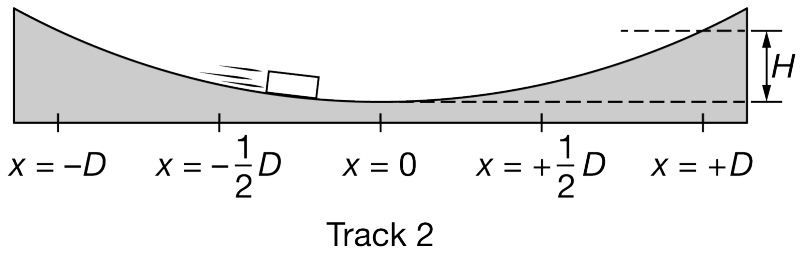
(a) Which of the graphs above, Graph 1 or Graph 2, could represent the kinetic energy of the block as a function of its height on the track, and which graph could represent the speed as a function of the block's height on the track? Justify both of your choices with physics principles.

(b) For the system consisting of the block and Earth, describe how the mechanical energy of the system changes, if at all, as the block moves from $x = +D$ to $x = 0$. Justify your answer.

frq2



(c) Graph 3 above incorrectly represents the horizontal position of the block as a function of time. Identify a feature of the graph that does not correctly represent the motion of the block. Justify your answer.



Note: Figure not drawn to scale.

(d) Track 2 is similar to Track 1, but the maximum height H on Track 2 is greater than h and is still much smaller than D, as shown above. The center of mass of the block oscillates between $x = -D$ and $x = +D$, the same as on Track 1. How does the period of the block's oscillation on Track 2 compare to the period of the block's oscillation on Track 1? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{H^2}{2Dg}}$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?

(f) When the block reaches $x = 0$ at the bottom of the circular track, is the force exerted by the track on the block greater than, less than, or equal to the weight of the block? Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

frq2

The response accurately includes **all** of the following criteria.

- 1 point is earned for a claim that only Graph 1 represents the speed as a function of height OR that only Graph 2 represents the kinetic energy as a function of height.
- 1 point is earned for a valid justification correctly relating speed to height and including a reference to a specific feature of the graph.
  Note: It acceptable to explain that speed is proportional to the square root of the kinetic energy.
- 1 point is earned for a valid justification that kinetic energy decreases linearly with increasing height and including a reference to a specific feature of the graph.

Example Response:

Graph 1 is the speed as a function of height and Graph 2 is the kinetic energy of the block as a function of height. Speed is equal to zero at h, but does not decrease linearly since the acceleration changes as the slope of the track changes. The kinetic energy however, changes linearly with height, since it is related to gravitational potential energy which changes linearly with height.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: The mechanical energy stays the same.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- 1 point is earned for a statement that the mechanical energy of the system consists of gravitational potential energy and kinetic energy of the block.
- 1 point is earned for a statement that no work is done on the system, so the mechanical energy is constant.

Example Response:

For this system, the total mechanical energy is both the kinetic energy of the block and the gravitational potential energy of the block-Earth system. Because there are no net external forces that do work on the system, the total mechanical energy of the system remains constant.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

frq2

The response accurately includes **both** of the following criteria.

- 1 point is earned for identifying that a maxima or minima on the graph indicates zero velocity,
  OR
  for identifying that the period on the graph is incorrect (2.0 seconds),
  OR
  for identifying that the graph should not have a zero slope at 1.0 seconds.
- 1 point is earned for a claim that the period should be longer (4.0 seconds) with no incorrect statements,
  OR
  for a claim that the next maxima should occur at 2.0 seconds with no incorrect statements,
  OR
  for a claim that the velocity should be zero (or flat slope) at only 0 and 2.0 seconds with no incorrect statements,
  OR
  for a claim that the graph should be "stretched" to twice its width along the *x*-axis (or similar) with no incorrect statements.

Example Response:

The period in this graph is incorrect – the period should be 4 seconds, since it takes 2 seconds just to go from $-D$ *to D, which is only half a period. Also, the velocity (the slope of the graph) should be zero at zero seconds and 2 seconds, not at 1 second.*

The problem states that the speed of the block is zero at $t = 0$ s *and* $t = 2$ s*, so the graph should have a maximum at* $t = 2$ s *and not* $t = 1$ s.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- 1 point is earned for a claim that the period of oscillation is shorter on Track 2.
- 1 point is earned for correct reasoning why the average speed or average acceleration is greater on Track 2 (connecting increased total mechanical energy to a greater average speed or connecting greater average acceleration to a steeper track).
- 1 point is earned for connecting greater average speed or greater average acceleration over the same distance to a shorter time.

Example Response:

The period is shorter on track 2 because the average speed is greater on track 2 because the total mechanical energy is larger since H is larger. Because the average speed is larger, over the same distance the block takes less time to travel from D to $-D$ *and back.*

Part E

frq2

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence of T on H, with an attempt at a justification.
- ☐ 1 point is earned for correctly indicating whether the dependence of T on H is consistent with the answer to part (d).

Example Response:

No this does not match my claim above. Because H is in the numerator, an increase in H should lead to an increase in T according to the equation. However, in part (d) I found that an increase in H leads to a shorter period (T).

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

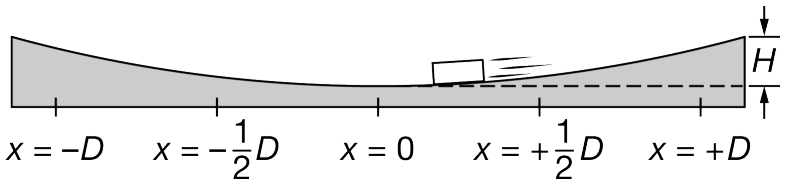
- ☐ 1 point is earned for a claim that the force exerted on the block by the track is greater than the weight.
- ☐ 1 point is earned for describing the net force in terms of the force of track on the block (or normal force) and weight only.
- ☐ 1 point is earned for describing that the net force is directed upward because the block is in circular motion.

Example Response:

The force exerted on the box by the track is greater than the weight at the lowest point, since the force from the track minus the weight of the block has to be equal to the centripetal force up.

frq2

18. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



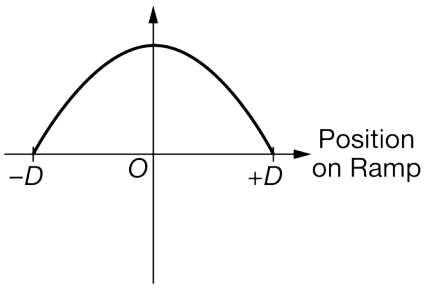
Track 1

Note: Figure not drawn to scale.

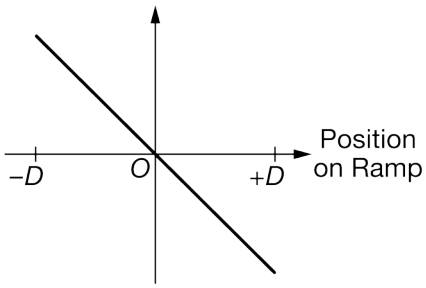
A block slides back and forth on a track with a slight circular curvature, where friction is negligible, as shown above. The maximum height H of the track at each end is much smaller than D.

The center of mass of the block oscillates between $x = -D$ and $x = +D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. At time $t = 0$ the block is released from rest at position $x = +D$. The period of the block's oscillation is 2.0 seconds.

Graph 1



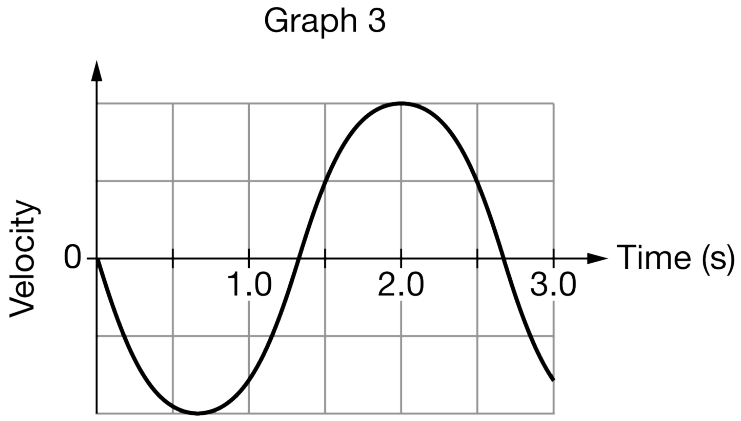
Graph 2



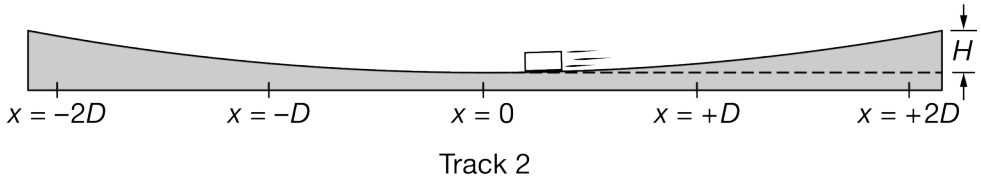
(a) Which of the graphs above, Graph 1 or Graph 2, could represent the block's kinetic energy as a function of its position on the circular track, and which graph could represent the horizontal component of the net force acting on the block as a function of its position on the track? Justify both of your choices with physics principles.

(b) Choose one of the graphs in part (a) above and explain how the graph shows evidence that a restoring force is acting on the block.

frq2



(c) Graph 3 above incorrectly shows the horizontal velocity of the block as a function of time. Identify a feature of the graph that does not correctly represent the motion of the block. Justify your answer.

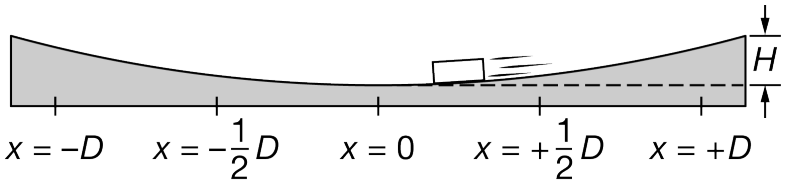


Note: Figure not drawn to scale.

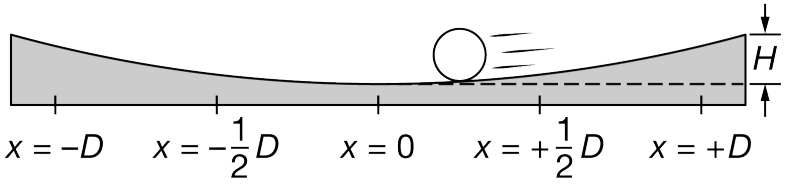
(d) Track 2, shown above, has a horizontal length that is greater than that of Track 1 and the same maximum height H. The block is released from rest at position $x = +2D$ Is the period of the block's oscillation on Track 2 greater than, less than, or equal to the period of oscillation on Track 1? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{h^2}{2dg}}$, where d is the maximum displacement of the block from equilibrium and h is the height of the track at $x = d$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?

frq2



Track 1



Track 3

Note: Figures not drawn to scale.

(f) On Track 1, shown above again for reference, friction is negligible and the center of mass of the block oscillates between a horizontal position of $+D$ and $-D$ with a period of 2.0 seconds. If a sphere with a mass equal to that of the block were to roll without slipping between $+D$ and $-D$ along Track 3 shown above, which is identical to Track 1 but has nonnegligible friction, would the period of the motion of the sphere be greater than, less than, or equal to 2.0 seconds? Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that only Graph 1 represents kinetic energy or that only Graph 2 represents the horizontal component of the net force.
- ☐ 1 point is earned for a valid justification relating the slope of the track to the horizontal force or describing the positive/negative values of the force on each side of the graph.
- ☐ 1 point is earned for a valid justification referring to the speed of the block and its kinetic energy and including a reference to a specific feature of the graph.

Example Response:

Graph 1 represents the kinetic energy of the block and Graph 2 represents the horizontal component of the net force. The kinetic energy is a maximum at $x = 0$ and a minimum at $x = -D$ and $x = +D$. The horizontal component of the net force is zero at $x = 0$ and the magnitude is a maximum at $x = -D$ and $x = +D$. The horizontal component of the net

frq2

force for small oscillations is linear and is proportional to displacement.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for identifying an appropriate feature of one of the graphs consistent with the answer in part (a).
- ☐ 1 point is earned for correctly explaining how a feature of the graph shows evidence of a restoring force.

Example Response:

Graph 2 shows evidence of a restoring force – a restoring force has a magnitude that is proportional to the displacement and opposite in direction to the displacement which is what graph 2 is showing.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for identifying that the period is incorrect,
  OR
  for identifying that the curve should intersect the horizontal axis at 1.0 seconds.
- ☐ 1 point is earned for a claim that the period is too long or that the graph should be compressed horizontally.

Example Response:

The period of the graph shown is incorrect. The problem states that the period is 2 seconds, but the graph shows a period that is longer than 2 seconds.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that the period of oscillation is longer on Track 2.
- ☐ 1 point is earned for correct reasoning why the average acceleration or average speed is less on Track 2.
- ☐ 1 point is earned for connecting the smaller average acceleration or average speed over the longer distance to a larger time.

Example Response:

The period on track 2 is longer. The block will have the same maximum speed but spread out over a much longer track means that the time to travel the length of the track and back is larger.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence of T on d, with an attempt at a justification.
- ☐ 1 point is earned for correctly indicating whether the dependence of T on d is consistent with the answer to part (d).
  Note: T should increase with d. If part (d) was answered correctly, the equation is inconsistent.

Example Response:

No this does not match my reasoning. In the equation when d is in the denominator, as d increases, T decreases, which means that if the ramp gets longer, the period should decrease, which is not what will happen.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

frq2

The response accurately includes **all** of the following criteria.

- 1 point is earned for a claim that the period of the sphere would be greater than the period of the block.
- 1 point is earned for a claim that maximum total kinetic energy of the sphere is equal to the maximum kinetic energy of the block,
  OR
  for a claim that the net force on the sphere will be less than that on the block because a frictional force is directed opposite the sphere's motion.
- 1 point is earned for recognizing that some portion of the sphere's kinetic energy is rotational,
  OR
  for recognizing that a smaller net force would result in a smaller average acceleration.

Example Response:

The period of the sphere would be greater. For both the block and the sphere the gravitational potential at the top are equal and so the kinetic energy as each passes through $x = 0$ *are equal. But for the sphere, some of that KE is turned into rotational kinetic energy, therefore it will be traveling, on average, slower than the block and so will take longer to travel the same distance.*

frq2

19. This question is a long free-response question. Show your work for each part of the question.

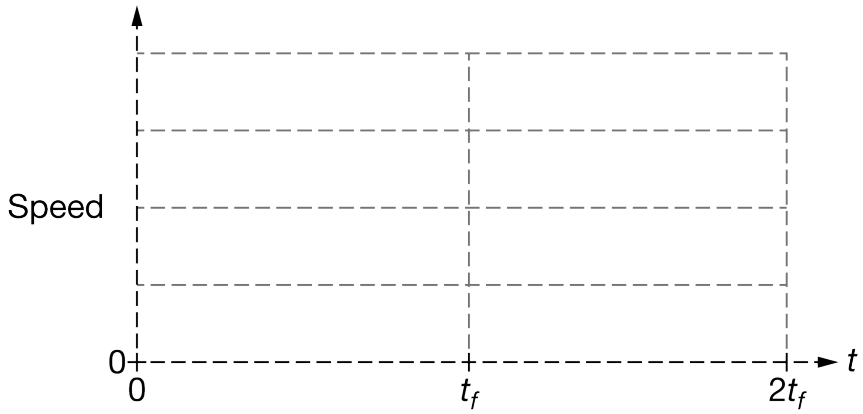
(12 points, suggested time 25 minutes)

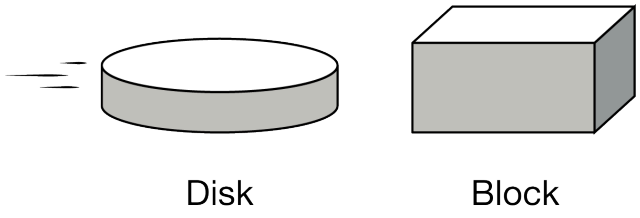
(a) A student of mass M_S, standing on a smooth surface, uses a stick to push a disk of mass M_D. The student exerts a constant horizontal force of magnitude F_H over the time interval from time $t = 0$ to $t = t_f$ while pushing the disk. Assume there is negligible friction between the disk and the surface.

i. Assuming the disk begins at rest, determine an expression for the final speed v_D of the disk relative to the surface. Express your answer in terms of F_H, t_f, M_S, M_D, and physical constants, as appropriate.

ii. Assume there is negligible friction between the student's shoes and the surface. After time t_f, the student slides with speed v_S. Derive an equation for the ratio v_D/v_S. Express your answer in terms of M_S, M_D, and physical constants, as appropriate.

(b) Assume that the student's mass is greater than that of the disk $(M_S > M_D)$. On the grid below, sketch graphs of the speeds of both the student and the disk as functions of time t between $t = 0$ and $t = 2t_f$. Assume that neither the disk nor the student collides with anything after $t = t_f$. On the vertical axis, label v_D and v_S. Label the graphs "S" and "D" for the student and the disk, respectively.





(c) The disk is now moving at a constant speed v_1 on the surface toward a block of mass M_B, which is at rest on the surface, as shown above. The disk and block collide head-on and stick together, and the center of mass of the disk-block system moves with speed v_{cm}.

frq2

i. Suppose the mass of the disk is much greater than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

ii. Suppose the mass of the disk is much less than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

iii. Now suppose that neither object's mass is much greater than the other but that they are not necessarily equal. Derive an equation for v_{cm}. Express your answer in terms of v_1, M_D, M_B, and physical constants, as appropriate.

iv. Consider the scenario from part (c)(i), where the mass of the disk was much greater than the mass of the block. Does your equation for v_{cm} from part (c)(iii) agree with your reasoning from part (c)(i) ?

____ Yes ____ No

Explain your reasoning by addressing why, according to your equation, v_{cm} becomes (or approaches) a certain value when M_D is much greater than M_B.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a correct answer $v_D = \frac{F_H t_f}{M_D}$ in part (a)(i).
- ☐ 1 point is earned for indicating the total momentum of the system is the same before and after the collision.
  Scoring Note: If the response only includes a correct final answer of $\frac{M_s}{M_D}$, the response earns this point but not the next point.
- ☐ 1 point is earned for correctly substituting the appropriate variables into a conservation of momentum equation
  AND

frq2

an answer in the form $\frac{v_D}{v_s} = \ldots$.

Scoring Notes:
This point can be earned only if the first point is earned.
The answer need not be correct to earn this point.

Example Response for part (a)(ii):

$p_i = p_f$
$0 = M_s v_s - M_D v_D$
$\frac{v_D}{v_s} = \frac{M_s}{M_D}$

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for two functions that are straight segments for $t < t_f$,
  AND
  begin at the origin,
  AND
  have two different positive slopes.
- ☐ 1 point is earned for two functions that are horizontal functions for $t > t_f$
  AND
  are continuous over the entire time range $0 < t < 2t_f$.
- ☐ 1 point is earned for labeling values on the vertical axis with $v_D > v_S$.
  OR
  The curve labeled D is greater than the curve labeled S for all $t > 0$.
  Scoring note: This point can still be earned if the labels are not on the vertical axis but clearly indicate that $v_D > v_S$.

Example Response:

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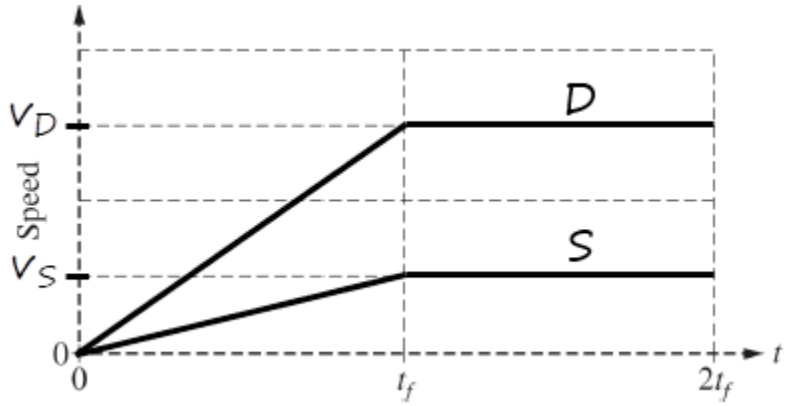


Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for stating or mathematically representing that if the disk is much more massive, then the block will have little effect on the motion of disk 1
  OR
  For stating or mathematically representing that when $M_D >> M_B$, $v_{cm} = v_1$.
- ☐ 1 point is earned for correct reasoning in part (c)(ii).
  Correct answer: When $M_D << M_B$, $v_{cm} = 0$
- ☐ 1 point is earned for using conservation of momentum in part (c)(iii).
- ☐ 1 point is earned for a correct answer in part (c)(iii), $v_{cm} = \frac{m_D v_1}{m_D + m_B}$.
- ☐ 1 point is earned for an attempt to use limiting-case reasoning or functional dependence with the equation in part (c)(iii).
- ☐ 1 point is earned for recognizing the equation from (c)(iii) reduces to a simpler form and the simplified form is correctly compared to their answer in (c)(i)

Example response for part (c)(ii)

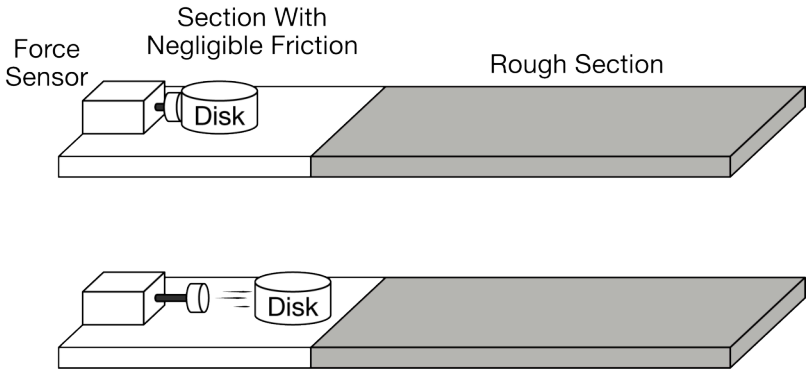
If the block is much more massive, then it will barely move when the disk collides and sticks to it.

Example response for part (c)(iv)

Yes. If M_B is very small, then the denominator of the equation simplifies to M_D, which then can cancel out of the equation leaving $v_{cm} = v_1$.

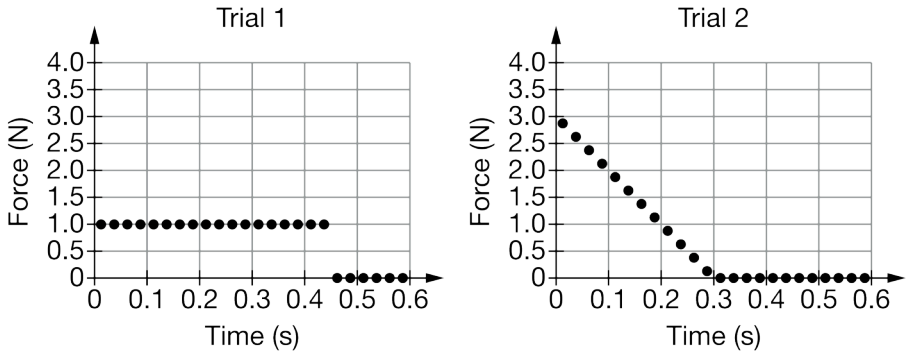
frq2

20. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Students use a force sensor to push a disk of mass m across a horizontal table. The force sensor pushes the disk over a short time interval. The magnitude of the force and the duration of the time interval can be set by the students. After being pushed by the sensor, the disk first slides along a surface with negligible friction and then enters a rough section where friction is not negligible, as shown above.

The students perform two trials in which the magnitude of the force exerted by the force sensor and the time interval are different. They create the graphs of force as a function of time shown below. The disk enters the rough section in each trial after $t = 0.5$ s.



(a) In which of the two trials, if either, is the change in momentum of the disk between 0 seconds and 0.5 seconds greater? Explain your reasoning.

(b) In both trials, the disk comes to rest in the rough section before reaching the edge of the table. In which trial, if either, does the disk travel a greater distance in the rough section? Explain your reasoning.

(c) The students repeat Trial 1 with a disk that is made from the same material as the original disk and has the same mass as the original disk but that has a larger radius (and therefore more surface area is in contact with the surface). The same force is exerted on the new disk over the same time interval as in Trial 1. Would the distance traveled in the rough section by the new disk be greater than, less than, or the same as the distance traveled by the original disk in Trial 1? Explain your reasoning.

Students derive the following equation, which may or may not be correct, to describe the distance D traveled by

frq2

the original disk on the rough section: $D = \frac{(F_{\text{avg}}{}^2)t^2}{2\mu mg}$, where F_{avg} is the average force exerted by the sensor, t is the time that the disk is in contact with the sensor, μ (mu) is the coefficient of kinetic friction, and m is the mass of the disk.

(d) Without algebraic manipulation of equations or attempting to derive this equation, indicate whether the above equation is consistent with your answer in part (b). Explain your reasoning.

A student measures the distance x_0 that the disk travels while in contact with the force sensor in Trial 1. The disk then travels a distance D along the rough section. The student claims that if a force with twice the magnitude of the original force was exerted on the disk for a distance x_0, the disk would travel a distance $2D$ in the rough section.

(e) Is the above claim correct or incorrect? Justify your answer without citing or manipulating equations.

The student finds the following equation on the Internet describing the relationship between the magnitude F of the force exerted on the disk, the distance x the disk is pushed, and the distance D it slides in the rough section: $D = C\frac{Fx}{\mu mg}$, where C is a constant with appropriate units.

(f) Indicate whether or not the equation above is consistent with your statement in part (e) relating D and x. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the impulse or $\mathrm{F(t)}$ or area under the curve is the impulse or change in momentum
  OR
  For recognizing that the acceleration can be determined from the force.
- ☐ 1 point is earned for correctly indicating the area is equal to the $\mathrm{F_{ave}(t)}$ or $\mathrm{F_t/2}$ or 0.45 Ns
  OR
  For addressing that the mass value is the same or need not be known.
- ☐ 1 point is earned for indicating that both disks have the same change in momentum
  OR
  For correct application of kinematics including the use of the time.

Example Response:

The area under each curve is the change in momentum, and since both graphs have the same area under the curve, $0.45\ Ns$*, both have the same change in momentum.*

frq2

OR

The change in momentum is the same for both pucks because the same average force was applied to the same mass for the same time frame causing the same launch velocity.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks.
- ☐ 1 point is earned for a correct relationship between the speed OR momentum OR kinetic energy and the distance traveled in the rough section consistent with their answer to part (a).

Example Response:

Both disks experience the same frictional force in both trials, so the acceleration will be the same for both disks, and since both disks start with the same initial speed, they will slide the same distance.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating both disks would have the same launch speed with the same force over the same time interval.
  OR
  For indicating that both disks will have the same kinetic energy OR momentum when a force is applied over the same time.
- ☐ 1 point is earned for indicating that the surface area of the disk does not affect the frictional force.
- ☐ 1 point is earned for indicating that the new block will travel the same distance before stopping.

Example Response:

Both disks will have the same launch speed because they experience the same force and have the same mass. The surface area of the disks does not affect the frictional force, so the new block will travel the same distance before stopping.

frq2

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an answer consistent with part (b), with an attempt to use the functional dependence of D on F_{avg} and/or t.
- ☐ 1 point is earned for correct reasoning (i.e., that $F_{\text{avg}}t$ is the same in both trials, so D is the same).

Example Response:

The equation is consistent with my reasoning in part (b). In the equation D is proportional to the square of the product of F_{avg} times t, which is the same for both trials, so D is the same.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: “Correct”.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a disk that is pushed with twice the force will have twice the original energy.
- ☐ 1 point is earned for a correct application of conservation of energy or work.
- ☐ 1 point is earned for indicating that since the work to stop the disk is also proportional to the distance to stop it, the disk will travel twice the distance before stopping.

Example Response:

A disk that is pushed from rest with twice the force for the same distance will have twice the work done one it, and so will have twice the original energy. The work done to stop the disk is equal to the energy that the disk has after being pushed. Work is also proportional to the distance to stop it; the new disk will travel twice the distance before stopping.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

0	1	2 ✓

The response accurately includes **both** of the following criteria.

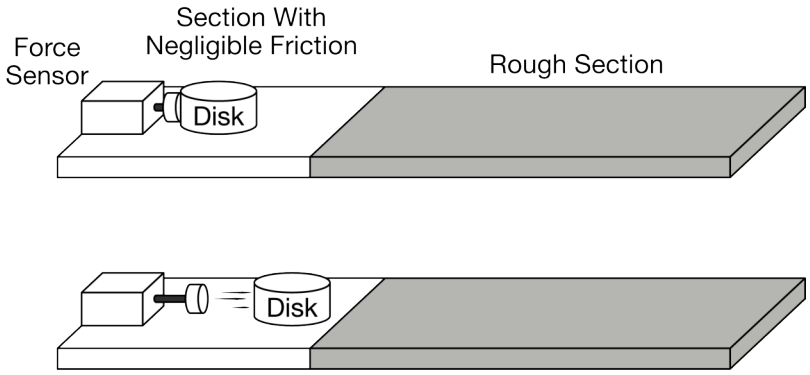
- ☐ 1 point is earned for indicating the equation is consistent with their statement in part (e) AND an attempt at a justification using functional dependence.
- ☐ 1 point is earned for correctly indicating D is proportional to F.

Example Response:

The given equation is consistent with my reasoning in part (e). Because D in the equation is directly proportional to F, pushing with twice the force will result in a distance D that is twice as large.

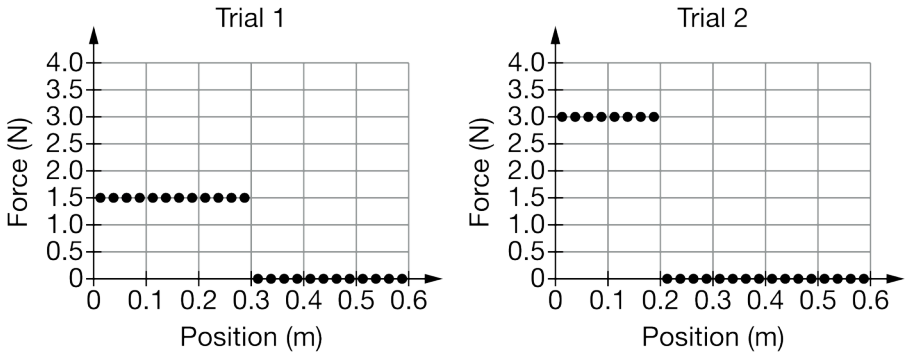
frq2

21. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Students use a force sensor to push a disk of mass m across a horizontal table. The force sensor pushes the disk over a short distance. The magnitude of the force and the distance the disk is pushed can be set by the students. After being pushed by the sensor, the disk first slides along a surface with negligible friction and then enters a rough section where friction is not negligible, as shown above.

The students perform two trials in which the magnitude of the force exerted by the force sensor and the distance the disk is pushed are different. They create the graphs of force as a function of position shown below. The disk enters the rough surface in each trial after moving 0.6 meters.



(a) In which of the two trials, if either, is the change in energy of the disk the greatest during the interval shown in the graphs? Explain your reasoning.

(b) In both trials, the disk comes to rest in the rough section before reaching the edge of the table. In which trial, if either, does the disk travel a shorter distance in the rough section? Explain your reasoning.

(c) The students repeat Trial 2 with a disk that is made from the same material as the original disk but that is less massive. The same constant force is exerted on the less massive disk over the same distance as in Trial 2. Would the distance traveled in the rough section by the less massive disk be greater than, less than, or the same as the distance traveled in the rough section by the original disk in Trial 2? Explain your reasoning.

Students derive the following equation, which may or may not be correct, to describe the distance D traveled by

frq2

the original disk in Trial 2 on the rough section: $D = \frac{Fx}{\mu mg}$, where F is the force exerted by the sensor, x is the distance over which the sensor exerts a force on the disk, μ (mu) is the coefficient of kinetic friction, and m is the mass of the disk.

(d) Without algebraic manipulation of equations or attempting to derive this equation, indicate whether the equation is consistent with your answer in part (b). Explain your reasoning.

A student measures the time t that the disk is in contact with the force sensor in Trial 2. The disk then travels a distance D along the rough section. The student claims that if the same force is exerted on the disk for a time $2t$, the disk would travel a distance $2D$ in the rough section.

(e) Is the above claim correct or incorrect? Justify your answer without citing or manipulating equations.

The student finds the following equation on the Internet describing the relationship between the time t the disk is pushed and the distance D it slides in the rough section: $D = C\frac{F^2t^2}{2\mu m^2 g}$, where C is a constant with appropriate units.

(f) Indicate whether or not the above equation is consistent with your statement in part (e) relating D and t. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that trial 2 has the graph with the greatest area under the curve
  OR
  For recognizing that the acceleration can be determined from the force.
- ☐ 1 point is earned for indicating the area represents the work done
  OR
  For addressing that the mass value is the same OR need not be known.
- ☐ 1 point is earned for relating the greatest work done to the greatest change in energy
  OR
  For correct application of kinematics including the use of the distance traveled.

Example Response:

Trial 2. The work done on the disk is represented by the area under the force vs position graph, so the graph with the largest area under the curve has the greatest work done on it. Graph 2 has the greatest area, and so the disk in trial 2 had the greatest amount of work done on it.

Part B

frq2

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks.
- ☐ 1 point is earned for indicating that the disk with the smaller launch speed will travel a shorter distance in the rough section.

Example Response:

Trial 1. The disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks. This means that the disk with the smaller speed after the push will travel a smaller distance.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a less massive disk would have a greater launch speed with the same force over the same distance.
- ☐ 1 point is earned for indicating that the distance traveled in the rough section, for a given speed, does not depend on the mass of the disk.
- ☐ 1 point is earned for indicating that the less massive disk will travel farther before stopping.

Example Response:

Farther than the original disk in trial 2. When pushed by the same force over the same distance, a less massive disk would have a larger launch speed. Since the mass of the disk doesn't affect the distance traveled through the rough section, the less massive disk will travel farther before stopping.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an answer consistent with part (b), with an attempt to use the functional dependence of D on F and/or x.
- ☐ 1 point is earned for correct reasoning (i.e., that Fx is greater in Trial 2, so D is greater).

Example Response:

This equation matches my reasoning in part (b). Since D is inversely proportional mass, if the mass increases, the distance D decreases, since m is in the numerator.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: “Incorrect”.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a disk that is pushed for twice the time will have twice the speed.
- ☐ 1 point is earned for a correct application of conservation of energy or work for the disk in the rough section.
- ☐ 1 point is earned for indicating that the stopping distance is proportional to kinetic energy, not speed.

Example Response:

A disk that is pushed from rest with the same force for twice the time will have twice the momentum. Since the mass of the disk doesn’t change, the second disk will have twice the speed. However, the stopping distance is proportional to the kinetic energy which is proportional to the velocity squared.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

0	1	2 ✓

The response accurately includes **both** of the following criteria.

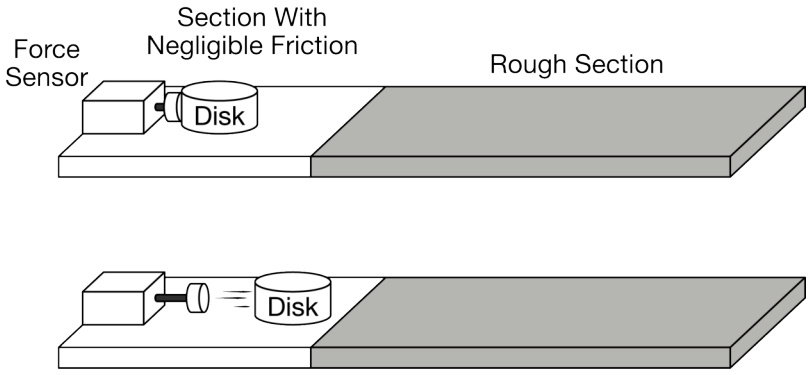
- ☐ 1 point is earned for indicating the equation is consistent (or inconsistent) with the statement in part (e) AND an attempt at a justification using functional dependence.
- ☐ 1 point is earned for indicating that the distance is proportional to *t* squared.

Example Response:

It is consistent. If t is doubled, the distance traveled should quadruple, because D is proportional to t squared.

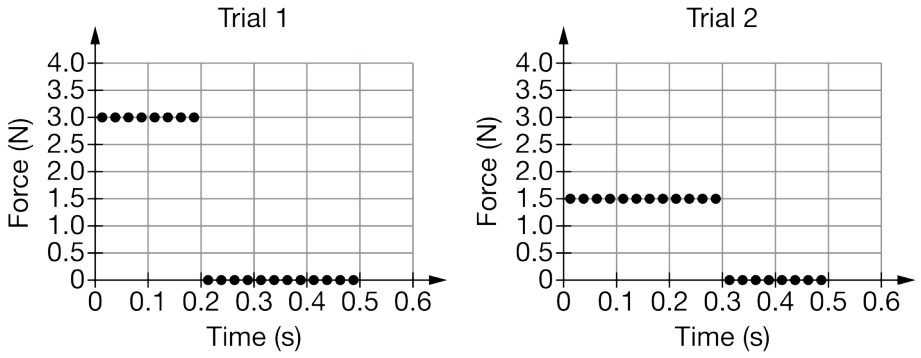
frq2

22. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Students use a force sensor to push a disk of mass m across a horizontal table. The force sensor pushes the disk with a force of constant magnitude over a short time interval. The magnitude of the force and the duration of the time interval can be set by the students. After being pushed by the sensor, the disk first slides along a surface with negligible friction and then enters a rough section where friction is not negligible, as shown above.

The students perform two trials in which the magnitude of the force exerted by the force sensor and the time interval are different. They create the graphs of force as a function of time shown below. The disk enters the rough surface in each trial after $t = 0.5$ s.



(a) In which of the two trials, if either, is the speed of the disk the greatest at time 0.5 seconds? Explain your reasoning.

(b) In both trials, the disk comes to rest in the rough section before reaching the edge of the table. In which trial, if either, does the disk travel a greater distance in the rough section? Explain your reasoning.

(c) The students repeat Trial 1 with a disk that is made from the same material as the original disk but that is more massive. The same force is exerted on the more massive disk over the same time interval as in Trial 1. Would the distance traveled in the rough section by the more massive disk be greater than, less than, or the same as the distance traveled by the original disk in Trial 1? Explain your reasoning.

Students derive the following equation, which may or may not be correct, to describe the distance D traveled by

frq2

the original disk on the rough section: $D = \frac{F^2t^2}{2\mu mg}$, where F is the force exerted by the sensor, t is the time that the disk is in contact with the sensor, μ (mu) is the coefficient of kinetic friction, and m is the mass of the disk.

(d) Without algebraic manipulation of equations or attempting to derive this equation, indicate whether the above equation is consistent with your answer in part (b). Explain your reasoning.

A student measures the distance x_0 that the disk travels while in contact with the force sensor in Trial 1. The disk then travels a distance D along the rough section. The student claims that if the same force is exerted on the disk for a distance $2x_0$, the disk would travel a distance $2D$ in the rough section.

(e) Is the above claim correct or incorrect? Justify your answer without citing or manipulating equations.

The student finds the following equation on the Internet describing the relationship between the distance x the disk is pushed and the distance D it slides in the rough section: $D = C\frac{Fx^2}{\mu mg}$, where C is a constant with appropriate units.

(f) Indicate whether or not the equation above is consistent with your statement in part (e) relating D and x. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that trial 1 has the graph with the greatest area under the curve OR impulse OR $F(t)$ OR addressing the numerical values of 0.45 Ns vs 0.6 Ns
  OR
  For recognizing that the acceleration can be determined from the force.
- ☐ 1 point is earned for indicating the area represents the change in momentum
  OR
  For addressing that the mass value is the same OR need not be known
- ☐ 1 point is earned for relating the greatest change in momentum to the greatest change in speed
  OR
  For correct application of kinematics including the use of the time.

Example Response:

Trial 1 has the biggest area under the curve, and since the area under a force vs time graph is impulse or change in momentum, the trial with the biggest area will have the biggest change in momentum. Both disks start at rest so the disk with the biggest change in momentum has the biggest change in speed.

OR

frq2

Because the mass of the puck in both trials is the same, the trial with the largest force has the largest acceleration. Since both pucks start at rest, the puck with the largest acceleration will have the largest speed when pushed for the same amount of time.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks.
- ☐ 1 point is earned for a correct relationship between the speed OR momentum OR kinetic energy and the distance traveled in the rough section consistent with their answer to part (a).

Example Response:

In both trials the disk experiences the same frictional force, so the acceleration is the same in both trials. The puck with the greatest launch speed travels the furthest before coming to rest because it will take a longer time to stop with the same acceleration.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a more massive disk would have a smaller launch speed OR momentum with the same force over the same time interval
  OR
  For indicating that a more massive disk will have less kinetic energy when a force is applied over the same time.
- ☐ 1 point is earned for indicating that the mass of the disk does not affect the distance traveled in the rough section.
- ☐ 1 point is earned for indicating that the more massive block will travel less far before stopping.

Example Response:

The more massive disk will have a smaller launch speed after being pushed with the same force for the same time. The pucks will have the same acceleration in the rough section of the track, regardless of the mass, so the more massive block

frq2

will not travel as far before stopping.

OR

The disk would have less kinetic energy over the same time frame, because the force would be applied over a smaller distance. Since the masses are equal, the F_f on the disks would be the same in the rough patch, for a disc with less kinetic energy entering the rough region, and experiencing the same F_f, the disk would not go as far.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an answer consistent with part (b), with an attempt to use the functional dependence of *D* on *F* and/or *t*.
- ☐ 1 point is earned for correct reasoning (i.e., that F_t is greater in trial 1, so *D* is greater).

Example Response:

The equation is consistent with my answer to part (b). D increases with the product F_t, so if this product increases, the distance D will also increase.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a disk that is pushed for twice the original distance will have twice the original energy.
- ☐ 1 point is earned for a correct application of conservation of energy or work.
- ☐ 1 point is earned for indicating that since the work to stop the disk is also proportional to the distance to stop it, the disk will travel twice the distance before stopping.

Example Response:

A disk that is pushed from rest for twice the original distance will have twice the work done on it and so will have twice the energy. The work done to stop the disk is equal to the energy that the disk has after being pushed. Work is also proportional to the distance to stop it; the disk will travel twice the distance before stopping.

frq2

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

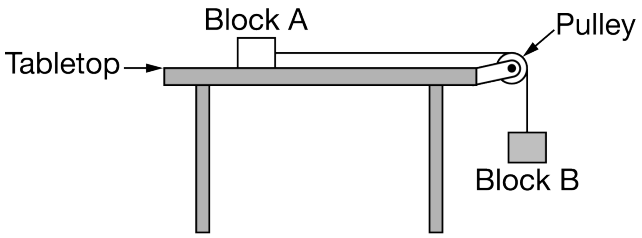
- ☐ 1 point is earned for indicating that the equation is not consistent (or consistent) with their statement in part (e) AND an attempt at justification using functional dependence.
- ☐ 1 point is earned for correctly indicating that D is proportional to x.

Example Response:

The given equation is not consistent with my statement in (e). In the equation D is directly proportional to x squared. But in part (e) I stated that if the distance x doubles, D will also double.

frq2

23. This question is a long free-response question. Show your work for each part of the question.



(12 points, suggested time 25 minutes)

This problem explores how the relative masses of two blocks affect the acceleration of the blocks. Block A, of mass m_A, rests on a horizontal tabletop. There is negligible friction between block A and the tabletop. Block B, of mass m_B, hangs from a light string that runs over a pulley and attaches to block A, as shown above. The pulley has negligible mass and spins with negligible friction about its axle. The blocks are released from rest.

(a)

i. Suppose the mass of block A is much greater than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

ii. Now suppose the mass of block A is much less than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

(b) Now suppose neither block's mass is much greater than the other, but that they are not necessarily equal. The dots below represent block A and block B, as indicated by the labels. On each dot, draw and label the forces (not components) exerted on that block after release. Represent each force by a distinct arrow starting on, and pointing away from, the dot.



Block A Block B

(c) Derive an equation for the acceleration of the blocks after release in terms of m_A, m_B, and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

(d) Consider the scenario from part (a)(ii), where the mass of block A is much less than the mass of block B. Does your equation for the acceleration of the blocks from part (c) agree with your reasoning in part (a)(ii)?

frq2

_____ Yes _____ No

Briefly explain your reasoning by addressing why, according to your equation, the acceleration becomes (or approaches) a certain value when m_A is much less than m_B.

(e) While the blocks are accelerating, the tension in the vertical portion of the string is T_1. Next, the pulley of negligible mass is replaced with a second pulley whose mass is not negligible. When the blocks are accelerating in this scenario, the tension in the vertical portion of the string is T_2. How do the two tensions compare to each other?

_____ $T_2 > T_1$ _____ $T_2 = T_1$ _____ $T_2 < T_1$

Briefly explain your reasoning.

Part A(i)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answers: "Zero", "small", "negligible", "much less than "g", or "$<< g$".

0	1	2 ✓

The response accurately includes **both** of the following criteria:

- ☐ For a correct answer and attempt at a consistent justification.
- ☐ For correct reasoning.

Example 1 Point Response:

Nearly zero. Because block A is much heavier than block B .

Example 2 Point Responses:

Very small. Because block A has a large inertia, it won't speed up much.

Close to zero because block B is so light that it can hardly budge block A .

Claim: The acceleration of the blocks is zero/small/negligible/ "$<< g$" .

Evidence: The mass of block A is much greater than the mass of block B .

Reasoning Statements: Very small. Because block A has a large inertia, it won't speed up much or close to zero because block B is so light that it can hardly budge block A .

Part A(ii)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

Correct Answer: g , $9.8\ \mathrm{m/s^2}$, or $10\ \mathrm{m/s^2}$ (or just 9.8 or 10)

0	1 ✓

The response does not accurately include:

- [ ] An indication that the acceleration is nearly equal to g and a correct justification.

Example Response:

Nearly equal to g . Because block B is almost in free fall.

$10\ \mathrm{m/s^2}$, because block A has negligible mass and the tension in the string is nearly zero.

Claim: The acceleration of the blocks is close to g .

Evidence: The mass of block A is much less than the mass of block B .There is negligible friction between block A and the tabletop. The pulley has negligible mass and spins with negligible friction about its axle.

Reasoning Statements: Nearly equal to g . Because block B is almost in free fall or $10\ \mathrm{m/s^2}$, because block A has negligible mass and the tension in the string is nearly zero.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: N , F_{N} , "normal force," F_{Table} , "table force," or any other label can be used to indicate the force is "normal" or comes from the table.

Note: F_{g} , F_{grav} , W , mg , $m_A g$, "gravity," "grav force," may be used.

Note: G or g are not acceptable.

Note: "tension," "string force," F_T, F_{tension}, F_{string}, F_S, T, or some other label indicating that the force comes from the string or from tension may be used.

Note: $m_B g$, F_{mB}, "force from block B " or other indications that the force is "created" by block B are NOT acceptable.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

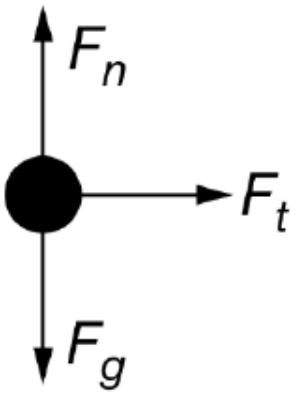
- [ ] A correct normal force on block A with acceptable label.
- [ ] Correct gravitational forces with acceptable label on both diagrams, and no extraneous forces on either

frq2

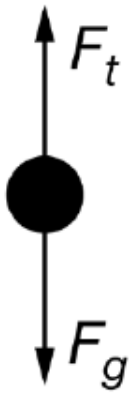
diagram.

- [ ] Correct tension forces with acceptable label on both diagrams.

Example Response:



Block A



Block B

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Writing a "whole system" equation is also sufficient to earn the first two points.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- [ ] For using separate Newton's second law equations for each block.
- [ ] For combining the equations with correct notation, including correctly using m_A and m_B , indicating that the same tension force acts on both blocks, *and* that they share the same acceleration.
- [ ] For a correct equation for a with supporting work.

Example Response:

frq2

$$\sum F_B = m_B a$$
$$m_B g - F_T = m_B a$$
$$\sum F_A = m_A a$$
$$F_T = m_A a$$
$$m_B g - m_A a = m_B a$$
$$m_B g = (m_B + m_A) a$$
$$a = \frac{m_B g}{(m_B + m_A)}$$

Part C - Alternate Solution

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: *Writing the correct whole-system equation is sufficient to earn the first two points of part (c).*

0	1	2	3 ✓

*The response accurately includes **three** of the following criteria.*

- ☐ *For writing a "whole-system" equation for the total mass that does not contain internal forces.*
- ☐ *For substituting the net force and system mass with correct quantities.*
- ☐ *For a correct equation for a with supporting work.*

Example Response:

$F_{\text{net}} = m_{\text{total}} a$

$m_B g = (m_A + m_B) a$

$a = \frac{m_B}{m_A + m_B} g$

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answer: "Yes"

frq2

Note: “No” is acceptable if the equation is inconsistent with the answer in (a)(ii).

0	1 ✓

The response accurately includes

- ☐ A valid reasoning that addresses the result in part (c) and the reasoning in part (a)(ii).

Example Response:

Claim:
Yes, the equation for the acceleration of the blocks from part (c) agrees with the reasoning in part (a)(ii).

OR
No, the equation for the acceleration of the blocks from part (c) does not agree with the reasoning in part (a)(ii).

Evidence:

The mass of block A is much less than the mass of block B .
$a = \frac{m_B}{m_A+m_B} g$ (derived as part (c) answer)

Reasoning Statements for “Yes” claim:
When m_A is much less than m_B , it can be neglected in the equation derived in part (c), giving an acceleration close to g as stated in (a)(ii).

Reasoning Statements for “No” claim, if the answer in part (a)(ii) is wrong:
When m_A is much less than m_B , it can be neglected in the equation derived in part (c), giving an acceleration close to g . This disagrees with the value of ___ stated in (a)(ii).

Reasoning Statements for “No” claim, if the answer in part (c) is wrong:

When m_A is much less than m_B , it can be neglected in the equation derived in part (c), giving an acceleration of ___. This disagrees with the value of g stated in (a)(ii).

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answer: $T_2 > T_1$.

Note: A maximum of 1 point can be earned for part (e) if an incorrect selection is made.

0	1	2 ✓

The response accurately includes **both** of the following criteria:

frq2

- For reasoning that the acceleration of both blocks is smaller.
- For doing any one of the following, consistent with the answer selection and Newton's second law for block B .
  - Concluding that a smaller acceleration implies that T_2 is greater than T_1
  - Concluding that an unchanged acceleration implies that T_2 is the same as T_1
  - Concluding that a larger acceleration implies that T_2 is less than T_1

Example Response:

Claim: $T_2 > T_1$

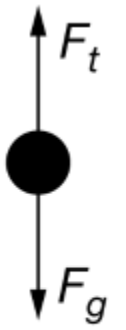
Evidence:

The pulleys spin with negligible friction about the axle.

The original pulley has negligible mass.

The second pulley's mass is not negligible.

$\overrightarrow{a} = \frac{\sum \vec{F}}{m}$



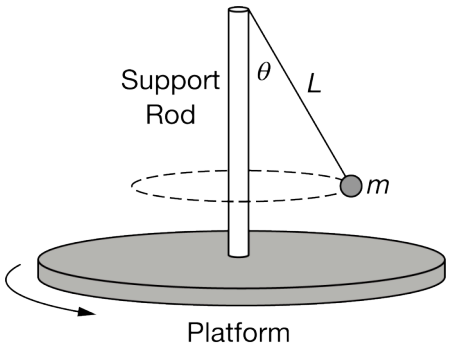
Block B

Reasoning Statements:

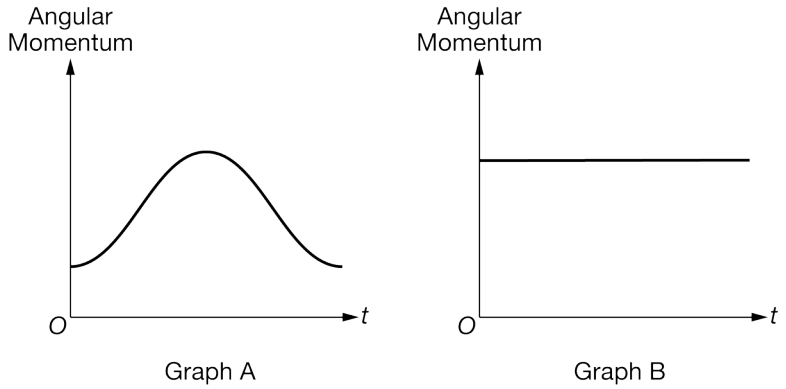
The rotational inertia of the second pulley results in a smaller acceleration for the blocks. Block B must have a smaller net force to have a smaller acceleration, so the rope tension must be larger than before (closer in magnitude to the gravitational force on block B).

frq2

24. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A vertical support rod is fixed at the center of a platform. A light string of length L is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass m, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed ω (omega), the string makes an angle θ (theta) with the rod.



(a) The platform rotates with a constant angular speed. Which of the above graphs correctly shows the magnitude of the angular momentum of the platform as a function of time t about a vertical axis at the center of the support rod? Briefly justify your answer.

(b) The platform rotates with a constant angular speed. Is the net torque exerted on the platform about its center clockwise, counterclockwise, or zero? Explain your reasoning.

(c) As the period of rotation becomes very small, what value does θ approach? Without writing any equations or formulas, justify your answer.

(d) A student derives an equation for the square of the period of revolution T of the sphere as a function of θ: $T^2 = \left(4\pi^2\right)\frac{L\cos\theta}{g}$. Indicate whether the equation is consistent with your answer in part (c) and explain why.

(e) In a series of experimental trials, a group of students changes L and adjusts T to keep θ constant. They record L and T for each trial. If the students use the equation given in part (d), what quantities could they graph to yield a

frq2

straight line whose slope could be used to find the acceleration due to gravity, g ?

(f) As the angular speed of the platform increases, does the period of revolution of the sphere increase, decrease, or stay the same? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for identifying the correct graph as graph B.
- ☐ 1 point is earned for a justification that angular momentum is directly proportional to the angular speed which remains constant.

Example Response:

Graph B is correct because the angular speed is constant, and therefore the angular momentum is constant.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that the net torque on the platform is zero.
- ☐ 1 point is earned for a correct statement relating the angular speed to angular acceleration.
- ☐ 1 point is earned for a correct statement relating net torque to angular acceleration.

Example Response:

The net torque on the platform is zero. Because the platform rotates with a constant angular speed, its angular acceleration is zero, and net external torque is equal to the rotational inertia times the angular acceleration, so if the angular acceleration is zero, the net external torque is also zero.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq2

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that correctly relates the period to the angular speed.
- ☐ 1 point is earned for a statement correctly relating the angular speed to the angle theta.

Example Response:

When the period is very small the platform is rotating quickly, making the angular speed very large. When the angular speed is very large, the angle theta is close to 90 degrees.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that is consistent with the reasoning in part (c).
- ☐ 1 point is earned for addressing functional dependence.
- ☐ 1 point is earned for a correct functional dependence argument.

Example Response:

The equation is consistent with part (c) because as theta gets close to 90 degrees, cos theta gets very small, and therefore the period gets very small.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement identifying a T term on one axis.

frq2

- [ ] 1 point is earned for a statement identifying any *L* term on the other axis.
- [ ] 1 point is earned for a correct power relationship for *T* and *L* (i.e., *T* squared and *L*).

Example Response:

They could graph T squared as a function of L.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

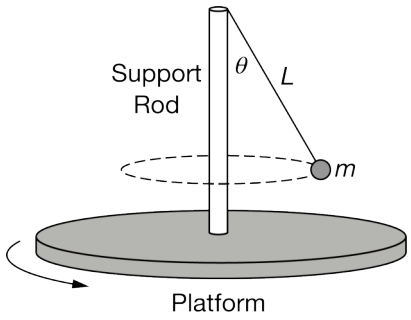
- [ ] 1 point is earned for a statement that the period of rotation decreases.
- [ ] 1 point is earned for a correct explanation of the relationship between the period and the angular speed of the platform.

Example Response:

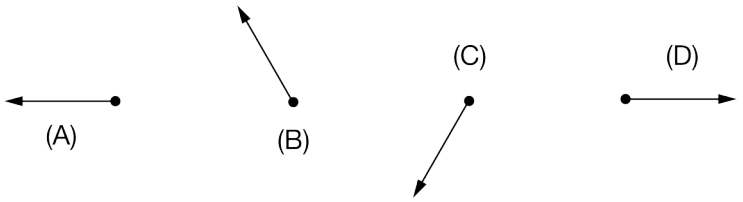
The period will decrease with angular speed because T = 2 pi / omega.

frq2

25. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A vertical support rod is fixed at the center of a platform. A light string of length L is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass m, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed ω (omega), the string makes an angle θ (theta) with the rod.



(a) Identify which of the above vectors represents the net force exerted on the sphere at the moment shown above. Briefly justify your answer.

(b) Is the magnitude of the tension in the string greater than, less than, or equal to the weight of the sphere? Explain your reasoning.

(c) The string can never be completely horizontal no matter how fast the platform is rotating. Without writing any equations or formulas, explain why.

One of the following two equations for the square of the tangential speed v of the sphere as a function of θ is consistent with the statement in part (c) that the string can never be horizontal as the platform is rotating.

Equation 1: $v^2 = \frac{gL(\sin^2\theta)}{(\cos\theta)}$ **Equation 2:** $v^2 = \frac{gL(\cos^2\theta)}{(\sin\theta)}$

(d) Indicate which equation is consistent with the statement in part (c), and explain why. Also explain why the other equation is not consistent with the statement in part (c).

(e) In a series of experimental trials, a group of students changes L and adjusts v to keep θ constant. They record L and v for each trial. Based on the equation in part (d) that you indicated as consistent with the statement in part (c), what quantities could the students graph to yield a straight line whose slope could be used to find the acceleration due to gravity, g ?

frq2

(f) Let I be the rotational inertia of the sphere-rod-platform system about a vertical axis through the center of the support rod. Does I increase, decrease, or stay the same when L increases (but θ stays constant)? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answer: (A), with a correct justification

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement identifying the correct net force vector (A)
- ☐ 1 point is earned for a justification that includes that the net force or acceleration will point toward the center of the circular path

Example Responses:

Figure A because the net force is toward the center of the sphere's circular motion.

OR

The forces acting on the sphere are the force of gravity and the force of tension. The force of gravity is directly toward the ground, and the force of tension is at an angle toward the top of the rod. When the forces are added together, the vertical component of the force of tension and the force of gravity cancel each other out, leaving only the horizontal component of the force of tension, which is demonstrated by vector A.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- ☐ 1 point is earned for a statement that the tension in the string is greater than the weight of the sphere
- ☐ 1 point is earned for an explanation referring to the components of the vectors
- ☐ 1 point is earned for a correct justification that addresses the sum of the vector components

Example Responses:

frq2

The tension is greater than the weight because the vertical component of the tension force must be equal to the weight. The tension also has a horizontal component, so the magnitude will be greater than either of the components.

OR

Explaining that

$\Sigma F_y = F_{Ty} - mg = 0$

$F_T \cos\theta = mg$

$F_T = mg/\cos\theta$

(Since $\cos\theta < 1$*), therefore* $F_T > mg$ *(or > "weight")*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement identifying forces and/or force components in the vertical direction
- ☐ 1 point is earned for a justification that includes that the weight of the sphere must be balanced by the string and so the vertical component of the tension can never be zero

Example Response:

The weight of the sphere is downward and will need to be balanced by the vertical component of the tension. So, there will always be a vertical component of tension.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Notes:

· If students switch sine and cosine (such that they use Equation 2) but then are completely consistent, they can earn 2 of 3 points in part (d).

· If students misinterpret theta as being zero when the string is horizontal, they can earn two of three points in part (d) for using Equation 2 consistently.

0	1	2	3 ✓

frq2

The response accurately includes **three** of the following criteria.

- [ ] 1 point is earned for a statement addressing functional dependence (regardless of the equation chosen)
- [ ] 1 point is earned for indicating that equation 1 shows that as theta goes to 90 degrees, *v* gets very large (infinite/undefined)
- [ ] 1 point is earned for correctly indicating a flaw in equation 2

Example Response:

Equation 1 is consistent with part (c) because as theta gets closer to 90 degrees, the speed gets very large. This shows that no matter how large *v* gets, theta will never get to 90 degrees. Equation 2 is not consistent because it has a finite value for theta equal to 90 degrees.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- [ ] 1 point is earned for a statement identifying any *v* term on one axis
- [ ] 1 point is earned for a statement identifying any *L* term on the other axis
- [ ] 1 point is earned for a statement identifying correct power relationships for *v* and *L* (i.e., *v* squared vs. *L*)

Example Response:

They could graph *v* squared on the *y*-axis as a function of *L* on the *x*-axis.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for a statement identifying that the rotational inertia will increase
- [ ] 1 point is earned for a statement that since the mass is further from the axis, the rotational inertia is bigger

Example Response:

frq2

I will increase because for a larger *L*, the distance of the mass from the axis will be larger, and rotational inertia increases with distance.