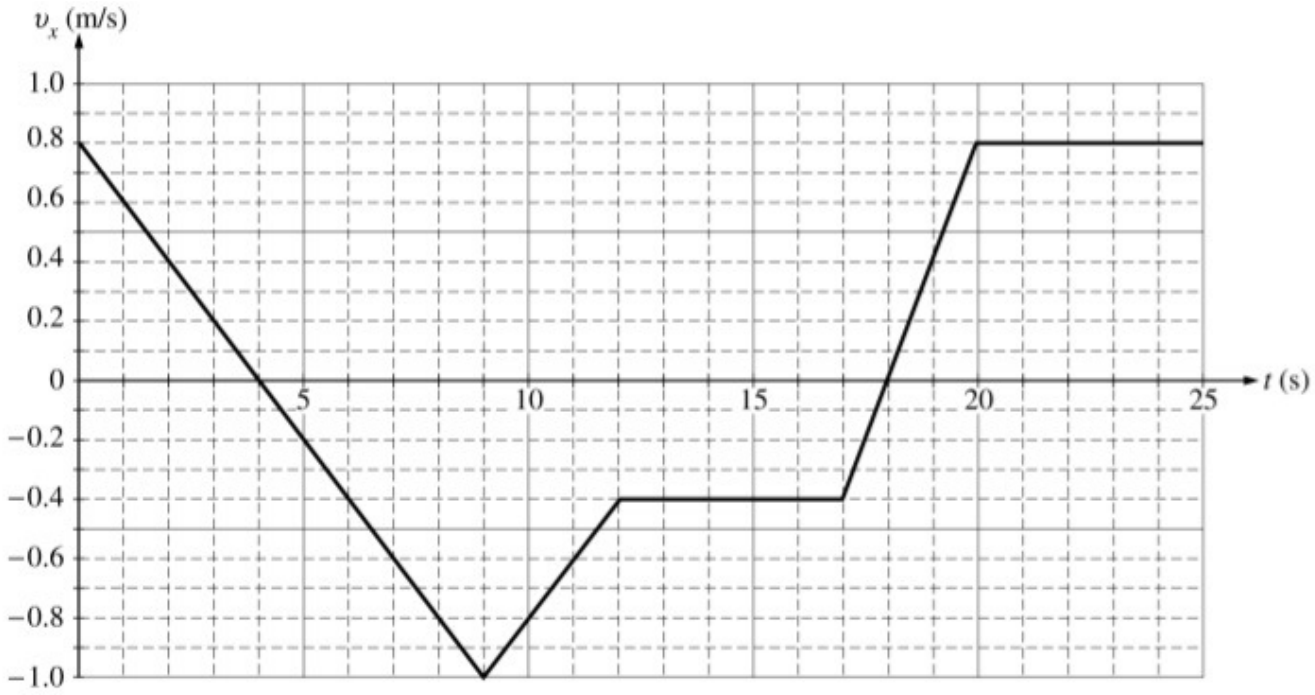
frq5

1. A 0.50 kg cart moves on a straight horizontal track. The graph of velocity v_x versus time t for the cart is given below.



a. Indicate every time t for which the cart is at rest.

b. Indicate every time interval for which the speed (magnitude of velocity) of the cart is increasing.

c. Determine the horizontal position x of the cart at $t = 9.0$ s if the cart is located at $x = 2.0$ m at $t = 0$.

d. On the axes below, sketch the acceleration a *versus* time t graph for the motion of the cart from $t = 0$ to $t = 25$ s.

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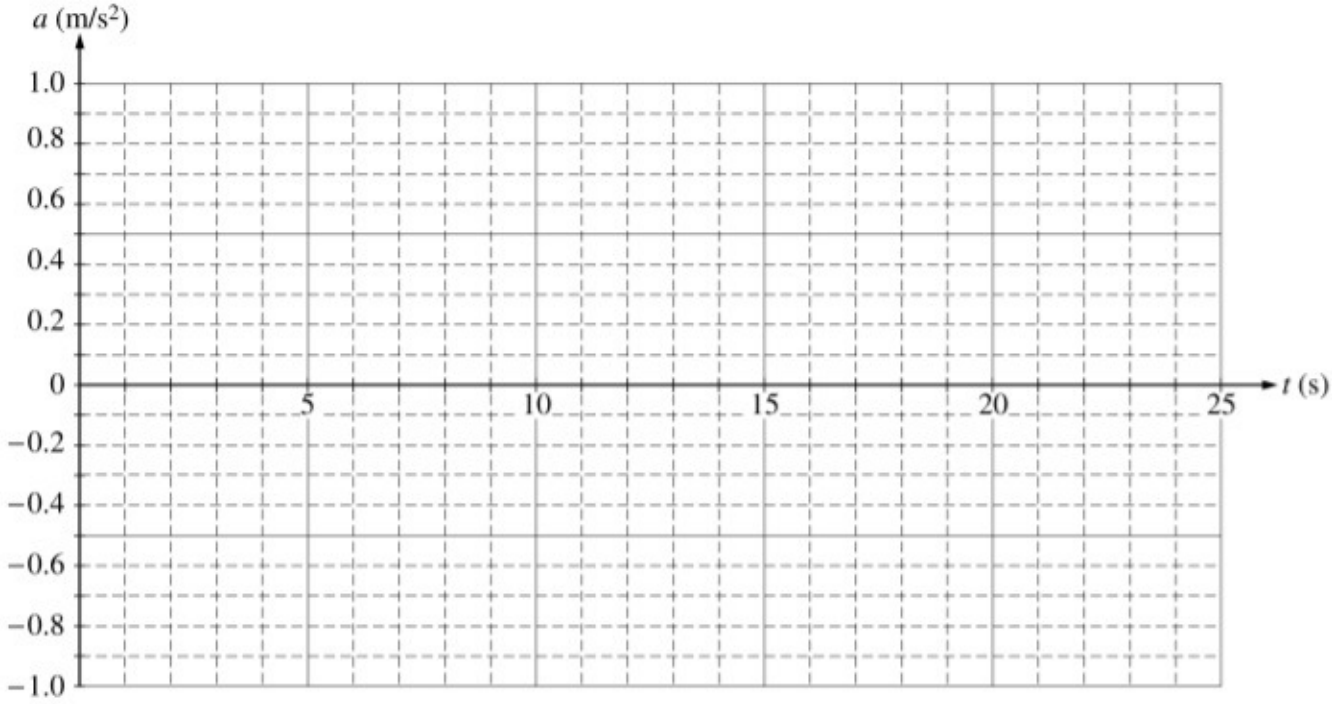


e. From $t = 25$ s until the cart reaches the end of the track, the cart continues with constant horizontal velocity. The cart leaves the end of the track and hits the floor, which is 0.40 m below the track. Neglecting air resistance, determine each of the following.

   i. The time from when the cart leaves the track until it first hits the floor

   ii. The horizontal distance from the end of the track to the point at which the cart first hits the floor

   iii. The kinetic energy of the cart immediately before it hits the floor

Part A

The cart is at rest at $t = 4$ s and 18 s.

1 point is earned: For indicating both times that the car is at rest and not including any incorrect times

0	1 ✓

The student response earns all of the following points:

The cart is at rest at $t = 4$ s and 18 s.

1 point is earned: For indicating both times that the car is at rest and not including any incorrect times

Part B

frq5

The speed of the cart is increasing during the intervals t = 4 to 9 s and 18 to 20 s.

2 points are earned: For each correct time interval one point was awarded.

One point was deducted for each incorrect interval, for a maximum two-point deduction.

0	1	2 ✓

The student response earns all of the following points:

The speed of the cart is increasing during the intervals t = 4 to 9 s and 18 to 20 s.

2 points are earned: For each correct time interval one point was awarded.

One point was deducted for each incorrect interval, for a maximum two-point deduction.

Part C

1 point is earned: For indicating that the change in position is equal to the area under the graph, or for using an appropriate kinematic equation with non-zero initial velocity

Δx = area OR, for example, $\Delta x = v_i t + \frac{1}{2}at^2$

1 point is earned: For correct substitution of values

$\Delta x = \frac{1}{2}(4\text{ s})(0.8\text{ m/s}) + \frac{1}{2}(5\text{ s})(-1\text{ m/s})$

OR $\Delta x = (0.8\text{ m/s})(9\text{ s}) + \frac{1}{2}(-0.2\text{ m/s}^2)(9\text{ s})^2$

Δx = –0.9 m

1 point is earned: For adding the initial position to Δx

$x = x0 + \Delta x$ = 2 m + (–0.9 m)

x = 1.1 m

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For indicating that the change in position is equal to the area under the graph, or for using an appropriate kinematic equation with non-zero initial velocity

Δx = area OR, for example, $\Delta x = v_i t + \frac{1}{2}at^2$

frq5

1 point is earned: For correct substitution of values

$\Delta x = \frac{1}{2}(4\text{ s})(0.8\text{ m/s}) + \frac{1}{2}(5\text{ s})(-1\text{ m/s})$

OR $\Delta x = (0.8\text{ m/s})(9\text{ s}) + \frac{1}{2}(-0.2\text{ m/s}^2)(9\text{ s})^2$

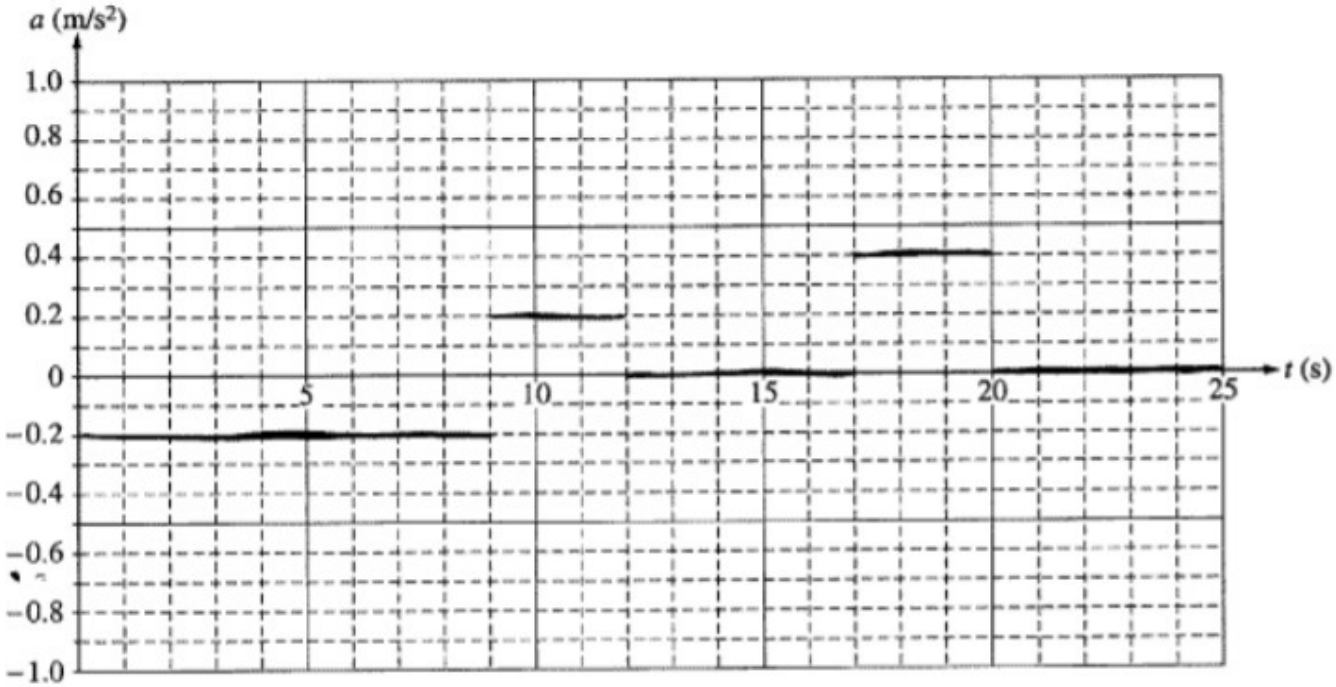
Δx = –0.9 m

1 point is earned: For adding the initial position to Δx

$x = x0 + \Delta x$ = 2 m + (–0.9 m)

x = 1.1 m

Part D



1 point is earned: For indicating that the acceleration is zero during the intervals of constant velocity, 12 to 17 s and 20 to 25 s

1 point is earned: For indicating that the acceleration is negative and constant from zero to 9 s, and positive and constant from 9 to 11 s and 17 to 20 s

1 point is earned: For having the correct values for two of the three intervals of non-zero accelerations:

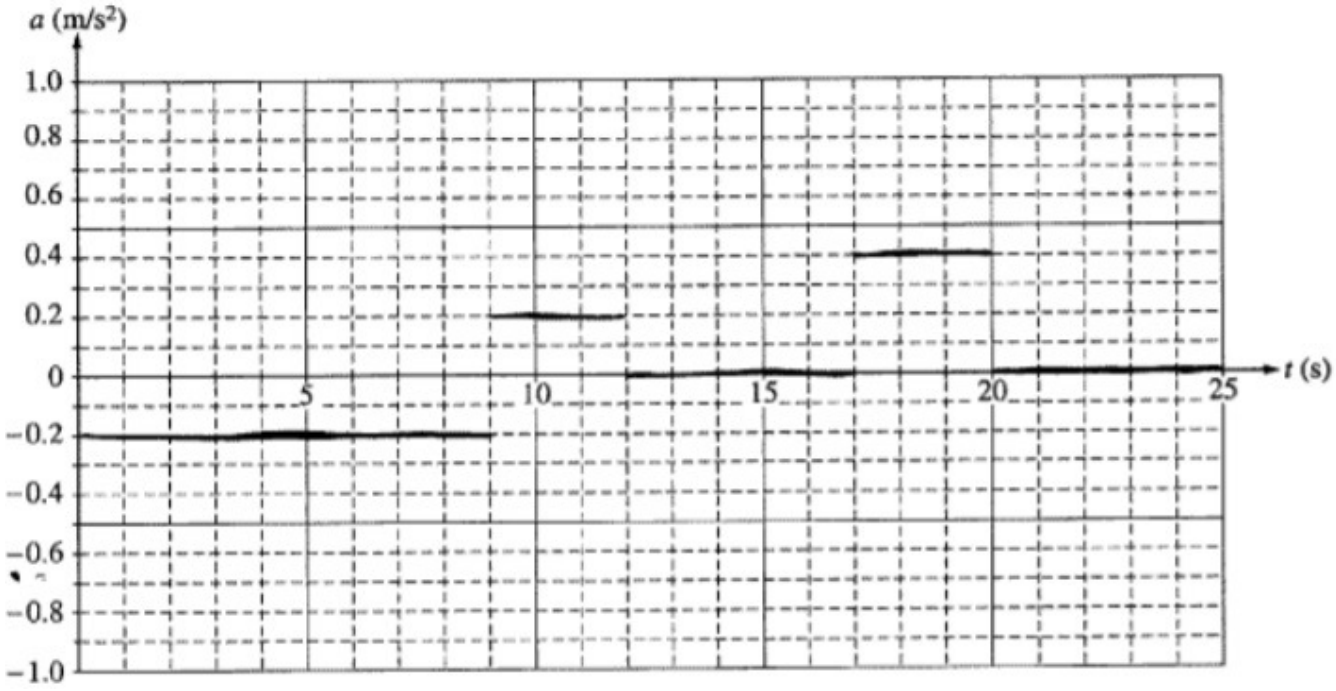
–0.20 m/s2, +0.20 m/s2, and +0.40 m/s2

✓

0	1	2	3

frq5

The student response earns all of the following points:



1 point is earned: For indicating that the acceleration is zero during the intervals of constant velocity, 12 to 17 s and 20 to 25 s

1 point is earned: For indicating that the acceleration is negative and constant from zero to 9 s, and positive and constant from 9 to 11 s and 17 to 20 s

1 point is earned: For having the correct values for two of the three intervals of non-zero accelerations:

–0.20 m/s2, +0.20 m/s2, and +0.40 m/s2

Part E

- **1 point is earned:** For using the correct kinematic equation, with the initial vertical velocity equal to zero

  $y = \frac{1}{2}gt^2$

  Solving for *t*:

  $t = \sqrt{2y/g}$

  $t = \sqrt{2(0.40\ \text{m})/10\ \text{m/s}^2}$

  $t = 0.28\ \text{s} (\text{or } 0.29\ \text{s using } g = 9.8\ \text{m/s}^2)$

- Using the kinematic equation for the horizontal motion:

  x = *vxt*

  1 point is earned: For using the correct value for the horizontal speed

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vx = 0.8 m/s

x = (0.8 m/s)(0.28 s)

1 point is earned: For the correct answer

x = 0.22 m (or 0.23 m using *g* = 9.8 m/s2)

- **2 points are earned:** For correctly applying conservation of energy

Kbottom = *Ktop* + *Utop*

$$K_{bottom} = \frac{1}{2}mv_x{}^2 + mgy$$
$$K_{bottom} = \frac{1}{2}(0.50\text{ kg})(0.8\text{ m/s})^2 + (0.50\text{ kg})(10\text{ m/s}^2)(0.40\text{ m})$$

1 point is earned: For the correct answer

Kbottom = 2.2 J (or 2.1 J using *g* = 9.8 m/s2)

In the absence of any of the above credit, one point was awarded for any recognition that the vertical velocity changes.

(Alternate solution)

1 point is earned: For any recognition that the vertical velocity changes.

Calculating the *y*-component of velocity at the bottom:

vy = vyt – *gt*

vy = 0 + (10 m/s2)(0.28 s) = 2.8 m/s

1 point is earned: For correctly calculating the total speed from the components

*v*2 = *vx*2 + *vy*2

$$v = \sqrt{(0.8\text{ m/s})^2 + (2.8\text{ m/s})^2}$$

v = 2.9 m/s

1 point is earned: For correctly calculating the final kinetic energy

$$K = \frac{1}{2}mv^2$$
$$K = \frac{1}{2}(0.50\text{ kg})(2.9\text{ m/s})^2$$

K = 2.2 J (or 2.1 J depending on the value of *g* used and amount of rounding in intermediate calculations)

frq5

0	1	2	3	4	5	6 ✓

The student response earns all of the following points:

- **1 point is earned:** For using the correct kinematic equation, with the initial vertical velocity equal to zero

  $y = \frac{1}{2}gt^2$

  Solving for *t*:

  $t = \sqrt{2y/g}$

  $t = \sqrt{2(0.40\text{ m})/10\text{ m/s}^2}$

  $t = 0.28\text{ s} (\text{or } 0.29\text{ s using } g = 9.8\text{ m/s}^2)$

- Using the kinematic equation for the horizontal motion:

  x = *vxt*

  1 point is earned: For using the correct value for the horizontal speed

  vx = 0.8 m/s

  x = (0.8 m/s)(0.28 s)

  1 point is earned: For the correct answer

  x = 0.22 m (or 0.23 m using *g* = 9.8 m/s2)

- **2 points are earned:** For correctly applying conservation of energy

  Kbottom = *Ktop* + *Utop*

  $K_{bottom} = \frac{1}{2}mv_x{}^2 + mgy$

  $K_{bottom} = \frac{1}{2}(0.50\text{ kg})(0.8\text{ m/s})^2 + (0.50\text{ kg})(10\text{ m/s}^2)(0.40\text{ m})$

  1 point is earned: For the correct answer

  Kbottom =2.2 J (or 2.1 J using *g* = 9.8 m/s2)

  (Alternate solution)

  1 point is earned: For any recognition that the vertical velocity changes.

  Calculating the *y*-component of velocity at the bottom:

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vy = vyt – gt

vy = 0 + (10 m/s2)(0.28 s) = 2.8 m/s

1 point is earned: For correctly calculating the total speed from the components

$v2 = vx2 + vy2$

$$v = \sqrt{(0.8\ \text{m/s})^2 + (2.8\ \text{m/s})^2}$$

v = 2.9 m/s

1 point is earned: For correctly calculating the final kinetic energy

$$K = \tfrac{1}{2}mv^2$$
$$K = \tfrac{1}{2}(0.50\ \text{kg})(2.9\ \text{m/s})^2$$

K = 2.2 J (or 2.1 J depending on the value of g used and amount of rounding in intermediate calculations)

frq5

2. A 20 kg box on a horizontal frictionless surface is moving to the right at a speed of 4.0 m/s. The box hits and remains attached to one end of a spring of negligible mass whose other end is attached to a wall. As a result, the spring compresses a maximum distance of 0.50 m, and the box then oscillates back and forth.

(a)

i. The spring does work on the box from the moment the box first hits the spring to the moment the spring first reaches its maximum compression. Indicate whether the work done by the spring is positive, negative, or zero.

____ Positive ____ Negative ____ Zero

Justify your answer.

ii. Calculate the magnitude of the work described in part i.

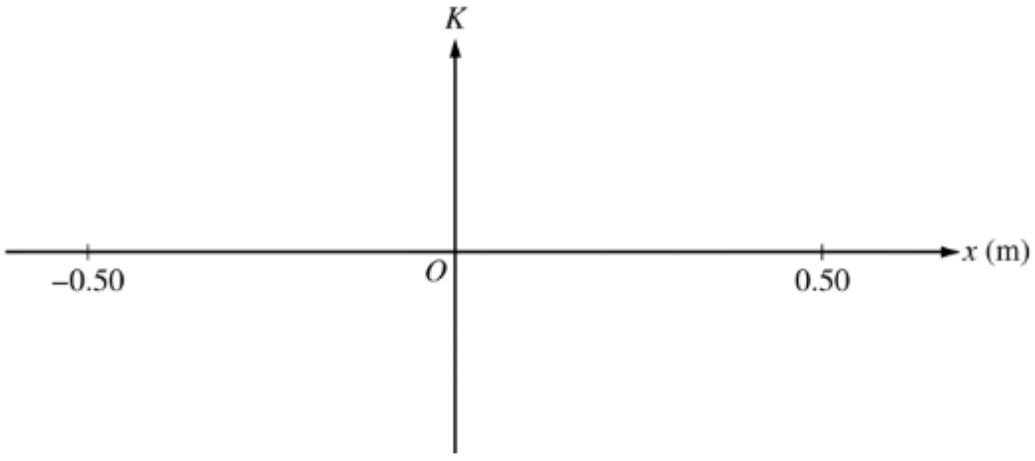
(b) Calculate the spring constant of the spring.

(c) Calculate the magnitude of the maximum acceleration of the box.

(d) Calculate the frequency of the oscillation of the box.

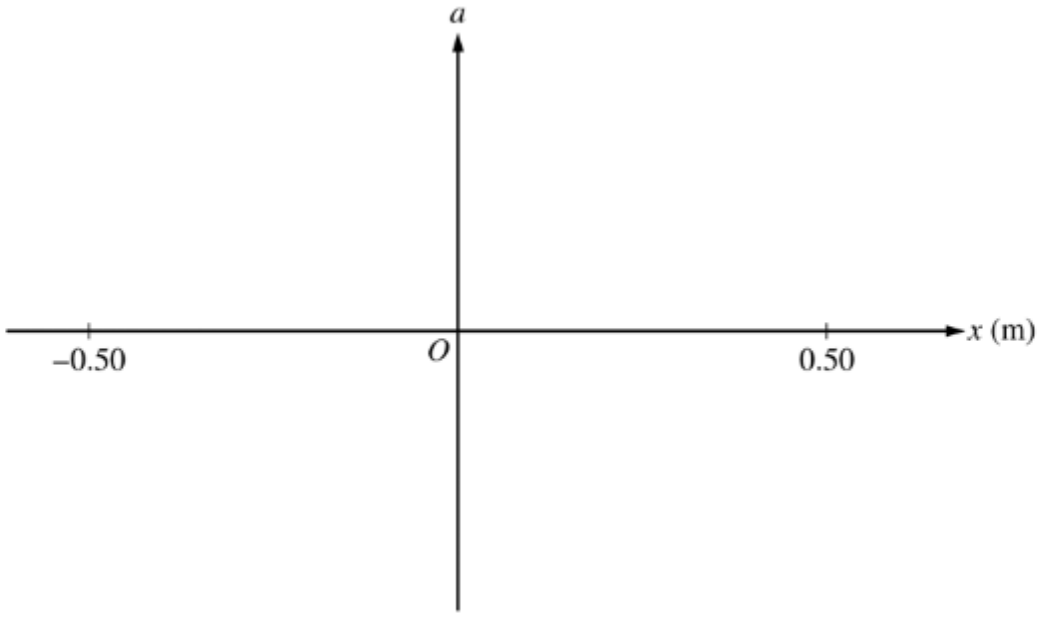
(e) Let x = 0 be the point where the box makes contact with the spring, with positive x directed toward the right.

i. On the axes below, sketch the kinetic energy *K* of the oscillating box as a function of *position* x for the range x = -0.50 m to x = +0.50 m.



ii. On the axes below, sketch the acceleration a of the oscillating box as a function of *position* x for the range x = -0.50 m to x = +0.50 m.

frq5



Part A

1 point is earned: For selecting "Negative"

1 point is earned: For a correct justification

Examples:

- The spring pushes on the box in a direction opposite to the box's motion, therefore the work done is negative.
- The box slows down, so its kinetic energy decreases. Therefore the work done is negative.

1 point is earned: For a correct expression relating to work to change in kinetic energy

$W = \Delta K$
$W = \frac{1}{2}m\left(v_2^2 - v_1^2\right) = -\frac{1}{2}mv_1^2$

1 point is earned: For substituting the correct values

$W = -\left(\frac{1}{2}\right)(20kg)(4.0m/s)^2$

1 point is earned: For a correct answer

W = 160 J (either positive or negative value acceptable)

frq5

0	1	2	3	4	5 ✓

The student response earns all of the following points:

1 point is earned: For selecting "Negative"

1 point is earned: For a correct justification

Examples:

- The spring pushes on the box in a direction opposite to the box's motion, therefore the work done is negative.
- The box slows down, so its kinetic energy decreases. Therefore the work done is negative.

1 point is earned: For a correct expression relating to work to change in kinetic energy

$W = \Delta K$
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1 point is earned: For substituting the correct values

$W = -\left(\frac{1}{2}\right)(20kg)(4.0m/s)^2$

1 point is earned: For a correct answer

W = 160 J (either positive or negative value acceptable)

Part B

1 point is earned: For a correct expression of conservation of energy

$K_1 = U_2$
$\frac{1}{2}mv_1^2 = \frac{1}{2}kx_2^2$

1 point is earned: For correct substitution of values consistent with the answer from part (a) ii

$(160J) = \left(\frac{1}{2}\right)k(0.50m)^2$
$k = 1280N/m$

0	1	2 ✓

The student response earns all of the following points:

frq5

1 point is earned: For a correct expression of conservation of energy

$K_1 = U_2$
$\frac{1}{2}mv_1^2 = \frac{1}{2}kx_2^2$

1 point is earned: For correct substitution of values consistent with the answer from part (a) ii

$(160J) = \left(\frac{1}{2}\right)k(0.50m)^2$
$k = 1280N/m$

Part C

1 point is earned: For a correct expression of the maximum acceleration

a = Fmax/m = kxmax/m

1 point is earned: For correct substitution consistent with the answer from part (b)

a = (1280 N/m)(0.50 m)/(20 kg)

a = 32 m/s2

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For a correct expression of the maximum acceleration

a = Fmax/m = kxmax/m

1 point is earned: For correct substitution consistent with the answer from part (b)

a = (1280 N/m)(0.50 m)/(20 kg)

a = 32 m/s2

Part D

1 point is earned: For a correct expression of the frequency

$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

1 point is earned: For correct substitution consistent with the answer from part (b)

$f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{(1280N/m)}{(20kg)}}$
$f = 1.27Hz$

frq5

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For a correct expression of the frequency

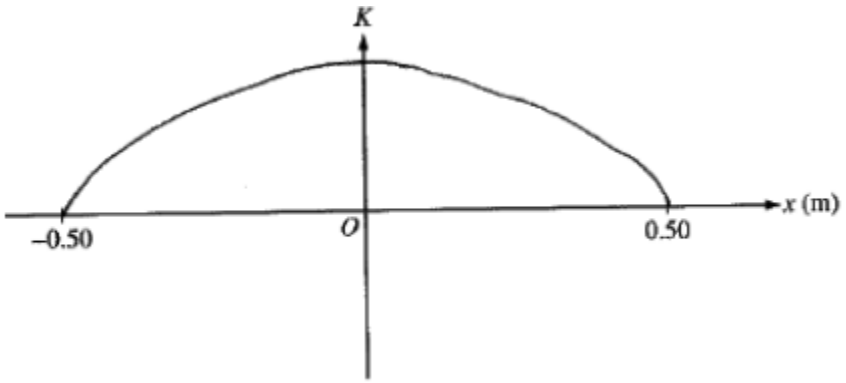
$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

1 point is earned: For correct substitution consistent with the answer from part (b)

$f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{(1280N/m)}{(20kg)}}$

$f = 1.27Hz$

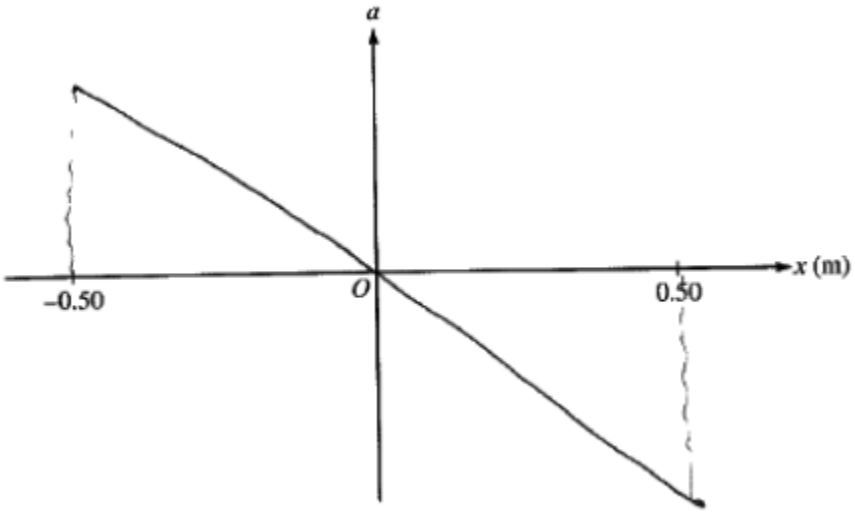
Part E



1 point is earned: For a graph that shows positive kinetic energy values and zero kinetic energy at -0.50 m and +0.50 m

1 point is earned: For a smooth curve that is concave down

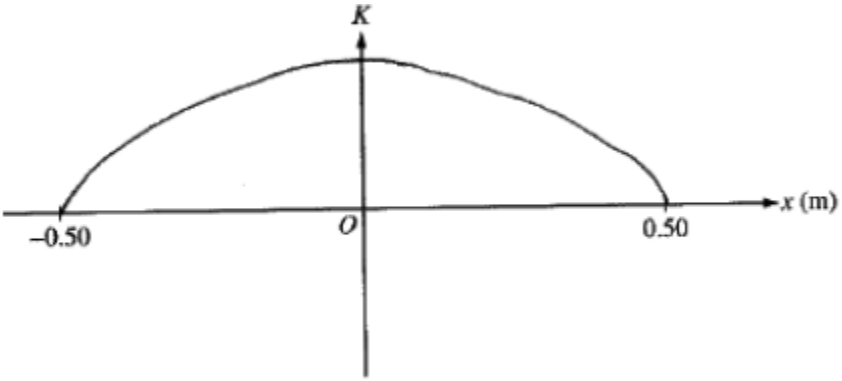
frq5



1 point is earned: For a straight line with negative slope that begins at -0.50 m, ends at +0.50 m, and passes through the origin

0	1	2	3 ✓

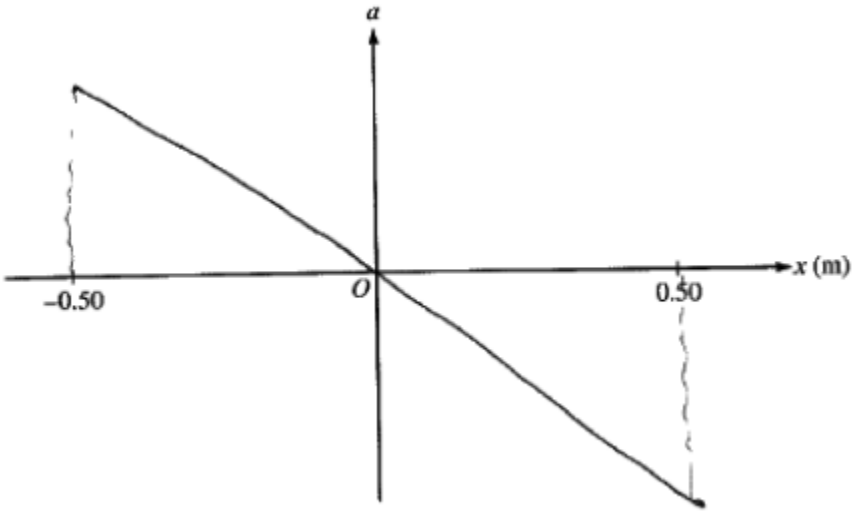
The student response earns all of the following points:



1 point is earned: For a graph that shows positive kinetic energy values and zero kinetic energy at -0.50 m and +0.50 m

1 point is earned: For a smooth curve that is concave down

frq5



1 point is earned: For a straight line with negative slope that begins at -0.50 m, ends at +0.50 m, and passes through the origin

Units

1 point is earned: For correct units on three out of four calculated answers

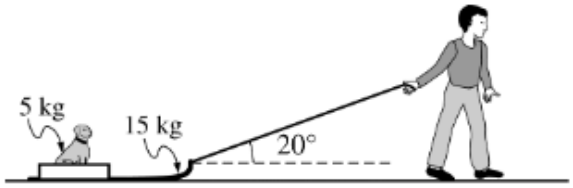
0	1 ✓

The student response earns all of the following points:

1 point is earned: For correct units on three out of four calculated answers

frq5

3.



A child pulls a 15 kg sled containing a 5.0 kg dog along a straight path on a horizontal surface. He exerts a force of 55 N on the sled at an angle of 20º above the horizontal, as shown in the figure above. The coefficient of friction between the sled and the surface is 0.22.

(a) On the dot below that represents the sled-dog system, draw and label a free-body diagram for the system as it is pulled along the surface.

(b) Calculate the normal force of the surface on the system.

(c) Calculate the acceleration of the system.

(d) Calculate the work done by the child's pulling force as the system moves a distance of 7.0 m.

(e) At some later time, the dog rolls off the side of the sled. The child continues to pull with the same force. On the axes below, sketch a graph of speed u versus time t for the sled. Include both the sled's travel with and without the dog on the sled. Clearly indicate with the symbol t_r the time at which the dog rolls off.



Part A

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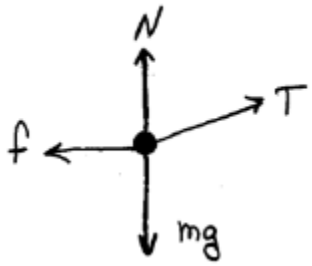


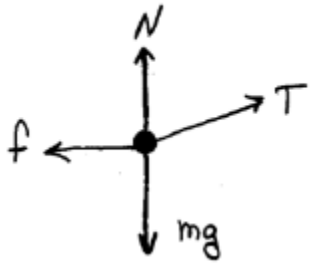
4 points are earned: For each of the forces above with arrow correctly drawn and labeled, 1 point was awarded 4 points

Notes:

The force vector T could be shown in component form or as one force, but not both. For extraneous forces, 1 point was deducted.

0	1	2	3	4 ✓

The student response earns all of the following points:



4 points are earned: For each of the forces above with arrow correctly drawn and labeled, 1 point was awarded 4 points

Notes:

The force vector T could be shown in component form or as one force, but not both. For extraneous forces, 1 point was deducted.

Part B

$\sum F_y = 0$
$N + T\sin\theta - mg = 0$

1 point is earned: For a correct force equation

N = mg - T sinθ

frq5

1 point is earned: For calculating the mg term correctly

1 point is earned: For calculating the term correctly
N = (15kg + 5kg) (9.8m/s2) - (55N) sin 20°

1 point is earned: For the correct numerical answer with units
N= 177N (181, N, if g = 10m/s2 is used)

0	1	2	3	4 ✓

The student response earns all of the following points:

$\sum F_y = 0$
$N + T \sin\theta - mg = 0$

1 point is earned: For a correct force equation

N = mg - T sinθ

1 point is earned: For calculating the mg term correctly

1 point is earned: For calculating the term correctly

N = (15kg + 5kg) (9.8m/s2) - (55N) sin 20°

1 point is earned: For the correct numerical answer with units

N= 177N (181, N, if g = 10m/s2 is used)

Part C

$\sum F_x = ma$

1 point is earned: For a correct Newton's second law equation involving horizontal forces

$T \cos\theta - f = ma$
$a = \frac{T\cos\theta - f}{m}$

1 point is earned: For the correct relationship between the frictional and normal forces

$f = \mu N$

1 point is earned: For correct calculation of the frictional force using the value of *N* from part (b)

frq5

$f = (0.22)(177N) = 38.9N\big(39.8N, if, g = 10m/s^2, is, used\big)$
$a = \frac{(55N)\cos 20^\circ - 38.9N}{20kg}$
$a = 0.64m/s^2\big(0.59m/s^2, if, g = 10m/s^2, is, used\big)$

0	1	2	3 ✓

The student response earns all of the following points:

$\sum F_x = ma$

1 point is earned: For a correct Newton's second law equation involving horizontal forces

$T\cos\theta - f = ma$
$a = \frac{T\cos\theta - f}{m}$

1 point is earned: For the correct relationship between the frictional and normal forces

$f = \mu N$

1 point is earned: For correct calculation of the frictional force using the value of *N* from part (b)

$f = (0.22)(177N) = 38.9N\big(39.8N, if, g = 10m/s^2, is, used\big)$
$a = \frac{(55N)\cos 20^\circ - 38.9N}{20kg}$
$a = 0.64m/s^2\big(0.59m/s^2, if, g = 10m/s^2, is, used\big)$

Part D

1 point is earned: For the correct work equation
W = (T cosθ) d

1 point is earned: For substituting the correct displacement
W = (55N) cos20° (7.0m)
W = 360J

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For the correct work equation

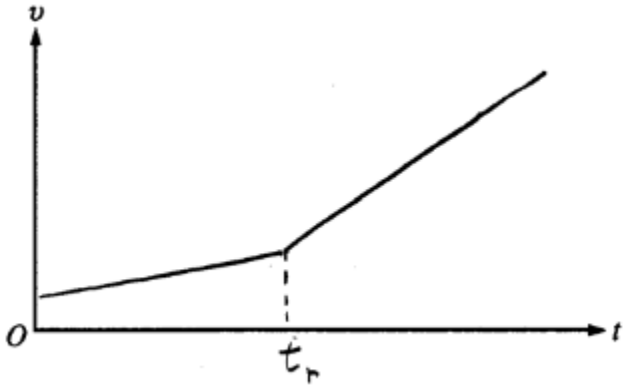
W = (T cosθ) d

frq5

1 point is earned: For substituting the correct displacement

W = (55N) cos20° (7.0m)
W = 360J

Part E

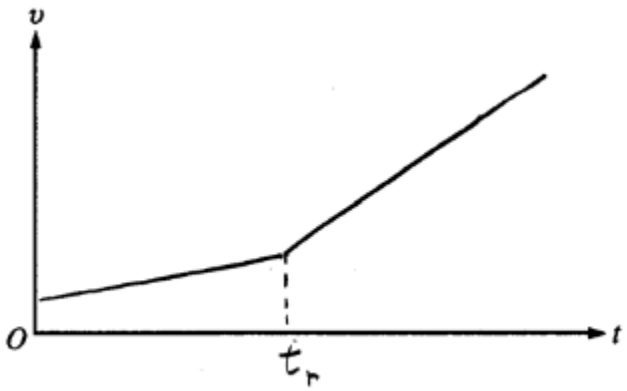


2 points are earned: For each section of the graph correctly drawn as above, 1 point was awarded.

Note: The second point was awarded only if the change at was abrupt.

0	1	2 ✓

The student response earns all of the following points:

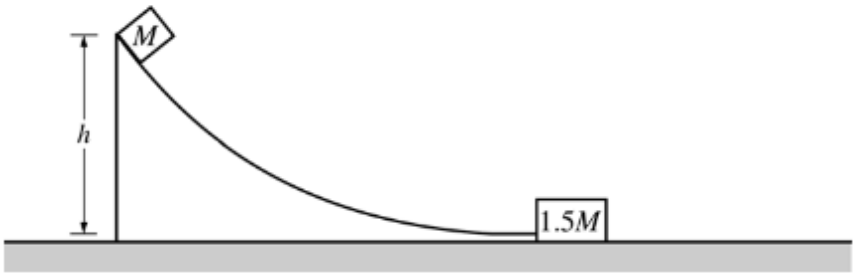


2 points are earned: For each section of the graph correctly drawn as above, 1 point was awarded.

Note: The second point was awarded only if the change at was abrupt.

frq5

4.



A small block of mass *M* is released from rest at the top of the curved frictionless ramp shown above. The block slides down the ramp and is moving with a speed $3.5v_0$ when it collides with a larger block of mass 1.5M at rest at the bottom of the incline. The larger block moves to the right at a speed $2v_0$ immediately after the collision. Express your answers to the following questions in terms of the given quantities and fundamental constants.

(a) Determine the height *h* of the ramp from which the small block was released.

(b) Determine the speed of the small block after the collision.

(c) The larger block slides a distance *D* before coming to rest. Determine the value of the coefficient of kinetic friction μ between the larger block and the surface on which it slides.

(d) Indicate whether the collision between the two blocks is elastic or inelastic. Justify your answer.

Part A

1 point is earned: For any use of conservation of energy

For example, initially the small block has only potential energy, and it is all converted to kinetic energy when it reaches the bottom of the ramp.

1 point is earned: For a correct expression for the initial potential energy

1 point is earned: For a correct expression for the kinetic energy at the bottom of the ramp

$Mgh = \frac{1}{2}M(3.5v_0)^2$

1 point is earned: For the correct answer

$h = \frac{3.5^2}{2}\frac{v_0^2}{g}$ or equivalent

✓

0	1	2	3	4

frq5

The student response earns all of the following points:

1 point is earned: For any use of conservation of energy

For example, initially the small block has only potential energy, and it is all converted to kinetic energy when it reaches the bottom of the ramp.

1 point is earned: For a correct expression for the initial potential energy

1 point is earned: For a correct expression for the kinetic energy at the bottom of the ramp

$Mgh = \frac{1}{2}M(3.5v_0)^2$

1 point is earned: For the correct answer

$h = \frac{3.5^2}{2}\frac{v_0^2}{g}$ or equivalent

Part B

1 point is earned: For any use of conservation of momentum

1 point is earned: For a correct expression for the initial momentum of the blocks

1 point is earned: For a correct expression for the final momentum of the blocks

$M(3.5v0) = Mv + (1.5M)(2v0)$

$v = 3.5v0 - 3v0$

1 point is earned: For the correct answer

$v = 0.5v0$



0	1	2	3	4

The student response earns all of the following points:

1 point is earned: For any use of conservation of momentum

1 point is earned: For a correct expression for the initial momentum of the blocks

1 point is earned: For a correct expression for the final momentum of the blocks

$M(3.5v0) = Mv + (1.5M)(2v0)$

$v = 3.5v0 - 3v0$

1 point is earned: For the correct answer

frq5

v = 0.5*v0*

Part C

1 point is earned: For a correct relationship between friction and the acceleration of the block

$$\sum F = ma = f_{fric}$$

1 point is earned: For a correct kinematic equation relating acceleration and distance that does not contain time

$$v_f^2 = v_i^2 - 2aD$$

1 point is earned: For using the correct initial speed of the block

$$0 = 4v_0^2 - 2aD$$
$$a = 2v_0^2/D$$

Substituting expressions for *a* and *f* into the first equation above

$$(1.5M)2v_0^2/D = \mu(1.5M)g$$

1 point is earned: For the correct answer

$$\mu = \frac{2v_0^2}{Dg}$$

Alternate solution

1 point is earned: *For any indication that the work done on the block as it slides is equal to its initial kinetic energy*

$$f_{fric}d = \frac{1}{2}mv_i^2$$

1 point is earned: *For a correct expression for the work done on the block*

1 point is earned: *For a correct expression for the initial kinetic energy of the block*

$$\mu(1.5M)gD = \frac{1}{2}(1.5M)(2v_0)^2$$

1 point is earned: *For the correct answer*

$$\mu = \frac{2v_0^2}{Dg}$$

0	1	2	3	4 ✓

The student response earns all of the following points:

1 point is earned: For a correct relationship between friction and the acceleration of the block

frq5

$\sum F = ma = f_{fric}$

1 point is earned: For a correct kinematic equation relating acceleration and distance that does not contain time

$v_f^2 = v_i^2 - 2aD$

1 point is earned: For using the correct initial speed of the block

$0 = 4v_0^2 - 2aD$
$a = 2v_0^2/D$

Substituting expressions for *a* and *f* into the first equation above

$(1.5M)2v_0^2/D = \mu(1.5M)g$

1 point is earned: For the correct answer

$\mu = \frac{2v_0^2}{Dg}$

Alternate solution

1 point is earned: *For any indication that the work done on the block as it slides is equal to its initial kinetic energy*

$f_{fric}d = \frac{1}{2}mv_i^2$

1 point is earned: *For a correct expression for the work done on the block*

1 point is earned: *For a correct expression for the initial kinetic energy of the block*

$\mu(1.5M)gD = \frac{1}{2}(1.5M)(2v_0)^2$

1 point is earned: *For the correct answer*

$\mu = \frac{2v_0^2}{Dg}$

Part D

1 point is earned: For indicating that the collision is inelastic

1 point is earned: For indicating that the reason it is inelastic is because the change in kinetic energy is not zero, or because kinetic energy is lost in the collision

1 point is earned: For showing that the change in kinetic energy is not zero

$\Delta K = K_f - K_i = \left[\frac{1}{2}M(0.5v_0)^2 + \frac{1}{2}(1.5M)(2v_0)^2\right] - \frac{1}{2}M(3.5v_0)^2$
$\Delta K = -3Mv_0^2$

frq5

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For indicating that the collision is inelastic

1 point is earned: For indicating that the reason it is inelastic is because the change in kinetic energy is not zero, or because kinetic energy is lost in the collision

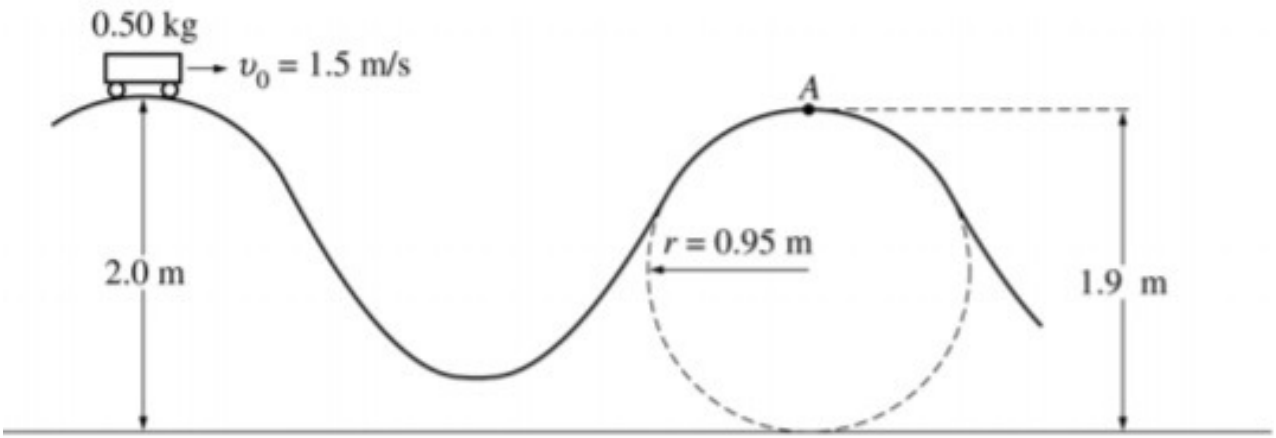
1 point is earned: For showing that the change in kinetic energy is not zero

$$\Delta K = K_f - K_i = \left[\tfrac{1}{2}M(0.5v_0)^2 + \tfrac{1}{2}(1.5M)(2v_0)^2\right] - \tfrac{1}{2}M(3.5v_0)^2$$

$$\Delta K = -3Mv_0^2$$

frq5

5.



A designer is working on a new roller coaster, and she begins by making a scale model. On this model, a car of total mass 0.50 kg moves with negligible friction along the track shown in the figure above. The car is given an initial speed v_0 = 1.5 m/s at the top of the first hill of height 2.0 m. Point *A* is located at a height of 1.9 m at the top of the second hill, the upper part of which is a circular arc of radius 0.95 m.

a. Calculate the speed of the car at point *A*.

b. On the figure of the car below, draw and label vectors to represent the forces on the car at point *A*.



c. Calculate the magnitude of the force of the track on the car at point *A*.

d. In order to stop the car at point A, some friction must be introduced. Calculate the work that must be done by the friction force in order to stop the car at point *A*.

e. Explain how to modify the track design to cause the car to lose contact with the track at point *A* before descending down the track. Justify your answer.

Part A

1 point is earned: For any indication of conservation of energy

1 point is earned: For a correct conservation equation for this situation

$\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_A^2 + mgh_A$

*v*2*A* = *v*2*0* +2*g*(*h*0 – *hA*)

1 point is earned: For the correct substitutions

*v*2*A* = (1.5 m/s)2 + 2(9.8 m/s2)(2.0 m – 1.9 m)

*v*2*A* = 4.2(m/s)2 (or 4.3(m/s)2 using *g* = 10 m/s2)

frq5

1 point is earned: For the correct answer

vA = 2.0 m/s (or 2.1 m/s if *v*2*A* was not rounded or if *g* = 10 m/s2 was used)

0	1	2	3	4 ✓

The student response earns four of the following points:

1 point is earned: For any indication of conservation of energy

1 point is earned: For a correct conservation equation for this situation

$\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_A^2 + mgh_A$

*v*2*A* = *v*2*0* +2*g*(*h*0 – *hA*)

1 point is earned: For the correct substitutions

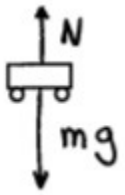
*v*2*A* = (1.5 m/s)2 + 2(9.8 m/s2)(2.0 m – 1.9 m)

*v*2*A* = 4.2(m/s)2 (or 4.3(m/s)2 using *g* = 10 m/s2)

1 point is earned: For the correct answer

vA = 2.0 m/s (or 2.1 m/s if *v*2*A* was not rounded or if *g* = 10 m/s2 was used)

Part B



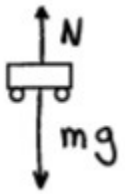
1 point is earned: For each correctly drawn and labeled force

One point earned for correct forces was deducted for having any extraneous forces

0	1	2 ✓

The student response earns two of the following points:

frq5



1 point is earned: For each correctly drawn and labeled force

One point earned for correct forces was deducted for having any extraneous forces

Part C

1 point is earned: For indicating that the sum of the forces equals the centripetal force

$\Sigma F = \frac{mv_A^2}{r}$

1 point is earned: For having the correct relative signs in the application of the above equation

$mg - N = \frac{mv_A^2}{r}$

1 point is earned: For solving for the normal force, substituting, and solving correctly

$N = mg - \frac{mv_A^2}{r}$

$N = (0.50\ \text{kg})(9.8\ \text{m/s}^2) - \frac{(0.50\ \text{kg})(4.21(\text{m/s})^2)}{0.95\ \text{m}}$

N = 2.7 N

0	1	2	3 ✓

The student response earns none of the following points:

1 point is earned: For indicating that the sum of the forces equals the centripetal force

$\Sigma F = \frac{mv_A^2}{r}$

1 point is earned: For having the correct relative signs in the application of the above equation

$mg - N = \frac{mv_A^2}{r}$

1 point is earned: For solving for the normal force, substituting, and solving correctly

$N = mg - \frac{mv_A^2}{r}$

$N = (0.50\ \text{kg})(9.8\ \text{m/s}^2) - \frac{(0.50\ \text{kg})(4.21(\text{m/s})^2)}{0.95\ \text{m}}$

frq5

N = 2.7 N

Part D

1 point is earned: For any indication that the work done must equal the kinetic energy at point A in the absence of friction

$W_{\text{friction}} = -\frac{1}{2}mv_A^2$

1 point is earned: For correct substitution

$W_{\text{friction}} = -\frac{1}{2}(0.50\ \text{kg})(4.2(\text{m/s})^2)$

1 point is earned: For the correct answer, including the negative sign

Wfriction = –1.1 J

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned: For any indication that the work done must equal the kinetic energy at point A in the absence of friction

$W_{\text{friction}} = -\frac{1}{2}mv_A^2$

1 point is earned: For correct substitution

$W_{\text{friction}} = -\frac{1}{2}(0.50\ \text{kg})(4.2(\text{m/s})^2)$

1 point is earned: For the correct answer, including the negative sign

Wfriction = –1.1 J

Part E

Method 1:

1 point is earned: For indicating that one should decrease the radius of the curve of the second hill

The cart will lose contact when $mg - \frac{mv_A^2}{r} \leq 0$. Decreasing the radius of the curve will cause the second term to increase and thus meet the condition

1 point is earned: For referring to the equation $N = mg - \frac{mv_A^2}{r}$

1 point is earned: For indicating that the cart will loose contact when $N \leq 0$

Method 2:

frq5

1 point is earned: For indicating that one should make the initial hill higher or the second hill lower

The cart will lose contact when $mg - \frac{mv_A^2}{r} \leq 0$. Increasing the difference in heights will cause the speed at *A* to increase, and thus the second term will increase and the condition will be met

1 point is earned: For referring to the equation $N = mg - \frac{mv_A^2}{r}$ and indicating that the cart will loose contact when *N* ≤ 0

1 point is earned: For indicating that the speed at *A* will increase

0	1	2	3 ✓

The student response earns three of the following points:

Method 1:

1 point is earned: For indicating that one should decrease the radius of the curve of the second hill

The cart will lose contact when $mg - \frac{mv_A^2}{r} \leq 0$. Decreasing the radius of the curve will cause the second term to increase and thus meet the condition

1 point is earned: For referring to the equation $N = mg - \frac{mv_A^2}{r}$

1 point is earned: For indicating that the cart will loose contact when *N* ≤ 0

Method 2:

1 point is earned: For indicating that one should make the initial hill higher or the second hill lower

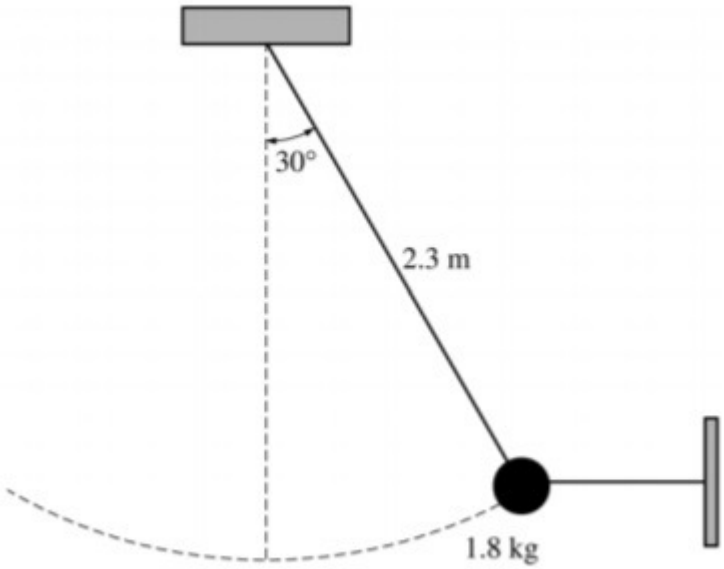
The cart will lose contact when $mg - \frac{mv_A^2}{r} \leq 0$. Increasing the difference in heights will cause the speed at *A* to increase, and thus the second term will increase and the condition will be met

1 point is earned: For referring to the equation $N = mg - \frac{mv_A^2}{r}$ and indicating that the cart will loose contact when *N* ≤ 0

1 point is earned: For indicating that the speed at *A* will increase

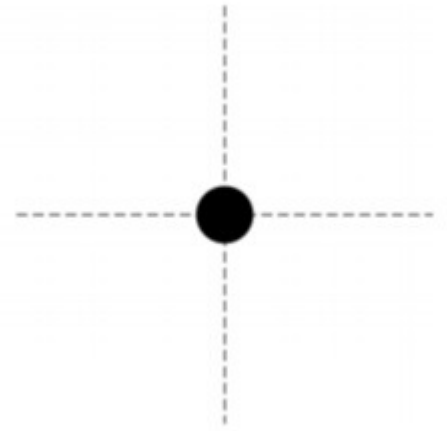
frq5

6.



A simple pendulum consists of a bob of mass 1.8 kg attached to a string of length 2.3 m. The pendulum is held at an angle of 30° from the vertical by a light horizontal string attached to a wall, as shown above.

a. On the figure below, draw a free-body diagram showing and labeling the forces on the bob in the position shown above.

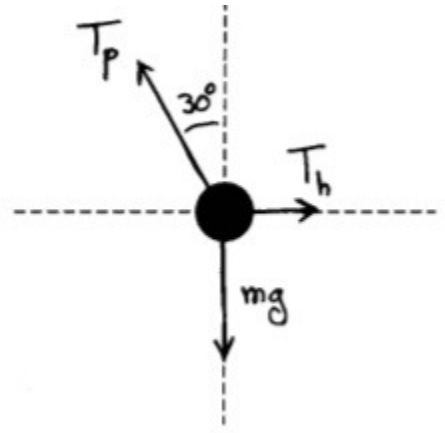


b. Calculate the tension in the horizontal string.

c. The horizontal string is now cut close to the bob, and the pendulum swings down. Calculate the speed of the bob at its lowest position

Part A

frq5



1 point is earned: For each correctly drawn and labeled tension, with arrowhead in right direction

One point was deducted for each of the following until score reached zero:

No force of gravity

Each extraneous force

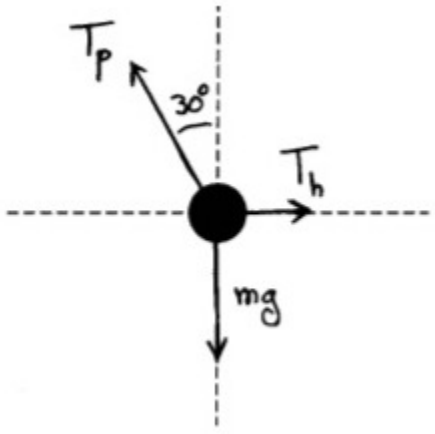
Any missing labels

Drawing all forces along correct lines with labels but no arrowheads received only one point.

Components of the tension in the pendulum string could be included in addition to or instead of the net tension, as long as they were clearly labeled as such.

0	1	2 ✓

The student response earns two of the following points:



1 point is earned: For each correctly drawn and labeled tension, with arrowhead in right direction

frq5

One point was deducted for each of the following until score reached zero:

No force of gravity

Each extraneous force

Any missing labels

Drawing all forces along correct lines with labels but no arrowheads received only one point.

Components of the tension in the pendulum string could be included in addition to or instead of the net tension, as long as they were clearly labeled as such.

Part B

1 point is earned: For any indication that the net force is zero

1 point is earned: For an attempt to determine the components of the tension in the pendulum string

1 point is earned: For correctly determining these components

Th = *Tp* sin 30°

mg = *Tp* cos 30°

$\frac{T_h}{mg} = \frac{\sin 30^\circ}{\cos 30^\circ} = \tan 30^\circ$

Th = tan 30°

Th = (1.8 kg)(9.8 m/s2) tan 30°

1 point is earned: For the correct answer with units

Th = 10 N

Alternate solution

2 points are earned: For indicating that the addition of the three force vectors gives a 30-60-90 right triangle

1 point is earned: For a correct trigonometric relationship between forces
Th / mg = tan 30°

1 point is earned: For the correct answer with units

Th = 10 N

If no forces were drawn in part (a), one point could be earned for writing the component equations T = *mg* sin θ and T = *mg* cos θ, and two points for $T = mg \tan \theta$.

frq5

0	1	2	3	4 ✓

The student response earns four of the following points:

1 point is earned: For any indication that the net force is zero

1 point is earned: For an attempt to determine the components of the tension in the pendulum string

1 point is earned: For correctly determining these components

Th = *Tp* sin 30°

mg = *Tp* cos 30°

$\frac{T_h}{mg} = \frac{\sin 30^\circ}{\cos 30^\circ} = \tan 30^\circ$

Th = tan 30°

Th = (1.8 kg)(9.8 m/s2) tan 30°

1 point is earned: For the correct answer with units

Th = 10 N

Alternate solution

2 points are earned: For indicating that the addition of the three force vectors gives a 30-60-90 right triangle

1 point is earned: For a correct trigonometric relationship between forces

Th / mg = tan 30°

1 point is earned: For the correct answer with units

Th = 10 N

If no forces were drawn in part (a), one point could be earned for writing the component equations T = *mg* sin θ and T = *mg* cos θ, and two points for $T = mg \tan \theta$.

Part C

1 point is earned: For any indication of conservation of energy

1 point is earned: For any indication of the need to use a change in height

$mgh_0 + \frac{1}{2}mv_0^2 = mgh_f + \frac{1}{2}mv_f^2$

1 point is earned: For setting *v*0 = 0

frq5

$\frac{1}{2}mv_f^2 = mg\Delta h$
$v_f = \sqrt{2g\Delta h}$
$\Delta h = L - L\cos 30^\circ$
$v_f = \sqrt{2gL(1 - \cos 30^\circ)}$
$v_f = \sqrt{2(9.8\ \text{m/s}^2)(2.3\ \text{m})(1 - \cos 30^\circ)}$

1 point is earned: For the correct answer, with units

vf = 2.5 m/s

A solution that used the kinematic equation *vf*2 = *v0*2 + 2*as* could only receive full credit if the student explained how the equation is equivalent to conservation of energy.

0	1	2	3	4 ✓

The student response earns four of the following points:

1 point is earned: For any indication of conservation of energy

1 point is earned: For any indication of the need to use a change in height

$mgh_0 + \frac{1}{2}mv_0^2 = mgh_f + \frac{1}{2}mv_f^2$

1 point is earned: For setting *v*0 = 0

$\frac{1}{2}mv_f^2 = mg\Delta h$
$v_f = \sqrt{2g\Delta h}$
$\Delta h = L - L\cos 30^\circ$
$v_f = \sqrt{2gL(1 - \cos 30^\circ)}$
$v_f = \sqrt{2(9.8\ \text{m/s}^2)(2.3\ \text{m})(1 - \cos 30^\circ)}$

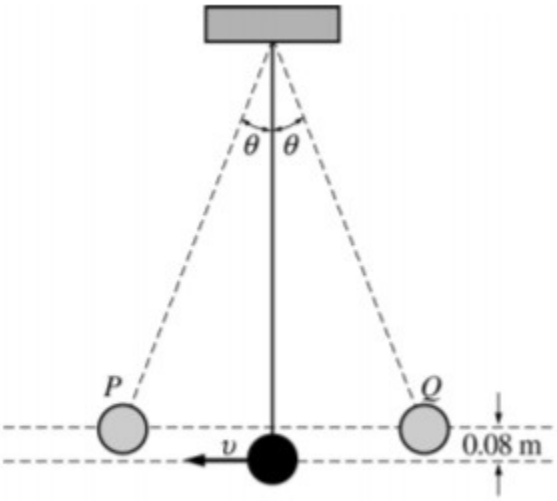
1 point is earned: For the correct answer, with units

vf = 2.5 m/s

A solution that used the kinematic equation *vf*2 = *v0*2 + 2*as* could only receive full credit if the student explained how the equation is equivalent to conservation of energy.

frq5

7. A simple pendulum consists of a bob of mass 0.085 kg attached to a string of length 1.5 m. The pendulum is raised to point Q, which is 0.08 m above its lowest position, and released so that it oscillates with small amplitude θ between the points P and Q as shown below.



Note: Figure not drawn to scale.

a. On the figures below, draw free-body diagrams showing and labeling the forces acting on the bob in each of the situations described.

i. When it is at point P



ii. When it is in motion at its lowest position

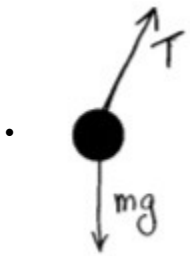


b. Calculate the speed u of the bob at its lowest position.

c. Calculate the tension in the string when the bob is passing through its lowest position.

d. Describe one modification that could be made to double the period of oscillation.

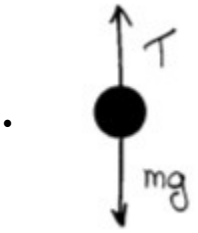
Part A



1 point is earned: For each correctly drawn and labeled force

frq5

One point was deducted if any extraneous forces were shown.

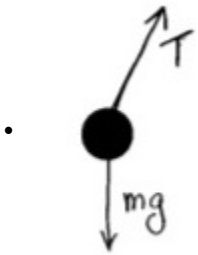


1 point is earned: For each correctly drawn and labeled force

One point was deducted if any extraneous forces were shown.

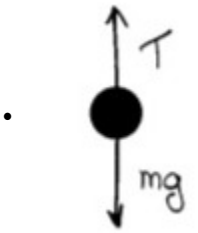
0	1	2	3	4 ✓

The student response earns four of the following points:



1 point is earned: For each correctly drawn and labeled force

One point was deducted if any extraneous forces were shown.



1 point is earned: For each correctly drawn and labeled force

One point was deducted if any extraneous forces were shown.

Part B

1 point is earned: For correctly applying conservation of energy

frq5

$\frac{1}{2}mv^2 = mg\Delta h$

$v = \sqrt{2g\Delta h}$

$v = \sqrt{2(9.8\ \text{m/s}^2)(0.08\ \text{m})} = \sqrt{1.568}\ \text{m/s}$

1 point is earned: For the correct answer

v = 1.3 m/s

0	1	2 ✓

The student response earns two of the following points:

1 point is earned: For correctly applying conservation of energy

$\frac{1}{2}mv^2 = mg\Delta h$

$v = \sqrt{2g\Delta h}$

$v = \sqrt{2(9.8\ \text{m/s}^2)(0.08\ \text{m})} = \sqrt{1.568}\ \text{m/s}$

1 point is earned: For the correct answer

v = 1.3 m/s

Part C

1 point is earned: For using ΣF = *m*a

1 point is earned: For a correct expression for the net force that is equated to a centripetal force

T – *mg* = *mv2/r*

T = *m*(*g* + *v2/r*)

T = (0.085 kg)[9.8 m/s2 + (1.3 m/s)2/(1.5 m)]

1 point is earned: For the correct answer with units

T = 0.93 N (or equivalent answer depending on rounding or use of *g* = 10 m/s2)

0	1	2	3 ✓

The student response earns three of the following points:

frq5

1 point is earned: For using $\Sigma F = ma$

1 point is earned: For a correct expression for the net force that is equated to a centripetal force

$T - mg = mv2/r$

$T = m(g + v2/r)$

$T = (0.085\ kg)[9.8\ m/s2 + (1.3\ m/s)2/(1.5\ m)]$

1 point is earned: For the correct answer with units

$T = 0.93\ N$ (or equivalent answer depending on rounding or use of $g = 10\ m/s2$)

Part D

1 point is earned: For a correct modification

For example: Make the string four times longer.

0	1 ✓

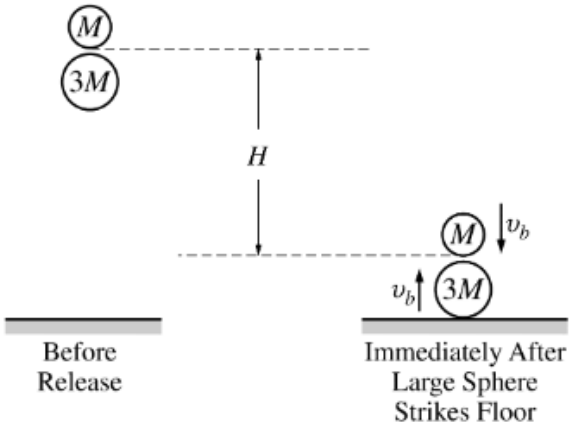
The student response earns one of the following points:

1 point is earned: For a correct modification

For example: Make the string four times longer.

frq5

8.



A small and a large sphere, of mass *M* and *3M* respectively, are arranged as shown on the left side of the figure above. The spheres are then simultaneously dropped from rest. When the large sphere strikes the floor, the spheres have fallen a height *H*. Assume air resistance is negligible. Express all answers in terms of *M, H*, and fundamental constants, as appropriate.

(a) Derive an expression for the speed v_b with which the large sphere strikes the floor.

Immediately after striking the floor, the large sphere is moving upward with speed v_b and collides head-on with the small sphere, which is moving downward with identical speed v_b at that instant. Immediately after the collision, the small sphere moves upward with speed v_S and the large sphere has speed v_L.

(b) Derive an equation that relates v_b, v_S, and v_L.

In this particular situation $v_L = 0$.

(c) Use your relationship from part (b) to determine the speed of the small sphere in terms of v_b.

(d) Indicate whether the collision is elastic. Justify your answer using your results from parts (b) and (c).

(e) Determine the height *h* that the small sphere rises above its lowest position, in terms of the original height *H*.

Part A

1 point is earned: For a statement that shows the conservation of energy for the large sphere

$\Delta U_{3M} = \Delta K_{3M}$
$3MgH = \frac{1}{2}(3M)v_b^2$

1 point is earned: For a correct answer (or equivalent expression for vb)

$v_b = \sqrt{2gH}$

Alternate solution

1 point is earned: *For using a proper kinematic approach*

frq5

$v_f^2 = v_0^2 + 2a\Delta y$
$v_b^2 = 2gH$

1 point is earned: *For a correct answer*

$v_b = \sqrt{2gH}$

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For a statement that shows the conservation of energy for the large sphere

$\Delta U_{3M} = \Delta K_{3M}$
$3MgH = \frac{1}{2}(3M)v_b^2$

1 point is earned: For a correct answer (or equivalent expression for vb)

$v_b = \sqrt{2gH}$

Alternate solution

1 point is earned: *For using a proper kinematic approach*

$v_f^2 = v_0^2 + 2a\Delta y$
$v_b^2 = 2gH$

1 point is earned: *For a correct answer*

$v_b = \sqrt{2gH}$

Part B

1 point is earned: For stating or showing the conservation of momentum applied to the collision

m1v1i + m2v2i = m1v1f + m2v2f

1 point is earned: For stating or showing that the spheres are initially traveling in opposite directions

3*Mvb* + *M*(-vb) = 3*M*vL + *M*vS

2vb = 3vL + vS

frq5

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For stating or showing the conservation of momentum applied to the collision

m1v1i + m2v2i = m1v1f + m2v2f

1 point is earned: For stating or showing that the spheres are initially traveling in opposite directions

3*Mvb* + *M*(-vb) = 3*M*vL + *M*vS

2vb = 3vL + vS

Part C

Substituting the given zero value into the answer from part (b)

2vb = 3vL + vS

2vb = 0 + vS

1 point is earned: For a correct answer

vS = 2vb

0	1 ✓

The student response earns all of the following points:

Substituting the given zero value into the answer from part (b)

2vb = 3vL + vS

2vb = 0 + vS

1 point is earned: For a correct answer

vS = 2vb

Part D

1 point is earned: For any correct attempt to compare total kinetic energy before the collision to total kinetic energy after the collision

frq5

Ki = K3Mi + KMi and Kf = K3Mf + KMf

1 point is earned: For correct substitutions of vb, the expression for vS from part (c), and the correct masses in the kinetic energy terms

$K_i = \frac{1}{2}(3M)(v_b)^2 + \frac{1}{2}(M)(-v_b)^2 = 2Mv_b^2$
$K_f = \frac{1}{2}(M)(2v_b)^2 = 2Mv_b^2$

1 point is earned: For correctly stating that the collision is elastic (or inelastic if consistent with the comparison of initial and final kinetic energies)

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For any correct attempt to compare total kinetic energy before the collision to total kinetic energy after the collision

Ki = K3Mi + KMi and Kf = K3Mf + KMf

1 point is earned: For correct substitutions of vb, the expression for vS from part (c), and the correct masses in the kinetic energy terms

$K_i = \frac{1}{2}(3M)(v_b)^2 + \frac{1}{2}(M)(-v_b)^2 = 2Mv_b^2$
$K_f = \frac{1}{2}(M)(2v_b)^2 = 2Mv_b^2$

1 point is earned: For correctly stating that the collision is elastic (or inelastic if consistent with the comparison of initial and final kinetic energies)

Part E

1 point is earned: For a statement of conservation of energy for ball *M* as it rises to the new height *h*

$U_{gf} = K_0$
$Mgh = \frac{1}{2}Mv_S^2$
$h = \frac{(2v_b)^2}{2g} = \frac{4v_b^2}{2g} = \frac{2\left(\sqrt{2gH}\right)^2}{g}$

1 point is earned: For a correct answer consistent with the expression for vb obtained in part (a)

h = 4H

Alternate solution

1 point is earned: *For using a correct kinematic approach to solve for maximum height of ball M*

frq5

$v_f^2 = v_0^2 + 2a\Delta y$

$v_0 = v_S = 2v_b = 2\sqrt{2gH}$

$0 = \left(2\left(\sqrt{2gH}\right)\right)^2 - 2gH$

1 point is earned: *For a correct answer consistent with the expression for vb obtained in part (a)*

h = 4H

Note: Both points are awarded for any correctly determined value of h without any written justification.

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For a statement of conservation of energy for ball *M* as it rises to the new height *h*

$U_{gf} = K_0$

$Mgh = \frac{1}{2}Mv_S^2$

$h = \frac{(2v_b)^2}{2g} = \frac{4v_b^2}{2g} = \frac{2\left(\sqrt{2gH}\right)^2}{g}$

1 point is earned: For a correct answer consistent with the expression for vb obtained in part (a)

h = 4H

Alternate solution

1 point is earned: *For using a correct kinematic approach to solve for maximum height of ball M*

$v_f^2 = v_0^2 + 2a\Delta y$

$v_0 = v_S = 2v_b = 2\sqrt{2gH}$

$0 = \left(2\left(\sqrt{2gH}\right)\right)^2 - 2gH$

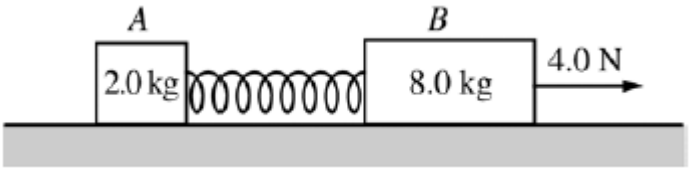
1 point is earned: *For a correct answer consistent with the expression for vb obtained in part (a)*

h = 4H

Note: Both points are awarded for any correctly determined value of h without any written justification.

frq5

9.

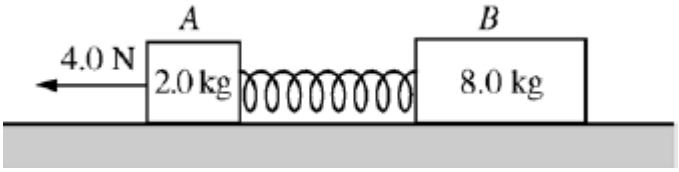


Block A of mass 2.0 kg and block B of mass 8.0 kg are connected as shown above by a spring of spring constant 80 N/m and negligible mass. The system is being pulled to the right across a horizontal frictionless surface by a horizontal force of 4.0 N, as shown, with both blocks experiencing equal constant acceleration.

(a) Calculate the force that the spring exerts on the 2.0 kg block.

(b) Calculate the extension of the spring.

The system is now pulled to the left, as shown below, with both blocks again experiencing equal constant acceleration.



(c) Is the magnitude of the acceleration greater than, less than, or the same as before?

____ Greater ____ Less ____ The same

Justify your answer.

(d) Is the amount the spring has stretched greater than, less than, or the same as before?

____ Greater ____ Less ____ The same

Justify your answer.

(e) In a new situation, the blocks and spring are moving together at a constant speed of 0.50 m s to the left.

Block A then hits and sticks to a wall. Calculate the maximum compression of the spring.

Part A

1 point is earned : For a correct application of Newton's 2nd law for the two-block system

$F = (m_A + m_B)a$

Note: Newton's 2nd law may be applied to each block separately to produce an equivalent solution.

1 point is earned : For a correct determination of the acceleration

frq5

$a = \frac{F}{(m_A+m_B)} = \frac{4.0\text{N}}{(2.0\text{kg}+8.0\text{kg})}$

a = 0.40 m/s 2

1 point is earned :For correct substitution of the acceleration into Newton's 2nd law for one of the blocks

F spring = m A a (or F-F spring = m B a)

1 point is earned :For the correct solution (consistent with the value of the acceleration found above)

F spring = (2.0 kg)(0.40 m/s 2)

F spring = 0.80 N

Note: A correct free-body diagram for each block could earn 1 point each.

0	1	2	3	4 ✓

The student response earns four of the following points:

1 point is earned : For a correct application of Newton's 2nd law for the two-block system

F = (m A + m B)a

Note: Newton's 2nd law may be applied to each block separately to produce an equivalent solution.

1 point is earned : For a correct determination of the acceleration

$a = \frac{F}{(m_A+m_B)} = \frac{4.0\text{N}}{(2.0\text{kg}+8.0\text{kg})}$

a = 0.40 m/s 2

1 point is earned :For correct substitution of the acceleration into Newton's 2nd law for one of the blocks

F spring = m A a (or F-F spring = m B a)

1 point is earned :For the correct solution (consistent with the value of the acceleration found above)

F spring = (2.0 kg)(0.40 m/s 2)

F spring = 0.80 N

Note: A correct free-body diagram for each block could earn 1 point each.

Part B

1 point is earned : For a correct expression relating spring force to extension

F spring = kx

frq5

1 point is earned : For the correct solution using the spring force from part (a)

$x = \frac{F_{spring}}{k} = \frac{0.80\text{N}}{80\text{N/m}}$

x = 0.010m

0	1	2 ✓

The student response earns two of the following points:

1 point is earned : For a correct expression relating spring force to extension

F spring = kx

1 point is earned : For the correct solution using the spring force from part (a)

$x = \frac{F_{spring}}{k} = \frac{0.80\text{N}}{80\text{N/m}}$

x = 0.010m

Part C

1 point is earned: For correctly indicating that the acceleration will be the same as before

2 points are earned: For a correct justification (only if the previous point was awarded)

Examples:

• Explaining that in both cases there is a 4.0 N force pulling a combined mass of 10 kg, and hence the acceleration will be the same in the two cases. (Note: One point was awarded when the student noted that either the net force acting on the system of two blocks or the mass of the system is unchanged. For full credit, the student must have noted that both the force and mass are the same in the two cases.)

• Applying Newton's 2nd law to each block and calculating an acceleration with the same value as in part (a).

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned: For correctly indicating that the acceleration will be the same as before

2 points are earned: For a correct justification (only if the previous point was awarded)

Examples:

frq5

• Explaining that in both cases there is a 4.0 N force pulling a combined mass of 10 kg, and hence the acceleration will be the same in the two cases. (Note: One point was awarded when the student noted that either the net force acting on the system of two blocks or the mass of the system is unchanged. For full credit, the student must have noted that both the force and mass are the same in the two cases.)

• Applying Newton's 2nd law to each block and calculating an acceleration with the same value as in part (a).

Part D

1 point is earned: For correctly indicating that the spring extension is greater than in part (b)

2 points are earned: For a correct justification (only if the previous point was awarded)

Examples:

• The spring force on the 8.0 kg block produces the same acceleration as the spring force on the 2.0 kg block in part (a); hence the spring force is greater than in part (a) so the extension is greater.

• Applying Newton's 2nd law to show that the new spring extension is 0.040 m.

Notes:

• A partial justification worth a single point may note that the spring is pulling on a larger mass than before, or may note that the force exerted by the spring is larger than before (without explaining why this force is larger).

• Students who answered part (c) by saying that the acceleration is greater could earn 2 points here by noting that the force exerted by the spring on block B must be larger in order to give the larger mass a greater acceleration.

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned: For correctly indicating that the spring extension is greater than in part (b)

2 points are earned: For a correct justification (only if the previous point was awarded)

Examples:

• The spring force on the 8.0 kg block produces the same acceleration as the spring force on the 2.0 kg block in part (a); hence the spring force is greater than in part (a) so the extension is greater.

• Applying Newton's 2nd law to show that the new spring extension is 0.040 m.

Notes:

• A partial justification worth a single point may note that the spring is pulling on a larger mass than before, or may note that the force exerted by the spring is larger than before (without explaining why this force is larger).

• Students who answered part (c) by saying that the acceleration is greater could earn 2 points here by noting that the force exerted by the spring on block B must be larger in order to give the larger mass a greater acceleration.

frq5

Part E

1 point is earned : For indicating that, after block A impacts the wall, mechanical energy is conserved

1 point is earned : For correctly applying conservation of energy, equating the energy immediately after block A hits the wall to the energy when the spring is at maximum compression

K before + U before = K after + U after

½m B v 2 + 0 = 0 + ½ kx 2

1 point is earned : For the correct solution

$$x = \sqrt{\frac{m_B v^2}{k}} = \sqrt{\frac{(80\text{kg})(050\text{m/s})^2}{80\text{N/m}}}$$

x = 0.16 m

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned : For indicating that, after block A impacts the wall, mechanical energy is conserved

1 point is earned : For correctly applying conservation of energy, equating the energy immediately after block A hits the wall to the energy when the spring is at maximum compression

K before + U before = K after + U after

½m B v 2 + 0 = 0 + ½ kx 2

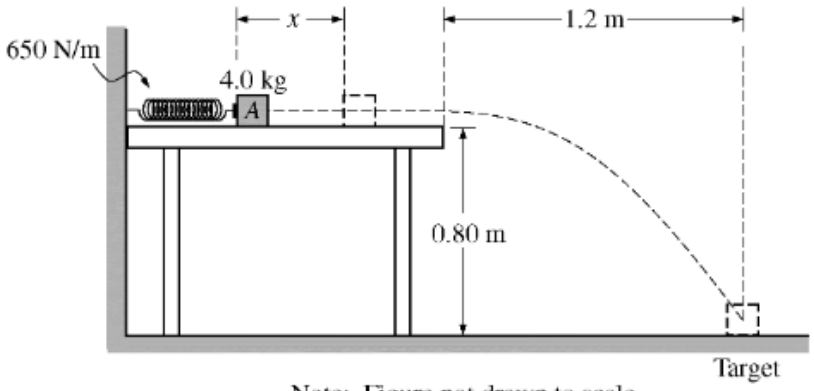
1 point is earned : For the correct solution

$$x = \sqrt{\frac{m_B v^2}{k}} = \sqrt{\frac{(80\text{kg})(050\text{m/s})^2}{80\text{N/m}}}$$

x = 0.16 m

frq5

10.



Note: Figure not drawn to scale.

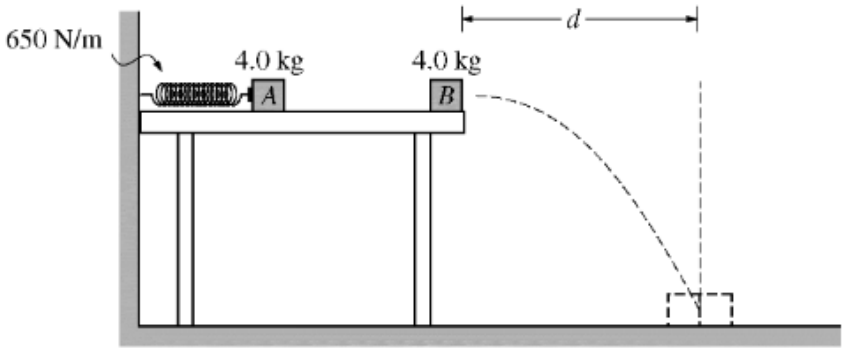
Block A of mass 4.0 kg is on a horizontal, frictionless tabletop and is placed against a spring of negligible mass and spring constant 650 N m. The other end of the spring is attached to a wall. The block is pushed toward the wall until the spring has been compressed a distance x, as shown above. The block is released and follows the trajectory shown, falling 0.80 m vertically and striking a target on the floor that is a horizontal distance of 1.2 m from the edge of the table. Air resistance is negligible.

(a) Calculate the time elapsed from the instant block A leaves the table to the instant it strikes the floor.

(b) Calculate the speed of the block as it leaves the table.

(c) Calculate the distance x the spring was compressed.

Block B, also of mass 4.0 kg, is now placed at the edge of the table. The spring is again compressed a distance x, and block A is released. As it nears the end of the table, it instantaneously collides with and sticks to block B. The blocks follow the trajectory shown in the figure below and strike the floor at a horizontal distance d from the edge of the table.



Note: Figure not drawn to scale.

frq5

(d) Calculate d if x is equal to the value determined in part (c).

(e) Consider the system consisting of the spring, the blocks, and the table. How does the total mechanical energy E_2 of the system just before the blocks leave the table compare to the total mechanical energy E_1 of the system just before block A is released?

____ $E_1 < E_2$ ____ $E_2 = E_1$ ____ $E_2 > E_1$

Justify your answer.

Part A

1 point is earned: For a correct relationship between the vertical distance and time

$h = \frac{1}{2}gt^2$

1 point is earned: For substitution of the vertical height and the acceleration due to gravity

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(0.80m)}{9.8m/s^2}}$

1 point is earned: For the correct answer

t = 0.40 s

Note: Credit was awarded for an alternate solution using

$$v_y^2 = 2gh$$

with appropriate substitutions to find the vertical velocity when the block lands, followed by substitution of this velocity into

$$v_y = gt$$

(or equivalent) to find the time

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For a correct relationship between the vertical distance and time

$h = \frac{1}{2}gt^2$

1 point is earned: For substitution of the vertical height and the acceleration due to gravity

frq5

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(0.80m)}{9.8m/s^2}}$

1 point is earned: For the correct answer

t = 0.40 s

Note: Credit was awarded for an alternate solution using

$$v_y^2 = 2gh$$

with appropriate substitutions to find the vertical velocity when the block lands, followed by substitution of this velocity into

$$v_y = gt$$

(or equivalent) to find the time

Part B

1 point is earned: For a correct relationship between the horizontal distance and time

x = *vt*

1 point is earned: For a consistent substitution of time from part (a) into the correct equation

$v = \frac{x}{t} = \frac{1.2m}{0.40s}$
$v = 3.0m/s$

0	1	2

The student response earns all of the following points:

1 point is earned: For a correct relationship between the horizontal distance and time

x = *vt*

1 point is earned: For a consistent substitution of time from part (a) into the correct equation

$v = \frac{x}{t} = \frac{1.2m}{0.40s}$
$v = 3.0m/s$

Part C

1 point is earned: For any statement of conservation of energy

1 point is earned: For correct use of appropriate energy equations

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

frq5

1 point is earned: For a consistent substitution of velocity from part (b) into the correct equation

$x = \sqrt{\frac{m}{k}}v = \sqrt{\frac{4kg}{650N/m}}(3.0m/s)$
$x = 0.24m$

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For any statement of conservation of energy

1 point is earned: For correct use of appropriate energy equations

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

1 point is earned: For a consistent substitution of velocity from part (b) into the correct equation

$x = \sqrt{\frac{m}{k}}v = \sqrt{\frac{4kg}{650N/m}}(3.0m/s)$
$x = 0.24m$

Part D

1 point is earned: For any statement of conservation of momentum

mAvi = (mA + mB)vf

1 point is earned: For substitution of both masses into the equation

1 point is earned: For substitution of the velocity from part (b) into the equation

$v_f = \left(\frac{m_A}{m_A+m_B}\right)v_i = \left(\frac{4kg}{4kg+4kg}\right)3.0m/s = 1.5m/s$

1 point is earned: For substitution of time from part (a) into a correct relationship between the horizontal distance and time

d = *vft* = (1.5 m/s)(0.40 s)

d = 0.60 m

0	1	2	3	4 ✓

The student response earns all of the following points:

frq5

1 point is earned: For any statement of conservation of momentum

mAvi = (mA + mB)vf

1 point is earned: For substitution of both masses into the equation

1 point is earned: For substitution of the velocity from part (b) into the equation

$v_f = \left(\frac{m_A}{m_A+m_B}\right)v_i = \left(\frac{4kg}{4kg+4kg}\right)3.0m/s = 1.5m/s$

1 point is earned: For substitution of time from part (a) into a correct relationship between the horizontal distance and time

d = *vft* = (1.5 m/s)(0.40 s)

d = 0.60 m

Part E

1 point is earned: For indicating that *E2* < *E1*

1 point is earned: For a correct justification stating one of the following:

- the kinetic energy (or energy) is transformed into other forms of energy during the collision (e.g., by reference to heat, internal energy, sound)
- the kinetic energy is not conserved in an inelastic collision
- a numerical calculation of the relevant energies

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For indicating that *E2* < *E1*

1 point is earned: For a correct justification stating one of the following:

- the kinetic energy (or energy) is transformed into other forms of energy during the collision (e.g., by reference to heat, internal energy, sound)
- the kinetic energy is not conserved in an inelastic collision
- a numerical calculation of the relevant energies

Units

1 point is earned: For correct units on all completed answers

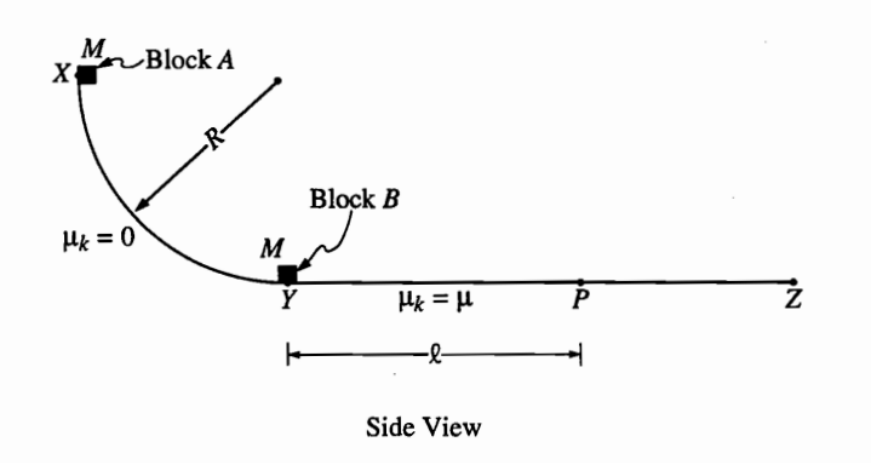
frq5

0	1 ✓

The student response earns all of the following points:

1 point is earned: For correct units on all completed answers

11.



Side View

A track consists of a frictionless arc *XY*, which is a quarter-circle of radius *R*, and a rough horizontal section *YZ*. Block *A* of mass *M* is released from rest at point *X,* slides down the curved section of the track, and collides instantaneously and in elastically with identical block *B* at point *Y*. The two blocks move together to the right, sliding past point *P*, which is a distance *L* from point *Y*. The coefficient of kinetic friction between the blocks and the horizontal part of the track is μ. Express your answers in terms of *M*, *L*, μ, *R*, and *g*.

(a) Determine the speed of block *A* just before it hits block *B*.

(b) Determine the speed of the combined blocks immediately after the collision.

(c) Determine the amount of kinetic energy lost due to the collision.

(d) The specific heat of the material used to make the blocks is *c*. Determine the temperature rise that results from the collision in terms of *c* and the other given quantities. (Assume that no energy is transferred to the track or to the air surrounding the blocks.)

(e) Determine the additional thermal energy that is generated as the blocks move from *Y* to *P*.

Part A

1 point is earned : For any indication that energy is conserved

$MgR = \frac{1}{2}Mv^2$

frq5

1 point is earned : v 2 = 2gR

1 point is earned : $v = \sqrt{2gR}$

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned : For any indication that energy is conserved

MgR = ½Mv 2

1 point is earned : v 2 = 2gR

1 point is earned : $v = \sqrt{2gR}$

Part B

1 point is earned : Momentum is conserved during the collision

1 point is earned : Mv i = 2Mv f

$v_f = \frac{Mv_i}{2M} = \frac{v_i}{2}$

But v i is the speed calculated in part (a).

1 point is earned : $v_f = \frac{\sqrt{2gR}}{2}$

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned : Momentum is conserved during the collision

1 point is earned : Mv i = 2Mv f

$v_f = \frac{Mv_i}{2M} = \frac{v_i}{2}$

But v i is the speed calculated in part (a).

1 point is earned : $v_f = \frac{\sqrt{2gR}}{2}$

Part C

frq5

1 point is earned : ΔK = K t - K f

1 point is earned : For the following two equations

$K_t = \frac{1}{2}Mv_i^2 = \frac{1}{2}M(2gR) = MgR$
$K_f = \frac{1}{2}(2M)v_f^2 - M\left(\frac{2gR}{4}\right) = \frac{MgR}{2}$

1 point is earned :

ΔK = MgR - MgR/2

ΔK = MgR/2

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned : ΔK = K t - K f

1 point is earned : For the following two equations

$K_t = \frac{1}{2}Mv_i^2 = \frac{1}{2}M(2gR) = MgR$
$K_f = \frac{1}{2}(2M)v_f^2 - M\left(\frac{2gR}{4}\right) = \frac{MgR}{2}$

1 point is earned :

ΔK = MgR - MgR/2

ΔK = MgR/2

Part D

The kinetic energy lost goes into heating the blocks.

1 point is earned: ΔK = Q

1 point is earned: Q = (2M)cΔT [This point also awarded for Q = Mc ΔT]

1 point is earned:

ΔT = ΔK/2Mc = MgR/2(2Mc)

ΔT = gR/4c

frq5

0	1	2	3 ✓

The student response earns three of the following points:

The kinetic energy lost goes into heating the blocks.

1 point is earned: $\Delta K = Q$

1 point is earned: $Q = (2M)c\Delta T$ [This point also awarded for $Q = Mc\ \Delta T$]

1 point is earned:

$\Delta T = \Delta K/2Mc = MgR/2(2Mc)$

$\Delta T = gR/4c$

Part E

As the blocks move from Y to P, a frictional force $Ff = \mu N$ acts on them.

The work W done by F, equals the additional thermal energy generated, Qadd.

1 point is earned: Qadd = W

1 point is earned: W = FfL

Substituting the expression $Ff = \mu N = \mu(2Mg)$:

$Qadd = \mu(2Mg)L$

1 point is earned: $= 2\ \mu MgL$

Alternate solution

The equation for frictional force causes the block to decelerate.

The equation showing the termal energy generated

Solving for Qadd

0	1	2	3 ✓

The student response earns three of the following points:

frq5

As the blocks move from Y to P, a frictional force Ff = μN acts on them.

The work W done by F, equals the additional thermal energy generated, Qadd.

1 point is earned: Qadd = W

1 point is earned: W = FfL

Substituting the expression Ff = μN = μ(2Mg):

Qadd = μ(2Mg)L

1 point is earned: = 2 μMgL

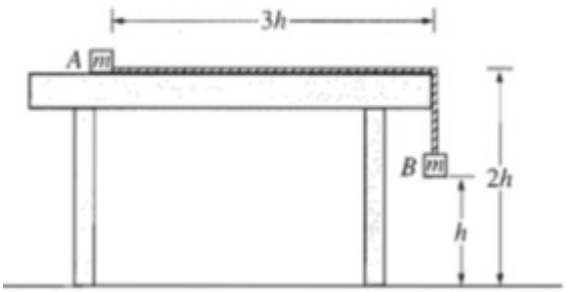
Alternate solution

The equation for frictional force causes the block to decelerate.

The equation showing the termal energy generated

Solving for Qadd

12.



Two small blocks, each of mass m, are connected by a string of constant length $4h$ and negligible mass. Block A is placed on a smooth tabletop as shown above, and block B hangs over the edge of the table. The tabletop is a distance $2h$ above the floor. Block B is then released from rest at a distance h above the floor at time $t = 0$. Express all algebraic answers in terms of h, m, and g.

(a) Determine the acceleration of block B as it descends.

(b) Block B strikes the floor and does not bounce. Determine the time t_1 at which block B strikes the floor.

(c) Describe the motion of block A from time $t = 0$ to the time when block B strikes the floor.

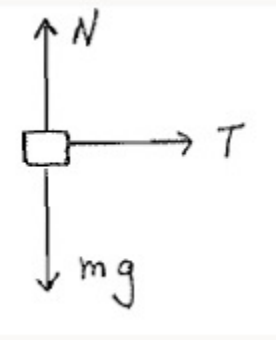
(d) Describe the motion of block A from the time block B strikes the floor to the time block A leaves the table.

(e) Determine the distance between the landing points of the two blocks.

Part A

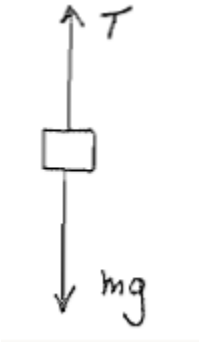
frq5

1 point is earned: For a force diagram or a correct application of Newton's second law for block A



Or $ma = T$

1 point is earned: For a force diagram or a correct application of Newton's second law for block B



Or $ma = mg-T$

Combining the two equation to eliminate T

$ma = mg - ma$

$2ma = mg$

1 point is earned: For the correct answer

$a = g/2$

Alternate solution

For treating the system as a one-body problem (of mass 2m), with the proper gravitational force acting

For correctly applying Newton's second law

$2ma = mg$

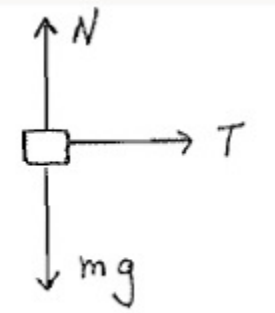
For the correct answer $a = g/2$

frq5

0	1	2	3 ✓

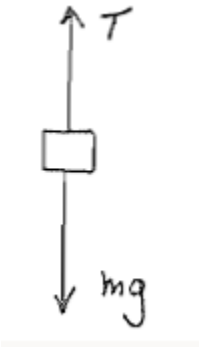
The student response earns three of the following points:

1 point is earned: For a force diagram or a correct application of Newton's second law for block A



Or $ma = T$

1 point is earned: For a force diagram or a correct application of Newton's second law for block B



Or $ma = mg-T$

Combining the two equation to eliminate T

$ma = mg - ma$

$2ma = mg$

1 point is earned: For the correct answer

$a = g/2$

Alternate solution

For treating the system as a one-body problem (of mass 2m), with the proper gravitational force acting

For correctly applying Newton's second law

frq5

$2ma = mg$

For the correct answer $a = g/2$

Part B

1 point is earned: For using the kinematic equation for distance

$s = s0 + v0t + \frac{1}{2}at2$

1 point is earned: For correctly substituting values, including the value of acceleration from part (a)

$h = \frac{1}{2}(g/2)t2$

1 point is earned: For the correct answer

$$t = 2\sqrt{\frac{h}{g}}$$

Alternate solution

For equating the work done as block B falls to the change in kinetic energy of the blocks during this time

$W = \Delta K$

$mgh = \frac{1}{2}(2m)v2$

For using the kinematic equation for speed to determine the velocity of the blocks just before block B strikes the floor

$v = v0 + at$

$v = gt/2$

For the correct answer

$$t = 2\sqrt{\frac{h}{g}}$$



0	1	2	3

The student response earns three of the following points:

1 point is earned: For using the kinematic equation for distance

$s = s0 + v0t + \frac{1}{2}at2$

1 point is earned: For correctly substituting values, including the value of acceleration from part (a)

frq5

h = ½(*g*/2)*t*2

1 point is earned: For the correct answer

$$t = 2\sqrt{\frac{h}{g}}$$

Alternate solution

For equating the work done as block B falls to the change in kinetic energy of the blocks during this time

W = *ΔK*

mgh= ½(2*m*)*v*2

For using the kinematic equation for speed to determine the velocity of the blocks just before block B strikes the floor

v = *v*0 + *at*

v = *gt*/2

For the correct answer

$$t = 2\sqrt{\frac{h}{g}}$$

Part C

1 point is earned: For indicating that block *A* accelerates across the table

1 point is earned: For also having no incorrect statements about the block's motion (This point was only awarded if the previous point was received.)

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For indicating that block *A* accelerates across the table

1 point is earned: For also having no incorrect statements about the block's motion (This point was only awarded if the previous point was received.)

Part D

1 point is earned: For any indication that block A is still in motion after block B strikes the floor

(e.g. referring to the block's velocity)

frq5

1 point is earned: For indicating that the block's velocity is constant

Note: One point was awarded for a statement that the velocity would decrease if friction was present

0	1	2 ✓

The student response earns two of the following points:

1 point is earned: For any indication that block A is still in motion after block B strikes the floor

(e.g. referring to the block's velocity)

1 point is earned: For indicating that the block's velocity is constant

Note: One point was awarded for a statement that the velocity would decrease if friction was present

Part E

Since block *B* falls straight to the floor and stops, the distance between the landing points is equal to the horizontal distance of block *A* from the edge of the table

1 point is earned: Determining the constant horizontal speed *v* at which block A travels:

For equating the work done as block *B* falls to the change in kinetic energy of the blocks during this time

$W = \Delta K$

$mgh = \frac{1}{2}(2m)v^2$

1 point is earned: For determining the velocity of the blocks just before block *B* strikes the floor

$$v = \sqrt{hg}$$

1 point is earned: For the correct kinematic equation for the horizontal distance *x* traveled by block A, with zero acceleration

$x = vt$

1 point is earned: Determining the time *t* during which block *A* falls:

For using the correct kinematic equation for the distance *y* that block *A* falls

$y = y_0 + v_{0y}t + \frac{1}{2}at^2$

1 point is earned: For substituting the appropriate values and solving for the time of fall

frq5

$$2h = \frac{1}{2}gt^2$$
$$t = 2\sqrt{\frac{h}{g}}$$
$$x = \sqrt{hg}\left(2\sqrt{\frac{h}{g}}\right)$$



0	1	2	3	4	5

The student response earns five of the following points:

Since block *B* falls straight to the floor and stops, the distance between the landing points is equal to the horizontal distance of block *A* from the edge of the table

1 point is earned: Determining the constant horizontal speed *v* at which block A travels:

For equating the work done as block *B* falls to the change in kinetic energy of the blocks during this time

W = ΔK

mgh = ½(2m)v2

1 point is earned: For determining the velocity of the blocks just before block *B* strikes the floor

$$v = \sqrt{hg}$$

1 point is earned: For the correct kinematic equation for the horizontal distance *x* traveled by block A, with zero acceleration

x = vt

1 point is earned: Determining the time *t* during which block *A* falls:

For using the correct kinematic equation for the distance *y* that block *A* falls

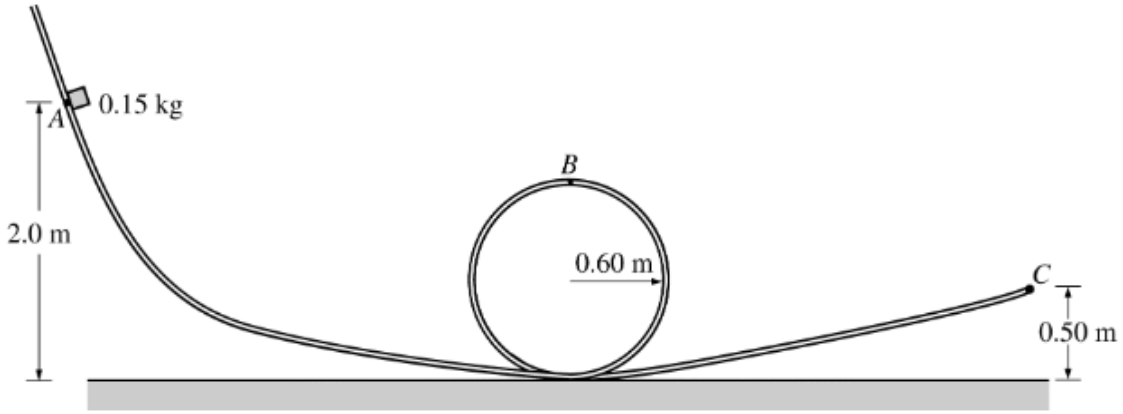
y = y0 + v0yt + ½at2

1 point is earned: For substituting the appropriate values and solving for the time of fall

$$2h = \frac{1}{2}gt^2$$
$$t = 2\sqrt{\frac{h}{g}}$$
$$x = \sqrt{hg}\left(2\sqrt{\frac{h}{g}}\right)$$

frq5

13.



A small block of mass 0.15 kg is placed at point *A* at a height 2.0 m above the bottom of a track, as shown in the figure above, and is released from rest. It slides with negligible friction down the track, around the inside of the loop of radius 0.60 m, and leaves the track at point *C* at a height 0.50 m above the bottom of the track.

(a) Calculate the speed of the block when it leaves the track at point *C*.

(b) On the figure below, draw and label the forces (not components) that act on the block when it is at the top of the loop at point *B*.

(c) Calculate the minimum speed the block can have at point *B* without losing contact with the track.

(d) Calculate the minimum height h_{min} above the bottom of the track at which the block can be released and still go around the loop without losing contact with the track.

Part A

1 point is earned: For a correct conservation of energy equation for this situation

$mgh_i = mgh_f + \frac{1}{2}mv_f^2$

$v_f = \sqrt{2g(h_i - h_f)}$

$v_f = \sqrt{2(9.8m/s^2)(2.0m - 0.50m)}$

1 point is earned: For the correct answer

vf = 5.4 m/s (or 5.5 using *g* = 10 m/s2)

frq5

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For a correct conservation of energy equation for this situation

$mgh_i = mgh_f + \frac{1}{2}mv_f^2$

$v_f = \sqrt{2g(h_i - h_f)}$

$v_f = \sqrt{2(9.8m/s^2)(2.0m - 0.50m)}$

1 point is earned: For the correct answer

vf = 5.4 m/s (or 5.5 using *g* = 10 m/s2)

Part B



1 point is earned: For correctly drawing and appropriately labeling the weight of the block

1 point is earned: For correctly drawing and appropriately labeling the normal force

1 point is earned: For no extraneous forces

0	1	2	3 ✓

The student response earns all of the following points:



1 point is earned: For correctly drawing and appropriately labeling the weight of the block

frq5

1 point is earned: For correctly drawing and appropriately labeling the normal force

1 point is earned: For no extraneous forces

Part C

At the top of the track, the net force on the block is the centripetal force

ma = mv2/r = mg + N

The condition for minimum speed is that the normal force is zero. **1 point is earned:** For a correct equation that can be solved for the minimum speed

$mv_{\text{min}}^2/r = mg$
$v_{\text{min}} = \sqrt{rg}$
$v_{\text{min}} = \sqrt{(0.60m)(9.8m/s^2)}$

1 point is earned: For the correct answer

vmin = 2.4 m/s

0	1	2 ✓

The student response earns all of the following points:

At the top of the track, the net force on the block is the centripetal force

ma = mv2/r = mg + N

The condition for minimum speed is that the normal force is zero. **1 point is earned:** For a correct equation that can be solved for the minimum speed

$mv_{\text{min}}^2/r = mg$
$v_{\text{min}} = \sqrt{rg}$
$v_{\text{min}} = \sqrt{(0.60m)(9.8m/s^2)}$

1 point is earned: For the correct answer

vmin = 2.4 m/s

Part D

1 point is earned: For a correct conservation of energy equation for this situation

$mgh_{\text{min}} = mg(2r) + \frac{1}{2}mv_{\text{min}}^2$
$h_{\text{min}} = 2r + \left(v_{\text{min}}^2/2g\right)$

frq5

1 point is earned: For correctly substituting the value of from part (c)

hmin = 2(0.60 m) + ((2.4 m/s)2 / 2(9.8 m/s2))

1 point is earned: For the correct answer

hmin = 1.5 m

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For a correct conservation of energy equation for this situation

$mgh_{\text{min}} = mg(2r) + \frac{1}{2}mv_{\text{min}}^2$
$h_{\text{min}} = 2r + \left(v_{\text{min}}^2/2g\right)$

1 point is earned: For correctly substituting the value of from part (c)

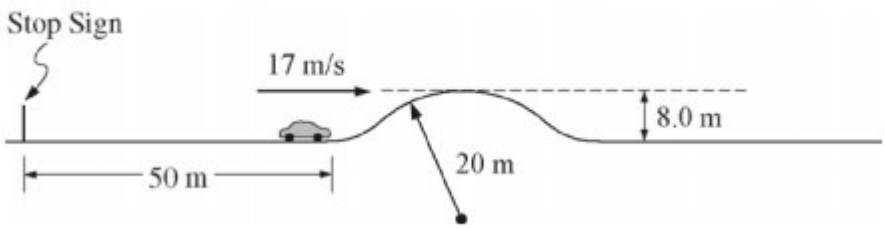
hmin = 2(0.60 m) + ((2.4 m/s)2 / 2(9.8 m/s2))

1 point is earned: For the correct answer

hmin = 1.5 m

frq5

14.



You are assigned to do some calculations for a movie stunt that involves a car on a straight road. The road, pictured above, has a hill that rises 8.0 m above the flat region. The top of the hill is a circular arc of radius 20 m. You need to determine whether a car traveling under certain conditions will lose contact with the road at the top of the hill. There is a stop sign 50 m from the beginning of the hill. You are to assume that a car of mass 1600 kg accelerates uniformly from rest at the stop sign, has a speed of when it reaches the beginning of the hill, and then coasts with the engine off. Assume energy losses due to friction and air resistance are negligible.

a. Calculate the magnitude of the acceleration of the car during the first 50 m.

b. Calculate the time it takes the car to reach the beginning of the hill

c. Calculate the magnitude of the net force required to accelerate the car during the first 50 m.

d. On the dot below that represents the car, draw and label the forces (not components) that act on the car at the top of the hill if it travels over the hill without losing contact.



e. Calculate the minimum speed the car must have at the top of the hill to momentarily lose contact with the road. If you need to draw anything other than what you have shown in part (d) to assist in your solution, use the space below. Do NOT add anything to the figure in part (d).

f. Calculate the speed the car must have at the beginning of the hill in order to have the speed at the top of the hill you calculated in part (e).

Part A

One point is earned for using the correct equation to solve for the acceleration

frq5

$$v^2 = v_0^2 + 2ax$$
$$v_0 = 0, \text{ so } a = \frac{v^2}{2x}$$
$$a = \frac{(17 \text{ m/s})^2}{(2)(50 \text{ m})}$$

One point is earned for the correct answer with units

a = 2.89 m/s2

0	1	2 ✓

The student earns all of the following points:

One point is earned for using the correct equation to solve for the acceleration

$$v^2 = v_0^2 + 2ax$$
$$v_0 = 0, \text{ so } a = \frac{v^2}{2x}$$
$$a = \frac{(17 \text{ m/s})^2}{(2)(50 \text{ m})}$$

One point is earned for the correct answer with units

a = 2.89 m/s2

Part B

One point is earned for using a correct expression to calculate the time

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

One point is earned for substituting values consistent with the answer from part (a)

$$x = 0 + (0)t + \frac{1}{2}at^2$$
$$t = \sqrt{\frac{2x}{a}}$$
$$t = \sqrt{\frac{(2)(50 \text{ m})}{(2.89 \text{ m/s}^2)}}$$
$$t = 5.88 \text{ s}$$

frq5

0	1	2 ✓

The student earns all of the following points:

One point is earned for using a correct expression to calculate the time

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

One point is earned for substituting values consistent with the answer from part (a)

$$x = 0 + (0)t + \frac{1}{2}at^2$$
$$t = \sqrt{\frac{2x}{a}}$$
$$t = \sqrt{\frac{(2)(50\text{ m})}{(2.89\text{ m/s}^2)}}$$
$$t = 5.88\text{ s}$$

Part C

One point is earned for using Newton's 2nd law

$F = ma$

One point is earned for substituting the correct value for m and a value for a consistent with part (a)

F = (1600 kg)(2.89 m/s2)

F = 4624 N

0	1	2 ✓

The student earns all of the following points:

One point is earned for using Newton's 2nd law

$F = ma$

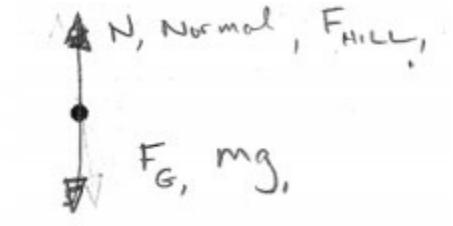
One point is earned for substituting the correct value for m and a value for a consistent with part (a)

F = (1600 kg)(2.89 m/s2)

frq5

$F = 4624$ N

Part D



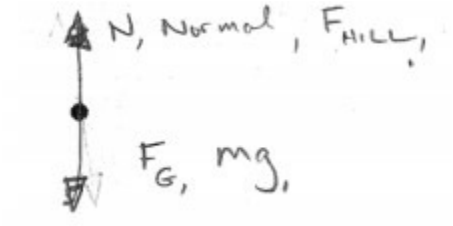
One point is earned for a correctly labeled vector representing the normal force

One point is earned for a correctly labeled vector representing the weight of the car

One point is earned for no incorrect or extraneous vectors or labeling

0	1	2	3 ✓

The student earns all of the following points:



One point is earned for a correctly labeled vector representing the normal force

One point is earned for a correctly labeled vector representing the weight of the car

One point is earned for no incorrect or extraneous vectors or labeling

Part E

One point is earned for equating the centripetal force exerted on the car to the net force

$$F_C = W + N$$

$$\frac{mv^2}{r} = mg + N$$

One point is earned for setting the normal force equal to zero

$$\frac{mv^2}{r} = mg + 0$$

frq5

Solving for v and substituting

$$v = \sqrt{rg}$$
$$v = \sqrt{(20\ \text{m})\left(9.8\ \text{m/s}^2\right)}$$

One point is earned for a correct answer with units

$v = 14$ m/s

0	1	2	3 ✓

The student earns all of the following points:

One point is earned for equating the centripetal force exerted on the car to the net force

$$F_C = W + N$$
$$\frac{mv^2}{r} = mg + N$$

One point is earned for setting the normal force equal to zero

$$\frac{mv^2}{r} = mg + 0$$

Solving for v and substituting

$$v = \sqrt{rg}$$
$$v = \sqrt{(20\ \text{m})\left(9.8\ \text{m/s}^2\right)}$$

One point is earned for a correct answer with units

$v = 14$ m/s

Part F

One point is earned for any indication of the use of conservation of energy

One point is earned for a correct expression for the conservation of energy

$$K_1 = K_2 + U_2$$
$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + mgh_2$$

One point is earned for correct substitution, with consistent with part (e)

frq5

$$v_1^2 = v_2^2 + 2gh_2$$
$$v_1 = \sqrt{v_2^2 + 2gh_2}$$
$$v_1 = \sqrt{(14\ \text{m/s})^2 + (2)\left(9.8\ \text{m/s}^2\right)(8.0\ \text{m})}$$
$$v_1 = 18.8\ \text{m/s}$$



0	1	2	3

The student earns all of the following points:

One point is earned for any indication of the use of conservation of energy

One point is earned for a correct expression for the conservation of energy

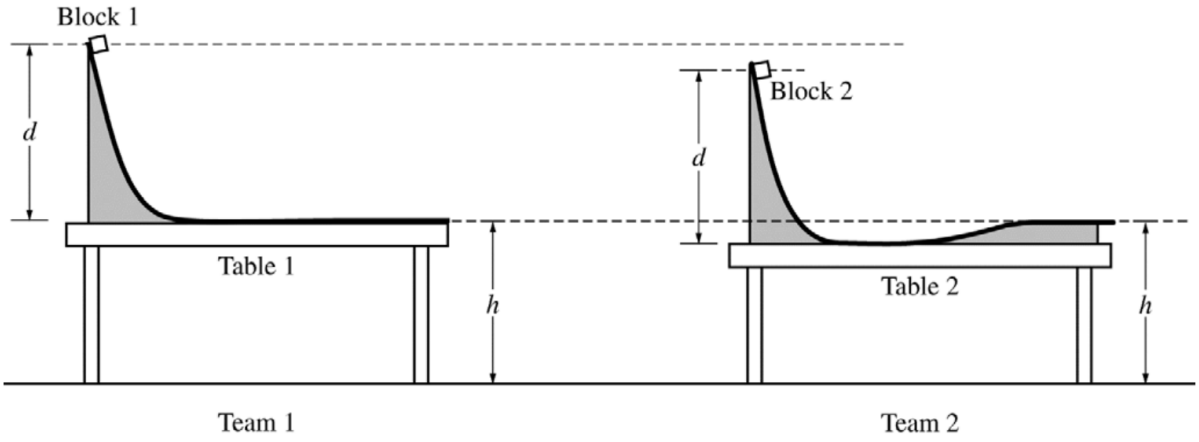
$$K_1 = K_2 + U_2$$
$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + mgh_2$$

One point is earned for correct substitution, with consistent with part (e)

$$v_1^2 = v_2^2 + 2gh_2$$
$$v_1 = \sqrt{v_2^2 + 2gh_2}$$
$$v_1 = \sqrt{(14\ \text{m/s})^2 + (2)\left(9.8\ \text{m/s}^2\right)(8.0\ \text{m})}$$
$$v_1 = 18.8\ \text{m/s}$$

frq5

15.



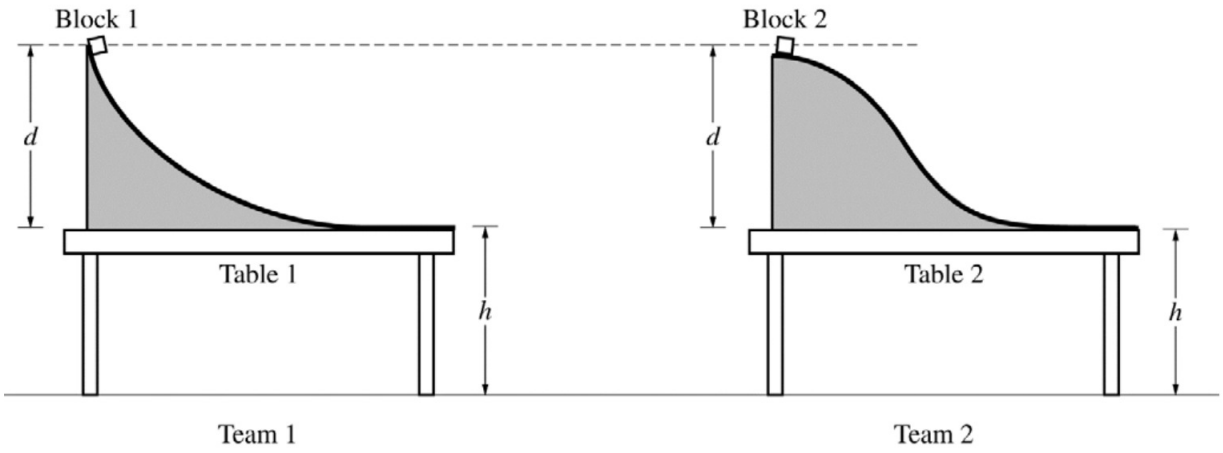
A physics class is asked to design a low-friction slide that will launch a block horizontally from the top of a lab table. Teams 1 and 2 assemble the slides shown above and use identical blocks 1 and 2, respectively. Both slides start at the same height d above the tabletop. However, team 2's table is lower than team 1's table. To compensate for the lower table, team 2 constructs the right end of the slide to rise above the tabletop so that the block leaves the slide horizontally at the same height h above the floor as does team 1's block (see figure above).

a. Both blocks are released from rest at the top of their respective slides. Do block 1 and block 2 land the same distance from their respective tables?

_____Yes _____No

Justify your answer.

In another experiment, teams 1 and 2 use tables and low-friction slides with the same height. However, the two slides have different shapes, as shown below.



b. Both blocks are released from rest at the top of their respective slides at the same time.

frq5

i. Which block, if either, lands farther from its respective table?

_____Block 1 _____ Block 2 ____ The two blocks land the same distance from their respective tables.

Briefly explain your reasoning without manipulating equations.

ii. Which block, if either, hits the floor first?

_____Block 1 _____ Block 2_____The two blocks hit the floor at the same time.

Briefly explain your reasoning without manipulating equations.

Part A

Correct answer: “No”

Note: If the wrong answer is selected, partial credit can be earned for the justification.

1 point is earned: For attempting to use conservation of energy to compare the two blocks

1 point is earned: For explicitly or implicitly indicating that the launch velocities are different

1 point is earned: For stating or implying that the time to reach the ground is the same for both blocks

Example: The amount of potential energy converted to kinetic energy is different for the two blocks. The potential energy is proportional to the change in height, which is smaller for block 2. Therefore, at the edge of the table, block 1 will have more kinetic energy than block 2, and hence a larger speed. The launches are both horizontal and from the same height, so the blocks will spend the same amount of time in the air. Because d = vt, the distances will be different for the two blocks (as the speeds are different).



0	1	2	3

Student response earns 3 of the following 3 point(s)

Correct answer: “No”

Note: If the wrong answer is selected, partial credit can be earned for the justification.

1 point is earned: For attempting to use conservation of energy to compare the two blocks

1 point is earned: For explicitly or implicitly indicating that the launch velocities are different

1 point is earned: For stating or implying that the time to reach the ground is the same for both blocks

Example: The amount of potential energy converted to kinetic energy is different for the two blocks. The potential energy is proportional to the change in height, which is smaller for block 2. Therefore, at the edge of

frq5

the table, block 1 will have more kinetic energy than block 2, and hence a larger speed. The launches are both horizontal and from the same height, so the blocks will spend the same amount of time in the air. Because d = vt, the distances will be different for the two blocks (as the speeds are different).

Part B

Part(i)

Correct answer: “The two blocks land the same distance from their respective tables.”

Note: If the wrong answer is selected, partial credit can be earned for the justification.

1 point is earned: For indicating that the change in potential energy from release to launch is the same for the two cases

1 point is earned: For an indication (explicit or implicit) that the launch velocities are the same

Part(ii)

Correct answer: “Block 1”

1 point is earned: For indicating that the average speed or velocity on the slide is higher for Team 1, OR that block 1 reaches its maximum speed in less time

1 point is earned: For a valid explanation of why the average speed or velocity is higher for team 1, OR why block 1 reaches its maximum speed in less time

Example: Because the ramp on Table 1 is initially steeper, block 1 has a higher average speed while it’s on the ramp so it launches off the table before block 2.

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Part(i)

Correct answer: “The two blocks land the same distance from their respective tables.”

Note: If the wrong answer is selected, partial credit can be earned for the justification.

1 point is earned: For indicating that the change in potential energy from release to launch is the same for the two cases

1 point is earned: For an indication (explicit or implicit) that the launch velocities are the same

Part(ii)

Correct answer: “Block 1”

1 point is earned: For indicating that the average speed or velocity on the slide is higher for Team 1, OR that block 1 reaches its maximum speed in less time

frq5

1 point is earned: For a valid explanation of why the average speed or velocity is higher for team 1, OR why block 1 reaches its maximum speed in less time

Example: Because the ramp on Table 1 is initially steeper, block 1 has a higher average speed while it's on the ramp so it launches off the table before block 2.

16. This question is a short free-response question. Show your work for each part of the question.

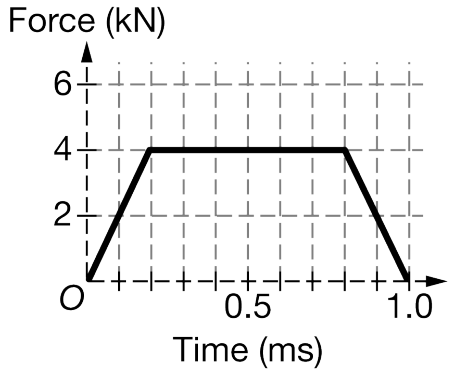
(7 points, suggested time 13 minutes)

A student in a physics lab drops a ball of mass $0.50\ \text{kg}$ from a height of $1.5\ \text{m}$ onto a force plate. The ball hits the force plate and rebounds upward. The student records the maximum height of the ball after it rebounds to be $1.1\ \text{m}$.

(a) The student uses their observations to correctly claim that the collision between the ball and the force plate is inelastic. Use the student's observations to justify their claim.

(b) Calculate the impulse exerted on the ball by the force plate.

The force plate records the following data for the magnitude of the force as a function of time for the $1\ \text{ms}$ the ball was in contact with the force sensor.



(c) Does the force plate data provide a reliable measurement of the impulse exerted on the ball by the force sensor? Justify your answer

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes the following criteria:

frq5

- [ ] For correctly relating the difference in maximum height of the ball to the kinetic energy of the ball before and after the collision.

Example Response:

If the collision between the ball and the floor were elastic, the speed just before the ball hits would be equal to the speed of the ball just after it leaves the floor. Because of conservation of energy, since air resistance is negligible, if the speed before and after the collision were the same the ball should return to its starting height. Because it returns to a lower height, we must conclude that there was energy lost in the collision, making the collision inelastic.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Scoring note: Responses do not have to correctly calculate the change in momentum. A response must show that the momentum (or speeds) have different directions, and that the difference in these two are the algebraic sum of the two magnitudes.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

- [ ] For using conservation of energy or work energy principle to attempt to find the speed of the ball just before the ball hits the force sensor.
- [ ] For using conservation of energy or work energy principle to attempt to find the speed of the ball just after the ball hits the force sensor.
- [ ] For correctly addressing the vector nature of momentum (or velocity)

Example Response:

$KE = U_g$

$\frac{1}{2}mv_i^2 = mgh$

$v_i = \sqrt{2gh} = \sqrt{2\left(10\ \frac{\text{m}}{\text{s}^2}\right)(1.5\ \text{m})} = 5.5\ \frac{\text{m}}{\text{s}}$

$U_g = KE$

$mgh = \frac{1}{2}mv_f^2$

$v_f = \sqrt{2gh} = \sqrt{2\left(10\ \frac{\text{m}}{\text{s}^2}\right)(1.1\ \text{m})} = 4.7\ \frac{\text{m}}{\text{s}}$

$\Delta p = mv_f - mv_i$

$\Delta p = (0.50\ \text{kg})(4.7\ \frac{\text{m}}{\text{s}}) - (0.50\ \text{kg})(-5.5\ \frac{\text{m}}{\text{s}})$

$\Delta p = 5.1\ \text{Ns}$

Part C

frq5

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: Both bars have the same angular momentum.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

- ☐ For attempting to calculate the area under the curve
- ☐ For comparing the area under the curve to the value of the change in momentum for part (b).
- ☐ For a correct justification that the impulse shown from the force sensor is not large enough to cause the ball to return to a height of 1.1 meters.

Example Response:

The area under the force as a function of time graph is equal to the impulse or change in momentum of the ball.

$\Delta p_{\text{observed}} = 5.1 \text{ Ns}$
$\Delta p_{\text{recorded}} = \frac{1}{2}(0.2 \text{ ms})(4 \text{ kN}) + (4 \text{ kN})(0.6 \text{ ms}) + \frac{1}{2}(0.2 \text{ ms})(4 \text{ kN})$
$\Delta p_{\text{recorded}} = 3.2 \text{ Ns}$

The change in momentum recorded by the force sensor is significantly smaller than the change in momentum determined from experimental data collected from the initial and final heights of the ball. It is likely that the force sensor reading cannot be trusted.

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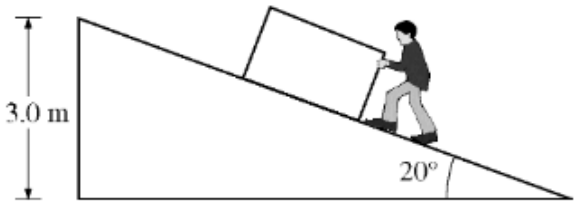
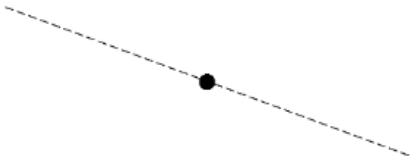
17.



A box is being pushed at constant speed up an inclined plane to a vertical height of 3.0 m above the ground, as shown in the figure above. The person exerts a force parallel to the plane. The mass m of the box is 50 kg, and the coefficient of kinetic friction km between the box and the plane is 0.30.

(a) On the dot below that represents the box, draw and label the forces (not components) acting on the box.



(b) Calculate the normal force of the plane on the box. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

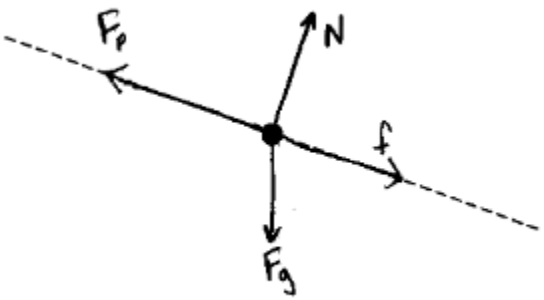
(c) Calculate the component of the force of gravity acting on the box that is parallel to the plane.

(d) Calculate the friction force between the plane and the box.

(e) Calculate the force applied by the person on the box.

(f) Calculate the work done by the person pushing the box, assuming the box is raised to the vertical height of 3.0 m.

Part A



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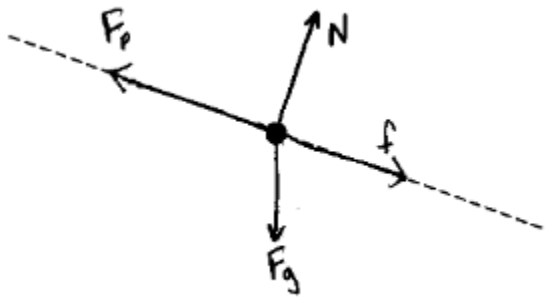
4 points are earned: For each correct force on the dot for which the vector drawn has the correct direction and a correct label such as those in the figure above, 1 point was earned.

No points were earned for an appropriately labeled force if the direction was incorrect. 1 earned point was deducted for each additional force or component.

0	1	2	3	4 ✓

The student response earns all of the following points:



4 points are earned: For each correct force on the dot for which the vector drawn has the correct direction and a correct label such as those in the figure above, 1 point was earned.

No points were earned for an appropriately labeled force if the direction was incorrect. 1 earned point was deducted for each additional force or component.

Part B

Applying Newton's second law to the forces perpendicular to the plane

$\Sigma F_{\perp} = ma_{\perp}$, where, $a_{\perp} = 0$
$N - Fg \cos\theta = 0$
$N - mg \cos\theta = 0$

1 point is earned: For a correction expression for the normal force

$N = mg \cos\theta$ (or $N = mg \sin\theta$, if 70° was clearly used for θ)

1 point is earned: For correctly substituting the values of m, g and θ into the correct expression

N = (50kg) (9.8m / s2) cos 20°

N = 460N (or, 470N, u sin g, g = 10m / s2)

frq5

0	1	2 ✓

The student response earns all of the following points:

Applying Newton's second law to the forces perpendicular to the plane

$\Sigma F_{\perp} = ma_{\perp}$, where, $a_{\perp} = 0$
$N - F_g \cos\theta = 0$
$N - mg \cos\theta = 0$

1 point is earned: For a correction expression for the normal force

$N = mg \cos\theta$ (or $N = mg \sin\theta$, if 70° was clearly used for θ)

1 point is earned: For correctly substituting the values of m, g and θ into the correct expression

N = (50kg) (9.8m / s2) cos 20°

N = 460N (or, 470N, u sin g, g = 10m / s2)

Part C

1 point is earned: For a correct expression for the component of the force of gravity parallel to the plane

$F_{g\parallel} = F_g \sin\theta = mg \sin\theta$ (or *mg* cosθ, if 70° was clearly used for θ)
$F_{g\parallel}$ = (50kg) (9.8m/s2) sin (20°)

1 point is earned: For a correct answer

$F_{g\parallel}$ = 168N (or 171 N using *g* = 10 m/s2)

Alternate solution

1 point is earned: *For indicating* $F_{g\parallel} = \sqrt{F_g{}^2 - F_{g\perp}{}^2}$

Fg = mg = (50kg) (10m/s2) = 500N
$F_{g\perp} = mg \cos\theta = N = 460N$

1 point is earned: *For a correct answer*

$F_{g\parallel}$ = 168N (or 171 N using *g* = 10 m/s2)

0	1	2 ✓

frq5

The student response earns all of the following points:

1 point is earned: For a correct expression for the component of the force of gravity parallel to the plane

$F_{g\parallel} = F_g \sin\theta = mg \sin\theta$ (or $mg \cos\theta$, if 70° was clearly used for θ)
$F_{g\parallel}$ = (50kg) (9.8m/s2) sin (20°)

1 point is earned: For a correct answer

$F_{g\parallel}$ = 168N (or 171 N using g = 10 m/s2)

Alternate solution

1 point is earned: *For indicating* $F_{g\parallel} = \sqrt{F_g{}^2 - F_{g\perp}{}^2}$

Fg = mg = (50kg) (10m/s2) = 500N
$F_{g\perp}$ = mg cosθ = N = 460N

1 point is earned: *For a correct answer*

$F_{g\parallel}$ = 168N (or 171 N using g = 10 m/s2)

Part D

1 point is earned: For a correct expression for the frictional force

$f = \mu KN$

1 point is earned: For correct substitution of μK and the value of N from part (b)

f = (0.30)(460 N)

f = 138 N (or 141 N using the g = 10 m/s2 result of N = 470 N from (b))

✓

0	1	2

The student response earns all of the following points:

1 point is earned: For a correct expression for the frictional force

$f = \mu KN$

1 point is earned: For correct substitution of μK and the value of N from part (b)

f = (0.30)(460 N)

f = 138 N (or 141 N using the g = 10 m/s2 result of N = 470 N from (b))

Part E

frq5

Applying Newton's second law to the forces parallel to the plane

$F_{net\|} = \sum F_{\|} = ma_{\|}$, where a‖ = 0 because the box is moving at constant speed

1 point is earned: For indicating Fnet‖ = 0

Fnet‖ = Fp - Fg‖ - f = 0

1 point is earned: For correct substitutions of the answers from parts (c) and (d) into a correct expression

Fp = Fg‖ + f
Fp = 168N + 138N
Fp = 306N (or 312N, u sin g, the, g = 10m/s2 results, of, Fg‖ = 171N, and, f = 141N)

0	1	2 ✓

The student response earns all of the following points:

Applying Newton's second law to the forces parallel to the plane

$F_{net\|} = \sum F_{\|} = ma_{\|}$, where a‖ = 0 because the box is moving at constant speed

1 point is earned: For indicating Fnet‖ = 0

Fnet‖ = Fp - Fg‖ - f = 0

1 point is earned: For correct substitutions of the answers from parts (c) and (d) into a correct expression

Fp = Fg‖ + f
Fp = 168N + 138N
Fp = 306N (or 312N, u sin g, the, g = 10m/s2 results, of, Fg‖ = 171N, and, f = 141N)

Part F

1 point is earned: For a correct expression relating work to force and distance

W = Fdcosφ, where φ is the angle between **F** and **d** (W = force times distance must be implied, not W = mgh. It was acceptable if φ was not explicitly included because cos cosφ = cos0° = 1.)

1 point is earned: For correctly substituting the force from part (e) and $d = \frac{3.0m}{\sin 20^\circ} or, 8.8m$

$W = 306N\left(\frac{3.0m}{\sin 20^\circ}\right)$
$W = 2648J\left(or 2737J, u \sin g, g = 10m/s^2 result, of, F_p = 312N\right)$

frq5

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For a correct expression relating work to force and distance

$W = Fd\cos\varphi$, where φ is the angle between **F** and **d** (W = force times distance must be implied, not $W = mgh$. It was acceptable if φ was not explicitly included because cos cosφ = cos0° = 1.)

1 point is earned: For correctly substituting the force from part (e) and $d = \frac{3.0m}{\sin 20^\circ} or, 8.8m$

$W = 306N\left(\frac{3.0m}{\sin 20^\circ}\right)$
$W = 2648J\left(or 2737J, u\sin g, g = 10m/s^2 result, of, F_p = 312N\right)$

Units

1 point is earned: For correct units on at least three answers and no incorrect units

0	1 ✓

The student response earns all of the following points:

1 point is earned: For correct units on at least three answers and no incorrect units

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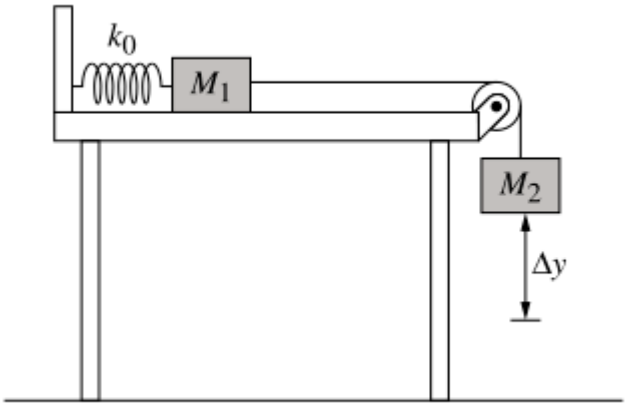
18. This question is a short free-response question. Show your work for each part of the question.



(7 points, suggested time 13 minutes)

Two blocks are connected by a string that passes over a pulley, as shown above. Block 1 is on a horizontal surface and is attached to a spring that is at its unstretched length. Frictional forces are negligible in the pulley's axle and between the block and the surface. Block 2 is released from rest and moves downward before momentarily coming to rest.

k_0 is the spring constant of the spring.
M_1 is the mass of block 1.
M_2 is the mass of block 2.
Δy is the distance block 2 moves before momentarily coming to rest.

(a)

i. Block 2 starts from rest and speeds up, then it slows down and momentarily comes to rest at a position below its initial position. In terms of only the forces directly exerted on block 2, explain why block 2 initially speeds up and explain why it slows down to a momentary stop.

ii. Derive an expression for the distance Δy that block 2 travels before momentarily coming to rest. Express your answer in terms of k_0, M_1, M_2, and physical constants, as appropriate.

(b) Indicate whether the total mechanical energy of the blocks-spring-Earth system changes as block 2 moves downward.

___ Changes ___ Does not change

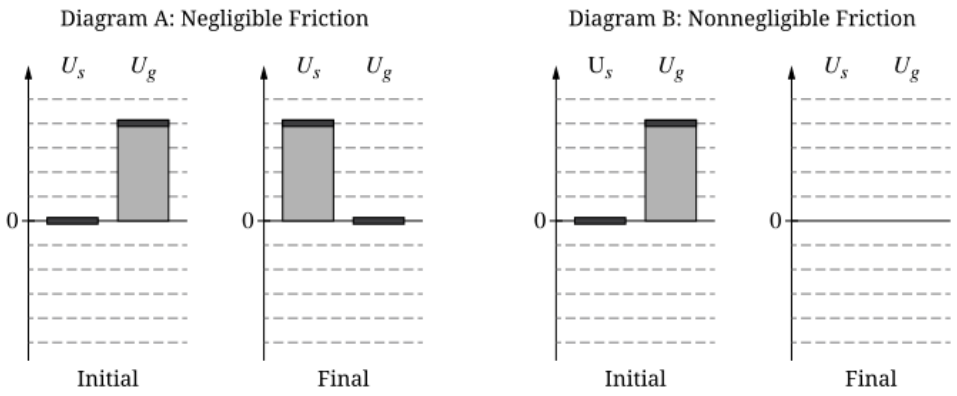
Briefly explain your reasoning.

Consider the system that includes the spring, Earth, both blocks, and the string, but not the surface. Let the initial state be when the blocks are at rest just before they start moving, and let the final state be when the blocks first

frq5

come momentarily to rest. Diagram A at left below is a bar chart that represents the energies in the scenario where there is negligible friction between block 1 and the surface.

The shaded-in bars in the energy bar charts represent the potential energy of the spring and the gravitational potential energy of the blocks-Earth system, U_s and U_g, respectively, in the initial and final states. Positive energy values are above the zero-point line ("0") and negative energy values are below the zero-point line.



(c) Complete diagram B (at right above) for the scenario in which friction is nonnegligible. The energies for the initial state are already provided. Shade in the energies in the final state using the same scale as in diagram A.

• Shaded regions should start at the solid line representing the zero-point line.
• Represent any energy that is equal to zero with a distinct line on the zero-point line.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** the following criteria.

- ☐ 1 point is earned for relating the net force to acceleration or change in velocity.
- ☐ 1 point is earned for a correct description of the relative magnitudes of the tension and gravitational forces during Block 2's motion – for both $T > W$ and $T < W$.

  Scoring Notes:

  · This point is not earned if the spring force is explicitly stated to be a force directly exerted on Block 2.

  · If the spring force is mentioned, it must be connected to the string tension.

  Example Response:

  The direction of the acceleration and the direction of the net force are the same. The block speeds up when the gravitational force is greater than the tension, and then slows down because the tension becomes larger than the gravitational force.
- ☐ 1 point is earned for an equation of conservation of energy that includes a gravitational potential energy

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term and a spring potential energy term.

- [ ] 1 point is earned for a correct expression for Δy that depends only on M_2, k_0, and g.

$\Delta y = \frac{2M_2 g}{k_0}$

Example Response:

$U_i = U_f, \quad (K_i = K_f = 0\ )$

$U_{\text{g},M_1} + M_2 g \Delta y = U_{\text{g},M_1} + \frac{1}{2} k_0 \Delta y^2$

$M_2 g \Delta y = \frac{1}{2} k_0 \Delta y^2$

$M_2 g = \frac{1}{2} k_0 \Delta y$

$\Delta y = \frac{2M_2 g}{k_0}$

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes the following criteria.

- [ ] 1 point is earned for selecting “Does not change” with a correct explanation.

Example Response:

There are no external forces on the system and no energy is dissipated by friction, so the total mechanical energy stays the same.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for a final bar chart where both U_{s} and U_{g} are positive.
- [ ] 1 point is earned for a final bar chart where the sum of U_{s} and U_{g} is less than 4 units.

Example Response:

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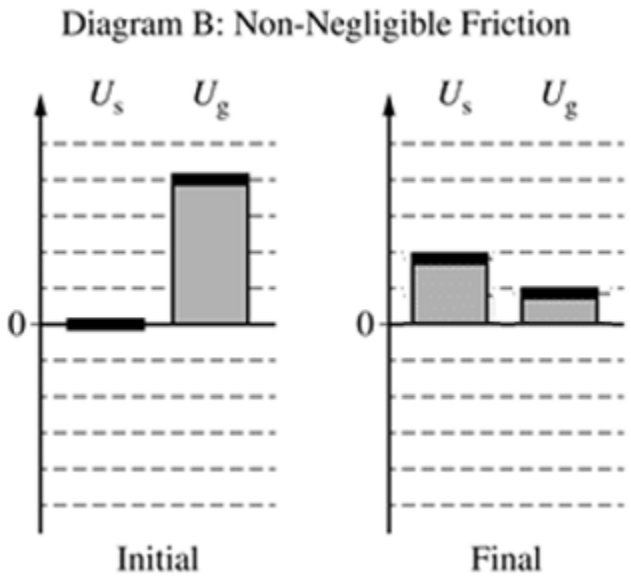
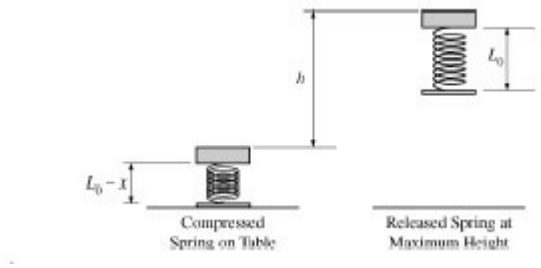


Diagram B: Non-Negligible Friction
U_s
U_g
0
Initial
U_s
U_g
0
Final

frq5

19.



In an experiment, students are to calculate the spring constant k of a vertical spring in a small jumping toy that initially rests on a table. When the spring in the toy is compressed a distance x from its uncompressed length L_0 and the toy is released, the top of the toy rises to a maximum height h above the point of maximum compression. The students repeat the experiment several times, measuring h with objects of various masses taped to the top of the toy so that the combined mass of the toy and added objects is m. The bottom of the toy and the spring each have negligible mass compared to the top of the toy and the objects taped to it.

(a) Derive an expression for the height h in terms of m, x, k, and fundamental constants.

With the spring compressed a distance x = 0.020 m in each trial, the students obtained the following data for different values of m.

	m (kg)	h (m)	
	0.020	0.49	
	0.030	0.34	
	0.040	0.28	
	0.050	0.19	
	0.060	0.18	

(b)

i. What quantities should be graphed so that the slope of a best-fit straight line through the data points can be used to calculate the spring constant k?

ii. Fill in one or both of the blank columns in the table with calculated values of your quantities, including units.

(c) On the axes below, plot your data and draw a best-fit straight line. Label the axes and indicate the scale.

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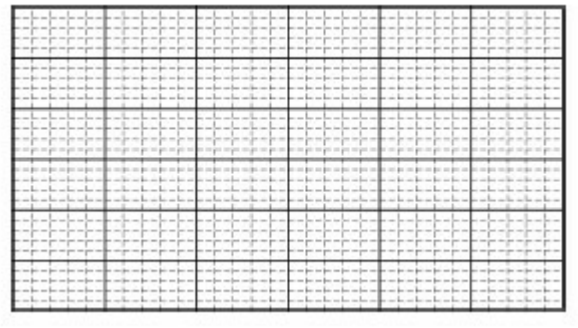


(d) Using your best-fit line, calculate the numerical value of the spring constant.

(e) Describe a procedure for measuring the height h in the experiment, given that the toy is only momentarily at that maximum height.

Part A

One point is earned for a correct statement of conservation of mechanical energy (which might be implied in the next statement)

One point is earned for correct energy expressions set equal

$mgh = 1/2\ kx2$

One point is earned for solving for h, consistent with the energy equation

$h = kx2/2mg$

0	1	2	3 ✓

The student earns all of the following points:

One point is earned for a correct statement of conservation of mechanical energy (which might be implied in the next statement)

One point is earned for correct energy expressions set equal

$mgh = 1/2\ kx2$

One point is earned for solving for h, consistent with the energy equation

$h = kx2/2mg$

Part B

Two points are earned for a correct combination of (1/m and h) or (1/h and m), with or without constants

frq5

$1/m\ (kg^{-1})$	m (kg)	h (m)	
50	0.020	0.49	
33	0.030	0.34	
25	0.040	0.28	
20	0.050	0.19	
17	0.060	0.18	

One point is earned for correctly filling in the table with the appropriate data

One point is earned for including the correct units in the table

0	1	2	3	4 ✓

The student earns all of the following points:

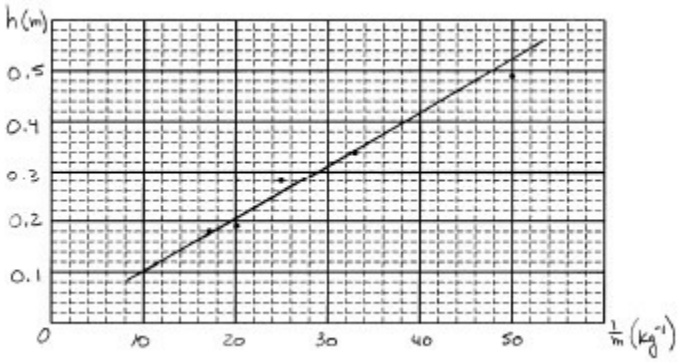
Two points are earned for a correct combination of (1/m and h) or (1/h and m), with or without constants

$1/m\ (kg^{-1})$	m (kg)	h (m)	
50	0.020	0.49	
33	0.030	0.34	
25	0.040	0.28	
20	0.050	0.19	
17	0.060	0.18	

One point is earned for correctly filling in the table with the appropriate data

One point is earned for including the correct units in the table

Part C



One point is earned for correctly plotting appropriate data (all five data points plotted correctly, assuming the data represented an inverse relationship between *m* and *h*)

One point is earned for correctly drawing a best-fit straight line (a single straight line with data points reasonably scattered above and below the line)

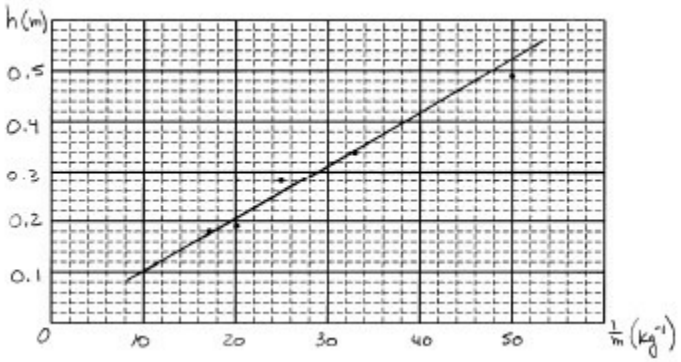
One point is earned for correctly labeling both axes

One point is earned for correctly indicating the scale on both axes

frq5

0	1	2	3	4 ✓

The student earns all of the following points:



One point is earned for correctly plotting appropriate data (all five data points plotted correctly, assuming the data represented an inverse relationship between *m* and *h*)

One point is earned for correctly drawing a best-fit straight line (a single straight line with data points reasonably scattered above and below the line)

One point is earned for correctly labeling both axes

One point is earned for correctly indicating the scale on both axes

Part D

One point is earned for correctly calculating the slope from points on the line

One point is earned for a correct numerical value of the spring constant

$$From (a), h = \frac{kx^2}{2mg}$$

, so the slope of the line =

$$\frac{kx^2}{2g}$$

$$k = \frac{2g(slope)}{x^2}$$

$$k = \frac{2(9.8 m/s^2)(1.07 \times 10^{-2} m \cdot kg)}{(0.02)^2}$$

$$k = 524 N/m \ (535 N/m \text{ using } g = 10 m/s^2)$$

frq5

0	1	2 ✓

The student earns all of the following points:

One point is earned for correctly calculating the slope from points on the line

One point is earned for a correct numerical value of the spring constant

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$$k = \frac{2(9.8 m/s^2)(1.07 \times 10^{-2} m \cdot kg)}{(0.02)^2}$$

$$k = 524 N/m \ (535 N/m \text{ using } g = 10 m/s^2)$$

Part E

Two points are earned for a correct, complete procedure

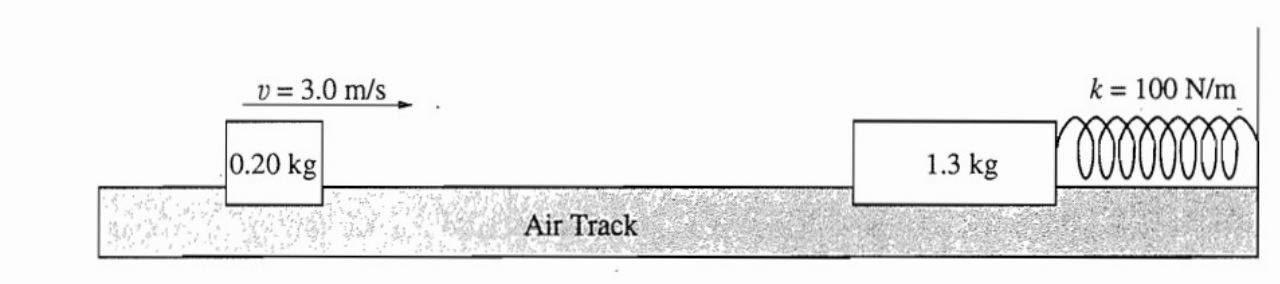
0	1	2 ✓

The student earns all of the following points:

Two points are earned for a correct, complete procedure

frq5

20.



As shown above, a 0.20-kilogram mass is sliding on a horizontal friction less air track with a speed of 3.0 meters per second when it instantaneously hits and sticks to a 1.3-kilogram mass initially at rest on the track. The 1.3-kilogram mass is connected to one end of a mass less spring, which has a spring constant of 100 newtons per meter. The other end of the spring is fixed.

(a) Determine the following for the 0.20-kilogram mass immediately before the impact.

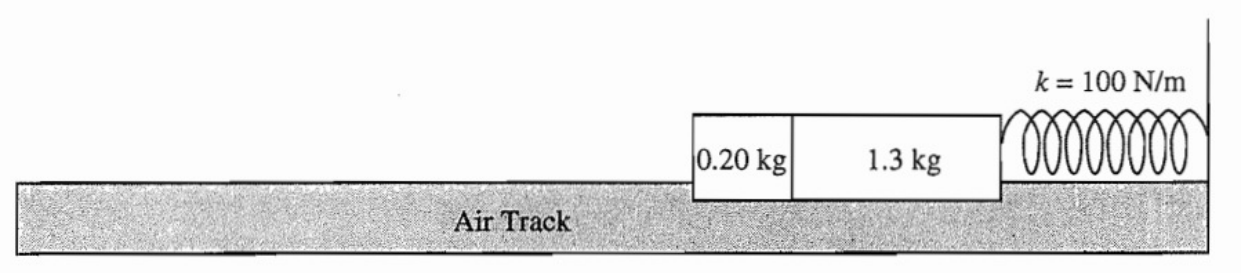
i. Its linear momentum

ii. Its kinetic energy

(b) Determine the following for the combined masses immediately after the impact

i. The linear momentum

ii. The kinetic energy



After the collision, the two masses undergo simple harmonic motion about their position at impact.

(c) Determine the amplitude of the harmonic motion.

(d) Determine the period of the harmonic motion.

Part A

(i)

1 point is earned: For a correct expression for momentum

p = mv

frq5

pi=(0.2kg)(3.0 m/s)

1 point is earned: For the correct answer

pi = 0.6kg * m/s

(ii)

1 point is earned: For a correct expression for kinetic energy

K = ½ mv2

Ki = ½(0.2 kg)(3.0 m/s)2

1 point is earned: For the correct answer

Ki = 0.9J

0	1	2	3	4 ✓

The student response earns four of the following points:

(i)

1 point is earned: For a correct expression for momentum

p = mv

pi=(0.2kg)(3.0 m/s)

1 point is earned: For the correct answer

pi = 0.6kg * m/s

(ii)

1 point is earned: For a correct expression for kinetic energy

K = ½ mv2

Ki = ½(0.2 kg)(3.0 m/s)2

1 point is earned: For the correct answer

Ki = 0.9J

Part B

(i)

frq5

1 point is earned : For any statement of conservation of momentum

p f = p i

1 point is earned : For the correct answer

p f = 0.6kg * m/s

(ii)

Knowing the final momentum, the final velocity can be calculated

p f = m f v f

v f = p f / m f

$= \frac{0.6kg \cdot m/s}{(1.3+0.2)kg}$

1 point is earned : For the correct value of the final velocity

v f = 0.4 m/s

Using the expression for kinetic energy

K f = ½m f v f 2

1 point is earned : For using the sum of the two masses

K f = ½(1.5kg)(0.4 m/s) 2

1 point is earned : For the correct answer

K f = 0.12J

0	1	2	3	4	5 ✓

The student response earns five of the following points:

(i)

1 point is earned: For any statement of conservation of momentum

pf = pi

1 point is earned: For the correct answer

pf = 0.6kg * m/s

(ii)

frq5

Knowing the final momentum, the final velocity can be calculated

pf = mfvf

vf = pf / mf

$= \frac{0.6kg \cdot m/s}{(1.3+0.2)kg}$

1 point is earned: For the correct value of the final velocity

vf = 0.4 m/s

Using the expression for kinetic energy

Kf = ½mfvf2

1 point is earned: For using the sum of the two masses

Kf = ½(1.5kg)(0.4 m/s)2

1 point is earned: For the correct answer

Kf = 0.12J

Part C

1 point is earned : For any statement of conservation of energy

The kinetic energy after the impact is equal to the maximum energy stored in the spring.

K f = ½ kA 2

Solving for the amplitude of the motion:

$A = \sqrt{2K_f/k}$

1 point is earned : For substituting the answer to (b)ii for the kinetic energy

$A = \sqrt{2(0.12J)/(100N/m)}$

1 point is earned : For the correct answer

A = 0.05 m

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned : For any statement of conservation of energy

frq5

The kinetic energy after the impact is equal to the maximum energy stored in the spring.

K f = ½ kA 2

Solving for the amplitude of the motion:

$A = \sqrt{2K_f/k}$

1 point is earned : For substituting the answer to (b)ii for the kinetic energy

$A = \sqrt{2(0.12J)/(100N/m)}$

1 point is earned : For the correct answer

A = 0.05 m

Part D

1 point is earned : For the correct expression for the period

$T = 2\pi\sqrt{m/k}$
$T = 2\pi\sqrt{(1.5kg)(100N/m)}$

1 point is earned : For the correct answer

T = 0.77 s

0	1	2 ✓

The student response earns two of the following points:

1 point is earned : For the correct expression for the period

$T = 2\pi\sqrt{m/k}$
$T = 2\pi\sqrt{(1.5kg)(100N/m)}$

1 point is earned : For the correct answer

T = 0.77 s

Units points

1 point is earned: For having both one correct energy unit and one correct momentum unit on answers in parts (a) and (b), and no incorrect units.

frq5

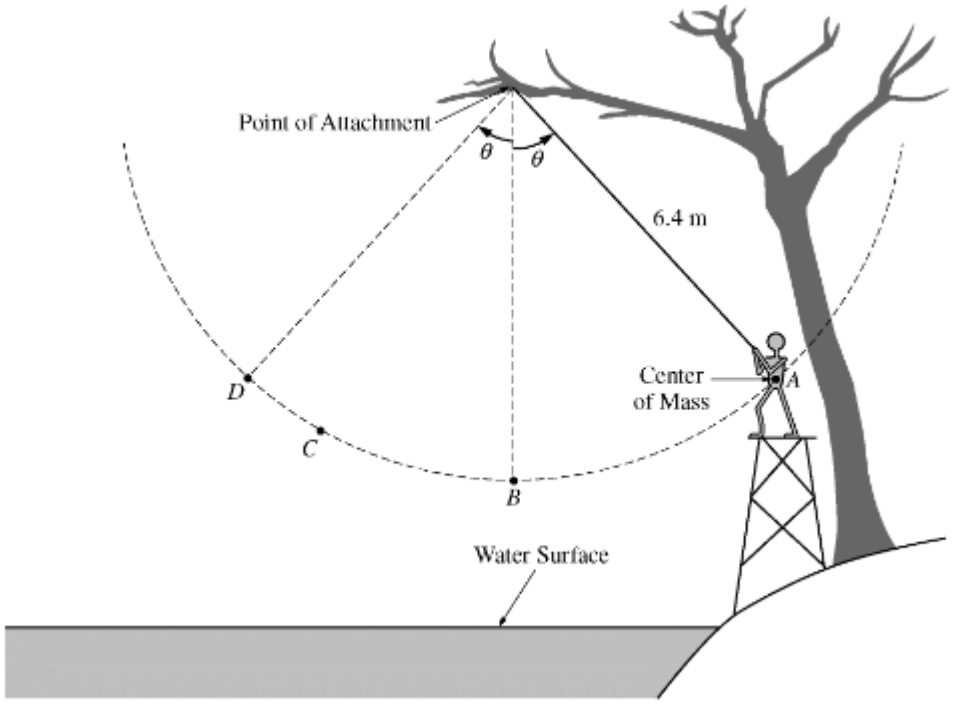
0	1 ✓

The student response earns one of the following points:

1 point is earned: For having both one correct energy unit and one correct momentum unit on answers in parts (a) and (b), and no incorrect units.

frq5

21.



Starting from rest at point A, a 50 kg person swings along a circular arc from a rope attached to a tree branch over a lake, as shown in the figure above. Point D is at the same height as point A. The distance from the point of attachment to the center of mass of the person is 6.4 m. Ignore air resistance and the mass and elasticity of the rope.

(a) The person swings two times, each time letting go of the rope at a different point.

i. On the first swing, the person lets go of the rope when first arriving at point C. Draw a solid line to represent the trajectory of the center of mass after the person releases the rope.

ii. A second time, the person lets go of the rope at point D. Draw a dashed line to represent the trajectory of the center of mass after the person releases the rope.

(b) The center of mass of the person standing on the platform is at point A, 4.1 m above the surface of the water. Calculate the gravitational potential energy when the person is at point A relative to when the person is at the surface of the water.

(c) The center of mass of the person at point B, the lowest point along the arc, is 2.4 m above the surface of the water. Calculate the person's speed at point B.

(d) Suppose that the person swings from the rope a third time, letting go of the rope at point B. Calculate R, the horizontal distance moved from where the person releases the rope at point B to where the person hits the water.

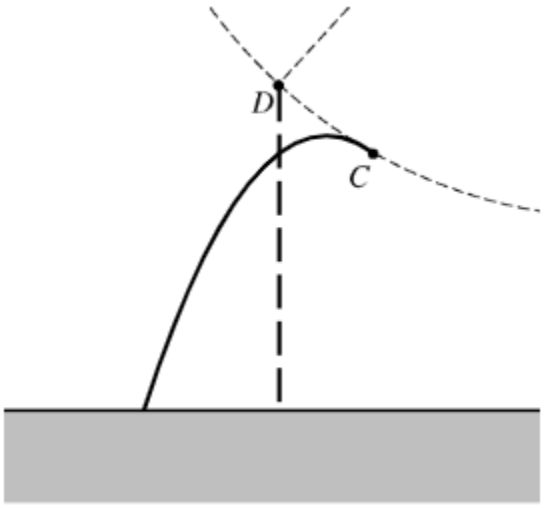
(e) If the person does not let go of the rope, how does the magnitude of the person's momentum p_C at point C compare with the magnitude of the person's momentum p_B at point B ?

frq5

____ $p_C > p_B$ ____ $p_C < p_B$ ____ $p_C = p_B$

Provide a physical explanation to justify your answer.

Part A



1 point is earned: For showing a parabolic path from point *C* that is tangent to the arc at *C*

1 point is earned: For a maximum height of the parabolic path that is lower than point *D*

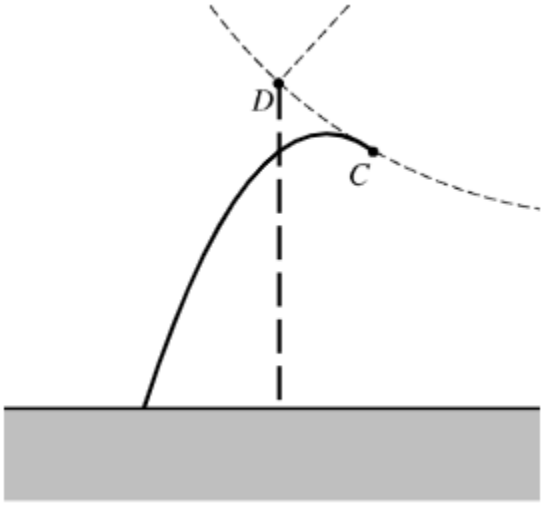
1 point is earned: For showing the person traveling straight downward from point *D*

Note: the two paths may, but need not, cross each other.

0	1	2	3 ✓

The student response earns all of the following points:

frq5



1 point is earned: For showing a parabolic path from point *C* that is tangent to the arc at *C*

1 point is earned: For a maximum height of the parabolic path that is lower than point *D*

1 point is earned: For showing the person traveling straight downward from point *D*

Note: the two paths may, but need not, cross each other

Part B

1 point is earned: For using the gravitational potential energy equation

$\Delta U_g = mgh = (50kg)\left(9.8m/s^2\right)(4.1m)$

1 point is earned: For a correct answer

$\Delta U_g = 2010J\left(or, 2050J, u \sin g, g = 10m/s^2\right)$

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For using the gravitational potential energy equation

$\Delta U_g = mgh = (50kg)\left(9.8m/s^2\right)(4.1m)$

1 point is earned: For a correct answer

$\Delta U_g = 2010J\left(or, 2050J, u \sin g, g = 10m/s^2\right)$

Part C

frq5

1 point is earned: For using an expression of conservation of energy

$U_{g1} = U_{g2} + K_2$
$mgh_1 = mgh_2 + \frac{1}{2}mv^2$

1 point is earned: For correctly substituting values

$gh_1 = gh_2 + \frac{1}{2}v^2$
$v = \sqrt{2g(h_1 - h_2)}$
$v = \sqrt{2(9.8m/s^2)(4.1m - 2.4m)}$

1 point is earned: For a correct answer

v = 5.8 m/s

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For using an expression of conservation of energy

$U_{g1} = U_{g2} + K_2$
$mgh_1 = mgh_2 + \frac{1}{2}mv^2$

1 point is earned: For correctly substituting values

$gh_1 = gh_2 + \frac{1}{2}v^2$
$v = \sqrt{2g(h_1 - h_2)}$
$v = \sqrt{2(9.8m/s^2)(4.1m - 2.4m)}$

1 point is earned: For a correct answer

v = 5.8 m/s

Part D

1 point is earned: For using a correct expression to solve for the time of fall

$\Delta y = v_0 t + \frac{1}{2}at^2$

1 point is earned: For correctly solving for the time of fall

frq5

$\Delta y = \frac{1}{2}at^2$

$t = \sqrt{\frac{2\Delta y}{a}}$

$t = \sqrt{\frac{2(2.4m)}{(9.8m/s^2)}}$

$t = 0.70s\left(or, 0.69s, u\sin g, g = 10m/s^2\right)$

In the horizontal direction, the acceleration is zero.

R = *vxt*

1 point is earned: For substituting the speed from part (c) into the constant-velocity equation for horizontal motion

R = (5.8 m/s)(0.70 s)

R = 4.1 m (or 4.0 m using either unrounded speed of 5.77 m/s or g = 10 m/s2)

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For using a correct expression to solve for the time of fall

$\Delta y = v_0 t + \frac{1}{2}at^2$

1 point is earned: For correctly solving for the time of fall

$\Delta y = \frac{1}{2}at^2$

$t = \sqrt{\frac{2\Delta y}{a}}$

$t = \sqrt{\frac{2(2.4m)}{(9.8m/s^2)}}$

$t = 0.70s\left(or, 0.69s, u\sin g, g = 10m/s^2\right)$

In the horizontal direction, the acceleration is zero.

R = *vxt*

1 point is earned: For substituting the speed from part (c) into the constant-velocity equation for horizontal motion

R = (5.8 m/s)(0.70 s)

R = 4.1 m (or 4.0 m using either unrounded speed of 5.77 m/s or g = 10 m/s2

Units

1 point is earned: For correct units in the calculated answers to parts (b), (c), and (d)

frq5

0	1 ✓

The student response earns all of the following points:

1 point is earned: For correct units in the calculated answers to parts (b), (c), and (d)

Part E

1 point is earned: For selecting " $pC < pB$ "

1 point is earned: For any indication that the speed at C is less than the speed at B

1 point is earned: For an explanation of why the speed decreases from B to C, which includes the following or equivalent statements:

by conservation of energy: U increases as the person moves from B to C, therefore K decreases, resulting in a decrease in speed

OR gravitational force opposes motion as the person moves from B to C, which results in a negative acceleration and a decrease in speed

OR by the impulse-momentum theorem: momentum decreases because the force of gravity provides impulse opposing the motion of the person

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For selecting " $pC < pB$ "

1 point is earned: For any indication that the speed at C is less than the speed at B

1 point is earned: For an explanation of why the speed decreases from B to C, which includes the following or equivalent statements:

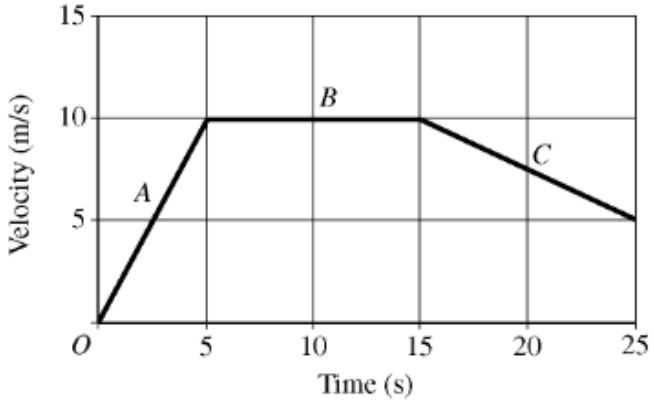
by conservation of energy: U increases as the person moves from B to C, therefore K decreases, resulting in a decrease in speed

OR gravitational force opposes motion as the person moves from B to C, which results in a negative acceleration and a decrease in speed

OR by the impulse-momentum theorem: momentum decreases because the force of gravity provides impulse opposing the motion of the person

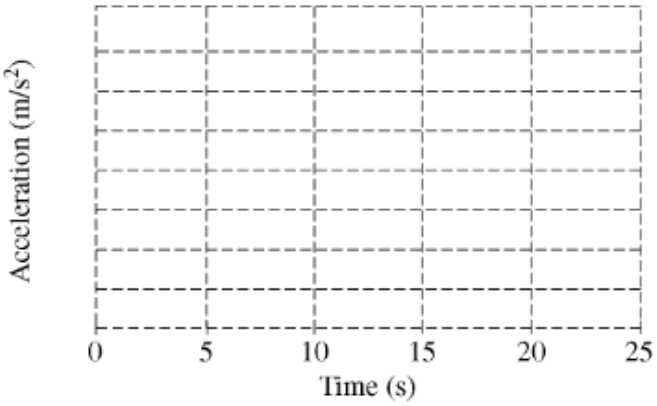
frq5

22.



A 0.40 kg object moves in a straight line under the action of a net force. The graph above shows the velocity as a function of time for the object during a 25 s interval. At time $t = 0$, the object is at the position $x = 0$.

(a) On the grid below, sketch a graph of the acceleration as a function of time for the object. Label the scale for the acceleration.



(b) Calculate the position of the object at t = 5.0 s.

(c) On which segment of the graph is the net force acting on the object zero?

____ A ____ B ____ C

Justify your answer.

(d) Calculate the net force on the object during the first 3.0 s of the motion.

(e) Calculate the amount of work done on the object by the net force during the first 15 s of the motion.

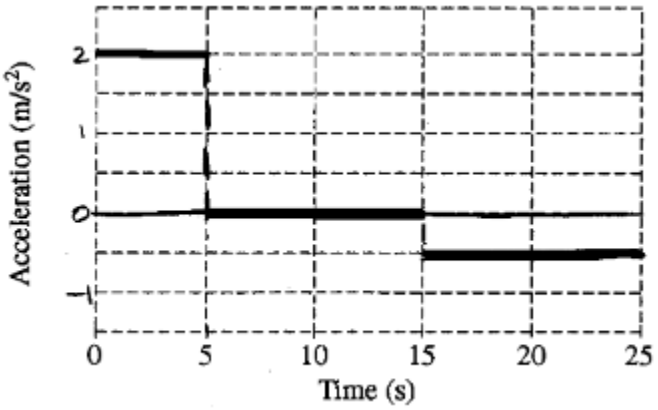
frq5

(f) For the interval t = 15 s to t = 25 s, is the work done on the object by the net force positive, negative, or zero?

____ Positive ____ Negative ____ Zero

Justify your answer.

Part A



1 point is earned: For using a linear scale on the vertical axis

1 point is earned: For showing horizontal lines in the three appropriate time frames

1 point is earned: For having the correct acceleration values for segments *B* and *C* (The correct value for segment *A* is accounted for in the answer point in part (d).)

a = $\Delta v / \Delta t$ (or the slope of the velocity versus time curve)

For segment A: a = (10m/s - 0m/s)/(5 s - 0 s) = 2 m/s2

For segment B: slope is zero, so acceleration is zero

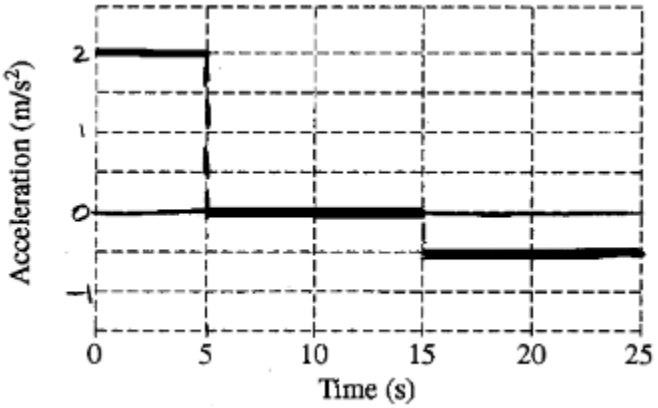
For segment C: a = (5m/s - 10m/s)/(25 s - 15 s) = -0.5 m/s2

✓

0	1	2	3

The student response earns all of the following points:

frq5



1 point is earned: For using a linear scale on the vertical axis

1 point is earned: For showing horizontal lines in the three appropriate time frames

1 point is earned: For having the correct acceleration values for segments *B* and *C* (The correct value for segment *A* is accounted for in the answer point in part (d).)

a = $\Delta v/\Delta t$ (or the slope of the velocity versus time curve)

For segment A: *a* = (10m/s - 0m/s)/(5 s - 0 s) = 2 m/s2

For segment B: slope is zero, so acceleration is zero

For segment C: *a* = (5m/s - 10m/s)/(25 s - 15 s) = -0.5 m/s2

Part B

x = *x0* + *v0t* + (1/2)at2, where *x0* = 0

1 point is earned: For substituting v0 = 0 and *t* = 5 s and into the equation above

1 point is earned: For substituting the value on the graph of the constant acceleration between 0 and 5 s or an explicit calculation of 2m/s2

x = (1/2)(2 m/s2)(5 s)2

x = 25 m

Alternate solution

Position is the area under the velocity versus time curve.

1 point is earned: *For correctly substituting into the expression for the area under the curve*

x = (1/2)(10 m/s)(5 s)

1 point is earned: *For the correct answer*

x = 25 m

frq5

0	1	2 ✓

The student response earns all of the following points:

$x = x0 + v0t + (1/2)at2$, where $x0 = 0$

1 point is earned: For substituting v0 = 0 and $t = 5$ s and into the equation above

1 point is earned: For substituting the value on the graph of the constant acceleration between 0 and 5 s or an explicit calculation of 2m/s2

$x = (1/2)(2\ m/s2)(5\ s)2$

$x = 25\ m$

Alternate solution

Position is the area under the velocity versus time curve.

1 point is earned: *For correctly substituting into the expression for the area under the curve*

$x = (1/2)(10\ m/s)(5\ s)$

1 point is earned: *For the correct answer*

$x = 25\ m$

Part C

1 point is earned: For selecting segment *B*

1 point is earned: For an appropriate justification

Example: Fnet = ma = m ($\Delta v/\ \Delta t$), the velocity is constant for this segment, $\Delta v = 0$; therefore, *Fnet* = 0.

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For selecting segment *B*

1 point is earned: For an appropriate justification

Example: Fnet = ma = m ($\Delta v/\ \Delta t$), the velocity is constant for this segment, $\Delta v = 0$; therefore, *Fnet* = 0.

frq5

Part D

1 point is earned: For using Newton's second law

Fnet = ma

Fnet = (0.40 kg)(2 m/s2)

1 point is earned: For the correct answer

Fnet = 0.80 N

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For using Newton's second law

Fnet = ma

Fnet = (0.40 kg)(2 m/s2)

1 point is earned: For the correct answer

Fnet = 0.80 N

Part E

1 point is earned: For using the work-energy theorem

W = FΔr cos θ

To move in a straight line, the net force must be in the direction of motion, so . *W* = *Fnetd*

1 point is earned: For substituting the value of the position determined in from part (b), which is equal to the displacement, to determine the work done for segment A

For segment A, *WA* = (0.8)(25 m) = 20 J

For segment B, *WB* = (0 N)(10 m/s)(10 s) = 0 J

Wtot = *WA* + *WB* = 20 J + 0 J = 20 J

Alternate solution

$W = \Delta K$

$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

frq5

1 point is earned: *For correct substitutions into the equation above*

$W = \frac{1}{2}(0.40kg)(10m/s)^2 - \frac{1}{2}(0.40kg)(0m/s)^2$

1 point is earned: *For the correct answer*

W = 20 J

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For using the work-energy theorem

W = F Δr cos θ

To move in a straight line, the net force must be in the direction of motion, so . *W = Fnetd*

1 point is earned: For substituting the value of the position determined in from part (b), which is equal to the displacement, to determine the work done for segment A

For segment A, *WA* = (0.8)(25 m) = 20 J

For segment B, *WB* = (0 N)(10 m/s)(10 s) = 0 J

Wtot = WA + WB = 20 J + 0 J = 20 J

Alternate solution

$W = \Delta K$
$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

1 point is earned: *For correct substitutions into the equation above*

$W = \frac{1}{2}(0.40kg)(10m/s)^2 - \frac{1}{2}(0.40kg)(0m/s)^2$

1 point is earned: *For the correct answer*

W = 20 J

Part F

1 point is earned: For selecting “Negative”

1 point is earned: For an appropriate explanation using kinematic or dynamic principles

Examples:

frq5

The velocity is decreasing, so the kinetic energy is also decreasing.

The acceleration is negative; therefore, the direction of the net force is opposite to the direction of the displacement.

1 point is earned: For connecting the explanation to the work done

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For selecting “Negative”

1 point is earned: For an appropriate explanation using kinematic or dynamic principles

Examples:

The velocity is decreasing, so the kinetic energy is also decreasing.

The acceleration is negative; therefore, the direction of the net force is opposite to the direction of the displacement.

1 point is earned: For connecting the explanation to the work done

Units

1 point is earned: For correct units on at least two parts with a calculated numerical answer and no incorrect units

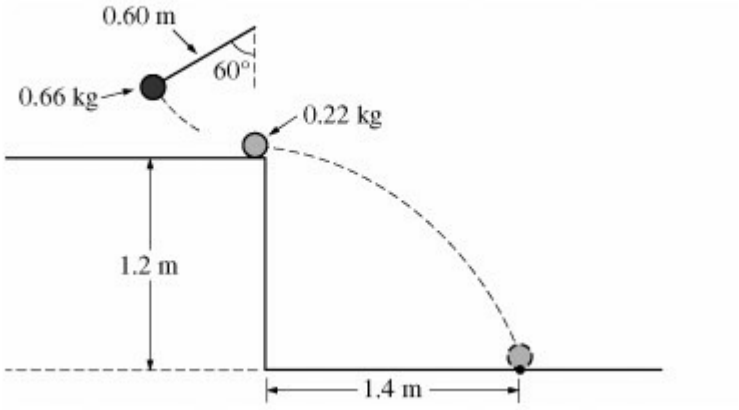
0	1 ✓

The student response earns all of the following points:

1 point is earned: For correct units on at least two parts with a calculated numerical answer and no incorrect units

frq5

23.



The experimental apparatus shown in the figure above contains a pendulum consisting of a 0.66 kg ball attached to a string of length 0.60 m. The pendulum is released from rest at an angle of 60 degrees and collides with a ball of mass 0.22 kg initially at rest at the edge of a table. The 0.22 kg ball hits the floor a distance of 1.4 m from the edge of the table.

a. Calculate the speed of the 0.66 kg ball just before the collision.

b. Calculate the speed of the 0.22 kg ball immediately after the collision.

c. Calculate the speed of the 0.66 kg ball immediately after the collision.

d. Indicate the direction of motion of the 0.66 kg ball immediately after the collision.

____ To the left ____ To the right

e. Calculate the height to which the 0.66 kg ball rises after the collision.

f. Based on your data, is the collision elastic?

____ Yes ____ No

Justify your answer.

Part A

One point is earned for work indicating use of conservation of energy

$U_{g1} = K_2$

$mgh_1 = \frac{1}{2}mv_2^2$

One point is earned for a correct expression for determining the vertical drop-distance

frq5

$$h = L - L\cos 60^\circ$$
$$h = 0.60m - (0.60m)\cos 60^\circ$$
$$h = 0.30m$$

One point is earned for substitution of a drop distance (other than 0.60 m) into a correct expression for the speed of the 0.66 kg ball just before the collision

$v = \sqrt{2gh} = \sqrt{2(9.8m/s^2)(0.30)}$
$v = 2.4m/s$

0	1	2	3 ✓

The student earns all of the following points:

One point is earned for work indicating use of conservation of energy

$U_{g1} = K_2$
$mgh_1 = \frac{1}{2}mv_2^2$

One point is earned for a correct expression for determining the vertical drop-distance

$$h = L - L\cos 60^\circ$$
$$h = 0.60m - (0.60m)\cos 60^\circ$$
$$h = 0.30m$$

One point is earned for substitution of a drop distance (other than 0.60 m) into a correct expression for the speed of the 0.66 kg ball just before the collision

$v = \sqrt{2gh} = \sqrt{2(9.8m/s^2)(0.30)}$
$v = 2.4m/s$

Part B

The speed immediately after the collision equals the horizontal speed of the 0.22 kg ball as it leaves the table, which is the horizontal distance it travels divided by the time it takes to fall.

One point is earned for a correct expression for the time it takes the 0.22 kg ball to fall

$$\triangle y = \frac{1}{2}gt^2$$
$$t = \sqrt{2 \triangle y/g}$$
$$t = \sqrt{2(1.2m)/(9.8m/s^2)}$$
$$t = 0.49s (or\ 0.50s \text{ using } g = 10m/s^2)$$

One point is earned for consistent substitution of the time into a correct expression for the speed of the 0.22 kg ball after

frq5

the collision

$v_{0.22} = x/t = (1.4m)/(0.49s)$

One point is earned for the correct numerical value of the speed

$$v_{0.22} = 2.8m/s \ (or \ 2.9m/s \text{ using } g = 10m/s^2)$$

0	1	2	3 ✓

The student earns all of the following points:

The speed immediately after the collision equals the horizontal speed of the 0.22 kg ball as it leaves the table, which is the horizontal distance it travels divided by the time it takes to fall.

One point is earned for a correct expression for the time it takes the 0.22 kg ball to fall

$$\triangle y = \tfrac{1}{2} g t^2$$
$$t = \sqrt{2 \triangle y / g}$$
$$t = \sqrt{2(1.2m)/(9.8m/s^2)}$$
$$t = 0.49s (or \ 0.50s \text{ using } g = 10m/s^2)$$

One point is earned for consistent substitution of the time into a correct expression for the speed of the 0.22 kg ball after the collision

$v_{0.22} = x/t = (1.4m)/(0.49s)$

One point is earned for the correct numerical value of the speed

$$v_{0.22} = 2.8m/s \ (or \ 2.9m/s \text{ using } g = 10m/s^2)$$

Part C

One point is earned for work indicating use of conservation of momentum

$m_{0.66} v_{0.66i} = m_{0.66} v_{0.66f} + m_{0.22} v_{0.22f}$

One point is earned for consistent substitution of the answer from part (a) as *v0.66i* in a correct expression for *v0.66f*

$v_{0.66f} = \frac{m_{0.66} v_{0.66i} - m_{0.22} v_{0.22f}}{m_{0.66}} = \frac{(0.66kg)(2.4m/s) - (0.22kg)(2.8m/s)}{(0.66kg)}$

frq5

One point is earned for the correct numerical value of the speed

$$v_{0.66f} = 1.5m/s \ (or \ 1.4m/s \text{ using } g = 10m/s^2)$$

0	1	2	3 ✓

The student earns all of the following points:

One point is earned for work indicating use of conservation of momentum

$m_{0.66}v_{0.66i} = m_{0.66}v_{0.66f} + m_{0.22}v_{0.22f}$

One point is earned for consistent substitution of the answer from part (a) as *v0.66i* in a correct expression for *v0.66f*

$v_{0.66f} = \frac{m_{0.66}v_{0.66i} - m_{0.22}v_{0.22f}}{m_{0.66}} = \frac{(0.66kg)(2.4m/s) - (0.22kg)(2.8m/s)}{(0.66kg)}$

One point is earned for the correct numerical value of the speed

$$v_{0.66f} = 1.5m/s \ (or \ 1.4m/s \text{ using } g = 10m/s^2)$$

Part D

One point is earned for indicating “To the right” as the direction of motion of the 0.66 kg ball after the collision

0	1 ✓

The student earns all of the following points:

One point is earned for indicating “To the right” as the direction of motion of the 0.66 kg ball after the collision

Part E

One point is earned for an appropriate conservation of energy statement

$K_1 = U_{g2}$
$\frac{1}{2}mv_{0.66f}^2 = mgh'$

One point is earned for consistent substitution of *v0.66f* from part (c) into a correct expression for h'

frq5

$h' = \frac{v_{0.66f}^2}{2g} = \frac{(1.5m/s)^2}{(2)(9.8m/s^2)}$
$h' = 0.11m$

0	1	2 ✓

The student earns all of the following points:

One point is earned for an appropriate conservation of energy statement

$K_1 = U_{g2}$
$\frac{1}{2}mv_{0.66f}^2 = mgh'$

One point is earned for consistent substitution of *v0.66f* from part (c) into a correct expression for *h'*

$h' = \frac{v_{0.66f}^2}{2g} = \frac{(1.5m/s)^2}{(2)(9.8m/s^2)}$
$h' = 0.11m$

Part F

One point is earned for any indication of comparing kinetic energies of the system before and after the collision

One point is earned for calculating the two values of kinetic energy

$K_i = \frac{1}{2}(0.66kg)\text{msup} = 1.9J$
$K_f = \frac{1}{2}(0.66kg)\text{msup} + \frac{1}{2}(0.22kg)(2.8m/s^2) = 1.6J$

One point is earned for indication that the difference in the kinetic energies means an inelastic collision

0	1	2	3 ✓

The student earns all of the following points:

One point is earned for any indication of comparing kinetic energies of the system before and after the collision

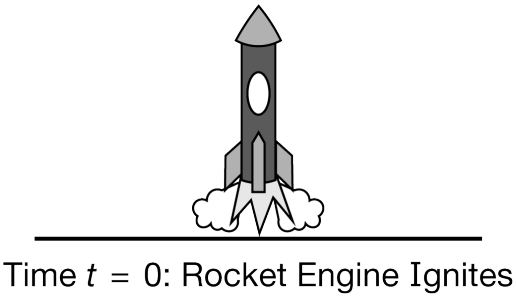
One point is earned for calculating the two values of kinetic energy

$K_i = \frac{1}{2}(0.66kg)\text{msup} = 1.9J$
$K_f = \frac{1}{2}(0.66kg)\text{msup} + \frac{1}{2}(0.22kg)(2.8m/s^2) = 1.6J$

One point is earned for indication that the difference in the kinetic energies means an inelastic collision

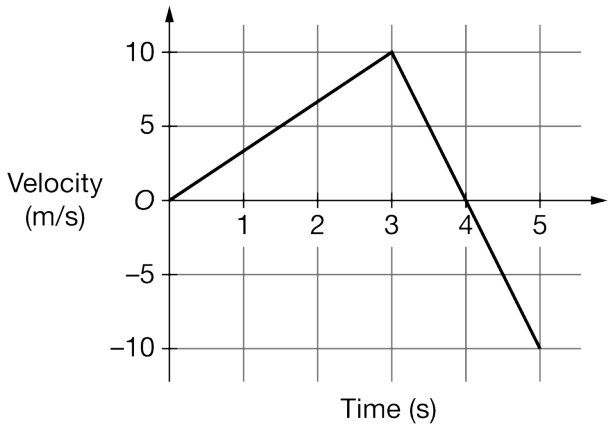
frq5

24. This question is a short free-response question. Show your work for each part of the question.



(7 points, suggested time 13 minutes)

A model rocket of constant mass 0.5 kg is launched vertically at time $t = 0$. The rocket's engine provides a constant force of magnitude F_0 for three seconds and then stops firing. The vertical velocity of the rocket as a function of time for the first 5 seconds of the flight is shown in the graph below.

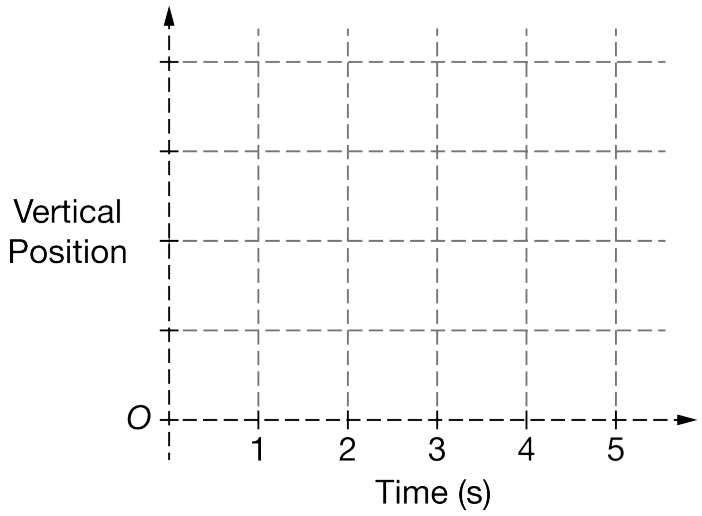


(a) Calculate F_0.

(b) Calculate the time at which the rocket hits Earth after being launched.

(c) On the axes below, sketch a graph of the vertical position of the rocket as a function of time for the first 5 seconds of the flight.

frq5



Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using the slope of the line from $t = 0$ to $t = 3$ seconds to find the acceleration of the rocket.
- ☐ 1 point is earned for using Newton's second law to correctly determine the force exerted by the engine.

Example Response for part (a):

Slope of the velocity graph from t=0 to t=3 seconds

$\text{Slope} = \frac{10\ \text{m/s}}{3\ \text{s}} = 3.3 \frac{\text{m}}{\text{s}^2}$

$\sum F_y = ma_y$

$F_{engine} - F_g = ma_y$

$F_{engine} = (0.5\ \text{kg})(3.3\ \frac{m}{s^2}) + (0.5\ \text{kg})(9.8\ \frac{\text{m}}{\text{s}^2}) = 6.6\ \text{N}$

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq5

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for identifying that the rocket rises for 4 seconds before reaching the maximum height.
- ☐ 1 point is earned for calculating the area under the velocity graph for the first 4 seconds to find the maximum height of the rocket:
  $\text{Max height} = (0.5)(3\text{ s})(10\text{ m/s}) + (0.5)(1\text{ s})(10\text{ m/s}) = 20\text{ meters}$
- ☐ 1 point is earned for finding the time the rocket takes to fall from its maximum height consistent with the maximum height found in the second point:
  $T_{fall} = 2\text{ seconds}$
  Scoring note: Time from launch back to ground is 6 seconds
- ☐ **Alternate Solution to part (b)**
  1 point is earned for identifying that the rocket undergoes the acceleration calculated in part (a) for the first 3 seconds.
- ☐ 1 point is earned for calculating the height reached at $t = 3\text{ seconds}$ either by using the acceleration from part (a) in a correct kinematic equation:
  $\Delta y = v_0 t + 0.5at^2$
  $\Delta y = 0 + 0.5(\frac{10}{3}\frac{\text{m}}{\text{s}^2})(3\text{ s})^2$
  $\Delta y = 15\text{ m in first 3 seconds}$
  OR by calculating the area under the velocity graph for the first 3 seconds.
- ☐ 1 point is earned for finding the time the rocket undergoes freefall acceleration until it hits the ground consistent with the distance found in the second point:
  $\Delta y = v_0 t + 0.5at^2$
  $-15\text{ m} = (10\ \frac{\text{m}}{\text{s}})t + 0.5(-9.8\frac{\text{m}}{\text{s}^2})t^2$
  $t = 3.04\text{ s}$
  Scoring note: Time from launch back to ground is 6 seconds.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

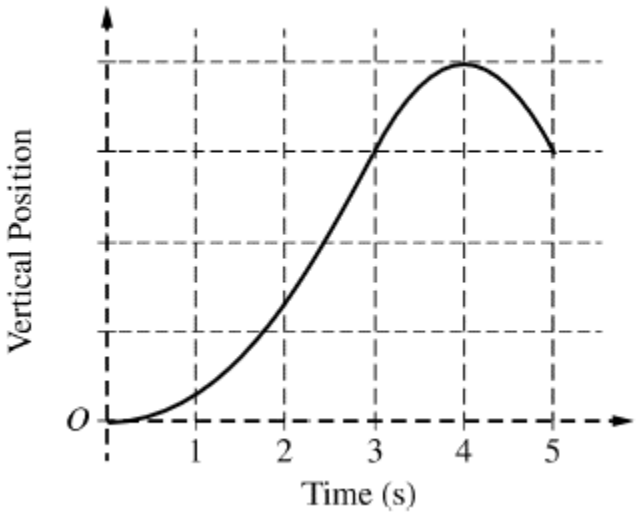
The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a graph that is concave-up from $t = 0$ to $t = 3$ seconds.

frq5

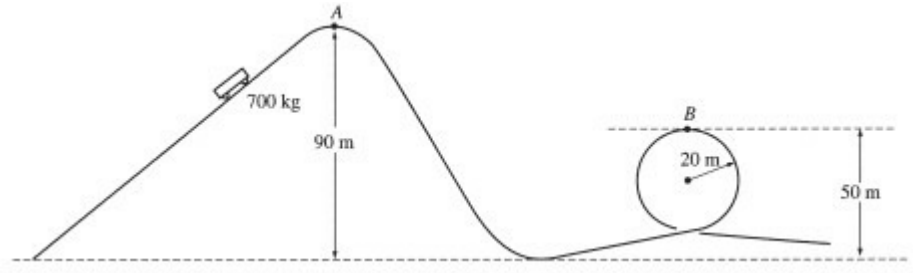
- [ ] 1 point is earned for a graph that is concave-down from $t = 3$ to $t = 5$ seconds **AND** has a maximum at 4 s.

Example Response for part (c):



frq5

25.



A roller coaster ride at an amusement park lifts a car of mass 700 kg to point *A* at a height of 90 m above the lowest point on the track, as shown above. The car starts from rest at point *A*, rolls with negligible friction down the incline and follows the track around a loop of radius 20 m. Point *B*, the highest point on the loop, is at a height of 50 m above the lowest point on the track.

(a)

i. Indicate on the figure the point *P* at which the maximum speed of the car is attained.

ii. Calculate the value v_{max} of this maximum speed.

(b) Calculate the speed v_B of the car at point *B*.

(c)

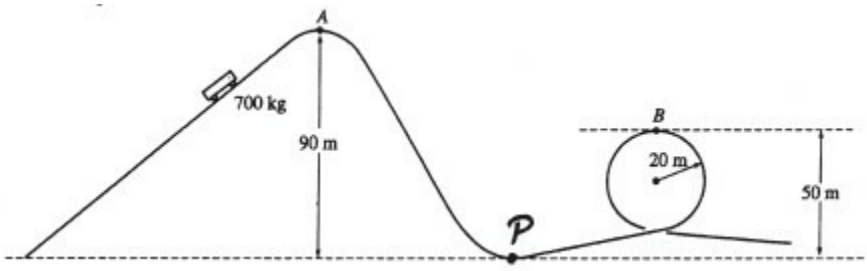
i. On the figure of the car below, draw and label vectors to represent the forces acting on the car when it is upside down at point *B*.



ii. Calculate the magnitude of all the forces identified in (c)i.

(d) Now suppose that friction is not negligible. How could the loop be modified to maintain the same speed at the top of the loop as found in (b)? Justify your answer.

Part A



frq5

One point is earned for correctly locating point P at the low point between the hill and the loop

One point is earned for any indication that energy is conserved.

One point is earned for setting the kinetic energy at P equal to the potential energy at A

$\frac{1}{2}mv_{\max} = mgy_A$
$v_{\max} = \sqrt{2gy_A}$

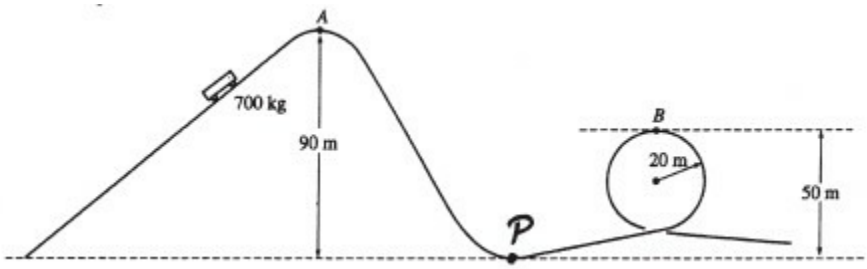
One point is earned for correct substitution

$v_{\max} = \sqrt{2(9.8m/s^2)(90m)}$

One point is earned for the correct answer
vmax = 42m/s (or 42.4m/s, u sin g, g = 10m/s2)

0	1	2	3	4 ✓

The student earns all of the following points:



One point is earned for correctly locating point P at the low point between the hill and the loop

One point is earned for any indication that energy is conserved.

One point is earned for setting the kinetic energy at P equal to the potential energy at A

$\frac{1}{2}mv_{\max} = mgy_A$
$v_{\max} = \sqrt{2gy_A}$

One point is earned for correct substitution

$v_{\max} = \sqrt{2(9.8m/s^2)(90m)}$

One point is earned for the correct answer

frq5

vmax = 42m/s (or 42.4m/s, u sin g, g = 10m/s2)

Part B

One point is earned for any indication that energy is conserved, equating the total energy at point *B* to the potential energy at point *A*

$\frac{1}{2}mv_B + mgy_B = mgy_A$

Solving for the speed at point *B*

$v_B = \sqrt{2g(y_A - y_B)}$

One point is earned for correct substitution

$v_B = \sqrt{2(9.8m/s^2)(90m - 50m)}$

One point is earned for the correct answer
vB = 28m/s (or 28.3m/s, u sin g, g = 10m/s2)

0	1	2	3 ✓

The student earns all of the following points:

One point is earned for any indication that energy is conserved, equating the total energy at point *B* to the potential energy at point *A*

$\frac{1}{2}mv_B + mgy_B = mgy_A$

Solving for the speed at point *B*

$v_B = \sqrt{2g(y_A - y_B)}$

One point is earned for correct substitution

$v_B = \sqrt{2(9.8m/s^2)(90m - 50m)}$

One point is earned for the correct answer

vB = 28m/s (or 28.3m/s, u sin g, g = 10m/s2)

frq5

Part C



One point is earned for each correctly drawn and labeled vector

One point is earned for no extraneous vectors, given that the two vectors above are drawn correctly.

One point is earned for determining the correct value of the weight
mg = (700kg) (9.8m/s2)
mg = 6860N (7000N, u sin g, g = 10m/s2)

One point is earned for indicating there is a centripetal force on the car, either separately or by setting the sum of the fores equal to the centripetal force.

$$N + mg = \frac{mv_B^2}{r}$$

$$N = \frac{mv_B^2}{r} - mg = \frac{(700kg)(28m/s^2)}{20m} - 6860N$$

One point is earned for the correct answer
N = 20, 580N (or 21,000 N, u sin g, g = 10m/s2)

✓

0	1	2	3	4	5	6

The student earns all of the following points:



One point is earned for each correctly drawn and labeled vector

One point is earned for no extraneous vectors, given that the two vectors above are drawn correctly.

One point is earned for determining the correct value of the weight

mg = (700kg) (9.8m/s2)
mg = 6860N (7000N, u sin g, g = 10m/s2)

One point is earned for indicating there is a centripetal force on the car, either separately or by setting the sum of the fores equal to the centripetal force.

frq5

$N + mg = \frac{mv_B^2}{r}$

$N = \frac{mv_B^2}{r} - mg = \frac{(700kg)(28m/s^2)}{20m} - 6860N$

One point is earned for the correct answer

N = 20, 580N (or 21,000 N, u sin g, g = 10m/s2)

Part D

Two points are earned for any clear description of both a modification that will lower the height of point *B* and a correct justification

0	1	2 ✓

The student earns all of the following points:

Two points are earned for any clear description of both a modification that will lower the height of point *B* and a correct justification

frq5

26.



The Sojourner rover vehicle shown in the sketch above was used to explore the surface of Mars as part of the Pathfinder mission in 1997. Use the data in the tables below to answer the questions that follow.

Mars Data	Sojourner Data
Radius: 0.53 x Earth's radius Mass: 0.11 x Earth's mass	Mass of Sojourner vehicle: 11.5 kg Wheel diameter: 0.13 m Stored energy available: 5.4 x 10^5J Power required for driving under average conditions: 10W Land speed: 6.7 x 10^{-3} m/s

1. Determine the acceleration due to gravity at the surface of Mars in terms of *g*, the acceleration due to gravity at the surface of Earth.
2. Calculate Sojournerís weight on the surface of Mars.
3. Assume that when leaving the Pathfinder spacecraft Sojourner rolls down a ramp inclined at 20° to the horizontal. The ramp must be lightweight but strong enough to support Sojourner. Calculate the minimum normal force that must be supplied by the ramp.
4. What is the net force on Sojourner as it travels across the Martian surface at constant velocity? Justify your answer.
5. Determine the maximum distance that Sojourner can travel on a horizontal Martian surface using its stored energy.
6. Suppose that 0.010% of the power for driving is expended against atmospheric drag as Sojourner travels on the Martian surface. Calculate the magnitude of the drag force.

Part A

1 point is earned: For a correct expression of Newton's Gravitation law

$$F = \frac{GM_1M_2}{R^2}$$

frq5

1 point is earned: For correct substitution/calculation in formula

$$gMars = \frac{GM_M}{R_M^2} = \frac{G(0.11)M_g}{(0.53R_g)^2} = \frac{0.11}{(0.53)^2}g_g$$

1 point is earned: For expressing *gM* in terms of *gg*

$$gM = \frac{0.11}{(0.53)^2}g_g$$

OR

$$gM = 0.39g_g$$

However, the third point for expressing *gM* in terms of *gg* is not awarded if no points have been awarded up to that part.

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned: For a correct expression of Newton's Gravitation law

$$F = \frac{GM_1M_2}{R^2}$$

1 point is earned: For correct substitution/calculation in formula

$$gMars = \frac{GM_M}{R_M^2} = \frac{G(0.11)M_g}{(0.53R_g)^2} = \frac{0.11}{(0.53)^2}g_g$$

1 point is earned: For expressing *gM* in terms of *gg*

$$gM = \frac{0.11}{(0.53)^2}g_g$$

OR

$$gM = 0.39g_g$$

However, the third point for expressing *gM* in terms of *gg* is not awarded if no points have been awarded up to that part.

Part B

1 point is earned: For the correct formula for weight: *W* = *Mg*

1 point is earned: For the correct value of *g* for Mars, where any value of *g* signifying Mars is acceptable, such as 0.39*gE*, 3.8 m/s2, *gMars*,

frq5

$$\frac{0.11}{(0.53)^2} g_g$$

, etc.

W = (11.5 kg)(3.8 m/s2) = 44 N

(Alternate solution)

2 points are earned: For correct calculation using

$$F = \frac{GM_1M_2}{R^2}$$

, where the student has calculated the mass of Mars, the radius of Mars and uses the correct mass of Sojourner.

(A student needs to have memorized the mass of the Earth and its radius to do this.)

0	1	2	3	4 ✓

The student response earns four of the following points:

1 point is earned: For the correct formula for weight: *W* = *Mg*

1 point is earned: For the correct value of *g* for Mars, where any value of *g* signifying Mars is acceptable, such as 0.39*gE*, 3.8 m/s2, *gMars*,

$$\frac{0.11}{(0.53)^2} g_g$$

, etc.

W = (11.5 kg)(3.8 m/s2) = 44 N

(Alternate solution)

2 points are earned: For correct calculation using

$$F = \frac{GM_1M_2}{R^2}$$

, where the student has calculated the mass of Mars, the radius of Mars and uses the correct mass of Sojourner.

(A student needs to have memorized the mass of the Earth and its radius to do this.)

Part C

1 point is earned: For indication that sine or cosine is needed as a necessary factor for computation of *N*

frq5

1 point is earned: For correct formula and calculation

$N = MgM \cos \theta$

N = (11.5 kg)(3.8 m/s2) cos 20° = 41 N

OR N = (11.5 kg)(3.8 m/s2) sin 70° = 41 N

0	1	2 ✓

The student response earns two of the following points:

1 point is earned: For indication that sine or cosine is needed as a necessary factor for computation of N

1 point is earned: For correct formula and calculation

$N = MgM \cos \theta$

N = (11.5 kg)(3.8 m/s2) cos 20° = 41 N

OR N = (11.5 kg)(3.8 m/s2) sin 70° = 41 N

Part D

1 point is earned: For correct statement that $Fnet = 0$

1 point is earned: For a correct reason such as $a = 0$, OR v = constant, OR Newton's First law

0	1	2 ✓

The student response earns two of the following points:

1 point is earned: For correct statement that $Fnet = 0$

1 point is earned: For a correct reason such as $a = 0$, OR v = constant, OR Newton's First law

Part E

1 point is earned: For a correct formula for power

$$P = \frac{W}{t}$$

1 point is earned: For a correct formula for distance

$d = vt$

frq5

1 point is earned: For a correct calculation of distance

$$d = \frac{vW}{P} = \frac{(6.7x10^{-3}m/s)(5.4x10^{5}J)}{10W}$$

d = 362 m

(Alternate solution)

1 point is earned: For correct formulas for work and power

W = *Fd* and *P* = *Fv*

1 point is earned: For combining these to yield a correct formula for the distance

$$d = \frac{vW}{P}$$

1 point is earned: For a correct calculation

$$d = \frac{(6.7x10^{-3}m/s)(5.4x10^{4}J)}{10W} = 362m$$

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned: For a correct formula for power

$$P = \frac{W}{t}$$

1 point is earned: For a correct formula for distance

d = *vt*

1 point is earned: For a correct calculation of distance

$$d = \frac{vW}{P} = \frac{(6.7x10^{-3}m/s)(5.4x10^{5}J)}{10W}$$

d = 362 m

(Alternate solution)

1 point is earned: For correct formulas for work and power

W = *Fd* and *P* = *Fv*

frq5

1 point is earned: For combining these to yield a correct formula for the distance

$$d = \frac{vW}{P}$$

1 point is earned: For a correct calculation

$$d = \frac{(6.7x10^{-3}m/s)(5.4x10^{4}J)}{10W} = 362m$$

Part F

1 point is earned: For correct formula *P* = *Fv*, so

1 point is earned: For correct substitution using the correct percentage of 0.01%

$$F = \frac{(0.0001)(10W)}{6.7x10^{-3}m/s}$$

1 point is earned: For correct calculation of force

F = 0.15 N (0.149 N and 0.1 N were acceptable)

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned: For correct formula *P* = *Fv*, so

1 point is earned: For correct substitution using the correct percentage of 0.01%

$$F = \frac{(0.0001)(10W)}{6.7x10^{-3}m/s}$$

1 point is earned: For correct calculation of force

F = 0.15 N (0.149 N and 0.1 N were acceptable)

frq5

27. This question is a short free-response question. Show your work for each part of the question.

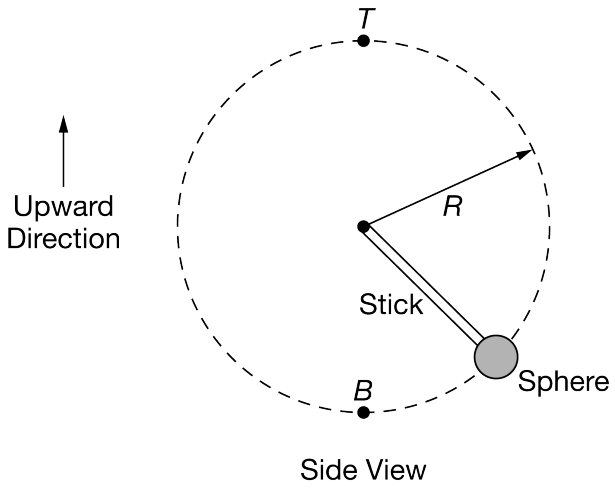


Figure 1

A sphere of mass M is attached to one end of a rigid stick of negligible mass. The other end of the stick is fixed in place so that the sphere can move in a circular path of radius R, as shown in Figure 1.

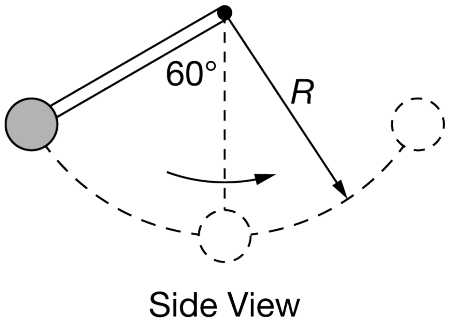


Figure 2

(a) The sphere is released from rest with the stick at an angle of 60 degrees from the vertical, as shown in Figure 2. Derive an expression for the speed of the sphere when it is at point B, the bottommost point of the circular path.

frq5

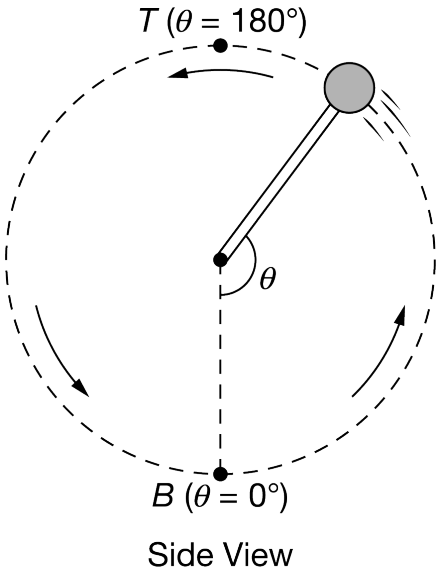
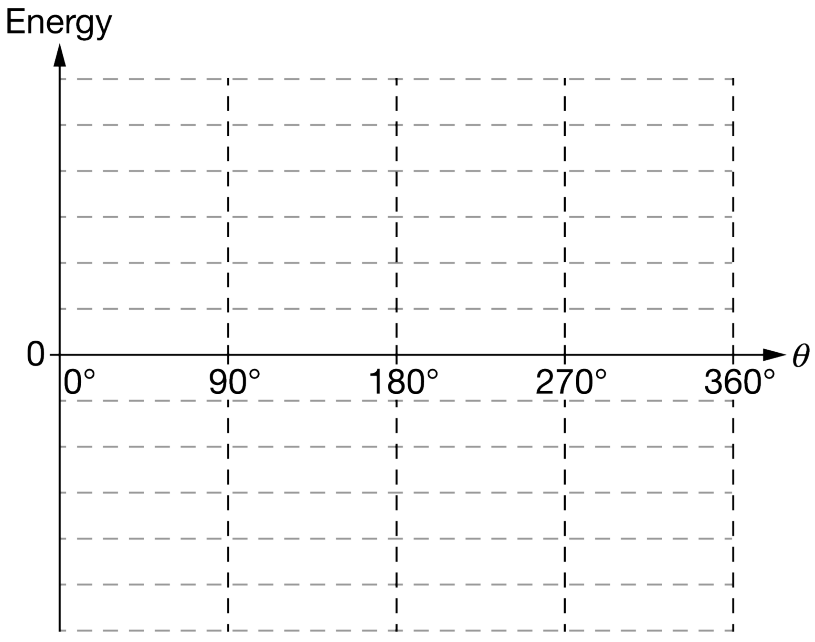


Figure 3

(b) An external force is exerted on the sphere to speed it up. After the force is removed, the sphere has enough speed to move in a complete circle, as shown in Figure 3. The angle θ is equal to zero when the sphere is at point B and is 180 degrees when the sphere is at point T, the topmost point of the circular path.

i. On the axes shown, sketch a possible graph of the potential energy of the sphere-Earth system as a function of θ. Label the graph "U."

ii. On the same axes, sketch a possible graph of the kinetic energy of the sphere as a function of the θ. Label this graph "K."



frq5

iii. When the sphere makes a complete revolution around its circular path, does it spend more time in the top half of the circle, more time in the bottom half of the circle, or the same amount of time in both halves of the circle?

Briefly state your reasoning.

Part (a)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

Student response accurately includes all of the following criteria.

- ☐ 1 point is earned for writing an equation expressing the conservation of energy.
- ☐ 1 point is earned for using the correct difference in height of $\frac{R}{2}$.
- ☐ 1 point is earned for solving the equation for the speed of the sphere at the lowest point and an answer consistent with the height difference used in the derivation.

Example Response:

$E_0 = E_f$
$U_{g0} = K_f$
$mgy_0 = \frac{1}{2}mv_f^2$
$Mg(R - R\cos\theta) = \frac{1}{2}Mv_f^2$

$R - R\cos\theta = R - \frac{R}{2} = \frac{R}{2}$

$\frac{MgR}{2} = \frac{1}{2}Mv_f^2$
$MgR = Mv_f^2$
$v_f = \sqrt{gR}$

Part (b)i

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes two of the following criteria.

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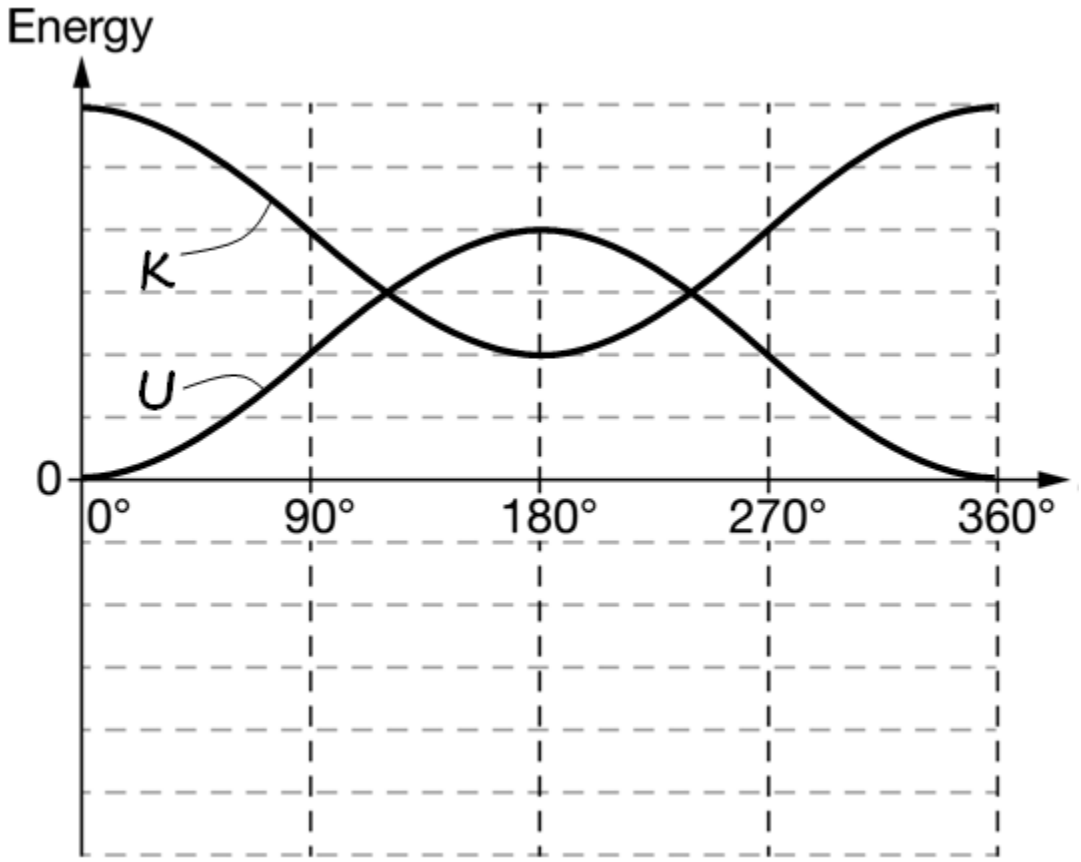
- [ ] [ ] 1 point is earned for a minima at 0° and 360° and a maximum at 180°.
- [ ] [ ] 1 point is earned for a graph proportional to a negative cosine function, which may or may not have a constant offset.

Example Response:



Part (b)ii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note:

The kinetic energy must be nonnegative for all angles to earn this point.

A kinetic energy graph with a minimum of zero can still earn this point, though in this case the sphere could not actually complete a full circle in a finite amount of time.

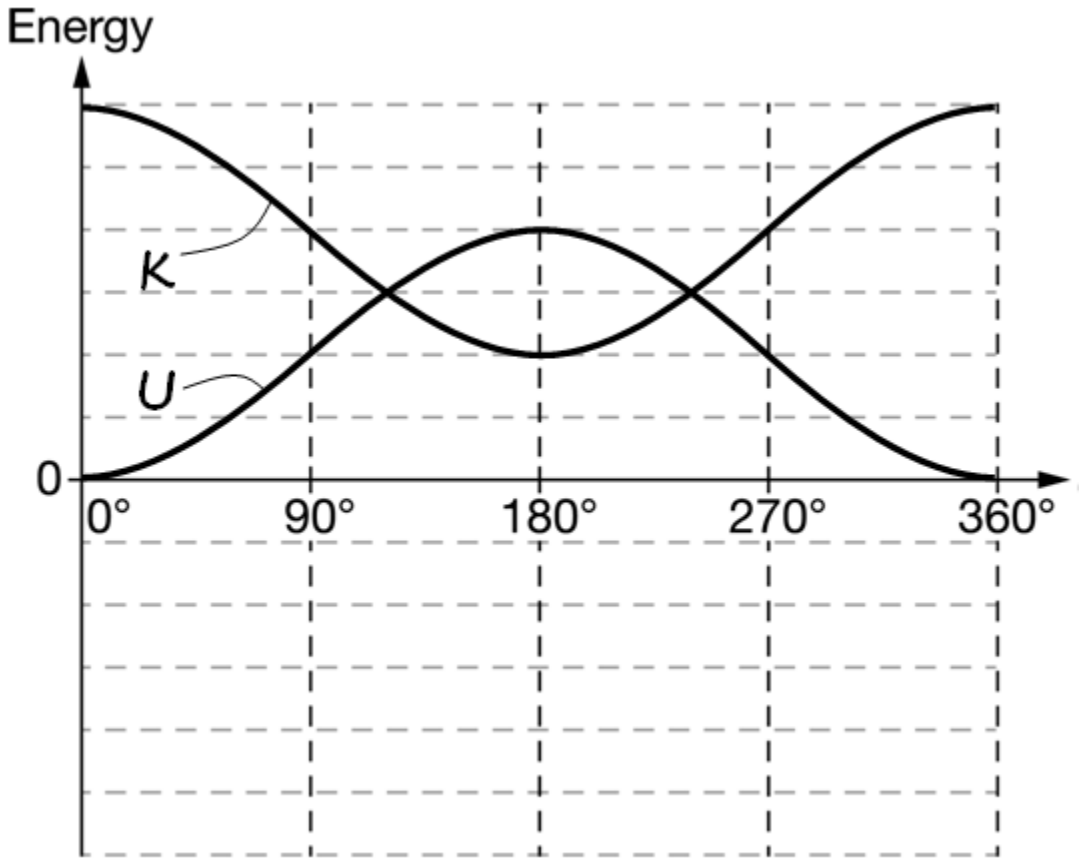
0	1 ✓

frq5

Student response accurately includes the following criteria.

- [ ] 1 point is earned for either a correct graph or a graph consistent with the graph in (a)(i) (i.e., the sum of the two graphs is a constant).

Example Response:



Note:

The kinetic energy must be nonnegative for all angles to earn this point.

A kinetic energy graph with a minimum of zero can still earn this point, though in this case the sphere could not actually complete a full circle in a finite amount of time.

Part (b)iii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: “Top half”

Note: If a wrong answer is given, the explanation is still graded for consistency with the graph in part (b)(ii)

frq5

0	1 ✓

Student response accurately includes the following criteria.

Correct answer: “Top half”

Note: If a wrong answer is given, the explanation is still graded for consistency with the graph in part (b)(ii)

- ☐ 1 point is earned for a correct explanation that mentions or implies that the sphere moves more slowly in the top half of the circular path.
- ☐ 1 point is earned for an explanation that mentions or implies that the sphere moves more slowly in the portion where the kinetic energy is least, as implied by the graph drawn in part (b)(ii).

Example Response:

The sphere spends more time in the top half of the circle because it is moving slower in the top half of the circle, and so it takes more time to cover the same distance.

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28. This question is a short free-response question. Show your work for each part of the question.

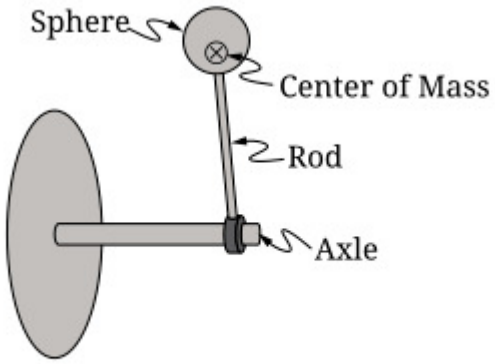

Figure 1

(7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a ⊗, as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video. Note: these frames are not equally spaced apart in time.

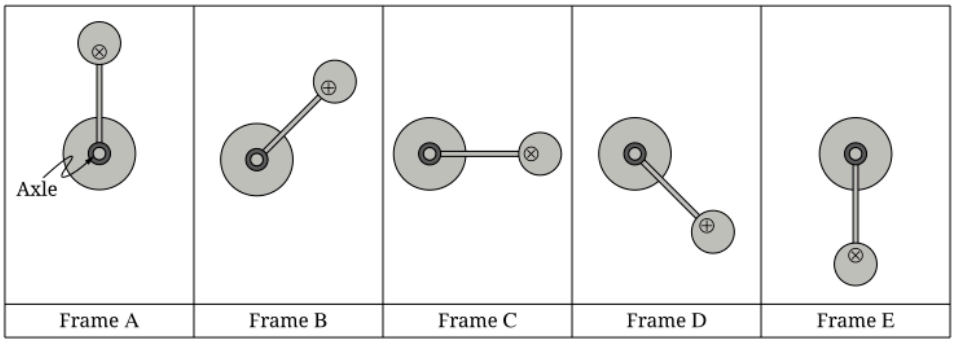

Figure 2

(a) Use the frames of the video shown in Figure 2 to answer the following questions.

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your

frq5

answer.

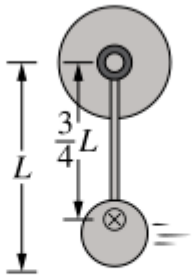

Figure 3

(b) The rod-sphere system has mass M and length L, and the center of mass is located a distance $\frac{3}{4}L$ from the axle, shown in Figure 3.

i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of M, L, and fundamental constants, as appropriate.

ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** the following criteria.

- ☐ 1 point is earned for indicating "Frame C" with correct reasoning about the magnitude of the torque being the greatest.
  Accept **one** of the following:
  - This is the instant when the lever arm is greatest.
  - This is when the angle between radius vector and weight force vector is most perpendicular.
- ☐ 1 point is earned for correctly relating torque and angular acceleration: $\alpha \propto \tau$.

frq5

- [ ] 1 point is earned for indicating "Frame E" with correct reasoning.
  Accept **one** of the following:
  - Work or energy (*e.g.*, this is when the maximum work has been done on the system by gravity).
  - Angular momentum (*e.g.*, the torque due to gravity is clockwise the entire time, causing the rod to gain angular momentum).
  - Kinematics (*e.g.*, the rod speeds up the entire time).

Example Response

The angular acceleration is greatest in Frame C because angular acceleration is proportional to torque, and in Frame C the gravitational force vector is directed perpendicular to the rod (lever arm) which means this is where the torque will be the greatest.

The rotational kinetic energy is greatest in Frame E because this is where the rod-sphere system has the greatest rotational speed since the torque has been in the same direction as the motion the entire time.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** the following criteria.

- [ ] 1 point is earned for a multistep derivation that begins with conservation of energy.
- [ ] 1 point is earned for indicating the change in height is equal to $\frac{3}{2}L$.
- [ ] 1 point is earned for an answer consistent with the height change indicated previously in the response.
  Scoring Note: A correct answer of $K_f = \frac{3}{2}MgL$ with no supporting work can earn only this point.
- [ ] 1 point is earned for indicating that the gravitational force is the external force that does work on the rod-sphere system.

Example Response

$E_i = E_f$
$U_{gi} + K_i = U_{gf} + K_f$
$\Delta K = U_{gi} - U_{gf}$
$\Delta K = Mg\Delta y$
$\Delta y = \frac{3L}{4} + \frac{3L}{4} = \frac{3}{2}L$
$\Delta K = \frac{3}{2}MgL$

The rod and sphere gain kinetic energy due to the positive work done by the gravitational force, which is an external force

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for the rod-sphere system.

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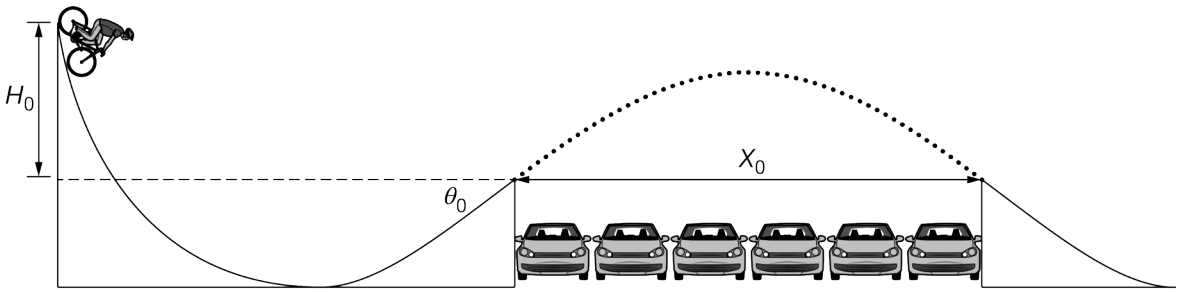
29. This question is a short free-response question. Show your work for each part of the question.



Note: Figure not drawn to scale.

(7 points, suggested time 13 minutes)

A stunt cyclist builds a ramp that will allow the cyclist to coast down the ramp and jump over several parked cars, as shown above. To test the ramp, the cyclist starts from rest at the top of the ramp, then leaves the ramp, jumps over six cars, and lands on a second ramp.

H_0 is the vertical distance between the top of the first ramp and the launch point.
θ_0 is the angle of the ramp at the launch point from the horizontal.
X_0 is the horizontal distance traveled while the cyclist and bicycle are in the air.
m_0 is the combined mass of the stunt cyclist and bicycle.

(a) Derive an expression for the distance X_0 in terms of H_0, θ_0, m_0, and physical constants, as appropriate.

(b) If the vertical distance between the top of the first ramp and the launch point were $2H_0$ instead of H_0, with no other changes to the first ramp, what is the maximum number of cars that the stunt cyclist could jump over?

Justify your answer, using the expression you derived in part (a).

(c) On the axes below, sketch a graph of the vertical component of the stunt cyclist's velocity as a function of time from immediately after the cyclist leaves the ramp to immediately before the cyclist lands on the second ramp. On the vertical axis, clearly indicate the initial and final vertical velocity components in terms of H_0, θ_0, m_0, and physical constants, as appropriate. Take the positive direction to be upward.

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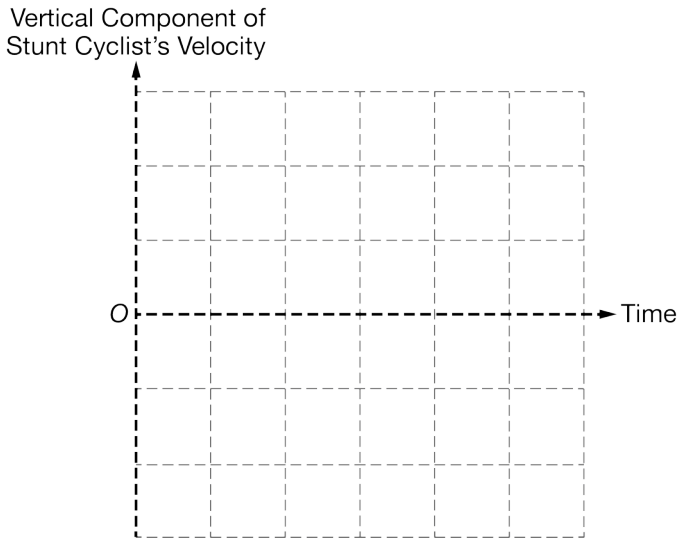

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for using conservation of energy to find the speed v of the bicycle as it leaves the ramp.
- ☐ 1 point is earned for using kinematics, vertical components, attempting to find the time the bicycle is in the air.
- ☐ 1 point is earned for a correct expression for X_0 in terms of given quantities.

Example Response:

$E_{top} = E_{bottom}$
$m_0 g H_0 = \frac{1}{2} m_0 v^2$
$v = \sqrt{2gH_0}$
$v_{fy} = v_{iy} + at$
$-v \sin\theta = v \sin\theta - gt$
$-2v \sin\theta = -gt$

$2 \sin\theta_0 \sqrt{2gH_0} = gt$
$t = \frac{2 \sin\theta_0 \sqrt{2gH_0}}{g}$
$X_0 = v_x t$
$X_0 = \cos\theta_0 \sqrt{2gH_0} \frac{2 \sin\theta_0 \sqrt{2gH_0}}{g}$
$X_0 = 4H_0 \cos\theta_0 \sin\theta_0$

Scoring Note:

Using the range equation to get $X_0 = 2H_0 \sin 2\theta_0$ is sufficient to earn the second and third points.

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Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an answer and justification that attempts to use the functional dependence of the horizontal distance on the initial height.
- ☐ 1 point is earned for an answer consistent with the expression derived in part (a).

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a linear graph with a constant negative slope.
- ☐ 1 point is earned for a graph that starts at v_y and ends at $-v_y$, using only allowed variables.

Example Response:

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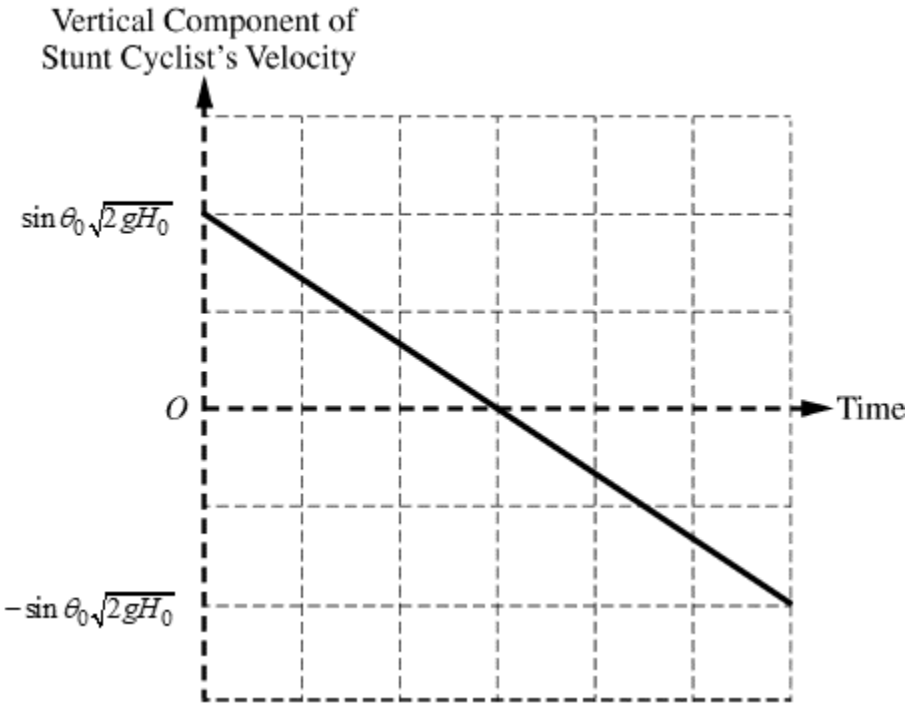
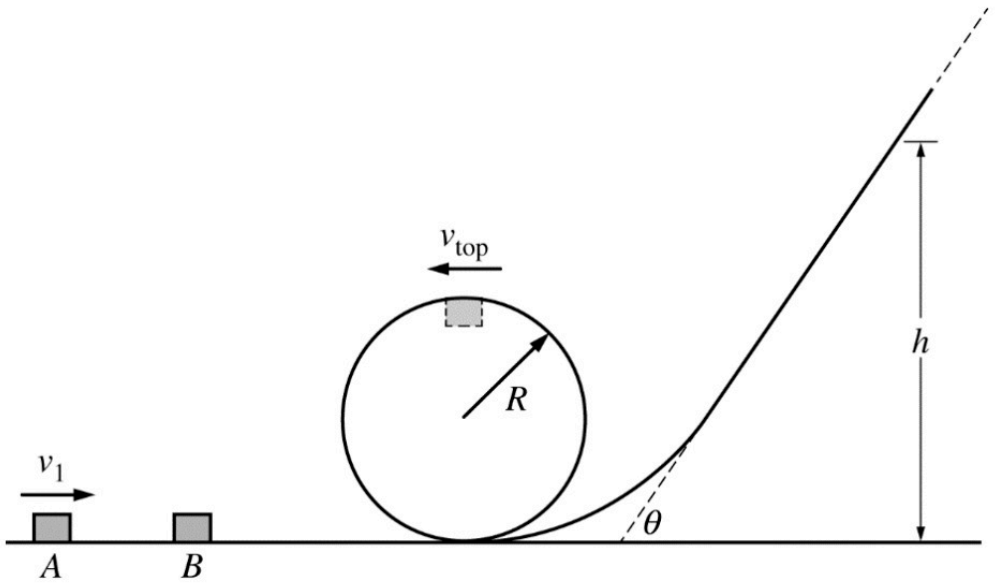

Vertical Component of
Stunt Cyclist's Velocity
$\sin\theta_0\sqrt{2gH_0}$
O
Time
$-\sin\theta_0\sqrt{2gH_0}$

frq5

30.



Note: Figure not drawn to scale.

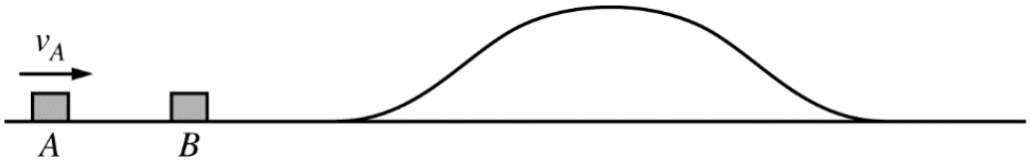
Two blocks A and B of equal mass m are on a frictionless track, as shown in the figure above. Block A, initially moving with speed v_1, has a perfectly elastic collision with block B. Block B has a speed v_1 immediately after the collision, and then it travels around a circular loop of radius R, where R is much larger than the size of the blocks. The speed of block B at the top of the loop is v_{top}. Block B then slides up a ramp until it comes momentarily to rest at a height h above the floor.

a. Derive an equation for the height *h* in terms of *m*, *R, v_{top}.* and *g,* as appropriate.

b. Suppose that the speed v_1 of block A is doubled. Will the final height of block *B* be greater than, less than, or equal to *2h*?

_____Greater than *2h* _____Less than *2h* _____ Equal to *2h*

Briefly explain how you arrived at your answer.



c. Blocks A and B (both of mass m) are moved to a different frictionless track, as shown above. Let v_A be the minimum initial speed for block A that allows block B to make it over the hump in the track. Block A is then replaced with a larger block, block C, which has mass 2m, as shown below.

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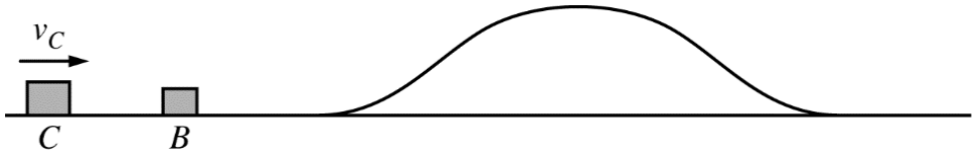

Block C makes a completely inelastic collision with block B so that both blocks stick together and travel along the track. What is the minimum initial speed v_C that block C must have so that the two-block system makes it over the hump? Express your answer in terms of v_A.

Part A

1 point is earned: For using an equation expressing the conservation of energy

$mgh = 2mgR + 1/2mv_{top}^2$ or $mgh = (1/2)mv_1^2$ or $mgh = (1/2)mv^2$

1 point is earned: For using two forms of energy when block B is at the top of the loop or using the initial kinetic energy

1 point is earned: For expressing the correct answer in terms of the listed quantities

$$h = \frac{2mgR + \frac{1}{2}mv_{top}^2}{mg}$$

$$h = 2R + \frac{v_{top}^2}{2g}$$

✓

0	1	2	3

Student response earns 3 of the following 3 point(s)

1 point is earned: For using an equation expressing the conservation of energy

$mgh = 2mgR + 1/2mv_{top}^2$ or $mgh = (1/2)mv_1^2$ or $mgh = (1/2)mv^2$

1 point is earned: For using two forms of energy when block B is at the top of the loop or using the initial kinetic energy

1 point is earned: For expressing the correct answer in terms of the listed quantities

frq5

$$h = \frac{2mgR + \frac{1}{2}mv_{top}^2}{mg}$$

$$h = 2R + \frac{v_{top}^2}{2g}$$

Part B

1 point is earned: For a correct answer “Greater than 2h ” with an explanation

No point is earned for a correct selection without an explanation. If an incorrect answer is selected, the following point for using conservation of energy can still be earned.

1 point is earned: For using an equation or semi-quantitative reasoning expressing the conservation of energy Note: “semi-quantitative reasoning” here includes reasoning in terms of proportionality, squared relationships, etc.

mgh = 1/2 mv_1^2

h = v_1^2/2g

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

1 point is earned: For a correct answer “Greater than 2h ” with an explanation

No point is earned for a correct selection without an explanation. If an incorrect answer is selected, the following point for using conservation of energy can still be earned.

1 point is earned: For using an equation or semi-quantitative reasoning expressing the conservation of energy Note: “semi-quantitative reasoning” here includes reasoning in terms of proportionality, squared relationships, etc.

mgh = 1/2 mv_1^2

h = v_1^2/2g

Part C

Correct answer: $V_C = 3/2V_A$

Note: there is no “answer point”, so no credit is awarded for an answer with no work shown

1 point is earned: For using momentum conservation to relate the speeds before and after the completely inelastic collision between blocks B and C

frq5

$2mv_C = (2m + m)V_{BC} = 3\ mv_{BC}$, where v_{BC} is the speed of blocks B and C after the collision

$2v_C = 2v_{BC}$

1 point is earned: For setting the speed of blocks B and C after the collision equal to the speed of block B after its elastic collision with block A, in order for blocks B and C to make it over the hump.

$2v_C = 3v_A$

$v_C = 3/2v_A$

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

Correct answer: $V_C = 3/2V_A$

Note: there is no “answer point”, so no credit is awarded for an answer with no work shown

1 point is earned: For using momentum conservation to relate the speeds before and after the completely inelastic collision between blocks B and C

$2mv_C = (2m + m)V_{BC} = 3\ mv_{BC}$, where v_{BC} is the speed of blocks B and C after the collision

$2v_C = 2v_{BC}$

1 point is earned: For setting the speed of blocks B and C after the collision equal to the speed of block B after its elastic collision with block A, in order for blocks B and C to make it over the hump.

$2v_C = 3v_A$

$v_C = 3/2v_A$

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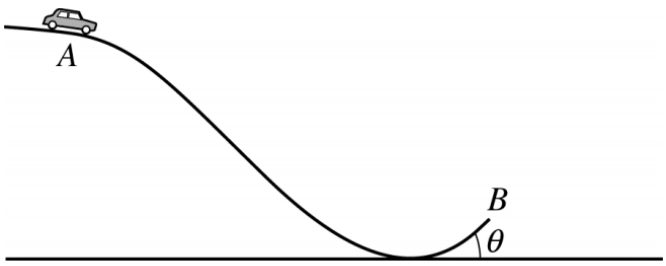
31.



A toy car coasts along the curved track shown above. The car has initial speed v_A when it is at point *A* at the top of the track, and the car leaves the track at point *B* with speed v_B at an angle θ above the horizontal. Assume that the rotational kinetic energy of the car's wheels and energy losses due to friction are both negligible.

a. Suppose the toy car is released from rest at point *A* ($v_A = 0$).

   i. After the car leaves the track and reaches the highest point in its trajectory it will be at a different height than it was at point *A*. Briefly explain why this is so.

   ii. Determine the speed of the car when it is at the highest point in its trajectory after leaving the track, in terms of v_B and θ. Briefly explain how you arrived at your answer.

b. Suppose the toy car is given an initial push so that it has nonzero speed at point *A*. Determine the speed Av of the car at point *A* such that the highest point in its trajectory after leaving the track is the same as its height at point *A*. Express your answer in terms of v_B and θ. Explain how you arrived at your answer.

Part A

Part(i)

1 point is earned: For indicating that the mechanical energy of the car-Earth system is constant between point A and the highest point in the car's trajectory

1 point is earned: For indicating that the car is still moving at its highest point and has some kinetic energy so the car-Earth system must have less gravitational potential energy, therefore must be at a lower height than it was at point A

Part(ii)

1 point is earned: For indicating that the velocity of the car at its highest point is equal to the horizontal speed at point B

1 point is earned: For stating that the speed at the highest point is cos v_B cosθ



0	1	2	3	4

Student response earns 4 of the following 4 point(s)

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Part(i)

1 point is earned: For indicating that the mechanical energy of the car-Earth system is constant between point A and the highest point in the car's trajectory

1 point is earned: For indicating that the car is still moving at its highest point and has some kinetic energy so the car-Earth system must have less gravitational potential energy, therefore must be at a lower height than it was at point A

Part(ii)

1 point is earned: For indicating that the velocity of the car at its highest point is equal to the horizontal speed at point B

1 point is earned: For stating that the speed at the highest point is cos v_B cosθ

Part B

1 point is earned: For implicitly or explicitly applying conservation of energy

1 point is earned: For implicitly or explicitly indicating that the gravitational potential energy is the same at the beginning and at the end

1 point is earned: For indicating that the speed is the same at the beginning and at the end, so $v_A = v_B \cos \theta$, or consistent with the answer in part (a-ii)

0	1	2	3 ✓

Student response earns 3 of the following 3 point(s)

1 point is earned: For implicitly or explicitly applying conservation of energy

1 point is earned: For implicitly or explicitly indicating that the gravitational potential energy is the same at the beginning and at the end

1 point is earned: For indicating that the speed is the same at the beginning and at the end, so $v_A = v_B \cos \theta$, or consistent with the answer in part (a-ii)

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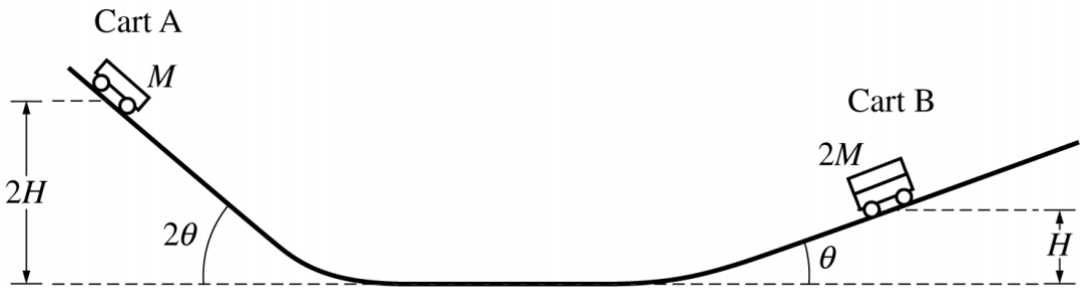
32.



Cart *A* has mass *M* and is released from rest at a height *2H* on a ramp making an angle 2θ with the horizontal, as shown above. Cart *B* has mass 2*M* and is released from rest at a height H on a ramp making an angle θ with the horizontal. The carts roll toward each other, have a head-on collision on the horizontal portion of the ramp, and stick together. The masses of the carts' wheels are negligible, as are any frictional or drag forces.

a. Indicate whether the carts remain at rest, move to the left, or move to the right after the collision.

____ Remain at rest ____ Move to the left ____ Move to the right

Explain how you arrived at your answer.

b. Consider the time interval from when the two carts are released until just after they collide.

i. For the system consisting of the two carts and Earth, indicate whether the total mechanical energy increases, decreases, or remains the same.

____ Increases ____ Decreases ____ Remains the same

Justify your answer.

ii. For the system consisting of only the two carts, indicate whether the total mechanical energy increases, decreases, or remains the same.

____ Increases ____ Decreases ____ Remains the same

Justify your answer.

Part A

1 point is earned: For any application of conservation of energy

1 point is earned: For correctly applying conservation of energy for each cart

1 point is earned: For any application of conservation of momentum

Example 1: The contributions of each cart to the potential energy of the carts-Earth system when the carts are at their points of release are the same, because one has twice the height and one has twice the mass of the other. All this potential energy is converted into kinetic energy when the carts reach the bottom of the ramp, so the

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carts have the same kinetic energy there. This means that the speed of cart A is 1.4 times the speed of Cart B. Before the collision, cart A has a momentum of about 1.4Mv to the right; cart B has a momentum of 2Mv to the left. The net momentum is to the left, so after the collision the carts move left

Example 2:

$$Mg(2H) = Mv_A^2/2; \quad v_A^2 = 4gH; \quad v_A = 2\sqrt{gH}$$
$$2Mg(H) = (2M)v_B^2/2; \quad v_B^2 = 2gH; \quad v_B = \sqrt{2gH}$$
$$M(2\sqrt{gH}) - 2M(\sqrt{2gH}) = (M+2M)v_F = 3Mv_F$$
$$v_F = \left(\tfrac{2-2\sqrt{2}}{3}\right)\sqrt{gH} = -\tfrac{0.83}{3}\sqrt{gH} = -0.28\sqrt{gH}$$

One earned point is deducted for any incorrect use of ramp angles.

0	1	2	3 ✓

Student response earns 3 of the following 3 point(s)

1 point is earned: For any application of conservation of energy

1 point is earned: For correctly applying conservation of energy for each cart

1 point is earned: For any application of conservation of momentum

Example 1: The contributions of each cart to the potential energy of the carts-Earth system when the carts are at their points of release are the same, because one has twice the height and one has twice the mass of the other. All this potential energy is converted into kinetic energy when the carts reach the bottom of the ramp, so the carts have the same kinetic energy there. This means that the speed of cart A is 1.4 times the speed of Cart B. Before the collision, cart A has a momentum of about 1.4Mv to the right; cart B has a momentum of 2Mv to the left. The net momentum is to the left, so after the collision the carts move left

Example 2:

$$Mg(2H) = Mv_A^2/2; \quad v_A^2 = 4gH; \quad v_A = 2\sqrt{gH}$$
$$2Mg(H) = (2M)v_B^2/2; \quad v_B^2 = 2gH; \quad v_B = \sqrt{2gH}$$
$$M(2\sqrt{gH}) - 2M(\sqrt{2gH}) = (M+2M)v_F = 3Mv_F$$
$$v_F = \left(\tfrac{2-2\sqrt{2}}{3}\right)\sqrt{gH} = -\tfrac{0.83}{3}\sqrt{gH} = -0.28\sqrt{gH}$$

One earned point is deducted for any incorrect use of ramp angles.

Part B

Part(i)

1 point is earned: For indicating that no energy is gained as the carts move down the ramps, or using the result of the

frq5

reasoning in part (a) regarding the energy for this part of the motion

1 point is earned: For indicating that energy is dissipated in the collision and a comparison of initial and final energies consistent with the students' assumptions about energy before the collision

Part(ii)

1 point is earned: For indicating that the initial mechanical energy of the carts is zero

1 point is earned: For a comparison with the energy after the collision consistent with previous responses

Example: When the carts are released, they have no mechanical energy, since they are not moving. After the collision, the carts have kinetic energy. The total mechanical energy of the cart-cart system increases.

Alternate solution

1 point is earned: For identifying gravitational force as the only external force doing work

1 point is earned: For correct use of the work-kinetic energy theorem

> Example: For the system of just the two carts, there is no initial mechanical energy, since the carts are at rest. The gravitational force exerted on the carts by Earth is the only external force doing work, and the work on the system is positive. So the kinetic energy of the system increases, which increases the total mechanical energy

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Part(i)

1 point is earned: For indicating that no energy is gained as the carts move down the ramps, or using the result of the reasoning in part (a) regarding the energy for this part of the motion

1 point is earned: For indicating that energy is dissipated in the collision and a comparison of initial and final energies consistent with the students' assumptions about energy before the collision

Part(ii)

1 point is earned: For indicating that the initial mechanical energy of the carts is zero

1 point is earned: For a comparison with the energy after the collision consistent with previous responses

Example: When the carts are released, they have no mechanical energy, since they are not moving. After the collision, the carts have kinetic energy. The total mechanical energy of the cart-cart system increases.

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Example: For the system of just the two carts, there is no initial mechanical energy, since the carts are at rest. The gravitational force exerted on the carts by Earth is the only external force doing work, and the work on the system is positive. So the kinetic energy of the system increases, which increases the total mechanical energy