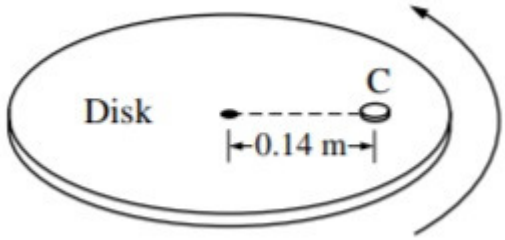
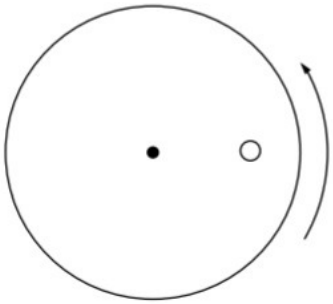
frq4

1.



A coin C of mass 0.0050 kg is placed on a horizontal disk at a distance of 0.14 m from the center, as shown above. The disk rotates at a constant rate in a counterclockwise direction as seen from above. The coin does not slip, and the time it takes for the coin to make a complete revolution is 1.5 s.

a. The figure below shows the disk and coin as viewed from above. Draw and label vectors on the figure below to show the instantaneous acceleration and linear velocity vectors for the coin when it is at the position shown.



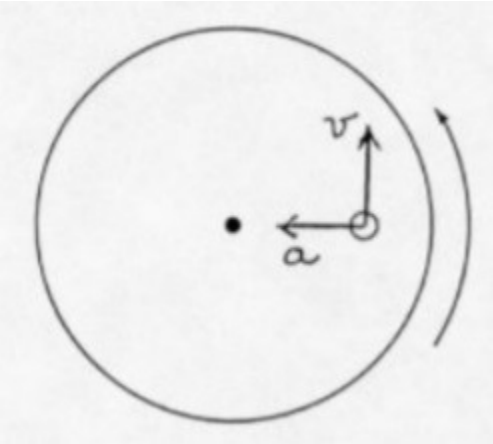
b. Determine the linear speed of the coin.

c. The rate of rotation of the disk is gradually increased. The coefficient of static friction between the coin and the disk is 0.50. Determine the linear speed of the coin when it just begins to slip.

d. If the experiment in part (c) were repeated with a second, identical coin glued to the top of the first coin, how would this affect the answer to part (c) ? Explain your reasoning.

Part A

frq4

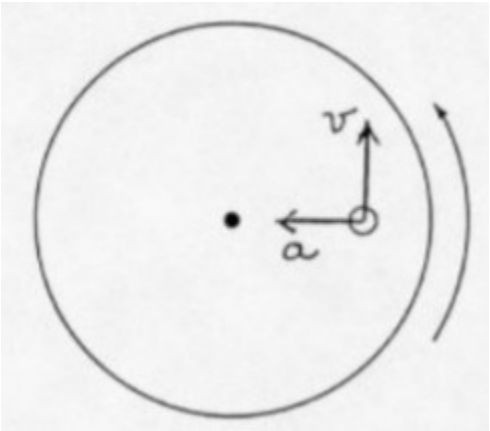


1 point is earned: For any properly labeled, vertical vector pointing upward

1 point is earned: For any properly labeled, horizontal vector pointing to the left

0	1	2 ✓

The student response earns two of the following points:



1 point is earned: For any properly labeled, vertical vector pointing upward

1 point is earned: For any properly labeled, horizontal vector pointing to the left

Part B

$$v = \frac{\text{circumference}}{\text{period}} = \frac{2\pi R}{T} = \frac{2\pi(0.14\text{ m})}{1.5\text{ s}} = 0.60\text{ m/s}$$

1 point is earned: For indicating use of circumference for distance traveled in one revolution, such as $\frac{\text{circumference}}{\text{period}}$ OR $\frac{2\pi}{T}$

1 point is earned: For correct answer

frq4

Answer ranges accepted: 0.187π → 0.190π m/s OR 0.58 → 0.60 m/s

0	1	2 ✓

The student response earns two of the following points:

$v = \frac{\text{circumference}}{\text{period}} = \frac{2\pi R}{T} = \frac{2\pi(0.14\text{ m})}{1.5\text{ s}} = 0.60\text{ m/s}$

1 point is earned: For indicating use of circumference for distance traveled in one revolution, such as $\frac{\text{circumference}}{\text{period}}$ OR $\frac{2\pi}{T}$

1 point is earned: For correct answer

Answer ranges accepted: 0.187π → 0.190π m/s OR 0.58 → 0.60 m/s

Part C

Ft = Fe

$\mu mg = \frac{mv^2}{r}$

$v = \sqrt{\mu rg} = \sqrt{(0.5)(0.14\text{ m})(9.8\text{ m/s}^2)} = 0.83\text{ m/s}$

1 point is earned: For correct expression of centripetal force

1 point is earned: For correct expression of frictional force

1 point is earned: For equating centripetal force to frictional force

1 point is earned: For correct answer

Answer range: 0.80 → 0.84 m/s

0	1	2	3	4 ✓

The student response earns four of the following points:

Ft = Fe

$\mu mg = \frac{mv^2}{r}$

$v = \sqrt{\mu rg} = \sqrt{(0.5)(0.14\text{ m})(9.8\text{ m/s}^2)} = 0.83\text{ m/s}$

1 point is earned: For correct expression of centripetal force

frq4

1 point is earned: For correct expression of frictional force

1 point is earned: For equating centripetal force to frictional force

1 point is earned: For correct answer

Answer range: 0.80 → 0.84 m/s

Part D

1 point is earned: For indicating no change to answer in part (c)

1 point is earned: For any reasonable explanation, such as the masses cancel

(No points awarded for explanation if it is without statement of no change.)

0	1	2 ✓

The student response earns two of the following points:

1 point is earned: For indicating no change to answer in part (c)

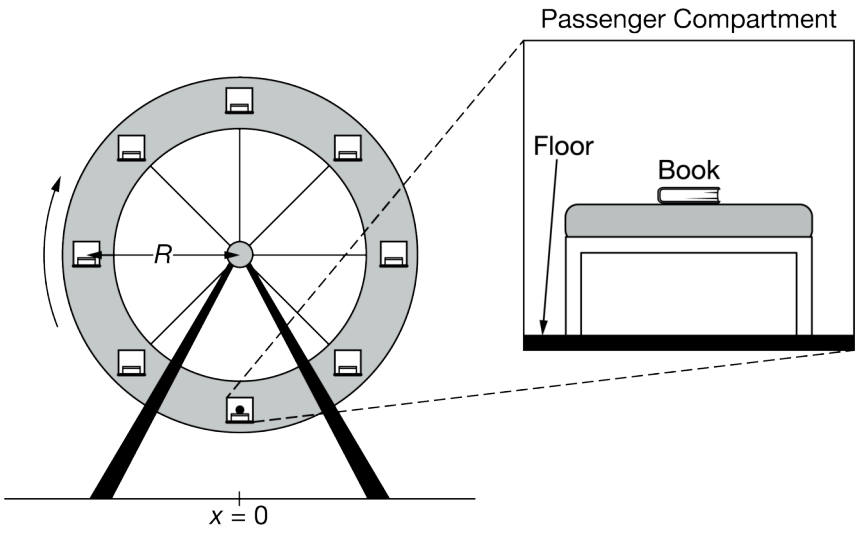
1 point is earned: For any reasonable explanation, such as the masses cancel

(No points awarded for explanation if it is without statement of no change.)

frq4

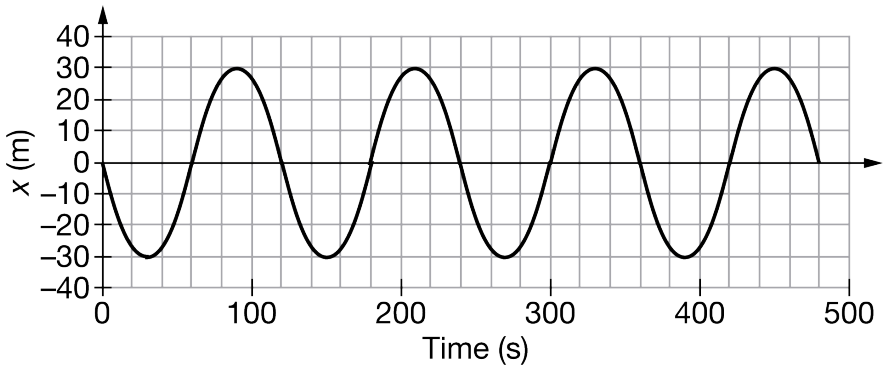
2. This question is a long free-response question. Show your work for each part of the question.

(12 points, suggested time 25 minutes)



A passenger compartment of a rotating amusement park ride contains a bench on which a book of mass m_b is placed, as indicated by the dot in the left figure above. The compartment moves with a constant angular speed about the center of the ride along a circular path of radius R. The bench remains horizontal throughout the compartment's motion. The right figure above shows a magnified view of the compartment.

The graph below shows the horizontal (x) component of the book's position as a function of time, where the $+x$ -direction is to the right.



(a)

i. Determine the period of revolution of the book.

ii. Calculate the tangential speed v_b (not the angular speed) of the book.

frq4

(b)

i. On the dot below, which represents the book, draw and label the forces (not components) that act on the book at the lowest point of its circular path. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



ii. At the lowest point of the circular path, the book is moving only in the horizontal direction. In what direction, if any, is the net vertical force on the book?

_____ Up _____ Down _____ No direction, since the net vertical force is equal to zero

Without deriving any equations, briefly explain your reasoning in terms of the book's motion.

(c) Derive an algebraic equation for the vertical force that the bench exerts on the book at the lowest point of the circular path in terms of the book's mass m_b, tangential speed v_b, radius R of the path, and physical constants, as appropriate. Do not substitute any numerical values for variables or physical constants.

(d) At the lowest point of the circular path, is the force that the bench exerts on the book greater than, less than, or equal to the weight of the book?

_____ Greater Than _____ Less Than _____ Equal to

Briefly explain how your answers in (b)ii and (c) support your selection.

Part A(i)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ An indication of the correct period of 120 seconds, 2 minutes, OR the equivalent.

Part A(ii)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

$(\pi/2)\ \mathrm{m/s}$ is acceptable.

$\pi R/(60\,\mathrm{s})$ or $(\pi R/60)\ \mathrm{m/s}$ can only earn 1 of the two points.

frq4

0	1	2 ✓

The response accurately includes both of the following criteria:

- ☐ For using the correct radius of $30\ \text{m}$ as found from the graph
- ☐ For substitution of the period, consistent with the answer in (a)(i), into a correct expression

Example Response:

$\text{Speed} = d/T = 2\pi R/T = 2\pi(30\ \text{m})/(120\ \text{s}) = 1.6\ \text{m/s}$

Part B(i)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes three of the following criteria.

- ☐ For a correctly drawn and labeled normal force
- ☐ For a correctly drawn and labeled gravitational force
- ☐ For drawing no extraneous forces, given that at least one force is drawn

Example Response:



Part B(ii)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: “Up”

Response cannot earn credit if incorrect selection is made.

frq4

0	1 ✓

The response accurately includes:

- [ ] An explicit or implicit assertion based on forces.

Example Response:

Because there is a centripetal force

Claim: The direction of the net vertical force on the book is up.

Evidence: The book is at the lowest point of its circular path. For an object in uniform circular motion, the centripetal force is always directed toward the center of the circular path.

Reasoning Statement: The net force is the centripetal force on the book, which must be upward toward the center of the book's circular path.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes three of the following criteria.

- [ ] For using Newton's 2nd law correctly or consistent with the diagram in (b)(i)
- [ ] For a correct expression for the centripetal acceleration
- [ ] For a correct final expression with no numbers substituted for values

Example Response:

$\Sigma \vec{F} = m\vec{a}$ (for the rotating system)

$F_{net} = ma_c = mv^2/r$

$a_c = mv^2/r = m_b v_b^2/R$

$F_{\text{net}} = m_b v_b^2/R = F_N - F_G$

frq4

$$F_N = m_b v_b^2/R + F_G$$
$$= m_b\left(v_b^2/R + g\right)$$

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answer: “Greater than”

A response with an incorrect selection consistent with parts (b)(ii) and (c) can earn 2 points.

0	1	2 ✓

The response accurately includes both of the following criteria:

- ☐ For correct reasoning with a selection consistent with the answer from (b)(ii) and (c)
- ☐ For explaining why the force from the bench is greater than the weight of the book.

Example Response:

Claim: At the lowest point of the circular path, the force that the bench exerts on the book is greater than the weight of the book.

Evidence: The direction of the net vertical force on the book is up. (Part (b)(ii) answer)

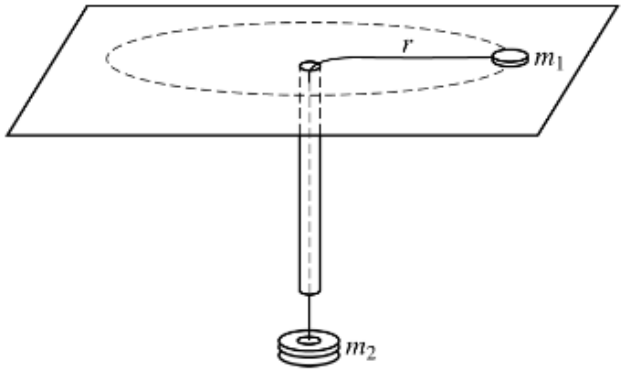
The book exerts a vertical force on the bench of magnitude $m_b\left(v_b^2/R + g\right)$. (Part (c) answer)

Reasoning Statements:

The upward force exerted by the bench must be greater than the downward gravitational force, in order for the net force to be upward as stated in the answer to (b)(ii). The magnitude $m_b\left(v_b^2/R + g\right)$ of the force exerted by the bench derived in part (c) is greater than the book’s weight $m_b g$.

frq4

3.



An experiment is performed using the apparatus above. A small disk of mass m_1 on a frictionless table is attached to one end of a string. The string passes through a hole in the table and an attached narrow, vertical plastic tube. An object of mass m_2 is hung at the other end of the string. A student holding the tube makes the disk rotate in a circle of constant radius r, while another student measures the period P.

(a) Derive the equation $P = 2\pi\sqrt{\frac{m_1 r}{m_2 g}}$ that relates P and m_2.

The procedure is repeated, and the period P is determined for four different values of m_2, where $m_1 = 0.012$ kg and $r = 0.80$ m. The data, which are presented below, can be used to compute an experimental value for g.

$m_2(kg)$	0.020	0.040	0.060	0.080
P (s)	1.40	1.05	0.80	0.75

(b) What quantities should be graphed to yield a straight line with a slope that could be used to determine g ?

(c) On the grid below, plot the quantities determined in part (b), label the axes, and draw the best-fit line to the data. You may use the blank rows above to record any values you may need to calculate.

frq4



(d) Use your graph to calculate the experimental value of *g*.

Part A

1 point is earned: For any indication that the centripetal force on the small disk equals the weight of the hanging objects

1 point is earned: For using the correct expression for the centripetal acceleration

$\frac{m_1 v^2}{r} = m_2 g$

1 point is earned: For the correct expression for the speed of the disk in terms of the period

$v = 2\pi r / P$

$\frac{m_1}{r}\left(\frac{2\pi r}{P}\right)^2 = m_2 g$

1 point is earned: For a correct expression relating *m2* and *P* in terms of the specified quantities

$\frac{4\pi^2 m_1 r}{P^2} = m_2 g$

$P = 2\pi\sqrt{\frac{m_1 r}{m_2 g}}$

0	1	2	3	4 ✓

The student response earns all of the following points:

1 point is earned: For any indication that the centripetal force on the small disk equals the weight of the hanging objects

frq4

1 point is earned: For using the correct expression for the centripetal acceleration

$\frac{m_1 v^2}{r} = m_2 g$

1 point is earned: For the correct expression for the speed of the disk in terms of the period

$v = 2\pi r / P$

$\frac{m_1}{r}\left(\frac{2\pi r}{P}\right)^2 = m_2 g$

1 point is earned: For a correct expression relating *m2* and *P* in terms of the specified quantities

$\frac{4\pi^2 m_1 r}{P^2} = m_2 g$

$P = 2\pi\sqrt{\frac{m_1 r}{m_2 g}}$

Part B

1 point is earned: For listing two quantities, one of which is correct, consistent with equation obtained in part (a)

1 point is earned: For having both quantities correct, consistent with equation obtained in part (a)

1 point is earned: For example: *1/P2* and *m2*, *P2* and *1/m2* ,*P* and $1/\sqrt{m_2}$

✓

0	1	2

The student response earns all of the following points:

1 point is earned: For listing two quantities, one of which is correct, consistent with equation obtained in part (a)

1 point is earned: For having both quantities correct, consistent with equation obtained in part (a)

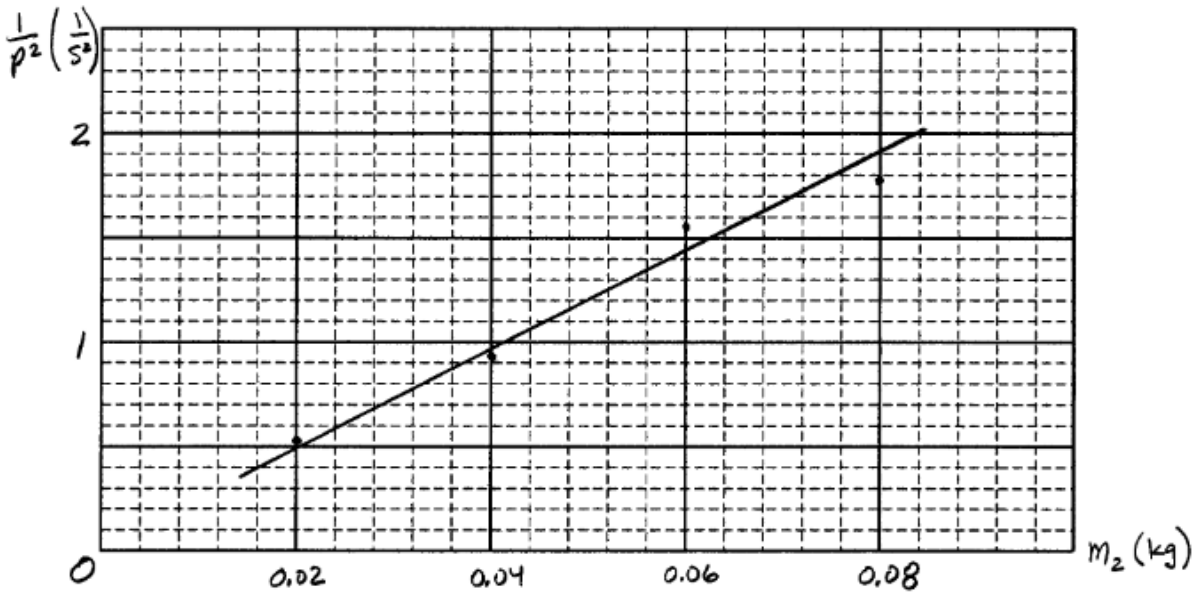
1 point is earned: For example: *1/P2* and *m2*, *P2* and *1/m2* ,*P* and $1/\sqrt{m_2}$

Part C

Using the example of *1/P2* and *m2*

m_2 (kg)	0.020	0.040	0.060	0.080
P (s)	1.40	1.05	0.80	0.75
$1/P^2$	0.51	0.91	1.6	1.8

frq4



1 point is earned: For correctly labeling both axes

1 point is earned: For correctly scaling both axes

1 point is earned: For a reasonably correct plotting of the data

1 point is earned: For a reasonably correct best-fit line

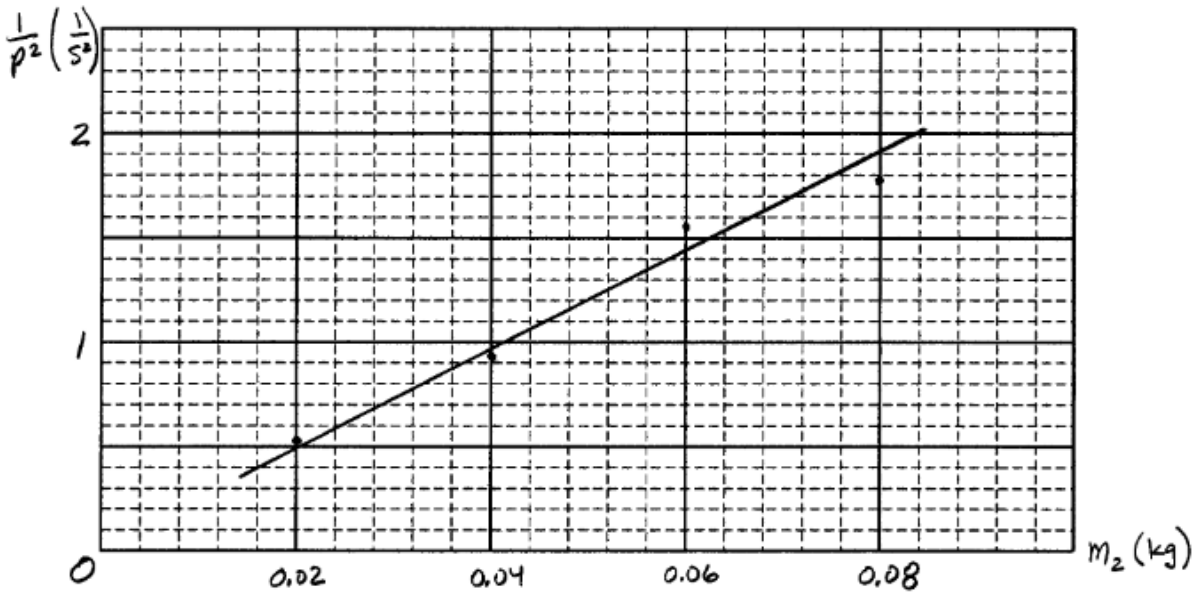
0	1	2	3	4 ✓

The student response earns all of the following points:

Using the example of *1/P2* and *m2*

m_2 (kg)	0.020	0.040	0.060	0.080
P (s)	1.40	1.05	0.80	0.75
$1/P^2$	0.51	0.91	1.6	1.8

frq4



1 point is earned: For correctly labeling both axes

1 point is earned: For correctly scaling both axes

1 point is earned: For a reasonably correct plotting of the data

1 point is earned: For a reasonably correct best-fit line

Part D

1 point is earned: For associating the slope of the line with the correct coefficient for the quantities graphed

For the example graph given, $\frac{1}{P^2} = \frac{m_2 g}{4\pi^2 m_1 r}$, so slope$= \frac{g}{4\pi^2 m_1 r}$

1 point is earned: For a correct method of calculating the slope, including substitution of points on the line

1 point is earned: For a correct substitution of given values (i.e., all values except the points from the line)

For example:

$$g = (slope)\left(4\pi^2 m_1 r\right) = \frac{\left(2s^{-2} - 0.4s^2\right)}{(0.084kg - 0.016kg)}\left(4\pi^2\right)(0.012kg)(0.80m)$$

1 point is earned: For a reasonable numerical value

1 point is earned: For correct units

g = 8.9 m/s2

✓

0	1	2	3	4	5

frq4

The student response earns all of the following points:

1 point is earned: For associating the slope of the line with the correct coefficient for the quantities graphed

For the example graph given, $\frac{1}{P^2} = \frac{m_2 g}{4\pi^2 m_1 r}$, so slope$= \frac{g}{4\pi^2 m_1 r}$

1 point is earned: For a correct method of calculating the slope, including substitution of points on the line

1 point is earned: For a correct substitution of given values (i.e., all values except the points from the line)

For example:

$g = (slope)\left(4\pi^2 m_1 r\right) = \frac{\left(2 s^{-2} - 0.4 s^2\right)}{(0.084 kg - 0.016 kg)}\left(4\pi^2\right)(0.012 kg)(0.80 m)$

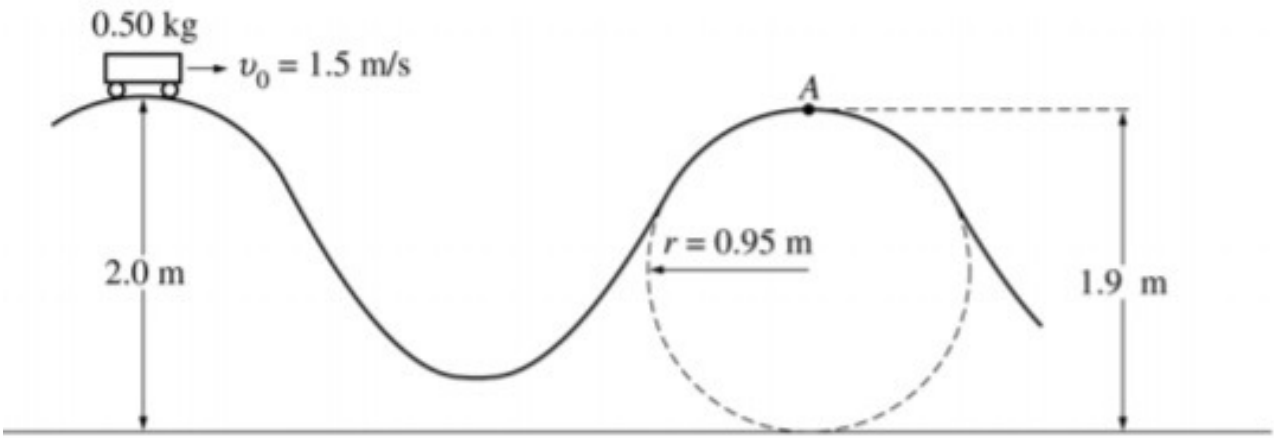
1 point is earned: For a reasonable numerical value

1 point is earned: For correct units

g = 8.9 m/s2

frq4

4.



A designer is working on a new roller coaster, and she begins by making a scale model. On this model, a car of total mass 0.50 kg moves with negligible friction along the track shown in the figure above. The car is given an initial speed v_0 = 1.5 m/s at the top of the first hill of height 2.0 m. Point *A* is located at a height of 1.9 m at the top of the second hill, the upper part of which is a circular arc of radius 0.95 m.

a. Calculate the speed of the car at point *A*.

b. On the figure of the car below, draw and label vectors to represent the forces on the car at point *A*.



c. Calculate the magnitude of the force of the track on the car at point *A*.

d. In order to stop the car at point A, some friction must be introduced. Calculate the work that must be done by the friction force in order to stop the car at point *A*.

e. Explain how to modify the track design to cause the car to lose contact with the track at point *A* before descending down the track. Justify your answer.

Part A

1 point is earned: For any indication of conservation of energy

1 point is earned: For a correct conservation equation for this situation

$\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_A^2 + mgh_A$

*v*2*A* = *v*2*0* +2*g*(*h*0 – *hA*)

1 point is earned: For the correct substitutions

*v*2*A* = (1.5 m/s)2 + 2(9.8 m/s2)(2.0 m – 1.9 m)

*v*2*A* = 4.2(m/s)2 (or 4.3(m/s)2 using *g* = 10 m/s2)

frq4

1 point is earned: For the correct answer

vA = 2.0 m/s (or 2.1 m/s if *v*2*A* was not rounded or if *g* = 10 m/s2 was used)

0	1	2	3	4 ✓

The student response earns four of the following points:

1 point is earned: For any indication of conservation of energy

1 point is earned: For a correct conservation equation for this situation

$\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_A^2 + mgh_A$

*v*2*A* = *v*2*0* +2*g*(*h*0 – *hA*)

1 point is earned: For the correct substitutions

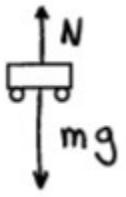
*v*2*A* = (1.5 m/s)2 + 2(9.8 m/s2)(2.0 m – 1.9 m)

*v*2*A* = 4.2(m/s)2 (or 4.3(m/s)2 using *g* = 10 m/s2)

1 point is earned: For the correct answer

vA = 2.0 m/s (or 2.1 m/s if *v*2*A* was not rounded or if *g* = 10 m/s2 was used)

Part B



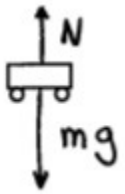
1 point is earned: For each correctly drawn and labeled force

One point earned for correct forces was deducted for having any extraneous forces

0	1	2 ✓

The student response earns two of the following points:

frq4



1 point is earned: For each correctly drawn and labeled force

One point earned for correct forces was deducted for having any extraneous forces

Part C

1 point is earned: For indicating that the sum of the forces equals the centripetal force

$\Sigma F = \frac{mv_A^2}{r}$

1 point is earned: For having the correct relative signs in the application of the above equation

$mg - N = \frac{mv_A^2}{r}$

1 point is earned: For solving for the normal force, substituting, and solving correctly

$N = mg - \frac{mv_A^2}{r}$

$N = (0.50\ \text{kg})(9.8\ \text{m/s}^2) - \frac{(0.50\ \text{kg})(4.21(\text{m/s})^2)}{0.95\ \text{m}}$

N = 2.7 N

0	1	2	3 ✓

The student response earns none of the following points:

1 point is earned: For indicating that the sum of the forces equals the centripetal force

$\Sigma F = \frac{mv_A^2}{r}$

1 point is earned: For having the correct relative signs in the application of the above equation

$mg - N = \frac{mv_A^2}{r}$

1 point is earned: For solving for the normal force, substituting, and solving correctly

$N = mg - \frac{mv_A^2}{r}$

$N = (0.50\ \text{kg})(9.8\ \text{m/s}^2) - \frac{(0.50\ \text{kg})(4.21(\text{m/s})^2)}{0.95\ \text{m}}$

frq4

N = 2.7 N

Part D

1 point is earned: For any indication that the work done must equal the kinetic energy at point A in the absence of friction

$W_{\text{friction}} = -\frac{1}{2}mv_A^2$

1 point is earned: For correct substitution

$W_{\text{friction}} = -\frac{1}{2}(0.50\ \text{kg})(4.2(\text{m/s})^2)$

1 point is earned: For the correct answer, including the negative sign

Wfriction = –1.1 J

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned: For any indication that the work done must equal the kinetic energy at point A in the absence of friction

$W_{\text{friction}} = -\frac{1}{2}mv_A^2$

1 point is earned: For correct substitution

$W_{\text{friction}} = -\frac{1}{2}(0.50\ \text{kg})(4.2(\text{m/s})^2)$

1 point is earned: For the correct answer, including the negative sign

Wfriction = –1.1 J

Part E

Method 1:

1 point is earned: For indicating that one should decrease the radius of the curve of the second hill

The cart will lose contact when $mg - \frac{mv_A^2}{r} \leq 0$. Decreasing the radius of the curve will cause the second term to increase and thus meet the condition

1 point is earned: For referring to the equation $N = mg - \frac{mv_A^2}{r}$

1 point is earned: For indicating that the cart will loose contact when $N \leq 0$

Method 2:

frq4

1 point is earned: For indicating that one should make the initial hill higher or the second hill lower

The cart will lose contact when $mg - \frac{mv_A^2}{r} \leq 0$. Increasing the difference in heights will cause the speed at *A* to increase, and thus the second term will increase and the condition will be met

1 point is earned: For referring to the equation $N = mg - \frac{mv_A^2}{r}$ and indicating that the cart will loose contact when *N* ≤ 0

1 point is earned: For indicating that the speed at *A* will increase

0	1	2	3 ✓

The student response earns three of the following points:

Method 1:

1 point is earned: For indicating that one should decrease the radius of the curve of the second hill

The cart will lose contact when $mg - \frac{mv_A^2}{r} \leq 0$. Decreasing the radius of the curve will cause the second term to increase and thus meet the condition

1 point is earned: For referring to the equation $N = mg - \frac{mv_A^2}{r}$

1 point is earned: For indicating that the cart will loose contact when *N* ≤ 0

Method 2:

1 point is earned: For indicating that one should make the initial hill higher or the second hill lower

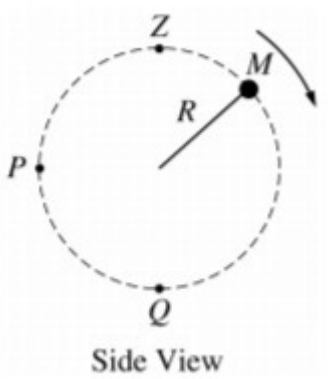
The cart will lose contact when $mg - \frac{mv_A^2}{r} \leq 0$. Increasing the difference in heights will cause the speed at *A* to increase, and thus the second term will increase and the condition will be met

1 point is earned: For referring to the equation $N = mg - \frac{mv_A^2}{r}$ and indicating that the cart will loose contact when *N* ≤ 0

1 point is earned: For indicating that the speed at *A* will increase

frq4

5.



A ball of mass M is attached to a string of length R and negligible mass. The ball moves clockwise in a vertical circle, as shown above. When the ball is at point P, the string is horizontal. Point Q is at the bottom of the circle and point Z is at the top of the circle. Air resistance is negligible. Express all algebraic answers in terms of the given quantities and fundamental constants.

a. On the figures below, draw and label all the forces exerted on the ball when it is at points P and Q, respectively.



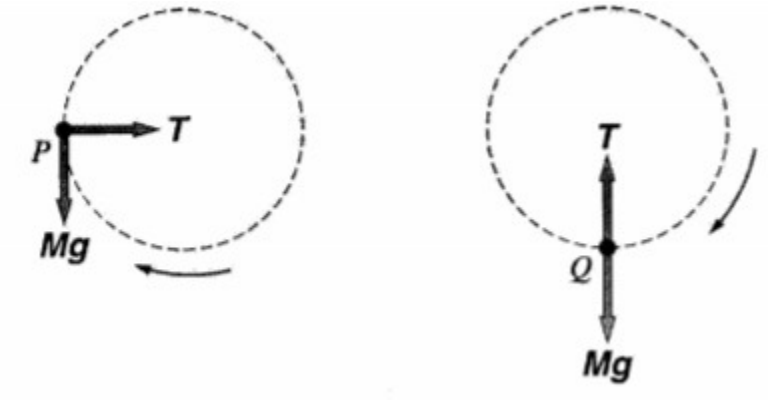
b. Derive an expression for v_{min} , the minimum speed the ball can have at point Z without leaving the circular path.

c. The maximum tension the string can have without breaking is T_{max} . Derive an expression for v_{max}, the maximum speed the ball can have at point Q without breaking the string.

d. Suppose that the string breaks at the instant the ball is at point P. Describe the motion of the ball immediately after the string breaks.

Part A

frq4



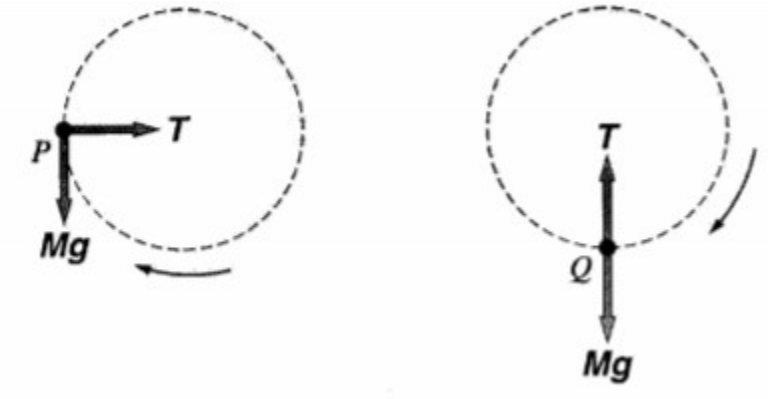
1 point is earned (total 4 points): For each correctly drawn and appropriately labeled force

One point was deducted for each incorrect force, to a maximum of the number of points earned for correct forces (i.e., lowest score awarded is zero).

The upward force in the figure on the right did not receive credit if it was labeled as the centripetal force, since in this case the net centripetal force is the sum of two physical forces (tension and weight).

0	1	2	3	4 ✓

The student response earns all of the following points:



1 point is earned (total 4 points): For each correctly drawn and appropriately labeled force

One point was deducted for each incorrect force, to a maximum of the number of points earned for correct forces (i.e., lowest score awarded is zero).

The upward force in the figure on the right did not receive credit if it was labeled as the centripetal force, since in this case the net centripetal force is the sum of two physical forces (tension and weight).

Part B

1 point is earned: For setting the net force equal to some physical force, such as gravity or a tension

frq4

1 point is earned: For correctly identifying that the only force is that due to gravity

Fnet = Ma = Mg

1 point is earned: For substituting the expression for centripetal acceleration

$\frac{Mv_{\text{min}}^2}{R} = Mg$

1 point is earned: For the correct answer

$v_{\text{min}} = \sqrt{Rg}$

Alternate Solution

1 point is earned: For using the expression for centripetal acceleration

$a = \frac{v^2}{R}$

2 points are earned: For recognizing that the condition for minimum speed at point *Z* is that the centripetal acceleration equals the acceleration due to gravity

$g = \frac{v^2}{R}$

1 point is earned: For the correct answer

$v_{\text{min}} = \sqrt{Rg}$

0	1	2	3	4 ✓

The student response earns all of the following points:

1 point is earned: For setting the net force equal to some physical force, such as gravity or a tension

1 point is earned: For correctly identifying that the only force is that due to gravity

Fnet = Ma = Mg

1 point is earned: For substituting the expression for centripetal acceleration

$\frac{Mv_{\text{min}}^2}{R} = Mg$

1 point is earned: For the correct answer

$v_{\text{min}} = \sqrt{Rg}$

Alternate Solution

frq4

1 point is earned: For using the expression for centripetal acceleration

$a = \frac{v^2}{R}$

2 points are earned: For recognizing that the condition for minimum speed at point *Z* is that the centripetal acceleration equals the acceleration due to gravity

$g = \frac{v^2}{R}$

1 point is earned: For the correct answer

$v_{\text{min}} = \sqrt{Rg}$

Part C

1 point is earned: For setting the net force equal to some physical force, such as gravity or a tension

1 point is earned: For indicating that the net force is the difference between the tension and gravity

1 point is earned: For indicating that the tension is larger than gravity

Ma = *T* – *Mg*

1 point is earned: For substituting the expression for centripetal acceleration

$M\frac{v_{\text{max}}^2}{R} = T_{\text{max}} - Mg$

1 point is earned: For the correct answer

$v_{\text{max}} = \sqrt{\frac{R}{M}(T_{\text{max}} - Mg)}$



0	1	2	3	4	5

The student response earns all of the following points:

1 point is earned: For setting the net force equal to some physical force, such as gravity or a tension

1 point is earned: For indicating that the net force is the difference between the tension and gravity

1 point is earned: For indicating that the tension is larger than gravity

Ma = *T* – *Mg*

1 point is earned: For substituting the expression for centripetal acceleration

$M\frac{v_{\text{max}}^2}{R} = T_{\text{max}} - Mg$

frq4

1 point is earned: For the correct answer

$v_{\max} = \sqrt{\frac{R}{M}(T_{\max} - Mg)}$

Part D

1 point is earned: For any indication that the velocity of the ball is straight up

1 point is earned: For any indication that the ball would be slowing down

0	1	2 ✓

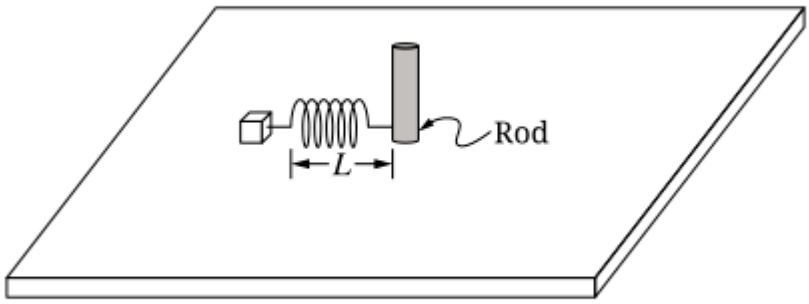
The student response earns all of the following points:

1 point is earned: For any indication that the velocity of the ball is straight up

1 point is earned: For any indication that the ball would be slowing down

frq4

6. This question is a long free-response question. Show your work for each part of the question.



(12 points, suggested time 25 minutes)

A small block of mass m_0 is attached to the end of a spring of spring constant k_0 that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length L, as shown. All frictional forces are negligible.

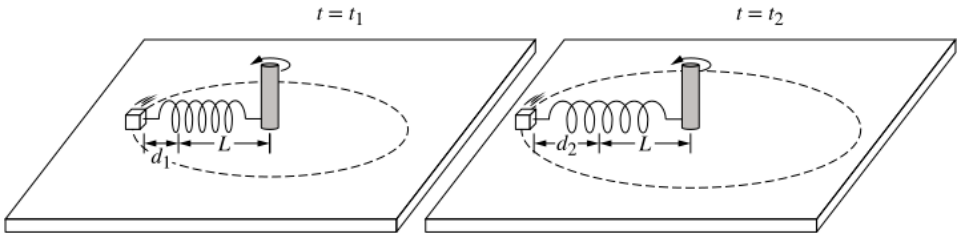
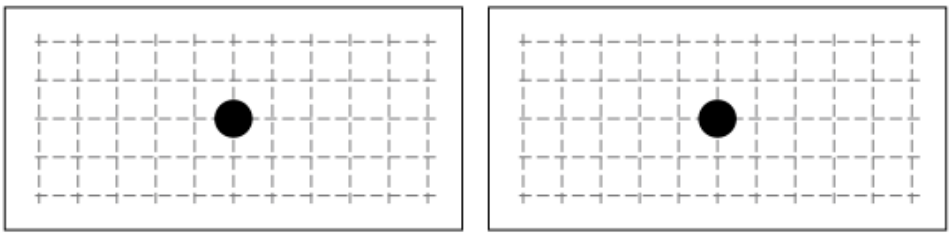


Figure 1 Figure 2

(a) At time $t = t_1$, the rod is spinning such that the block moves in a circular path with a constant tangential speed v_1 and the spring is stretched a distance d_1 from the spring's unstretched length, as shown in Figure 1. At time $t = t_2$, the rod is spinning such that the block moves in a circular path with a constant tangential speed v_2 and the spring is stretched a distance d_2 from the spring's unstretched length, where $d_2 > d_1$, as shown in Figure 2.

i. On the following dots, which represent the block at the locations shown in Figure 1 and Figure 2, draw the force that is exerted on the block by the spring at times $t = t_1$ and $t = t_2$. The spring force must be represented by a distinct arrow starting on, and pointing away from, the dot.
Note: Draw the relative lengths of the vectors to reflect the relative magnitudes of the forces exerted by the

spring at both times.



ii. Referencing d_1 and d_2, describe your reasoning for drawing the arrows the length that you did in part (a)(i).

iii. Is the tangential speed v_1 of the block at time $t = t_1$ greater than, less than, or equal to the tangential speed v_2 of the block at time $t = t_2$?

_____ $v_1 > v_2$ _____ $v_1 < v_2$ _____ $v_1 = v_2$

Justify your answer without using equations.

(b) Consider a scenario where the block travels in a circular path where the spring is stretched a distance d from its unstretched length L.

i. Determine an expression for the magnitude of the net force F_{net} exerted on the block. Express your answer in terms of m_0, k_0, L, d, and fundamental constants, as appropriate.

ii. Derive an equation for the tangential speed v of the block. Express your answer in terms of m_0, k_0, L, d, and fundamental constants, as appropriate.

(c) Does your equation for the tangential speed v of the block from part (b)(ii) agree with your reasoning from part (a) ?

____ Yes ____ No

Explain your reasoning.

frq4

Part A

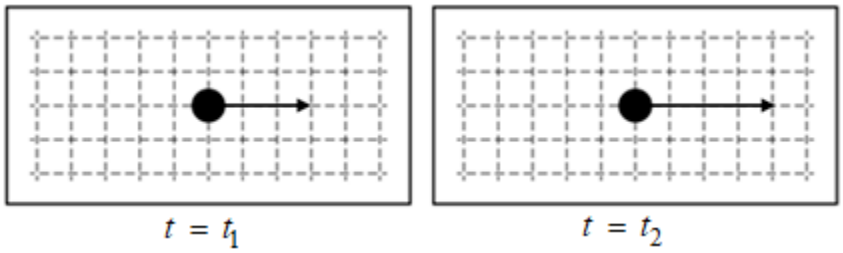
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** the following criteria.

- ☐ 1 point is earned for drawing rightward arrows in both diagrams.
- ☐ 1 point is earned for the length of the arrow at $t = t_2$ being longer than the arrow at $t = t_1$.
  Scoring Notes:
  - A maximum of 1 point can be earned if extraneous unlabeled arrows are drawn.
  - A maximum of 1 point can be earned if incorrect labeled forces are drawn.
- ☐ 1 point is earned for an explanation that refers to the difference in the stretch length and indicates that the magnitude of the spring force is (or is not) related to the stretch length, consistent with the force diagram drawn in part (a)(i).
- ☐ 1 point is earned for a correct selection with an attempt at a relevant justification, or a selection and justification consistent with the response in part (a)(ii).
- ☐ 1 point is earned for indicating that the spring force is the net force.
  Scoring Note: Stating $F = kx$ earns this point.
- ☐ 1 point is earned for indicating that the net force is related to the speed (or acceleration).
  Scoring Note: The relationship does not need to be defined to earn this point.

Example Response



The spring force arrow drawn at $t = t_2$ is longer because the spring is stretched a greater distance at that time and the spring force is related to the stretch distance.

_____ $v_1 > v_2$ X $v_1 < v_2$ _____ $v_1 = v_2$

The net force is the spring force. When the spring is stretched a greater length, the spring force is greater, so the net force is greater, and therefore the tangential speed is greater at $t = t_2$.

Part B

frq4

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** the following criteria.

- ☐ 1 point is earned for the correct answer: $F_{\text{net}} = k_0 d$
  Scoring Notes:
  - An answer of kx does not earn this point.
  - Points for part (b)(i) may be earned if correct in (b)(ii).
- ☐ 1 point is earned for a multistep derivation that begins with Newton's second law: $\Sigma F = ma$.
- ☐ 1 point is earned for **one** of the following:
  - Substituting kx for force into Newton's second law
  - Substituting $\frac{v^2}{r}$ for acceleration into Newton's second law
  - Substituting $(L + d)$ for the radius
- ☐ 1 point is earned for the consistent answer in terms of the given variables: $v = \sqrt{\frac{k_0 d(L+d)}{m_0}}$.
  Scoring Notes:
  - Subscripts for m and k are not required to earn this point.
  - Points in (b)(ii) can be earned if correct in (b)(i).

Example Response

$F_{\text{net}} = \Sigma F = F_s$
$F_{\text{net}} = \Sigma F = k_0 d$

$\Sigma F = ma_c$
$kx = \frac{mv^2}{r}$
$k_0 d = \frac{m_0 v^2}{L+d}$

$v = \sqrt{\frac{k_0 d(L+d)}{m_0}}$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq4

0	1	2 ✓

The response accurately includes **both** the following criteria.

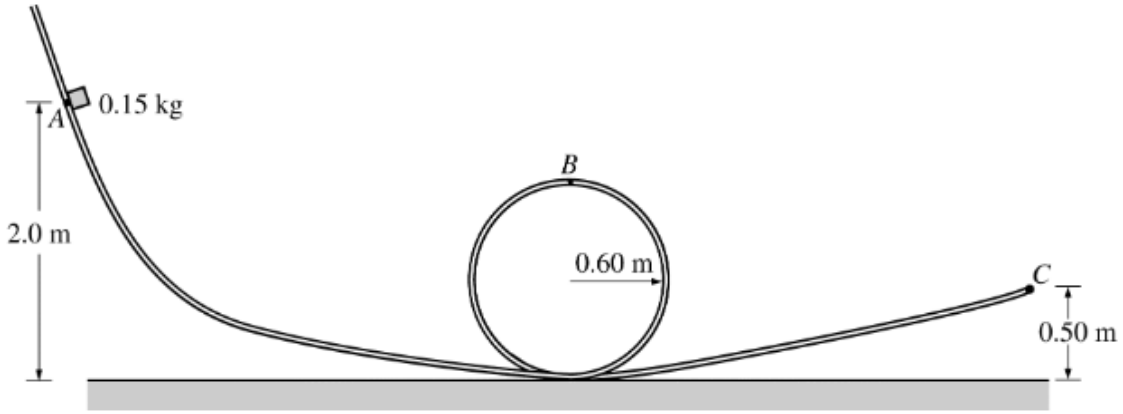
- ☐ 1 point is earned for an answer that attempts to use functional dependence to relate the tangential speed with stretched distance.
  Scoring Note: It is not necessary to use the functional dependence correctly to earn this point.
- ☐ 1 point is earned for a correct explanation for why the derived equation in part (b)(ii) does or does not support the reasoning in part (a).

Example Response

My equation from part (b)(ii) agrees with my reasoning in part (a). The tangential speed of the block as it travels in a horizontal circle is related to the distance the spring is stretched. The greater the tangential speed of the block, the greater distance the spring is stretched. The equation shows this because the d is in the numerator.

frq4

7.



A small block of mass 0.15 kg is placed at point *A* at a height 2.0 m above the bottom of a track, as shown in the figure above, and is released from rest. It slides with negligible friction down the track, around the inside of the loop of radius 0.60 m, and leaves the track at point *C* at a height 0.50 m above the bottom of the track.

(a) Calculate the speed of the block when it leaves the track at point *C*.

(b) On the figure below, draw and label the forces (not components) that act on the block when it is at the top of the loop at point *B*.

(c) Calculate the minimum speed the block can have at point *B* without losing contact with the track.

(d) Calculate the minimum height h_{min} above the bottom of the track at which the block can be released and still go around the loop without losing contact with the track.

Part A

1 point is earned: For a correct conservation of energy equation for this situation

$mgh_i = mgh_f + \frac{1}{2}mv_f^2$

$v_f = \sqrt{2g(h_i - h_f)}$

$v_f = \sqrt{2(9.8m/s^2)(2.0m - 0.50m)}$

1 point is earned: For the correct answer

vf = 5.4 m/s (or 5.5 using *g* = 10 m/s2)

frq4

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For a correct conservation of energy equation for this situation

$mgh_i = mgh_f + \frac{1}{2}mv_f^2$

$v_f = \sqrt{2g(h_i - h_f)}$

$v_f = \sqrt{2(9.8m/s^2)(2.0m - 0.50m)}$

1 point is earned: For the correct answer

vf = 5.4 m/s (or 5.5 using *g* = 10 m/s2)

Part B



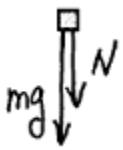
1 point is earned: For correctly drawing and appropriately labeling the weight of the block

1 point is earned: For correctly drawing and appropriately labeling the normal force

1 point is earned: For no extraneous forces

0	1	2	3 ✓

The student response earns all of the following points:



1 point is earned: For correctly drawing and appropriately labeling the weight of the block

frq4

1 point is earned: For correctly drawing and appropriately labeling the normal force

1 point is earned: For no extraneous forces

Part C

At the top of the track, the net force on the block is the centripetal force

ma = mv2/r = mg + N

The condition for minimum speed is that the normal force is zero. **1 point is earned:** For a correct equation that can be solved for the minimum speed

$mv_{\text{min}}^2/r = mg$
$v_{\text{min}} = \sqrt{rg}$
$v_{\text{min}} = \sqrt{(0.60m)(9.8m/s^2)}$

1 point is earned: For the correct answer

vmin = 2.4 m/s

0	1	2 ✓

The student response earns all of the following points:

At the top of the track, the net force on the block is the centripetal force

ma = mv2/r = mg + N

The condition for minimum speed is that the normal force is zero. **1 point is earned:** For a correct equation that can be solved for the minimum speed

$mv_{\text{min}}^2/r = mg$
$v_{\text{min}} = \sqrt{rg}$
$v_{\text{min}} = \sqrt{(0.60m)(9.8m/s^2)}$

1 point is earned: For the correct answer

vmin = 2.4 m/s

Part D

1 point is earned: For a correct conservation of energy equation for this situation

$mgh_{\text{min}} = mg(2r) + \frac{1}{2}mv_{\text{min}}^2$
$h_{\text{min}} = 2r + \left(v_{\text{min}}^2/2g\right)$

frq4

1 point is earned: For correctly substituting the value of from part (c)

hmin = 2(0.60 m) + ((2.4 m/s)2 / 2(9.8 m/s2))

1 point is earned: For the correct answer

hmin = 1.5 m

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For a correct conservation of energy equation for this situation

$mgh_{\text{min}} = mg(2r) + \frac{1}{2}mv_{\text{min}}^2$
$h_{\text{min}} = 2r + \left(v_{\text{min}}^2/2g\right)$

1 point is earned: For correctly substituting the value of from part (c)

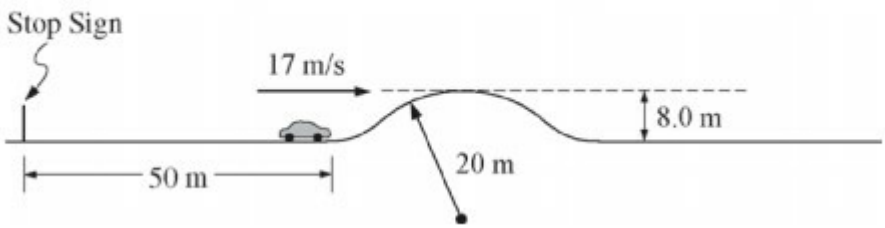
hmin = 2(0.60 m) + ((2.4 m/s)2 / 2(9.8 m/s2))

1 point is earned: For the correct answer

hmin = 1.5 m

frq4

8.



You are assigned to do some calculations for a movie stunt that involves a car on a straight road. The road, pictured above, has a hill that rises 8.0 m above the flat region. The top of the hill is a circular arc of radius 20 m. You need to determine whether a car traveling under certain conditions will lose contact with the road at the top of the hill. There is a stop sign 50 m from the beginning of the hill. You are to assume that a car of mass 1600 kg accelerates uniformly from rest at the stop sign, has a speed of when it reaches the beginning of the hill, and then coasts with the engine off. Assume energy losses due to friction and air resistance are negligible.

a. Calculate the magnitude of the acceleration of the car during the first 50 m.

b. Calculate the time it takes the car to reach the beginning of the hill

c. Calculate the magnitude of the net force required to accelerate the car during the first 50 m.

d. On the dot below that represents the car, draw and label the forces (not components) that act on the car at the top of the hill if it travels over the hill without losing contact.

e. Calculate the minimum speed the car must have at the top of the hill to momentarily lose contact with the road. If you need to draw anything other than what you have shown in part (d) to assist in your solution, use the space below. Do NOT add anything to the figure in part (d).

f. Calculate the speed the car must have at the beginning of the hill in order to have the speed at the top of the hill you calculated in part (e).

Part A

One point is earned for using the correct equation to solve for the acceleration

frq4

$$v^2 = v_0^2 + 2ax$$

$$v_0 = 0, \text{ so } a = \frac{v^2}{2x}$$

$$a = \frac{(17 \text{ m/s})^2}{(2)(50 \text{ m})}$$

One point is earned for the correct answer with units

a = 2.89 m/s2

0	1	2 ✓

The student earns all of the following points:

One point is earned for using the correct equation to solve for the acceleration

$$v^2 = v_0^2 + 2ax$$

$$v_0 = 0, \text{ so } a = \frac{v^2}{2x}$$

$$a = \frac{(17 \text{ m/s})^2}{(2)(50 \text{ m})}$$

One point is earned for the correct answer with units

a = 2.89 m/s2

Part B

One point is earned for using a correct expression to calculate the time

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

One point is earned for substituting values consistent with the answer from part (a)

$$x = 0 + (0)t + \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2x}{a}}$$

$$t = \sqrt{\frac{(2)(50 \text{ m})}{(2.89 \text{ m/s}^2)}}$$

$$t = 5.88 \text{ s}$$

frq4

0	1	2 ✓

The student earns all of the following points:

One point is earned for using a correct expression to calculate the time

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

One point is earned for substituting values consistent with the answer from part (a)

$$x = 0 + (0)t + \frac{1}{2}at^2$$
$$t = \sqrt{\frac{2x}{a}}$$
$$t = \sqrt{\frac{(2)(50\text{ m})}{(2.89\text{ m/s}^2)}}$$
$$t = 5.88\text{ s}$$

Part C

One point is earned for using Newton's 2nd law

$F = ma$

One point is earned for substituting the correct value for m and a value for a consistent with part (a)

F = (1600 kg)(2.89 m/s2)

F = 4624 N

0	1	2 ✓

The student earns all of the following points:

One point is earned for using Newton's 2nd law

$F = ma$

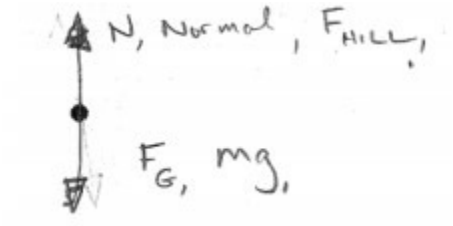
One point is earned for substituting the correct value for m and a value for a consistent with part (a)

F = (1600 kg)(2.89 m/s2)

frq4

F = 4624 N

Part D



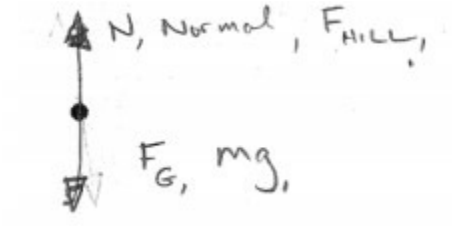
One point is earned for a correctly labeled vector representing the normal force

One point is earned for a correctly labeled vector representing the weight of the car

One point is earned for no incorrect or extraneous vectors or labeling

0	1	2	3 ✓

The student earns all of the following points:



One point is earned for a correctly labeled vector representing the normal force

One point is earned for a correctly labeled vector representing the weight of the car

One point is earned for no incorrect or extraneous vectors or labeling

Part E

One point is earned for equating the centripetal force exerted on the car to the net force

$$F_C = W + N$$

$$\frac{mv^2}{r} = mg + N$$

One point is earned for setting the normal force equal to zero

$$\frac{mv^2}{r} = mg + 0$$

frq4

Solving for v and substituting

$$v = \sqrt{rg}$$
$$v = \sqrt{(20\ \text{m})\left(9.8\ \text{m/s}^2\right)}$$

One point is earned for a correct answer with units

v = 14 m/s

0	1	2	3 ✓

The student earns all of the following points:

One point is earned for equating the centripetal force exerted on the car to the net force

$$F_C = W + N$$
$$\frac{mv^2}{r} = mg + N$$

One point is earned for setting the normal force equal to zero

$$\frac{mv^2}{r} = mg + 0$$

Solving for v and substituting

$$v = \sqrt{rg}$$
$$v = \sqrt{(20\ \text{m})\left(9.8\ \text{m/s}^2\right)}$$

One point is earned for a correct answer with units

v = 14 m/s

Part F

One point is earned for any indication of the use of conservation of energy

One point is earned for a correct expression for the conservation of energy

$$K_1 = K_2 + U_2$$
$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + mgh_2$$

One point is earned for correct substitution, with consistent with part (e)

frq4

$$v_1^2 = v_2^2 + 2gh_2$$
$$v_1 = \sqrt{v_2^2 + 2gh_2}$$
$$v_1 = \sqrt{(14\ \text{m/s})^2 + (2)\left(9.8\ \text{m/s}^2\right)(8.0\ \text{m})}$$
$$v_1 = 18.8\ \text{m/s}$$



0	1	2	3

The student earns all of the following points:

One point is earned for any indication of the use of conservation of energy

One point is earned for a correct expression for the conservation of energy

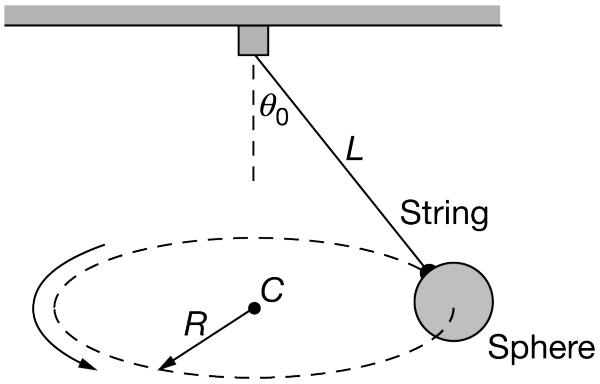
$$K_1 = K_2 + U_2$$
$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + mgh_2$$

One point is earned for correct substitution, with consistent with part (e)

$$v_1^2 = v_2^2 + 2gh_2$$
$$v_1 = \sqrt{v_2^2 + 2gh_2}$$
$$v_1 = \sqrt{(14\ \text{m/s})^2 + (2)\left(9.8\ \text{m/s}^2\right)(8.0\ \text{m})}$$
$$v_1 = 18.8\ \text{m/s}$$

frq4

9. This question is a long free-response question. Show your work for each part of the question.



(12 points, suggested time 25 minutes)

A small sphere of mass M is suspended by a string of length L. The sphere is made to move in a horizontal circle of radius R at a constant speed, as shown above. The center of the circle is labeled point C, and the string makes an θ_0 with the vertical.

(a) Two students are discussing the motion of the sphere and make the following statements.

Student 1: None of the forces exerted on the sphere are in the direction of point C, the center of the circular path. Therefore, I don't see how there can be a centripetal force exerted on the sphere to make it move in a circle.

Student 2: I see another problem. The tension force exerted by the string is at an angle from the vertical. Therefore, its vertical component must be less than the weight Mg of the sphere. That means the net force on the sphere has a downward vertical component, and the sphere should move downward as well as moving around in a circle.

i. What is one aspect of Student 1's reasoning that is incorrect?

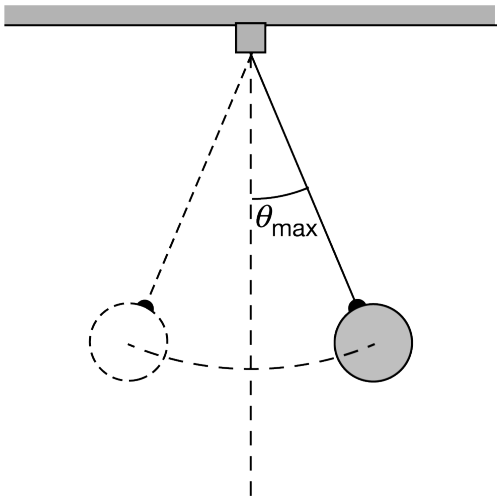
ii. What is one aspect of Student 2's reasoning that is incorrect?

(b)

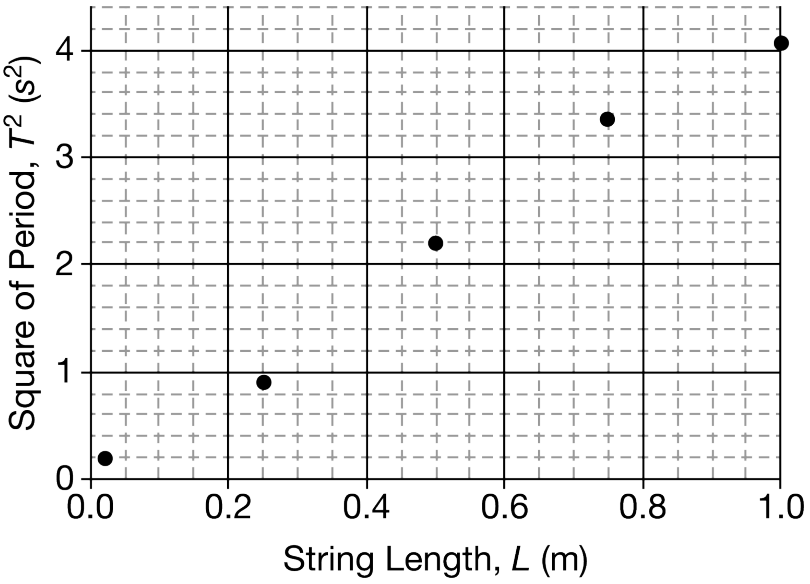
i. Derive an equation for the magnitude of the net force exerted on the sphere. Express your answer in terms of M, θ_0, and physical constants, as appropriate.

ii. Describe one aspect or step in your derivation of part (b)(i) that can be correctly linked to your answer to either part (a)(i) or part (a)(ii).

frq4

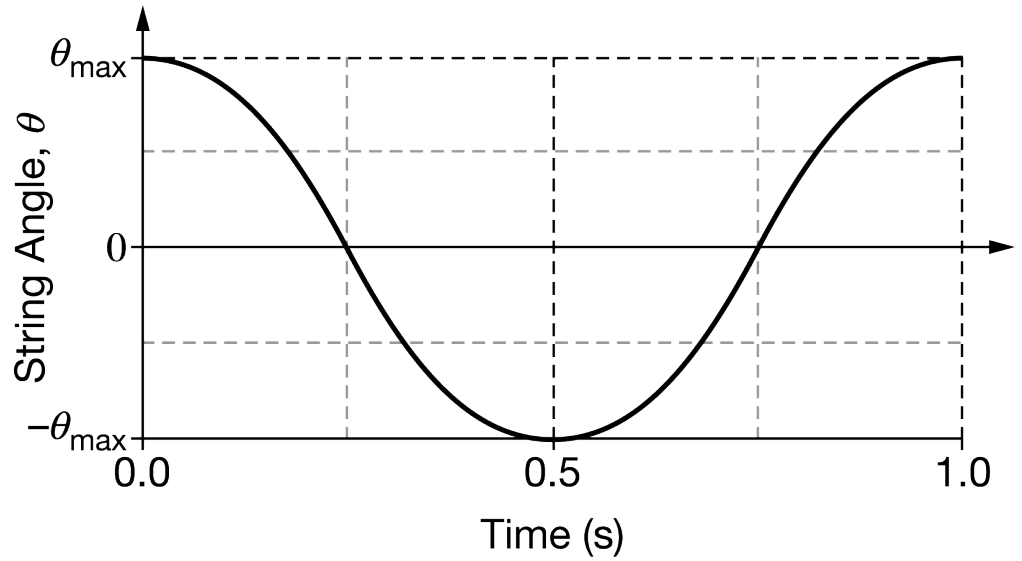


Instead of moving in a horizontal circle, the sphere now moves in a vertical plane so that it is a simple pendulum, as shown above. The maximum angle θ_{max} that the string makes from the vertical can be assumed to be small. The graph below shows data for the square of the pendulum period T as a function of string length L.

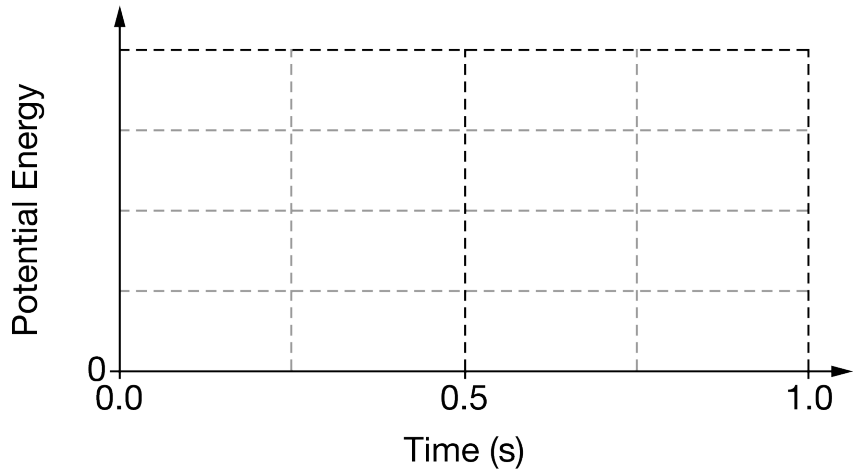


(c) On the graph above, draw a best-fit line for the data. Then use the line to calculate a numerical value for the gravitational acceleration g.

frq4

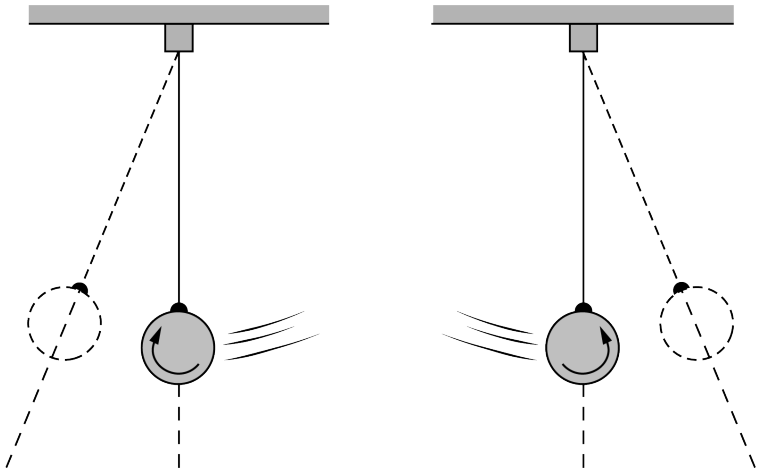


(d) The graph above shows the angle θ from the vertical as a function of time for the pendulum. On the axes below, sketch a graph of the gravitational potential energy of the sphere-Earth system for the same time interval. Take the zero of potential energy to be when the potential energy has its minimum value.



(e) As the sphere swings back and forth, it must also rotate a small amount during each swing. The figures below indicate the direction that the sphere rotates as it is swinging in each direction.

frq4



In order for the sphere's rotation to change direction, a torque must be exerted on the sphere. When the sphere is at its maximum rightward displacement, what is the direction of the torque exerted on the sphere with respect to the point of attachment between the sphere and string?

_____ Clockwise _____ Counterclockwise

Briefly state why the torque is in the direction you indicated.

Part A(i)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes a claim (explicit or implied) that the sphere can have a centripetal force without any individual force pointing toward the center of its circular path.

Example Response:

The centripetal force is provided by the net force on an object.

The tension force from the string has a horizontal component, which provides the centripetal force.

Part A(ii)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

frq4

The response accurately includes a claim (explicit or implied) that the tension force is or can be larger than Mg

Example Response:

The tension force is larger than Mg.

The tension force is larger than Mg, allowing the vertical component to be equal to Mg and the net force to have zero vertical component, as it must.

Part B(i)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes all three of the following criteria:

- ☐ Using Newton's second law for vertical force components.
- ☐ Writing the horizontal component of string tension in terms of angle.
- ☐ Substituting a correct tension force and writing an answer in terms of the stated quantities.

Example Response:

$\sum F_y = 0$
$F_T \cos\theta - Mg = 0$ or $F_T \cos\theta = Mg$

$F_{\text{net}} = F_x = F_T \sin\theta$

$F_T \cos\theta = Mg,\ \text{so}\ \ F_T = Mg/\cos\theta$
$F_{\text{net}} = F_T \sin\theta = \left(\frac{Mg}{\cos\theta}\right)\sin\theta\ \ \text{or}\ \ Mg\tan\theta$

Part B(ii)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes a correct statement linking the derivation to the answer in either (a)(i) or (a)(ii).

Example Response:

The net force F_{net} is in the horizontal direction and equals the horizontal component $F_T \sin\theta$, as indicated by the

frq4

statement $F_{\text{net}} = F_T \sin\theta$ in the derivation.

The tension force was shown to be $F_T = Mg/\cos\theta$, which is greater than Mg as stated in the answer to (a)(ii).

Part C

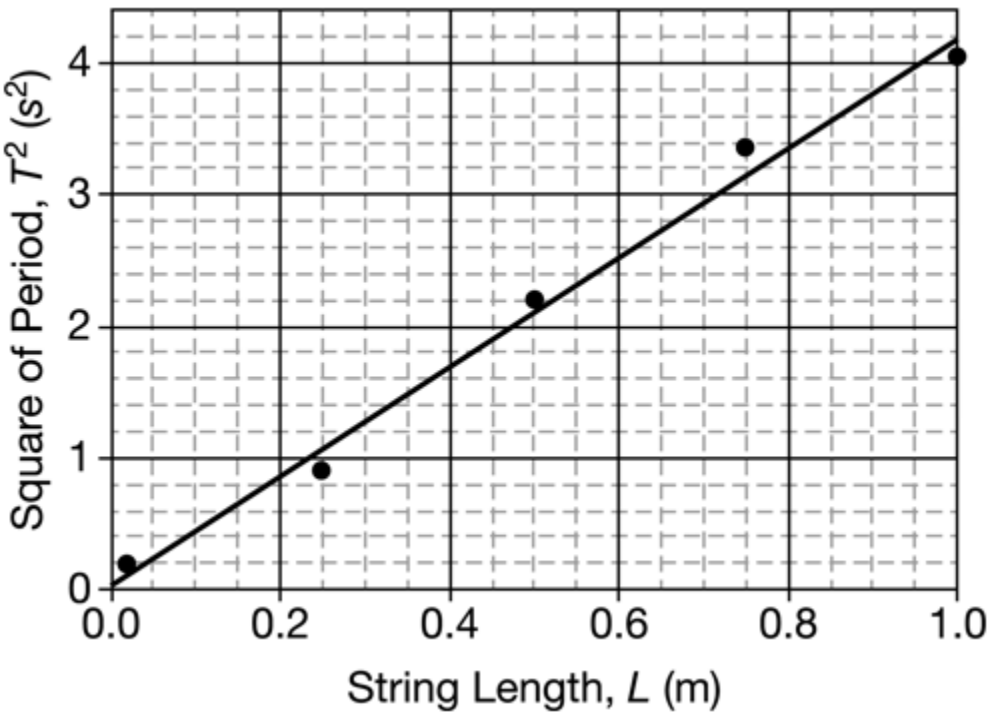
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes all three of the following criteria:

- ☐ Drawing a reasonable straight line of best fit.
- ☐ Correctly calculating the slope or its inverse, using points on the line drawn.
- ☐ A calculation of *g* consistent with the calculated slope or slope inverse.

Example Response:



$\text{Slope} = \frac{\Delta(T^2)}{\Delta L} = \frac{4.15-1.3}{1.0-0.3} = 4.07\ \text{s}^2/\text{m}$

$T = 2\pi\sqrt{L/g}$ so $T^2 = 4\pi^2 L/g$

$\text{Slope} = 4\pi^2/g$

$g = 4\pi^2/\text{Slope} = 4\pi^2/\left(4.07\ \text{s}^2/\text{m}\right) = 9.7\ \text{m/s}^2$

frq4

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

The maxima should be equal in energy but may have any energy value.

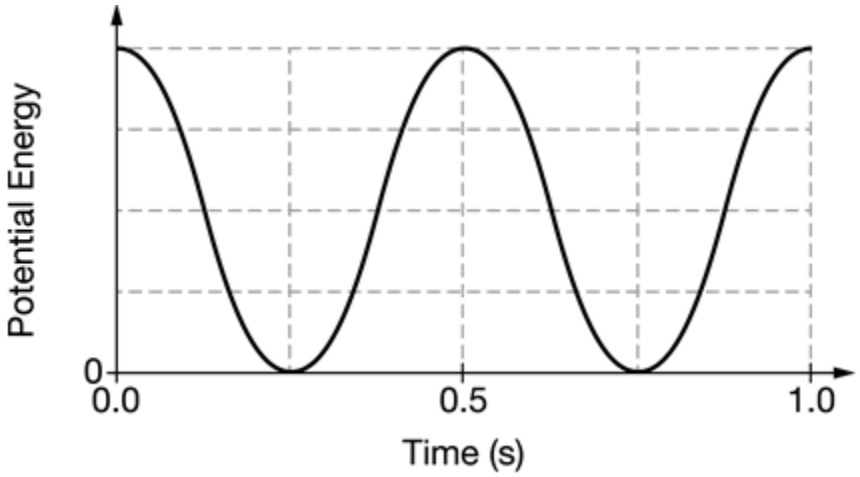
A graph with maxima at 0 and 1 s, and a minimum at 0.5 s, can earn the second point only.

0	1	2 ✓

The response accurately includes both of the following criteria:

- ☐ A graph with equal maxima at 0, 0.5 s, and 1 s, and minima of zero at 0.25 s and 0.75 s.
- ☐ A graph that has even symmetry (mirror symmetry) about time $t = 0.5$ s.

Example Response:



Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If the incorrect selection is made, the response is not graded.

0	1 ✓

The response accurately includes a correct selection AND correct reasoning in terms of the changing direction of the sphere's rotation.

Example Response:

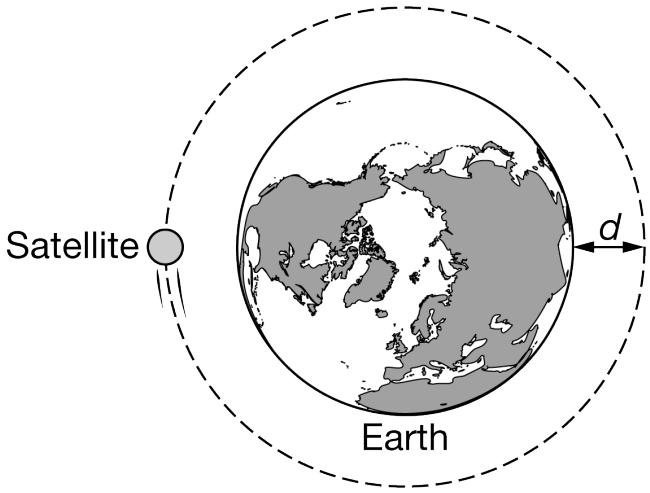
frq4

The correct answer is clockwise.

At the sphere's maximum rightward displacement, the sphere's rotation is changing from counterclockwise (swinging to the right) to clockwise (swinging to the left). So the torque must be clockwise.
At the sphere's maximum rightward displacement, the gravitational force (taken to act at the center of the sphere) exerts a clockwise torque about the point of attachment to the string.

frq4

10. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface, as shown above.

An identical satellite is set into orbit around a new planet. The new planet has the same radius as the radius of Earth but is significantly more massive than Earth. The satellite orbits the same distance d from the surface of the new planet.

(a) Is the speed of the satellite orbiting the new planet greater than, less than, or the same as the speed of the satellite orbiting Earth? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

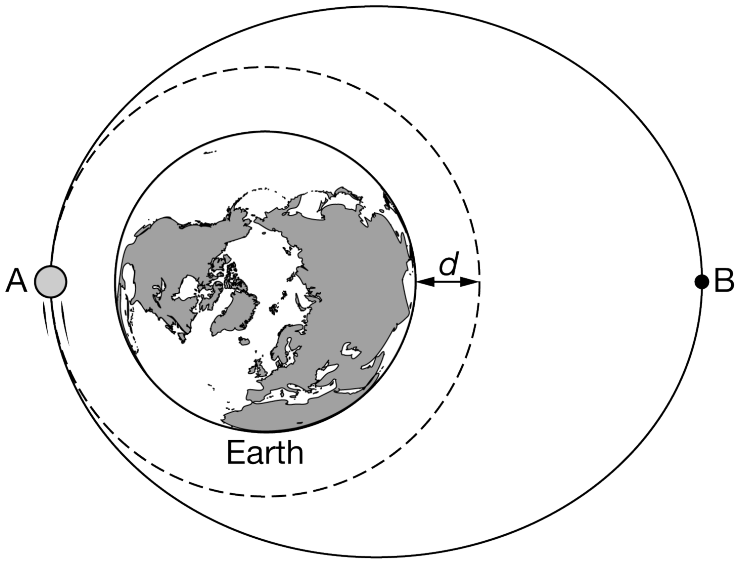
$$\frac{MT^2}{R^3} = C$$

In the equation above

M is the mass of a planet;
T is the orbital period of a satellite in a circular orbit;
R is the distance from the planet's center to the satellite; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

frq4



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point A to point B, and state the following:

Student 1: The satellite is moving faster at point A because the satellite's angular momentum is greater at point A than at point B.

Student 2: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is higher at point B than at point A.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

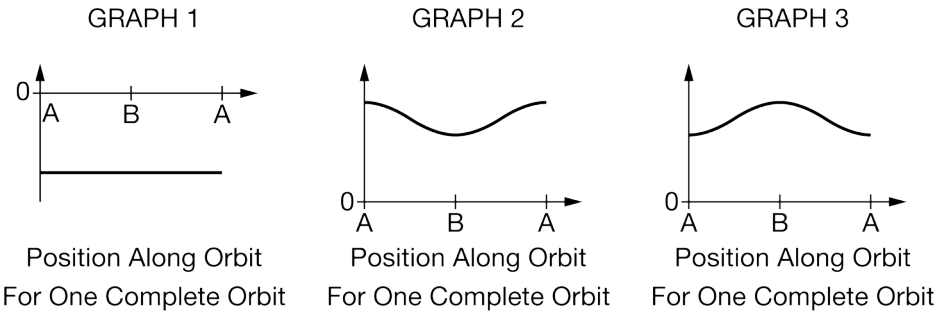
(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite

frq4

as it completes one elliptical orbit, starting and ending at point A.



(h) Which of the above graphs best represents the magnitude of the gravitational field at the location of the satellite as it orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the total mechanical energy of the satellite-Earth system as the satellite orbits? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the satellite around the new planet will orbit with a faster speed.
- ☐ 1 point is earned for a correct explanation in terms of the relationship between gravitational force and centripetal force (or acceleration).

Example Response:

There is greater gravitational force acting on the satellite around the new planet. The increased gravitational force means that the centripetal force $m\frac{v^2}{R}$ *(or acceleration* $\frac{v^2}{R}$*) is greater. Since R is the same in both cases, v must be greater for the satellite orbiting the new planet.*

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

frq4

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a consistent answer AND an attempt at reasoning using functional dependence.
- ☐ 1 point is earned for correctly relating the increased M to a decreased T.
- ☐ 1 point is earned for correctly explaining how a decreased T increases v.

Example Response:

Yes, this is consistent with my reasoning. Since the mass of the new planet is larger, the time it will take to go in an orbit must decrease because M is inversely proportional to T squared. Since the new satellite travels the same circumference orbit, if the second satellite takes less time to go around, it must be going faster.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	✓ 2

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the force was applied in the satellite's direction of motion.
- ☐ 1 point is earned for a correct explanation relating the direction of force with the change in speed.

Example Response:

A force in the direction of the satellite's motion would make the satellite move faster

OR

A force in the direction of the satellite's motion would do positive work on the satellite.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	✓ 1

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

frq4

Student 1 is correct in saying that the satellite is moving faster at A

OR

The first statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is incorrect in saying that the satellite's angular momentum is larger at A than at B

OR

The second statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that the potential energy of the satellite-Earth system is higher at B than at A

OR

The second statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq4

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is incorrect in saying that the satellite is moving faster at B

OR

The first statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 2.
- ☐ 1 point is earned for a correct explanation relating field strength to distance.

Example Response:

Graph 2. The field is larger where the satellite is closer to the planet, so it is larger at A than at B.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 1.

frq4

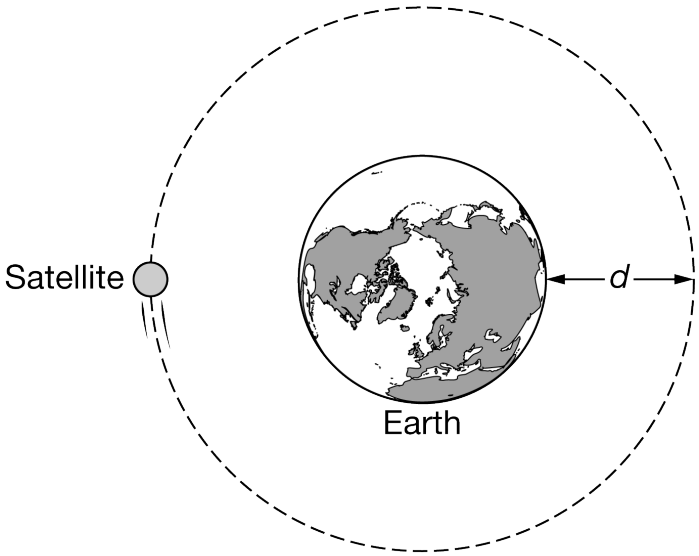
- [ ] 1 point is earned for a correct explanation of why the total mechanical energy is constant in terms of net external force.

Example Response:

Graph 1. The total mechanical energy is constant for the satellite-Earth system because there are no external forces on the system.

frq4

11. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface, as shown above.

An identical satellite is set into orbit around a new planet. The new planet's radius is the same as the radius of Earth but is significantly more massive than Earth. The satellite orbits the new planet with the same orbital period.

(a) Is the distance of the satellite from the surface of the new planet greater than, less than, or equal to d ? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

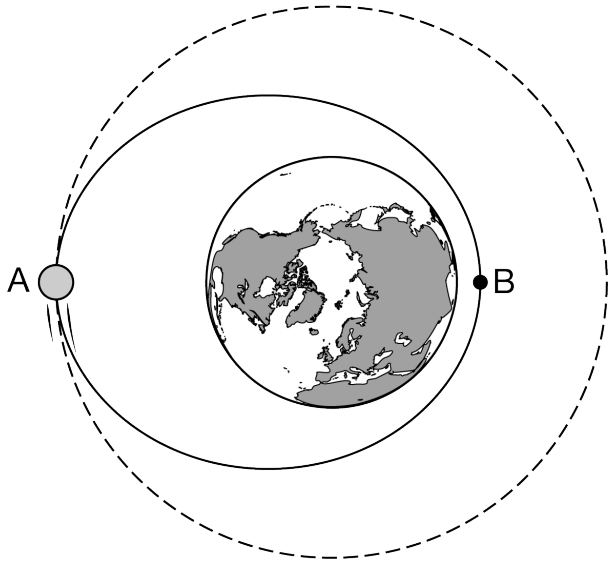
$\frac{Rv^2}{M} = C$

In the equation above

R is the distance from a planet's center to a satellite in a circular orbit;
v is the speed of the satellite;
M is the mass of the planet; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

frq4



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point A to point B, and state the following:

Student 1: The satellite is moving faster at point A because the total mechanical energy of the satellite-Earth system is constant.

Student 2: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is higher at point B than at point A.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

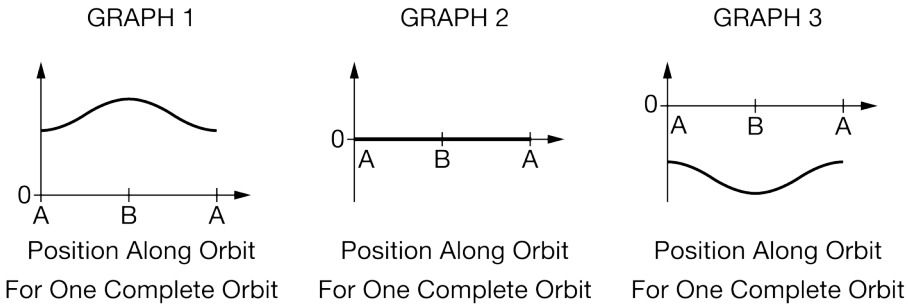
(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite

frq4

as it completes one elliptical orbit, starting and ending at point A.



(h) Which of the above graphs best represents the potential energy of the satellite-Earth system as the satellite orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the torque about the center of Earth exerted on the satellite by Earth's gravitational force as the satellite orbits? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the satellite around the new planet will orbit at a greater distance.
- ☐ 1 point is earned for correctly explaining an increasing distance from the planet to satellite is caused by an increasing speed to maintain a constant period.
  OR
  For correctly explaining an increased distance from the planet to decrease the gravitational force or acceleration on the satellite.

Example Response:

For the satellite to maintain the same period, it must orbit at a greater distance from the new planet because the satellite's orbital speed will be now be greater than before.

OR

"*The distance of the satellite must be further since increasing mass increases the force of gravity keeping the satellite in orbit, to maintain the same period, you would need to be further from the planet to decrease the gravitational force from the larger mass.*

Part B

frq4

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a consistent answer AND an attempt at reasoning using functional dependence.
- ☐ 1 point is earned for using $v = \frac{2\pi R}{T}$ or arguing that *R* must increase if *T* remains the same.
- ☐ 1 point is earned for correctly relating the increase in mass of the planet increases the distance from the planet to satellite.

Example Response:

Yes, this matches with my reasoning in part (a). The mass of the new planet is greater, which means that for C to stay constant the top of the fraction must also be greater. Since v is $\frac{2\pi R}{T}$ *and T is constant, R must be larger for the speed to be larger.*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the force was applied opposite to the satellite's direction of motion.
- ☐ 1 point is earned for a correct explanation relating the direction of force with the changing speed.

Example Response:

The force was applied opposite to the motion of the satellite's direction of motion because this would make the satellite move slower which does negative work on the satellite.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

frq4

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the total mechanical energy of the satellite-Earth system is constant

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is incorrect in saying that the satellite is moving faster at A

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that the satellite is moving faster at B

frq4

OR

The first statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is incorrect in saying that the potential energy of the satellite-Earth system is higher at B than at A

OR

The second statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 3.
- ☐ 1 point is earned for a correct explanation relating potential energy to distance.

Example Response:

Graph 3. The potential energy is higher where the satellite is farther from the planet, so it is higher at A than at B.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq4

0	1	2 ✓

The response accurately includes **both** of the following criteria.

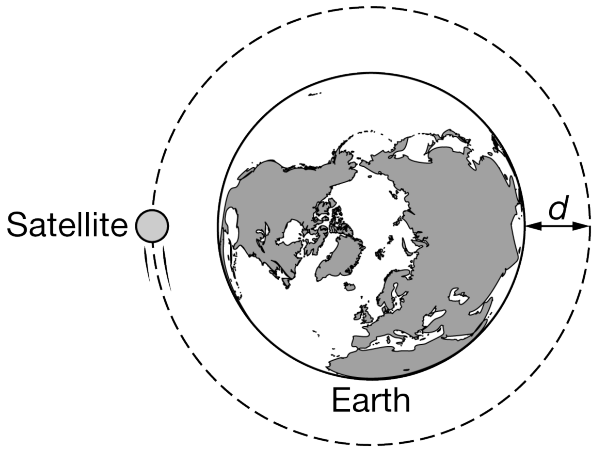
- ☐ 1 point is earned for indicating graph 2.
- ☐ 1 point is earned for a correct explanation why the torque is zero.

Example Response:

Graph 2. The torque is zero since the angle between $\vec{F}$ *and* $\vec{r}$ *is 180 degrees.*

frq4

12. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface.

An identical satellite is set into orbit around a new planet. The new planet's radius is the same as the radius of Earth but the new planet is significantly more massive than Earth. The satellite orbits the new planet with the same speed v.

(a) Is the distance of the satellite from the surface of the new planet greater than, less than, or equal to d ? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

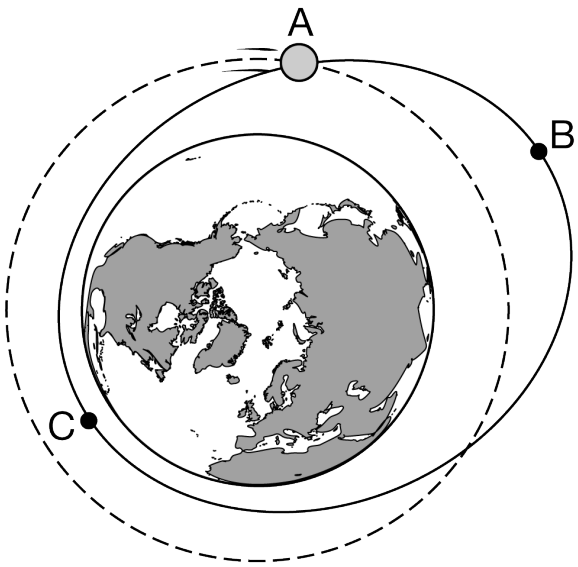
$$\frac{R^3}{MT^2} = C$$

In the equation above

R is the distance from a planet's center to a satellite in a circular orbit;
M is the mass of the planet;
T is the orbital period of the satellite; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

frq4



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit. In the modified orbit, the satellite's speed at point A, after the thrusters were fired, is the same as it was in the original orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point B to point C, and state the following:

Student 1: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is negative.

Student 2: The satellite is moving faster at point C because the total mechanical energy of the satellite-Earth system is greater at point C than at point B.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

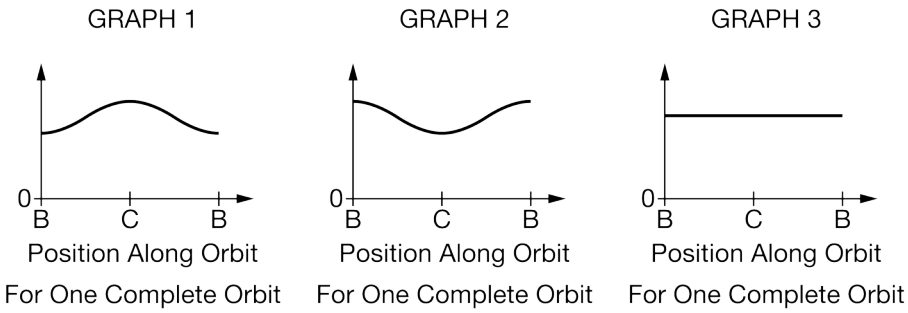
(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is

frq4

incorrect, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite as it completes one elliptical orbit, starting and ending at point B.



(h) Which of the above graphs best represents the magnitude of the acceleration of the satellite as it orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the magnitude of the angular momentum of the satellite with respect to the center of Earth as the satellite orbits? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the satellite around the new planet will orbit at a greater distance.
- ☐ 1 point is earned for a correct explanation in terms of centripetal and gravitational force or acceleration.

Example Response:

Equating the centripetal acceleration v^2/r *with the gravitational acceleration* GM/r^2, *we see that* v^2 *is proportional to* M/r. *So a bigger mass M means that the distance must be greater as well.*

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq4

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for an answer that is consistent with part (a) AND an attempt at reasoning using functional dependence.
- ☐ 1 point is earned for using $v = \frac{2\pi R}{T}$ or for arguing that *R* must increase if *v* remains the same.
- ☐ 1 point is earned for correctly relating the increase in mass of the planet increases the distance from the planet to the satellite.

Example Response:

Yes, it matches my reasoning. Since v is $\frac{2\pi R}{T}$ *and v is the same, if R increases then T must also increase. Since M and T both increase, R must increase to keep C constant.*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the force was applied perpendicular to the satellite's motion.
- ☐ 1 point is earned for a correct explanation relating the direction of force with change in speed.

Example Response:

The force was applied perpendicular to the satellite's motion because this force did not change the speed of the satellite.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

frq4

☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the potential energy of the satellite-Earth system is negative

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is incorrect in saying that the satellite is moving faster at B

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that the satellite is moving faster at C

OR

frq4

The first statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is incorrect in saying that the total mechanical energy of the satellite-Earth system is greater at C than at B.

OR

The second statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 1.
- ☐ 1 point is earned for a correct explanation that the gravitational field or acceleration increases as distance decreases.

Example Response:

Graph 1. The acceleration (or gravitational field) is greater closer to Earth, so it is greater at C than at B.

OR

Graph 1. The centripetal acceleration v^2/r is greater at C because v is greater, and r is smaller than at B.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq4

0	1	2 ✓

The response accurately includes **both** of the following criteria.

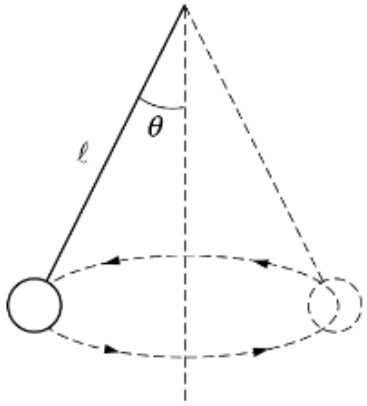
- ☐ 1 point is earned for indicating graph 3.
- ☐ 1 point is earned for a correct explanation why the angular momentum is constant throughout the motion.

Example Response:

Graph 3. The angular momentum is constant because the torque on the satellite is zero.

frq4

13.



A ball attached to a string of length l swings in a horizontal circle, as shown above, with a constant speed. The string makes an angle θ with the vertical, and T is the magnitude of the tension in the string. Express your answers to the following in terms of the given quantities and fundamental constants.

a. On the figure below, draw and label vectors to represent all the forces acting on the ball when it is at the position shown in the diagram. The lengths of the vectors should be consistent with the relative magnitudes of the forces.



b. Determine the mass of the ball.

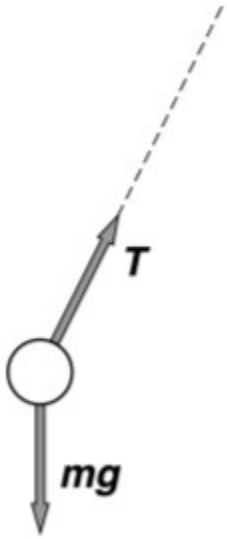
c. Determine the speed of the ball.

d. Determine the frequency of revolution of the ball.

e. Suppose that the string breaks as the ball swings in its circular path. Qualitatively describe the trajectory of the ball after the string breaks but before it hits the ground.

Part A

frq4



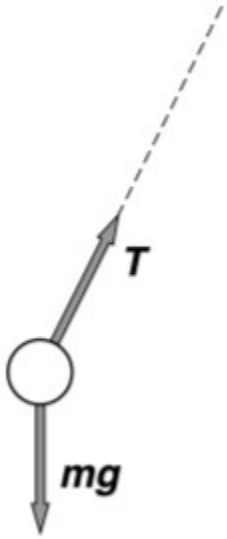
2 points are earned: For each correctly drawn and appropriately labeled force

1 point is earned: For no incorrect forces

The horizontal and vertical components of the tension were acceptable if properly labeled

0	1	2	3 ✓

The student response earns all of the following points:



2 points are earned: For each correctly drawn and appropriately labeled force

1 point is earned: For no incorrect forces

frq4

The horizontal and vertical components of the tension were acceptable if properly labeled

Part B

1 point is earned: For indicating that the sum of the vertical forces is zero

$\sum Fy = 0$

1 point is earned: For correctly substituting the applicable forces

$T \cos \theta - mg = 0$

1 point is earned: For the correct answer

$m = \frac{T \cos \theta}{g}$

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For indicating that the sum of the vertical forces is zero

$\sum Fy = 0$

1 point is earned: For correctly substituting the applicable forces

$T \cos \theta - mg = 0$

1 point is earned: For the correct answer

$m = \frac{T \cos \theta}{g}$

Part C

The centripetal force is supplied by the horizontal component of the tension

$Fc = T \sin \theta$

1 point is earned: For using a correct expression for the centripetal force *Fc*

$\frac{mv^2}{r} = T \sin \theta$

1 point is earned: For eliminating the tension or the mass using the answer to part (b)

$\frac{mv^2}{r} = mg\frac{\sin \theta}{\cos \theta} = mg \tan \theta$ OR $\frac{T \cos \theta v^2}{gr} = T \sin \theta$

1 point is earned: For using the substitution $r = l \sin \theta$

frq4

$\frac{mv^2}{\diamondsuit \sin\theta} = mg\tan\theta$ OR $\frac{T\cos\theta v^2}{g\diamondsuit \sin\theta} = T\sin\theta$

1 point is earned: For the correct answer

$v = \sqrt{g\diamondsuit \sin\theta\tan\theta}$ OR $\sqrt{\frac{g\diamondsuit \sin^2\theta}{\cos\theta}}$

0	1	2	3	4 ✓

The student response earns all of the following points:

The centripetal force is supplied by the horizontal component of the tension

Fc = *T* sin θ

1 point is earned: For using a correct expression for the centripetal force *Fc*

$\frac{mv^2}{r} = T\sin\theta$

1 point is earned: For eliminating the tension or the mass using the answer to part (b)

$\frac{mv^2}{r} = mg\frac{\sin\theta}{\cos\theta} = mg\tan\theta$ OR $\frac{T\cos\theta v^2}{gr} = T\sin\theta$

1 point is earned: For using the substitution *r* = *l* sin θ

$\frac{mv^2}{\diamondsuit \sin\theta} = mg\tan\theta$ OR $\frac{T\cos\theta v^2}{g\diamondsuit \sin\theta} = T\sin\theta$

1 point is earned: For the correct answer

$v = \sqrt{g\diamondsuit \sin\theta\tan\theta}$ OR $\sqrt{\frac{g\diamondsuit \sin^2\theta}{\cos\theta}}$

Part D

1 point is earned: For using a correct expression for period or frequency

$T = \frac{2\pi r}{v}$ OR $f = \frac{v}{2\pi r}$

1 point is earned: For substituting the velocity from part (c)

1 point is earned: For the correct answer (unsimplified acceptable)

$f = \frac{\sqrt{gl\sin\theta\tan\theta}}{2\pi l\sin\theta}$ OR $\frac{1}{2\pi}\sqrt{\frac{g\tan\theta}{l\sin\theta}}$ OR $\frac{1}{2\pi}\sqrt{\frac{g}{l\cos\theta}}$

frq4

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For using a correct expression for period or frequency

$T = \frac{2\pi r}{v}$ OR $f = \frac{v}{2\pi r}$

1 point is earned: For substituting the velocity from part (c)

1 point is earned: For the correct answer (unsimplified acceptable)

$f = \frac{\sqrt{gl \sin\theta \tan\theta}}{2\pi l \sin\theta}$ OR $\frac{1}{2\pi}\sqrt{\frac{g \tan\theta}{l \sin\theta}}$ OR $\frac{1}{2\pi}\sqrt{\frac{g}{l \cos\theta}}$

Part E

1 point is earned: For any indication that the initial velocity of the ball is horizontal

1 point is earned: For any indication that the subsequent trajectory is parabolic

0	1	2 ✓

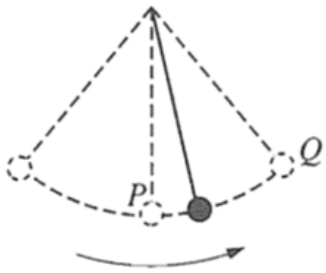
The student response earns all of the following points:

1 point is earned: For any indication that the initial velocity of the ball is horizontal

1 point is earned: For any indication that the subsequent trajectory is parabolic

frq4

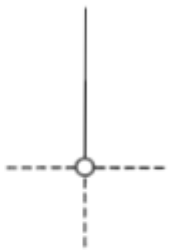
14.



A heavy ball swings at the end of a string as shown above, with negligible air resistance. Point *P* is the lowest point reached by the ball in its motion, and point *Q* is one of the two highest points.

(a) On the following diagrams draw and label vectors that could represent the velocity and acceleration of the ball at points *P* and *Q*. If a vector is zero, explicitly state this fact. The dashed lines indicate horizontal and vertical directions.

i. Point *P*



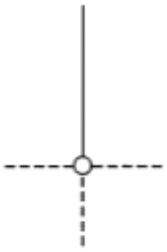
ii. Point *Q*



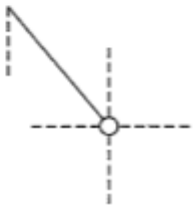
(b) After several swings, the string breaks. The mass of the string and air resistance are negligible. On the following diagrams, sketch the path of the ball if the break occurs when the ball is at point *P* or point *Q*. In each case, briefly describe the motion of the ball after the break.

frq4

i. Point P

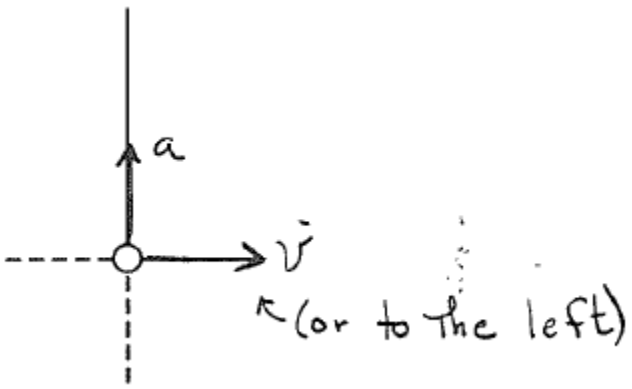


ii. Point Q



Part A

(i)



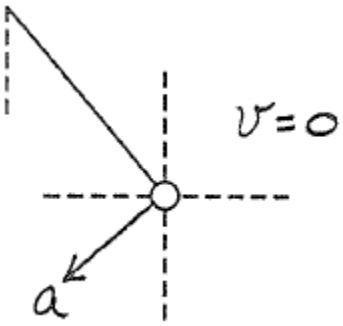
1 point is earned: For a correctly drawn velocity vector, directed along the horizontal line, either to the right or the left

1 point is earned: For the correctly drawn acceleration vector, directed upward along the string

Vectors must be labeled, and have an arrowhead to show their direction

frq4

(ii)



1 point is earned: For indicating that the velocity is zero

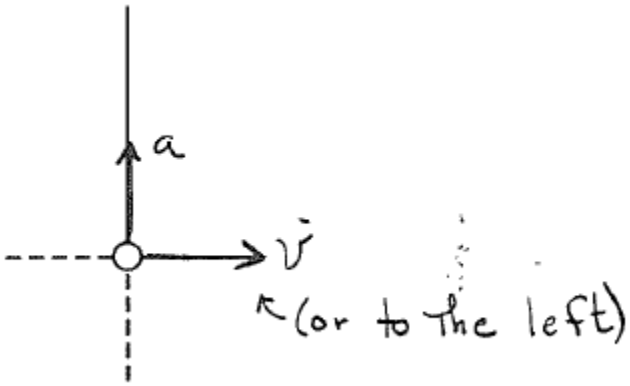
2 points are earned: For the correctly drawn acceleration vector, pointing down and to the left, perpendicular to the string

One of these points was awarded if the vector was in the proper quadrant, not perpendicular to the string, and not on either the vertical or horizontal lines

0	1	2	3	4	5 ✓

The student response earns five of the following points:

(i)



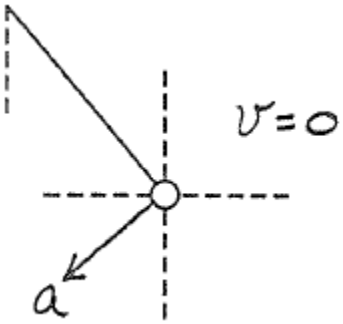
1 point is earned: For a correctly drawn velocity vector, directed along the horizontal line, either to the right or the left

1 point is earned: For the correctly drawn acceleration vector, directed upward along the string

Vectors must be labeled, and have an arrowhead to show their direction

(ii)

frq4



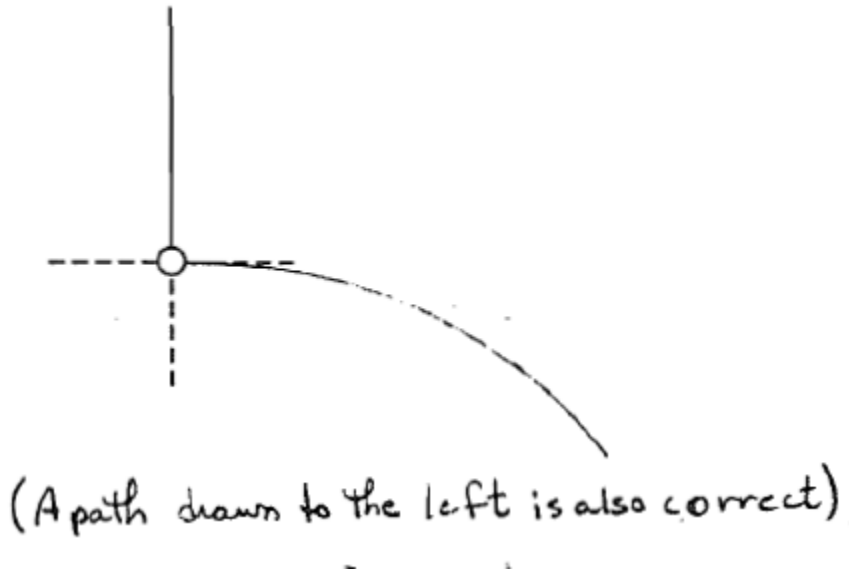
1 point is earned: For indicating that the velocity is zero

2 points are earned: For the correctly drawn acceleration vector, pointing down and to the left, perpendicular to the string

One of these points was awarded if the vector was in the proper quadrant, not perpendicular to the string, and not on either the vertical or horizontal lines

Part B

(i)



1 point is earned: For indicating that the path is initially horizontal (via figure or words)

1 point is earned: For indicating that the horizontal velocity is constant

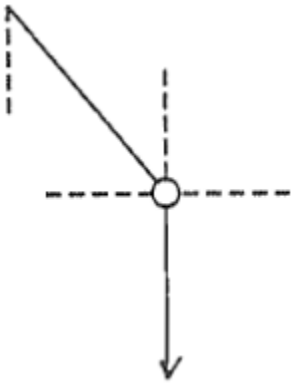
1 point is earned: For indicating that the vertical motion is under constant acceleration or free-fall

The last two points were also awarded for describing the path as a parabola, or indicating that the ball is a projectile

A maximum of 2 points were awarded for a correct figure with no description or a correct description with no path drawn on the figure

(ii)

frq4



1 point is earned: For correctly indicating that the ball falls straight down

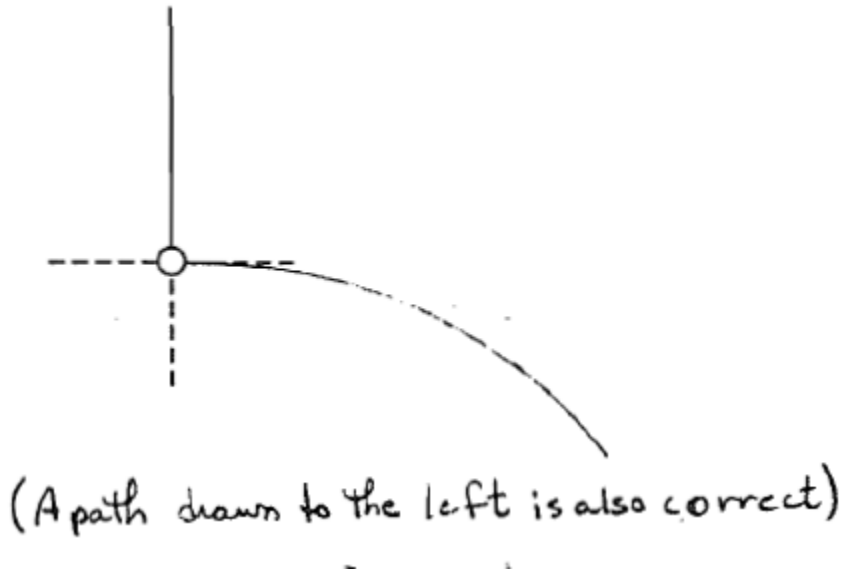
1 point is earned: For indicating that the vertical motion is under constant acceleration or free-fall

A maximum of 1 point was awarded for a correct figure with no description a correct description with no path drawn on the figure

0	1	2	3	4	5 ✓

The student response earns five of the following points:

(i)



1 point is earned: For indicating that the path is initially horizontal (via figure or words)

1 point is earned: For indicating that the horizontal velocity is constant

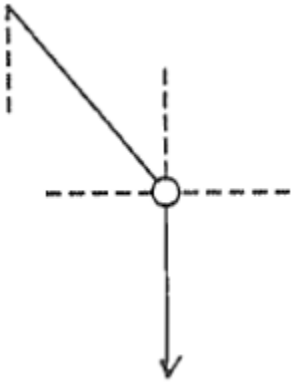
1 point is earned: For indicating that the vertical motion is under constant acceleration or free-fall

frq4

The last two points were also awarded for describing the path as a parabola, or indicating that the ball is a projectile

A maximum of 2 points were awarded for a correct figure with no description or a correct description with no path drawn on the figure

(ii)



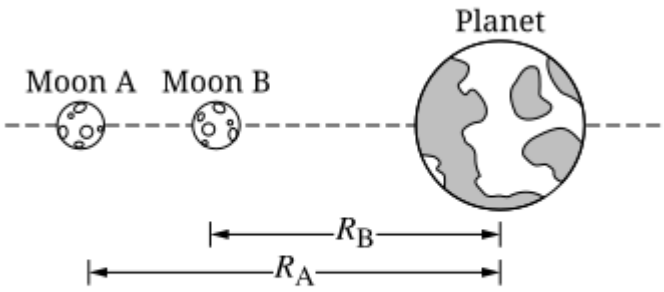
1 point is earned: For correctly indicating that the ball falls straight down

1 point is earned: For indicating that the vertical motion is under constant acceleration or free-fall

A maximum of 1 point was awarded for a correct figure with no description a correct description with no path drawn on the figure

frq4

15. This question is a long free-response question. Show your work for each part of the question.



Note: Figure not drawn to scale.

(12 points, suggested time 25 minutes)

Two identical moons, Moon A and Moon B, orbit a planet. The mass m_0 of each moon is significant, but less than the mass m_p of the planet. At some point in their orbits, the planet and the two moons are aligned as shown in the figure.

(a) The following dots represent the two moons when they are at the locations shown in the previous figure. On each dot, draw and label the forces (not components) exerted on Moon A and on Moon B. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.



(b) Consider the net gravitational force exerted on each moon due to the planet and the other moon.

frq4

i. Justify why the magnitude of the net force exerted on Moon A could be larger than the magnitude of the net force exerted on Moon B.

ii. Justify why the magnitude of the net force exerted on Moon B could be larger than the magnitude of the net force exerted on Moon A.

(c) Derive expressions for both of the following quantities. Express your answers in terms of m_0, m_p, R_A, R_B, and physical constants, as appropriate.

• The net force F_A exerted on Moon A

• The net force F_B exerted on Moon B

(d)

i. Could the expressions in part (c) support your reasoning in part (b)(i) ?

___ Yes ___ No

Explain your reasoning.

ii. Could the expressions in part (c) support your reasoning in part (b)(ii) ?

___ Yes ___ No

Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for two forces directed to the right on Moon A, correctly labeled, with no extraneous forces.
- ☐ 1 point is earned for two horizontal forces in opposite directions on Moon B. The labels on the forces must be correct and distinguishable from each other, with no extraneous forces.

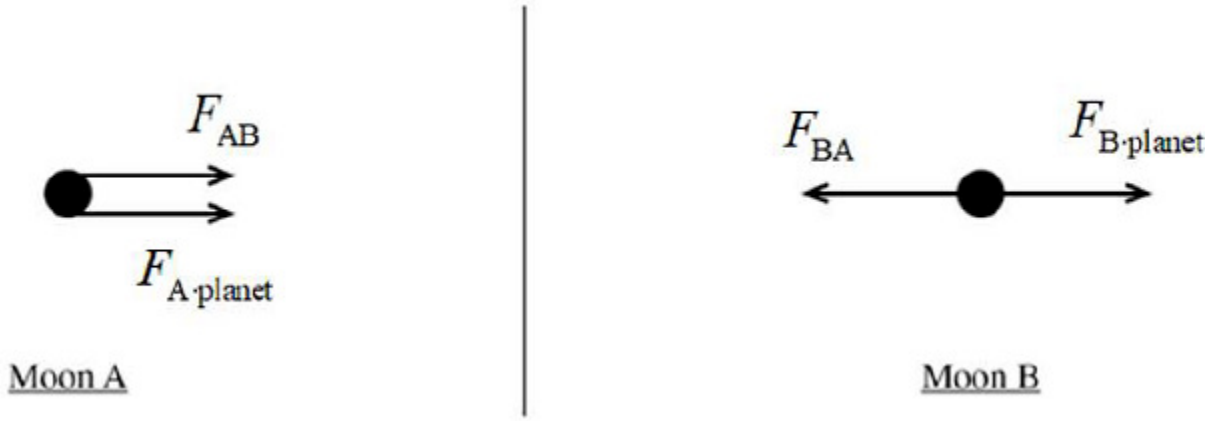
Scoring Notes:

· Acceptable labels for forces include: F_AB, $F_\text{A on B}$, F_planet, F_moon, F_0, F_p, $F_{g\text{A}}$, etc.

· Maximum 1 point can be earned if the arrows do not start on the dot.

Example Response:

frq4



Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** the following criteria.

- ☐ 1 point is earned for indicating the force vectors on Moon A point in the same direction and therefore the magnitudes add, while the force vectors on Moon B point in opposite directions and the magnitudes are to be subtracted.
  Example Response:
  The force vectors on Moon A point in the same direction and therefore add, while the force vectors on Moon B point in opposite directions and will subtract.
- ☐ 1 point is earned for a justification based on the inverse relation between gravitational force and distance.
  Scoring Note: This point may be earned at any point in part (b).
- ☐ 1 point is earned for indicating Moon B is closer to the planet than Moon A and therefore the gravitational force exerted by the planet is larger for Moon B than for Moon A.
  Example Response:
  The gravitational force is greater for objects that are closer together, and Moon B is closer to the planet than Moon A, so the gravitational force from the planet is greater for Moon B than for Moon A.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** the following criteria.

frq4

- [ ] 1 point is earned for using the law of gravitation at least once.
- [ ] 1 point is earned for substituting in correct distances in all expressions.
- [ ] 1 point is earned for substituting in correct masses in all expressions.
- [ ] 1 point is earned for correctly adding terms for ΣF_{A} and subtracting terms for ΣF_{B}.

Scoring Note: An overall negative sign on either force is acceptable.

Example Response:

Moon A

$F_g = G\frac{m_1 m_2}{r^2}$

$\Sigma F_{\mathrm{A}} = F_{\mathrm{A\cdot planet}} + F_{\mathrm{AB}}$

$\Sigma F_{\mathrm{A}} = G\frac{m_0 m_p}{{R_{\mathrm{A}}}^2} + G\frac{{m_0}^2}{(R_{\mathrm{A}} - R_{\mathrm{B}})^2}$

$\Sigma F_{\mathrm{A}} = Gm_0\left(\frac{m_p}{{R_{\mathrm{A}}}^2} + \frac{m_0}{(R_{\mathrm{A}} - R_{\mathrm{B}})^2}\right)$

Moon B

$F_g = G\frac{m_1 m_2}{r^2}$

$\Sigma F_{\mathrm{B}} = F_{\mathrm{B\cdot planet}} - F_{\mathrm{BA}}$

$\Sigma F_{\mathrm{B}} = G\frac{m_0 m_p}{{R_{\mathrm{B}}}^2} - G\frac{{m_0}^2}{(R_{\mathrm{A}} - R_{\mathrm{B}})^2}$

$\Sigma F_{\mathrm{B}} = Gm_0\left(\frac{m_p}{{R_{\mathrm{B}}}^2} - \frac{m_0}{(R_{\mathrm{A}} - R_{\mathrm{B}})^2}\right)$

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** the following criteria.

- [ ] 1 point is earned for addressing the functional dependence expressed in the equations from part (c).
  Scoring Notes: It is not necessary to use the functional dependence correctly to earn this point. This point may be earned at any point in part (d).
- [ ] 1 point is earned for a correct explanation for why the expressions in part (c) either support or do not support the reasoning consistent with response in part (b).
  Example Response:
  Yes. For the net force on Moon A, both force terms have the same sign, so they add, while for the net force on Moon B the two terms have opposite signs, so they have a cancelling effect.
- [ ] 1 point is earned for a correct explanation for why the expressions in part (c) either support or do not support the reasoning consistent with response in part (b).

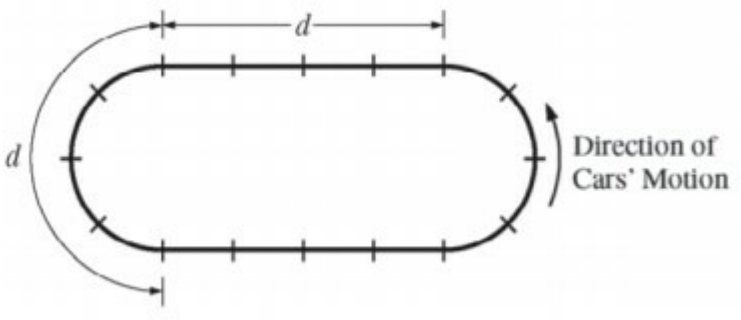
frq4

Example Response:

The gravitational force has an inverse relationship with distance. If Moon A is very far away, ΣF_{A} *from part (c). will be small. If Moon B is close to the planet while Moon A is far away, the force toward the planet would be big while the force toward the moon would be small such that* ΣF_{B} *could be larger than* ΣF_{A}.

frq4

16.



The figure above represents a racetrack with semicircular sections connected by straight sections. Each section has length d, and markers along the track are spaced $d/4$ apart. Two people drive cars counterclockwise around the track, as shown. Car X goes around the curves at constant speed v_c, increases speed at constant acceleration for half of each straight section to reach a maximum speed of $2v_c$, then brakes at constant acceleration for the other half of each straight section to return to speed v_c. Car Y also goes around the curves at constant speed v_c, increases speed at constant acceleration for one-fourth of each straight section to reach the same maximum speed $2v_c$, stays at that speed for half of each straight section, then brakes at constant acceleration for the remaining fourth of each straight section to return to speed v_c.

(a) On the figures below, draw an arrow showing the direction of the net force on each of the cars at the positions noted by the dots. If the net force is zero at any position, label the dot with 0.



(b)

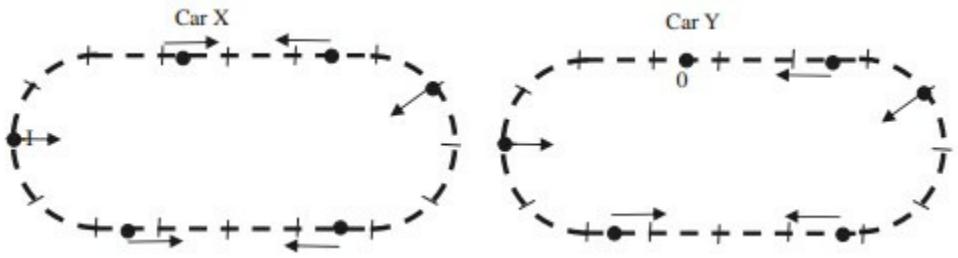
i. Indicate which car, if either, completes one trip around the track in less time, and justify your answer quantitatively without using equations.

ii. Justify your answer about which car, if either, completes one trip around the track in less time quantitatively with appropriate equations.

(c) Explain how your equations in part (b) ii reexpress your reasoning in part (b) i. Do not simply refer to any final results of your calculations, but instead indicate how terms in your equations correspond to concepts in your qualitative explanation.

Part A

frq4



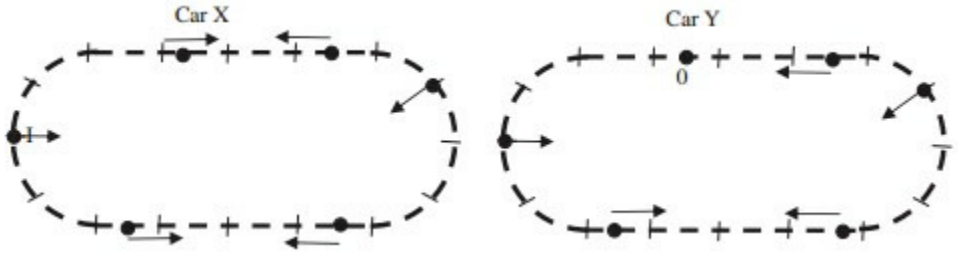
1 point is earned for correct directions of the net forces at all the locations on the semicircular sections (i.e., all directed generally toward the center of the circle)

1 point is earned for correct directions of the net forces at all the locations on the bottom straightaways (i.e., directed toward the center of the segment)

1 point is earned for correct directions of the net forces at all locations on the top straightaway (i.e., both rightmost arrows directed toward the left, the left one for car X directed toward the right, and the left one for car Y equal to zero)

0	1	2	3 ✓

The student response earns all of the following points:



1 point is earned for correct directions of the net forces at all the locations on the semicircular sections (i.e., all directed generally toward the center of the circle)

1 point is earned for correct directions of the net forces at all the locations on the bottom straightaways (i.e., directed toward the center of the segment)

1 point is earned for correct directions of the net forces at all locations on the top straightaway (i.e., both rightmost arrows directed toward the left, the left one for car X directed toward the right, and the left one for car Y equal to zero)

Part B

1 point is earned for realizing that the difference in time is only on the straightaways

1 point is earned for correct reasoning leading to Car Y taking a shorter time on the straightaways

Example:

Car *X* takes longer to accelerate and does not spend any time traveling at top speed. Car *Y* accelerates over a shorter time

frq4

and spends time going at top speed. So car *Y* must cover the straightaways in a shorter time. Curves take the same time, so car *Y* must overall take a shorter time.

The time to travel each curve is *d/vc*. Answers can be expressed in terms of *d/vc* or *tc* = *d/v*c or some other defined unit of time. The calculations below will use *t*c = *d/v*c.

1 point is earned for stating that the time to travel each curve is d/vc

1 point is earned for correct kinematics expressions that allow determination of the time it takes for one segment of acceleration on the straightaways

$Example: D = v_c t_1 + \frac{1}{2} a t_1^2, a = (2v_c - v_c)/t_1 = v_c/t_1$

1 point is earned for work that shows an understanding of how to determine the time that car X and car Y each spend accelerating

1 point is earned for work that shows an understanding of how to determine the time that car Y spends at constant speed

1 point is earned for correctly determining the total straightaway times for each car

Calculating the time for car X to travel one straightaway:

$$\frac{d}{2} = v_c t_1 + \frac{1}{2} a t_1^2, a = (2v_c - v_c)/t_1 = v_c/t_1$$
$$t_1 = \frac{d}{3v_c} = \frac{t_c}{3}, \text{total time is } \frac{2t_c}{3}$$

Calculating the time for car Y to travel one straightaway:

Doing the calculation shown above using the distance of acceleration gives the result that one section of acceleration takes a time *tc*/6.

The time for car Y to travel one constant speed section on the straightaway is (*d*/2)/2*vc* = (*tc*/4)

Adding three segments to get the total time for one straightaway gives 7*tc*/12.

The calculations show that car *Y* takes less time on a straightaway, and both cars take the same time on the curves, so car *Y* overall takes less time.

✓

0	1	2	3	4	5	6	7

The student response earns all of the following points:

1 point is earned for realizing that the difference in time is only on the straightaways

1 point is earned for correct reasoning leading to Car Y taking a shorter time on the straightaways

Example:

frq4

Car *X* takes longer to accelerate and does not spend any time traveling at top speed. Car *Y* accelerates over a shorter time and spends time going at top speed. So car *Y* must cover the straightaways in a shorter time. Curves take the same time, so car *Y* must overall take a shorter time.

The time to travel each curve is *d/vc*. Answers can be expressed in terms of *d/vc* or *tc* = *d/v*c or some other defined unit of time. The calculations below will use *t*c = *d/v*c.

1 point is earned for stating that the time to travel each curve is d/vc

1 point is earned for correct kinematics expressions that allow determination of the time it takes for one segment of acceleration on the straightaways

$Example: D = v_c t_1 + \frac{1}{2} a t_1^2, a = (2v_c - v_c)/t_1 = v_c/t_1$

1 point is earned for work that shows an understanding of how to determine the time that car X and car Y each spend accelerating

1 point is earned for work that shows an understanding of how to determine the time that car Y spends at constant speed

1 point is earned for correctly determining the total straightaway times for each car

Calculating the time for car X to travel one straightaway:

$$\frac{d}{2} = v_c t_1 + \frac{1}{2} a t_1^2, a = (2v_c - v_c)/t_1 = v_c/t_1$$
$$t_1 = \frac{d}{3v_c} = \frac{t_c}{3}, \text{total time is } \frac{2t_c}{3}$$

Calculating the time for car Y to travel one straightaway:

Doing the calculation shown above using the distance of acceleration gives the result that one section of acceleration takes a time *tc*/6.

The time for car Y to travel one constant speed section on the straightaway is (*d*/2)/2*vc* = (*tc*/4)

Adding three segments to get the total time for one straightaway gives 7*tc*/12.

The calculations show that car *Y* takes less time on a straightaway, and both cars take the same time on the curves, so car *Y* overall takes less time.

Part C

1 point is earned for linking math to one aspect of qualitative reasoning that explains the difference in times

1 point is earned for linking math to all other qualitative reasoning that explains the difference in times

Examples:

The only difference in the calculations for the time of one segment of linear acceleration is the difference in distances. That shows that car *X* takes longer to accelerate. The equation(*d*/2)/2*vc* = (*tc*/4) corresponds to car *Y* traveling for a time at top speed.

Substituting *a* = *vc*/t1 into the displacement equation in part (b) ii gives D = (3/2)*vct1*. This shows that a car takes less

frq4

time to reach its maximum speed when it accelerates over a shorter distance. This means Car *Y* reaches its maximum speed more quickly and therefore spends more time at its maximum speed than Car *X* does, as argued in part (b) i.

0	1	2 ✓

The student response earns all of the following points:

1 point is earned for linking math to one aspect of qualitative reasoning that explains the difference in times

1 point is earned for linking math to all other qualitative reasoning that explains the difference in times

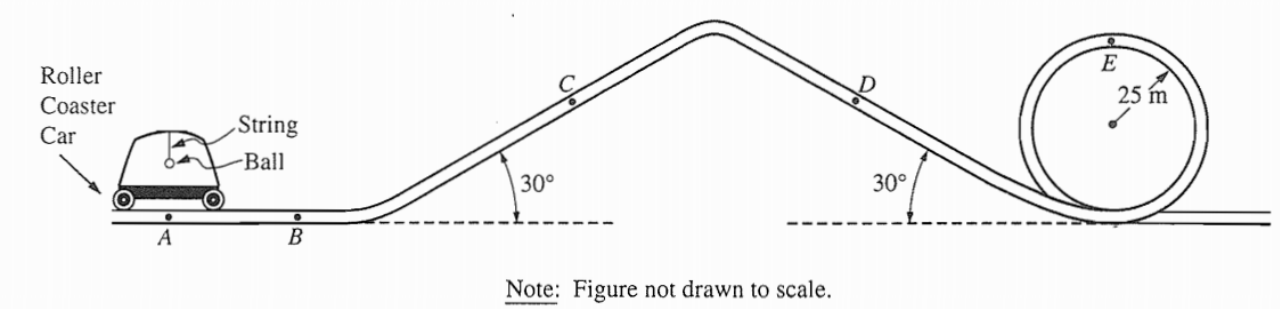
Examples:

The only difference in the calculations for the time of one segment of linear acceleration is the difference in distances. That shows that car *X* takes longer to accelerate. The equation(d/2)/2vc = (tc/4) corresponds to car *Y* traveling for a time at top speed.

Substituting $a = vc$/t1 into the displacement equation in part (b) ii gives D = (3/2)$vct1$. This shows that a car takes less time to reach its maximum speed when it accelerates over a shorter distance. This means Car *Y* reaches its maximum speed more quickly and therefore spends more time at its maximum speed than Car *X* does, as argued in part (b) i.

frq4

17.



Part of the track of an amusement park roller coaster is shaped as shown above. A safety bar is oriented lengthwise along the top of each car. In one roller coaster car, a small 0.10-kilogram ball is suspended from this bar by a short length of light, inextensible string.

(a) Initially, the car is at rest a point A.

i. On the diagram to the right, draw and label all the forces acting on the 0.10-kilogram ball. 

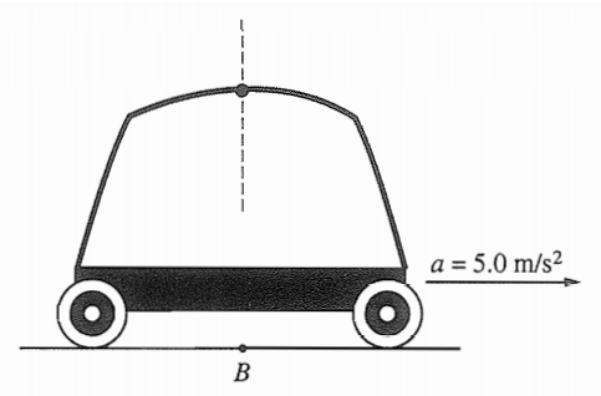
ii. Calculate the tension in the string.

The car is then accelerated horizontally, goes up a 30^o incline, goes down a 30^o incline, and then goes around a vertical circular loop of radius 25 meters. For each of the four situations described in parts (b) to (e), do all three of the following. In each situation, assume that the ball has stopped swinging back and forth.

• Determine the horizontal component T_h of the tension in the string in newtons and record your answer in the space provided.

• Determine the vertical component T_v of the tension in the string in newtons and record your answer in the space provided.

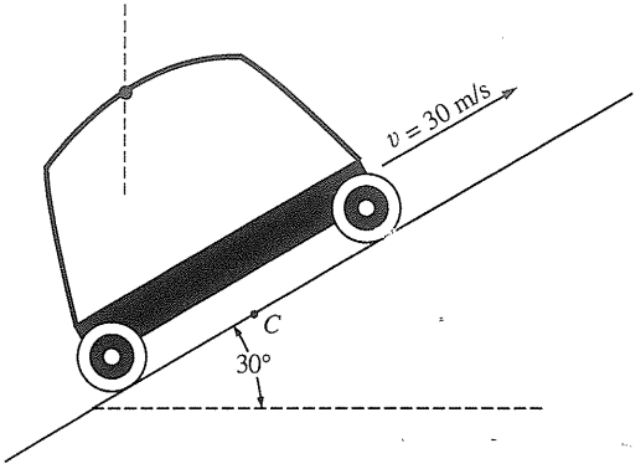
• Show on the adjacent diagram the approximate direction of the string with respect to the vertical. The dashed line shows the vertical in each situation.



frq4

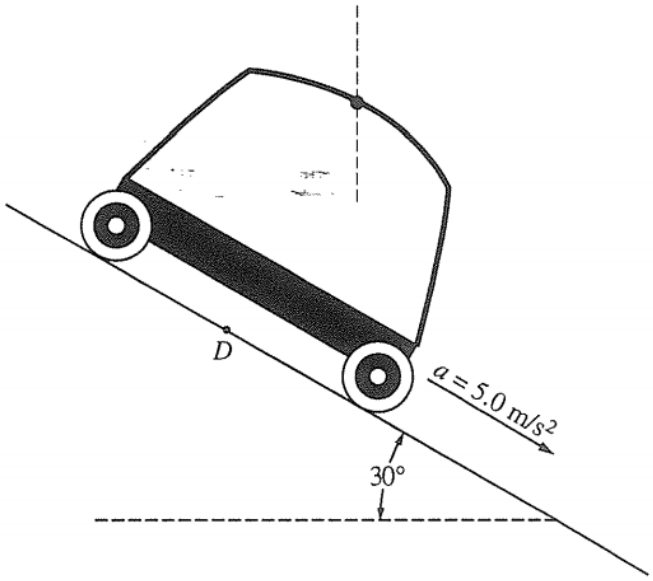
(b) The car is at point B moving horizontally to the right with an acceleration of 5.0 m/s^2.

T_h =_____N T_v =_____N



(c) The car is at point C and is being pulled up the 30^o incline with a constant speed of 30 m/s

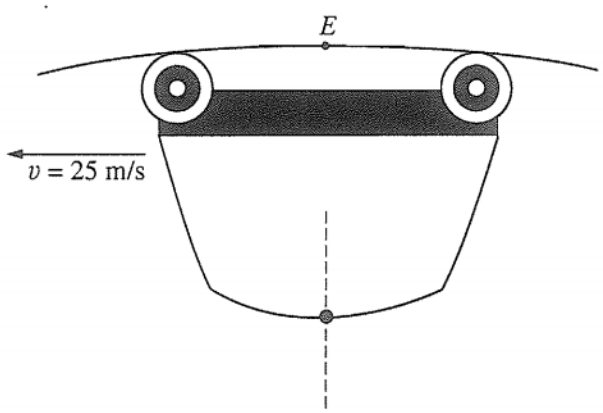
T_h =_____N T_v =_____N



(d) The car is at point D moving down the 30^o incline with an acceleration of 5.0 m/s^2.

T_h =_____N T_v =_____N

frq4



(e) The car is at point E moving upside down with an instantaneous speed of 25 m/s and no tangential acceleration at the top of the vertical loop of radius 25 meters.

T_h =_____N T_v =_____N

Part A(i,ii)

(i)



1 point is earned: For one correctly drawn and labeled force

1 point is earned: For a second correctly drawn and labeled force and no additional forces.

(ii)

Applying Newton's second law to the forces drawn above:

T = mg

T = (0.10 kg)(10 m/s2)

1 point is earned: For a correct answer

T = 1N

frq4

0	1	2	3 ✓

The student response earns three of the following points:



(i)

1 point is earned: For one correctly drawn and labeled force

1 point is earned: For a second correctly drawn and labeled force and no additional forces.

(ii)

Applying Newton's second law to the forces drawn above:

T = mg

T = (0.10 kg)(10 m/s2)

1 point is earned: For a correct answer

T = 1N

Part B

The horizontal component of the tension supplies the horizontal acceleration.

Tb = ma

Tb = (0.10kg)(5.0 m/s2)

1 point is earned: For a correct answer

Tb = 0.5N

The vertical component of the tension is equal to the weight of the ball, as in (a)ii.

1 point is earned: For a correct answer

frq4

Tv = 1N

1 point is earned: For correctly drawing the string hanging down at some angle between vertical and horizontal, and away from the direction of the acceleration.

0	1	2	3 ✓

The student response earns three of the following points:

The horizontal component of the tension supplies the horizontal acceleration.

Tb = ma

Tb = (0.10kg)(5.0 m/s2)

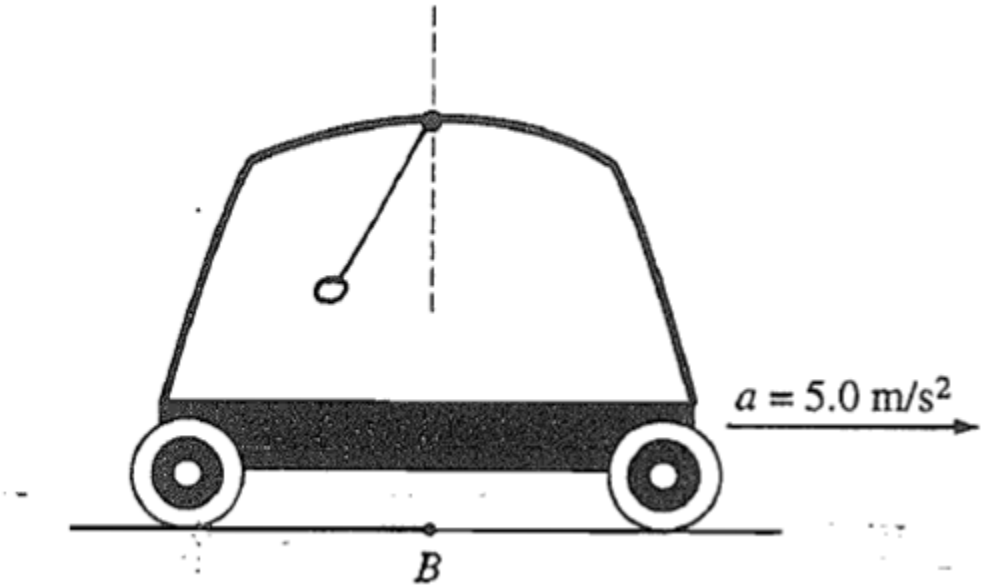
1 point is earned: For a correct answer

Tb = 0.5N

The vertical component of the tension is equal to the weight of the ball, as in (a)ii.

1 point is earned: For a correct answer

Tv = 1N



1 point is earned: For correctly drawing the string hanging down at some angle between vertical and horizontal, and away from the direction of the acceleration.

frq4

Part C

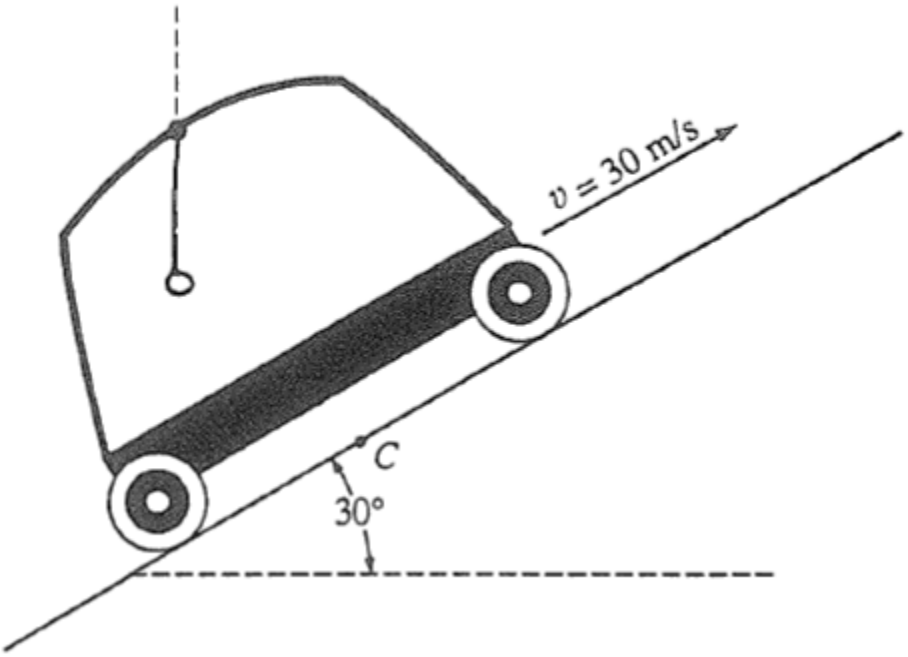
Since there is no acceleration, the sum of the forces must be zero, so the tension is equal and opposite to the weight of the ball.

1 point is earned: For a correct answer

Tb = zero

1 point is earned: For a correct answer

Tv = 1N



1 point is earned: For correctly drawing the string downward

0	1	2	3 ✓

The student response earns three of the following points:

Since there is no acceleration, the sum of the forces must be zero, so the tension is equal and opposite to the weight of the ball.

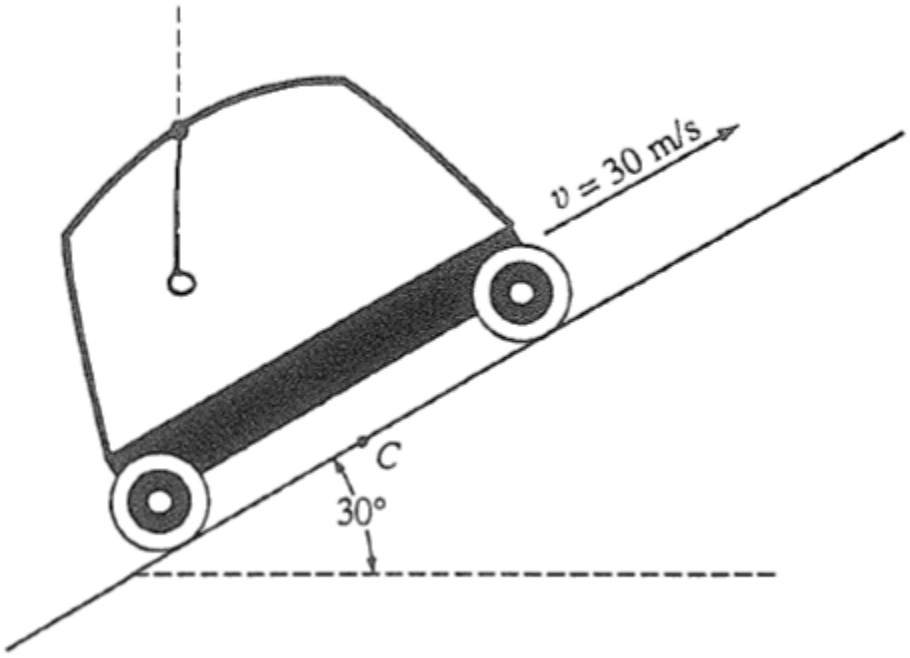
1 point is earned: For a correct answer

Tb = zero

1 point is earned: For a correct answer

frq4

Tv = 1N



1 point is earned: For correctly drawing the string downward

Part D

The horizontal component of the tension is responsible for the horizontal component of the acceleration. Applying Newton's second law:

Tb = ma cos θ, where θ is the angle between the acceleration and horizontal

Tb = (0.10kg)(5.0 m/s2)cos 30o.

1 point is earned: For a correct answer

Tb = 0.43N

The verticle component of the tension counteracts only part of the gravitational force, resulting in a vertical component of the acceleration.

Applying Newton's secondlaw:

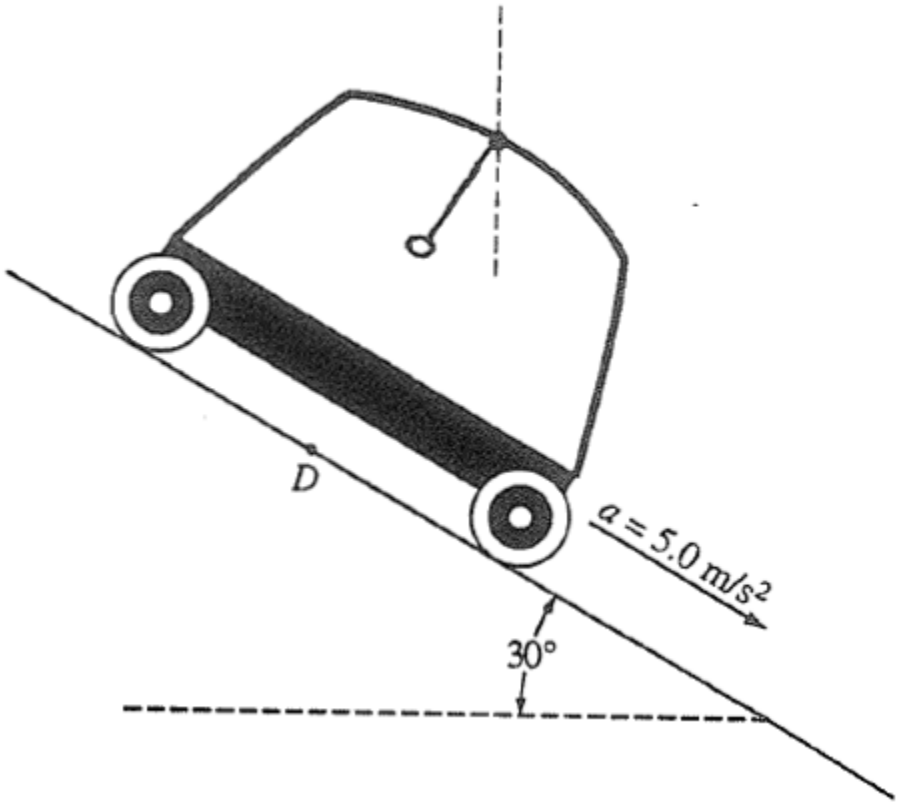
Tv = mg - ma sin θ

Tv = (0.10 kg)(10 m/s2)-(0.10 kg)(5.0 m/s2)sin 30o

1 point is earned: For a correct answer

Tv = 0.75 N

frq4



1 point is earned: For correctly drawing the string hanging down at some angle between vertical and horizontal, and away from the direction of acceleration

0	1	2	3 ✓

The student response earns three of the following points:

The horizontal component of the tension is responsible for the horizontal component of the acceleration. Applying Newton's second law:

Tb = ma cos θ, where θ is the angle between the acceleration and horizontal

Tb = (0.10kg)(5.0 m/s2)cos 30o.

1 point is earned: For a correct answer

Tb = 0.43N

The verticle component of the tension counteracts only part of the gravitational force, resulting in a vertical component of the acceleration.

Applying Newton's secondlaw:

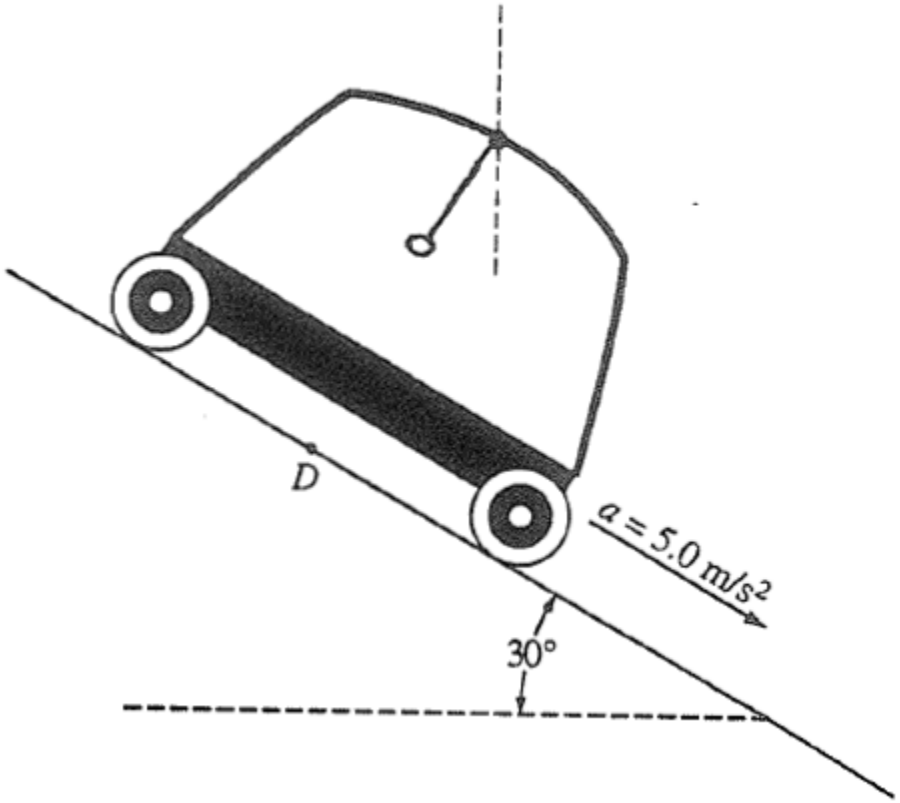
frq4

Tv = mg - ma sin θ

Tv = (0.10 kg)(10 m/s2)-(0.10 kg)(5.0 m/s2)sin 30o

1 point is earned: For a correct answer

Tv = 0.75 N



1 point is earned: For correctly drawing the string hanging down at some angle between vertical and horizontal, and away from the direction of acceleration

Part E

Since there is no horizontal acceleration, there is no horizontal component of the tension

1 point is earned: For a correct answer

Tb = Zero

Assuming for the moment that the string is hanging downward, the centripetal force is the difference between the gravitational force and the tension.Applying Newton's second law.

Solving for the vertical component of the tension:

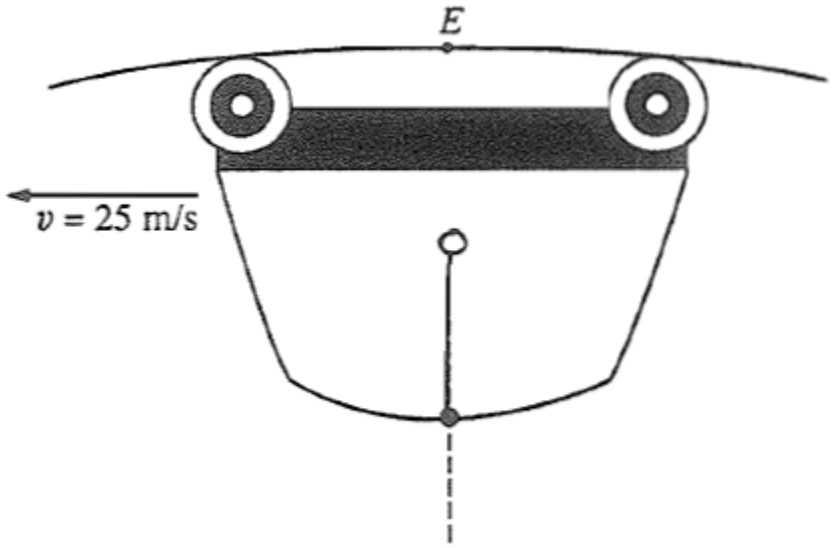
Tv = mg - mv2/r

Tv = -1.5N, i.e. the string is actually pulling down on theball

frq4

1 point is earned: For a correct answer

Tv = 1.5N



1 point is earned: For correctly drawing the string vertically upward

0	1	2	3 ✓

The student response earns three of the following points:

Since there is no horizontal acceleration, there is no horizontal component of the tension

1 point is earned: For a correct answer

Tb = Zero

Assuming for the moment that the string is hanging downward, the centripetal force is the difference between the gravitational force and the tension.Applying Newton's second law.

Solving for the vertical component of the tension:

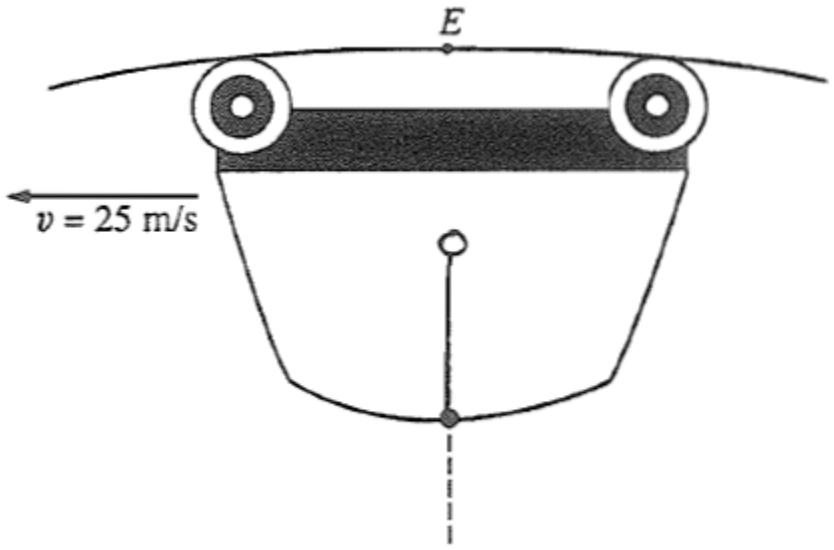
Tv = mg - mv2/r

Tv = -1.5N, i.e. the string is actually pulling down on theball

1 point is earned: For a correct answer

Tv = 1.5N

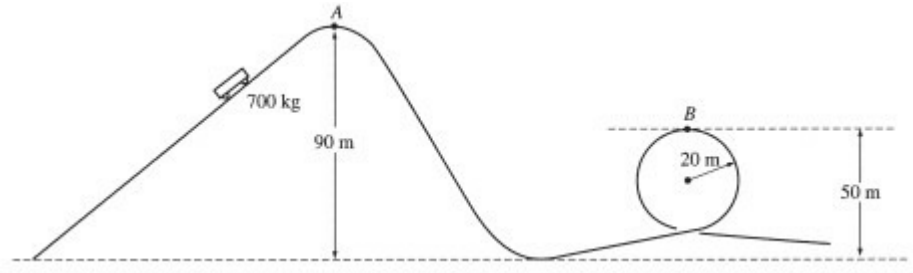
frq4



1 point is earned: For correctly drawing the string vertically upward

frq4

18.



A roller coaster ride at an amusement park lifts a car of mass 700 kg to point *A* at a height of 90 m above the lowest point on the track, as shown above. The car starts from rest at point *A*, rolls with negligible friction down the incline and follows the track around a loop of radius 20 m. Point *B*, the highest point on the loop, is at a height of 50 m above the lowest point on the track.

(a)

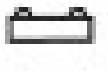
i. Indicate on the figure the point *P* at which the maximum speed of the car is attained.

ii. Calculate the value v_{max} of this maximum speed.

(b) Calculate the speed v_B of the car at point *B*.

(c)

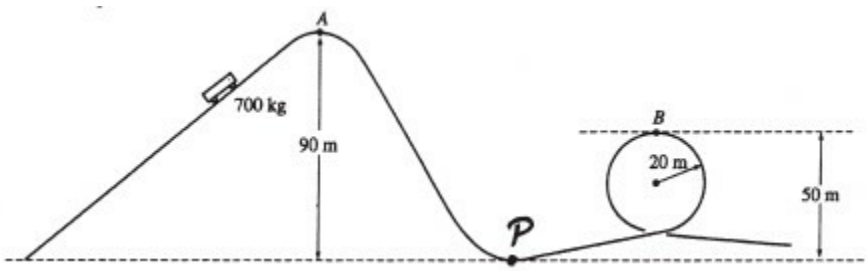
i. On the figure of the car below, draw and label vectors to represent the forces acting on the car when it is upside down at point *B*.



ii. Calculate the magnitude of all the forces identified in (c)i.

(d) Now suppose that friction is not negligible. How could the loop be modified to maintain the same speed at the top of the loop as found in (b)? Justify your answer.

Part A



frq4

One point is earned for correctly locating point P at the low point between the hill and the loop

One point is earned for any indication that energy is conserved.

One point is earned for setting the kinetic energy at P equal to the potential energy at A

$\frac{1}{2}mv_{\text{max}} = mgy_A$
$v_{\text{max}} = \sqrt{2gy_A}$

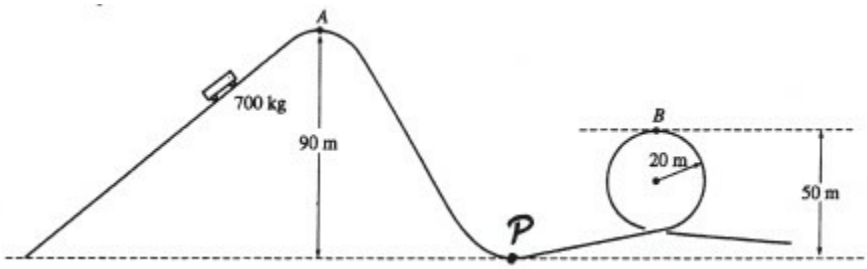
One point is earned for correct substitution

$v_{\text{max}} = \sqrt{2(9.8m/s^2)(90m)}$

One point is earned for the correct answer
vmax = 42m/s (or 42.4m/s, u sin g, g = 10m/s2)

0	1	2	3	4 ✓

The student earns all of the following points:



One point is earned for correctly locating point P at the low point between the hill and the loop

One point is earned for any indication that energy is conserved.

One point is earned for setting the kinetic energy at P equal to the potential energy at A

$\frac{1}{2}mv_{\text{max}} = mgy_A$
$v_{\text{max}} = \sqrt{2gy_A}$

One point is earned for correct substitution

$v_{\text{max}} = \sqrt{2(9.8m/s^2)(90m)}$

One point is earned for the correct answer

frq4

vmax = 42m/s (or 42.4m/s, u sin g, g = 10m/s2)

Part B

One point is earned for any indication that energy is conserved, equating the total energy at point *B* to the potential energy at point *A*

$\frac{1}{2}mv_B + mgy_B = mgy_A$

Solving for the speed at point *B*

$v_B = \sqrt{2g(y_A - y_B)}$

One point is earned for correct substitution

$v_B = \sqrt{2(9.8m/s^2)(90m - 50m)}$

One point is earned for the correct answer
vB = 28m/s (or 28.3m/s, u sin g, g = 10m/s2)

0	1	2	3 ✓

The student earns all of the following points:

One point is earned for any indication that energy is conserved, equating the total energy at point *B* to the potential energy at point *A*

$\frac{1}{2}mv_B + mgy_B = mgy_A$

Solving for the speed at point *B*

$v_B = \sqrt{2g(y_A - y_B)}$

One point is earned for correct substitution

$v_B = \sqrt{2(9.8m/s^2)(90m - 50m)}$

One point is earned for the correct answer

vB = 28m/s (or 28.3m/s, u sin g, g = 10m/s2)

frq4

Part C



One point is earned for each correctly drawn and labeled vector

One point is earned for no extraneous vectors, given that the two vectors above are drawn correctly.

One point is earned for determining the correct value of the weight
mg = (700kg) (9.8m/s2)
mg = 6860N (7000N, u sin g, g = 10m/s2)

One point is earned for indicating there is a centripetal force on the car, either separately or by setting the sum of the fores equal to the centripetal force.

$$N + mg = \frac{mv_B^2}{r}$$

$$N = \frac{mv_B^2}{r} - mg = \frac{(700kg)(28m/s^2)}{20m} - 6860N$$

One point is earned for the correct answer
N = 20, 580N (or 21,000 N, u sin g, g = 10m/s2)

0	1	2	3	4	5	6 ✓

The student earns all of the following points:



One point is earned for each correctly drawn and labeled vector

One point is earned for no extraneous vectors, given that the two vectors above are drawn correctly.

One point is earned for determining the correct value of the weight

mg = (700kg) (9.8m/s2)
mg = 6860N (7000N, u sin g, g = 10m/s2)

One point is earned for indicating there is a centripetal force on the car, either separately or by setting the sum of the fores equal to the centripetal force.

frq4

$N + mg = \frac{mv_B^2}{r}$

$N = \frac{mv_B^2}{r} - mg = \frac{(700kg)(28m/s^2)}{20m} - 6860N$

One point is earned for the correct answer

N = 20, 580N (or 21,000 N, u sin g, g = 10m/s2)

Part D

Two points are earned for any clear description of both a modification that will lower the height of point *B* and a correct justification

0	1	2 ✓

The student earns all of the following points:

Two points are earned for any clear description of both a modification that will lower the height of point *B* and a correct justification

frq4

19.



The Sojourner rover vehicle shown in the sketch above was used to explore the surface of Mars as part of the Pathfinder mission in 1997. Use the data in the tables below to answer the questions that follow.

Mars Data	Sojourner Data
Radius: 0.53 x Earth's radius Mass: 0.11 x Earth's mass	Mass of Sojourner vehicle: 11.5 kg Wheel diameter: 0.13 m Stored energy available: 5.4 x 10^5J Power required for driving under average conditions: 10W Land speed: 6.7 x 10^{-3} m/s

1. Determine the acceleration due to gravity at the surface of Mars in terms of *g*, the acceleration due to gravity at the surface of Earth.
2. Calculate Sojournerís weight on the surface of Mars.
3. Assume that when leaving the Pathfinder spacecraft Sojourner rolls down a ramp inclined at 20° to the horizontal. The ramp must be lightweight but strong enough to support Sojourner. Calculate the minimum normal force that must be supplied by the ramp.
4. What is the net force on Sojourner as it travels across the Martian surface at constant velocity? Justify your answer.
5. Determine the maximum distance that Sojourner can travel on a horizontal Martian surface using its stored energy.
6. Suppose that 0.010% of the power for driving is expended against atmospheric drag as Sojourner travels on the Martian surface. Calculate the magnitude of the drag force.

Part A

1 point is earned: For a correct expression of Newton's Gravitation law

$$F = \frac{GM_1M_2}{R^2}$$

frq4

1 point is earned: For correct substitution/calculation in formula

$$gMars = \frac{GM_M}{R_M^2} = \frac{G(0.11)M_g}{(0.53R_g)^2} = \frac{0.11}{(0.53)^2}g_g$$

1 point is earned: For expressing *gM* in terms of *gg*

$$gM = \frac{0.11}{(0.53)^2}g_g$$

OR

$$gM = 0.39g_g$$

However, the third point for expressing *gM* in terms of *gg* is not awarded if no points have been awarded up to that part.

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned: For a correct expression of Newton's Gravitation law

$$F = \frac{GM_1M_2}{R^2}$$

1 point is earned: For correct substitution/calculation in formula

$$gMars = \frac{GM_M}{R_M^2} = \frac{G(0.11)M_g}{(0.53R_g)^2} = \frac{0.11}{(0.53)^2}g_g$$

1 point is earned: For expressing *gM* in terms of *gg*

$$gM = \frac{0.11}{(0.53)^2}g_g$$

OR

$$gM = 0.39g_g$$

However, the third point for expressing *gM* in terms of *gg* is not awarded if no points have been awarded up to that part.

Part B

1 point is earned: For the correct formula for weight: *W* = *Mg*

1 point is earned: For the correct value of *g* for Mars, where any value of *g* signifying Mars is acceptable, such as 0.39*gE*, 3.8 m/s2, *gMars*,

frq4

$$\frac{0.11}{(0.53)^2} g_g$$

, etc.

W = (11.5 kg)(3.8 m/s2) = 44 N

(Alternate solution)

2 points are earned: For correct calculation using

$$F = \frac{GM_1M_2}{R^2}$$

, where the student has calculated the mass of Mars, the radius of Mars and uses the correct mass of Sojourner.

(A student needs to have memorized the mass of the Earth and its radius to do this.)

0	1	2	3	4 ✓

The student response earns four of the following points:

1 point is earned: For the correct formula for weight: *W* = *Mg*

1 point is earned: For the correct value of *g* for Mars, where any value of *g* signifying Mars is acceptable, such as 0.39*gE*, 3.8 m/s2, *gMars*,

$$\frac{0.11}{(0.53)^2} g_g$$

, etc.

W = (11.5 kg)(3.8 m/s2) = 44 N

(Alternate solution)

2 points are earned: For correct calculation using

$$F = \frac{GM_1M_2}{R^2}$$

, where the student has calculated the mass of Mars, the radius of Mars and uses the correct mass of Sojourner.

(A student needs to have memorized the mass of the Earth and its radius to do this.)

Part C

1 point is earned: For indication that sine or cosine is needed as a necessary factor for computation of *N*

frq4

1 point is earned: For correct formula and calculation

$N = MgM \cos \theta$

N = (11.5 kg)(3.8 m/s2) cos 20° = 41 N

OR N = (11.5 kg)(3.8 m/s2) sin 70° = 41 N

0	1	2 ✓

The student response earns two of the following points:

1 point is earned: For indication that sine or cosine is needed as a necessary factor for computation of N

1 point is earned: For correct formula and calculation

$N = MgM \cos \theta$

N = (11.5 kg)(3.8 m/s2) cos 20° = 41 N

OR N = (11.5 kg)(3.8 m/s2) sin 70° = 41 N

Part D

1 point is earned: For correct statement that $Fnet = 0$

1 point is earned: For a correct reason such as $a = 0$, OR v = constant, OR Newton's First law

0	1	2 ✓

The student response earns two of the following points:

1 point is earned: For correct statement that $Fnet = 0$

1 point is earned: For a correct reason such as $a = 0$, OR v = constant, OR Newton's First law

Part E

1 point is earned: For a correct formula for power

$$P = \frac{W}{t}$$

1 point is earned: For a correct formula for distance

$d = vt$

frq4

1 point is earned: For a correct calculation of distance

$$d = \frac{vW}{P} = \frac{(6.7x10^{-3}m/s)(5.4x10^{5}J)}{10W}$$

d = 362 m

(Alternate solution)

1 point is earned: For correct formulas for work and power

W = *Fd* and *P* = *Fv*

1 point is earned: For combining these to yield a correct formula for the distance

$$d = \frac{vW}{P}$$

1 point is earned: For a correct calculation

$$d = \frac{(6.7x10^{-3}m/s)(5.4x10^{4}J)}{10W} = 362m$$

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned: For a correct formula for power

$$P = \frac{W}{t}$$

1 point is earned: For a correct formula for distance

d = *vt*

1 point is earned: For a correct calculation of distance

$$d = \frac{vW}{P} = \frac{(6.7x10^{-3}m/s)(5.4x10^{5}J)}{10W}$$

d = 362 m

(Alternate solution)

1 point is earned: For correct formulas for work and power

W = *Fd* and *P* = *Fv*

frq4

1 point is earned: For combining these to yield a correct formula for the distance

$$d = \frac{vW}{P}$$

1 point is earned: For a correct calculation

$$d = \frac{(6.7x10^{-3}m/s)(5.4x10^{4}J)}{10W} = 362m$$

Part F

1 point is earned: For correct formula $P = Fv$, so

1 point is earned: For correct substitution using the correct percentage of 0.01%

$$F = \frac{(0.0001)(10W)}{6.7x10^{-3}m/s}$$

1 point is earned: For correct calculation of force

F = 0.15 N (0.149 N and 0.1 N were acceptable)

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned: For correct formula $P = Fv$, so

1 point is earned: For correct substitution using the correct percentage of 0.01%

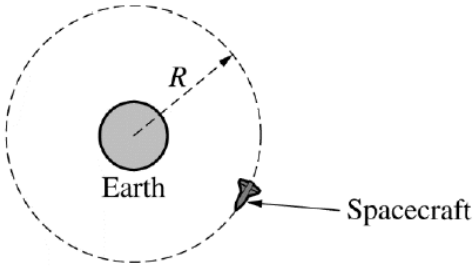
$$F = \frac{(0.0001)(10W)}{6.7x10^{-3}m/s}$$

1 point is earned: For correct calculation of force

F = 0.15 N (0.149 N and 0.1 N were acceptable)

frq4

20.

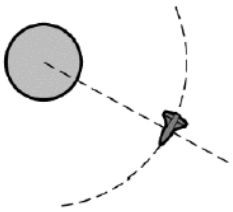


Note: Figure not drawn to scale.

A spacecraft of mass *m* is in a clockwise circular orbit of radius *R* around Earth, as shown in the figure above.

The mass of Earth is M_E.

a. In the figure below, draw and label the forces (not components) that act on the spacecraft. Each force must be represented by a distinct arrow starting on, and pointing away from, the spacecraft.



Note: Figure not drawn to scale.

b. i. Derive an equation for the orbital period *T* of the spacecraft in terms of *m, M_E, R*, and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

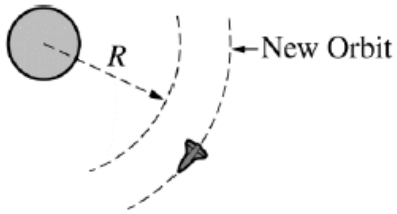
ii. A second spacecraft of mass *2m* is placed in a circular orbit with the same radius R. Is the orbital period of the second spacecraft greater than, less than, or equal to the orbital period of the first spacecraft?

____Greater than ____Less than ____Equal to

Briefly explain your reasoning.

c. The first spacecraft is moved into a new circular orbit that has a radius greater than *R*, as shown in the figure below.

frq4



Note: Figure not drawn to scale.

Is the speed of the spacecraft in the new orbit greater than, less than, or equal to the original speed?

____Greater than ____Less than ____Equal to

Briefly explain your reasoning.

Part A

1 point is earned: For an arrow directed toward Earth's center

1 point is earned: For a correct label on the arrow representing the gravitational force, where the arrow is pointing toward Earth's center

Note: A maximum of 1 point may be earned if extraneous forces are present.

0	1	2 ✓

Student response earns all of the following points

1 point is earned: For an arrow directed toward Earth's center

1 point is earned: For a correct label on the arrow representing the gravitational force, where the arrow is pointing toward Earth's center

Note: A maximum of 1 point may be earned if extraneous forces are present.

Part B

PART i

1 point is earned: For using (or implying) Newton's second law and equating the centripetal force to the gravitational force: $F_g = ma = \frac{mv^2}{R} \quad \frac{GmM_E}{R^2} = \frac{mv^2}{R}$

1 point is earned: For explicitly or implicitly determining that the speed of the spacecraft is: $v = \frac{2\pi R}{T}$

frq4

1 point is earned: For a correct answer algebraically equivalent to: $T = \sqrt{\frac{4\pi^2 R^3}{GM_E}}$

Note: It is acceptable to leave answer in terms of T2 : $T^2 = \frac{4\pi^2 R^3}{GM_E}$

PART ii

Correct answer: “Equal to”

Note: For an incorrect answer consistent with part (b)(i), the explanation is still graded

for consistency with part (b)(i).

1 point is earned: For a correct explanation that the period of the spacecraft does not depend on the

spacecraft mass (or only depends on the mass of Earth and the radius of the orbit)

OR an explanation consistent with the answer from (b)(i)

Note: The explanation must be consistent with the checked answer.

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

PART i

1 point is earned: For using (or implying) Newton’s second law and equating the centripetal force to the gravitational force:$F_g = ma = \frac{mv^2}{R} \quad \frac{GmM_E}{R^2} = \frac{mv^2}{R}$

1 point is earned: For explicitly or implicitly determining that the speed of the spacecraft is:$v = \frac{2\pi R}{T}$

1 point is earned: For a correct answer algebraically equivalent to: $T = \sqrt{\frac{4\pi^2 R^3}{GM_E}}$

Note: It is acceptable to leave answer in terms of T2 :$T^2 = \frac{4\pi^2 R^3}{GM_E}$

PART ii

Correct answer: “Equal to”

Note: For an incorrect answer consistent with part (b)(i), the explanation is still graded

for consistency with part (b)(i).

1 point is earned: For a correct explanation that the period of the spacecraft does not depend on the

spacecraft mass (or only depends on the mass of Earth and the radius of the orbit)

frq4

OR an explanation consistent with the answer from (b)(i)

Note: The explanation must be consistent with the checked answer.

Part C

Correct answer: “Less than”

Note: If the wrong selection is made, the explanation is not graded.

1 point is earned: For a correct explanation of why speed decreases with increasing orbital radius

Example:

Derivation step in (b)(i) shows that speed decreases with increasing R.

0	1 ✓

Student response earns 1 of the following 1 point(s)

Correct answer: “Less than”

Note: If the wrong selection is made, the explanation is not graded.

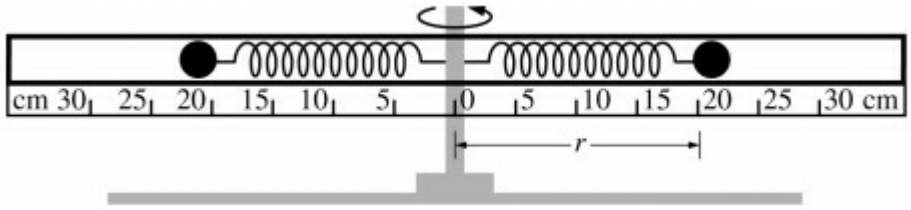
1 point is earned: For a correct explanation of why speed decreases with increasing orbital radius

Example:

Derivation step in (b)(i) shows that speed decreases with increasing R.

frq4

21.



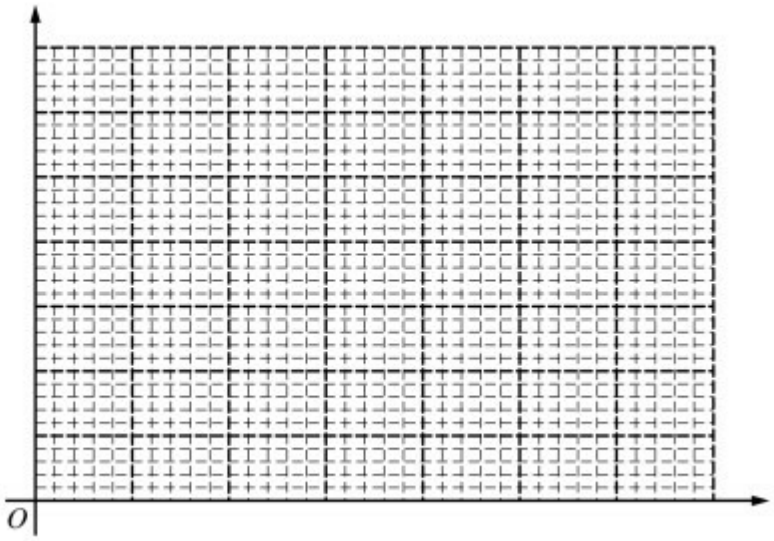
The apparatus shown above consists of two identical springs of negligible mass, each with spring constant k = 20.4 N/m and each attached at one end to a vertical rotating pole. Identical small spheres of mass m are attached to the other ends of the springs. The spheres are constrained to horizontal motion by horizontal guides of negligible friction, each of which has a ruler below it for measuring the radial position r of the sphere. The system can be manually rotated about the pole's axis. In a lab experiment, a student adjusts the rotational speed so that the spheres move to a desired radius r. For each such value of r, the student measures the rotational period T. The student's partially completed data table is shown below. The length of each unstretched spring is L = 0.15 m.

Trial	Radial Position r (m)	Period T (s)	Acceleration a_c $\left(\text{m/s}^2\right)$	Spring Force F_{spring} (N)
1	0.300	0.440	61.2	3.06
2	0.270	0.475	47.2	2.45
3	0.240	0.530		
4	0.210	0.570	25.5	1.22
5	0.180	0.795	11.2	0.61

(a) Calculate the missing values from the table above. Show your work in the space below

(b) On the axes below, plot the data points for the spring force F_{spring} as a function of the acceleration a_c. Label the axes, including the scale. Draw a straight line that best represents the data

frq4

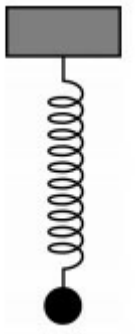


(c)

i. Calculate the slope of your line.

ii. Indicate what the slope calculated in part (c)-i represents.

(d) One sphere and one spring are removed from the rotation apparatus. They are hung vertically so that the sphere is now suspended from the spring, as shown below.



i. Describe a procedure you could use, and the measurements you would make, to verify the value obtained in part (c) using the setup shown above.

ii. Show how you would use the measurements described in part (d)-i to verify the value obtained in part (c).

Part A

frq4

Trial	Radial Position r (m)	Period T (s)	Acceleration a_c $\left(\text{m/s}^2\right)$	Spring Force F_{spring} (N)
1	0.300	0.440	61.2	3.06
2	0.270	0.475	47.2	2.45
3	0.240	0.530	33.7	1.84
4	0.210	0.570	25.5	1.22
5	0.180	0.795	11.2	0.61

1 point is earned for using a correct expression for the centripetal acceleration

$a_c = \frac{v^2}{r}$

1 point is earned for substituting an expression for speed in terms of period

$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{(4\pi^2)(0.240m)}{(0.530s)^2}$

1 point is earned for a correct answer for the centripetal acceleration

ac = 33.7 m/s2

1 point is earned for using a correct expression for the magnitude of the spring force

*F*spring = *kx*

1 point is earned for substituting the correct distance for the stretch of the spring

*F*spring = (20.4 N/m)(0.240 m - 0.15 m)

Correct answer

*F*spring = 1.84 N

0	1	2	3	4	5 ✓

The student response earns all of the following points:

frq4

Trial	Radial Position r (m)	Period T (s)	Acceleration a_c $\left(\text{m/s}^2\right)$	Spring Force F_{spring} (N)
1	0.300	0.440	61.2	3.06
2	0.270	0.475	47.2	2.45
3	0.240	0.530	33.7	1.84
4	0.210	0.570	25.5	1.22
5	0.180	0.795	11.2	0.61

1 point is earned for using a correct expression for the centripetal acceleration

$a_c = \frac{v^2}{r}$

1 point is earned for substituting an expression for speed in terms of period

$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{(4\pi^2)(0.240m)}{(0.530s)^2}$

1 point is earned for a correct answer for the centripetal acceleration

ac = 33.7 m/s2

1 point is earned for using a correct expression for the magnitude of the spring force

*F*spring = *kx*

1 point is earned for substituting the correct distance for the stretch of the spring

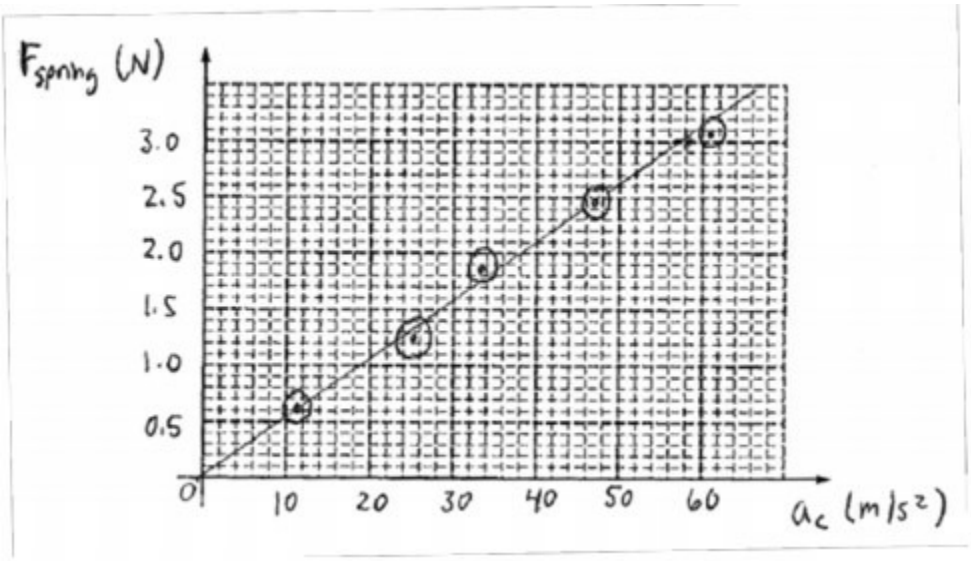
*F*spring = (20.4 N/m)(0.240 m - 0.15 m)

Correct answer

*F*spring = 1.84 N

Part B

frq4



1 point is earned for properly labeling both axes with variables and units

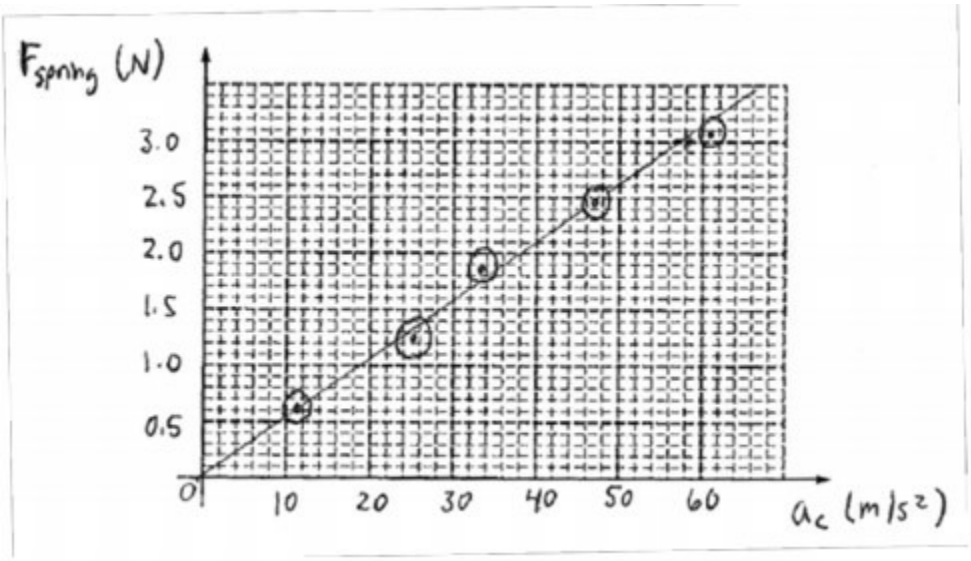
1 point is earned for using and labeling a proper scale on both axes, to use approximately half the grid

1 point is earned for correctly plotting the data points

1 point is earned for drawing a straight line that best represents the data

0	1	2	3	4 ✓

The student response earns all of the following points:



frq4

1 point is earned for properly labeling both axes with variables and units

1 point is earned for using and labeling a proper scale on both axes, to use approximately half the grid

1 point is earned for correctly plotting the data points

1 point is earned for drawing a straight line that best represents the data

Part C

1 point is earned for calculating the slope using points on the straight line in part (b)

$slope = \frac{(3.0N - 0)}{(58m/s^2 - 0)}$

1 point is earned for a correct answer with correct units

slope = 0.052 kg

1 point is earned for indicating the slope is the mass of each sphere

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned for calculating the slope using points on the straight line in part (b)

$slope = \frac{(3.0N - 0)}{(58m/s^2 - 0)}$

1 point is earned for a correct answer with correct units

slope = 0.052 kg

1 point is earned for indicating the slope is the mass of each sphere

Part D

1 point is earned for a procedure that could be used to verify the value found in part (c)

1 point is earned for indicating the measurements that must be made

- Equilibrium Example: Let the sphere hang in equilibrium. Measure how far the spring has stretched from its unstretched length.
- Oscillation Example: You can pull down and release the sphere so that it oscillates and measure the period of oscillation.

frq4

1 point is earned for a correct indication of how the measurements can be used to confirm the value from part (c)

- Equilibrium Example: Once you know the stretch of the spring and you already know the spring constant, you can set the gravitational force equal to the spring force (*F*spring = *mg* = *kx*) and solve for the mass of the sphere.
- Oscillation Example: With the period of oscillation and the spring constant known, you could use the period of a spring equation $\left(T_s = 2\pi\sqrt{k/m}\right)$ to determine the mass of the sphere.

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned for a procedure that could be used to verify the value found in part (c)

1 point is earned for indicating the measurements that must be made

- Equilibrium Example: Let the sphere hang in equilibrium. Measure how far the spring has stretched from its unstretched length.
- Oscillation Example: You can pull down and release the sphere so that it oscillates and measure the period of oscillation.

1 point is earned for a correct indication of how the measurements can be used to confirm the value from part (c)

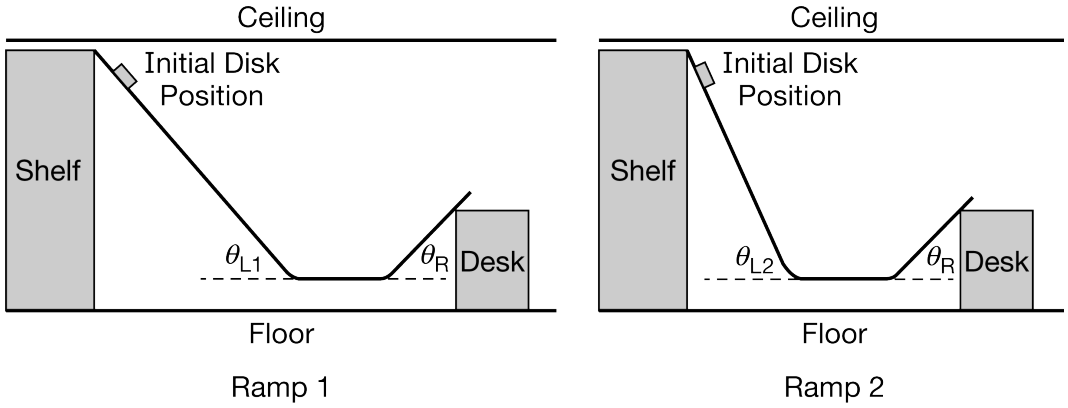
- Equilibrium Example: Once you know the stretch of the spring and you already know the spring constant, you can set the gravitational force equal to the spring force (*F*spring = *mg* = *kx*) and solve for the mass of the sphere.
- Oscillation Example: With the period of oscillation and the spring constant known, you could use the period of a spring equation $\left(T_s = 2\pi\sqrt{k/m}\right)$ to determine the mass of the sphere.

frq4

22. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)

A class of physics students is investigating projectile motion using a variety of equipment. Assume air resistance can be ignored in all cases.

One group constructs a long ramp as shown in Ramp 1 below. In their experiment, they release a disk from the left side of the ramp and allow the disk to launch off the right side. The ramp is smooth enough that friction can be ignored. The left side of the ramp is supported by the top of a shelf, and the other side is supported by the top of a desk. The left and right sides of the ramp are at angles θ_{L1} and θ_{R} with the horizontal respectively.

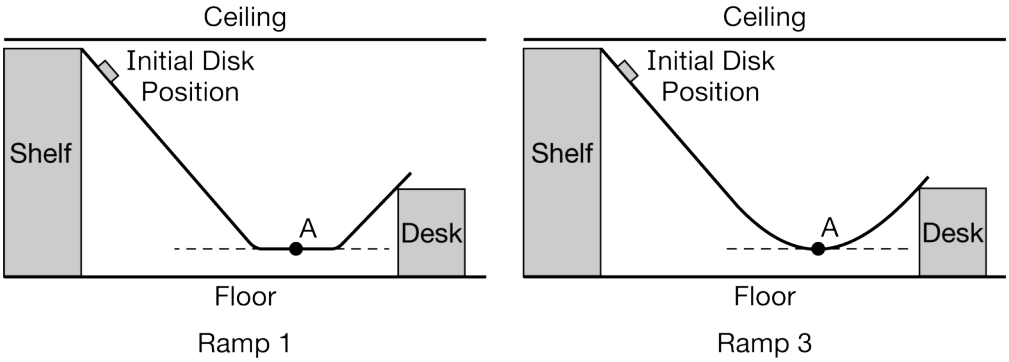


Note: Figures not drawn to scale.

The students release a disk from Ramp 1 and measure the maximum height it reaches after leaving the right side of the ramp. The students then increase the angle of the left ramp so that $\theta_{\mathrm{L1}} < \theta_{\mathrm{L2}}$, and they keep the top of the ramp against the shelf as shown in the Ramp 2 configuration above so that the disk is released at the same height.

(a) Will the disk launched from Ramp 2 reach a maximum height that is greater than, less than, or equal to the maximum height reached by the disk launched from Ramp 1? Justify your answer.

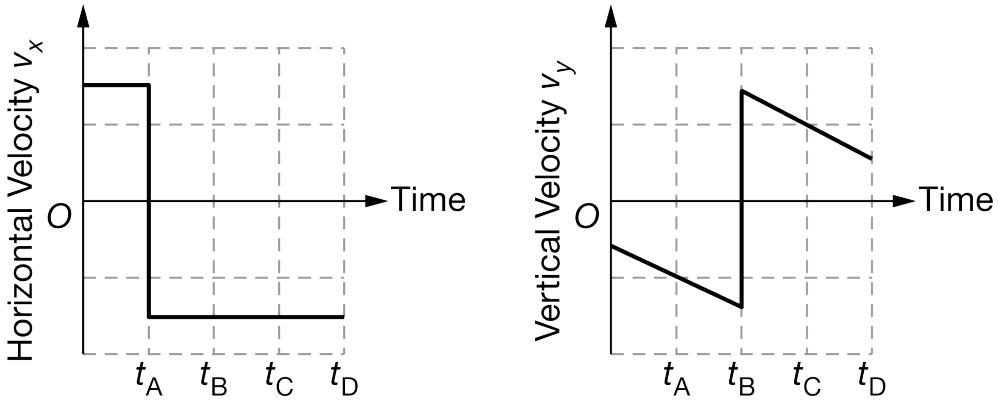
frq4



Note: Figures not drawn to scale.

Next, the students adjust the ramp into the configuration labeled Ramp 3 in which the lowest part of the ramp has been shaped into a circular curve. The lowest point of Ramp 3 is at the same height as the lowest section of Ramp 1. Both ramps are shown above.

(b) On which ramp, Ramp 1 or Ramp 3, is the force exerted by the ramp on the disk at point A greater? Explain your answer.



Another group uses a device that can launch a steel ball. The group launches the ball and it bounces off the ceiling, floor, and wall several times though not necessarily in that order. Each collision can be modeled as elastic, and friction between the ball and the surfaces is negligible. The graphs above show the horizontal and vertical components of the velocity of the ball during some time interval in the ball's motion. The positive direction is taken to be upward.

(c) During the time interval shown in the graphs, the ball has two collisions. In a clear, coherent, paragraph-length response, identify the times when each of these collisions occur using the times indicated on the graph, and explain how you know whether each collision is with the wall, the ceiling, or the floor.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq4

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the disk has the same speed (or kinetic energy) as it leaves the right end of both ramps because the energy of the disk-Earth system is conserved.
- ☐ 1 point is earned for indicating that the vertical component of the velocity at the right end is the same, since both are launched at the same angle, so the disk reaches the same maximum height for both ramps.

Example Response:

The disk has the same change in height as it travels from the left end to the right end of the ramp, so the same amount of potential energy is converted to kinetic energy. Thus, the disk leaves the right end of both ramps at the same speed. Since it also leaves at the same angle, it will travel the same projectile path and have the same maximum height in both cases.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that at point A on ramp 1, the block is moving at a constant velocity, so the net force is zero. Thus, the force by ramp 1 is equal to the disk's weight.
- ☐ 1 point is earned for indicating that at point A on ramp 3, the disk is moving in a circular path and thus has an upward net force.
- ☐ 1 point is earned for concluding that at point A on ramp 3, the normal force must be greater than the disk's weight, and thus greater than it was on ramp 1.

Example Response:

The disk on ramp 1 is traveling at constant velocity on the flat section. Thus, the normal force is equal to the weight of the disk. The disk on ramp 3 is executing circular motion, with an upward acceleration. Thus, the net force must be upward, and thus the normal force on ramp 3 is greater than the disk's weight. Thus, the normal force exerted on the block is greater for ramp 3.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq4

0	1	2	3	4	5 ✓

The response accurately includes **all** of the following criteria.

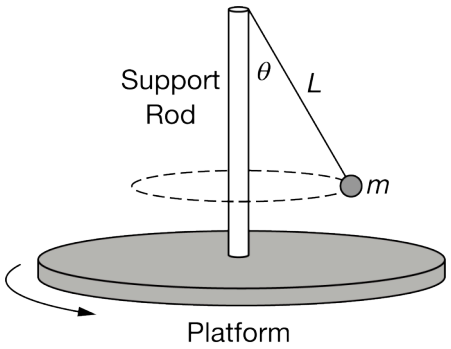
- ☐ 1 point is earned for correctly identifying the collision with a wall at time t_A.
- ☐ 1 point is earned for indicating that a collision with the wall will exert a force on the ball, accelerating it
  OR
  For indicating that the force changes the horizontal velocity of the ball
  OR
  For indicating that there is a change in the direction of the horizontal velocity.
- ☐ 1 point is earned for correctly identifying the collision with the floor at time t_B.
- ☐ 1 point is earned for For indicating that a collision with the floor will exert a force on the ball, accelerating it
  OR
  For indicating that the force changes the vertical velocity of the ball
  OR
  For indicating that there is a change in the direction of the vertical velocity.
- ☐ 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

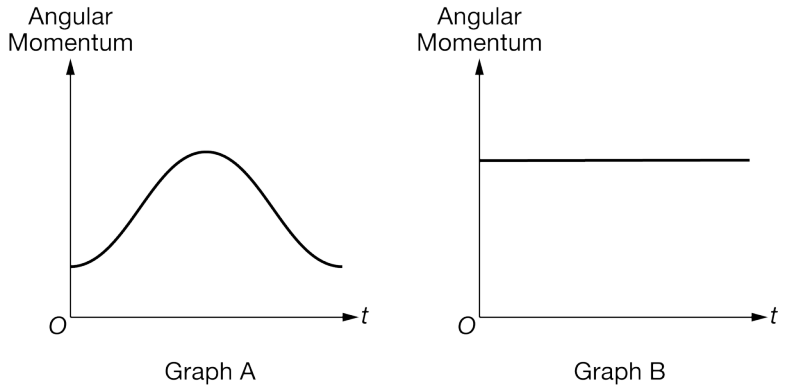
Each surface exerts a normal force; thus, the ceiling and floor exert vertical forces and the walls exert horizontal forces. When a net force is applied, there is an acceleration and a change in velocity. Thus, when there is a horizontal force by the wall there is a change in the horizontal velocity, which happens at time t_A, so the ball collides with the wall at t_A. When the vertical velocity changes from negative to positive at time t_B, there is an upward force, which only happens when the ball collides with the floor. So, the ball collides with the floor at t_B.

frq4

23. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A vertical support rod is fixed at the center of a platform. A light string of length L is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass m, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed ω (omega), the string makes an angle θ (theta) with the rod.



(a) The platform rotates with a constant angular speed. Which of the above graphs correctly shows the magnitude of the angular momentum of the platform as a function of time t about a vertical axis at the center of the support rod? Briefly justify your answer.

(b) The platform rotates with a constant angular speed. Is the net torque exerted on the platform about its center clockwise, counterclockwise, or zero? Explain your reasoning.

(c) As the period of rotation becomes very small, what value does θ approach? Without writing any equations or formulas, justify your answer.

(d) A student derives an equation for the square of the period of revolution T of the sphere as a function of θ: $T^2 = \left(4\pi^2\right)\frac{L\cos\theta}{g}$. Indicate whether the equation is consistent with your answer in part (c) and explain why.

(e) In a series of experimental trials, a group of students changes L and adjusts T to keep θ constant. They record L and T for each trial. If the students use the equation given in part (d), what quantities could they graph to yield a

frq4

straight line whose slope could be used to find the acceleration due to gravity, g ?

(f) As the angular speed of the platform increases, does the period of revolution of the sphere increase, decrease, or stay the same? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for identifying the correct graph as graph B.
- ☐ 1 point is earned for a justification that angular momentum is directly proportional to the angular speed which remains constant.

Example Response:

Graph B is correct because the angular speed is constant, and therefore the angular momentum is constant.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that the net torque on the platform is zero.
- ☐ 1 point is earned for a correct statement relating the angular speed to angular acceleration.
- ☐ 1 point is earned for a correct statement relating net torque to angular acceleration.

Example Response:

The net torque on the platform is zero. Because the platform rotates with a constant angular speed, its angular acceleration is zero, and net external torque is equal to the rotational inertia times the angular acceleration, so if the angular acceleration is zero, the net external torque is also zero.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq4

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that correctly relates the period to the angular speed.
- ☐ 1 point is earned for a statement correctly relating the angular speed to the angle theta.

Example Response:

When the period is very small the platform is rotating quickly, making the angular speed very large. When the angular speed is very large, the angle theta is close to 90 degrees.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that is consistent with the reasoning in part (c).
- ☐ 1 point is earned for addressing functional dependence.
- ☐ 1 point is earned for a correct functional dependence argument.

Example Response:

The equation is consistent with part (c) because as theta gets close to 90 degrees, cos theta gets very small, and therefore the period gets very small.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement identifying a T term on one axis.

frq4

- [ ] 1 point is earned for a statement identifying any *L* term on the other axis.
- [ ] 1 point is earned for a correct power relationship for *T* and *L* (i.e., *T* squared and *L*).

Example Response:

They could graph T squared as a function of L.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

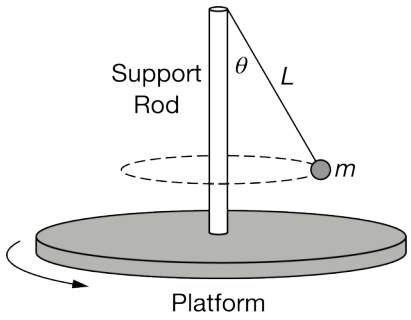
- [ ] 1 point is earned for a statement that the period of rotation decreases.
- [ ] 1 point is earned for a correct explanation of the relationship between the period and the angular speed of the platform.

Example Response:

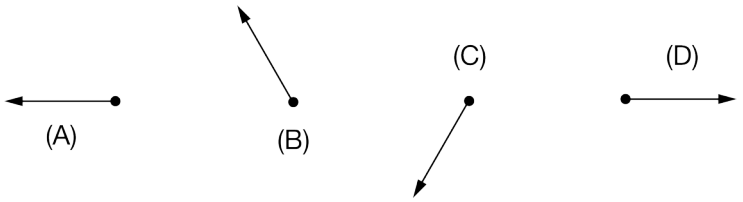
The period will decrease with angular speed because T = 2 pi / omega.

frq4

24. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A vertical support rod is fixed at the center of a platform. A light string of length L is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass m, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed ω (omega), the string makes an angle θ (theta) with the rod.



(a) Identify which of the above vectors represents the net force exerted on the sphere at the moment shown above. Briefly justify your answer.

(b) Is the magnitude of the tension in the string greater than, less than, or equal to the weight of the sphere? Explain your reasoning.

(c) The string can never be completely horizontal no matter how fast the platform is rotating. Without writing any equations or formulas, explain why.

One of the following two equations for the square of the tangential speed v of the sphere as a function of θ is consistent with the statement in part (c) that the string can never be horizontal as the platform is rotating.

Equation 1: $v^2 = \frac{gL(\sin^2\theta)}{(\cos\theta)}$ **Equation 2:** $v^2 = \frac{gL(\cos^2\theta)}{(\sin\theta)}$

(d) Indicate which equation is consistent with the statement in part (c), and explain why. Also explain why the other equation is not consistent with the statement in part (c).

(e) In a series of experimental trials, a group of students changes L and adjusts v to keep θ constant. They record L and v for each trial. Based on the equation in part (d) that you indicated as consistent with the statement in part (c), what quantities could the students graph to yield a straight line whose slope could be used to find the acceleration due to gravity, g ?

frq4

(f) Let I be the rotational inertia of the sphere-rod-platform system about a vertical axis through the center of the support rod. Does I increase, decrease, or stay the same when L increases (but θ stays constant)? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answer: (A), with a correct justification

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement identifying the correct net force vector (A)
- ☐ 1 point is earned for a justification that includes that the net force or acceleration will point toward the center of the circular path

Example Responses:

Figure A because the net force is toward the center of the sphere's circular motion.

OR

The forces acting on the sphere are the force of gravity and the force of tension. The force of gravity is directly toward the ground, and the force of tension is at an angle toward the top of the rod. When the forces are added together, the vertical component of the force of tension and the force of gravity cancel each other out, leaving only the horizontal component of the force of tension, which is demonstrated by vector A.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- ☐ 1 point is earned for a statement that the tension in the string is greater than the weight of the sphere
- ☐ 1 point is earned for an explanation referring to the components of the vectors
- ☐ 1 point is earned for a correct justification that addresses the sum of the vector components

Example Responses:

frq4

The tension is greater than the weight because the vertical component of the tension force must be equal to the weight. The tension also has a horizontal component, so the magnitude will be greater than either of the components.

OR

Explaining that

$\Sigma F_y = F_{Ty} - mg = 0$

$F_T \cos\theta = mg$

$F_T = mg/\cos\theta$

(Since $\cos\theta < 1$*), therefore* $F_T > mg$ *(or > "weight")*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement identifying forces and/or force components in the vertical direction
- ☐ 1 point is earned for a justification that includes that the weight of the sphere must be balanced by the string and so the vertical component of the tension can never be zero

Example Response:

The weight of the sphere is downward and will need to be balanced by the vertical component of the tension. So, there will always be a vertical component of tension.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Notes:

· If students switch sine and cosine (such that they use Equation 2) but then are completely consistent, they can earn 2 of 3 points in part (d).

· If students misinterpret theta as being zero when the string is horizontal, they can earn two of three points in part (d) for using Equation 2 consistently.

0	1	2	3 ✓

frq4

The response accurately includes **three** of the following criteria.

- [ ] 1 point is earned for a statement addressing functional dependence (regardless of the equation chosen)
- [ ] 1 point is earned for indicating that equation 1 shows that as theta goes to 90 degrees, *v* gets very large (infinite/undefined)
- [ ] 1 point is earned for correctly indicating a flaw in equation 2

Example Response:

Equation 1 is consistent with part (c) because as theta gets closer to 90 degrees, the speed gets very large. This shows that no matter how large *v* gets, theta will never get to 90 degrees. Equation 2 is not consistent because it has a finite value for theta equal to 90 degrees.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- [ ] 1 point is earned for a statement identifying any *v* term on one axis
- [ ] 1 point is earned for a statement identifying any *L* term on the other axis
- [ ] 1 point is earned for a statement identifying correct power relationships for *v* and *L* (i.e., *v* squared vs. *L*)

Example Response:

They could graph *v* squared on the *y*-axis as a function of *L* on the *x*-axis.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for a statement identifying that the rotational inertia will increase
- [ ] 1 point is earned for a statement that since the mass is further from the axis, the rotational inertia is bigger

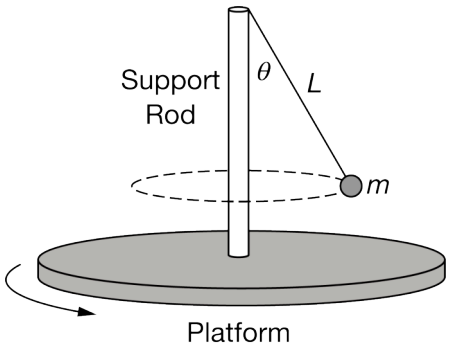
Example Response:

frq4

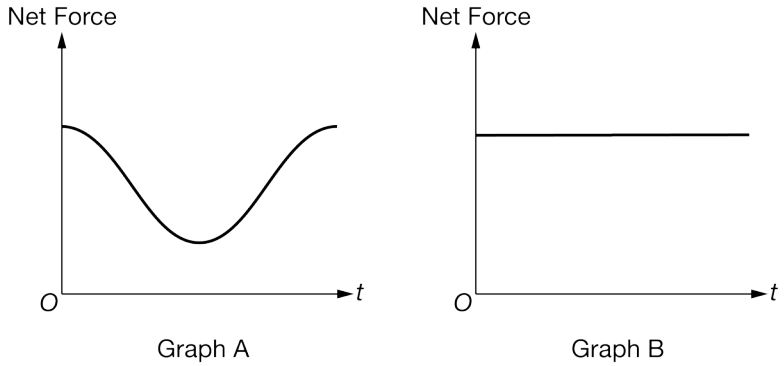
I will increase because for a larger *L*, the distance of the mass from the axis will be larger, and rotational inertia increases with distance.

frq4

25. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A vertical support rod is fixed at the center of a platform. A light string of length L is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass m, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed ω (omega), the string makes an angle θ (theta) with the rod.



(a) The platform rotates with a constant angular speed. Which of the above graphs correctly shows the magnitude of the net force exerted on the sphere as a function of time t ? Briefly justify your answer.

(b) Is the magnitude of the tension in the string greater than, less than, or equal to the centripetal force exerted on the sphere? Explain your reasoning.

(c) The string is nearly vertical when the sphere's tangential speed is very small. Without writing any equations or formulas, explain why.

(d) A student derives an equation for the square of the tangential speed v of the sphere as a function of θ : $v^2 = gL(\tan\theta)(\sin\theta)$. Indicate whether the equation is consistent with the statement in part (c) and explain why.

(e) In a series of experimental trials, a group of students adjusts v and measures θ while keeping all other variables constant. They record v and θ for each trial. If the students use the equation given in part (d), what quantities could they graph to yield a straight line whose slope could be used to find the acceleration due to gravity, g ?

frq4

(f) Let I be the rotational inertia of the sphere-rod-platform system about a vertical axis through the center of the support rod. Does I increase, decrease, or stay the same when θ decreases? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement identifying graph B as correctly describing the magnitude of the net force on the sphere as a function of time.
- ☐ 1 point is earned for a justification that the net force is constant because the speed is constant.

Example Response:

Graph B is correct because the magnitude of the net force on an object in uniform circular motion is constant.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that the magnitude of the tension is greater than the centripetal force exerted on the sphere.
- ☐ 1 point is earned for an explanation referring to the components of the vectors.
- ☐ 1 point is earned for a correct justification that addresses the sum of the vector components.

Example Response:

The tension is greater than the centripetal force because the horizontal component of the tension force must be equal to the centripetal force. The tension also has a vertical component, so the magnitude will be greater than either of the components.

OR

Explaining that

frq4

$\sum F_x = F_{Tx} = mv^2/r$ *where* mv^2/r *is the "centripetal force."*

$F_T \sin\theta = mv^2/r$

$F_T = mv^2/(r\sin\theta)$

(Since $\sin\theta < 1$*), therefore* $F_T > mv^2/r$ *(or >* *"centripetal force")*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that when the speed is very small the centripetal force is very small.
- ☐ 1 point is earned for a justification that includes that if the horizontal force is very small the angle of the tension with the vertical is very small.

Example Response:

A small speed means a small centripetal acceleration. The horizontal component of the tension is equal to the centripetal force, so if it is very small, then the tension will be almost vertical, and the angle will be very small.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that the equation is consistent with the reasoning in part (c).
- ☐ 1 point is earned for addressing the functional dependence.
- ☐ 1 point is earned for a correct functional dependence argument.

Example Response:

It is consistent because as theta goes to zero, both trigonometric terms go to zero, and therefore the speed goes to zero. This matches the statement that the string is nearly vertical when the speed is small.

Part E

frq4

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for identifying any *v* term on one axis.
- ☐ 1 point is earned for identifying any theta term on the other axis.
- ☐ 1 point is earned for a correct power relationship between *v* and trig functions of theta (for example, *v* squared vs. $(\tan\theta)(\sin\theta)$.

Example Response:

They could graph v squared as a function of tan theta sin theta.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the rotational inertia of the sphere-rod-platform-system decreases.
- ☐ 1 point is earned for a justification that includes that the rotational inertia decreases with decreasing distance of the mass from the axis.

Example Response:

I decreases because as theta decreases, the mass gets closer to the axis of rotation and rotational inertia increases with distance from the axis.

frq4

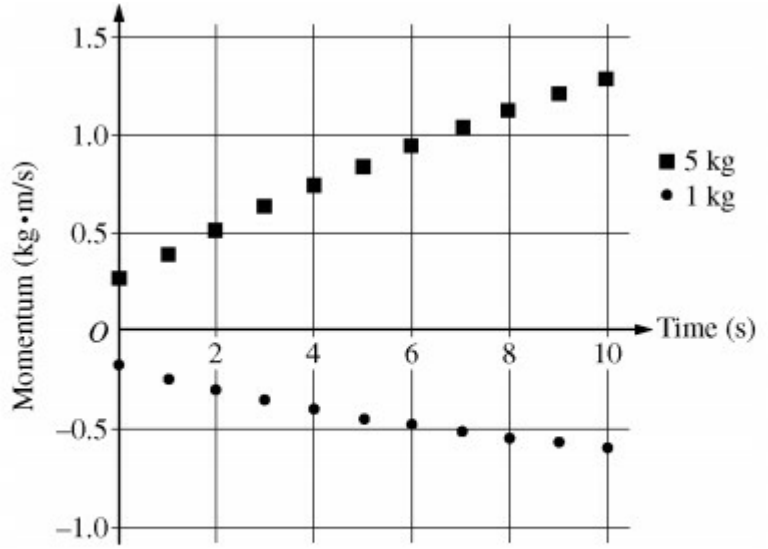
26.



Top View

The figure above represents two carts, with magnets attached, that make up a system. The mass of one cart and magnet is 1 kg and the mass of the other is 5 kg. The carts are initially at rest on a frictionless track. They are released from rest and exert a repulsive force on each other. The track is not quite horizontal, with the right side slightly lower than the left side.

The speeds of the carts are measured over a 10 s interval. The graph below shows the momentum of the two carts as a function of time for this interval as they move along the x-axis.



a. Based on the graph, were the measurements started at the instant the carts were released?

   Justify your answer.

b. Calculate the magnitude of the external force exerted on the system.

c. Suppose the experiment is repeated with different carts, so that the masses of cart plus magnet are 2 kg and 4 kg. Would your answer to part (b) be different with the new masses?

   Justify your answer.

Part A

1 point is earned for a correct, justified response about the state of motion at time $t = 0$

frq4

For example: No. The carts have a non-zero momentum at $t = 0$, so this cannot be the instant they were released.

0	1 ✓

The student response earns all of the following points:

1 point is earned for a correct, justified response about the state of motion at time $t = 0$

For example: No. The carts have a non-zero momentum at $t = 0$, so this cannot be the instant they were released.

Part B

1 point is earned for using the relationship between force and momentum, or Newton's second law and the definitions of acceleration and momentum

$F = \Delta p/\Delta t, or, F = ma, a = \Delta v/\Delta t, and, p = mv$

1 point is earned for recognizing that the force causes a change in momentum or a change in velocity

1 point is earned for using reasonable values of momenta from the graph to calculate the momenta of the carts

$F_5 = \Delta p/\Delta t = (1.25kg \cdot m/s - 0.25kg \cdot m/s)/10s = 0.10N$
$F_1 = \Delta p/\Delta t = (-0.6kg \cdot m/s - -0.2kg \cdot m/s)/10s = -0.04N$

1 point is earned for recognizing that the external force is the sum of the two forces on the cart

Fext = *F5* + *F1* = 0.10N - 0.04N = 0.06N

Calculating only the force on one of the carts earns a maximum of 1 point

0	1	2	3	4 ✓

The student response earns all of the following points:

1 point is earned for using the relationship between force and momentum, or Newton's second law and the definitions of acceleration and momentum

$F = \Delta p/\Delta t, or, F = ma, a = \Delta v/\Delta t, and, p = mv$

1 point is earned for recognizing that the force causes a change in momentum or a change in velocity

1 point is earned for using reasonable values of momenta from the graph to calculate the momenta of the carts

frq4

$F_5 = \Delta p/\Delta t = (1.25kg \cdot m/s - 0.25kg \cdot m/s)/10s = 0.10N$
$F_1 = \Delta p/\Delta t = (-0.6kg \cdot m/s - -0.2kg \cdot m/s)/10s = -0.04N$

1 point is earned for recognizing that the external force is the sum of the two forces on the cart

Fext = *F5* + *F1* = 0.10N - 0.04N = 0.06N

Calculating only the force on one of the carts earns a maximum of 1 point

Part C

2 points are earned for a correct, justified response about the effect of changing the masses of the carts

For example: No. The calculations in part (b) account for the effect of the external force on the motion of the entire system. The changes in individual masses do not change the total mass of the system, so the net effect on the system remains the same. In the absence of other correct reasoning, recognizing that the mass is the same for both situations earned 1 point.

0	1	2 ✓

The student response earns all of the following points:

2 points are earned for a correct, justified response about the effect of changing the masses of the carts

For example: No. The calculations in part (b) account for the effect of the external force on the motion of the entire system. The changes in individual masses do not change the total mass of the system, so the net effect on the system remains the same. In the absence of other correct reasoning, recognizing that the mass is the same for both situations earned 1 point.