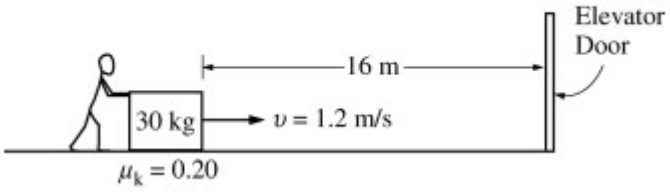
frq3

1.



Beginning at time $t = 0$, a student exerts a horizontal force on a box of mass 30 kg, causing it to move at 1.2 m/s toward an elevator door located 16 m away, as shown above. The coefficient of kinetic friction u_k between the box and the floor is 0.20.

(a) On the dot below that represents the box, draw and label the forces (not components) that act on the box as it moves at constant speed.

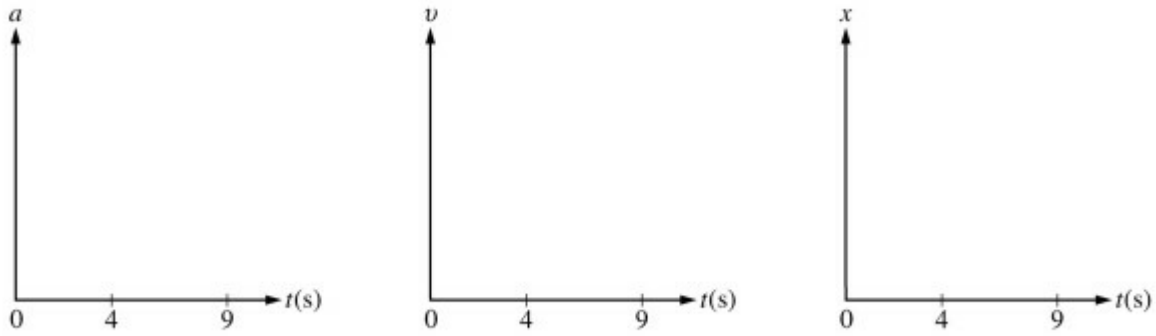
(b) Calculate the magnitude of the horizontal force the student must exert on the box in order to keep it moving at 1.2 m/s. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

At $t = 4.0$ s, the elevator door opens and remains open for 5.0 s. The student immediately exerts a larger constant force on the box and the front of the box reaches the elevator door just as it starts to close.

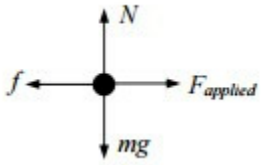
(c) Calculate the magnitude of the new force that the student exerts.

(d) On the axes below, sketch graphs of the acceleration a, velocity v, and position x of the box versus time t between $t = 0$ and the time the front of the box reaches the elevator.

frq3



Part A

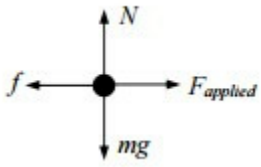


The line of action of each force vector must pass through the dot.

One point is earned for each correctly drawn and labeled force

0	1	2	3	4 ✓

The student earns all of the following points:



The line of action of each force vector must pass through the dot.

One point is earned for each correctly drawn and labeled force

Part B

To move the box at constant speed the applied force must equal the frictional force.

frq3

$\sum F = F_{applied} - f = ma = 0$

$F_{applied} = f$

One point is earned for a correct expression relating the frictional and normal forces

$f = \mu N$

One point is earned for a correct expression for the normal force

$N = mg$
$f = \mu mg$

One point is earned for consistent substitution and evaluation

$$f = (0.20)(30kg)(9.8m/s^2)$$
$$f = 58.8N \ (or \ 60N \text{ using } g = 10m/s^2)$$

0	1	2	3 ✓

The student earns all of the following points:

To move the box at constant speed the applied force must equal the frictional force.

$\sum F = F_{applied} - f = ma = 0$

$F_{applied} = f$

One point is earned for a correct expression relating the frictional and normal forces

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One point is earned for consistent substitution and evaluation

$$f = (0.20)(30kg)(9.8m/s^2)$$
$$f = 58.8N \ (or \ 60N \text{ using } g = 10m/s^2)$$

Part C

frq3

One point is earned for correctly determining the distance between the box and the elevator at $t = 4$ s

$d = 16m - (1.2m/s)(4s) = 11.2m$

One point is earned for consistent substitution into a correct kinematic equation to determine the box's acceleration

$d = v_0t + (1/2)at^2$
$a = 2(d - v_0t)/t^2 = 2((11.2m) - (1.2m/s)(5/s))/(5s)^2 = 0.42m/s^2$

One point is earned for an application of Newton's second law to find the new applied force

$F'_{applied} - f = ma$
$F'_{applied} = ma + f$

One point is earned for consistent substitution of values into Newton's second law

$$F'_{applied} = (30kg)(0.42m/s^2) + 58.8N$$
$$F'_{applied} = 71.4N \ (or\ 72.6N \text{ using } g = 10m/s^2$$

0	1	2	3	4 ✓

The student earns all of the following points:

One point is earned for correctly determining the distance between the box and the elevator at $t = 4$ s

$d = 16m - (1.2m/s)(4s) = 11.2m$

One point is earned for consistent substitution into a correct kinematic equation to determine the box's acceleration

$d = v_0t + (1/2)at^2$
$a = 2(d - v_0t)/t^2 = 2((11.2m) - (1.2m/s)(5/s))/(5s)^2 = 0.42m/s^2$

One point is earned for an application of Newton's second law to find the new applied force

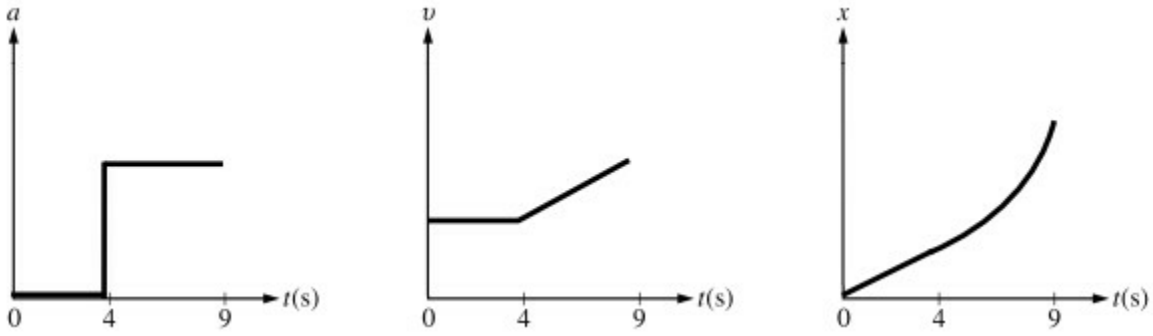
$F'_{applied} - f = ma$
$F'_{applied} = ma + f$

One point is earned for consistent substitution of values into Newton's second law

$$F'_{applied} = (30kg)(0.42m/s^2) + 58.8N$$
$$F'_{applied} = 71.4N \ (or\ 72.6N \text{ using } g = 10m/s^2$$

Part D

frq3



One point is earned for indicating a zero initial acceleration

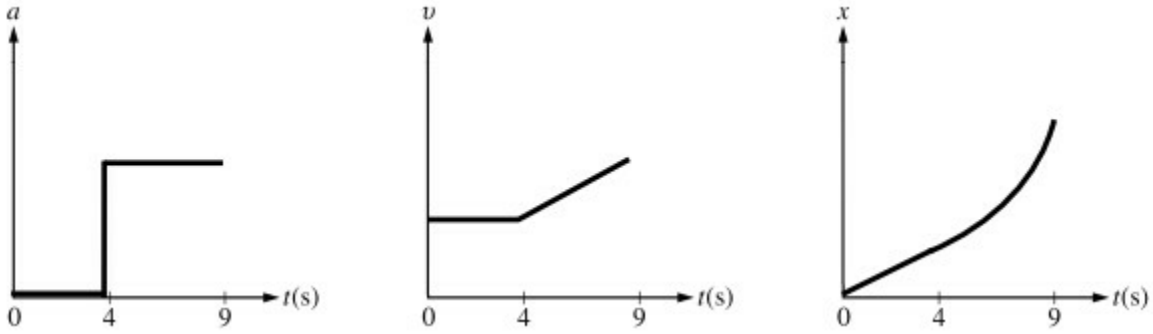
One point is earned for showing zero acceleration until 4 s, then a change to a non-zero, positive constant until 9 s

One point is earned for a velocity graph that is a non-zero, positive constant until 4 s, then linearly increasing until 9 s

One point is earned for a distance graph that is linearly increasing until 4 s, then appropriately curved until 9 s

0	1	2	3	4 ✓

The student earns all of the following points:



One point is earned for indicating a zero initial acceleration

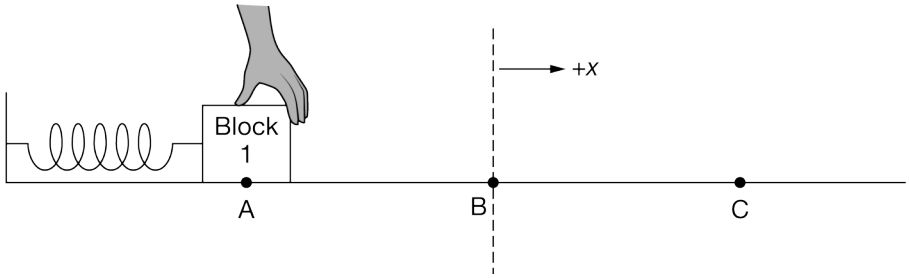
One point is earned for showing zero acceleration until 4 s, then a change to a non-zero, positive constant until 9 s

One point is earned for a velocity graph that is a non-zero, positive constant until 4 s, then linearly increasing until 9 s

One point is earned for a distance graph that is linearly increasing until 4 s, then appropriately curved until 9 s

frq3

2. This question is a short free-response question. Show your work for each part of the question.



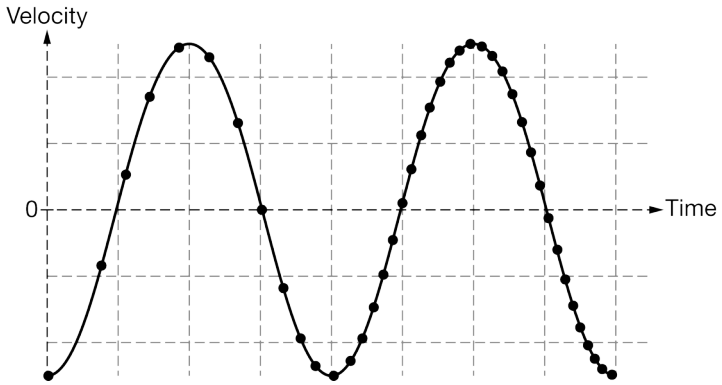
(7 points, suggested time 13 minutes)

Block 1 is at rest on a horizontal surface and is connected to a wall by an ideal spring. Friction between block 1 and the surface is negligible. Block 1 is held at rest at point A, to the left of point B which is the equilibrium position of the spring-block system, as shown in the figure. Block 1 is then released and allowed to oscillate. Some time later, block 1 is momentarily at rest at point C. Consider the positive horizontal direction to be toward the right.

(a) The dot below represents block 1 just after it is released. On the dot, draw an arrow indicating the direction of the net force exerted on the block just after it is released. The arrow should start on, and point away from, the dot.

●

Net Force on Block 1



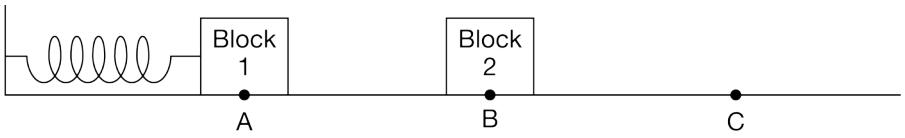
After the block has been oscillating on the table, a motion detector is turned on and the velocity of block 1 as a

frq3

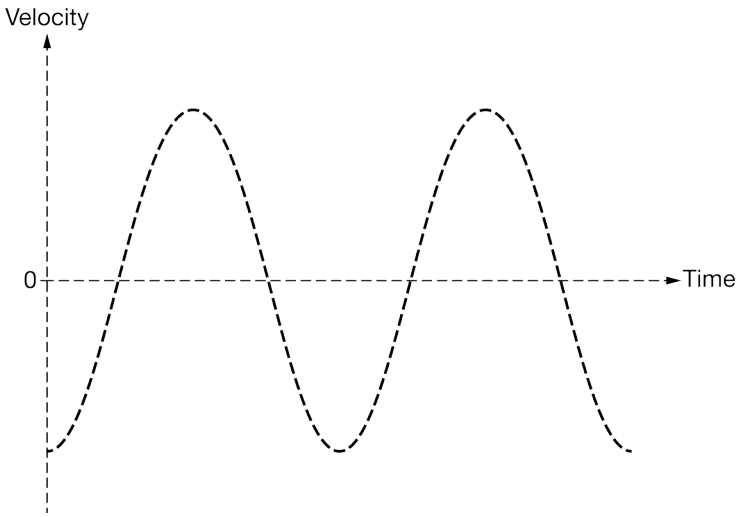
function of time is measured and plotted in a graph. The students correctly draw a best-fit curve that represents the collected data, as shown above.

(b) A student looking at the graph of velocity as a function of time makes the following claim.
"*The motion sensor was turned on when the block was at point A and was moving to the left*."

Does the data in this graph support the student's claim about the location AND the direction of the block's motion when the motion sensor was turned on? For any incorrect claims, if any, state the correct position and/or direction of motion. Justify your answer.

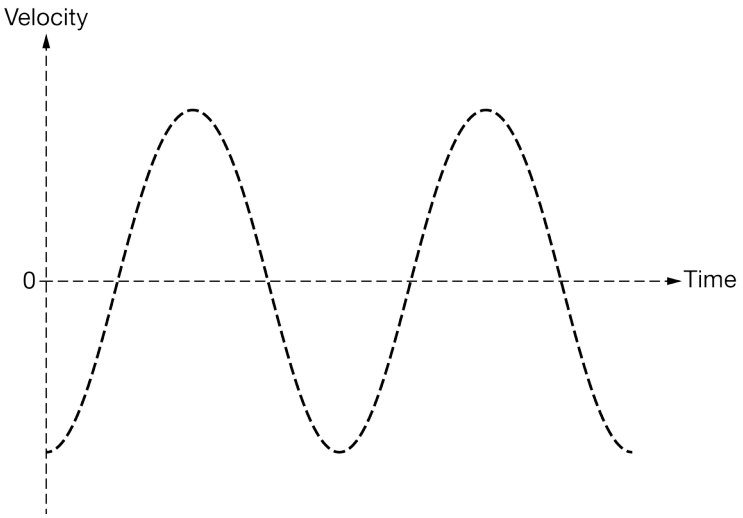


(c) Block 1 is stopped, and again block 1 is held at rest at point A. A second identical block, block 2, is placed at point B. Block 1 is released from rest, and then collides with and sticks to Block 2. After the collision, the two blocks oscillate together on the spring. On the graph below, sketch a curve to indicate the new velocity as a function of time of the two blocks after the collision. For reference, the dashed curve represents the students' best-fit curve of the velocity as a function of time described in part (b).



Note: Do any scratch (practice) work on the graph below. Sketches made below will NOT be graded.

frq3



Part A

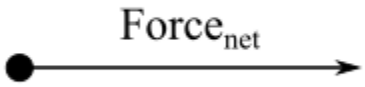
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: The amount of current that flows through each resistor must be indicated to earn the second point of part (a).

0	1 ✓

The response accurately includes **one** of the following criteria:

- ☐ For a net force arrow that is labeled and points horizontally to the right.



Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

frq3

The response accurately includes **three** of the following criteria.

- ☐ For a correct claim that the motion sensor was started when the block was at point B.
- ☐ For a correct justification of why the block is not at point B **OR** For a correct justification of why the block is at point C
- ☐ For a correct justification of why the block is moving to the left.

Example Response:

At time $t = 0$ s the block has a maximum velocity, which means that it is traveling through the equilibrium position, which is position B. Since the velocity is negative, and to the right was defined as positive, the block must be moving to the left through position B.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

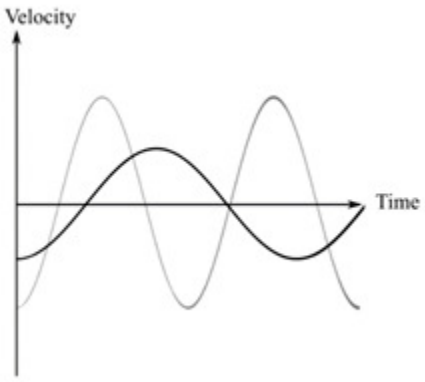
Scoring note: The starting point of this graph is not required to be correct to earn these points. Any phase shift is acceptable for this graph.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

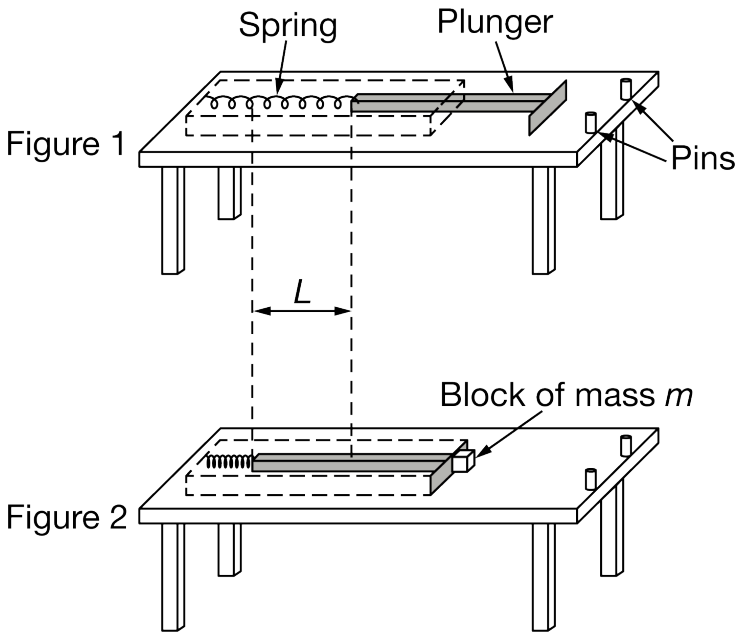
- ☐ For a sine or cosine graph
- ☐ For a graph with a smaller maximum velocity
- ☐ For a sinusoidal graph with a longer period

Example Response:

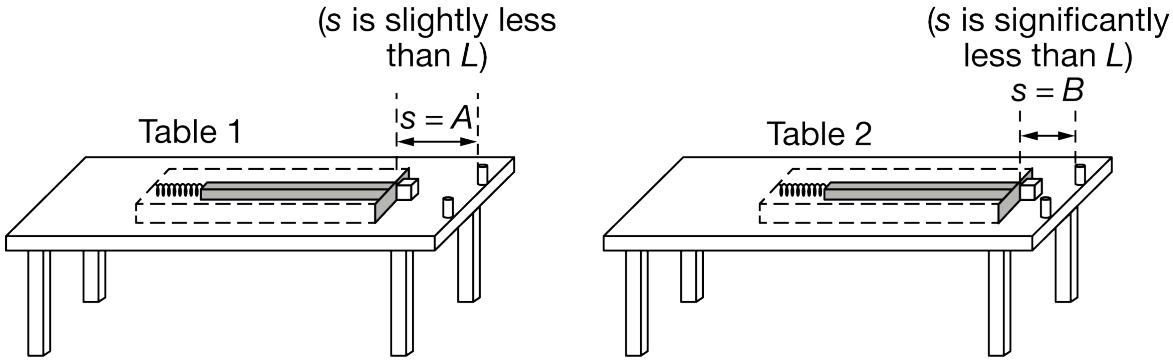


frq3

3. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A spring-loaded launcher can be securely attached to a tabletop. When the spring is at its uncompressed length, the plunger is in the position shown above in Figure 1. A small wood block can be pressed against the end of the plunger, compressing the spring a distance L, as shown in Figure 2. When the block is released, the plunger pushes the block along the tabletop. Two pins are attached to the edge of the table to prevent the plunger from extending beyond the table. When the plunger hits the pins, the block then leaves the table. There is nonnegligible friction between the block and the tabletop.



The launcher can be moved closer to or farther from the edge of the tabletop. Students set up tables 1 and 2, as shown above, with tabletops that are made of the same material. On table 1, the launcher is positioned so that the distance $s = A$ between the launcher and the edge of the tabletop is slightly less than L. On table 2, the launcher is positioned so that the distance $s = B$ between the launcher and the edge of the tabletop is significantly less than L. On both tables, the block is released from rest and loses contact with the plunger at the moment the plunger reaches the pins. Tables 1 and 2 are the same height.

frq3

k is the spring constant of the spring.
m is the mass of the block.
L is the distance the spring is compressed.
μ (mu) is the coefficient of kinetic friction between the block and the tabletop.
s is the distance between the launcher and the edge of the tabletop.

Assume the spring is ideal, the plunger has negligible mass, the block loses contact with the plunger immediately after it reaches the end of the table, and there is non-negligible friction between the tabletops and the blocks.

(a) Without manipulating equations, explain why the block launched from table 1 could land farther from the table than the block launched from table 2 does.

(b) Does the block launched from table 1 spend more, less, or the same amount of time in the air than the block launched from table 2 does? Explain your reasoning.

(c) Consider the Earth-block system. Determine the change, if any, in the total mechanical energy of the system from the instant the block leaves the table to the instant immediately before it reaches the ground.

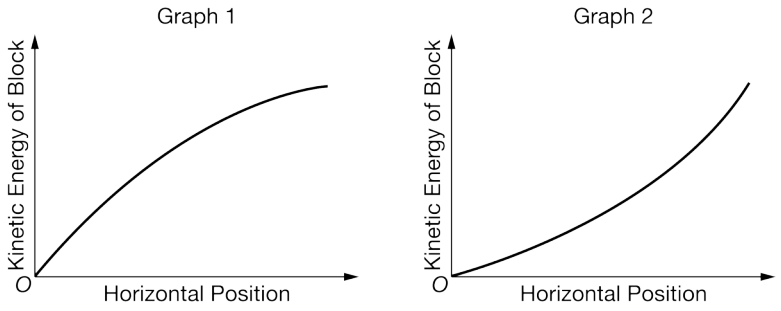
The students correctly derive an equation for the horizontal distance D from the edge of the tabletop that the block lands, in terms of s (the distance between the launcher and the edge of the tabletop):

$$D = t\sqrt{\frac{k}{m}(2Ls - s^2) - 2\mu g s},$$

where t is the time interval between the instant the block leaves the table and the instant immediately before it reaches the ground.

(d) Does this equation for D support your argument from part (a) that the block launched from table 1 could land farther from the table than the block launched from table 2? Briefly explain why or why not.

(e) The students now use the same setups to launch blocks that are identical to each other but more massive than the original blocks. The blocks are made of the same material as the original blocks. Does this change in the mass of the blocks make it more likely or less likely that the block launched from table 1 goes farther than the block launched from table 2 does? Briefly explain your reasoning.



(f) Two students each try to sketch a graph of the kinetic energy of the block as a function of its position from launch to the time the block reaches the edge of the table. Their graphs are shown above. Which graph best represents the kinetic energy of the block? Justify your selection.

frq3

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for applying work/energy or momentum/impulse to explain why launch speed could be greater for Table 1.
- ☐ 1 point is earned for indicating that a larger *s* enables the spring to exert a force on the block for a longer time or distance, or that a larger *s* corresponds to more elastic potential energy being transferred to the block.
- ☐ 1 point is earned for indicating that the block could gain more speed and/or kinetic energy and/or momentum on Table 1.

Example Response:

The distance s is greater for table 1 than table 2 which means that the force that the spring exerts on the block is exerted over a longer distance, allowing the force to do more work on the block. Since the spring force does more work on the block, the change in kinetic energy could be greater, meaning the block could be launched at a greater speed from table 1. The block from table 1 will travel a larger horizontal distance because it was launched with a greater horizontal speed.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating the same time.
- ☐ 1 point is earned for indicating that both blocks will have the same initial vertical speed or zero initial vertical speed.
- ☐ 1 point is earned for indicating that either:
  only the vertical motion determines the fall time,
  OR
  The vertical displacement is the same in both cases
  OR
  The vertical acceleration is the same in both cases.

Example Response:

frq3

The blocks launched from table 1 and table 2 will spend the same amount of time in the air because they have an initial vertical velocity of zero and start from the same height above the floor.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: just "zero" earns this point.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for indicating there is no change in the total mechanical energy.

Example Response:

After the block leaves the table the total mechanical energy does not change.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for addressing the functional dependence of *D* on *s*.
- ☐ 1 point is earned for correctly indicating how a specific term in the expression changes with increasing *s*.
- ☐ 1 point is earned for indicating that an increase in s could result in an increase in *D*, or in the right-hand-side of the equation
  Note: If a student answers "no" the third point is awarded for indicating that an increase in s results in a decrease in *D*, or in the right-hand side of the equation.

Example Response:

Yes. The closer s is to L, the greater the $(2Ls - L\hat{}2)$ *part of the equation would be. If that term of the equation is greater, D will be greater.*

OR

Yes. Because s is always less than L, Ls *is greater than* $s\hat{}2$*. So as s increases, the equation increases.*

Part E

frq3

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating "Less likely", with an attempt at an explanation.
- ☐ 1 point is earned for noting that the larger mass increases the force of friction.
- ☐ 1 point is earned for indicating that a larger frictional force will dissipate more energy (or result in a smaller net force on the block/acceleration of the block), which results in a lower launch speed, and therefore smaller distance.
  Note: Part (e) can be answered quantitatively. See alternate solution below.

It is less likely that the block in 1 will go farther. The larger mass means that the force of friction is larger, so the net force on the block is less, resulting in a lower launch speed. On table 1, the frictional force is exerted over a longer distance, and so has a bigger effect on reducing the speed compared to table 2.

OR

Less likely. The first term is multiplied by k/m; so this term decreases with bigger m. The second term doesn't change with m. So, the difference between the two terms is smaller, reducing the effect of increasing s on the distance traveled.

Part E – Alternate Solution

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

*The response accurately includes **all** of the following criteria.*

- ☐ *1 point is earned for indicating "less likely", with an attempt at an explanation.*
- ☐ *1 point is earned for addressing the functional dependence of D on m in the given equation.*
- ☐ *1 point is earned for explaining how the right side of the equation becomes smaller with reference to both terms in the substitution, therefore, D becomes smaller.*

Example Response:

Less likely. The first term is multiplied by k/m; so this term decreases with bigger m. The second term doesn't change with m. So, the difference between the two terms is smaller, reducing the effect of increasing s on the distance traveled."

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

0	1	2 ✓

The response accurately includes **both** of the following criteria.

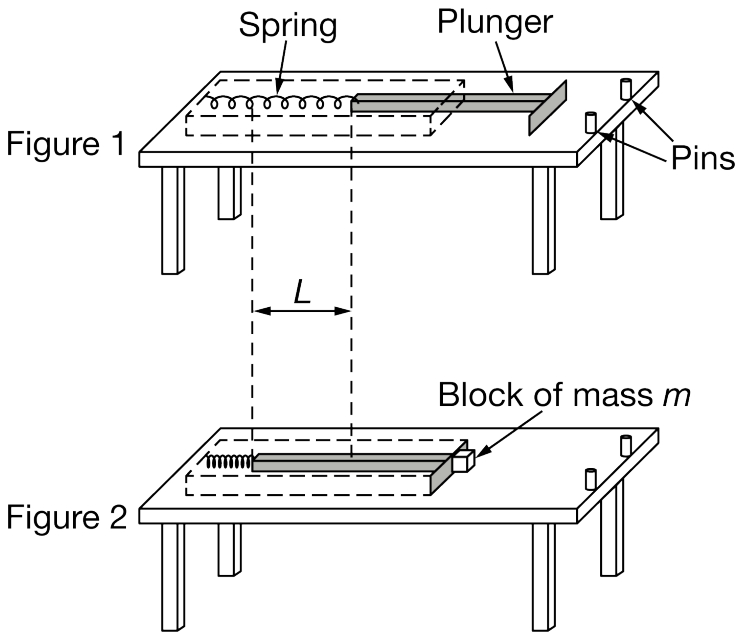
- ☐ 1 point is earned for indicating that Graph 1 is the correct graph, with an attempt at an explanation.
- ☐ 1 point is earned for correctly addressing the decreasing slope of the curve for large values of horizontal position.

Example Response:

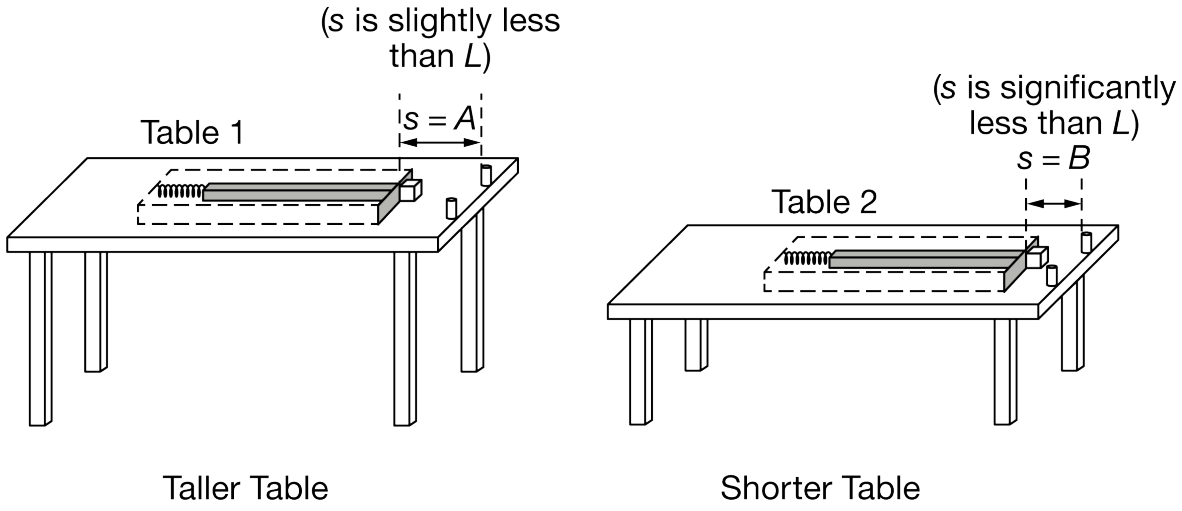
Graph 1 is the correct graph. The slope decreases with increasing position. This makes sense because, as the spring expands, it exerts a smaller force on the block. The smaller force means that the kinetic energy increases by a smaller amount for a given change in position.

frq3

4. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A spring-loaded launcher can be securely attached to a tabletop. When the spring is at its uncompressed length, the plunger is in the position shown above in Figure 1. A small wood block can be pressed against the end of the plunger, compressing the spring a distance L as shown in Figure 2. When the block is released, the plunger pushes the block along the tabletop. Two pins are attached to the edge of the table to prevent the plunger from extending beyond the table. When the plunger hits the pins, the block then leaves the table. There is nonnegligible friction between the block and the tabletop.



The launcher can be moved closer to or farther from the edge of the tabletop. Students set up tables 1 and 2, as shown above, with tabletops that are made of the same material. On table 1, which is taller than table 2, the

frq3

launcher is positioned so that the distance $s = A$ between the launcher and the edge of the tabletop is slightly less than L. On table 2, the launcher is positioned so that the distance $s = B$ between the launcher and the edge of the tabletop is significantly less than L. On both tables, the block is released from rest and loses contact with the plunger at the moment the plunger reaches the pins.

k is the spring constant of the spring.
m is the mass of the block.
L is the distance the spring is compressed.
μ (mu) is the coefficient of kinetic friction between the block and the tabletop.
s is the distance between the launcher and the edge of the tabletop.

Assume the spring is ideal, the plunger has negligible mass, the block loses contact with the plunger immediately after it reaches the end of the table, and there is non-negligible friction between the tabletops and the blocks.

(a) Without manipulating equations, explain why the block launched from table 2 could have a greater speed when it leaves the table than the block launched from table 1 does.

(b) Does the block launched from table 2 spend more, less, or the same amount of time in the air than the block launched from table 1 does? Explain your reasoning.

(c) Consider the system consisting of only the block. Determine the change, if any, in the total mechanical energy of the system from the instant the block leaves the table to the instant immediately before it reaches the ground.

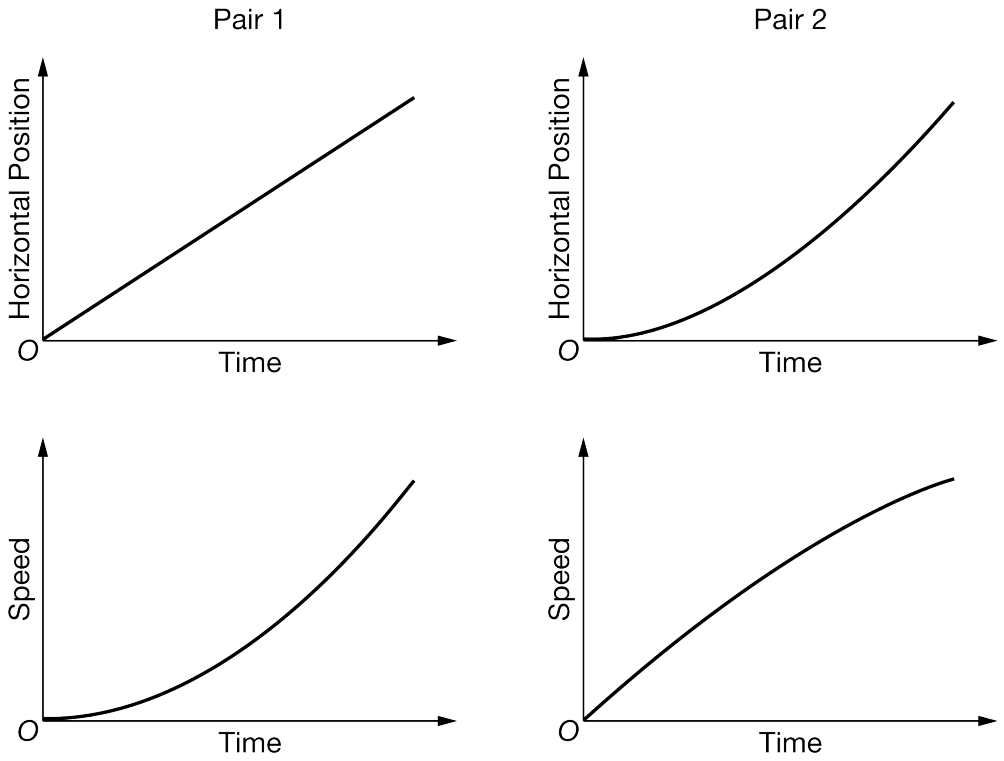
The students correctly derive an equation for the speed v of the block when it is at the edge of the tabletop, in terms of s (the distance between the launcher and the edge of the tabletop):

$v = \sqrt{\frac{k}{m}(2Ls - s^2) - 2\mu g s}$.

(d) Does this equation for v support your argument from part (a) that the block launched from table 2 could have a larger speed when it leaves the table than the block on table 1 does? Briefly explain why or why not.

(e) The students now repeat the scenario with identical setups except that the coefficient of kinetic friction between the blocks and the table is larger. Does this change in the coefficient of friction make it more likely or less likely that the block launched from table 1 has a greater speed when it leaves the table than the block on table 2 does? Briefly explain your reasoning.

frq3



(f) Two students each try to sketch a graph of the horizontal position and speed of the block as a function of time while the block is being pushed by the plunger. Their graphs are shown above. In which pair of graphs are the two graphs consistent with each other? Justify your selection.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for attempting to apply work/energy or momentum/impulse to explain why launch speed could be greater for Table 2.
- ☐ 1 point is earned for indicating that a larger *s* for the block on Table 1 enables friction to exert a force on the block for a longer time or distance, or that a larger *s* corresponds to more energy being dissipated.
- ☐ 1 point is earned for correctly applying work/energy or momentum/impulse to explain why launch speed could be greater for Table 2.

Example Response:

The distance s is greater for table 1 than table 2 which means that the force of friction from the table can be exerted for a

frq3

longer distance, allowing the friction force to dissipate more energy. Since the friction force dissipates more energy, the reduction in kinetic energy due to friction is greater, meaning the block will be launched at a smaller speed from table 1.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the block launched from Table 2 will spend less time in the air.
- ☐ 1 point is earned for indicating that both blocks will have the same initial vertical speed or zero initial vertical speed.
- ☐ 1 point is earned for indicating that either:
  only the vertical motion determines the fall time,
  OR
  the vertical displacement is different for the two cases.

Example Response:

The block launched from table 2 will spend less time in the air because both blocks have an initial vertical velocity of zero, but the block on table 2 starts from a lower height above the floor.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for indicating that the change is mgh.

Example Response:

mgh

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for addressing the functional dependence of v on s.
- ☐ 1 point is earned for correctly indicating how a specific term in the expression changes with increasing s.
- ☐ 1 point is earned for indicating that an increase in s could result in a decrease in v, or in the right-hand-side of the equation.
  Scoring note: If a student answers "no" the third point is awarded for indicating that an increase in s results in an increase in v, or in the right-hand side of the equation.

Example Response:

Yes, because the equation indicates that v could increase as s decreases like my argument in part (a). In the equation, the expression under the radical could increase when s decreases because there are two "s" terms with negative signs. If the number under the radical increases, v will increase.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating "Less likely", with an attempt at an explanation.
- ☐ 1 point is earned for noting that the larger coefficient of friction results in a larger frictional force.
- ☐ 1 point is earned for indicating that a larger frictional force will dissipate more energy (or result in a smaller net force on the block/acceleration of the block), which results in a lower launch speed.

Example Response:

It is less likely that the block in 1 will have a greater launch speed. The larger coefficient of friction means that the force of friction is larger, so the net force on the block is less, resulting in a lower launch speed. On table 1, the frictional force is exerted over a longer distance, and so has a bigger effect on reducing the speed compared to table 2.

Part E-Alternate Solution

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

0	1	2	3 ✓

*The response accurately includes **all** of the following criteria.*

- ☐ *1 point is earned for indicating "less likely" with an attempt at an explanation.*
- ☐ *1 point is earned for addressing the functional dependence of v on the coefficient of friction (μ) in the given "correct" equation.*
- ☐ *1 point is earned for explaining how the right side of the equation becomes smaller with reference to both terms in the substitution, therefore v becomes smaller.*

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating pair 2, with an attempt at an explanation.
- ☐ 1 point is earned for either correctly ruling out the inconsistent position and time graphs in pair 1,
  OR
  For indicating that the speed graph in pair 2 is consistent with the position graph (the speed graph corresponds to the slope of the position graph).

Example Response:

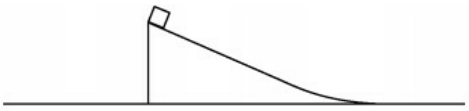
Pair 2 has the two consistent graphs. The slope of the position graph is initially zero, and then increases, which is the behavior of the speed graph.

OR

Pair 2 has the two consistent graphs. In pair 1, the position graph is a straight line, so the speed graph should be constant.

frq3

5.



The figure above shows part of a system consisting of a block at the top of an inclined plane that rests on a table, which is located on Earth. The block and plane are at rest when the block is released. In trial 1 there is no friction between the block and the plane or between the plane and the table. In trial 2 the plane is fixed to the table so it cannot move, but there is still no friction between the block and the plane.

Indicate whether the speed of the block relative to the table when the block reaches the bottom of the plane is greater in trial 1 or trial 2. Justify your answer in a clear, coherent, paragraph-length explanation.

Part 1

1 point is earned for any mention of energy conversion

1 point is earned f or indicating that the same amount of potential energy is converted to kinetic energy in each trial

1 point is earned for indicating that the kinetic energy is shared between the block and plane in trial 1

1 point is earned for indicating that the block has all the kinetic energy in trial 2

1 point is earned for a description of the motion of the plane in each trial

1 point is earned for using momentum or forces to explain the motion in each trial

1 point is earned for a coherent argument that leads to a correct conclusion

For example: The speed of the block is greater in trial 2. Since gravity is the only force doing work in both trials, energy is conserved. The potential energy of the block at the top of the plane is converted into kinetic energy. In trial 1 the plane is free to move, so if the block ends up going to the right, the plane has to move to the left to conserve momentum.



0	1	2	3	4	5	6	7

The student response earns all of the following points:

1 point is earned for any mention of energy conversion

1 point is earned f or indicating that the same amount of potential energy is converted to kinetic energy in each trial

1 point is earned for indicating that the kinetic energy is shared between the block and plane in trial 1

1 point is earned for indicating that the block has all the kinetic energy in trial 2

1 point is earned for a description of the motion of the plane in each trial

frq3

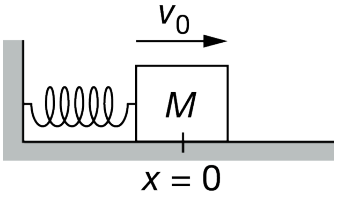
1 point is earned for using momentum or forces to explain the motion in each trial

1 point is earned for a coherent argument that leads to a correct conclusion

For example: The speed of the block is greater in trial 2. Since gravity is the only force doing work in both trials, energy is conserved. The potential energy of the block at the top of the plane is converted into kinetic energy. In trial 1 the plane is free to move, so if the block ends up going to the right, the plane has to move to the left to conserve momentum.

frq3

6. This question is a long free-response question. Show your work for each part of the question.

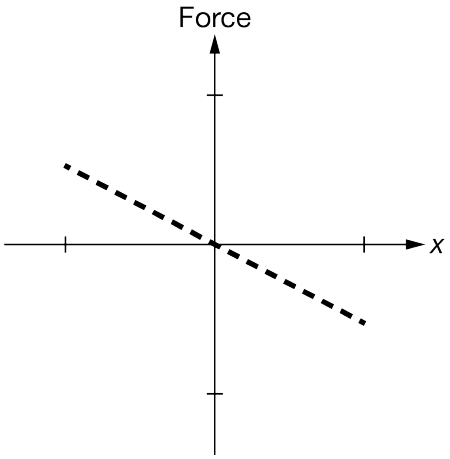


A block of mass M is attached to a spring of negligible mass and can slide on a horizontal surface along the x -direction, as shown above. There is friction present between the block and the surface. The spring exerts no force on the block when the center of the block is located at $x = 0$. At the instant shown above, the block is located at $x = 0$ and moving toward the right with speed $v = v_0$.

(a) For the instant shown above, with the block located at $x = 0$ and moving to the right, predict the direction of the net force on the block. If the net force is zero, select "The net force is zero."

____ To the left ____ The net force is zero ____ To the right

Briefly justify your prediction.



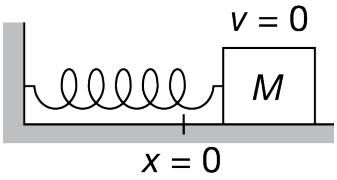
(b) The above graph indicates the force exerted on the block by the spring as a function of position x, with the positive direction toward the right. On the same graph, draw a graph of the net force on the block, as a function of position x, for situations where the block is moving toward the right.

(c) Is the graph you drew in part (b) consistent with your prediction in part (a) ?

____ Yes ____ No

Provide claims for why the answers to parts (a) and (b) are or are not consistent.

frq3



(d) In the figure above, the block of mass M has come to rest at a location to the right of the spring's equilibrium position. The spring has a spring constant k, and the coefficients of kinetic and static friction between the block and the surface are μ_k and μ_s, respectively.

i. On the dot below, which represents the block at rest and to the right of $x = 0$, draw and label the forces (not components) that act on the block. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



ii. Derive an expression for the maximum displacement possible for the block that will allow the block to still remain at rest. Express your answer in terms of M, k, μ_k, μ_s, and physical constants, as appropriate.

Part (a)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

Student response accurately includes the following criteria.

- ☐ 1 point is earned for statingthat the friction force is opposite the direction of motion.

Example Response:

["To the left" is selected.]

Because in this case the block is at x = 0, the spring force on the block is 0. The normal force and the gravitational force add to 0. Because the block is moving to the right, there is a friction force to the left. Therefore, the net force on the block is equal to the frictional force and is pointed to the left..

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Part (b)

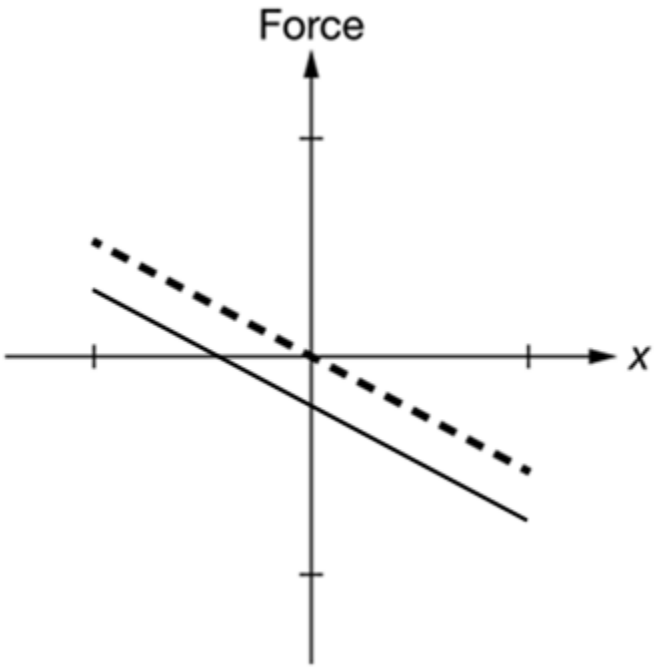
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

Student response accurately includes all of the following criteria.

- ☐ 1 point is earned for indicating a negative net force at zero displacement.
- ☐ 1 point is earned for creating a graph that is either above the spring force everywhere or below the spring force everywhere (so friction force is always in the same direction).
- ☐ 1 point is earned for creating a graph that is a straight line and parallel to the spring force graph.

Example Response:



Part (c)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: For this part, the selection of “Yes” or “No” is not graded alone. It must be consistent with the explanation in order to receive the second point.

frq3

0	1	2 ✓

Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for providing a claim that addresses the graph at an x-value of zero.
- ☐ 1 point is earned for providing a claim that is consistent with the answers to parts (a) and (b) AND with the selection of "Yes" or "No" in part (c).

Example Response:

The graph indicates a negative net force at zero displacement, consistent with the leftward net force predicted in part (a).

Part (d)i

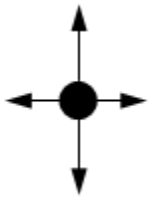
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

Student response accurately includes all of the following criteria.

- ☐ 1 point is earned for indicating a downward gravitational force and an upward normal force.
- ☐ 1 point is earned for indicating a spring force to the left.
- ☐ 1 point is earned for indicating a friction force to the right.

Example Response:



Part (d)ii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

frq3

Student response accurately includes all of the following criteria.

- [ ] 1 point is earned for equating the spring force and the friction force: $F_{\text{spring}} = F_f$
- [ ] 1 point is earned for providing correct expressions representing the spring force and the friction force: $kx_{\max} = \mu_S F_n = \mu_S Mg$
- [ ] 1 point is earned for correctly solving for the maximum displacement in terms of the listed quantities: $x_{\max} = \mu_S Mg/k$

Example Response:

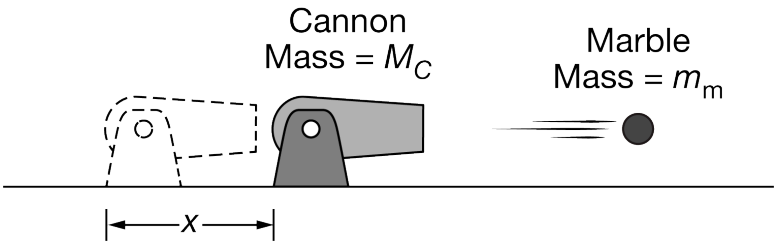
$\Sigma \overrightarrow{F} = m\overrightarrow{a}$
$\Sigma F_x = ma_x$
$F_{fs} - F_s = ma_x$
$F_{fs} - F_s = M(0)$
$F_{fs} - F_s = 0$
$F_s = F_{fs}$
$kx = \mu_s F_n$
$kx_{\max} = \mu_s F_n$
$x_{\max} = \frac{\mu_s F_n}{k}$

$\Sigma \overrightarrow{F} = m\overrightarrow{a}$
$\Sigma F_y = ma_y$
$F_n - F_g = ma_y$
$F_n - F_g = M(0)$
$F_n - F_g = 0$
$F_n = F_g$
$F_n = mg$
$F_n = Mg$

$x_{\max} = \frac{\mu_s Mg}{k}$

frq3

7. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch marbles of mass m_{m} horizontally from a toy cannon of mass M_C, where M_C is always greater than m_{m}, as shown above. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students use a new marble with a different mass launched at the same speed v relative to the ground. The students record the distance x the cannon slides backward for each launch.

(a) Without manipulating equations, briefly describe what happens to the distance x that the cannon slides backward as students increase the mass of the marble. Explain your reasoning.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\mathrm{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) Without manipulating or stating equations, briefly describe whether it would take more work to stop the marble or to stop the cannon after the marble has been launched. Justify your answer.

(d) Students correctly derive the following equation relating the work W done on an object initially at rest, the momentum p of the object, and the mass m of the object: $W = \frac{p^2}{2m}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble-cannon system during the short time interval in which the marble is accelerated inside the cannon and make the following claims:

Student 1: The cannon is pushing the marble forward, and the marble is pushing back with equal and opposite force. Since the work done by the cannon cancels out the work done by the marble, the mechanical energy of the system is constant.

Student 2: The mechanical energy of the system increases while the marble is being accelerated. The marble gains kinetic energy to the right, and the cannon gains an equal amount of kinetic energy to the left.

(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

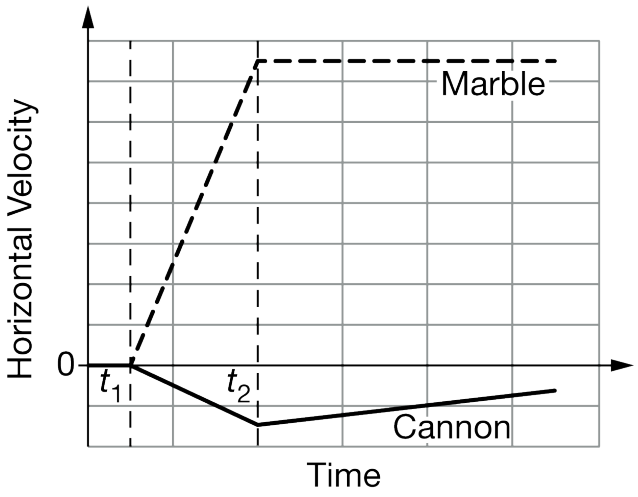
(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is

frq3

incorrect, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity of the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Provide three pieces of evidence that together explain why the magnitude of the rate of change of the cannon's velocity is less than the magnitude of the rate of change of the marble's velocity during the time interval from t_1 to t_2.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to indicate that a larger marble mass means a larger cannon speed
  OR
  For using a combination of Newton's 3rd law leading to Newton's 2nd law to indicate that a larger marble mass means a larger cannon speed.
- ☐ 1 point is earned for explaining why the larger cannon speed results in sliding a greater distance.

frq3

Example Response:

The initial momentum of the marble-cannon system is zero, so it will be zero after the marble is launched as well. A greater marble mass then means that the backward cannon speed is greater. Since the acceleration due to friction is the same regardless of cannon speed, a greater cannon speed means that the cannon will slide a greater distance.

OR

While the marble is in the cannon, the force on the marble has the same magnitude as the force on the cannon (according to Newton's 3rd law). If the final speed of the marble is the same each time but the mass of the marble increases, the net force on the marble must increase (according to Newton's 2nd law) and, therefore, the net force on the cannon must increase as well (the force of friction remains constant) giving the cannon a larger final speed resulting in a greater distance.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence between the mass m_m and x.
- ☐ 1 point is earned for a claim that as m_m in the equation increases, x in the equation also increases.

Example Response:

Yes, the equation matches that as the mass of the marble increases the slide distance of the cannon also increases. The mass of the marble is in the numerator of the equation so if the numerator increases the slide distance will also increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim that the marble requires more work to stop, with attempt at a justification.
- ☐ 1 point is earned for a valid justification.

Example Response:

The marble requires more work to stop because the marble has a greater kinetic energy thus a greater change in kinetic

frq3

energy in order to stop.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim consistent with the answer to (c).
- ☐ 1 point is earned for a claim that both the marble and cannon have equal momentum when the marble is launched and for recognizing that the marble's smaller mass in the denominator means it has more work done on it.

Example Response:

Yes. By conservation of momentum, the cannon and marble have the same magnitude of momentum immediately after the launch. Since p is the same, but m is smaller for the marble, the kinetic energy is greater for the marble since m is in the denominator, which means that the marble had more work done on it.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1's statement that is correct is the marble is pushing back with equal and opposite force

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 1's statement that is incorrect is work done by the cannon cancels out the work done by the marble and mechanical energy is constant.

OR

The second statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If a student considered the cannon to include a spring launching mechanism, then the first statement would be incorrect. A student who states "none" in part (g) and "both" in part (h) earns full credit for both parts.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2's statement that is correct is mechanical energy increases while the marble is being accelerated.

OR

The first statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student's 2 statement that is incorrect is the cannon gains an equal amount of kinetic energy to the left.

OR

The second statement

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Though the slope change in the graph occurs at t_2 (rather than "in the interval from t_1 to t_2 "), a student who indicates the effect of force of friction by discussing the slope change at t_2 is awarded this point.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

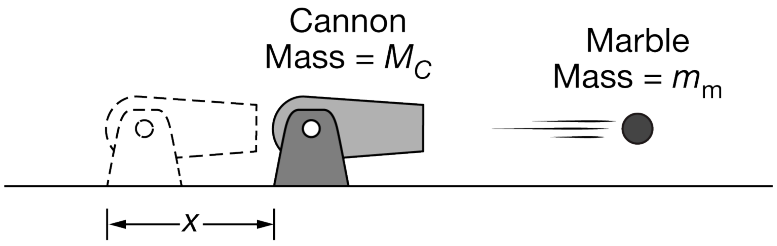
- ☐ 1 point is earned for recognizing that the same force is exerted on both the marble and cannon.
- ☐ 1 point is earned for indicating that the larger mass of the cannon results in its acceleration being less.
- ☐ 1 point is earned for indicating the force of friction acting opposite the cannon's direction of motion will further reduce the acceleration of the cannon.

Example Response:

Even the though the force on the cannon and the marble is the same due to Newton's third law, the cannon's acceleration is smaller. So, its velocity increases less than the marble because the cannon's mass is larger. The cannon also has a frictional force opposing its motion, which decreases the net force and therefore the acceleration.

frq3

8. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch identical marbles of mass m_{m} horizontally from a toy cannon of varying mass M_C, where M_C is always greater than m_{m}, as shown above. The students can attach weights to the cannon to change M_C. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students vary the mass of the cannon and launch a marble with the same speed v relative to the ground. The students record the distance x that the cannon slides backward.

(a) Without manipulating equations, explain how the distance x that the cannon slides backward changes, if at all, when marbles are launched from more massive cannons.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\text{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) If constant horizontal forces were exerted over the same distance on the cannon and on the marble after the launch, which object would require a force with a greater magnitude to completely stop it? Without manipulating or referencing equations, briefly explain your reasoning.

(d) Students correctly derive the following equation relating the force F exerted on an object initially at rest over a distance d, the mass m of the object, and the final momentum p of the object: $F = \frac{p^2}{2md}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble after it has been launched by the cannon but before it reaches the ground. Both students agree that air resistance is negligible and make the following claims.

Student 1: The mechanical energy of the marble is constant. There are no external forces exerted on the marble, so its energy does not change.

Student 2: The mechanical energy decreases as the marble falls. This happens because the gravitational force is doing work on the marble.

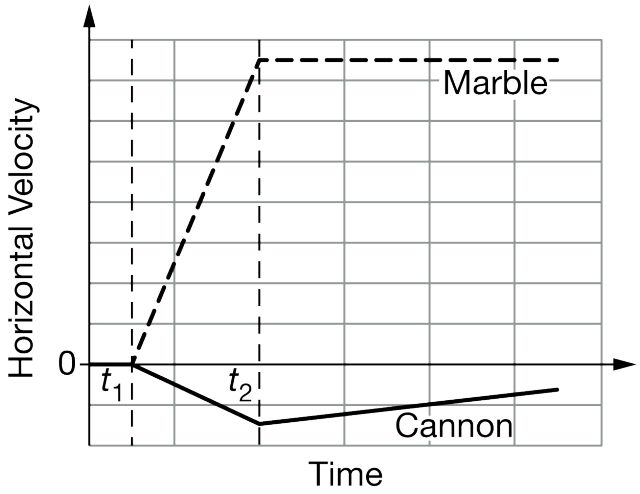
(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

frq3

(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity of the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Describe the motion of the center of mass of the marble-cannon system during the time interval from t_1 to t_2. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Students may earn the points for a correct discussion regarding the force of friction. A more massive cannon leads to a greater normal force increasing the force of friction. This would cause the net force on the cannon to the left to be smaller while the marble was in the cannon (giving it an initially smaller speed once the marble has left the cannon) resulting in a smaller distance traveled.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to indicate that a larger cannon mass means a smaller cannon speed

frq3

OR

For using a combination of Newton's 3rd law leading to Newton's 2nd law to indicate that a larger cannon mass means a smaller cannon speed.

☐ 1 point is earned for explaining why the smaller cannon speed results in sliding a shorter distance.

Example Response:

The initial momentum of the marble-cannon system is zero, so it will be zero after the marble is launched as well. A more massive cannon then means that the backward cannon speed is less. A smaller cannon speed means that the cannon will slide a shorter distance.

OR

While the marble is in the cannon, the force on the marble has the same magnitude as the force on the cannon (according to Newton's 3rd law). If the mass of the marble stays constant but the mass of the cannon increases, the cannon will have smaller acceleration and smaller speed (according to Newton's 2nd law) resulting in a smaller distance.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

☐ 1 point is earned for using functional dependence between the mass and the distance x.

☐ 1 point is earned for a claim that as M_c in the equation increases, x in the equation decreases.

Example Response:

Yes, the equation matches the prediction that as the mass of the cannon increases the distance the cannon slides backwards decreases. From the equation, the mass of the cannon and the distance it slides backwards are inversely proportional. The mass of the cannon is in the denominator of the equations so as the mass increases the distance the cannon slides backwards will decreases.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

frq3

- ☐ 1 point is earned for a claim that it would take more force to stop the marble, with an attempt at a justification.
- ☐ 1 point is earned for a valid justification using either work/energy or momentum.

Example Response:

It would take more force to stop the marble because the change in kinetic energy to stop the marble is greater so the work on the marble needs to be greater.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim consistent with the answer to (c).
- ☐ 1 point is earned for a claim that both the marble and cannon have equal momentum when the marble is launched and for recognizing that the marble's smaller mass in the denominator means it requires more force to stop.

Example Response:

Yes. By conservation of momentum, the cannon and marble have the same magnitude of momentum immediately after the launch. Since p is the same, but m is smaller for the marble, the force is greater for the marble since m is in the denominator.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a statement of "None".

Example Response:

None

Part F

frq3

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating both statements are incorrect.

Example Response:

Both parts of Student 1's statement are incorrect.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

The gravitational force is doing work on the marble

OR

The second statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

frq3

Example Response:

The mechanical energy decreases as the marble falls.

OR

The first statement

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

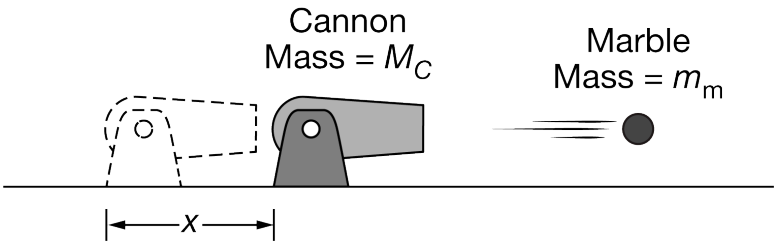
- ☐ 1 point is earned for indicating the center of mass moves to the right.
- ☐ 1 point is earned for indicating that there is an external force on the system due to friction.
- ☐ 1 point is earned for indicating that the net force on the system is needed to move the center on mass.

Example Response:

During the launch, the only horizontal external force exerted on the marble-cannon system is the frictional force exerted on the cannon. The friction force is directed to the right, so the net force on the system is to the right. This means the center of mass of the system is accelerating to the right.

frq3

9. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch identical marbles of mass m_{m} horizontally from a toy cannon of mass M_C, where $M_C > m_{\text{m}}$, as shown above. The cannon can be adjusted to change the launch speed v of the marble relative to the ground. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students vary the launch speed v and record the distance x the cannon slides backward for each launch speed.

(a) Without manipulating equations, briefly describe what happens to the distance x that the cannon slides backward as students increase the launch speed. Explain your reasoning.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\text{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) Without manipulating or stating equations, briefly describe whether the cannon or the marble has more kinetic energy immediately after the marble leaves the cannon. Justify your answer.

(d) Students correctly derive an equation relating the kinetic energy K, the momentum p, and the mass m of an object: $K = \frac{p^2}{2m}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble after it has been launched by the cannon but before it reaches the ground. Both students agree that air resistance is negligible and make the following claims.

Student 1: There are two external forces exerted on the marble while it is in the air, the gravitational force and the force exerted by the cannon. Therefore, the mechanical energy of the marble changes while it is in the air.

Student 2: The cannon does not exert a force on the marble after the launch. Therefore, the mechanical energy of the marble is constant.

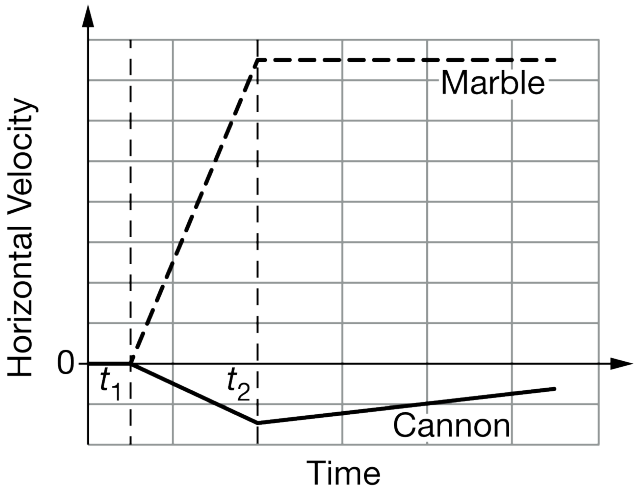
(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

frq3

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity for the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Describe how to find the change in the distance between the center of mass of the cannon and the center of mass of the marble from time t_1 to time t_2. Assume that before time t_1 the center of mass of the marble is at the same position as the center of mass of the cannon.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to indicate that a larger marble speed means a larger cannon speed
  OR
  For using a combination of Newton's 3rd law leading to Newton's 2nd law to indicate that a larger marble speed means a larger cannon speed.
- ☐ 1 point is earned for correctly explaining why the larger cannon speed results in sliding a greater distance.

frq3

Example Response:

The initial momentum of the marble-cannon system is zero, so it will be zero after the marble is launched as well. A greater marble speed then means that the backward cannon speed is greater. Since the acceleration due to friction is the same regardless of cannon speed, a greater cannon speed means that the cannon will slide a greater distance.

OR

While the marble is in the cannon, the force on the marble has the same magnitude as the force on the cannon (according to Newton's 3rd law). If the mass of the marble stays constant but the final speed of the marble increases, the net force on the marble must increase (according to Newton's 2nd law) and, therefore, the net force on the cannon must increase as well (the force of friction remains constant) giving the cannon a larger final speed resulting in a greater distance.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence.
- ☐ 1 point is earned for a claim that as v in the equation increases, x in the equation also increases.

Example Response:

Yes, the equation matches the prediction that as the velocity of the marble increases the distance the cannon moves backward also increases. In the equation as velocity increases the distance would increase. Since the velocity is in the numerator of the equation, as the numerator of a fraction increases the distance x would increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim that the marble has more kinetic energy, with an attempt at a justification.
- ☐ 1 point is earned for a valid justification.

Example Response:

By conservation of momentum, mass and velocity are inversely proportional. Since velocity is squared when determining

frq3

kinetic energy, a greater speed increases the kinetic energy by a larger factor than a smaller mass decreases the energy.

OR

The cannon and marble exert forces of equal magnitude on each other. However, the marble will move a greater distance than the cannon while these forces are exerted. So, the marble has more work done on it, and therefore a greater kinetic energy.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim consistent with the answer to (c).
- ☐ 1 point is earned for a claim that both the marble and cannon have equal momentum when the marble is launched and for recognizing that the marble's smaller mass in the denominator means it has more kinetic energy.

Example Response:

Yes. By conservation of momentum, the cannon and marble have the same magnitude of momentum immediately after the launch. Since p is the same, but m is smaller for the marble, the kinetic energy is greater for the marble since m is in the denominator.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

The correct statement from student 1 is that mechanical energy of the marble changes while it is in the air

OR

The second statement

frq3

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

The incorrect statement for student 1 is that the cannon exerts a force on the marble while it is in the air

OR

The first statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

The correct statement for student 2 is that the cannon no longer exerts a force on the marble

OR

The first statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

The incorrect statement for student 2 is that the mechanical energy of the marble is constant

OR

The second statement

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for recognizing that displacement can be calculated by finding the area under a *v* vs. *t* graph.
- ☐ 1 point is earned for recognizing that the total displacement will be represented by combining the areas under each *v* vs. *t* graph.
- ☐ 1 point is earned for indicating that, for the cannon, either the area (which is negative) is to be subtracted
  OR
  the absolute value of the area is to be added, to the area under the marble graph.

Example Response:

The area under the velocity vs. time graph represents the displacement of an object. To find the change in the distance, subtract the areas of the two graphs between t_1 *and* t_2.

frq3

10. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)

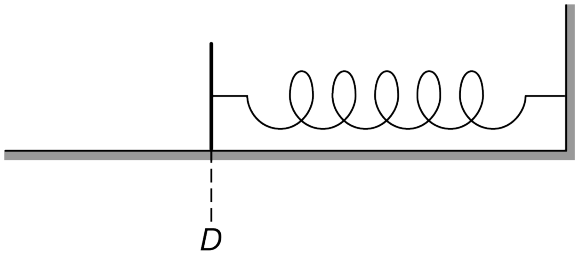


Figure 1. Spring at Relaxed Length

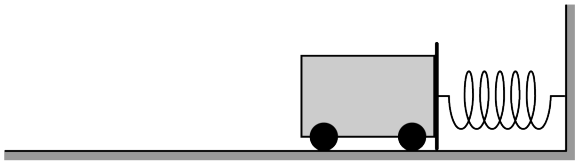


Figure 2. Cart Held Against Plate

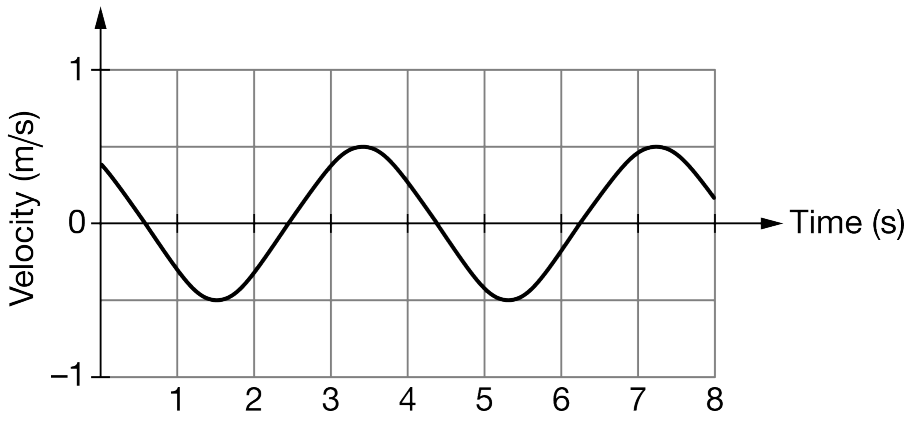
Students intend to use a cart-spring system to study motion. A vertical plate with negligible mass is attached to a horizontal spring of spring constant k. When the spring is at its relaxed length, the plate is at position D as indicated in Figure 1 above. A cart is glued to the plate. The cart is initially held so that the spring is compressed as shown in Figure 2 above. The maximum force the glue can exert is $F_{\max}$. Assume friction is negligible.

The students plan to release the cart from rest so that the cart oscillates. They discuss whether the cart could detach from the plate during the cart's motion if the glue is not strong enough. One student makes the following claim:

"If the spring is stiffer, the force exerted by the spring will be greater for the same compression. This means that the cart could come loose as the spring expands, before it reaches D."

(a) Which underlined phrase or phrases are ***correct***, if either? If neither phrase is ***correct***, write "none."

(b) Which underlined phrase or phrases are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



frq3

The cart is removed, and the plate is coated with very strong glue. The cart rolls to the right with positive velocity v_{cart}, collides with and sticks to the plate, and then oscillates back and forth at the end of the spring. The graph above shows the velocity of the cart as a function of time after it has completed several oscillations.

(c) Explain how this graph could be used along with the known mass of the cart to find the impulse exerted on the cart by the spring during the time interval between 4 seconds and 6 seconds.

(d) The collision is repeated with a lighter cart. Again, the cart moves with velocity v_{cart}, collides with and sticks to the plate, and oscillates back and forth. In a clear, coherent paragraph-length response, explain how a graph of the cart's velocity as a function of time would differ from the one shown above if the cart were lighter. Provide a physical reason why each difference would occur.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Responses:

The student is correct in saying that if the spring is stiffer, the force exerted by the spring will be greater for the same compression.

OR

The first statement

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

frq3

The student is incorrect in saying that the cart could come loose as the spring expands, before it reaches D.

OR

The second statement

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the difference in the two velocities from the graph is needed
  OR
  indicating that the force multiplied by the change in time would give the impulse.
- ☐ 1 point is earned for indicating that multiplying the difference in velocity by the mass will give the impulse or change in momentum
  OR
  providing a correct way to solve for acceleration and stating that the acceleration multiplied by the mass would yield the average force.

Example Responses:

Subtract the velocity at 4 seconds from the velocity at 6 seconds, as read from the graph. Multiplying the velocity difference by the cart mass gives the change in momentum, or impulse.

OR

Impulse is equal to force multiplied by change in time. Use the graph to find the change in velocities from 4 seconds to 6 seconds. Divide this difference by the time (2 seconds) to determine the average acceleration. Multiply the acceleration by the cart's mass to determine the average force applied to the cart. Multiply the average force by the difference in time to determine impulse.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the period would be different (not necessarily shorter).

frq3

- 1 point is earned for indicating that the period would be shorter because less mass is oscillating.
- 1 point is earned for indicating that the amplitude of the velocity graph would not change
  OR
  stating that that no other factors change.
- 1 point is earned for a correct justification that the amplitude of the velocity graph will not change.
- 1 point is earned for an additional correct statement about a change in the oscillation of the spring-cart system, even if it is not related to the velocity graph. Examples include:
  · The frequency would be larger
  · The energy of the system will be smaller
  · The amplitude of the position vs time graph would be smaller (the spring would compress less)
  · The acceleration of the cart would change
- 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

With a less massive cart, the period shown in the graph would be shorter, since period increases with mass. The amplitude of the velocity graph would stay the same because the maximum velocity would be the same as the velocity of the cart at impact, and that has not changed. Changing the mass of the cart would affect the period. If the mass decreases, the period would also decrease. The spring would compress less with a lighter car.

frq3

11. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)

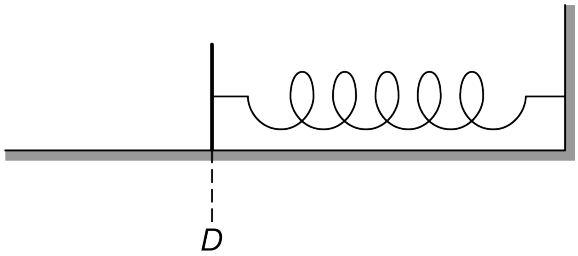


Figure 1. Spring at Relaxed Length

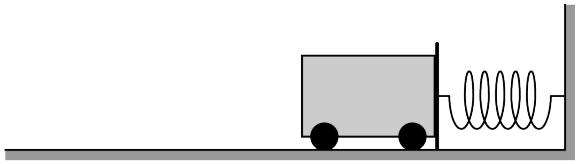


Figure 2. Cart Held Against Plate

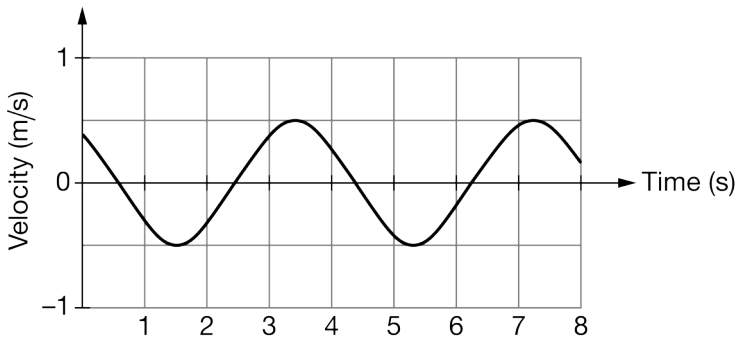
Students intend to use a cart-spring system to study motion. A vertical plate with negligible mass is attached to a horizontal spring of spring constant k. When the spring is at its relaxed length, the plate is at position D as indicated in Figure 1 above. A cart is glued to the plate. The cart is initially held so that the spring is compressed as shown in Figure 2 above. The maximum force the glue can exert is $F_{\max}$. Assume friction is negligible.

The students plan to release the cart from rest so that the cart oscillates. They discuss whether the cart could detach from the plate during the cart's motion if the glue is not strong enough. One student makes the following claim:

"The cart could never detach from the plate because the cart and the plate exert equal and opposite forces on each other, so the forces on the cart add to zero."

(a) Which underlined phrase or phrases are ***correct***, if either? If neither phrase is ***correct***, write "none."

(b) Which underlined phrase or phrases are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



frq3

The cart is removed, and the plate is coated with very strong glue. The cart rolls to the right with positive velocity v_{cart}, collides with and sticks to the plate, and then oscillates back and forth at the end of the spring. The graph above shows the velocity of the cart as a function of time after it has completed several oscillations.

(c) Explain how this graph could be used to find the maximum compression of the spring as it oscillates.

(d) The plate of negligible mass is replaced with a plate with mass similar to the cart's mass, and the collision is repeated. Again, the cart moves with velocity v_{cart}, collides with and sticks to the plate, and oscillates back and forth. In a clear, coherent paragraph-length response, explain how a graph of the cart's velocity as a function of time would differ from the one shown above if the plate's mass were not negligible. Provide a physical reason why each difference would occur.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Responses:

The student is correct in saying that the cart and the plate exert equal and opposite forces on each other.

OR

The first statement

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Responses:

The student is incorrect in saying that the forces on the cart add to zero.

frq3

OR

The second statement

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that finding the area somewhere under the curve is needed
  OR
  indicating that energy is conserved during oscillation.
- ☐ 1 point is earned for indicating that the area for one quarter of a cycle is needed
  OR
  using $v_{\max}$ from the graph to solve for $x_{\max}$ by setting maximum kinetic energy equal to maximum spring potential energy.

Example Responses:

Take the area under the curve for a quarter of a period.

OR

Energy is conserved during the oscillation, therefore Kinetic Energy at equilibrium = Spring Potential energy at the extremes. By setting those 2 values equal to each other, and using the max velocity from the graph, we can solve for the maximum distance (x) and determine the maximum distance the spring is compressed.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for recognizing that the period would be different (not necessarily longer).
- ☐ 1 point is earned for indicating that the period would be longer because there is more mass.
- ☐ 1 point is earned for recognizing that kinetic energy is lost in an inelastic collision.
- ☐ 1 point is earned for a claim that the amplitude of the graph will different (not necessarily less).

frq3

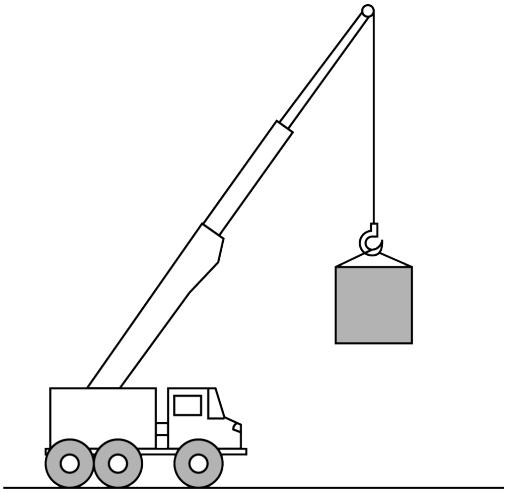
- 1 point is earned for indicating that the amplitude will be less because the total energy of the cart-plate-spring system is less
  OR
  indicating that the amplitude will be less because a greater mass means a smaller velocity because of conservation of momentum.
- 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

With an additional mass due to the plate, the period would be longer since period increases with mass. Some of the initial kinetic energy of the cart will be dissipated in the completely inelastic collision with the plate if the plate has nonzero mass, so there will be less kinetic energy immediately after the collision. With less kinetic energy at zero spring displacement, there is less total mechanical energy that can be converted to spring potential energy. So, the maximum spring potential energy will be less and therefore the amplitude will be less as well.

frq3

12. This question is a long free-response question. Show your work for each part of the question.

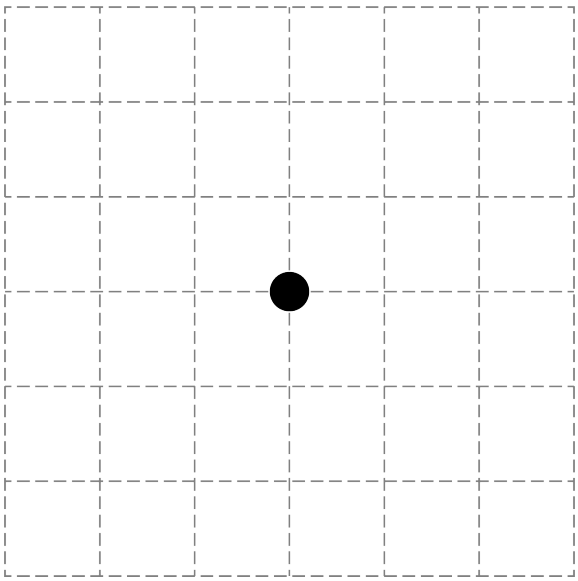


(12 points, suggested time 25 minutes)

A crane and box are both initially at rest. At time $t = 0\ \text{s}$, the crane begins to drive forward at a constant speed of $0.5\ \frac{\text{m}}{\text{s}}$, while also lifting the box with an upward acceleration of $1\ \frac{\text{m}}{\text{s}^2}$. The box does not swing while being lifted by the crane.

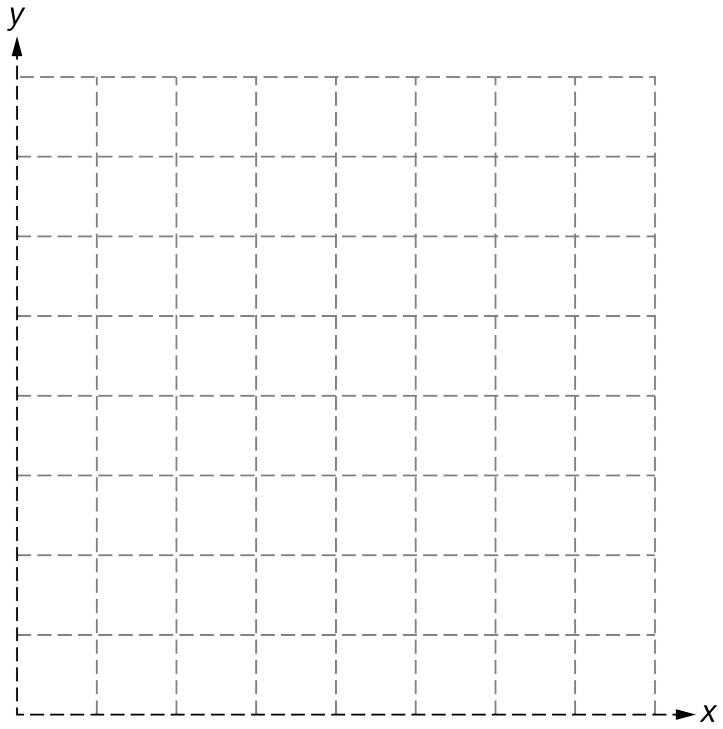
(a) On the dot below, which represents the box, draw and label the forces (not components) that are exerted on the box during the scenario described above. Represent each force by a distinct arrow starting on, and pointing away from, the dot.

Note: Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces.



frq3

(b) On the grid below, sketch the shape of the path taken by the box as it is lifted by the crane as viewed by a stationary observer.



Justify the shape of the path you drew above.

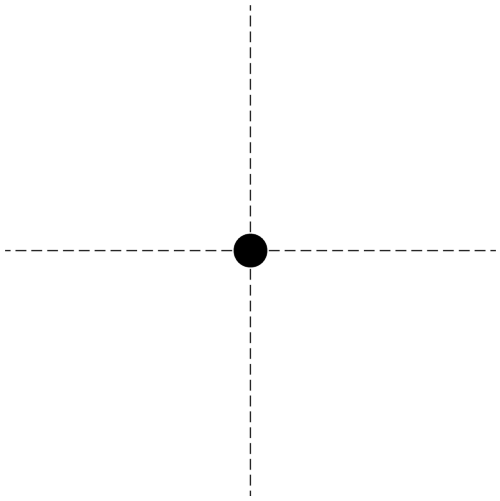
(c) Assume that, at $t = 0$ s, the horizontal position of the box is $x = 0$ and it has a vertical position of $y = 0$. Derive an equation that describes the vertical position y of the box as a function of the horizontal position x of the box.

(d) Does your equation from part (c) agree with your sketch in part (b)? Justify your response.

At time $t = t_1$, the crane begins to accelerate forward, reaching a new constant velocity at time $t = t_2$. The box accelerates upward during the entire time interval $t_1 < t < t_2$.

(e) The dot below represents the box during the time interval $t_1 < t < t_2$. On the dot, draw an arrow indicating the direction of the net force exerted on the block during this interval. The arrow should start on, and point away from, the dot.

frq3



Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Scoring notes:

· G or g are not acceptable labels for the force of gravity

· A maximum of two points can be earned if there are any extraneous forces.

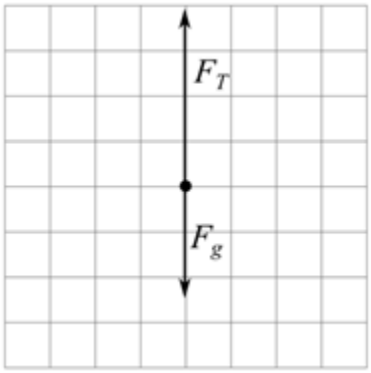
0	1	2	3 ✓

The response accurately includes **one** of the following criteria:

- ☐ For an arrow directed upwards labeled Tension (or F_T)
- ☐ For an arrow directed downwards labeled Force of Gravity (or mg, or F_g).
- ☐ For an arrow labeled tension that is larger than the arrow labeled Force of Gravity.

Example Response:

frq3



Part B

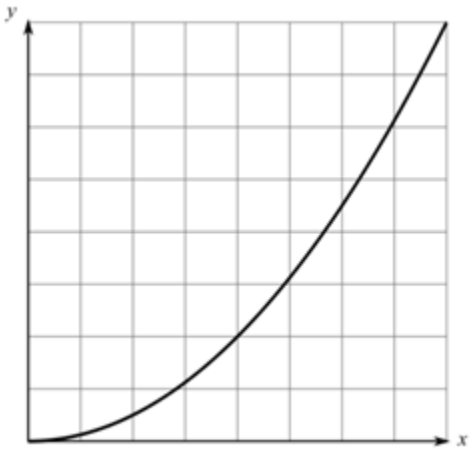
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

- ☐ For a sketch showing a non-linear path
- ☐ For a sketch that is concave up
- ☐ For a justification that addresses that the box is moving at a constant speed in the horizontal direction and accelerating in the vertical direction which leads to a concave up parabola.

Example Response:



Since the box is moving at a constant speed in the horizontal direction and accelerating in the vertical direction, the path that the box takes should look like a concave up parabola

Part C

frq3

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

- ☐ For a kinematic equation correctly relating the horizontal position of the box to the horizontal component of the speed of the box and the time
  $v_x = \frac{x}{t}$
  $0.5\ \text{m/s} = \frac{x}{t}$
  $t = \frac{x}{0.5\ \text{m/s}}$
- ☐ For a kinematic equation correctly relating the vertical position of the box as a function of the vertical acceleration of the box and the time
  $y = \frac{1}{2}a_y t^2$
  $y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)t^2$
- ☐ For correctly combining the vertical and horizontal motions of the box into a single equation.
  $y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)\left(\frac{x}{0.5\ \text{m/s}}\right)^2$
  $y = \left(2\ \text{m}^{-1}\right)x^2$

Example Response:

$v_x = \frac{x}{t}$
$0.5\ \text{m/s} = \frac{x}{t}$
$t = \frac{x}{0.5\ \text{m/s}}$

$y = \frac{1}{2}a_y t^2$
$y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)t^2$

$y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)\left(\frac{x}{0.5\ \text{m/s}}\right)^2$
$y = \left(2\ \text{m}^{-1}\right)x^2$

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

frq3

The response accurately includes **both** of the following criteria.

- ☐ For an attempt to use functional dependence to describe how the equation and the sketch in part B relate
- ☐ For correctly stating that the vertical position is proportional to the horizontal position squared and so the shape of the path traveled by the box should be a parabola.

Example Response:

Yes. My equation in part (c) describes a parabola with a minimum at the origin, which matches my sketch in part (b).

Part E

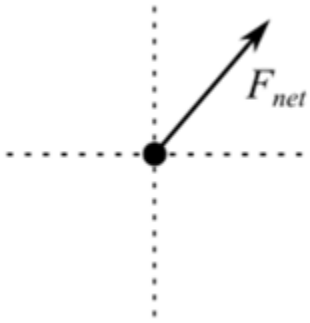
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes the following criteria:

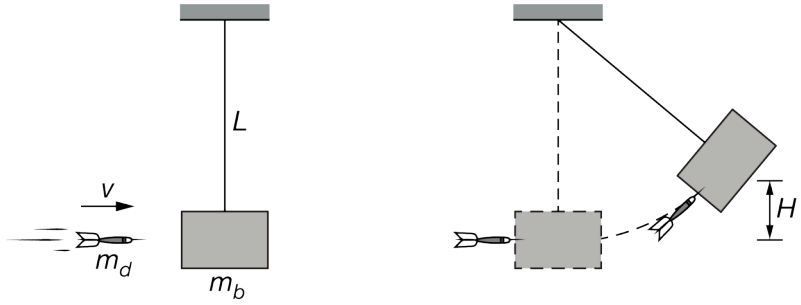
- ☐ One point for an arrow labeled ΣF or net force, pointing upwards and to the right.

Example Response:



frq3

13. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown at left above. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown at right above. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, if either, experiences a net horizontal force of larger magnitude during the collision. If the net horizontal force is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the experiment was repeated with a dart with the same horizontal speed but with mass smaller than m_d, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the motion of the dart-block system at the instant it first reaches a height of $H/2$, and they make the following statements:

Student 1: When the dart-block system is at a height of $H/2$, the kinetic energy of the system is half of its maximum value, and therefore the speed of the system is half of its maximum speed.

Student 2: When the dart-block system reaches a height of $H/2$, the centripetal force is equal to the tension in the string. Therefore, the acceleration of the system must point toward the fixed end of the string.

(c) Which underlined phrase or phrases, if either, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if either, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if either, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

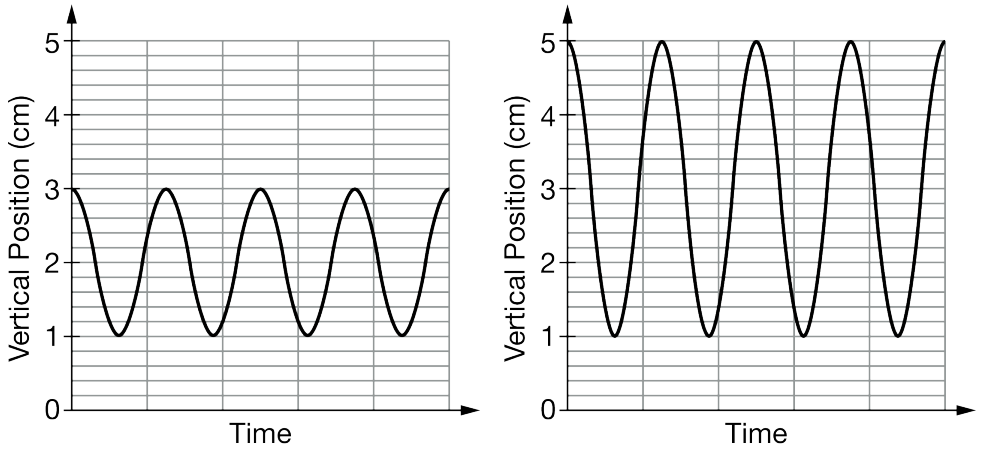
(f) Which underlined phrase or phrases, if either, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

frq3

(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{v}{1+\frac{m_b}{m_d}}\right)^2$. Explain how this equation supports your claim in part (b).

(h) Could the masses of the dart and block both be changed so that the maximum height *H* reached by the dart-block system would remain the same as in the original setup? Justify your answer by referring to the equation in part (g).

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart mass or the string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a statement that both objects experience the same net force because of Newton's third law.

frq3

Example Response:

Both the dart and the block experience the same force because they exert equal and opposite forces on each other because of Newton's third law.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the final height is smaller.
- ☐ 1 point is earned for a statement of conservation of momentum to indicate that the final speed of the dart-block system is smaller **AND** an indication of conservation of energy leading to a smaller height.

Example Response:

Because the momentum of the dart before the collision must be equal to the momentum of the dart-block system after the collision, a smaller dart mass (a smaller dart momentum) will lead to a smaller dart-block speed after the collision (a smaller dart-block momentum). Because the dart-block is traveling slower after the collision, it has less kinetic energy which means that the dart-block will not swing as high.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Responses:

Student 1 is correct in saying that the kinetic energy of the system is half of its maximum at $H/2$.

OR

The first statement

Part D

frq3

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Responses:

Student 1 is incorrect in saying that the speed of the system is half of its maximum speed

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating student 2 has no correct statements.

Example Response:

None

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating both of Student 2’s statements are incorrect.

frq3

Example Responses:

Student 2 is incorrect in saying that centripetal force is equal to the tension in the string, AND incorrect in saying that the acceleration of the system must point toward the center of the circle.

OR

Both statements

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement including functional dependence of the variables.
- ☐ 1 point is earned for correctly asserting that as the mass of the dart decreases in the equation *H* also decreases.

Example Response:

This equation matches my reasoning in part (b)– if the mass of the dart decreases, the height of the swing decreases because a decrease in the mass of the dart means an increase in the ratio $(block\ mass)/(dart\ mass)$*, which is in the denominator of the expression.*

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for “yes”, with an attempt at explanation.
- ☐ 1 point is earned for a statement that both either increase or decrease.
- ☐ 1 point is earned for a statement indicating the factor in increase/decrease (or ratio of masses) is the same.

Example Response:

Yes, if the masses were both doubled, the maximum height would still be H – it works as long as the masses both increase by the same factor .

frq3

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

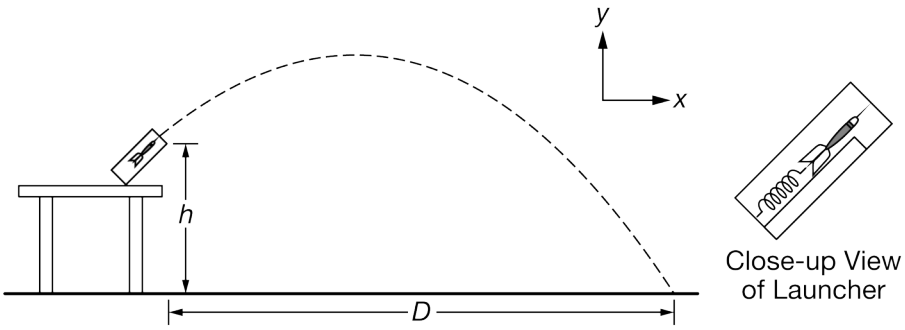
- ☐ 1 point is earned for recognizing that the amplitude of the graph for group 2 is larger.
- ☐ 1 point is earned for a statement discussing the dependence of amplitude on dart mass.
- ☐ 1 point is earned for indicating that the mass of the dart could be larger.

Example Response:

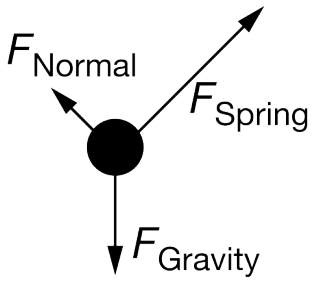
If group 2 used a dart with a larger mass, the dart would have had more momentum, and so the dart-block system would have gone higher (making the amplitude larger), but the string length is the same and so the period would still be the same.

frq3

14. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)

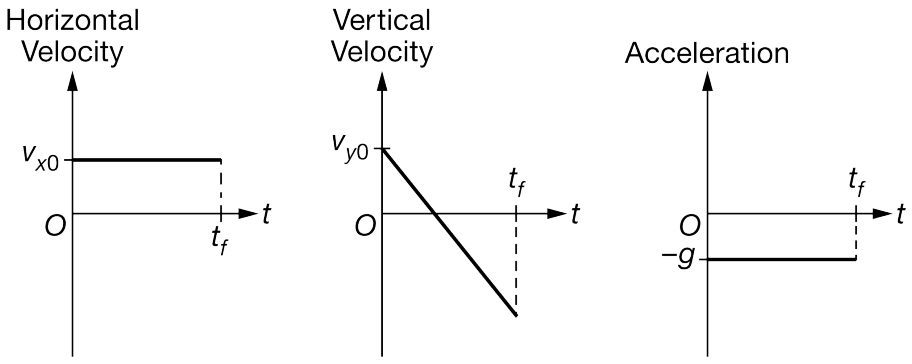


A launcher is used to launch a dart with initial speed v from a table at an angle, as shown above. The dart exits the end of the launcher at a height h above the floor and lands a horizontal distance D from the end of the launcher. A close-up view is included that shows the dart in the launcher immediately before being launched. Inside the launcher, the dart rests on a platform with negligible friction, and a compressed spring is released to push the dart out of the launcher. Assume air resistance is negligible.



(a) A diagram of the forces exerted on the dart while it is being launched is shown above. Describe what changes would need to be made to the diagram to make it a correct diagram for the dart when it is in the air (after it has left the launcher but before it has reached the ground). Assume air resistance is negligible.

The three graphs shown below represent properties of the motion of the dart from the instant it is launched at time $t = 0$ until immediately before it hits the floor at time t_f.

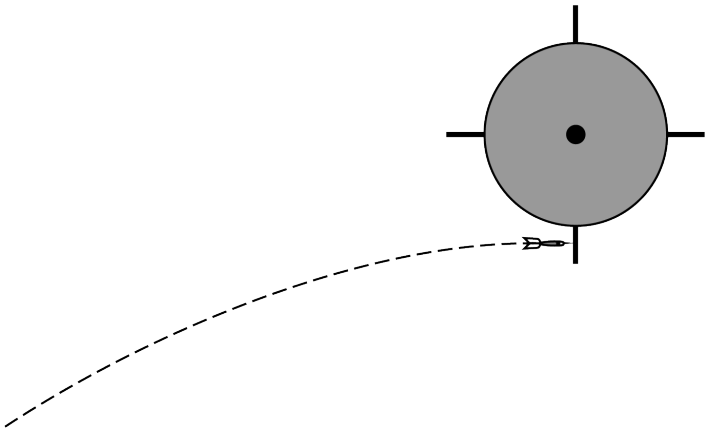


frq3

(b) Assuming the values of v_{x0}, v_{y0}, t_f, and g are known, which of the above graphs could lead to a determination of the distance D, and how could a student use the chosen graph to determine D ?

(c) After the dart passes the highest point of its trajectory, the angle between the velocity vector of the dart and the horizontal (the x-axis) continues to increase until the dart strikes the ground. In a clear, coherent, paragraph-length response, explain the following:

- why the angle between the dart's velocity and the horizontal continues to increase and
- why, no matter how large the height h of the end of the launcher is, the angle between the dart's velocity and the horizontal can never be 90 degrees.



The dart is now launched so that it collides and sticks to a wheel at the instant its velocity is completely horizontal, as shown above. The wheel is free to rotate about a pivot at its center that has negligible friction.

(d) Indicate whether the linear momentum of the dart-wheel system will be constant during the collision. Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating that the normal force and spring force would be removed.

Example Response:

When the dart is in the air, the normal force and the spring force should be removed from the diagram.

Part B

frq3

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for selecting the horizontal velocity graph.
- ☐ 1 point is earned for indicating area under the velocity graph can be used to find distance.

Example Response:

The students can determine the distance the dart travels horizontally by looking at the area under the horizontal velocity graph. This will be the horizontal displacement of the dart.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the horizontal velocity component remains constant.
- ☐ 1 point is earned for indicating that the vertical velocity component increases.
- ☐ 1 point is earned for indicating that the resultant velocity is the vector sum of the two components (this point is for mentioning/evaluating the resultant, rather than the two individual components).
- ☐ 1 point is earned for explaining that the angle can never be 90 degrees because there is always a horizontal component (or for indicating that the velocity vector can never be totally vertical).
- ☐ 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

The horizontal component of the velocity remains constant because there is no acceleration in the horizontal direction. The vertical component of the velocity will increase. The total velocity is the vector sum of the two velocity components. Since there is always a horizontal component, the angle can never be 90 degrees, so the total velocity cannot ever be totally vertical.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

0	1	2 ✓

The response accurately includes **both** of the following criteria.

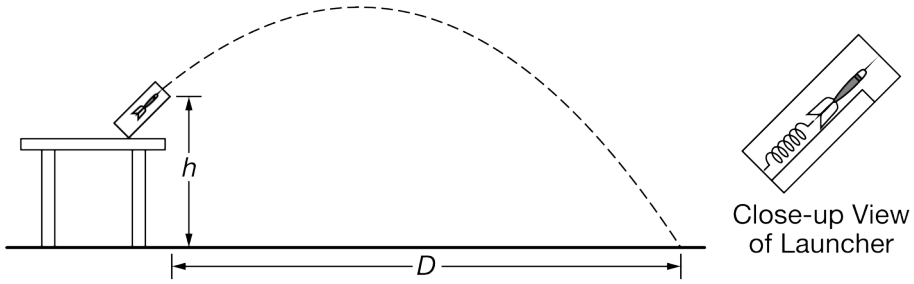
- ☐ 1 point is earned for indicating the linear momentum will not be constant, with an attempt at justification.
- ☐ 1 point is earned for indicating that there is an external force on the wheel (or that the dart will slow down and the wheel will have no net linear momentum).

Example Response:

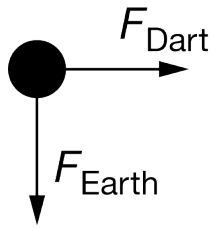
The linear momentum of the system will not be constant because there is an external force on the wheel, exerted at the wheel axle.

frq3

15. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)

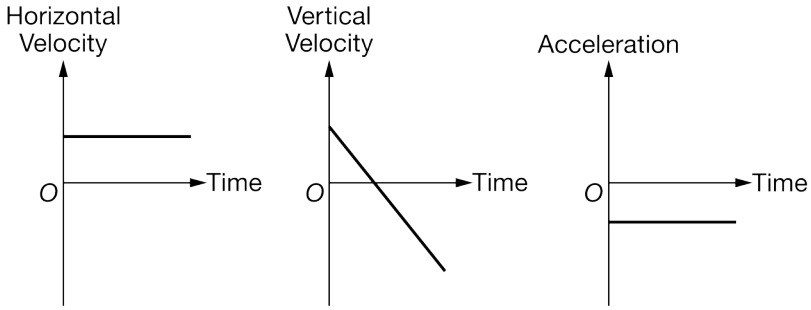


A launcher is used to launch a dart with initial speed v from a table at an angle, as shown above. The dart exits the end of the launcher at a height h above the floor and lands a horizontal distance D from the end of the launcher. A close-up view is included that shows the dart in the launcher immediately before being launched. Inside the launcher, the dart rests on a platform with negligible friction, and a compressed spring is released to push the dart out of the launcher. Assume air resistance is negligible.



(a) A student creates the incorrect diagram shown above of the forces exerted on the dart at an instant in time after it has been launched but before it reaches the floor. Indicate what aspect of the diagram is incorrect. Assume air resistance is negligible.

A student produces the three possibly incorrect graphs shown below representing properties of the motion of the dart after it is launched.

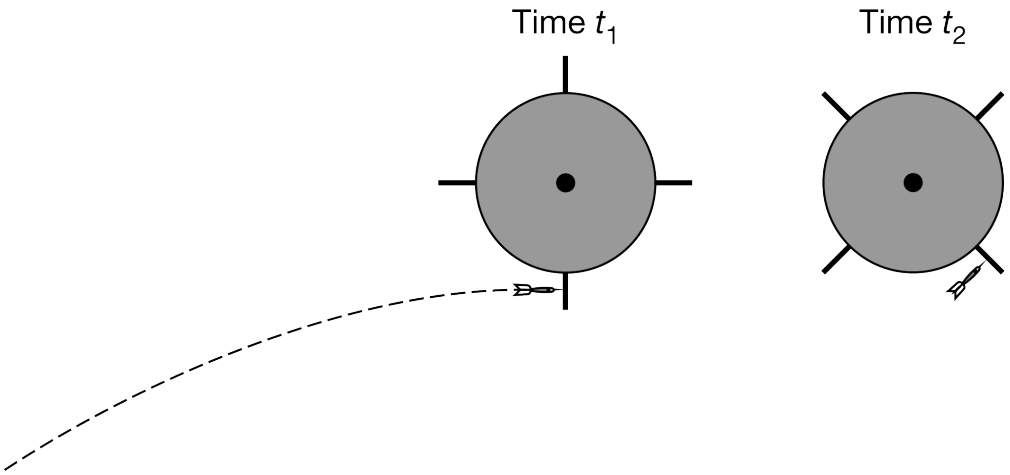


(b) Another student claims that the horizontal velocity graph is not correct because the dart is accelerating, and therefore the horizontal velocity is not constant. Is the student's claim valid? Justify your answer.

frq3

(c) From the instant the dart is launched until it reaches the highest point in its trajectory, the magnitude of the dart's velocity decreases. In a clear, coherent, paragraph-length response, explain the following:

why the magnitude of the dart's velocity decreases until the dart reaches its highest point and
why the magnitude of the dart's velocity does not decrease to zero.



The dart is now launched so that it collides with and sticks to a wheel at time t_1, which is the instant the dart's velocity is completely horizontal, as shown above at left. The wheel is initially at rest and is free to rotate about a pivot at its center that has negligible friction. A short time later, at time t_2, the dart-wheel system is still rotating in a counterclockwise direction, as shown above at right.

(d) At time t_2, is the angular speed of the dart-wheel system about the pivot increasing, decreasing, or constant? Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating the horizontal force should not be included.

Example Response:

There shouldn't be a horizontal force exerted on the dart.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the claim is not valid, with an attempt at an explanation.
- ☐ 1 point is earned for indicating the acceleration or force is vertical, so the horizontal velocity does not change.
  OR
  For indicating that the acceleration or force is not horizontal, so the horizontal velocity does not change.

Example Response:

The claim is not valid. The acceleration is vertical only, so the horizontal velocity does not change.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that velocity has two components.
- ☐ 1 point is earned for indicating that the horizontal velocity component remains constant while vertical velocity component decreases.
- ☐ 1 point is earned for indicating that the resultant velocity decreases because it is the vector sum of the horizontal and vertical components (point is for mentioning/evaluating the resultant, rather than the two individual components).
- ☐ 1 point is earned for indicating that the magnitude cannot be zero because there is always a nonzero horizontal component.
- ☐ 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

The velocity includes both the horizontal and vertical components. The horizontal component stays constant while the vertical component decreases. So, the resultant velocity decreases because it is the resultant of the vertical and horizontal components. But the total velocity can never be zero since there is always a horizontal component that is nonzero.

Part D

frq3

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

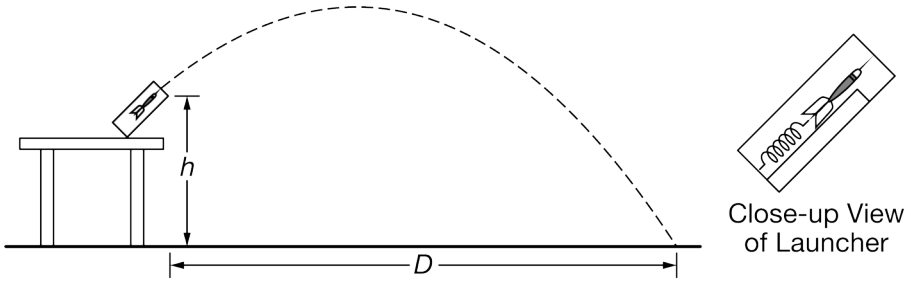
- ☐ 1 point is earned for indicating that the angular speed is decreasing, with an attempt at an explanation.
- ☐ 1 point is earned for indicating that there is a net torque exerted by the gravitational force on the dart, opposite the direction of rotation (or, clockwise).

Example Response:

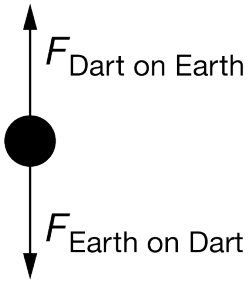
The angular speed is decreasing because there is a net torque exerted by the gravitational force on the dart, opposite to the direction of the rotation.

frq3

16. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)

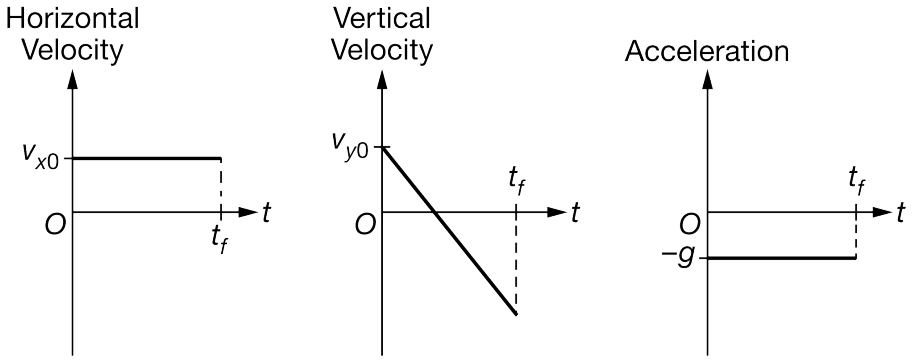


A launcher is used to launch a dart with initial speed v from a table at an angle, as shown above. The dart exits the end of the launcher at a height h above the floor and lands a horizontal distance D from the end of the launcher. A close-up view is included that shows the dart in the launcher immediately before being launched. Inside the launcher, the dart rests on a platform with negligible friction, and a compressed spring is released to push the dart out of the launcher. Assume air resistance is negligible.



(a) A student creates the incorrect diagram shown above of the forces exerted on the dart at an instant in time after it has been launched but before it reaches the floor. Indicate what aspect of the diagram is incorrect. Assume air resistance is negligible.

The three graphs shown below represent properties of the motion of the dart from the instant it is launched at time $t = 0$ until immediately before it hits the floor at time t_f.

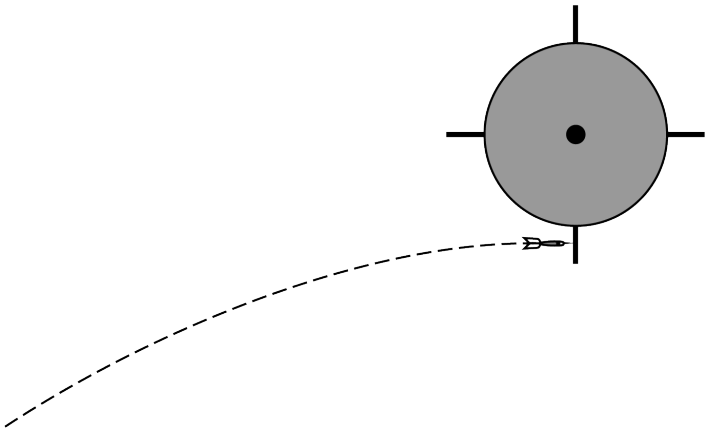


frq3

(b) Assuming the values of v_{x0}, v_{y0}, t_f, and g are known, which of the above graphs could lead to a determination of the height h, and how could a student use the chosen graph to determine h ?

(c) After the dart passes the highest point of its trajectory, the angle between the velocity vector of the dart and the acceleration vector of the dart decreases until the dart strikes the ground. In a clear, coherent, paragraph-length response, explain the following:

- why the angle between the dart's velocity and acceleration continues to decrease and
- why, no matter how large the height h of the end of the launcher is, the angle between the dart's velocity and acceleration can never reach zero degrees.



The dart is now launched so that it collides and sticks to a wheel at the instant its velocity is completely horizontal, as shown above. The wheel is free to rotate about a pivot at its center that has negligible friction.

(d) Indicate whether the angular momentum of the dart-wheel system about the pivot will be constant during the collision. Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating the force exerted by the dart on Earth should not be included.

Example Response:

The force of the dart on Earth does not belong on a free-body diagram of the dart.

Part B

frq3

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for selecting the vertical velocity graph.
- ☐ 1 point is earned for indicating area under the velocity graph can be used to find *h*.

Example Response:

The vertical velocity graph can be used to find h. The total area under the curve on the vertical velocity graph will be the vertical displacement of the dart. The absolute value of this area is equal to h.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the acceleration is downward.
- ☐ 1 point is earned for indicating that the vertical velocity component increases while the horizontal velocity component remains constant.
- ☐ 1 point is earned for a statement that the resultant velocity is the vector sum of the two components (this point is for mentioning/evaluating the resultant, rather than the two individual components).
- ☐ 1 point is earned for explaining that the angle can never be zero degrees since there is always a horizontal component (or for indicating that the velocity vector can never be totally vertical).
- ☐ 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

The acceleration of the dart is downward and constant, equal in magnitude to g. The vertical velocity increases, while the horizontal velocity remains constant because there is no acceleration in the horizontal direction. The total velocity is the vector sum (using the Pythagorean theorem) of the two velocities. Therefore, the angle can never be zero, since there is always a horizontal component.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

0	1	2 ✓

The response accurately includes **both** of the following criteria.

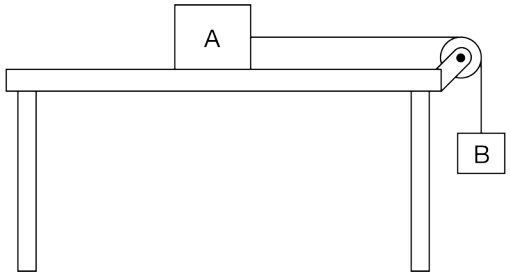
- ☐ 1 point is earned for indicating the angular momentum will be constant, with an attempt at justification.
- ☐ 1 point is earned for indicating that there is no net torque on the system.

Example Response:

The angular momentum is constant since there is no net torque on the system.

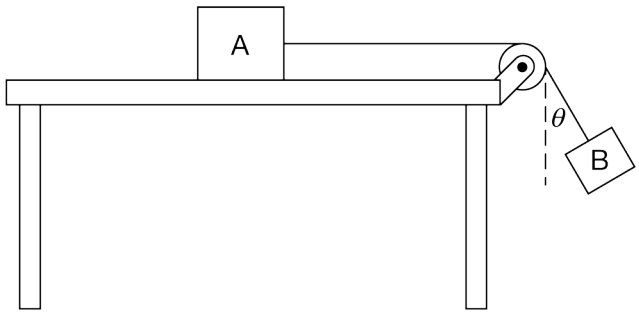
frq3

17. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)



Block A with mass m_A rests on a horizontal tabletop, while block B with mass m_B hangs from a string of negligible mass that passes over a pulley and attaches to block A, as shown above. The pulley has negligible mass and rotates with negligible friction about its axle. The blocks are at rest due to friction between block A and the tabletop. The tension in the string while the blocks are at rest has magnitude T_1, and the coefficient of static friction between block A and the tabletop is μ (mu).

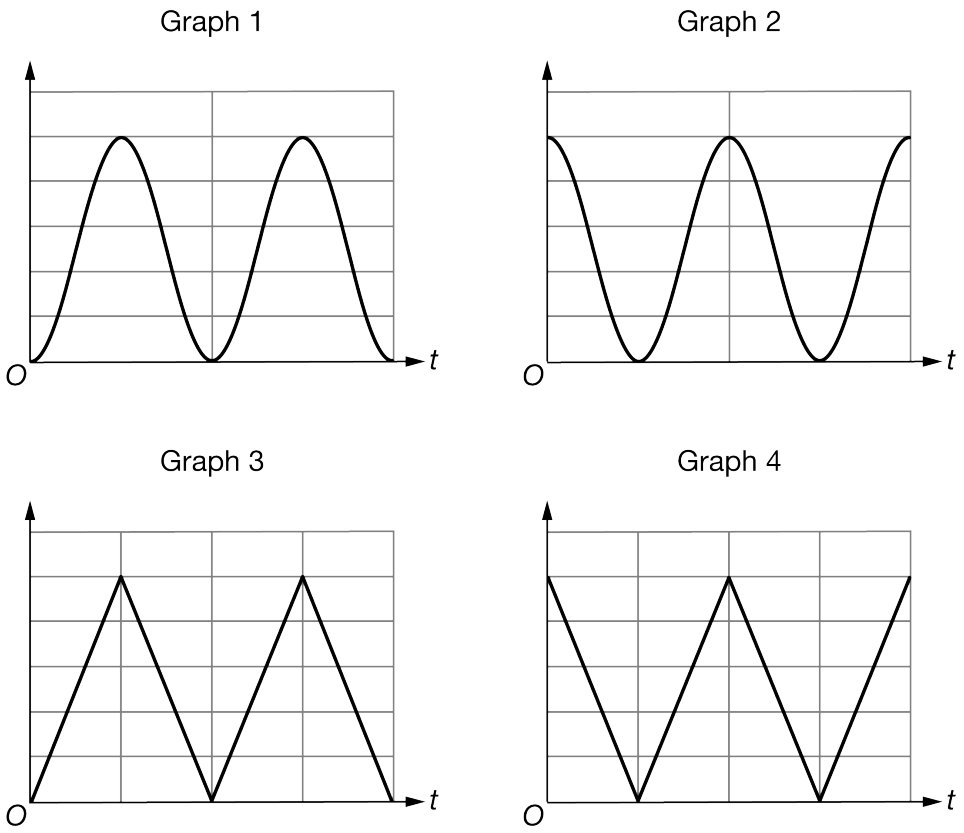
(a) What is the magnitude of the frictional force acting on block A? Justify your answer.



Note: Figure not drawn to scale.

Block B is displaced from the vertical such that the string makes a small angle θ with the vertical, as shown above. The string still has nonzero tension when block B is then released from rest.

frq3



(b) Block A is held at rest while block B swings back and forth. Which of the graphs shown above best represents the kinetic energy of block B as a function of time *t* after it is released from rest at time $t = 0$? Explain your reasoning.

Block B is again displaced from the vertical such that the string makes a small angle θ with the vertical. Block A is not held in place, and block B is released from rest.

(c) In the original scenario in which block B hung at rest, block A remained at rest. In this new scenario in which block A is not held in place, block A begins accelerating to the right across the tabletop at the instant block B passes through the bottom of its swing. In a clear, coherent paragraph-length response, explain in terms of the net force on block A why block A remained at rest in the original scenario but accelerates in this new scenario.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response does not accurately include

- ☐ 1 point is earned for a valid justification of why the friction force is either T_1 or $m_B g$.

frq3

Example Response:

The horizontal forces on block A must sum to zero because block A is not accelerating, so the friction force must be equal to T_1.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- ☐ 1 point is earned for indicating that Graph 1 is the correct graph for kinetic energy as a function of time.
- ☐ 1 point is earned for a justification that the initial speed of the block is zero and so the initial kinetic energy of the block is zero.
- ☐ 1 point is earned for a statement that includes that the kinetic energy as a function of time is not linear because it is based on a sinusoidal function.

Example Response:

Graph 1. The block is released from rest, so the kinetic energy must be zero at t = 0. This eliminates graphs 2 and 4. Kinetic energy is proportional to velocity-squared, and the velocity is a sinusoidal function. The square of a sinusoid is depicted in graph 1, not graph 3.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that object B is moving at the bottom in circular motion.
- ☐ 1 point is earned for indicating that since Block B is moving in circular motion, the acceleration or net force is toward the center of the circle or upward when at the lowest point.
- ☐ 1 point is earned for indicating that the tension is larger than when Block B is at rest
  OR
  larger than $m_B g$, or larger than T_1.
- ☐ 1 point is earned for indicating that the new string tension is greater than the maximum possible static friction on block A.
- ☐ 1 point is earned for indicating that since block A is accelerating to the right, the net force must be to the

frq3

right.

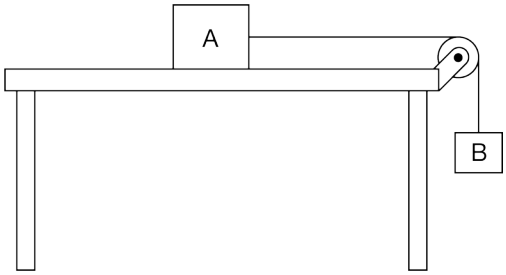
- [ ] 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

When block B is swinging and is at the bottom of its motion, it is in circular motion and has an upward acceleration and net force. Therefore, the tension in the string is now greater than $m_B g$ *or* T_1*. Because block A accelerates to the right, there is a net force to the right, which must result from the tension being greater than the maximum possible static friction force.*

frq3

18. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)



Block A with mass m_A rests on a horizontal tabletop, while block B with mass m_B hangs from a string of negligible mass that passes over a pulley and attaches to block A, as shown above. The pulley has negligible mass and rotates with negligible friction about its axle. The blocks are at rest due to friction between block A and the tabletop. The coefficient of static friction between block A and the tabletop is μ (mu), where $\mu < 1$.

(a) Determine the magnitude of the tension in the string. Briefly explain your reasoning.

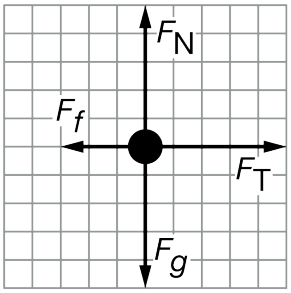


Figure 1

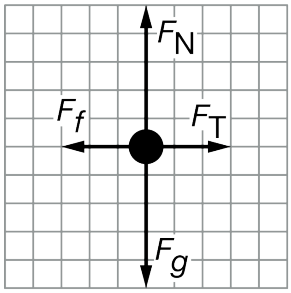


Figure 2

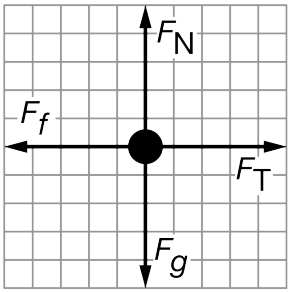


Figure 3

(b) Three students draw the free-body diagrams shown above to represent the forces acting on block A while the two-block system is at rest. Which figure correctly depicts the forces and their relative magnitudes? Explain your reasoning.

(c) A student slowly adds mass to the top of block B. When a total mass m_0 has been added to the top of block B, block A begins to accelerate across the tabletop. In a clear, coherent, paragraph-length response, explain in terms of the net force on block A why block A remained at rest even as mass was added to block B, but block A accelerates once a total mass m_0 was added to block B.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

frq3

The response accurately includes

- [ ] 1 point is earned for a valid explanation for why the tension is equal to the weight.

Example Response:

When block A is at rest, block B is at rest. So, the tension in the string must be equal to the weight of block B, $T_1 = m_B g$.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- [ ] 1 point is earned for indicating that Figure 2 is the correct free body diagram.
- [ ] 1 point is earned for an explanation which addresses that the forces must add to zero because the system is in equilibrium.
- [ ] 1 point is earned for explanation which addresses that the force of friction must be less than the normal force.

Example Response:

The system is at rest, so the forces must add to zero, and F_f *and* F_T *must be equal in magnitude. Friction also must be less than the normal force because* $\mu < 1$ *and the maximum force of friction is mu times the normal force. Therefore, Figure 2 correctly depicts the forces on Block A.*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- [ ] 1 point is earned for indicating that as the mass was added to the top of block B, the tension in the string increased.
- [ ] 1 point is earned for indicating that while block A was not moving, the static friction force was equal to the tension.
- [ ] 1 point is earned for indicating that the static friction force is not constant but can vary up to a maximum value.
- [ ] 1 point is earned for indicating that block A moved as soon as the tension was greater than the maximum

frq3

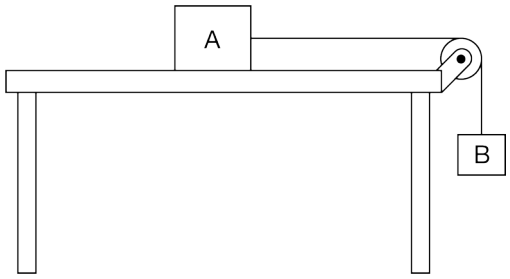
static friction force.

- 1 point is earned for indicating that when the tension on block A is greater than the static friction force, the block experiences a net force and accelerates to the right.
- 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

As mass was added to the top of block B, the tension in the string was increasing. As the tension was increasing, the static friction force was increasing, to keep block A at rest. Once the additional mass reached a value m_0, the tension increased past the maximum value of static friction. Since the forward tension force is greater now than the friction force, block A accelerates to the right.

19. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)



Block A with mass m_A rests on a horizontal tabletop, while block B with mass m_B hangs from a string of negligible mass that passes over a pulley and attaches to block A, as shown above. The pulley has negligible mass and rotates with negligible friction about its axle. The blocks are at rest due to friction between block A and the tabletop. The tension in the string while the blocks are at rest has magnitude T_1, and the coefficient of static friction between block A and the tabletop is μ (mu).

(a) A student claims the magnitude of the frictional force acting on block A must be equal to $\mu m_A g$. Briefly explain why the student's claim is incorrect.

(b) Block B is given a quick downward push so that block A begins accelerating across the tabletop and block B accelerates toward the floor. The tension in the string while the two-block system accelerates is T_2. How does T_2 compare to T_1 ? Explain your reasoning.

(c) After block B falls a distance , the blocks have speed v_1. In a second trial, the pulley of negligible mass is replaced with a different pulley whose mass is not negligible, and the process described in part (b) is repeated. After block B again falls a distance d, the blocks have speed v_2. In a clear, coherent, paragraph-length response, explain how v_1 and v_2 compare.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a valid explanation of why the student is incorrect.

Example Response:

The student is incorrect because $\mu m_A g$ is the maximum static friction force, and the static friction force could be less than that value.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that T_2 is less than T_1.
- ☐ 1 point is earned for indicating that when the block is at rest, the tension in the string, $T_1$is equal to the weight of block B.
- ☐ 1 point is earned for indicating that when the block is accelerating downward, the net force must be downward, so T_2 must be less than the weight of block B.

Example Response:

If block B is accelerating downward, the net force must be downward. That means the gravitational force on B is greater than the tension, T_2. When the block was at rest, $T_1$was equal to the force of gravity on B, so therefore T_2 is less than T_1.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

frq3

- [ ] 1 point is earned for indicating that the block-pulley system has the same amount of work done on it by gravity in both cases
  OR
  For indicating that there is the same gravitational potential energy change in both cases.
- [ ] 1 point is earned for indicating that the blocks-pulley system has the same amount of energy dissipated by friction in both cases.
- [ ] 1 point is earned for indicating that both systems have the same total kinetic energy change.
- [ ] 1 point is earned for indicating that when the pulley has mass, there is rotational kinetic energy and thus the blocks have less translational energy.
- [ ] 1 point is earned for concluding that the blocks will have less speed when the pulley has nonnegligible mass.
- [ ] 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

The gravitational potential energy of blocks-pulley-earth system is reduced as Block B falls, so the kinetic energy and thermal energy of the system are increased by an equivalent amount. The overall change in kinetic energy is the same whether or not the pulley has mass, since the same amount of thermal energy is produced by friction when Block A moves a distance d. When the pulley has mass, some of the kinetic energy will be in the form of rotational kinetic energy, so less translational kinetic energy can be acquired by the blocks and they will therefore have less speed.

Part C - Alternate solution

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- [ ] *1 point is earned for applying Newton's second law and recognizing that the pulley's mass increases the mass of the accelerating system.*
- [ ] *1 point is earned for recognizing that the translational acceleration of the system is reduced when the pulley has mass.*
- [ ] *1 point is earned for recognizing that mass in the pulley gives it rotational inertia.*
- [ ] *1 point is earned for indicating that the torque required to turn the pulley affects the translational motion of the blocks.*
- [ ] *1 point is earned for concluding that the blocks have less speed when the pulley has more mass.*
- [ ] *1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.*

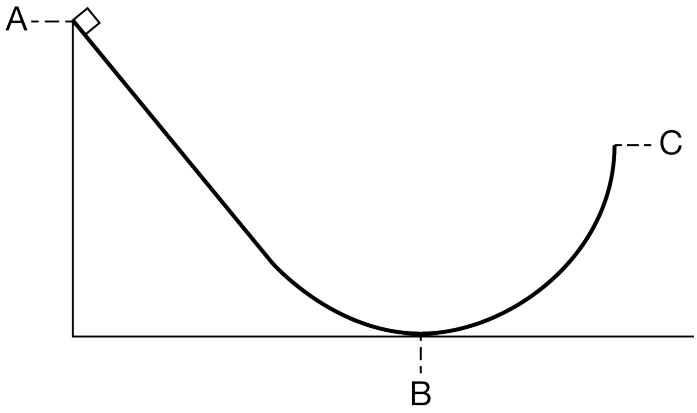
Example Response:

frq3

When the pulley has mass, it has rotational inertia. To begin rotating, it must experience a net torque. This will affect the tensions on the blocks. Block B will have a greater upward tension than in the first case, the net force it experiences will be less, and its acceleration will be less. Because of this, when it falls a distance d, it will have less speed.

frq3

20. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)



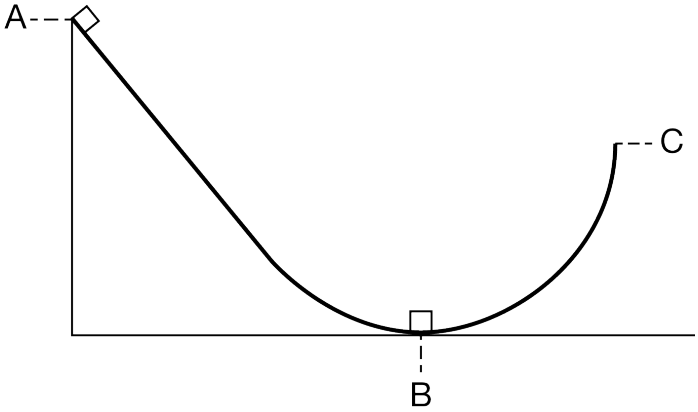
The figure above shows a track consisting of a straight section that connects to a circular section. The bottommost point of the track is at location B, and the track ends at location C, where the track is vertical. A block is released from rest at location A. There is negligible friction between the track and the box.

(a) Is the magnitude of the normal force exerted on the block by the track when the block is at location B greater than, less than, or equal to the normal force immediately before the block reaches location C? Justify your answer.

(b) In a coherent, paragraph-length response, describe the relative angle between the direction of the normal force exerted on the block by the track and the direction of the block's acceleration at the following times.

Immediately after the block is released at location A
When the block is at location B
Immediately before the block reaches location C

Specifically, indicate whether the normal force is parallel to the acceleration, perpendicular to the acceleration, or neither, at each location. Use physical principles to justify your responses.



(c) The scenario is repeated, but an identical block is at rest at location B, as shown above. When the block at

frq3

location A is released from rest, it slides down the track and collides with and sticks to the identical block. The two blocks then continue sliding up the ramp. Is the maximum height reached by the two blocks greater than, less than, or equal to the maximum height reached by the block in the original scenario? Briefly explain your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the normal force at B is greater than it is immediately before C.
- ☐ 1 point is earned for a correct explanation that correctly uses either or both of the following effects:
  · At B, the speed *v* is greater so the net force $F_{\text{net}} = mv^2/r$ is greater
  · At B, the normal force must "overcome" the downward weight force, so the normal force $F_n = F_{\text{net}} + mg$ is greater.

Example Response:

At B, the speed v is greater, so the net force mv^2/r is greater, resulting in a greater normal force than at just before location C.

OR

At B, the normal force must "overcome" the downward weight force, so the normal force $(= F_{\text{net}} + mg)$ is greater than at just before location C.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the normal force is always perpendicular to the surface of the track.
- ☐ 1 point is earned for relating the acceleration to the net force.
- ☐ 1 point is earned for indicating the acceleration at location A is along the ramp and thus perpendicular to the normal force.
- ☐ 1 point is earned for indicating that at location B the acceleration is parallel to the normal force.

frq3

- [ ] 1 point is earned for indicating that at position just before location C the acceleration is neither perpendicular nor parallel to the normal force.
- [ ] 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

The normal force is always perpendicular to the track surface. At location A, the block is accelerating from rest in a straight line down the ramp, so the acceleration is along the ramp and thus perpendicular to the normal force.

At B, the acceleration is upward, so it is in the same direction as the normal force.

Just before C, the forces of gravity (downward) and the normal force (perpendicular to the track) by vector summation result in the net force, which is in the direction of the acceleration. –The acceleration therefore is at an angle to the normal force, that is neither parallel nor perpendicular.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for indicating the height of the two blocks is less than that of the original block.
- [ ] 1 point is earned for a correct explanation of why the height is less than the original block.

Example Response:

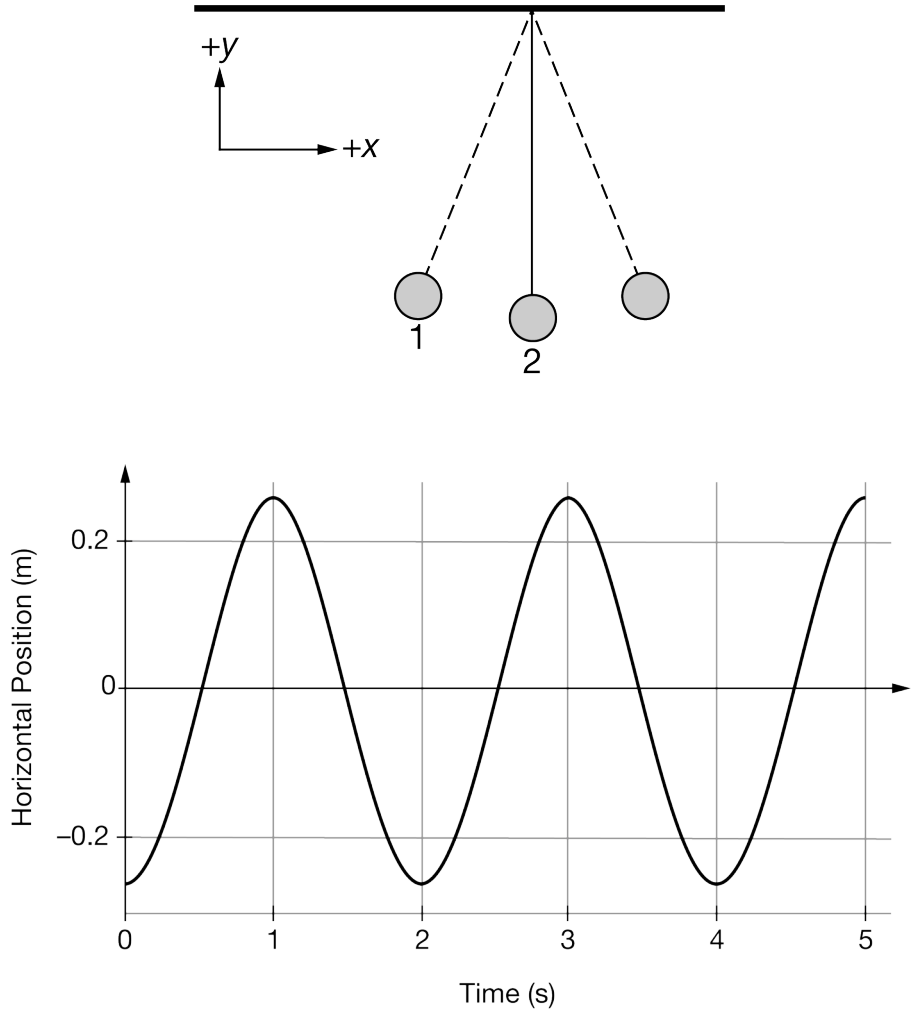
In an inelastic collision linear momentum is constant and kinetic energy is not constant. The two-block system has less energy than the first block had at the same position. All of the kinetic energy at that point can now be converted to gravitational potential energy. Which means that the two-block system will not go as high as the first block.

OR

By conservation of momentum, the speed of the two blocks after the collision is half the speed of the original block. Since the blocks are going slower, they will not go as high as the original block did.

frq3

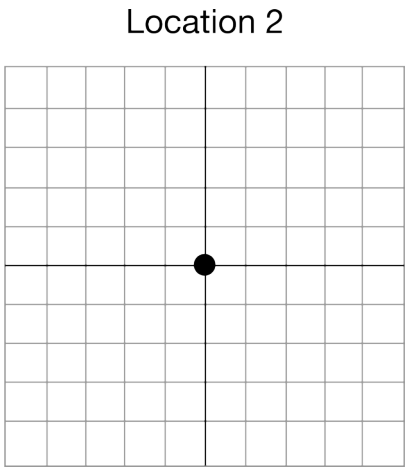
21. This question is a short free-response question. Show your work for each part of the question.



One end of a string is secured to the ceiling of a classroom, and the other end of the string is attached to a sphere of mass $0.2\ \text{kg}$ so that a pendulum is established. The sphere is raised at an angle of 15° above the sphere's equilibrium position and then released from rest so that the pendulum oscillates, as shown in the figure. Location 1 is shown in the figure. A graph of the pendulum's horizontal position as a function of time is shown in the graph.

(a) The dot below represents the sphere at location 2. Draw a free-body diagram that shows and labels the forces (not components) exerted on the sphere at the location. Draw the relative lengths of all vectors to reflect the relative magnitudes of all forces.

frq3



(b) Calculate the maximum kinetic energy of the sphere as it oscillates.

The sphere is raised to a new angle above the sphere's equilibrium position that is less than the angle described for part (a). The sphere is released from rest so that the pendulum oscillates.

(c) Explain how the motion of the sphere changes, if at all, based on the appropriate evidence.

Part (a)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

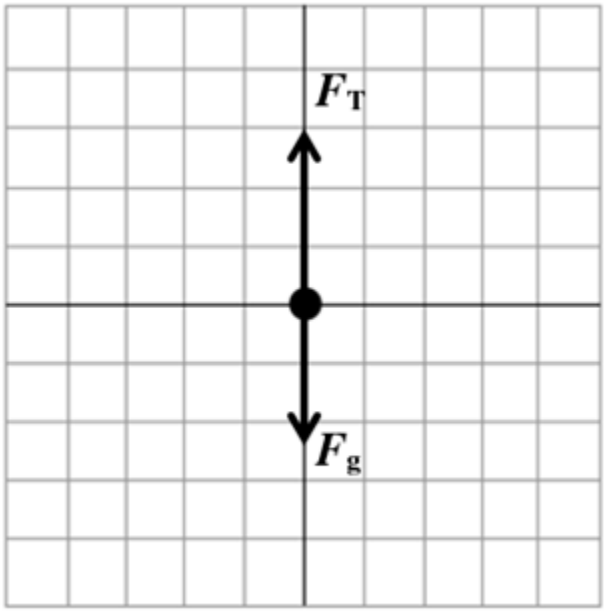
0	1	2 ✓

Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for drawing an upward tension force and a downward force due to gravity
- ☐ 1 point is earned for drawing an upward tension force that has a larger magnitude than the force due to gravity

Example Response:

frq3



Part (b)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

Student response accurately includes one of the following criteria.

Correct Answer: 0.068 J

- 1 point is earned for using the period of oscillation from the graph to determine the length of the string,
  Example Response:
  $T_p = 2\pi\sqrt{\frac{\diamondsuit}{g}}$
  $\frac{T_p}{2\pi} = \sqrt{\frac{\diamondsuit}{g}}$
  $\frac{{T_p}^2}{4\pi^2} = \frac{\diamondsuit}{g}$
  $\diamondsuit = \frac{g{T_p}^2}{4\pi^2}$
  $\diamondsuit = \frac{(10\,\text{N/kg})(2.0\,\text{s})^2}{4\pi^2}$
  $\diamondsuit \approx 1.0\,\text{m}$
- 1 point is earned for using the correct trigonometric relationship to determine the maximum height of the sphere from its lowest position.
  Example Response:

frq3

$\Delta y = \diamondsuit - \diamondsuit \cos(\theta)$
$\Delta y = (1.0\,\text{m}) - (1.0\,\text{m})\cos(15^\text{o})$
$\Delta y \approx 0.034\,\text{m}$

- [ ] 1 point is earned for correctly applying the conservation of energy to obtain an expression for the maximum kinetic energy of the sphere.

Example Response:
$E_0 = E_f$
$U_{g0} + U_{s0} + K_0 + W_{\text{External}} = U_{gf} + U_{sf} + K_f$
$U_{g0} = K_f$
$K_f = mgy_0$
$K_f = (0.2\,\text{kg})(10\,\text{N/kg})(0.034\,\text{m})$
$K_f \approx 0.068\,\text{J}$

Part (c)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

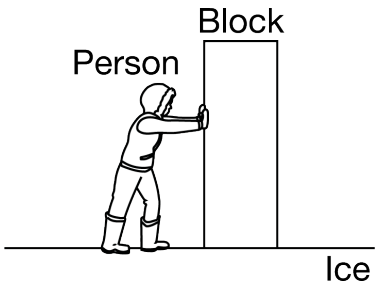
0	1	2 ✓

Student response accurately includes both of the following criteria.

- [ ] 1 point is earned for stating that the period of oscillation will remain the same.
- [ ] 1 point is earned for stating that the period of oscillation for a pendulum only depends on the gravitational field strength (acceleration due to gravity) of the sphere at its location and the length of the pendulum.

frq3

22. This question is a short free-response question. Show your work for each part of the question.



A person pushes a large block on a horizontal ice surface in a straight line to the right with constant speed, as shown above. The mass of the block is $10\ \text{kg}$ and frictional forces between the block and the ice are negligible. However, the block has a wide cross-sectional area such that air resistance acting on the block cannot be neglected. The opposite is true for the person: air resistance on the person is negligible, but the person's shoes do not slip on the ice. The table shows the force exerted by the person on the block for several values of constant speed.

Force of person's push (N)	**Constant speed of block** $(\text{m/s}) \pm 0.05$
20	1.25
40	2.51
60	3.73
80	5.00

(a) A student claims that the data show that the magnitude of the force of air resistance is proportional to the speed of the object, within experimental uncertainty. Use physics principles to explain whether the claim is correct.

(b) As the person pushes the block, the person moves with the same constant speed as the block. Consider the scenario in which the person's speed is $5.00\ \text{m/s}$.

i. On the dot below, which represents the person, draw and label the forces (not components) exerted on the person. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



frq3

ii. The person now stops and releases the block. Determine the magnitude and direction of the block's acceleration at the instant the block is released.

Part (a)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

Student response accurately includes all of the following criteria.

- ☐ 1 point is earned for arguing that the acceleration is zero, so that the push force equals the force of air resistance in magnitude.
- ☐ 1 point is earned for performing a mathematical analysis with the numbers to show that the magnitude of the force of air resistance is proportional to the speed.
- ☐ 1 point is earned for addressing the uncertainty in the speed measurements.

Example Response:

The block moves at a constant velocity, so the acceleration of and the net force on the block are both zero. The horizontal forces of the person's push (given in the table) and of air resistance must therefore be equal in magnitude at each speed. Multiplying each force value by a proportionality constant of $\frac{5\ \text{m/s}}{80\ \text{N}}$ *gives results that agree with each speed value within the* $\pm 0.05\ \text{m/s}$ per *experimental uncertainty for the speeds:* $1.25,\ 2.50,\ 3.75,\ \text{and}\ 5.00\ \text{m/s}$

Claim, Evidence, Reasoning Statements:

Claim: The results from the table agree with each speed value within the uncertainty of $\pm 0.05\ \text{m/s}$.

Evidence: The block moves at a constant velocity, so the acceleration of and the net force on the block are both zero.

Reasoning Statements: The horizontal forces of the person's push (given in the table) and of air resistance must therefore be equal in magnitude at each speed. Multiplying each force value by a proportionality constant of $\frac{5\ \text{m/s}}{80\ \text{N}}$.

Part (b)i

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

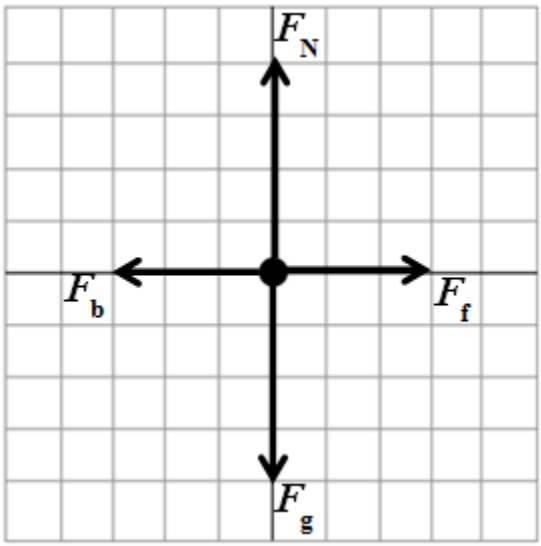
Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for drawing and labeling the normal force upward and gravitational force downward.

frq3

- [ ] 1 point is earned for drawing the force exerted by the block to the left and the frictional force to the right.

Example Response:



Part (b)ii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes both of the following criteria.

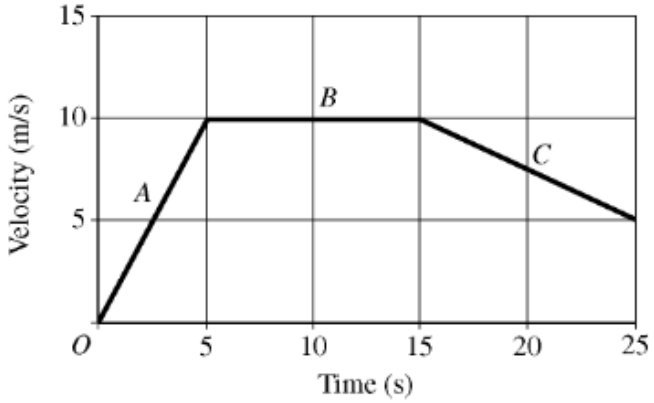
- [ ] 1 point is earned for stating the direction of the acceleration.
- [ ] 1 point is earned for determining the magnitude of the acceleration.

Example Response:

The direction of the acceleration is to the left. The magnitude of the acceleration is: $(80\ \text{N})/(10\ \text{kg}) = 8\ \text{m/s}^2$

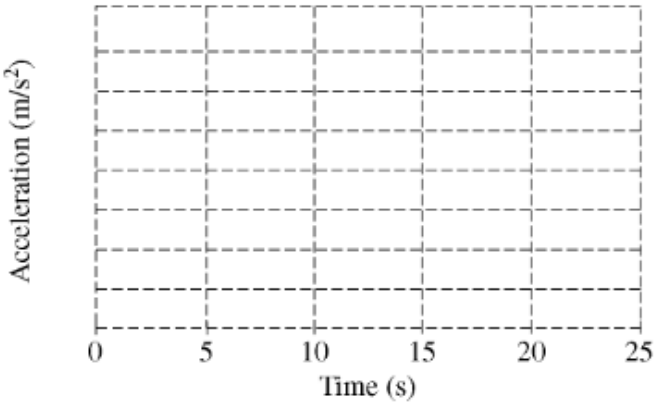
frq3

23.



A 0.40 kg object moves in a straight line under the action of a net force. The graph above shows the velocity as a function of time for the object during a 25 s interval. At time $t = 0$, the object is at the position $x = 0$.

(a) On the grid below, sketch a graph of the acceleration as a function of time for the object. Label the scale for the acceleration.



(b) Calculate the position of the object at t = 5.0 s.

(c) On which segment of the graph is the net force acting on the object zero?

____ A ____ B ____ C

Justify your answer.

(d) Calculate the net force on the object during the first 3.0 s of the motion.

(e) Calculate the amount of work done on the object by the net force during the first 15 s of the motion.

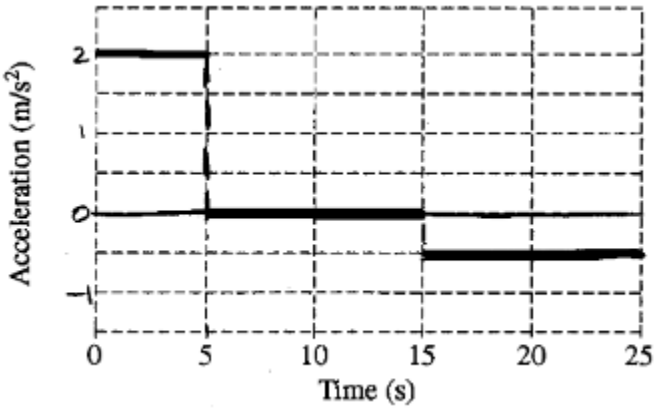
frq3

(f) For the interval t = 15 s to t = 25 s, is the work done on the object by the net force positive, negative, or zero?

____ Positive ____ Negative ____ Zero

Justify your answer.

Part A



1 point is earned: For using a linear scale on the vertical axis

1 point is earned: For showing horizontal lines in the three appropriate time frames

1 point is earned: For having the correct acceleration values for segments *B* and *C* (The correct value for segment *A* is accounted for in the answer point in part (d).)

a = $\Delta v / \Delta t$ (or the slope of the velocity versus time curve)

For segment A: a = (10m/s - 0m/s)/(5 s - 0 s) = 2 m/s2

For segment B: slope is zero, so acceleration is zero

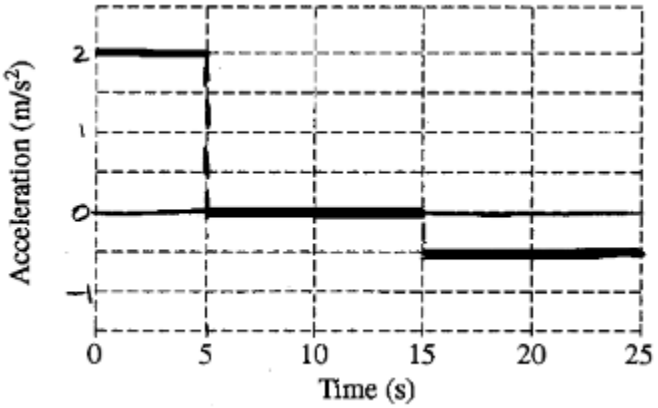
For segment C: a = (5m/s - 10m/s)/(25 s - 15 s) = -0.5 m/s2

✓

0	1	2	3

The student response earns all of the following points:

frq3



1 point is earned: For using a linear scale on the vertical axis

1 point is earned: For showing horizontal lines in the three appropriate time frames

1 point is earned: For having the correct acceleration values for segments *B* and *C* (The correct value for segment *A* is accounted for in the answer point in part (d).)

a = Δ*v*/ Δ*t* (or the slope of the velocity versus time curve)

For segment A: *a* = (10m/s - 0m/s)/(5 s - 0 s) = 2 m/s2

For segment B: slope is zero, so acceleration is zero

For segment C: *a* = (5m/s - 10m/s)/(25 s - 15 s) = -0.5 m/s2

Part B

x = *x0* + *v0t* + (1/2)at2, where *x0* = 0

1 point is earned: For substituting v0 = 0 and *t* = 5 s and into the equation above

1 point is earned: For substituting the value on the graph of the constant acceleration between 0 and 5 s or an explicit calculation of 2m/s2

x = (1/2)(2 m/s2)(5 s)2

x = 25 m

Alternate solution

Position is the area under the velocity versus time curve.

1 point is earned: *For correctly substituting into the expression for the area under the curve*

x = (1/2)(10 m/s)(5 s)

1 point is earned: *For the correct answer*

x = 25 m

frq3

0	1	2 ✓

The student response earns all of the following points:

$x = x0 + v0t + (1/2)at2$, where $x0 = 0$

1 point is earned: For substituting v0 = 0 and $t = 5$ s and into the equation above

1 point is earned: For substituting the value on the graph of the constant acceleration between 0 and 5 s or an explicit calculation of 2m/s2

$x = (1/2)(2\ m/s2)(5\ s)2$

$x = 25\ m$

Alternate solution

Position is the area under the velocity versus time curve.

1 point is earned: *For correctly substituting into the expression for the area under the curve*

$x = (1/2)(10\ m/s)(5\ s)$

1 point is earned: *For the correct answer*

$x = 25\ m$

Part C

1 point is earned: For selecting segment *B*

1 point is earned: For an appropriate justification

Example: Fnet = ma = m ($\Delta v/\ \Delta t$), the velocity is constant for this segment, $\Delta v = 0$; therefore, *Fnet* = 0.

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For selecting segment *B*

1 point is earned: For an appropriate justification

Example: Fnet = ma = m ($\Delta v/\ \Delta t$), the velocity is constant for this segment, $\Delta v = 0$; therefore, *Fnet* = 0.

frq3

Part D

1 point is earned: For using Newton's second law

Fnet = ma

Fnet = (0.40 kg)(2 m/s2)

1 point is earned: For the correct answer

Fnet = 0.80 N

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For using Newton's second law

Fnet = ma

Fnet = (0.40 kg)(2 m/s2)

1 point is earned: For the correct answer

Fnet = 0.80 N

Part E

1 point is earned: For using the work-energy theorem

W = FΔr cos θ

To move in a straight line, the net force must be in the direction of motion, so . *W* = *Fnetd*

1 point is earned: For substituting the value of the position determined in from part (b), which is equal to the displacement, to determine the work done for segment A

For segment A, *WA* = (0.8)(25 m) = 20 J

For segment B, *WB* = (0 N)(10 m/s)(10 s) = 0 J

Wtot = *WA* + *WB* = 20 J + 0 J = 20 J

Alternate solution

$W = \Delta K$

$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

frq3

1 point is earned: *For correct substitutions into the equation above*

$W = \frac{1}{2}(0.40kg)(10m/s)^2 - \frac{1}{2}(0.40kg)(0m/s)^2$

1 point is earned: *For the correct answer*

W = 20 J

✓

0	1	2

The student response earns all of the following points:

1 point is earned: For using the work-energy theorem

W = F Δr cos θ

To move in a straight line, the net force must be in the direction of motion, so . *W = Fnetd*

1 point is earned: For substituting the value of the position determined in from part (b), which is equal to the displacement, to determine the work done for segment A

For segment A, *WA* = (0.8)(25 m) = 20 J

For segment B, *WB* = (0 N)(10 m/s)(10 s) = 0 J

Wtot = WA + WB = 20 J + 0 J = 20 J

Alternate solution

$W = \Delta K$
$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

1 point is earned: *For correct substitutions into the equation above*

$W = \frac{1}{2}(0.40kg)(10m/s)^2 - \frac{1}{2}(0.40kg)(0m/s)^2$

1 point is earned: *For the correct answer*

W = 20 J

Part F

1 point is earned: For selecting “Negative”

1 point is earned: For an appropriate explanation using kinematic or dynamic principles

Examples:

frq3

The velocity is decreasing, so the kinetic energy is also decreasing.

The acceleration is negative; therefore, the direction of the net force is opposite to the direction of the displacement.

1 point is earned: For connecting the explanation to the work done

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For selecting “Negative”

1 point is earned: For an appropriate explanation using kinematic or dynamic principles

Examples:

The velocity is decreasing, so the kinetic energy is also decreasing.

The acceleration is negative; therefore, the direction of the net force is opposite to the direction of the displacement.

1 point is earned: For connecting the explanation to the work done

Units

1 point is earned: For correct units on at least two parts with a calculated numerical answer and no incorrect units

0	1 ✓

The student response earns all of the following points:

1 point is earned: For correct units on at least two parts with a calculated numerical answer and no incorrect units

frq3

24.



The Sojourner rover vehicle shown in the sketch above was used to explore the surface of Mars as part of the Pathfinder mission in 1997. Use the data in the tables below to answer the questions that follow.

Mars Data	Sojourner Data
Radius: 0.53 x Earth's radius Mass: 0.11 x Earth's mass	Mass of Sojourner vehicle: 11.5 kg Wheel diameter: 0.13 m Stored energy available: 5.4 x 10^5J Power required for driving under average conditions: 10W Land speed: 6.7 x 10^{-3} m/s

1. Determine the acceleration due to gravity at the surface of Mars in terms of *g*, the acceleration due to gravity at the surface of Earth.
2. Calculate Sojournerís weight on the surface of Mars.
3. Assume that when leaving the Pathfinder spacecraft Sojourner rolls down a ramp inclined at 20° to the horizontal. The ramp must be lightweight but strong enough to support Sojourner. Calculate the minimum normal force that must be supplied by the ramp.
4. What is the net force on Sojourner as it travels across the Martian surface at constant velocity? Justify your answer.
5. Determine the maximum distance that Sojourner can travel on a horizontal Martian surface using its stored energy.
6. Suppose that 0.010% of the power for driving is expended against atmospheric drag as Sojourner travels on the Martian surface. Calculate the magnitude of the drag force.

Part A

1 point is earned: For a correct expression of Newton's Gravitation law

$$F = \frac{GM_1M_2}{R^2}$$

frq3

1 point is earned: For correct substitution/calculation in formula

$$gMars = \frac{GM_M}{R_M^2} = \frac{G(0.11)M_g}{(0.53R_g)^2} = \frac{0.11}{(0.53)^2}g_g$$

1 point is earned: For expressing *gM* in terms of *gg*

$$gM = \frac{0.11}{(0.53)^2}g_g$$

OR

$$gM = 0.39g_g$$

However, the third point for expressing *gM* in terms of *gg* is not awarded if no points have been awarded up to that part.

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned: For a correct expression of Newton's Gravitation law

$$F = \frac{GM_1M_2}{R^2}$$

1 point is earned: For correct substitution/calculation in formula

$$gMars = \frac{GM_M}{R_M^2} = \frac{G(0.11)M_g}{(0.53R_g)^2} = \frac{0.11}{(0.53)^2}g_g$$

1 point is earned: For expressing *gM* in terms of *gg*

$$gM = \frac{0.11}{(0.53)^2}g_g$$

OR

$$gM = 0.39g_g$$

However, the third point for expressing *gM* in terms of *gg* is not awarded if no points have been awarded up to that part.

Part B

1 point is earned: For the correct formula for weight: *W* = *Mg*

1 point is earned: For the correct value of *g* for Mars, where any value of *g* signifying Mars is acceptable, such as 0.39*gE*, 3.8 m/s2, *gMars*,

frq3

$$\frac{0.11}{(0.53)^2} g_g$$

, etc.

W = (11.5 kg)(3.8 m/s2) = 44 N

(Alternate solution)

2 points are earned: For correct calculation using

$$F = \frac{GM_1M_2}{R^2}$$

, where the student has calculated the mass of Mars, the radius of Mars and uses the correct mass of Sojourner.

(A student needs to have memorized the mass of the Earth and its radius to do this.)

0	1	2	3	4 ✓

The student response earns four of the following points:

1 point is earned: For the correct formula for weight: *W* = *Mg*

1 point is earned: For the correct value of *g* for Mars, where any value of *g* signifying Mars is acceptable, such as 0.39*gE*, 3.8 m/s2, *gMars*,

$$\frac{0.11}{(0.53)^2} g_g$$

, etc.

W = (11.5 kg)(3.8 m/s2) = 44 N

(Alternate solution)

2 points are earned: For correct calculation using

$$F = \frac{GM_1M_2}{R^2}$$

, where the student has calculated the mass of Mars, the radius of Mars and uses the correct mass of Sojourner.

(A student needs to have memorized the mass of the Earth and its radius to do this.)

Part C

1 point is earned: For indication that sine or cosine is needed as a necessary factor for computation of *N*

frq3

1 point is earned: For correct formula and calculation

$N = MgM \cos \theta$

N = (11.5 kg)(3.8 m/s2) cos 20° = 41 N

OR N = (11.5 kg)(3.8 m/s2) sin 70° = 41 N

0	1	2 ✓

The student response earns two of the following points:

1 point is earned: For indication that sine or cosine is needed as a necessary factor for computation of N

1 point is earned: For correct formula and calculation

$N = MgM \cos \theta$

N = (11.5 kg)(3.8 m/s2) cos 20° = 41 N

OR N = (11.5 kg)(3.8 m/s2) sin 70° = 41 N

Part D

1 point is earned: For correct statement that $Fnet = 0$

1 point is earned: For a correct reason such as $a = 0$, OR v = constant, OR Newton's First law

0	1	2 ✓

The student response earns two of the following points:

1 point is earned: For correct statement that $Fnet = 0$

1 point is earned: For a correct reason such as $a = 0$, OR v = constant, OR Newton's First law

Part E

1 point is earned: For a correct formula for power

$$P = \frac{W}{t}$$

1 point is earned: For a correct formula for distance

$d = vt$

frq3

1 point is earned: For a correct calculation of distance

$$d = \frac{vW}{P} = \frac{(6.7x10^{-3}m/s)(5.4x10^{5}J)}{10W}$$

d = 362 m

(Alternate solution)

1 point is earned: For correct formulas for work and power

W = *Fd* and *P* = *Fv*

1 point is earned: For combining these to yield a correct formula for the distance

$$d = \frac{vW}{P}$$

1 point is earned: For a correct calculation

$$d = \frac{(6.7x10^{-3}m/s)(5.4x10^{4}J)}{10W} = 362m$$

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned: For a correct formula for power

$$P = \frac{W}{t}$$

1 point is earned: For a correct formula for distance

d = *vt*

1 point is earned: For a correct calculation of distance

$$d = \frac{vW}{P} = \frac{(6.7x10^{-3}m/s)(5.4x10^{5}J)}{10W}$$

d = 362 m

(Alternate solution)

1 point is earned: For correct formulas for work and power

W = *Fd* and *P* = *Fv*

frq3

1 point is earned: For combining these to yield a correct formula for the distance

$$d = \frac{vW}{P}$$

1 point is earned: For a correct calculation

$$d = \frac{(6.7x10^{-3}m/s)(5.4x10^{4}J)}{10W} = 362m$$

Part F

1 point is earned: For correct formula *P* = *Fv*, so

1 point is earned: For correct substitution using the correct percentage of 0.01%

$$F = \frac{(0.0001)(10W)}{6.7x10^{-3}m/s}$$

1 point is earned: For correct calculation of force

F = 0.15 N (0.149 N and 0.1 N were acceptable)

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned: For correct formula *P* = *Fv*, so

1 point is earned: For correct substitution using the correct percentage of 0.01%

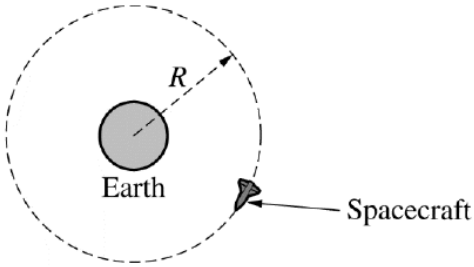
$$F = \frac{(0.0001)(10W)}{6.7x10^{-3}m/s}$$

1 point is earned: For correct calculation of force

F = 0.15 N (0.149 N and 0.1 N were acceptable)

frq3

25.

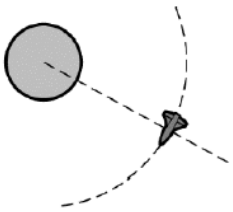


Note: Figure not drawn to scale.

A spacecraft of mass *m* is in a clockwise circular orbit of radius *R* around Earth, as shown in the figure above.

The mass of Earth is M_E.

a. In the figure below, draw and label the forces (not components) that act on the spacecraft. Each force must be represented by a distinct arrow starting on, and pointing away from, the spacecraft.



Note: Figure not drawn to scale.

b. i. Derive an equation for the orbital period *T* of the spacecraft in terms of *m, M_E, R*, and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

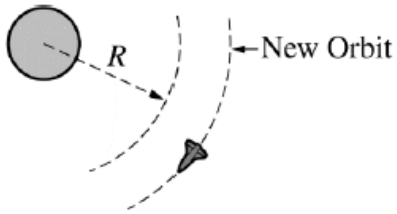
ii. A second spacecraft of mass *2m* is placed in a circular orbit with the same radius R. Is the orbital period of the second spacecraft greater than, less than, or equal to the orbital period of the first spacecraft?

____Greater than ____Less than ____Equal to

Briefly explain your reasoning.

c. The first spacecraft is moved into a new circular orbit that has a radius greater than *R*, as shown in the figure below.

frq3



Note: Figure not drawn to scale.

Is the speed of the spacecraft in the new orbit greater than, less than, or equal to the original speed?

____Greater than ____Less than ____Equal to

Briefly explain your reasoning.

Part A

1 point is earned: For an arrow directed toward Earth's center

1 point is earned: For a correct label on the arrow representing the gravitational force, where the arrow is pointing toward Earth's center

Note: A maximum of 1 point may be earned if extraneous forces are present.

0	1	2 ✓

Student response earns all of the following points

1 point is earned: For an arrow directed toward Earth's center

1 point is earned: For a correct label on the arrow representing the gravitational force, where the arrow is pointing toward Earth's center

Note: A maximum of 1 point may be earned if extraneous forces are present.

Part B

PART i

1 point is earned: For using (or implying) Newton's second law and equating the centripetal force to the gravitational force: $F_g = ma = \frac{mv^2}{R} \quad \frac{GmM_E}{R^2} = \frac{mv^2}{R}$

1 point is earned: For explicitly or implicitly determining that the speed of the spacecraft is: $v = \frac{2\pi R}{T}$

frq3

1 point is earned: For a correct answer algebraically equivalent to: $T = \sqrt{\frac{4\pi^2 R^3}{GM_E}}$

Note: It is acceptable to leave answer in terms of T2 : $T^2 = \frac{4\pi^2 R^3}{GM_E}$

PART ii

Correct answer: “Equal to”

Note: For an incorrect answer consistent with part (b)(i), the explanation is still graded

for consistency with part (b)(i).

1 point is earned: For a correct explanation that the period of the spacecraft does not depend on the

spacecraft mass (or only depends on the mass of Earth and the radius of the orbit)

OR an explanation consistent with the answer from (b)(i)

Note: The explanation must be consistent with the checked answer.

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

PART i

1 point is earned: For using (or implying) Newton’s second law and equating the centripetal force to the gravitational force:$F_g = ma = \frac{mv^2}{R} \quad \frac{GmM_E}{R^2} = \frac{mv^2}{R}$

1 point is earned: For explicitly or implicitly determining that the speed of the spacecraft is:$v = \frac{2\pi R}{T}$

1 point is earned: For a correct answer algebraically equivalent to: $T = \sqrt{\frac{4\pi^2 R^3}{GM_E}}$

Note: It is acceptable to leave answer in terms of T2 :$T^2 = \frac{4\pi^2 R^3}{GM_E}$

PART ii

Correct answer: “Equal to”

Note: For an incorrect answer consistent with part (b)(i), the explanation is still graded

for consistency with part (b)(i).

1 point is earned: For a correct explanation that the period of the spacecraft does not depend on the

spacecraft mass (or only depends on the mass of Earth and the radius of the orbit)

frq3

OR an explanation consistent with the answer from (b)(i)

Note: The explanation must be consistent with the checked answer.

Part C

Correct answer: “Less than”

Note: If the wrong selection is made, the explanation is not graded.

1 point is earned: For a correct explanation of why speed decreases with increasing orbital radius

Example:

Derivation step in (b)(i) shows that speed decreases with increasing R.

0	1 ✓

Student response earns 1 of the following 1 point(s)

Correct answer: “Less than”

Note: If the wrong selection is made, the explanation is not graded.

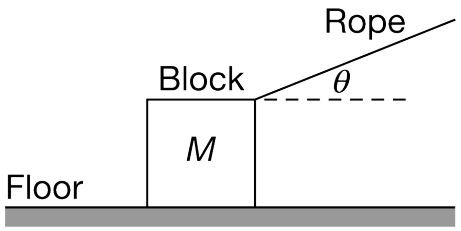
1 point is earned: For a correct explanation of why speed decreases with increasing orbital radius

Example:

Derivation step in (b)(i) shows that speed decreases with increasing R.

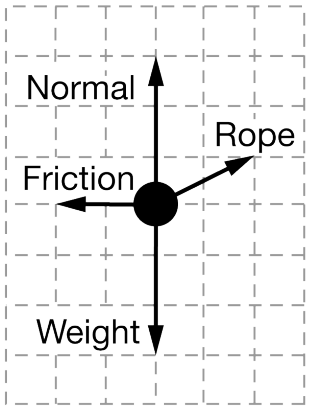
frq3

26. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

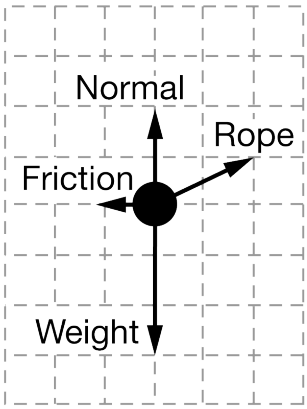


A block of mass M is pulled by a rope that makes an angle θ (theta) above the horizontal, as shown above. The coefficient of kinetic friction between the block and floor is μ (mu).

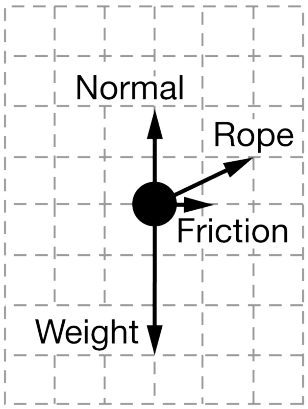
The block is initially at rest and starts accelerating to the right along the floor. Three students each draw a diagram to indicate all of the forces that are exerted on the block as it moves. In the students' diagrams, which are shown below, the lengths of the arrows indicate the relative magnitudes of the forces.



Student 1's Diagram



Student 2's Diagram



Student 3's Diagram

Only one of the students has drawn a diagram that could correctly represent the forces. The other two students' diagrams are incorrect.

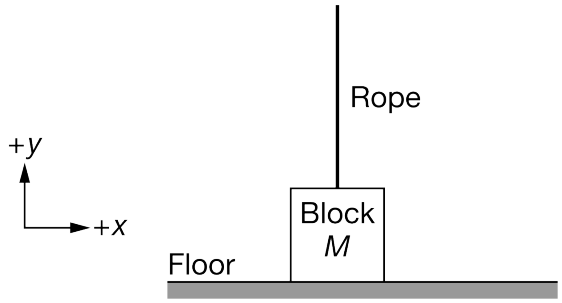
(a) Which student has drawn a diagram that could correctly represent the forces exerted on the block?

(b) For the incorrect student diagrams, explain why the diagram could not represent the forces exerted on the block. If there is more than one specific reason a diagram is incorrect, identify each of the reasons.

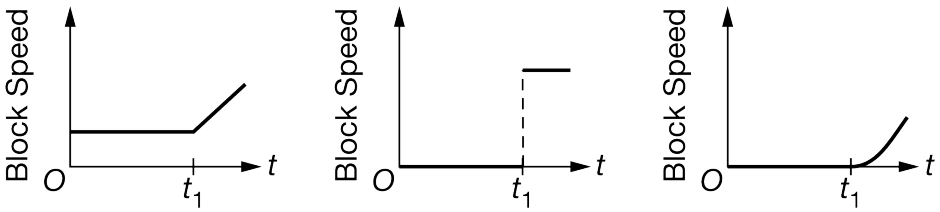
(c) Briefly explain why slightly increasing the force F exerted by the rope on the block (without changing the angle θ) would cause a decrease in the friction force.

(d) The friction force f is $f = \mu(W - F\sin\theta)$, where W is the weight of the block and F is the force exerted by the rope. Explain why this equation is consistent with the claim that an increase in F would cause a decrease in the friction force.

frq3



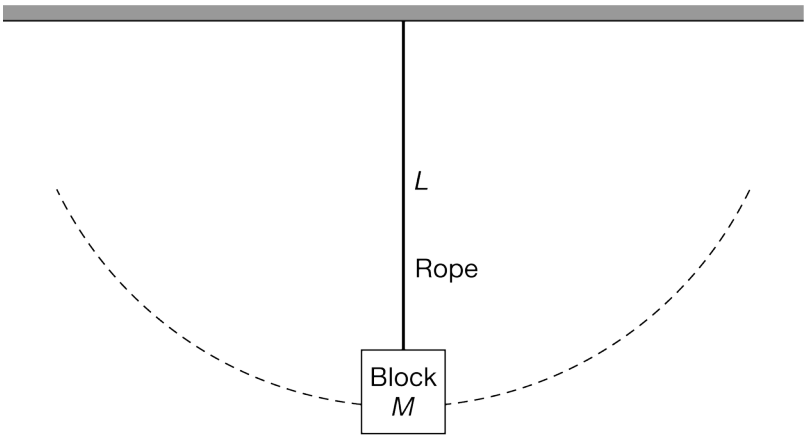
The rope is now attached to the center of the top of the block, as shown above. The force exerted by the rope is initially zero newtons at time $t = 0$. The force steadily increases, becomes equal to the block's weight W at time t_1, and continues to increase further. Three students sketch the graphs shown below to represent the speed of the block as a function of time t. One of the graphs is consistent with this scenario, while the other graphs are not consistent with this scenario.



Graph 1 Graph 2 Graph 3

(e) Which graph is consistent with the scenario in which the force exerted by the rope on the block steadily increases?

(f) Identify all features of the other two graphs that make them inconsistent with this scenario. Explain your reasoning.



Note: Figure not drawn to scale.

frq3

With one end of the rope still attached to the block, the other end of the rope is attached to a ceiling to create a pendulum with length L, where L is much greater than the dimensions of the block. A second identical pendulum is located on the Moon, where the acceleration due to gravity is less than that on Earth. Both pendulums are displaced by the same small angle and released from rest.

(g) Does the pendulum on Earth or the pendulum on the Moon have a greater period of oscillation? Briefly explain your choice without reference to equations, but by describing the restoring forces exerted on and motion of each block.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating that Student 2 drew a correct diagram.

Example Response:

The diagram for student 2 is correct.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that Student 1's diagram has equal left and right forces and thus would result in a constant horizontal velocity or no horizontal acceleration.
- ☐ 1 point is earned for indicating that Student 1's diagram would result in an upward acceleration component since the sum of the upward components is greater than the force of gravity.
- ☐ 1 point is earned for indicating that Student 3's diagram has friction in the incorrect direction.

Example Response:

Student 3 has correctly balanced the vertical forces but has the friction in the same direction as the motion. Student 1 has vertical components in the upward direction that are greater than gravity, so the block would have an upward acceleration. Also, student 1 has equal horizontal components which would not cause an acceleration to the right.

frq3

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that as the pulling force increases, its vertical component increases, decreasing the normal force on the block.
- ☐ 1 point is earned for indicating that since the friction force is proportional to the normal force, the friction force decreases.

Example Response:

As the pulling force increases, its vertical component increases, decreasing the normal force on the block. Since the friction force is proportional to the normal force, the friction force decreases.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for any attempt to use a functional dependence between the friction force and the force *F*.
- ☐ 1 point is earned for indicating that the term with F is subtracted, or that the expression $(W - F\sin\theta)$ decreases when *F* increases.

Example Response:

In the given expression for friction, there is a term that depends on the pulling force F, but that term is subtracted from the weight term. So as F increases, friction decreases.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for identifying Graph 3 as consistent with the scenario in which the force exerted by the rope on the block steadily increases.

Example Response:

Graph 3 is consistent with the scenario in which the force exerted by the rope on the block steadily increases.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that graph 1 shows an initial non-zero speed even though the initial speed is zero.
  Note: If it was indicated that there is ONLY a horizontal component of the velocity before the vertical force of the rope acts on the block (from the previous part of the question), this point can be earned for thinking that graph 1 is correct in the first segment but graphs 2 & 3 are not correct in the first segment.
- ☐ 1 point is earned for indicating that the speed after time in graph 1 is incorrect because the upward force is not constant, so the acceleration (slope of speed graph) should not be constant.
- ☐ 1 point is earned for indicating that t_1 graph 2 is incorrect because of the discontinuity in the graph at t_1.
- ☐ 1 point is earned for indicating that graph 2 is incorrect because the speed is not constant after time t_1, it increases since there is a net force.

Example Response:

Graph 1 is incorrect because the speed should be zero before time t_1 and after t_1 it should have an increasing slope since the net force is increasing. Graph 2 is incorrect because, although the speed is initially zero, the speed is not constant after time t_1 because there is an upward net force.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: A response *can* mention the equation for a pendulum, and still get full credit as long as the points described below are earned.

frq3

0	1	2 ✓

The response accurately includes **both** of the following criteria.

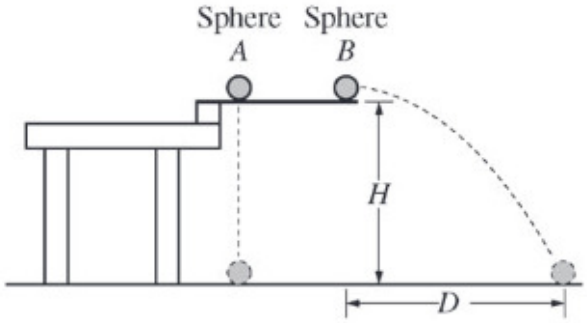
- ☐ 1 point is earned for indicating that the pendulum on the Earth has a greater restoring force at maximum displacement from equilibrium.
- ☐ 1 point is earned for indicating that since the two pendulums have the same length, the amplitude of the motion is the same, and thus the pendulum on the moon, with a smaller acceleration, will take longer to complete once cycle.

Example Response:

Since both pendulums have the same length, they will both have the same amplitude. But the pendulum on the Earth will have a greater restoring force and acceleration, so it will take less time to return to equilibrium. Thus, the pendulum on the moon will have a greater period.

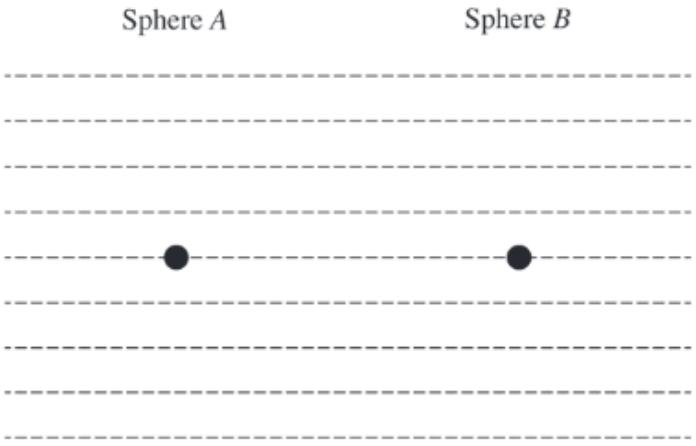
frq3

27.



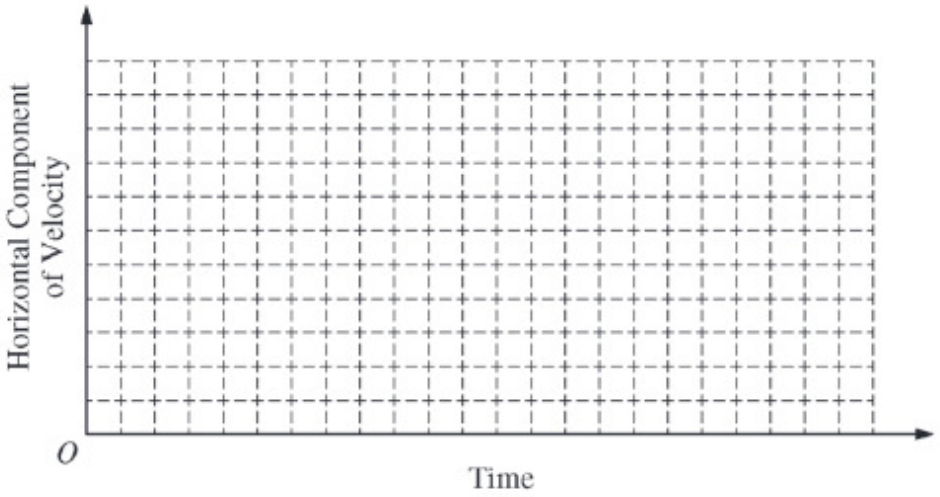
Two identical spheres are released from a device at time t = 0 from the same height H, as shown above. Sphere A has no initial velocity and falls straight down. Sphere B is given an initial horizontal velocity of magnitude v_0 and travels a horizontal distance D before it reaches the ground. The spheres reach the ground at the same time t_f, even though sphere B has more distance to cover before landing. Air resistance is negligible.

(a) The dots below represent spheres A and B. Draw a free-body diagram showing and labeling the forces (not components) exerted on each sphere at time $\frac{t_f}{2}$.



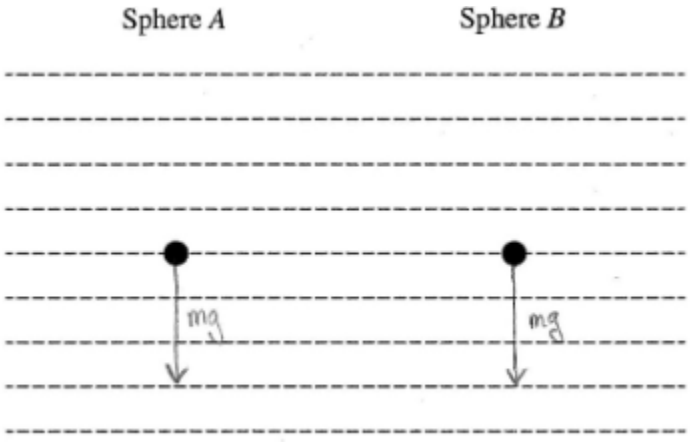
(b) On the axes below, sketch and label a graph of the horizontal component of the velocity of sphere A and of sphere B as a function of time.

frq3



(c) In a clear, coherent, paragraph-length response, explain why the spheres reach the ground at the same time even though they travel different distances. Include references to your answers to parts (a) and (b).

Part A

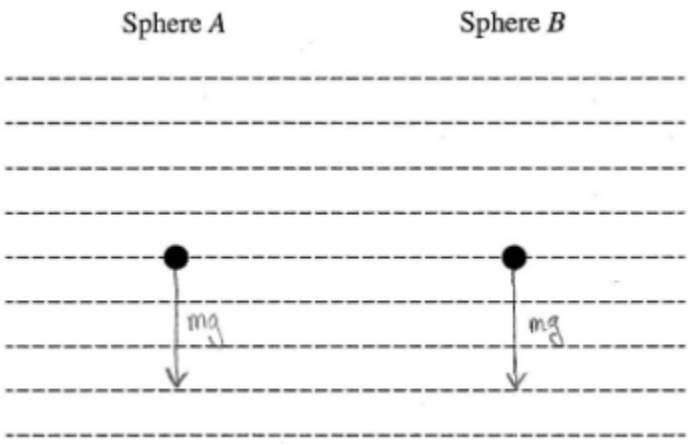


1 point is earned: For sketching only one force pointing straight down from each sphere and indicating that this force represents the force of gravity

0	1 ✓

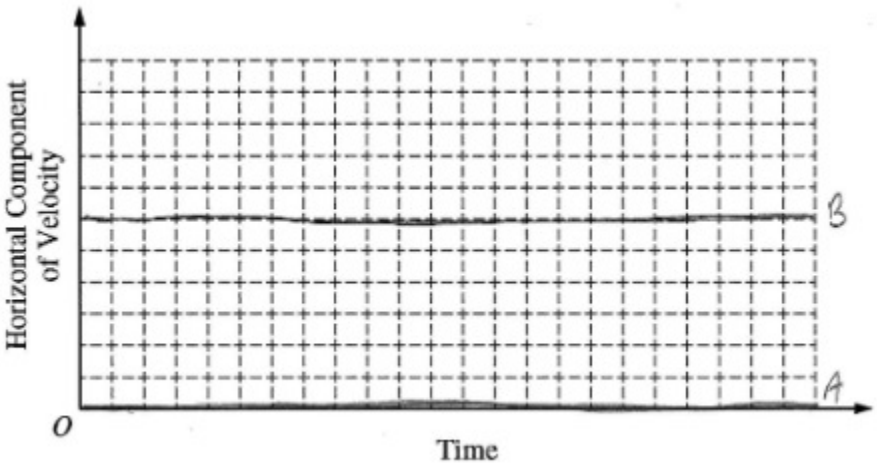
The student response earns all of the following points:

frq3



1 point is earned: For sketching only one force pointing straight down from each sphere and indicating that this force represents the force of gravity

Part B

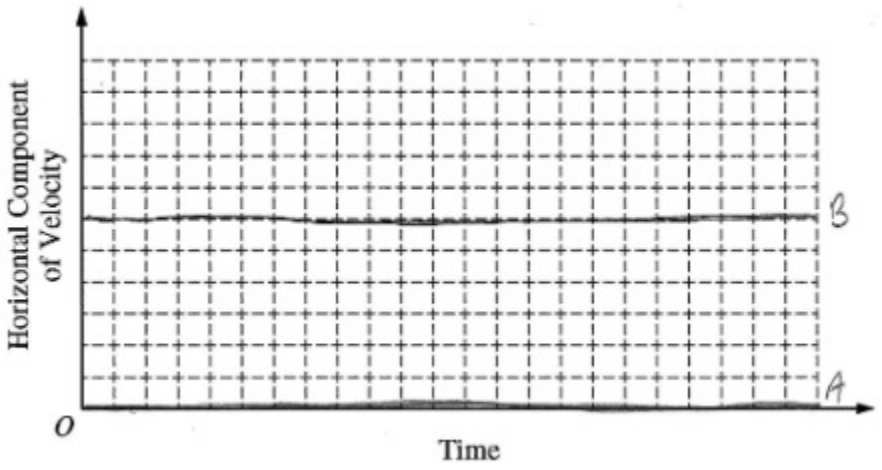


1 point is earned: For sketching a horizontal line at zero velocity for sphere *A*, and sketching a horizontal line at some non-zero velocity for sphere ***B***

0	1 ✓

The student response earns all of the following points:

frq3



1 point is earned: For sketching a horizontal line at zero velocity for sphere *A*, and sketching a horizontal line at some non-zero velocity for sphere ***B***

Part C

1 point is earned: For indicating that the difference in horizontal motion does not affect the vertical motion of the spheres

1 point is earned: For indicating that both spheres start with the same vertical velocity

1 point is earned: For indicating that both spheres have the same vertical acceleration

1 point is earned: For indicating that falling the same height would take the same time

1 point is earned: For no incorrect or irrelevant statements

0	1	2	3	4	5 ✓

The student response earns all of the following points:

1 point is earned: For indicating that the difference in horizontal motion does not affect the vertical motion of the spheres

1 point is earned: For indicating that both spheres start with the same vertical velocity

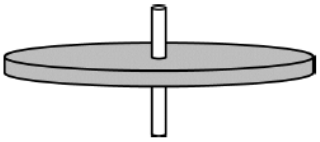
1 point is earned: For indicating that both spheres have the same vertical acceleration

1 point is earned: For indicating that falling the same height would take the same time

1 point is earned: For no incorrect or irrelevant statements

frq3

28.

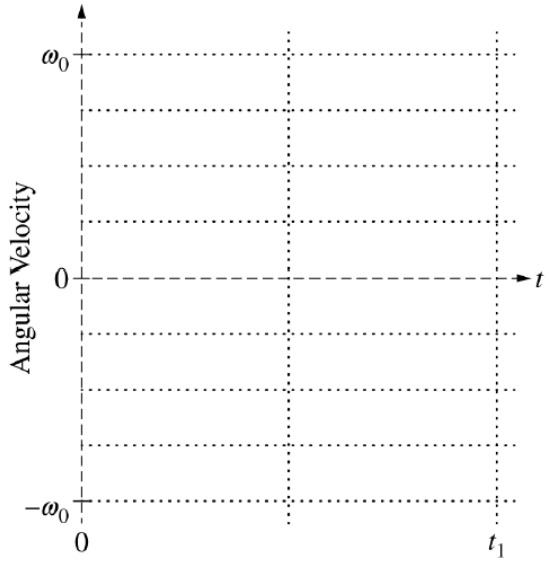


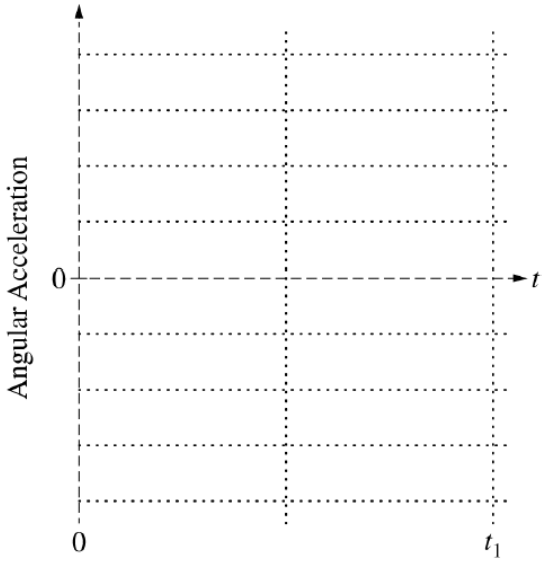
The disk shown above spins about the axle at its center. A student's experiments reveal that, while the disk is spinning, friction between the axle and the disk exerts a constant torque on the disk.

a. At time $t = 0$ the disk has an initial counterclockwise (positive) angular velocity ω_0.The disk later comes to rest at time $t = t_1$.

   i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time t from $t = 0$ until the disk comes to rest at time $t = t_1$.

   ii. On the grid at right below, sketch the disk's angular acceleration as a function of time t from $t = 0$ until the disk comes to rest at time $t = t_1$.





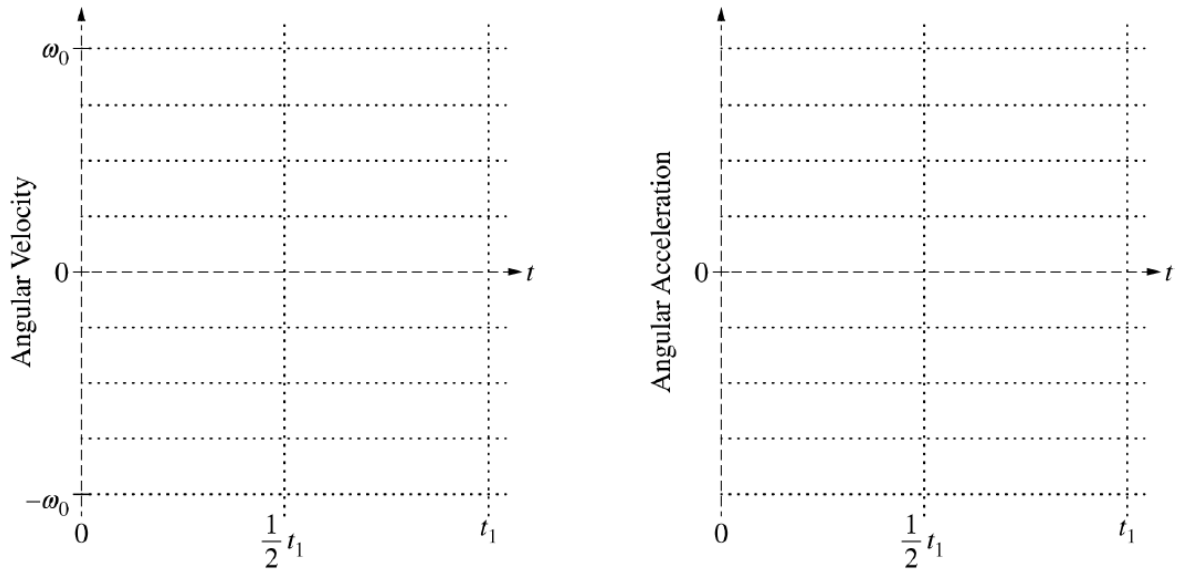
b. The magnitude of the frictional torque exerted on the disk is τ_0. Derive an equation for the rotational inertia I of the disk in terms of τ_0, ω_0, t_1, and physical constants, as appropriate.

c. In another experiment, the disk again has an initial positive angular velocity ω_0 at time $t = 0$. At time $t = \frac{1}{2}t_1$, the student starts dripping oil on the contact surface between the axle and the disk to reduce the friction. As time passes, more and more oil reaches that contact surface, reducing the friction even further.

frq3

i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time from $t = 0$ to $t = t_1$, which is the time at which the disk came to rest in part (a).

ii. On the grid at right below, sketch the disk's angular acceleration as a function of time from $t = 0$ to $t = t_1$.



d. The student is trying to mathematically model the magnitude τ of the torque exerted by the axle on the disk when the oil is present at times $t > \frac{1}{2}t_1$. The student writes down the following two equations, each of which includes a positive constant (C_1 or C_2) with appropriate units.

$$(1)\ \tau = C_1\left(t - \tfrac{1}{2}t_1\right) \text{ (for } t > \tfrac{1}{2}t_1\text{)}$$
$$(2)\ \tau = \frac{C_2}{(t+\frac{1}{2}t_1)} \text{ (for } t > \tfrac{1}{2}t_1\text{)}$$

Which equation better mathematically models this experiment?

____Equation (1) ____Equation (2)

Briefly explain why the equation you selected is plausible and why the other equation is not plausible.

Part A

PART i

1 point is earned: For a curve that has an angular velocity of +ω0 at time t = 0 and decreases to zero at time t = t1

1 point is earned: For a curve that is a straight line with a negative slope showing the angular velocity approaching zero (can be a positive slope, if the initial angular velocity on the graph is negative)

frq3

PART ii

1 point is earned: For a curve that is negative for the entire time

1 point is earned: For a curve that is a constant nonzero function

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

PART i

1 point is earned: For a curve that has an angular velocity of +ω0 at time t = 0 and decreases to zero at time t = t1

1 point is earned: For a curve that is a straight line with a negative slope showing the angular velocity approaching zero (can be a positive slope, if the initial angular velocity on the graph is negative)

PART ii

1 point is earned: For a curve that is negative for the entire time

1 point is earned: For a curve that is a constant nonzero function

Part B

1 point is earned: For using an equation for the rotational version of Newton's second law

1 point is earned: For using an appropriate rotational kinematics equation α = Δω/Δt

1 point is earned: For a correct answer in terms of the listed quantities, derived from first principles $I = \frac{\tau_0 t_1}{\omega_0}$

Note: This point is still earned if there is a minus sign

Alternate solution using angular momentum and rotational impulse:

1 point is earned: For defining and using angular momentum L = iω

1 point is earned: For using rotational impulse ΔL= τΔt

1 point is earned: For a correct answer in terms of the listed quantities, derived from first principles $I = \frac{\tau_0 t_1}{\omega_0}$

Note: This point is still earned if there is a minus sign

0	1	2	3 ✓

frq3

Student response earns 0 of the following 3 point(s)

1 point is earned: For using an equation for the rotational version of Newton's second law

1 point is earned: For using an appropriate rotational kinematics equation $\alpha = \Delta\omega/\Delta t$

1 point is earned: For a correct answer in terms of the listed quantities, derived from first principles $I = \frac{\tau_0 t_1}{\omega_0}$

Note: This point is still earned if there is a minus sign

Alternate solution using angular momentum and rotational impulse:

1 point is earned: For defining and using angular momentum $L = i\omega$

1 point is earned: For using rotational impulse $\Delta L = \tau\Delta t$

1 point is earned: For a correct answer in terms of the listed quantities, derived from first principles $I = \frac{\tau_0 t_1}{\omega_0}$

Note: This point is still earned if there is a minus sign

Part C

PART i

1 point is earned: For curve with a clear change of slope or curvature at ½t1 and showing a decrease in speed thereafter

1 point is earned: For a curve that indicates slowing at a decreasing rate between times ½ t1 and t1

1 point is earned: For a curve that does not reach zero at or before time t1

PART ii

1 point is earned: For a curve with decreasing magnitude between times ½ t1 and t1

Note: The acceleration may reach zero at or before time t1. If so, it must remain zero for

the remaining time.

0	1	2	3	4	5 ✓

Student response earns 5 of the following 5 point(s)

PART i

1 point is earned: For curve with a clear change of slope or curvature at ½t1 and showing a decrease in speed thereafter

1 point is earned: For a curve that indicates slowing at a decreasing rate between times ½ t1 and t1

1 point is earned: For a curve that does not reach zero at or before time t1

frq3

PART ii

1 point is earned: For a curve with decreasing magnitude between times ½ t1 and t1

Note: The acceleration may reach zero at or before time t1. If so, it must remain zero for

the remaining time.

Part D

Correct answer: “Equation (2)”

Note: If the wrong selection is made, the explanation is not graded.

1 point is earned: For stating that Equation (2) is plausible because the frictional torque decreases with increasing time, whereas in Equation (1) torque increases with increasing time

Examples:

Equation (2) because ◈ decreases. In Equation (1), it doesn’t.

Equation (2) is plausible because the frictional torque decreases as more oil reaches the contact surface over time. Equation (1) is not plausible because it shows friction increasing as more oil reaches the surface over time.

0	1 ✓

Student response earns 1 of the following 1 point(s)

Correct answer: “Equation (2)”

Note: If the wrong selection is made, the explanation is not graded.

1 point is earned: For stating that Equation (2) is plausible because the frictional torque decreases with increasing time, whereas in Equation (1) torque increases with increasing time

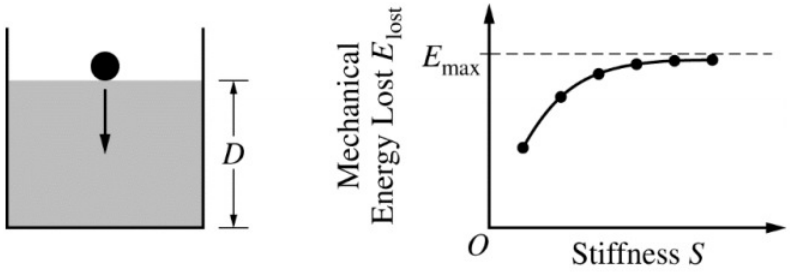
Examples:

Equation (2) because ◈ decreases. In Equation (1), it doesn’t.

Equation (2) is plausible because the frictional torque decreases as more oil reaches the contact surface over time. Equation (1) is not plausible because it shows friction increasing as more oil reaches the surface over time.

frq3

29.



Food scientists have created a new oil. At room temperature, the oil is a liquid. As the oil gets colder, however, it stiffens (thickens) into a sticky gel. To explore the properties of the oil, the scientists fill a container with the oil to a height D, as shown in the figure above on the left. They drop a small steel ball of mass M from rest at the top of the oil. Using video to capture the ball's motion, the scientists calculate E_{lost} , the mechanical energy lost by the ball-Earth system from the time the ball enters the oil to the time just before the ball strikes the bottom of the container. The scientists also define the "stiffness" S of the oil as a quantity proportional to the force required to move a rod through the oil at a standard constant speed. The scientists calculate E_{lost} and S at several different temperatures, ranging from room temperature to the lowest temperature at which the ball still falls through the oil. The graph above on the right shows E_{lost} as a function of S for the calculated data points and a best-fit curve.

a. Give a physical reason why the curve in the graph would not reach the vertical (E_{lost}) axis even if the scientists had taken data over a broader range of temperatures.

b. As S increases, E_{lost} approaches a maximum value labeled E_{max} on the graph above. Write an equation for E_{max} in terms of *M, D*, and physical constants, as appropriate.

   Justify your answer.

c. One of the scientists, in trying to represent the relationship between the oil stiffness and the mechanical energy lost, writes down the equation $2\ E_{lost} = CS^2$, where *C* is a constant with appropriate units. Another scientist points out that this equation cannot be correct. Give two reasons why the equation cannot be correct.

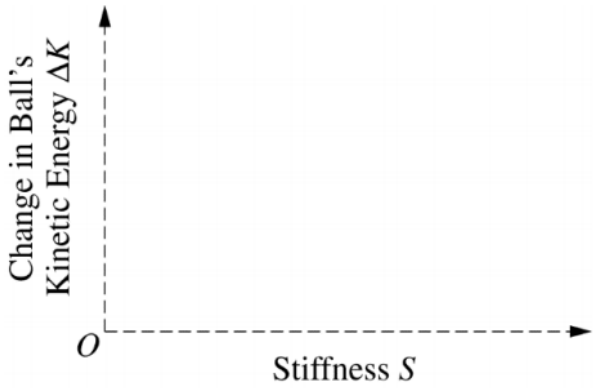
d. Further attempting to model the ball's motion, the scientists write the following equation for the time t the ball takes to fall through the oil: $t = Z/S$, where Z is a constant with appropriate units. Is this equation plausible—in other words, does it make physical sense?

   ____ Plausible ____ Not plausible

   Briefly explain your reasoning.

e. ΔK is the change in kinetic energy of the ball between the time it is released from rest and the time just before the ball strikes the bottom of the container. On the axes below, sketch ΔK as a function of S, the oil stiffness.

frq3



Part A

1 point is earned: For indicating (explicitly or implicitly) that intersecting the vertical axis corresponds to zero stiffness

1 point is earned: For correctly indicating that no measurement of stiffness could be zero

Example: The vertical axis is where S = 0 , indicating a substance puts up no resistance to getting stirred. But even at room temperature the oil has some stiffness. Hence, the lowest measured stiffness is positive, not zero, and the graph reflects this.

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

1 point is earned: For indicating (explicitly or implicitly) that intersecting the vertical axis corresponds to zero stiffness

1 point is earned: For correctly indicating that no measurement of stiffness could be zero

Example: The vertical axis is where S = 0 , indicating a substance puts up no resistance to getting stirred. But even at room temperature the oil has some stiffness. Hence, the lowest measured stiffness is positive, not zero, and the graph reflects this.

Part B

1 point is earned: For a correct equation: E_{max} = MgD OR E_{max} = -MgD

1 point is earned: For stating that the maximum energy that can be lost is equal to the initial energy of the system

1 point is earned: For stating that the initial energy of the system is the initial gravitational potential energy

Example: The maximum energy that can be lost is equal to the initial energy, which is all gravitational potential energy: MgD

Alternate solution for the last 2 points

frq3

1 point is earned: For a statement or the application of mechanical energy as the sum of kinetic and potential energies

1 point is earned: For combining the change in the gravitational potential energy with a small change in kinetic energy as the ball falls to make a valid argument for E_{max}

> Example: In a very stiff oil that the ball can barely fall through, it gains essentially no kinetic energy. In this case the overall change in mechanical energy, $\Delta K + \Delta U$, is approximately equal to ΔU because ΔK is essentially zero.
>
> So, $E_{max} = |\Delta U| = MgD$.

0	1	2	3 ✓

Student response earns 3 of the following 3 point(s)

1 point is earned: For a correct equation: $E_{max} = MgD$ OR $E_{max} = -MgD$

1 point is earned: For stating that the maximum energy that can be lost is equal to the initial energy of the system

1 point is earned: For stating that the initial energy of the system is the initial gravitational potential energy

Example: The maximum energy that can be lost is equal to the initial energy, which is all gravitational potential energy: MgD

Alternate solution for the last 2 points

1 point is earned: For a statement or the application of mechanical energy as the sum of kinetic and potential energies

1 point is earned: For combining the change in the gravitational potential energy with a small change in kinetic energy as the ball falls to make a valid argument for E_{max}

> Example: In a very stiff oil that the ball can barely fall through, it gains essentially no kinetic energy. In this case the overall change in mechanical energy, $\Delta K + \Delta U$, is approximately equal to ΔU because ΔK is essentially zero.
>
> So, $E_{max} = |\Delta U| = MgD$.

Part C

2 points are earned: For each of the following, up to a maximum of 2 points

1 point for each valid, independent statement that the equation does not match the graph (maximum of 2 points).

1 point for a statement that demonstrates that E_{lost} cannot increase indefinitely.

> Example statements (1 point each, any two could be combined for 2 points): The equation indicates that E_{lost} increases indefinitely with S, but the graph has an upper limit for E_{lost}. The equation represents a parabola (or is concave-up), which does not match the graph. It doesn't make physical sense that E_{lost} could be more than the available potential energy MgD.

frq3

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

2 points are earned: For each of the following, up to a maximum of 2 points

1 point for each valid, independent statement that the equation does not match the graph (maximum of 2 points).

1 point for a statement that demonstrates that E_{lost} cannot increase indefinitely.

> Example statements (1 point each, any two could be combined for 2 points): The equation indicates that E_{lost} increases indefinitely with S, but the graph has an upper limit for E_{lost}. The equation represents a parabola (or is concave-up), which does not match the graph. It doesn't make physical sense that E_{lost} could be more than the available potential energy MgD.

Part D

Correct answer: "Not plausible"

Note: The explanation can earn the first point if the incorrect selection is made.

1 point is earned: For a statement or implication that the equation indicates that t increases as S decreases

1 point is earned: For a statement or implication that t should be larger as S increases

Note: If only one of the above statements is written explicitly, and "Not plausible" is selected, the second point is earned by implication.

> Example 1 (2 points): "Not plausible". According to the equation, a less stiff oil makes the sphere take more time to reach the bottom.
>
> Example 2 (2 points): "Not plausible". A greater stiffness means that the ball should take a longer time to fall through the oil.
>
> Example 3 (1 point): "Plausible". The equation says that bigger S means bigger time, which agrees with the idea that a thicker oil resists the ball's motion more.

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

Correct answer: "Not plausible"

Note: The explanation can earn the first point if the incorrect selection is made.

frq3

1 point is earned: For a statement or implication that the equation indicates that t increases as S decreases

1 point is earned: For a statement or implication that t should be larger as S increases

Note: If only one of the above statements is written explicitly, and "Not plausible" is selected, the second point is earned by implication.

> Example 1 (2 points): "Not plausible". According to the equation, a less stiff oil makes the sphere take more time to reach the bottom.
>
> Example 2 (2 points): "Not plausible". A greater stiffness means that the ball should take a longer time to fall through the oil.
>
> Example 3 (1 point): "Plausible". The equation says that bigger S means bigger time, which agrees with the idea that a thicker oil resists the ball's motion more.

Part E

1 point is earned: For drawing a concave-upward curve that approaches the horizontal at higher S

1 point is earned: For having the maximum value of ΔK at the lowest graphed value of S, which can be S = 0 or a positive value

Note: This point is not earned if the graph indicates that ΔK approaches infinity as S approaches zero.

1 point is earned: For high values of S, the graph approaches and does not cross the horizontal axis

Note: This point is earned if the graph touches the horizontal axis without crossing it.

0	1	2	3 ✓

Student response earns 3 of the following 3 point(s)

1 point is earned: For drawing a concave-upward curve that approaches the horizontal at higher S

1 point is earned: For having the maximum value of ΔK at the lowest graphed value of S, which can be S = 0 or a positive value

Note: This point is not earned if the graph indicates that ΔK approaches infinity as S approaches zero.

1 point is earned: For high values of S, the graph approaches and does not cross the horizontal axis

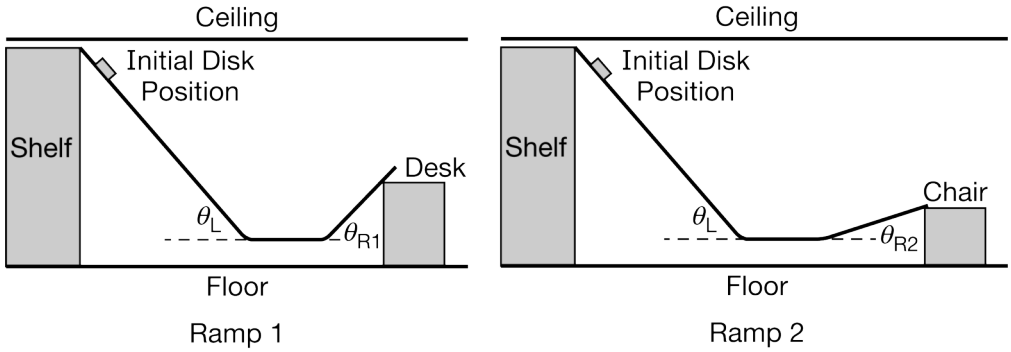
Note: This point is earned if the graph touches the horizontal axis without crossing it.

frq3

30. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)

A class of physics students is investigating projectile motion using a variety of equipment. Assume air resistance can be ignored in all cases.

One group constructs a long ramp as shown in Ramp 1 below. In their experiment, they release a disk from the left side of the ramp and allow the disk to launch off the right side. The ramp is smooth enough that friction can be ignored. The left side of the ramp is supported by the top of a shelf, and the other side is supported by the top of a desk. The left and right sides of the ramp are at angles θ_{L} and θ_{R1} with the horizontal respectively.

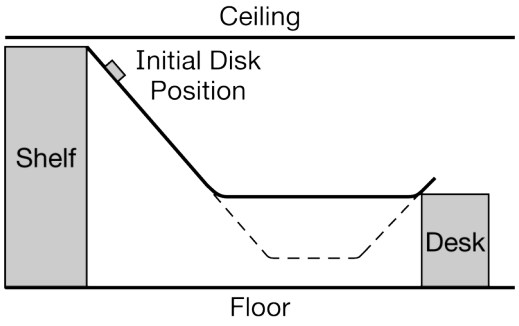


Note: Figures not drawn to scale.

The students release a disk and measure the maximum height it reaches after leaving the right side of the ramp. The students then adjust the ramp so that $\theta_{\mathrm{R2}} < \theta_{\mathrm{R1}}$ and the height of the right side of the ramp is lower as shown in the Ramp 2 configuration above. Assume the length of the right section of the ramp is the same in both cases.

(a) Will the disk launched from Ramp 2 reach a maximum height that is greater than, less than, or equal to the maximum height reached by the disk launched from Ramp 1? Justify your answer.

frq3

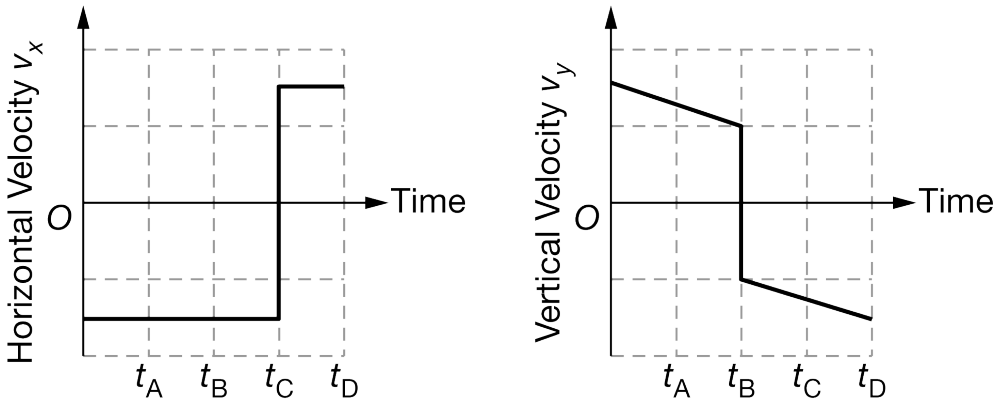


Ramp 3

Note: Figure not drawn to scale.

Finally, the students adjust the ramp into the configuration labeled Ramp 3 above in which the horizontal section of the ramp is at a greater height than in Ramp 1, which is represented by the dotted line. The students release the disk on Ramp 3.

(b) On which ramp, Ramp 1 or Ramp 3, if either, does the disk spend more time on the horizontal section of the ramp before launching off the right side? Explain your answer.



One group uses a device that can launch a steel ball. The group launches the ball and it bounces off the ceiling, floor, and wall several times though not necessarily in that order. Each collision can be modeled as elastic, and friction between the ball and the surfaces is negligible. The graphs above show the horizontal and vertical components of the velocity of the ball during some time interval in the ball's motion. The positive direction is taken to be upward.

(c) During the time interval shown in the graphs, the ball has two collisions. In a clear, coherent, paragraph-length response, identify the times when these collisions occur using the times indicated on the graph, and explain how you know whether each collision is with the wall, the ceiling, or the floor.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the disk has a greater speed as it leaves the right end of ramp 2 because more potential energy was converted to kinetic energy.
- ☐ 1 point is earned for stating that disk on ramp 2 will not go as high because it will have less potential energy (or disk on ramp 1 has more potential energy) due to conservation of mechanical energy
  OR
  For stating that disk on ramp 2 will not go as high because disk on ramp 2 has a lower initial vertical velocity (or disk on ramp 1 has a higher initial vertical velocity).

Example Response:

The disk has a greater speed when leaving the right end of ramp 2. A greater speed and smaller launch angle means that the horizontal velocity component at launch is greater for the disk on ramp 2. So the disk launched form ramp 2 will have a greater kinetic energy at the apex, and therefore less potential energy (in the disk-Earth system) and a lower height.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the speed of the disk on the flat section is less for ramp 3 since less potential energy has been converted to kinetic energy.
- ☐ 1 point is earned for indicating the flat section of the ramp is longer in ramp 3, so there is a greater distance to travel.
- ☐ 1 point is earned for concluding that since the speed on the flat section of ramp 3 is less and the distance is greater, the time must be greater as well.

Example Response:

For ramp 3, the change in potential energy of the disk-Earth system is less, so the disk has less kinetic energy and thus less speed on the flat section. Since the flat section of ramp 3 is also longer, and the disk is moving more slowly, the disk will spend more time on the flat section of ramp 3 than on the flat section of ramp 1.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

0	1	2	3	4	5 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for correctly identifying the collision with the ceiling at time t_B.
- ☐ 1 point is earned for indicating that a collision with the ceiling will exert a force on the ball, accelerating it
  OR
  For saying that the force changes the vertical velocity of the ball
  OR
  For saying that the direction of the vertical velocity changes.
- ☐ 1 point is earned for correctly identifying the collision with a wall at time t_C.
- ☐ 1 point is earned for indicating that a collision with the wall will exert a force on the ball, accelerating it
  OR
  For saying that the force changes the horizontal velocity of the ball
  OR
  For saying that the direction of the horizontal velocity changes.
- ☐ 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

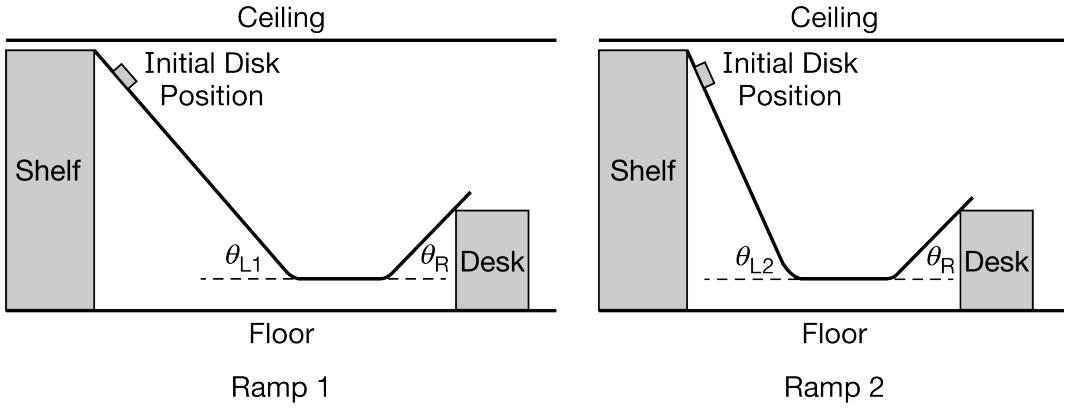
Each surface exerts a normal force; thus, the ceiling and floor exert vertical forces and the walls exert horizontal forces. When a net force is applied, there is an acceleration and a change in velocity. Thus, when there is a horizontal force exerted by the wall there is a change in the horizontal velocity, which happens at time t_C, so, the ball collides with the wall at t_C. When the vertical velocity changes from positive to negative at time t_B, there is a downward force, which only happens when the ball collides with the ceiling. So, the ball collides with the ceiling at t_B.

frq3

31. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)

A class of physics students is investigating projectile motion using a variety of equipment. Assume air resistance can be ignored in all cases.

One group constructs a long ramp as shown in Ramp 1 below. In their experiment, they release a disk from the left side of the ramp and allow the disk to launch off the right side. The ramp is smooth enough that friction can be ignored. The left side of the ramp is supported by the top of a shelf, and the other side is supported by the top of a desk. The left and right sides of the ramp are at angles θ_{L1} and θ_{R} with the horizontal respectively.

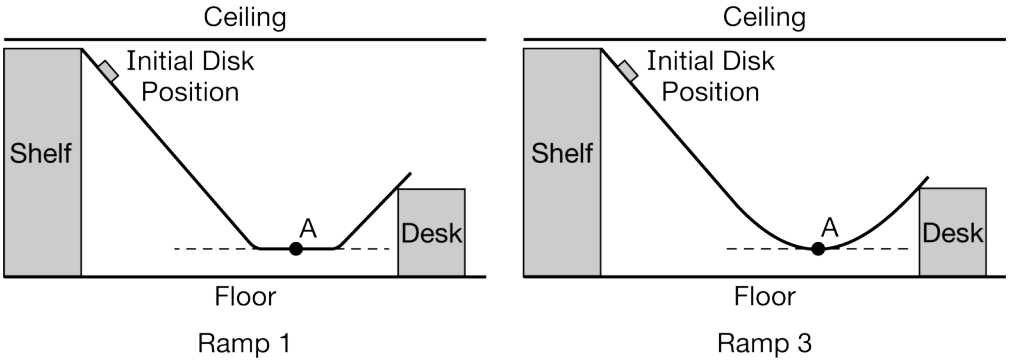


Note: Figures not drawn to scale.

The students release a disk from Ramp 1 and measure the maximum height it reaches after leaving the right side of the ramp. The students then increase the angle of the left ramp so that $\theta_{\mathrm{L1}} < \theta_{\mathrm{L2}}$, and they keep the top of the ramp against the shelf as shown in the Ramp 2 configuration above so that the disk is released at the same height.

(a) Will the disk launched from Ramp 2 reach a maximum height that is greater than, less than, or equal to the maximum height reached by the disk launched from Ramp 1? Justify your answer.

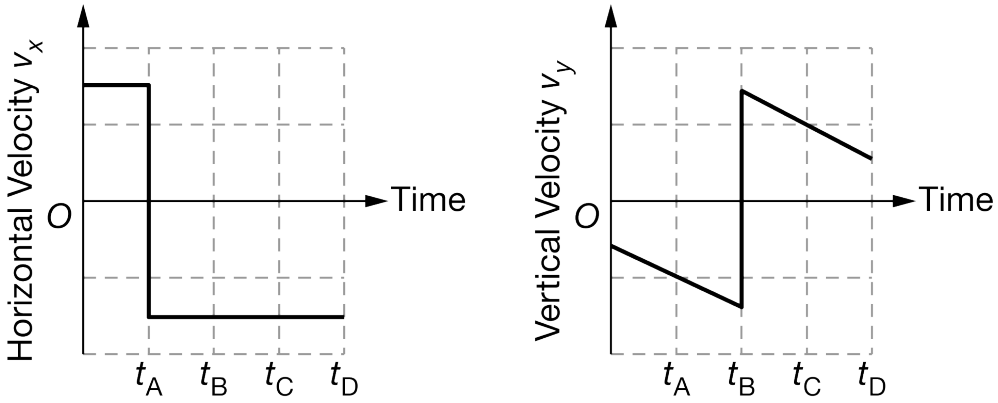
frq3



Note: Figures not drawn to scale.

Next, the students adjust the ramp into the configuration labeled Ramp 3 in which the lowest part of the ramp has been shaped into a circular curve. The lowest point of Ramp 3 is at the same height as the lowest section of Ramp 1. Both ramps are shown above.

(b) On which ramp, Ramp 1 or Ramp 3, is the force exerted by the ramp on the disk at point A greater? Explain your answer.



Another group uses a device that can launch a steel ball. The group launches the ball and it bounces off the ceiling, floor, and wall several times though not necessarily in that order. Each collision can be modeled as elastic, and friction between the ball and the surfaces is negligible. The graphs above show the horizontal and vertical components of the velocity of the ball during some time interval in the ball's motion. The positive direction is taken to be upward.

(c) During the time interval shown in the graphs, the ball has two collisions. In a clear, coherent, paragraph-length response, identify the times when each of these collisions occur using the times indicated on the graph, and explain how you know whether each collision is with the wall, the ceiling, or the floor.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the disk has the same speed (or kinetic energy) as it leaves the right end of both ramps because the energy of the disk-Earth system is conserved.
- ☐ 1 point is earned for indicating that the vertical component of the velocity at the right end is the same, since both are launched at the same angle, so the disk reaches the same maximum height for both ramps.

Example Response:

The disk has the same change in height as it travels from the left end to the right end of the ramp, so the same amount of potential energy is converted to kinetic energy. Thus, the disk leaves the right end of both ramps at the same speed. Since it also leaves at the same angle, it will travel the same projectile path and have the same maximum height in both cases.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that at point A on ramp 1, the block is moving at a constant velocity, so the net force is zero. Thus, the force by ramp 1 is equal to the disk's weight.
- ☐ 1 point is earned for indicating that at point A on ramp 3, the disk is moving in a circular path and thus has an upward net force.
- ☐ 1 point is earned for concluding that at point A on ramp 3, the normal force must be greater than the disk's weight, and thus greater than it was on ramp 1.

Example Response:

The disk on ramp 1 is traveling at constant velocity on the flat section. Thus, the normal force is equal to the weight of the disk. The disk on ramp 3 is executing circular motion, with an upward acceleration. Thus, the net force must be upward, and thus the normal force on ramp 3 is greater than the disk's weight. Thus, the normal force exerted on the block is greater for ramp 3.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

0	1	2	3	4	5 ✓

The response accurately includes **all** of the following criteria.

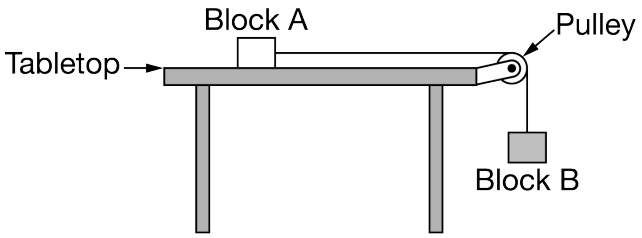
- ☐ 1 point is earned for correctly identifying the collision with a wall at time t_A.
- ☐ 1 point is earned for indicating that a collision with the wall will exert a force on the ball, accelerating it
  OR
  For indicating that the force changes the horizontal velocity of the ball
  OR
  For indicating that there is a change in the direction of the horizontal velocity.
- ☐ 1 point is earned for correctly identifying the collision with the floor at time t_B.
- ☐ 1 point is earned for For indicating that a collision with the floor will exert a force on the ball, accelerating it
  OR
  For indicating that the force changes the vertical velocity of the ball
  OR
  For indicating that there is a change in the direction of the vertical velocity.
- ☐ 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

Each surface exerts a normal force; thus, the ceiling and floor exert vertical forces and the walls exert horizontal forces. When a net force is applied, there is an acceleration and a change in velocity. Thus, when there is a horizontal force by the wall there is a change in the horizontal velocity, which happens at time t_A, so the ball collides with the wall at t_A. When the vertical velocity changes from negative to positive at time t_B, there is an upward force, which only happens when the ball collides with the floor. So, the ball collides with the floor at t_B.

frq3

32. This question is a long free-response question. Show your work for each part of the question.



(12 points, suggested time 25 minutes)

This problem explores how the relative masses of two blocks affect the acceleration of the blocks. Block A, of mass m_A, rests on a horizontal tabletop. There is negligible friction between block A and the tabletop. Block B, of mass m_B, hangs from a light string that runs over a pulley and attaches to block A, as shown above. The pulley has negligible mass and spins with negligible friction about its axle. The blocks are released from rest.

(a)

i. Suppose the mass of block A is much greater than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

ii. Now suppose the mass of block A is much less than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

(b) Now suppose neither block's mass is much greater than the other, but that they are not necessarily equal. The dots below represent block A and block B, as indicated by the labels. On each dot, draw and label the forces (not components) exerted on that block after release. Represent each force by a distinct arrow starting on, and pointing away from, the dot.



(c) Derive an equation for the acceleration of the blocks after release in terms of m_A, m_B, and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

(d) Consider the scenario from part (a)(ii), where the mass of block A is much less than the mass of block B. Does your equation for the acceleration of the blocks from part (c) agree with your reasoning in part (a)(ii)?

frq3

_____ Yes _____ No

Briefly explain your reasoning by addressing why, according to your equation, the acceleration becomes (or approaches) a certain value when m_A is much less than m_B.

(e) While the blocks are accelerating, the tension in the vertical portion of the string is T_1. Next, the pulley of negligible mass is replaced with a second pulley whose mass is not negligible. When the blocks are accelerating in this scenario, the tension in the vertical portion of the string is T_2. How do the two tensions compare to each other?

_____ $T_2 > T_1$ _____ $T_2 = T_1$ _____ $T_2 < T_1$

Briefly explain your reasoning.

Part A(i)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answers: "Zero", "small", "negligible", "much less than "g", or "$<< g$".

0	1	2 ✓

The response accurately includes **both** of the following criteria:

- ☐ For a correct answer and attempt at a consistent justification.
- ☐ For correct reasoning.

Example 1 Point Response:

Nearly zero. Because block A is much heavier than block B .

Example 2 Point Responses:

Very small. Because block A has a large inertia, it won't speed up much.

Close to zero because block B is so light that it can hardly budge block A .

Claim: The acceleration of the blocks is zero/small/negligible/ "$<< g$" .

Evidence: The mass of block A is much greater than the mass of block B .

Reasoning Statements: Very small. Because block A has a large inertia, it won't speed up much or close to zero because block B is so light that it can hardly budge block A .

Part A(ii)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq3

Correct Answer: g , $9.8\ \mathrm{m/s^2}$, or $10\ \mathrm{m/s^2}$ (or just 9.8 or 10)

0	1 ✓

The response does not accurately include:

- ☐ An indication that the acceleration is nearly equal to g and a correct justification.

Example Response:

Nearly equal to g . Because block B is almost in free fall.

$10\ \mathrm{m/s^2}$, because block A has negligible mass and the tension in the string is nearly zero.

Claim: The acceleration of the blocks is close to g .

Evidence: The mass of block A is much less than the mass of block B .There is negligible friction between block A and the tabletop. The pulley has negligible mass and spins with negligible friction about its axle.

Reasoning Statements: Nearly equal to g . Because block B is almost in free fall or $10\ \mathrm{m/s^2}$, because block A has negligible mass and the tension in the string is nearly zero.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: N , F_{N} , "normal force," F_{Table} , "table force," or any other label can be used to indicate the force is "normal" or comes from the table.

Note: F_{g} , F_{grav} , W , mg , $m_A g$, "gravity," "grav force," may be used.

Note: G or g are not acceptable.

Note: "tension," "string force," F_T, F_{tension}, F_{string}, F_S, T, or some other label indicating that the force comes from the string or from tension may be used.

Note: $m_B g$, F_{mB}, "force from block B " or other indications that the force is "created" by block B are NOT acceptable.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

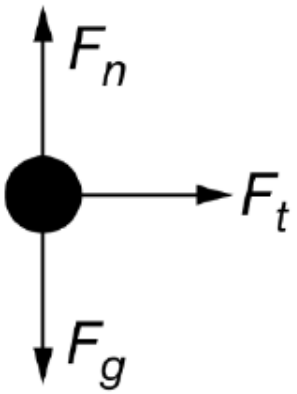
- ☐ A correct normal force on block A with acceptable label.
- ☐ Correct gravitational forces with acceptable label on both diagrams, and no extraneous forces on either

frq3

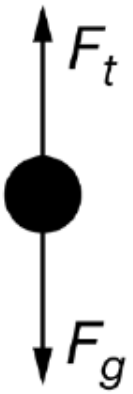
diagram.

- [ ] Correct tension forces with acceptable label on both diagrams.

Example Response:



Block A



Block B

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Writing a "whole system" equation is also sufficient to earn the first two points.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- [ ] For using separate Newton's second law equations for each block.
- [ ] For combining the equations with correct notation, including correctly using m_A and m_B , indicating that the same tension force acts on both blocks, *and* that they share the same acceleration.
- [ ] For a correct equation for a with supporting work.

Example Response:

frq3

$$\sum F_B = m_B a$$
$$m_B g - F_T = m_B a$$
$$\sum F_A = m_A a$$
$$F_T = m_A a$$
$$m_B g - m_A a = m_B a$$
$$m_B g = (m_B + m_A) a$$
$$a = \frac{m_B g}{(m_B + m_A)}$$

Part C - Alternate Solution

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: *Writing the correct whole-system equation is sufficient to earn the first two points of part (c).*

0	1	2	3 ✓

*The response accurately includes **three** of the following criteria.*

- ☐ *For writing a "whole-system" equation for the total mass that does not contain internal forces.*
- ☐ *For substituting the net force and system mass with correct quantities.*
- ☐ *For a correct equation for a with supporting work.*

Example Response:

$F_{\text{net}} = m_{\text{total}} a$

$m_B g = (m_A + m_B) a$

$a = \frac{m_B}{m_A + m_B} g$

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answer: "Yes"

frq3

Note: “No” is acceptable if the equation is inconsistent with the answer in (a)(ii).

0	1 ✓

The response accurately includes

- ☐ A valid reasoning that addresses the result in part (c) and the reasoning in part (a)(ii).

Example Response:

Claim:
Yes, the equation for the acceleration of the blocks from part (c) agrees with the reasoning in part (a)(ii).

OR
No, the equation for the acceleration of the blocks from part (c) does not agree with the reasoning in part (a)(ii).

Evidence:

The mass of block A is much less than the mass of block B .
$a = \frac{m_B}{m_A+m_B} g$ (derived as part (c) answer)

Reasoning Statements for “Yes” claim:
When m_A is much less than m_B , it can be neglected in the equation derived in part (c), giving an acceleration close to g as stated in (a)(ii).

Reasoning Statements for “No” claim, if the answer in part (a)(ii) is wrong:
When m_A is much less than m_B , it can be neglected in the equation derived in part (c), giving an acceleration close to g . This disagrees with the value of ___ stated in (a)(ii).

Reasoning Statements for “No” claim, if the answer in part (c) is wrong:

When m_A is much less than m_B , it can be neglected in the equation derived in part (c), giving an acceleration of ___. This disagrees with the value of g stated in (a)(ii).

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answer: $T_2 > T_1$.

Note: A maximum of 1 point can be earned for part (e) if an incorrect selection is made.

0	1	2 ✓

The response accurately includes **both** of the following criteria:

frq3

- For reasoning that the acceleration of both blocks is smaller.
- For doing any one of the following, consistent with the answer selection and Newton's second law for block B .
  - Concluding that a smaller acceleration implies that T_2 is greater than T_1
  - Concluding that an unchanged acceleration implies that T_2 is the same as T_1
  - Concluding that a larger acceleration implies that T_2 is less than T_1

Example Response:

Claim: $T_2 > T_1$

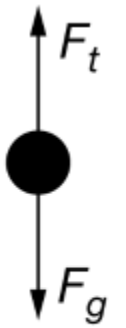
Evidence:

The pulleys spin with negligible friction about the axle.

The original pulley has negligible mass.

The second pulley's mass is not negligible.

$$\vec{a} = \frac{\sum \vec{F}}{m}$$



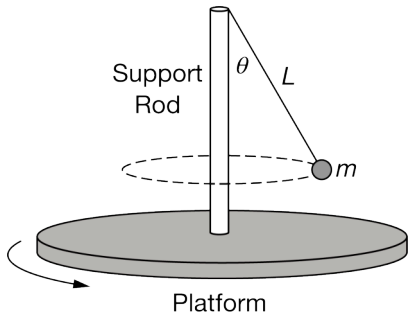
Block B

Reasoning Statements:

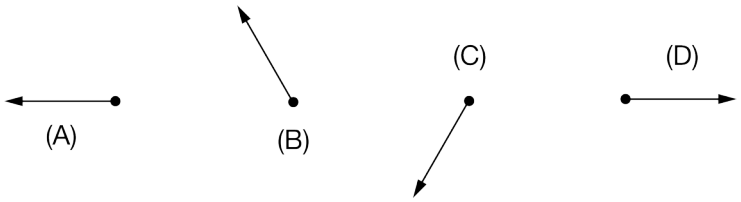
The rotational inertia of the second pulley results in a smaller acceleration for the blocks. Block B must have a smaller net force to have a smaller acceleration, so the rope tension must be larger than before (closer in magnitude to the gravitational force on block B).

frq3

33. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A vertical support rod is fixed at the center of a platform. A light string of length L is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass m, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed ω (omega), the string makes an angle θ (theta) with the rod.



(a) Identify which of the above vectors represents the net force exerted on the sphere at the moment shown above. Briefly justify your answer.

(b) Is the magnitude of the tension in the string greater than, less than, or equal to the weight of the sphere? Explain your reasoning.

(c) The string can never be completely horizontal no matter how fast the platform is rotating. Without writing any equations or formulas, explain why.

One of the following two equations for the square of the tangential speed v of the sphere as a function of θ is consistent with the statement in part (c) that the string can never be horizontal as the platform is rotating.

Equation 1: $v^2 = \frac{gL(\sin^2\theta)}{(\cos\theta)}$ **Equation 2:** $v^2 = \frac{gL(\cos^2\theta)}{(\sin\theta)}$

(d) Indicate which equation is consistent with the statement in part (c), and explain why. Also explain why the other equation is not consistent with the statement in part (c).

(e) In a series of experimental trials, a group of students changes L and adjusts v to keep θ constant. They record L and v for each trial. Based on the equation in part (d) that you indicated as consistent with the statement in part (c), what quantities could the students graph to yield a straight line whose slope could be used to find the acceleration due to gravity, g ?

frq3

(f) Let I be the rotational inertia of the sphere-rod-platform system about a vertical axis through the center of the support rod. Does I increase, decrease, or stay the same when L increases (but θ stays constant)? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answer: (A), with a correct justification

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement identifying the correct net force vector (A)
- ☐ 1 point is earned for a justification that includes that the net force or acceleration will point toward the center of the circular path

Example Responses:

Figure A because the net force is toward the center of the sphere's circular motion.

OR

The forces acting on the sphere are the force of gravity and the force of tension. The force of gravity is directly toward the ground, and the force of tension is at an angle toward the top of the rod. When the forces are added together, the vertical component of the force of tension and the force of gravity cancel each other out, leaving only the horizontal component of the force of tension, which is demonstrated by vector A.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- ☐ 1 point is earned for a statement that the tension in the string is greater than the weight of the sphere
- ☐ 1 point is earned for an explanation referring to the components of the vectors
- ☐ 1 point is earned for a correct justification that addresses the sum of the vector components

Example Responses:

frq3

The tension is greater than the weight because the vertical component of the tension force must be equal to the weight. The tension also has a horizontal component, so the magnitude will be greater than either of the components.

OR

Explaining that

$\Sigma F_y = F_{Ty} - mg = 0$

$F_T \cos\theta = mg$

$F_T = mg/\cos\theta$

(Since $\cos\theta < 1$*), therefore* $F_T > mg$ *(or > "weight")*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement identifying forces and/or force components in the vertical direction
- ☐ 1 point is earned for a justification that includes that the weight of the sphere must be balanced by the string and so the vertical component of the tension can never be zero

Example Response:

The weight of the sphere is downward and will need to be balanced by the vertical component of the tension. So, there will always be a vertical component of tension.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Notes:

· If students switch sine and cosine (such that they use Equation 2) but then are completely consistent, they can earn 2 of 3 points in part (d).

· If students misinterpret theta as being zero when the string is horizontal, they can earn two of three points in part (d) for using Equation 2 consistently.

0	1	2	3 ✓

frq3

The response accurately includes **three** of the following criteria.

- [ ] 1 point is earned for a statement addressing functional dependence (regardless of the equation chosen)
- [ ] 1 point is earned for indicating that equation 1 shows that as theta goes to 90 degrees, *v* gets very large (infinite/undefined)
- [ ] 1 point is earned for correctly indicating a flaw in equation 2

Example Response:

Equation 1 is consistent with part (c) because as theta gets closer to 90 degrees, the speed gets very large. This shows that no matter how large *v* gets, theta will never get to 90 degrees. Equation 2 is not consistent because it has a finite value for theta equal to 90 degrees.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- [ ] 1 point is earned for a statement identifying any *v* term on one axis
- [ ] 1 point is earned for a statement identifying any *L* term on the other axis
- [ ] 1 point is earned for a statement identifying correct power relationships for *v* and *L* (i.e., *v* squared vs. *L*)

Example Response:

They could graph *v* squared on the *y*-axis as a function of *L* on the *x*-axis.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for a statement identifying that the rotational inertia will increase
- [ ] 1 point is earned for a statement that since the mass is further from the axis, the rotational inertia is bigger

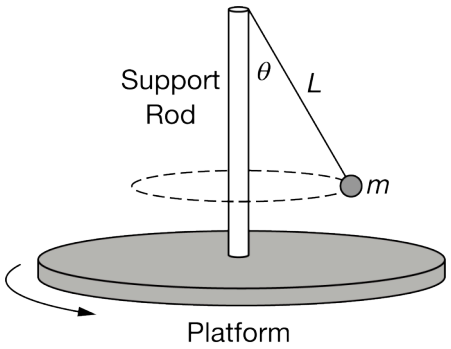
Example Response:

frq3

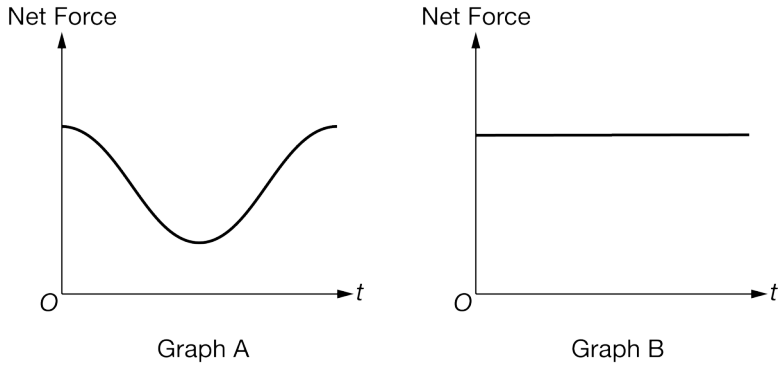
I will increase because for a larger *L*, the distance of the mass from the axis will be larger, and rotational inertia increases with distance.

frq3

34. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A vertical support rod is fixed at the center of a platform. A light string of length L is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass m, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed ω (omega), the string makes an angle θ (theta) with the rod.



(a) The platform rotates with a constant angular speed. Which of the above graphs correctly shows the magnitude of the net force exerted on the sphere as a function of time t ? Briefly justify your answer.

(b) Is the magnitude of the tension in the string greater than, less than, or equal to the centripetal force exerted on the sphere? Explain your reasoning.

(c) The string is nearly vertical when the sphere's tangential speed is very small. Without writing any equations or formulas, explain why.

(d) A student derives an equation for the square of the tangential speed v of the sphere as a function of θ : $v^2 = gL(\tan\theta)(\sin\theta)$. Indicate whether the equation is consistent with the statement in part (c) and explain why.

(e) In a series of experimental trials, a group of students adjusts v and measures θ while keeping all other variables constant. They record v and θ for each trial. If the students use the equation given in part (d), what quantities could they graph to yield a straight line whose slope could be used to find the acceleration due to gravity, g ?

frq3

(f) Let I be the rotational inertia of the sphere-rod-platform system about a vertical axis through the center of the support rod. Does I increase, decrease, or stay the same when θ decreases? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement identifying graph B as correctly describing the magnitude of the net force on the sphere as a function of time.
- ☐ 1 point is earned for a justification that the net force is constant because the speed is constant.

Example Response:

Graph B is correct because the magnitude of the net force on an object in uniform circular motion is constant.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that the magnitude of the tension is greater than the centripetal force exerted on the sphere.
- ☐ 1 point is earned for an explanation referring to the components of the vectors.
- ☐ 1 point is earned for a correct justification that addresses the sum of the vector components.

Example Response:

The tension is greater than the centripetal force because the horizontal component of the tension force must be equal to the centripetal force. The tension also has a vertical component, so the magnitude will be greater than either of the components.

OR

Explaining that

frq3

$\sum F_x = F_{Tx} = mv^2/r$ *where* mv^2/r *is the "centripetal force."*

$F_T \sin\theta = mv^2/r$

$F_T = mv^2/(r\sin\theta)$

(Since $\sin\theta < 1$*), therefore* $F_T > mv^2/r$ *(or >* *"centripetal force")*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that when the speed is very small the centripetal force is very small.
- ☐ 1 point is earned for a justification that includes that if the horizontal force is very small the angle of the tension with the vertical is very small.

Example Response:

A small speed means a small centripetal acceleration. The horizontal component of the tension is equal to the centripetal force, so if it is very small, then the tension will be almost vertical, and the angle will be very small.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that the equation is consistent with the reasoning in part (c).
- ☐ 1 point is earned for addressing the functional dependence.
- ☐ 1 point is earned for a correct functional dependence argument.

Example Response:

It is consistent because as theta goes to zero, both trigonometric terms go to zero, and therefore the speed goes to zero. This matches the statement that the string is nearly vertical when the speed is small.

Part E

frq3

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for identifying any *v* term on one axis.
- ☐ 1 point is earned for identifying any theta term on the other axis.
- ☐ 1 point is earned for a correct power relationship between *v* and trig functions of theta (for example, *v* squared vs. $(\tan\theta)(\sin\theta)$.

Example Response:

They could graph v squared as a function of tan theta sin theta.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

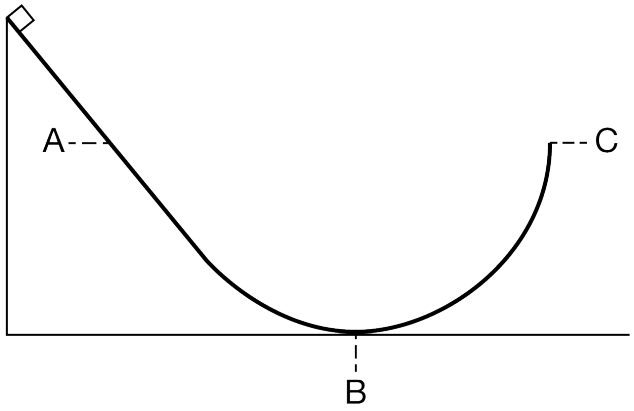
- ☐ 1 point is earned for a statement that the rotational inertia of the sphere-rod-platform-system decreases.
- ☐ 1 point is earned for a justification that includes that the rotational inertia decreases with decreasing distance of the mass from the axis.

Example Response:

I decreases because as theta decreases, the mass gets closer to the axis of rotation and rotational inertia increases with distance from the axis.

frq3

35. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)



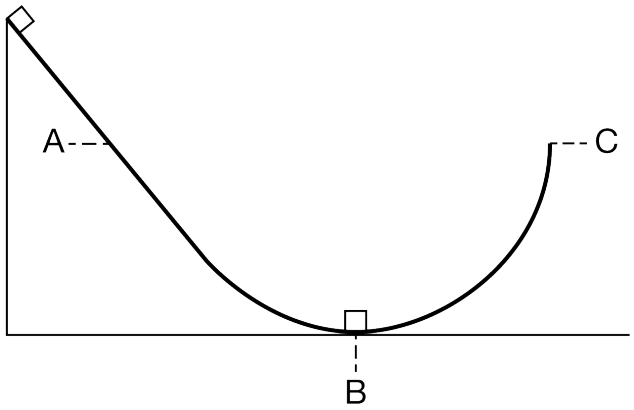
The figure above shows a track consisting of a straight section that connects to a circular section. Three locations are indicated: location A on the straight section of the track, location B at the bottommost point of the track, and location C at the point where the track is vertical. Locations A and C are at the same height. A block is released from rest at the top of the straight section of the track. There is negligible friction between the track and the block.

(a) Is the magnitude of the normal force exerted on the block by the track at location A greater than, less than, or equal to the normal force at location B? Justify your answer.

(b) In a coherent, paragraph-length response, describe the relative angle between the direction of the block's velocity and the direction of the block's acceleration at the following times.

When the block is at location A
When the block is at location B
Immediately before the block reaches location C

Specifically, indicate whether the velocity is parallel to the acceleration, perpendicular to the acceleration, or neither, at each location. Use physical principles to justify your responses.



frq3

(c) The scenario is repeated, but an identical block is at rest at location B, as shown above. When the block at the top of the straight section of the track is released from rest, it slides down the track and collides elastically with the identical block and comes to rest. The identical block then begins sliding up the ramp. Is the maximum height reached by the identical block greater than, less than, or equal to the maximum height reached by the block in the original scenario? Briefly explain your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the normal force at A is less than at B.
- ☐ 1 point is earned for a correct explanation including both locations A and B (e.g., at A the normal force is less than the weight, but at B it is greater than the weight).

Example Response:

At A, the block's acceleration is along the straight track, so the normal force must cancel the component of the gravitational force normal to the track and is therefore less in magnitude than the block's weight. At B, the block's acceleration and net force are upward, so the upward normal force must be greater than the block's weight.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the velocity is always parallel to the track.
- ☐ 1 point is earned for relating the acceleration to the net force.
- ☐ 1 point is earned for indicating the acceleration at location A is along the track and thus parallel to velocity.
- ☐ 1 point is earned for indicating that at location B the velocity is perpendicular to the acceleration.
- ☐ 1 point is earned for indicating that just before location C the acceleration is neither perpendicular nor parallel to the normal force.
- ☐ 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the

frq3

paragraph-length response.

Example Response:

The velocity of the box is along the track.

At location A, the box is accelerating down the ramp because there is a net force exerted on the box down the ramp. The net force and acceleration always have the same direction. If the net force is down the track then the acceleration is down the track. So that makes the velocity and acceleration in the same direction at location A, therefore parallel.

At location B, the bottommost point, the box is moving in a circle. Here the velocity is still along the track. Since location B is the bottommost point of a circle, the net force and acceleration would be directed toward the center of that circle. So that makes the velocity and acceleration perpendicular at location B.

Just before location C, the box is both moving in a circle and slowing down as it rises above the ground. The velocity is in the direction of motion, which is upward. Since the box is slowing down, the acceleration must have a downward component. The acceleration would also have a component toward the center of the block's circular path. So, the acceleration is neither parallel nor perpendicular to the upward velocity at just before location C.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the maximum height reached by the second block is the same as that of the original block.
- ☐ 1 point is earned for a correct explanation of why the second block reaches the same height.

Example Response:

Since the collision is elastic and the first block comes to rest, the second block has the same kinetic energy that the first block had just before the collision, which is the same as the first block's kinetic energy at the bottom of the ramp in the original scenario.