frq7

1. A 20 kg box on a horizontal frictionless surface is moving to the right at a speed of 4.0 m/s. The box hits and remains attached to one end of a spring of negligible mass whose other end is attached to a wall. As a result, the spring compresses a maximum distance of 0.50 m, and the box then oscillates back and forth.

(a)

i. The spring does work on the box from the moment the box first hits the spring to the moment the spring first reaches its maximum compression. Indicate whether the work done by the spring is positive, negative, or zero.

____ Positive ____ Negative ____ Zero

Justify your answer.

ii. Calculate the magnitude of the work described in part i.

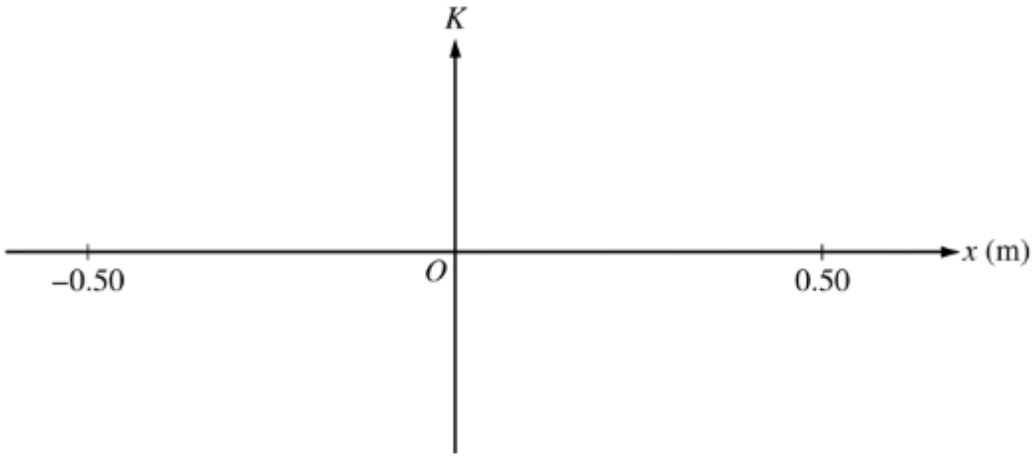
(b) Calculate the spring constant of the spring.

(c) Calculate the magnitude of the maximum acceleration of the box.

(d) Calculate the frequency of the oscillation of the box.

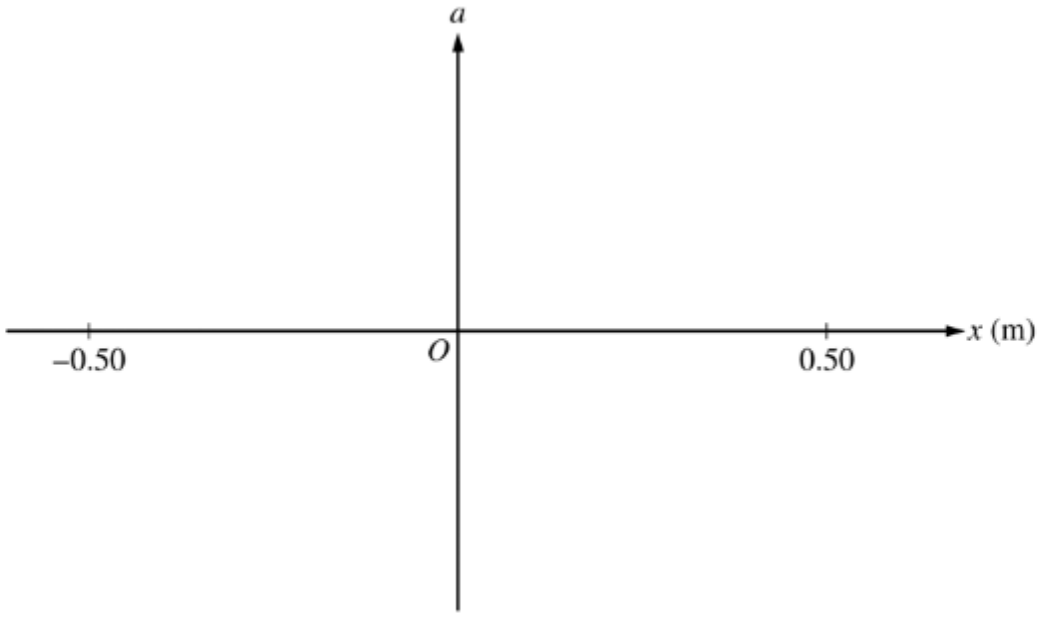
(e) Let x = 0 be the point where the box makes contact with the spring, with positive x directed toward the right.

i. On the axes below, sketch the kinetic energy *K* of the oscillating box as a function of *position* x for the range x = -0.50 m to x = +0.50 m.



ii. On the axes below, sketch the acceleration a of the oscillating box as a function of *position* x for the range x = -0.50 m to x = +0.50 m.

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Part A

1 point is earned: For selecting "Negative"

1 point is earned: For a correct justification

Examples:

- The spring pushes on the box in a direction opposite to the box's motion, therefore the work done is negative.
- The box slows down, so its kinetic energy decreases. Therefore the work done is negative.

1 point is earned: For a correct expression relating to work to change in kinetic energy

$W = \Delta K$
$W = \frac{1}{2}m\left(v_2^2 - v_1^2\right) = -\frac{1}{2}mv_1^2$

1 point is earned: For substituting the correct values

$W = -\left(\frac{1}{2}\right)(20kg)(4.0m/s)^2$

1 point is earned: For a correct answer

W = 160 J (either positive or negative value acceptable)

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0	1	2	3	4	5 ✓

The student response earns all of the following points:

1 point is earned: For selecting "Negative"

1 point is earned: For a correct justification

Examples:

- The spring pushes on the box in a direction opposite to the box's motion, therefore the work done is negative.
- The box slows down, so its kinetic energy decreases. Therefore the work done is negative.

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1 point is earned: For substituting the correct values

$W = -\left(\frac{1}{2}\right)(20kg)(4.0m/s)^2$

1 point is earned: For a correct answer

W = 160 J (either positive or negative value acceptable)

Part B

1 point is earned: For a correct expression of conservation of energy

$K_1 = U_2$
$\frac{1}{2}mv_1^2 = \frac{1}{2}kx_2^2$

1 point is earned: For correct substitution of values consistent with the answer from part (a) ii

$(160J) = \left(\frac{1}{2}\right)k(0.50m)^2$
$k = 1280N/m$

0	1	2 ✓

The student response earns all of the following points:

frq7

1 point is earned: For a correct expression of conservation of energy

$K_1 = U_2$
$\frac{1}{2}mv_1^2 = \frac{1}{2}kx_2^2$

1 point is earned: For correct substitution of values consistent with the answer from part (a) ii

$(160J) = \left(\frac{1}{2}\right)k(0.50m)^2$
$k = 1280N/m$

Part C

1 point is earned: For a correct expression of the maximum acceleration

a = Fmax/m = kxmax/m

1 point is earned: For correct substitution consistent with the answer from part (b)

a = (1280 N/m)(0.50 m)/(20 kg)

a = 32 m/s2

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For a correct expression of the maximum acceleration

a = Fmax/m = kxmax/m

1 point is earned: For correct substitution consistent with the answer from part (b)

a = (1280 N/m)(0.50 m)/(20 kg)

a = 32 m/s2

Part D

1 point is earned: For a correct expression of the frequency

$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

1 point is earned: For correct substitution consistent with the answer from part (b)

$f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{(1280N/m)}{(20kg)}}$
$f = 1.27Hz$

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0	1	2 ✓

The student response earns all of the following points:

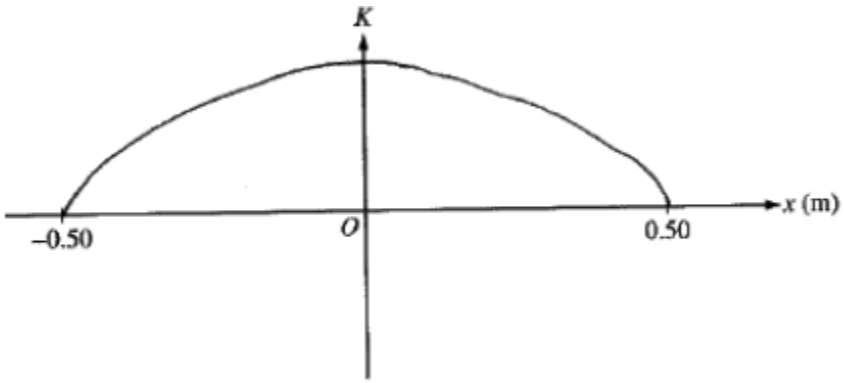
1 point is earned: For a correct expression of the frequency

$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

1 point is earned: For correct substitution consistent with the answer from part (b)

$f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{(1280N/m)}{(20kg)}}$
$f = 1.27Hz$

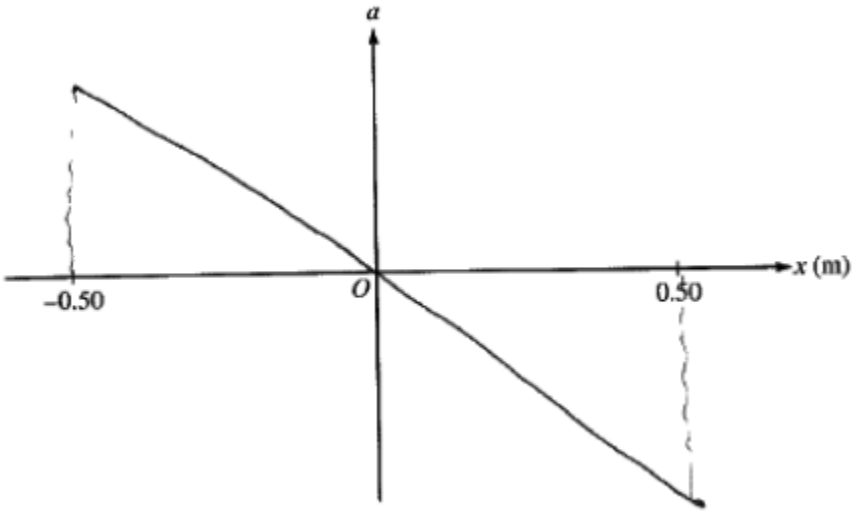
Part E



1 point is earned: For a graph that shows positive kinetic energy values and zero kinetic energy at -0.50 m and +0.50 m

1 point is earned: For a smooth curve that is concave down

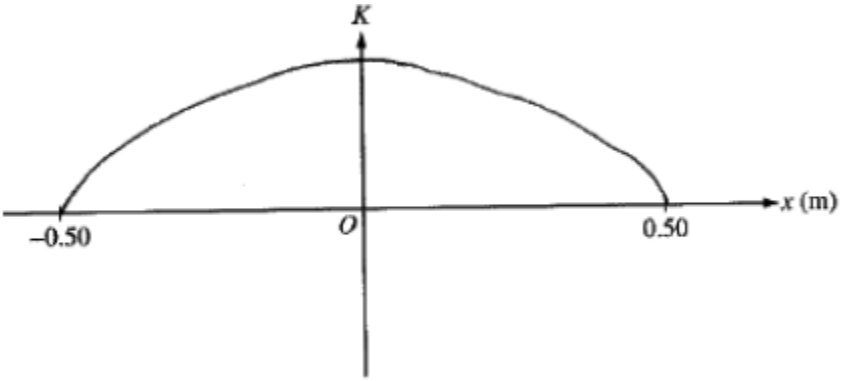
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1 point is earned: For a straight line with negative slope that begins at -0.50 m, ends at +0.50 m, and passes through the origin

0	1	2	3 ✓

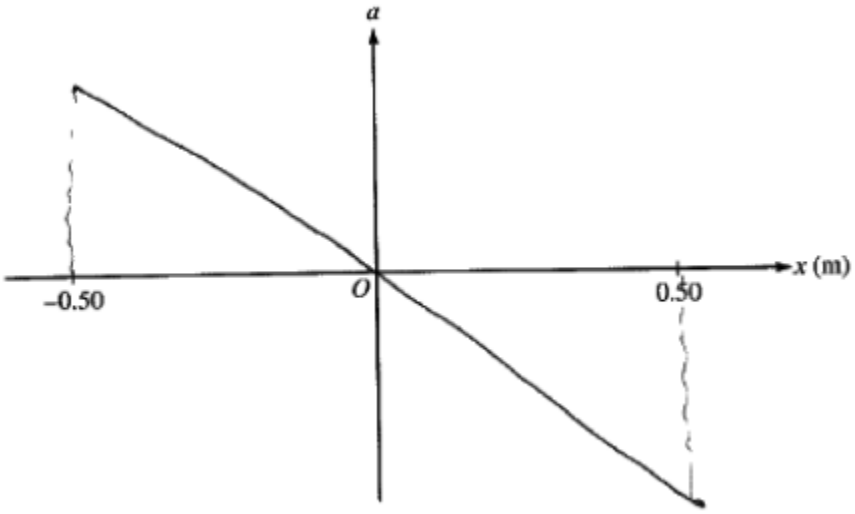
The student response earns all of the following points:



1 point is earned: For a graph that shows positive kinetic energy values and zero kinetic energy at -0.50 m and +0.50 m

1 point is earned: For a smooth curve that is concave down

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1 point is earned: For a straight line with negative slope that begins at -0.50 m, ends at +0.50 m, and passes through the origin

Units

1 point is earned: For correct units on three out of four calculated answers

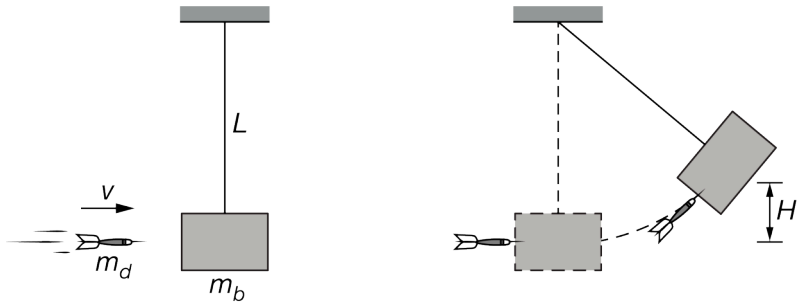
0	1 ✓

The student response earns all of the following points:

1 point is earned: For correct units on three out of four calculated answers

frq7

2. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown at left above. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown at right above. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, the dart or the block, if either, experiences an impulse of larger magnitude during the collision. If the impulse is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the speed of the dart as it embeds itself into the block is greater than *v*, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the motion of the dart-block system at the instant it reaches height *H*. The students make the following statements:

Student 1: The speed of the system at height *H* is zero. Since it is moving in circular motion and centripetal acceleration is given by $a_c = \frac{v^2}{r}$, the acceleration must also be zero.

Student 2: The acceleration of the system at height *H* can't be zero, or the system would not swing back down. The acceleration must be equal to *g* at height *H*.

(c) Which underlined phrase or phrases, if either, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if either, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if either, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases, if either, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{m_d v}{m_d + m_b}\right)^2$. Explain how this

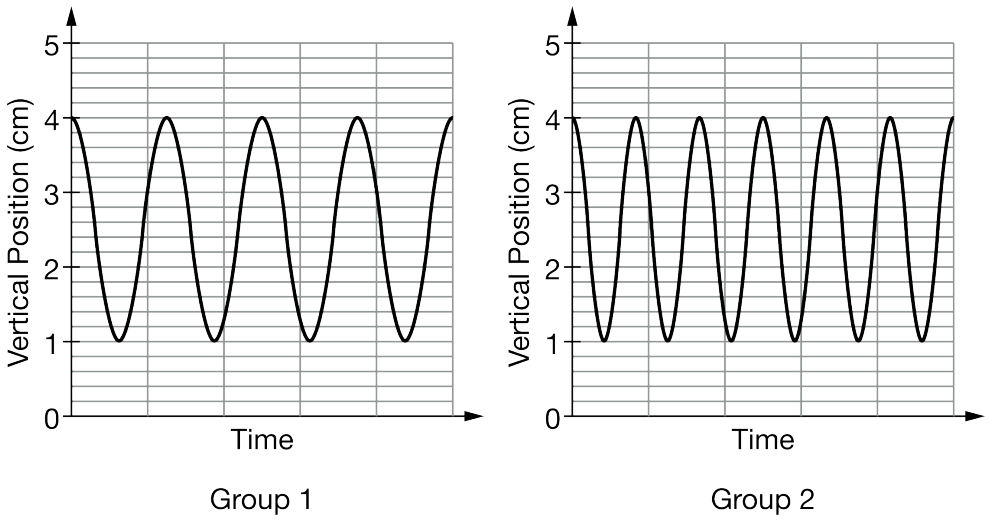
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equation supports your claim in part (b).

The students perform several experimental trials. In each trial, a different speed v is used, and the resulting maximum height reached by the center of mass of the dart-block system is measured.

(h) Which quantities could the students graph to produce a straight line whose slope can be used to find the acceleration due to gravity, g ?

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart mass, initial dart speed, block mass, or string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- 1 point is earned for a correct explanation, in terms of force & time (using Newton's third law)
  OR
  a correct explanation in terms of change in momentum (using conservation of momentum).

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Example Response:

Both the dart and the block experience the same impulse because they exert equal and opposite forces on each other because of Newton's third law, and the forces are exerted for equal times.

OR

By conservation of momentum, the decrease in the momentum of the dart must be equal and opposite to the increase in the momentum of the block, and therefore the impulse is the same in magnitude for both objects.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the final height is greater.
- ☐ 1 point is earned for a statement of conservation of momentum to indicate that the final speed of the dart-block system is greater **AND** an indication of conservation of energy leading to a greater height.

Example Response:

Because the momentum of the dart before the collision must be equal to the momentum of the dart-block system after the collision, a higher dart speed (a higher dart momentum) will lead to a higher dart-block speed after the collision (a higher dart-block momentum). Because the dart-block is traveling faster after the collision, it has more kinetic energy which means that the dart-block will swing higher.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Responses:

Student 1 is correct in saying that the speed at height H is zero

OR

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The first statement.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Responses:

Student 1 is incorrect in saying that the acceleration is zero at the height H

OR

The second statement.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Responses:

Student 2 is correct in saying the acceleration of the system at height H can't be zero or the system would not swing back down

OR

The first statement.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq7

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Responses:

Student 2 is incorrect in saying that the acceleration is g at height H

OR

The second statement.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement including functional dependence of the variables.
- ☐ 1 point is earned for correctly asserting that as the speed increases in the equation *H* also increases .

Example Response:

This equation matches my reasoning in part (b) – if the speed of the dart increases, the height of the swing increases because v is in the numerator.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for plotting a variant of *v* on one axis.

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- [ ] 1 point is earned for plotting a variant of *H* on the other axis.
- [ ] 1 point is earned for a correct power relationship between the two variables, i.e., *H* vs. *v* squared.

Example Response:

Students could graph H as a function of v squared.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

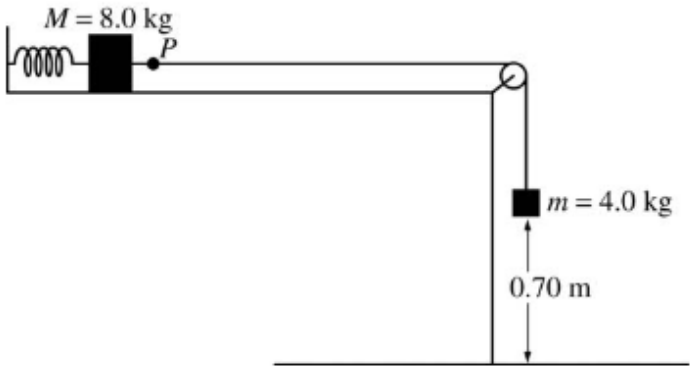
- [ ] 1 point is earned for recognizing that the period in the second graph is shorter.
- [ ] 1 point is earned for a statement about the dependence of the period on the length of the pendulum.
- [ ] 1 point is earned for indicating that the length of the pendulum must be shorter for a shorter period.

Example Response:

Since the period is shorter for group 2, and the period is related to the length, if the period became shorter with no other changes, the length of the string must have been shortened.

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3.



An ideal spring of unstretched length 0.20 m is placed horizontally on a frictionless table as shown above. One end of the spring is fixed and the other end is attached to a block of mass M = 8.0 kg. The 8.0 kg block is also attached to a massless string that passes over a small frictionless pulley. A block of mass m = 4.0 kg hangs from the other end of the string. When this spring-and-blocks system is in equilibrium, the length of the spring is 0.25 m and the 4.0 kg block is 0.70 m above the floor.

(a) On the figures below, draw free-body diagrams showing and labeling the forces on each block when the system is in equilibrium.

M = 8.0 kg

m = 4.0 kg

(b) Calculate the tension in the string.

(c) Calculate the force constant of the spring.

The string is now cut at point P.

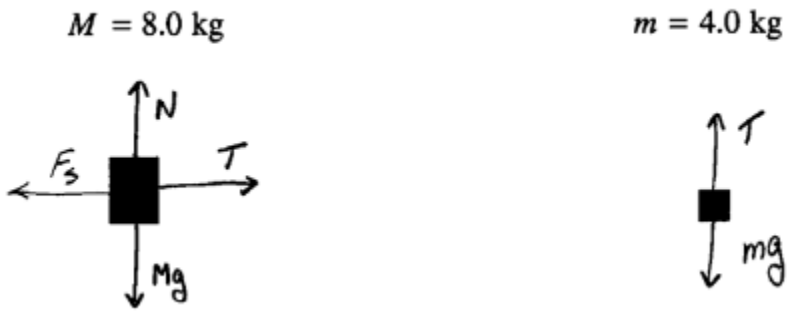
(d) Calculate the time taken by the 4.0 kg block to hit the floor.

(e) Calculate the frequency of oscillation of the 8.0 kg block.

(f) Calculate the maximum speed attained by the 8.0 kg block.

Part A

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For the 4 kg mass:

1 point is earned: For two correctly labeled vertical vectors, one up and one down, and no horizontal vectors

For the 8 kg mass:

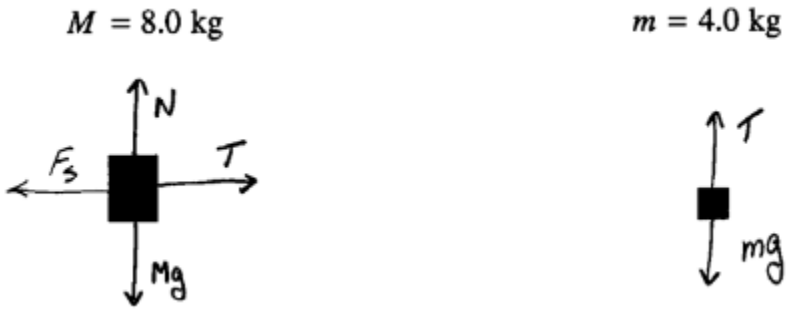
1 point is earned: For two correctly labeled vertical vectors, one up and one down

1 point is earned: For two correctly labeled horizontal vectors, one left and one right

Note: Labels could be in words, symbols, or correct numerical values. The two masses were considered independently. It was not necessary to indicate that the tension forces had the same magnitudes or that the weights were different.

0	1	2	3 ✓

The student response earns all of the following points:



For the 4 kg mass:

1 point is earned: For two correctly labeled vertical vectors, one up and one down, and no horizontal vectors

For the 8 kg mass:

1 point is earned: For two correctly labeled vertical vectors, one up and one down

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1 point is earned: For two correctly labeled horizontal vectors, one left and one right

Note: Labels could be in words, symbols, or correct numerical values. The two masses were considered independently. It was not necessary to indicate that the tension forces had the same magnitudes or that the weights were different.

Part B

1 point is earned: For a correct approach using Newton's 2nd law and the static equilibrium condition for the 4 kg mass that leads to a relationship between tension and weight

$T = mg$

T = (4.0 kg)(9.8 m/s2)

1 point is earned: For the correct answer

T = 39 N (40 N using g = 10m/s2)

0	1	2

The student response earns all of the following points:

1 point is earned: For a correct approach using Newton's 2nd law and the static equilibrium condition for the 4 kg mass that leads to a relationship between tension and weight

$T = mg$

T = (4.0 kg)(9.8 m/s2)

1 point is earned: For the correct answer

T = 39 N (40 N using g = 10m/s2)

Part C

1 point is earned: For a correct application of Newton's 2nd law and the static equilibrium condition for the 8 kg mass leading to a relationship between tension from part (b) and spring force
T = Fs = kΔx
k = T / Δx

1 point is earned: For using the correct displacement of the spring from equilibrium
Δx = 0.25m - 0.20m = 0.05m

1 point is earned: For a correct calculation leading to a positive value of k using the tension from (b)

k = 39 N/0.05 m

k = 780 N/m (800 N/m using 40 N from part (b))

frq7

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For a correct application of Newton's 2nd law and the static equilibrium condition for the 8 kg mass leading to a relationship between tension from part (b) and spring force

T = Fs = kΔx
k = T / Δx

1 point is earned: For using the correct displacement of the spring from equilibrium

Δx = 0.25m - 0.20m = 0.05m

1 point is earned: For a correct calculation leading to a positive value of k using the tension from (b)

k = 39 N/0.05 m

k = 780 N/m (800 N/m using 40 N from part (b))

Part D

1 point is earned: For a correct kinematic approach for an accelerating system applied to the 4 kg mass

$y = \frac{1}{2}gt^2$
$t = \sqrt{2y/g}$
$t = \sqrt{2(0.70m)(9.8m/s^2)}$

1 point is earned: For the correct answer

t = 0.38 s (0.37 s using *g* = 10 m/s2)

Note: An alternate approach using conservation of energy to determine the speed at the bottom and then use of a kinematic equation for time could also earn full credit.

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For a correct kinematic approach for an accelerating system applied to the 4 kg mass

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$y = \frac{1}{2}gt^2$

$t = \sqrt{2y/g}$

$t = \sqrt{2(0.70m)(9.8m/s^2)}$

1 point is earned: For the correct answer

t = 0.38 s (0.37 s using *g* = 10 m/s2)

Note: An alternate approach using conservation of energy to determine the speed at the bottom and then use of a kinematic equation for time could also earn full credit.

Part E

1 point is earned: For a correct approach to calculating the frequency (*f or* ω) of a mass-spring system

$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} OR, \omega = \sqrt{\frac{k}{m}}$

$f = \frac{1}{2\pi}\sqrt{\frac{780N/m}{8.0kg}} OR, \omega = \sqrt{\frac{780N/m}{8.0kg}}$

1 point is earned: For a correct value of frequency *(f or* ω *)* consistent with the value of *k* from part (c)

f = 1.6 Hz OR ω = 10 rad/s

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For a correct approach to calculating the frequency (*f or* ω) of a mass-spring system

$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} OR, \omega = \sqrt{\frac{k}{m}}$

$f = \frac{1}{2\pi}\sqrt{\frac{780N/m}{8.0kg}} OR, \omega = \sqrt{\frac{780N/m}{8.0kg}}$

1 point is earned: For a correct value of frequency *(f or* ω *)* consistent with the value of *k* from part (c)

f = 1.6 Hz OR ω = 10 rad/s

Part F

1 point is earned: For using conservation of energy, setting the spring potential energy equal to the kinetic energy of the block

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$\frac{1}{2}mv^2 = \frac{1}{2}kA^2$

$v = \sqrt{\frac{k}{m}}A$

$v = \sqrt{\frac{780N/m}{8.0kg}}(0.05m)$

1 point is earned: For a correct calculation of speed consistent with the value of *k* from part (c) and the correct displacement from equilibrium

v = 0.49 m/s (0.50 ms using 800 N/m from part (c))

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For using conservation of energy, setting the spring potential energy equal to the kinetic energy of the block

$\frac{1}{2}mv^2 = \frac{1}{2}kA^2$

$v = \sqrt{\frac{k}{m}}A$

$v = \sqrt{\frac{780N/m}{8.0kg}}(0.05m)$

1 point is earned: For a correct calculation of speed consistent with the value of *k* from part (c) and the correct displacement from equilibrium

v = 0.49 m/s (0.50 ms using 800 N/m from part (c))

(Global)

1 point is earned: For correct units and a reasonable number of digits in all numerical answers obtained (must have at least one final numerical answer to earn this point)

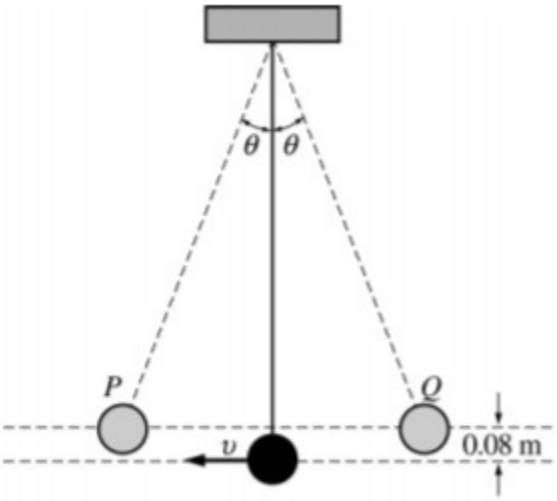
0	1 ✓

The student response earns all of the following points:

1 point is earned: For correct units and a reasonable number of digits in all numerical answers obtained (must have at least one final numerical answer to earn this point)

frq7

4. A simple pendulum consists of a bob of mass 0.085 kg attached to a string of length 1.5 m. The pendulum is raised to point Q, which is 0.08 m above its lowest position, and released so that it oscillates with small amplitude θ between the points P and Q as shown below.



Note: Figure not drawn to scale.

a. On the figures below, draw free-body diagrams showing and labeling the forces acting on the bob in each of the situations described.

i. When it is at point P



ii. When it is in motion at its lowest position

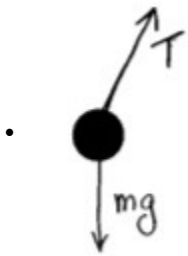


b. Calculate the speed u of the bob at its lowest position.

c. Calculate the tension in the string when the bob is passing through its lowest position.

d. Describe one modification that could be made to double the period of oscillation.

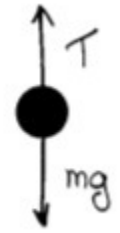
Part A



1 point is earned: For each correctly drawn and labeled force

frq7

One point was deducted if any extraneous forces were shown.

- 

1 point is earned: For each correctly drawn and labeled force

One point was deducted if any extraneous forces were shown.

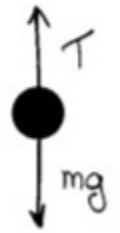
0	1	2	3	4 ✓

The student response earns four of the following points:

- 

1 point is earned: For each correctly drawn and labeled force

One point was deducted if any extraneous forces were shown.

- 

1 point is earned: For each correctly drawn and labeled force

One point was deducted if any extraneous forces were shown.

Part B

1 point is earned: For correctly applying conservation of energy

frq7

$\frac{1}{2}mv^2 = mg\Delta h$

$v = \sqrt{2g\Delta h}$

$v = \sqrt{2(9.8\ \text{m/s}^2)(0.08\ \text{m})} = \sqrt{1.568}\ \text{m/s}$

1 point is earned: For the correct answer

v = 1.3 m/s

0	1	2 ✓

The student response earns two of the following points:

1 point is earned: For correctly applying conservation of energy

$\frac{1}{2}mv^2 = mg\Delta h$

$v = \sqrt{2g\Delta h}$

$v = \sqrt{2(9.8\ \text{m/s}^2)(0.08\ \text{m})} = \sqrt{1.568}\ \text{m/s}$

1 point is earned: For the correct answer

v = 1.3 m/s

Part C

1 point is earned: For using ΣF = *m*a

1 point is earned: For a correct expression for the net force that is equated to a centripetal force

T – *mg* = *mv2/r*

T = *m*(*g* + *v2/r*)

T = (0.085 kg)[9.8 m/s2 + (1.3 m/s)2/(1.5 m)]

1 point is earned: For the correct answer with units

T = 0.93 N (or equivalent answer depending on rounding or use of *g* = 10 m/s2)

0	1	2	3 ✓

The student response earns three of the following points:

frq7

1 point is earned: For using $\Sigma F = ma$

1 point is earned: For a correct expression for the net force that is equated to a centripetal force

$T - mg = mv2/r$

$T = m(g + v2/r)$

$T = (0.085\ kg)[9.8\ m/s2 + (1.3\ m/s)2/(1.5\ m)]$

1 point is earned: For the correct answer with units

$T = 0.93\ N$ (or equivalent answer depending on rounding or use of $g = 10\ m/s2$)

Part D

1 point is earned: For a correct modification

For example: Make the string four times longer.

0	1 ✓

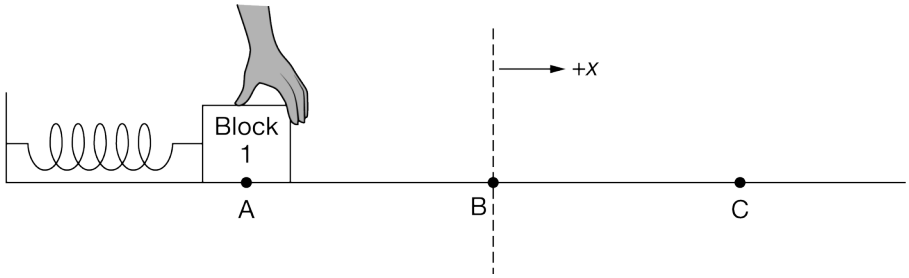
The student response earns one of the following points:

1 point is earned: For a correct modification

For example: Make the string four times longer.

frq7

5. This question is a short free-response question. Show your work for each part of the question.



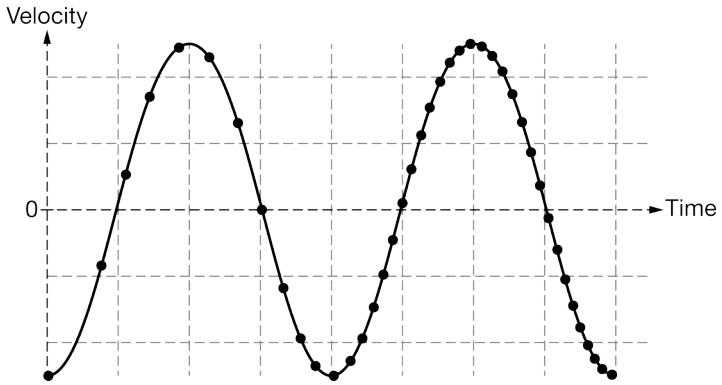
(7 points, suggested time 13 minutes)

Block 1 is at rest on a horizontal surface and is connected to a wall by an ideal spring. Friction between block 1 and the surface is negligible. Block 1 is held at rest at point A, to the left of point B which is the equilibrium position of the spring-block system, as shown in the figure. Block 1 is then released and allowed to oscillate. Some time later, block 1 is momentarily at rest at point C. Consider the positive horizontal direction to be toward the right.

(a) The dot below represents block 1 just after it is released. On the dot, draw an arrow indicating the direction of the net force exerted on the block just after it is released. The arrow should start on, and point away from, the dot.



Net Force on Block 1



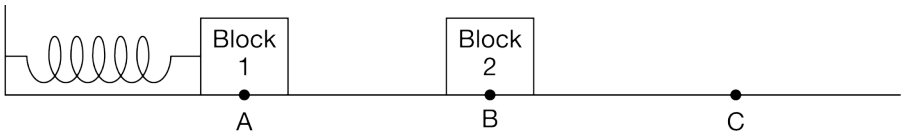
After the block has been oscillating on the table, a motion detector is turned on and the velocity of block 1 as a

frq7

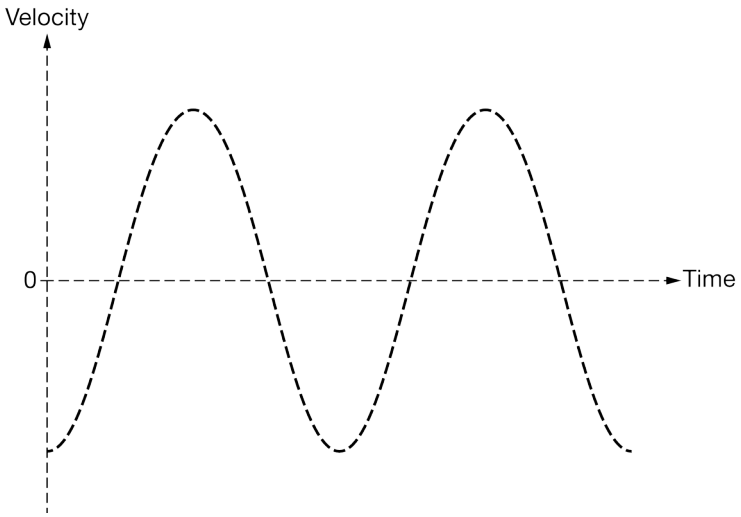
function of time is measured and plotted in a graph. The students correctly draw a best-fit curve that represents the collected data, as shown above.

(b) A student looking at the graph of velocity as a function of time makes the following claim.
"*The motion sensor was turned on when the block was at point A and was moving to the left*."

Does the data in this graph support the student's claim about the location AND the direction of the block's motion when the motion sensor was turned on? For any incorrect claims, if any, state the correct position and/or direction of motion. Justify your answer.

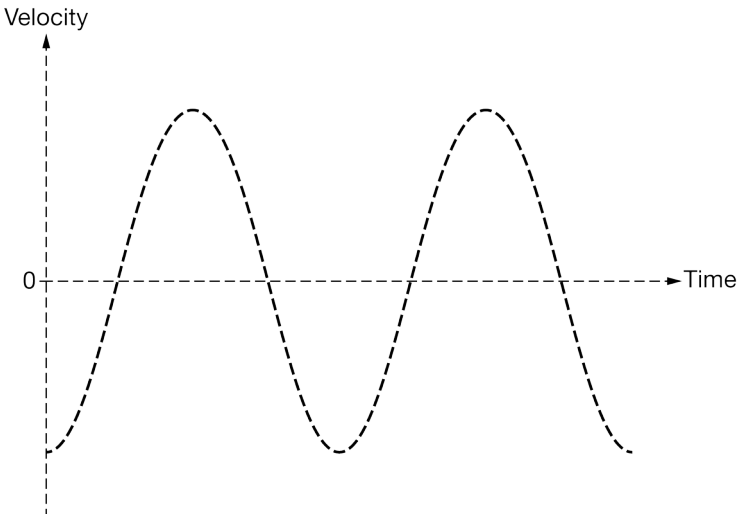


(c) Block 1 is stopped, and again block 1 is held at rest at point A. A second identical block, block 2, is placed at point B. Block 1 is released from rest, and then collides with and sticks to Block 2. After the collision, the two blocks oscillate together on the spring. On the graph below, sketch a curve to indicate the new velocity as a function of time of the two blocks after the collision. For reference, the dashed curve represents the students' best-fit curve of the velocity as a function of time described in part (b).



Note: Do any scratch (practice) work on the graph below. Sketches made below will NOT be graded.

frq7



Part A

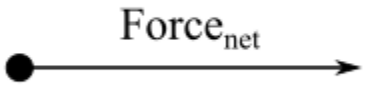
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: The amount of current that flows through each resistor must be indicated to earn the second point of part (a).

0	1 ✓

The response accurately includes **one** of the following criteria:

- ☐ For a net force arrow that is labeled and points horizontally to the right.



Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

frq7

The response accurately includes **three** of the following criteria.

- ☐ For a correct claim that the motion sensor was started when the block was at point B.
- ☐ For a correct justification of why the block is not at point B **OR** For a correct justification of why the block is at point C
- ☐ For a correct justification of why the block is moving to the left.

Example Response:

At time $t = 0$ s the block has a maximum velocity, which means that it is traveling through the equilibrium position, which is position B. Since the velocity is negative, and to the right was defined as positive, the block must be moving to the left through position B.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

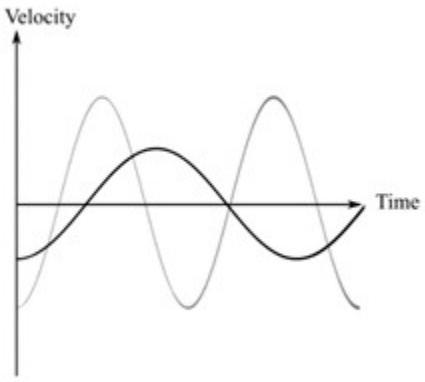
Scoring note: The starting point of this graph is not required to be correct to earn these points. Any phase shift is acceptable for this graph.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

- ☐ For a sine or cosine graph
- ☐ For a graph with a smaller maximum velocity
- ☐ For a sinusoidal graph with a longer period

Example Response:

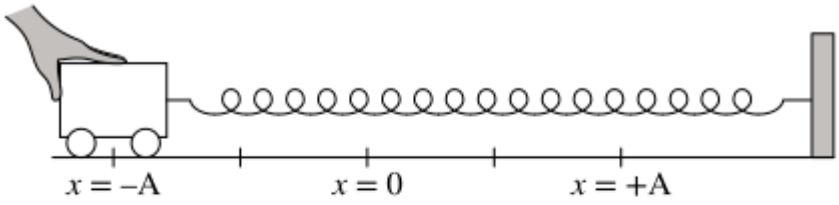


frq7

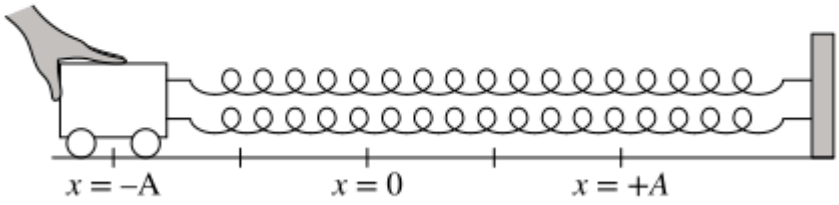
6. This question is a short free-response question. Show your work for each part of the question.

(7 points, suggested time 13 minutes)

The following two scenarios describe a cart attached to either one or two identical springs which are at their relaxed length when the cart is at position $x = 0$. In each scenario, the cart is pulled to position $x = -A$, is released from rest, and oscillates with negligible friction between $x = +A$ and $x = -A$.



Scenario 1



Scenario 2

In Scenario 1, the cart is attached to the wall with one spring as shown. The total mechanical energy of the cart-spring system in Scenario 1 is E_1.

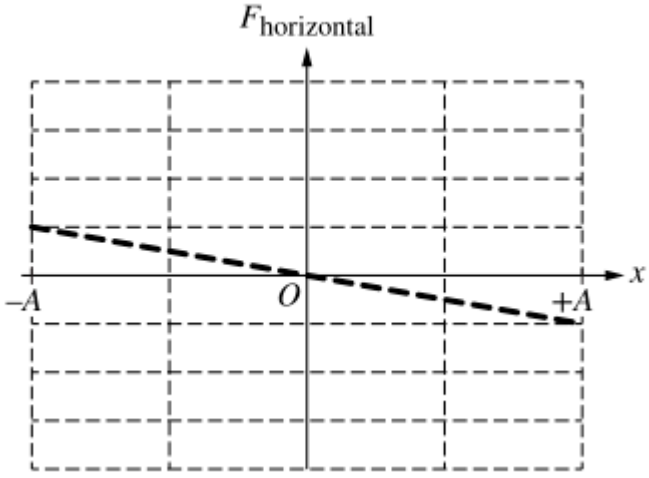
(a) For Scenario 1, derive an expression for the kinetic energy of the cart in terms of E_1 when the cart is at position $x = +\frac{A}{2}$.

(b) As shown in the figure for Scenario 2, a second identical spring is attached between the same cart and the wall before the cart is pulled to position $x = -A$.

i. Determine the total mechanical energy of the cart-springs system in Scenario 2 in terms of E_1.

ii. The maximum speed of the cart in Scenario 1 is v_1 and the maximum speed of the cart in Scenario 2 is v_2. Calculate the numerical value of the ratio $\frac{v_2}{v_1}$.

frq7



A student correctly sketches a graph of the horizontal component of the net force exerted on the cart by the spring in Scenario 1 as a function of position, as shown above with a dashed line.

(c) On the same graph above, using a solid line, sketch the horizontal component of the net force exerted on the cart in Scenario 2 with two springs attached.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an attempt at relating the mechanical energy of the system at $x = \frac{1}{2}A$ and $x = A$
- ☐ 1 point is earned for using $E = K + U$ to determine the kinetic energy and writing the correct value of the kinetic energy in terms of E_1 :$K_{A/2} = \frac{3}{4}E_1$

Example Response:

$E_1 = U_A = \frac{1}{2}kA^2$

$U_{A/2} = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \left(\frac{1}{2}\right)^2\left[\frac{1}{2}kA^2\right] = \left(\frac{1}{4}\right)U_A = \left(\frac{1}{4}\right)E_1$

$E = K + U, \quad \text{so} \quad K = E - U$

$K_{A/2} = E - U_{A/2} = E_1 - \frac{1}{4}E_1 = \frac{3}{4}E_1$

$K_{A/2} = \frac{3}{4}E_1$

Part B

frq7

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for writing $2E_1$
- ☐ 1 point is earned for relating the maximum speed of the cart to the square root of the total energy of the cart-springs system
- ☐ 1 point is earned for a numerical value consistent with part (b)(i) OR (b)(ii)

Example Response:

$E = K + U = K_{\text{max}} + 0 = K_{\text{max}} \quad (K = K_{\text{max}} \text{ when } U = 0)$

$K = \frac{1}{2}mv^2, \text{ so } K \propto v^2 \text{ or } v_{\text{max}} \propto \sqrt{K_{\text{max}}} = \sqrt{E}$

$\frac{v_2}{v_1} = \sqrt{\frac{E_2}{E_1}} = \sqrt{\frac{2E_1}{E_1}} = \sqrt{2}$

$\frac{v_2}{v_1} = \sqrt{2}$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

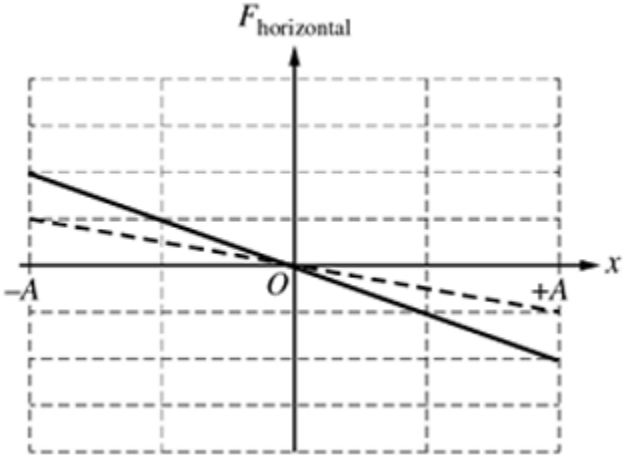
0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a straight line that passes through the origin
- ☐ 1 point is earned for a line or curve that has double the values at $-A$ and at $+A$

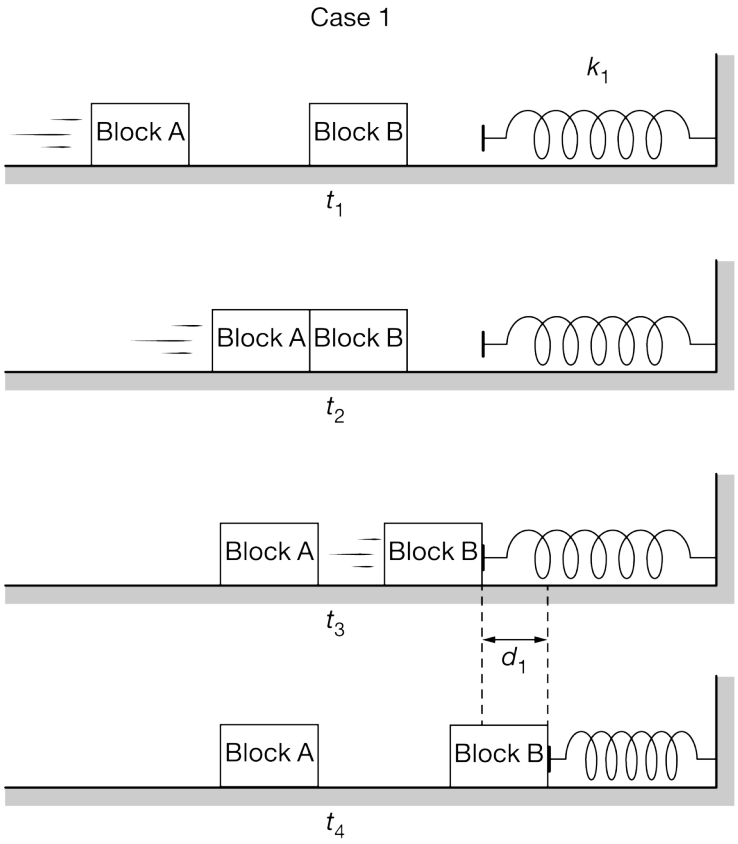
Example Response:

frq7



frq7

7. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



The figure above shows Case 1 of an experiment, in which the following events occur.

- At time t_1, block A slides to the right along a horizontal surface toward identical block B, which is at rest.
- At time t_2, block A collides elastically with block B and block A comes to rest.
- At time t_3, the right side of block B reaches an ideal, horizontal spring with negligible mass and spring constant k_1. The spring is initially at its relaxed (uncompressed) length and fixed to a wall.
- At time t_4, the spring is compressed a maximum distance d_1 and block B is instantaneously at rest.

Assume there is negligible friction between each block and the surface. For parts (a)-(d), consider the system consisting of block A, block B, and the spring.

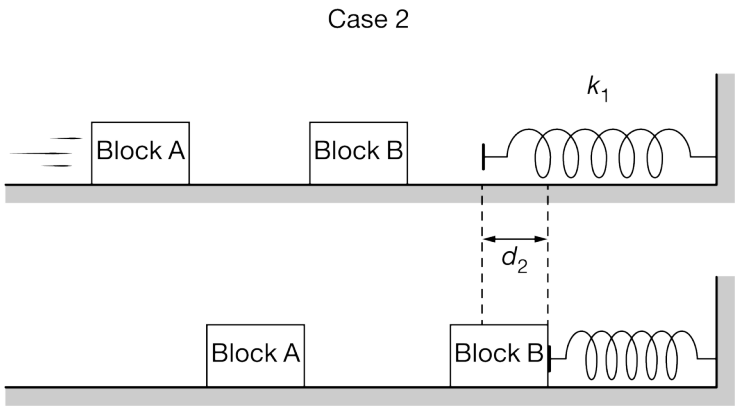
(a) Indicate whether the linear momentum of the two-block–spring system is either constant or not constant during the time interval from t_1 to immediately before t_3, and explain why.

(b) Indicate whether the mechanical energy of the two-block–spring system is either constant or not constant during the time interval from t_1 to immediately before t_3, and explain why.

frq7

(c) Indicate whether the linear momentum of the two-block–spring system is either constant or not constant during the time interval t_3 from to t_4, and explain why.

(d) Indicate whether the mechanical energy of the two-block–spring system is either constant or not constant during the time interval from t_3 to t_4, and explain why.



In Case 2 shown above, the setup is identical to Case 1 except that the speed of block A before colliding with block B is greater than in Case 1. Block A again comes to rest immediately after colliding with block B. The spring compresses a maximum distance d_2.

(e) How does the new maximum spring compression d_2 compare to d_1, the spring compression in Case 1? (Is d_2 greater than d_1, the same as d_1, or less than d_1 ?)

(f) Briefly explain your reasoning for part (e).

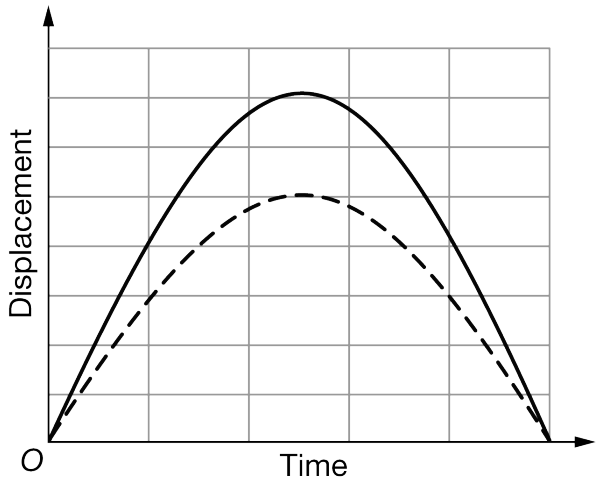
Students attempt to determine a general equation for the maximum compression distance d of a spring with spring constant k. Blocks A and B each have mass m and block A has initial speed v. The students come up with the following two equations.

Equation 1: $d = C_1\sqrt{\frac{m}{vk}}$ **Equation 2:** $d = C_2\sqrt{\frac{m}{k}}v$

In the two equations, C_1 and C_2 are positive constants with appropriate units.

(g) Without algebraic manipulation of equations or deriving the correct equation, explain which equation better supports your explanation for the compression distance in part (f).

frq7



(h) The graph above shows the displacement of the blocks while block B is in contact with the spring. The solid line corresponds to either Case 1 or Case 2, while the dashed line corresponds to the other case. Does the dashed-line curve correspond to Case 1 (slower block A) or Case 2 (faster block A)? Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that linear momentum is constant, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the linear momentum is constant.

Example Response:

No net external forces are exerted on the two-block–spring system, so the momentum of the system is constant.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

frq7

- [ ] 1 point is earned for indicating that mechanical energy is constant, with an attempt at an explanation.
- [ ] 1 point is earned for a correct explanation of why the mechanical energy is constant.

Example Response:

The mechanical energy of the system is constant because the collision is elastic.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for indicating that linear momentum is not constant, with an attempt at an explanation.
- [ ] 1 point is earned for a correct explanation of why the linear momentum is not constant.

Example Response:

The linear momentum is not constant, because a net external force is exerted on the two-block–spring system.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for indicating that mechanical energy is constant, with an attempt at an explanation.
- [ ] 1 point is earned for a correct explanation of why the mechanical energy is constant.

Example Response:

The mechanical energy is constant because no work is being done on the system by an external force.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq7

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for stating the compression distance is larger.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that block A transfers all of its kinetic energy to block B (i.e., block B has more kinetic energy when it reaches the spring than in Case 1).
- ☐ 1 point is earned for indicating that the greater kinetic energy means a larger compression because the elastic potential energy is equal to the initial kinetic energy.

Example Response:

Block A transfers all of its kinetic energy to block B in the elastic collision, so block B has more kinetic energy than in case 1. This means that there is more spring potential energy when the spring is compressed, so it is compressed a larger amount.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If part (g) is answered with a correct interpretation of Equation 1 consistent with an incorrect response in part (f), both points can be earned in part (g).

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that Equation 2 best models the compression with an attempt at an explanation that addresses functional dependence.
- ☐ 1 point is earned for a correct explanation that addresses the dependence of d on v.

frq7

Example Response:

Since v is proportional to d in equation 2, as v increases d will increase, as predicted in part (f).

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

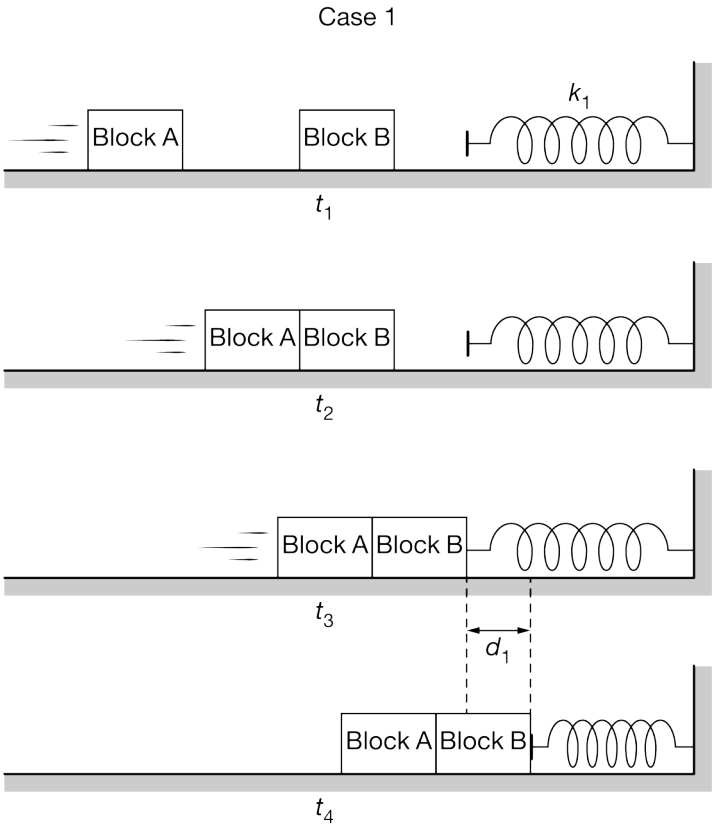
- ☐ 1 point is earned for indicating the dashed line corresponds to Case 1, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation regarding the slope of the displacement vs. time graph
  OR
  For a correct explanation addressing the amplitude (energy).

Example Response:

The dashed-line has an initially smaller slope representing a smaller initial speed of block B. Both graphs have zero slope where the block B momentarily stops at the spring's maximum displacement. The maximum displacement is smaller for the initially slower Block B. Therefore, the dashed-line graph corresponds to Case 1.
OR
The smaller maximum displacement for the dashed line is consistent with less energy being stored in the spring. This would indicate that block B had less initial kinetic energy when it collided with the spring corresponding to Case 1.

frq7

8. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



The figure above shows Case 1 of an experiment, in which the following events occur.

- At time t_1, block A slides to the right along a horizontal surface toward identical block B, which is at rest.
- At time t_2, block A collides with and sticks to block B.
- At time t_3, block A and block B are still stuck together and the right side of block B reaches an ideal, horizontal spring with negligible mass and spring constant k_1. The spring is initially at its relaxed (uncompressed) length and fixed to a wall.
- At time t_4, the spring is compressed a maximum distance d_1 and the blocks are instantaneously at rest.

Assume there is negligible friction between each block and the surface.

A student observes the experiment and makes the following claim.

"*Since some mechanical energy is dissipated when the blocks stick together, the momentum of the two-block system decreases.*"

(a) Indicate whether the claim is ***correct*** or ***incorrect*** and explain why.

frq7

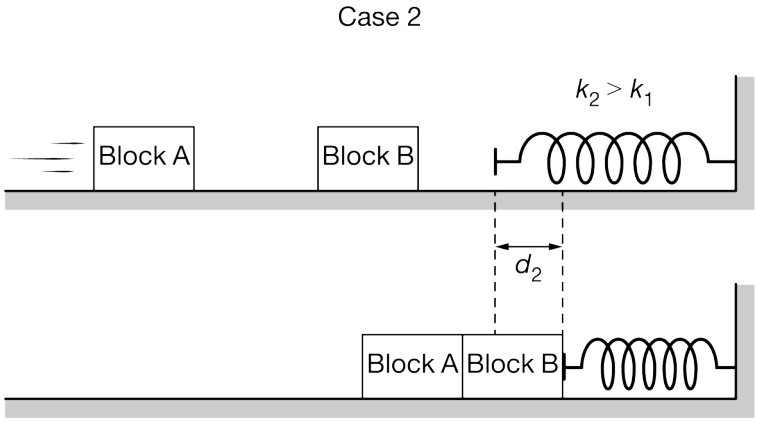
Another student makes the following claim about the collision between block A and block B.

"*When the blocks collide the total momentum of the two-block system doesn't change, so the kinetic energy of the system must also be constant.*"

(b) Indicate whether the claim is ***correct*** or ***incorrect*** and explain why.

(c) During the time interval from t_3 to t_4, is the magnitude of the acceleration of the two-block system increasing, decreasing, or constant? Briefly explain why.

(d) During the time interval from t_3 to t_4, does the spring do positive, negative, or zero work on the two-block system? Briefly explain your answer.



In Case 2 shown above, the setup is identical to Case 1 except that a stiffer spring is used. The spring in Case 2 has a spring constant k_2 that is greater than k_1. Block A slides with the same speed as in Case 1 and the spring compresses a maximum distance d_2.

(e) How does the new maximum spring compression distance d_2 in Case 2 compare to d_1 in Case 1? (Is d_2 greater than d_1, the same as d_1, or less than d_1 ?)

(f) Briefly explain your reasoning for part (e).

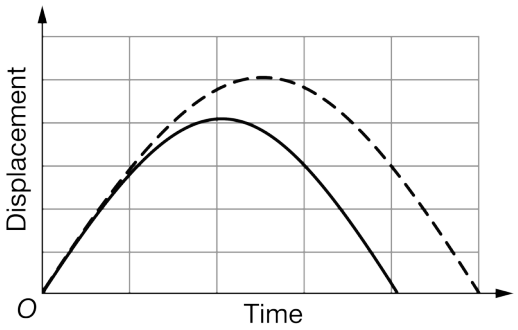
Students attempt to determine a general equation for the maximum compression distance d of a spring with spring constant k. Block A has mass m_A and initial speed v, and block B has mass m_B. The students come up with the following two equations.

Equation 1: $d = \frac{C_1 m_A v}{\sqrt{(k)(m_A+m_B)}}$ **Equation 2:** $d = C_2\left(k\sqrt{m_A + m_B}\right)v$

In the two equations, C_1 and C_2 are positive constants with appropriate units.

(g) Without algebraic manipulation of equations or deriving the correct equation, explain which equation better supports your explanation for the compression distance in part (f).

frq7



(h) The graph above shows the displacement of the blocks while block B is in contact with the spring. The solid line corresponds to either Case 1 or Case 2, while the dashed line corresponds to the other case. Does the solid-line curve correspond to Case 1 (with a smaller spring constant) or Case 2 (with a larger spring constant)? Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the claim is incorrect, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the momentum does not change.

Example Response:

The claim is incorrect. The momentum does not change because no net external force is exerted on the two-block system.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the claim is incorrect, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation.

frq7

Example Response:

The claim is incorrect. Energy is dissipated during the collision and therefore the kinetic energy decreases.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the magnitude of the acceleration is increasing, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation.

Example Response:

The force exerted by the spring increases with increasing compression.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the spring does negative work on the two-block system, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why negative work is done.

Example Response:

The spring does negative work on the two-block system because the spring exerts a net external force opposite the direction of motion of the system.

OR

The spring force does negative work on the two-block system because the force exerted from the spring decreases the kinetic energy of the system.

Part E

frq7

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for stating the compression distance is smaller.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the two-block system has the same kinetic energy before reaching the spring as in Case 1 (or that the maximum elastic potential energy in the spring is the same as in Case 1).
- ☐ 1 point is earned for indicating that the elastic potential energy is directly related to both spring constant and compression distance, or that a greater spring constant means a smaller compression distance for the same potential energy.

Example Response:

The two-block system has the same kinetic energy when it reaches the spring. All the kinetic energy is converted to spring potential energy. Since the spring constant is now greater, the maximum compression distance must be less in order to have the same amount of potential energy at maximum compression.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If part (g) is answered with a correct interpretation of Equation 2 consistent with an incorrect response in part (f), both points can be earned in part (g).

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that Equation 1 best models the compression with an attempt at an

frq7

explanation that addresses functional dependence.

- [ ] 1 point is earned for a correct explanation that addresses the dependence of d on k.

Example Response:

Since k is in the denominator of Equation 1, as k increases d will decrease, as predicted in part (f).

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

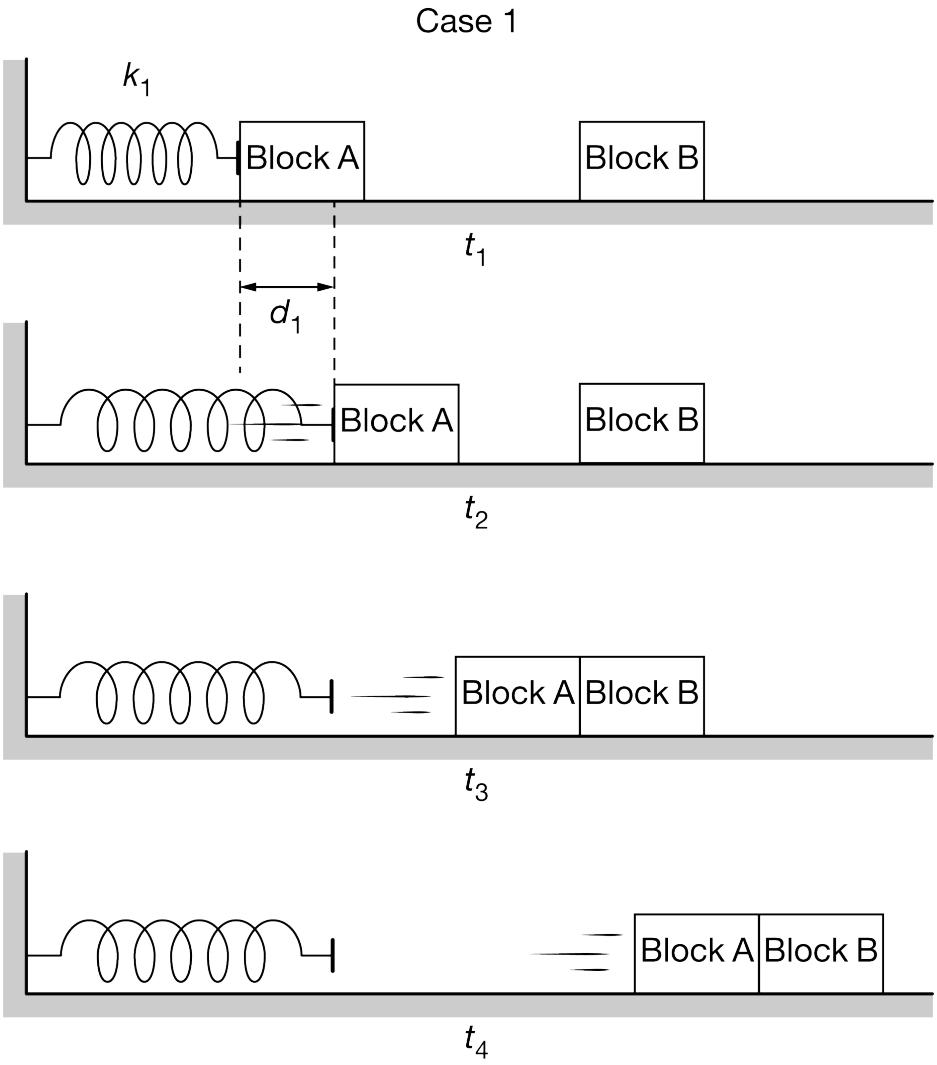
- [ ] 1 point is earned for indicating the solid line corresponds to Case 2, with an attempt at an explanation.
- [ ] 1 point is earned for a correct explanation addressing either the amplitude or period (or both).

Example Responses:

The smaller maximum displacement for the solid line is consistent with the larger spring constant in case 2.
OR
The period of an object oscillating on a spring is shorter when the spring constant is larger, so the solid line represents case 2.

frq7

9. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



The figure above shows Case 1 of an experiment. Initially, block A is held in place while in contact with a horizontal spring with spring constant k_1 that is compressed a distance d_1 and fixed to a wall. The following events occur.

- At time t_1, block A is released from rest.
- At time t_2, the ideal spring reaches its uncompressed length, and block A leaves the spring.
- At time t_3, block A collides with and sticks to identical block B, which is initially at rest.
- At time t_4, block A and block B are sliding together along the surface.

Assume there is negligible friction between each block and the surface.

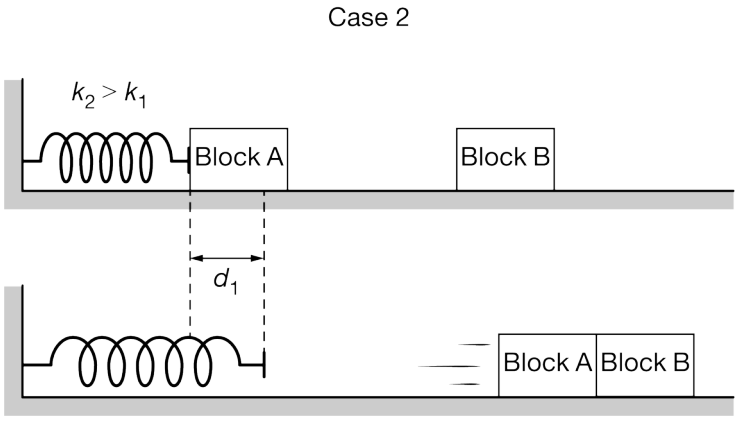
frq7

(a) Indicate whether the speed of the center of mass of the two-block system is either constant or not constant during the time interval from t_1 to t_2, and explain why.

(b) Indicate whether the mechanical energy of the two-block system is either constant or not constant during the time interval from t_1 to t_2, and explain why.

(c) Indicate whether the speed of the center of mass of the two-block system is either constant or not constant during the time interval from t_2 to t_4, and explain why.

(d) Indicate whether the mechanical energy of the two-block system is either constant or not constant during the time interval from t_2 to t_4, and explain why.



In Case 2 shown above, the setup is identical to Case 1 except that a stiffer spring is used. The spring in Case 2 has a spring constant k_2 that is greater than k_1. The spring is compressed the same distance, d_1, before block A is released. As in Case 1, the blocks stick together when they collide.

(e) Is the speed of the blocks after the collision in Case 2 greater than, less than, or the same as the speed of the blocks after the collision in Case 1?

(f) Briefly explain your reasoning for part (e).

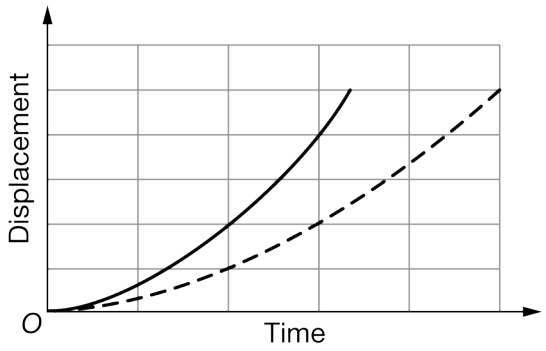
Students attempt to determine a general equation for the speed v of the blocks after the collision. A spring with spring constant k is initially compressed a distance d, block A has mass m_A, and block B has mass m_B. The students come up with the following two equations.

Equation 1: $v = C_1\left(\frac{m_B+m_A}{\sqrt{km_A}}\right)d$ **Equation 2:** $v = C_2\left(\frac{\sqrt{km_A}}{m_A+m_B}\right)d$

In the two equations, C_1 and C_2 are positive constants with appropriate units.

(g) Without algebraic manipulation of equations or deriving the correct equation, explain which equation better supports your explanation for the speed of the two blocks in part (f).

frq7



The graph above shows the displacement of block A while it is in contact with the spring for Case 1 and Case 2. The solid line corresponds to either Case 1 or Case 2, while the dashed line corresponds to the other case.

(h) Does the solid-line curve best correspond to Case 1 (with a smaller spring constant) or to Case 2 (with a larger spring constant)? Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the speed of the center of mass is not constant, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the center of mass speed is not constant.

Example Response:

The speed of the center of mass is not constant, because a net external force is exerted on the two-block system.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

frq7

- [ ] 1 point is earned for indicating that mechanical energy is not constant, with an attempt at an explanation.
- [ ] 1 point is earned for a correct explanation of why the mechanical energy is not constant.

Example Response:

The mechanical energy is not constant, because the spring is doing work on the two-block system

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for indicating that the speed of the center of mass of the system is constant, with an attempt at an explanation.
- [ ] 1 point is earned for a correct explanation of why the speed of the center of mass of the system is constant.

Example Response:

The speed of the center of mass of the system is constant, because no net external force is exerted on the two-block system.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for indicating that mechanical energy is not constant, with an attempt at an explanation.
- [ ] 1 point is earned for a correct explanation of why the mechanical energy is not constant.

Example Response:

The mechanical energy is not constant, because the collision is inelastic and therefore the kinetic energy decreases.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq7

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for stating the speed in Case 2 is greater than that in Case 1.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for indicating that more energy is transferred to block A by the spring.
- [ ] 1 point is earned for indicating that a larger kinetic energy means larger initial speed for block A before the collision with block B and, therefore, a larger final speed for the two blocks after the collision.

Example Response:

Because the spring constant is greater in case 2, more energy is transferred to block A by the spring and block A has more kinetic energy than in case 1. The larger kinetic energy means a larger initial speed for block A before the collision with block B, so there is more momentum and a greater final speed.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If (g) is answered with a correct interpretation of Equation 1 consistent with an incorrect response in part (f), both points can be earned in part (g)

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for indicating that Equation 2 best models the compression with an attempt at an explanation that addresses functional dependence.
- [ ] 1 point is earned for a correct explanation that addresses the dependence of v on k.

frq7

Example Response:

Since v and k are directly related (k is in the numerator of the expression), a larger k means that v will be larger, as predicted in part (f).

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

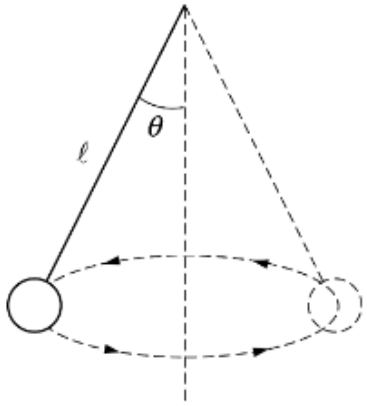
- ☐ 1 point is earned for indicating the solid line corresponds to Case 2, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation that the block will spend less time in contact with the spring with a stiffer spring constant (or that the speed/slope will be larger).

Example Response:

The solid line has a greater slope at the end, which means a greater speed, consistent with the larger spring constant in case 2.
OR
The solid line corresponds to a shorter period of a block attached to a spring, consistent with the larger spring constant in case 2.

frq7

10.



A ball attached to a string of length l swings in a horizontal circle, as shown above, with a constant speed. The string makes an angle θ with the vertical, and T is the magnitude of the tension in the string. Express your answers to the following in terms of the given quantities and fundamental constants.

a. On the figure below, draw and label vectors to represent all the forces acting on the ball when it is at the position shown in the diagram. The lengths of the vectors should be consistent with the relative magnitudes of the forces.



b. Determine the mass of the ball.

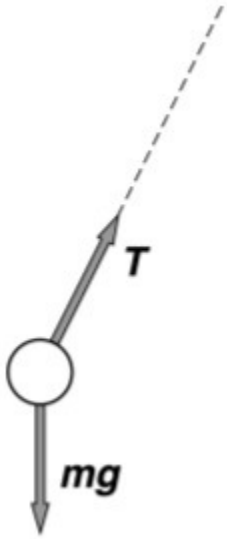
c. Determine the speed of the ball.

d. Determine the frequency of revolution of the ball.

e. Suppose that the string breaks as the ball swings in its circular path. Qualitatively describe the trajectory of the ball after the string breaks but before it hits the ground.

Part A

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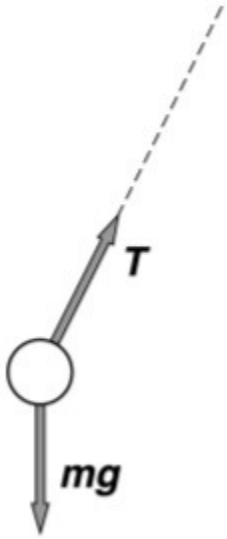
2 points are earned: For each correctly drawn and appropriately labeled force

1 point is earned: For no incorrect forces

The horizontal and vertical components of the tension were acceptable if properly labeled

0	1	2	3 ✓

The student response earns all of the following points:



2 points are earned: For each correctly drawn and appropriately labeled force

1 point is earned: For no incorrect forces

frq7

The horizontal and vertical components of the tension were acceptable if properly labeled

Part B

1 point is earned: For indicating that the sum of the vertical forces is zero

$\sum Fy = 0$

1 point is earned: For correctly substituting the applicable forces

$T \cos \theta - mg = 0$

1 point is earned: For the correct answer

$m = \frac{T \cos \theta}{g}$

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For indicating that the sum of the vertical forces is zero

$\sum Fy = 0$

1 point is earned: For correctly substituting the applicable forces

$T \cos \theta - mg = 0$

1 point is earned: For the correct answer

$m = \frac{T \cos \theta}{g}$

Part C

The centripetal force is supplied by the horizontal component of the tension

$Fc = T \sin \theta$

1 point is earned: For using a correct expression for the centripetal force *Fc*

$\frac{mv^2}{r} = T \sin \theta$

1 point is earned: For eliminating the tension or the mass using the answer to part (b)

$\frac{mv^2}{r} = mg\frac{\sin \theta}{\cos \theta} = mg \tan \theta$ OR $\frac{T \cos \theta v^2}{gr} = T \sin \theta$

1 point is earned: For using the substitution $r = l \sin \theta$

frq7

$\frac{mv^2}{\diamondsuit \sin\theta} = mg\tan\theta$ OR $\frac{T\cos\theta v^2}{g\diamondsuit \sin\theta} = T\sin\theta$

1 point is earned: For the correct answer

$v = \sqrt{g\diamondsuit \sin\theta\tan\theta}$ OR $\sqrt{\frac{g\diamondsuit \sin^2\theta}{\cos\theta}}$

0	1	2	3	4 ✓

The student response earns all of the following points:

The centripetal force is supplied by the horizontal component of the tension

Fc = *T* sin θ

1 point is earned: For using a correct expression for the centripetal force *Fc*

$\frac{mv^2}{r} = T\sin\theta$

1 point is earned: For eliminating the tension or the mass using the answer to part (b)

$\frac{mv^2}{r} = mg\frac{\sin\theta}{\cos\theta} = mg\tan\theta$ OR $\frac{T\cos\theta v^2}{gr} = T\sin\theta$

1 point is earned: For using the substitution *r* = *l* sin θ

$\frac{mv^2}{\diamondsuit \sin\theta} = mg\tan\theta$ OR $\frac{T\cos\theta v^2}{g\diamondsuit \sin\theta} = T\sin\theta$

1 point is earned: For the correct answer

$v = \sqrt{g\diamondsuit \sin\theta\tan\theta}$ OR $\sqrt{\frac{g\diamondsuit \sin^2\theta}{\cos\theta}}$

Part D

1 point is earned: For using a correct expression for period or frequency

$T = \frac{2\pi r}{v}$ OR $f = \frac{v}{2\pi r}$

1 point is earned: For substituting the velocity from part (c)

1 point is earned: For the correct answer (unsimplified acceptable)

$f = \frac{\sqrt{gl\sin\theta\tan\theta}}{2\pi l\sin\theta}$ OR $\frac{1}{2\pi}\sqrt{\frac{g\tan\theta}{l\sin\theta}}$ OR $\frac{1}{2\pi}\sqrt{\frac{g}{l\cos\theta}}$

frq7

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For using a correct expression for period or frequency

$T = \frac{2\pi r}{v}$ OR $f = \frac{v}{2\pi r}$

1 point is earned: For substituting the velocity from part (c)

1 point is earned: For the correct answer (unsimplified acceptable)

$f = \frac{\sqrt{gl \sin\theta \tan\theta}}{2\pi l \sin\theta}$ OR $\frac{1}{2\pi}\sqrt{\frac{g \tan\theta}{l \sin\theta}}$ OR $\frac{1}{2\pi}\sqrt{\frac{g}{l \cos\theta}}$

Part E

1 point is earned: For any indication that the initial velocity of the ball is horizontal

1 point is earned: For any indication that the subsequent trajectory is parabolic

0	1	2 ✓

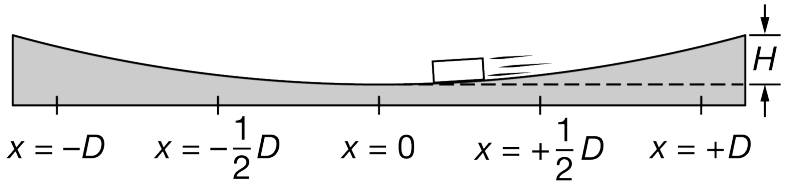
The student response earns all of the following points:

1 point is earned: For any indication that the initial velocity of the ball is horizontal

1 point is earned: For any indication that the subsequent trajectory is parabolic

frq7

11. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

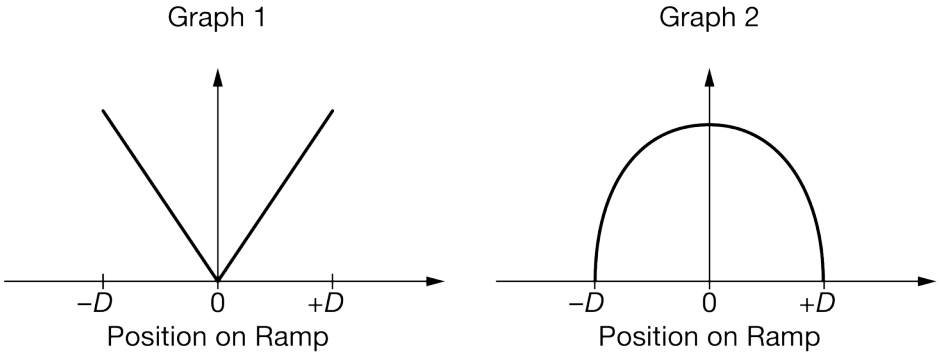


Track 1

Note: Figure not drawn to scale.

A block slides back and forth without any appreciable loss of energy on a track with a slight circular curvature, as shown above. The maximum height H of the track at each end is much smaller than D.

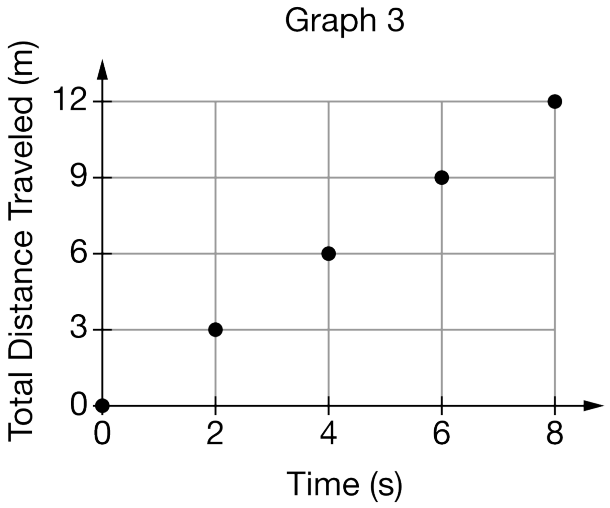
The center of mass of the block oscillates between $x = -D$ and $x = +D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. The block undergoes two full oscillations in 8.0 seconds.



(a) Which of the graphs above, Graph 1 or Graph 2, could represent the magnitude of the horizontal component of the block's acceleration as a function of its position on the ramp, and which graph could represent the magnitude of the block's momentum as a function of its position on the ramp? Justify both your choices with physics principles.

(b) For the system consisting of only the block, describe how the mechanical energy of the system changes, if at all, as the block moves from $x = 0$ to $x = -D$. Justify your answer.

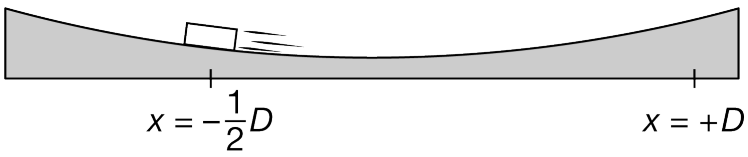
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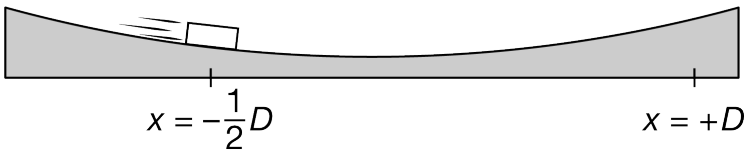
(c) Graph 3 above shows the total horizontal distance traveled by the block as a function of time as it completes two oscillations. Using the graph, determine the distance D between the midpoint of the ramp and the maximum position of the block on the ramp. Explain how you arrived at your answer.

(d) Suppose the entire setup was moved to the Moon, where the gravitational field strength is less than that on Earth. The center of mass of the block again oscillates between $x = -D$ and $x = +D$. Would the period of the block's oscillation on the Moon be greater than, less than, or equal to the period of oscillation on Earth? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{D^2}{2Hg}}$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?



Block on Track 2 at Time $t = T_1$



Block on Track 2 at Time $t = T_2$

Note: Figures not drawn to scale.

(f) Track 2 is identical to Track 1 except there is nonnegligible friction between the block and Track 2. The block

frq7

is released from rest on Track 2 at position $x = +D$. At time $t = T_1$, the block moves to the left, passing through position $x = -\frac{1}{2}D$, as shown above. After the block reverses direction near $x = -D$, the block moves to the right, passing through position $x = -\frac{1}{2}D$ at time $t = T_2$. At which time, T_1 or T_2, if either, is the magnitude of the tangential acceleration of the block greater? Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that only Graph 1 represents acceleration or that only Graph 2 represents momentum.
- ☐ 1 point is earned for a valid justification correctly relating force to acceleration and including a reference to a specific feature of the graph.
- ☐ 1 point is earned for a valid justification relating velocity to momentum and including a reference to a specific feature of the graph.

Example Response:

Graph 1 represents the magnitude of the cart's horizontal acceleration, and Graph 2 represents the block's momentum. The horizontal component of the acceleration will look like the net force graph, which should be a restoring force, and so should be proportional to D. The momentum of the cart graph will look like the velocity graph which has a maximum at $x = 0$ *and is zero at D and* $-D$.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for any statement that between $x = 0$ and $x = -D$, the kinetic or mechanical energy decreases.
- ☐ 1 point is earned for a clear indication that the mechanical energy of the system consists of only the kinetic energy of the block.

Example Response:

frq7

The mechanical energy of the block is only kinetic energy, so between $x = 0$ and $x = -D$, the mechanical energy (kinetic energy), decreases.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an attempt at relating the distance traveled to the amplitude.
- ☐ 1 point is earned for a correct answer: $D = 1.5\ \text{m}$.

Example Response:

D is equal to 1.5 meters. In two complete oscillations the block travels $8D$, in 8 seconds. From the graph we can see that the total distance traveled in 8 seconds is 12 meters, so the distance D is $(12\ \text{m})/8 = 1.5\ \text{m}$.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that the period of oscillation is longer on the moon.
- ☐ 1 point is earned for correct physical reasoning why the average speed or average acceleration is less on the moon.
- ☐ 1 point is earned for connecting smaller average speed or smaller average acceleration over the same distance to a longer time.

Example Response:

The period of oscillation on the moon would be greater. The average speed of the box on the moon would be smaller because the acceleration due to gravity is smaller, so for a smaller average speed, over the same distance, the time to oscillate would be larger.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq7

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence of *T* on *g*, with an attempt at justification.
- ☐ 1 point is earned for correctly indicating whether the dependence of *T* on *g* is consistent with the answer to part (d).
  Note: *T* should increase when *g* decreases. If part (d) was answered correctly, the equation is consistent.

Example Response:

Yes, this makes sense with my reasoning. Since the acceleration due to gravity, g, is in the denominator, when g is smaller, (like it is on the Moon), the period T is larger.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

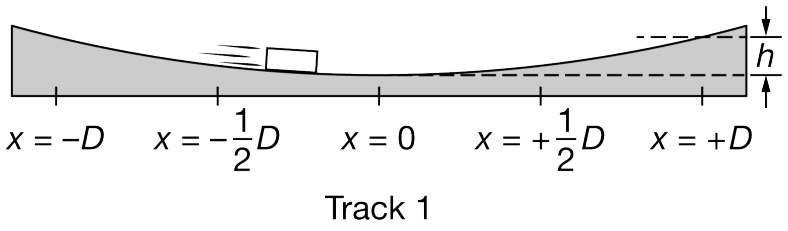
- ☐ 1 point is earned for a claim that the acceleration is greater at T_1.
- ☐ 1 point is earned for stating that both forces (component of gravity and friction) act on the block.
- ☐ 1 point is earned for correctly relating the sum of the forces on the block to the acceleration of the block at both T_1 OR T_2.

Example Response:

The tangential acceleration is greater at time T_1. At time T_1 both the component of gravity along the track and the friction force are in the same direction, while at T_2, the force of friction is in the opposite direction, so the force along the track is less at T_2, or greater at T_1. Since the force along the track is greater at T_1, the tangential acceleration of the block is greater at T_1.

frq7

12. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

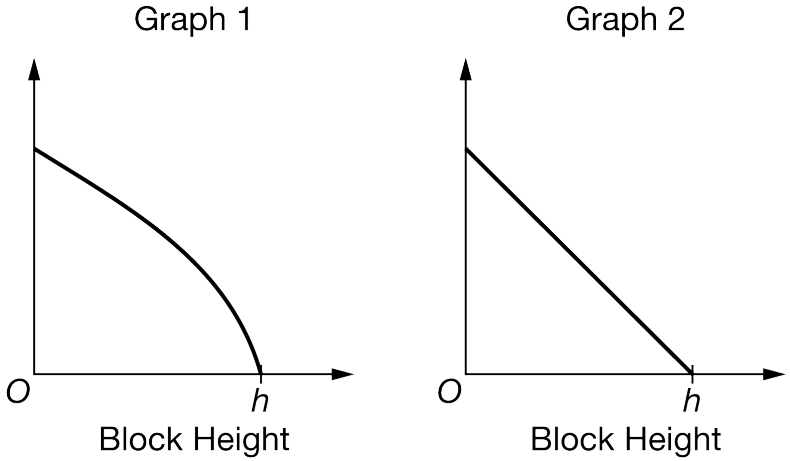


Track 1

Note: Figure not drawn to scale.

A block slides back and forth on a track with a slight circular curvature, where friction is negligible, as shown above. The maximum height h of the track at each end is much smaller than D.

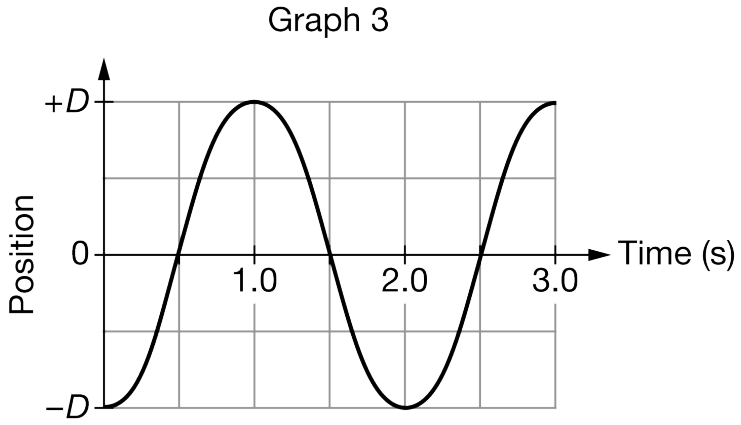
The center of mass of the block oscillates between $x = -D$ and $x = +D$, reaching a maximum height h every time the block is at $x = +D$ or $x = -D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. At time $t = 0$, the speed of the block is zero. The next time that the block's speed is zero is at time $t = 2.0$ s.



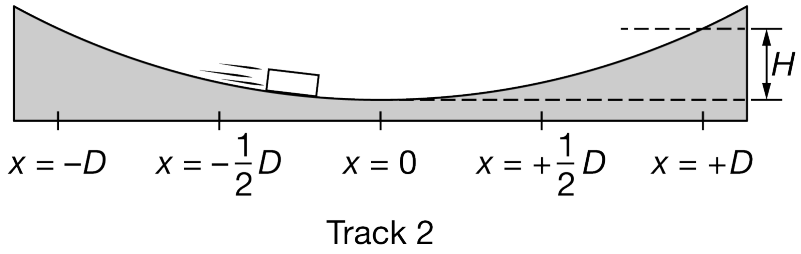
(a) Which of the graphs above, Graph 1 or Graph 2, could represent the kinetic energy of the block as a function of its height on the track, and which graph could represent the speed as a function of the block's height on the track? Justify both of your choices with physics principles.

(b) For the system consisting of the block and Earth, describe how the mechanical energy of the system changes, if at all, as the block moves from $x = +D$ to $x = 0$. Justify your answer.

frq7



(c) Graph 3 above incorrectly represents the horizontal position of the block as a function of time. Identify a feature of the graph that does not correctly represent the motion of the block. Justify your answer.



Note: Figure not drawn to scale.

(d) Track 2 is similar to Track 1, but the maximum height H on Track 2 is greater than h and is still much smaller than D, as shown above. The center of mass of the block oscillates between $x = -D$ and $x = +D$, the same as on Track 1. How does the period of the block's oscillation on Track 2 compare to the period of the block's oscillation on Track 1? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{H^2}{2Dg}}$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?

(f) When the block reaches $x = 0$ at the bottom of the circular track, is the force exerted by the track on the block greater than, less than, or equal to the weight of the block? Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

frq7

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that only Graph 1 represents the speed as a function of height OR that only Graph 2 represents the kinetic energy as a function of height.
- ☐ 1 point is earned for a valid justification correctly relating speed to height and including a reference to a specific feature of the graph.
  Note: It acceptable to explain that speed is proportional to the square root of the kinetic energy.
- ☐ 1 point is earned for a valid justification that kinetic energy decreases linearly with increasing height and including a reference to a specific feature of the graph.

Example Response:

Graph 1 is the speed as a function of height and Graph 2 is the kinetic energy of the block as a function of height. Speed is equal to zero at h, but does not decrease linearly since the acceleration changes as the slope of the track changes. The kinetic energy however, changes linearly with height, since it is related to gravitational potential energy which changes linearly with height.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: The mechanical energy stays the same.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the mechanical energy of the system consists of gravitational potential energy and kinetic energy of the block.
- ☐ 1 point is earned for a statement that no work is done on the system, so the mechanical energy is constant.

Example Response:

For this system, the total mechanical energy is both the kinetic energy of the block and the gravitational potential energy of the block-Earth system. Because there are no net external forces that do work on the system, the total mechanical energy of the system remains constant.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

frq7

The response accurately includes **both** of the following criteria.

- 1 point is earned for identifying that a maxima or minima on the graph indicates zero velocity,
  OR
  for identifying that the period on the graph is incorrect (2.0 seconds),
  OR
  for identifying that the graph should not have a zero slope at 1.0 seconds.
- 1 point is earned for a claim that the period should be longer (4.0 seconds) with no incorrect statements,
  OR
  for a claim that the next maxima should occur at 2.0 seconds with no incorrect statements,
  OR
  for a claim that the velocity should be zero (or flat slope) at only 0 and 2.0 seconds with no incorrect statements,
  OR
  for a claim that the graph should be "stretched" to twice its width along the *x*-axis (or similar) with no incorrect statements.

Example Response:

The period in this graph is incorrect – the period should be 4 seconds, since it takes 2 seconds just to go from $-D$ to D, which is only half a period. Also, the velocity (the slope of the graph) should be zero at zero seconds and 2 seconds, not at 1 second.

The problem states that the speed of the block is zero at $t = 0$ s and $t = 2$ s, so the graph should have a maximum at $t = 2$ s and not $t = 1$ s.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- 1 point is earned for a claim that the period of oscillation is shorter on Track 2.
- 1 point is earned for correct reasoning why the average speed or average acceleration is greater on Track 2 (connecting increased total mechanical energy to a greater average speed or connecting greater average acceleration to a steeper track).
- 1 point is earned for connecting greater average speed or greater average acceleration over the same distance to a shorter time.

Example Response:

The period is shorter on track 2 because the average speed is greater on track 2 because the total mechanical energy is larger since H is larger. Because the average speed is larger, over the same distance the block takes less time to travel from D to $-D$ and back.

Part E

frq7

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence of T on H, with an attempt at a justification.
- ☐ 1 point is earned for correctly indicating whether the dependence of T on H is consistent with the answer to part (d).

Example Response:

No this does not match my claim above. Because H is in the numerator, an increase in H should lead to an increase in T according to the equation. However, in part (d) I found that an increase in H leads to a shorter period (T).

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

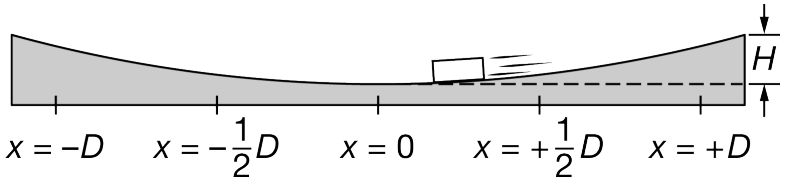
- ☐ 1 point is earned for a claim that the force exerted on the block by the track is greater than the weight.
- ☐ 1 point is earned for describing the net force in terms of the force of track on the block (or normal force) and weight only.
- ☐ 1 point is earned for describing that the net force is directed upward because the block is in circular motion.

Example Response:

The force exerted on the box by the track is greater than the weight at the lowest point, since the force from the track minus the weight of the block has to be equal to the centripetal force up.

frq7

13. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

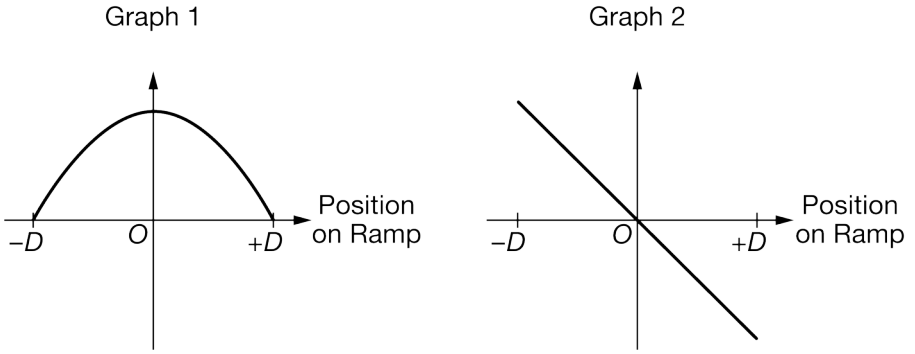


Track 1

Note: Figure not drawn to scale.

A block slides back and forth on a track with a slight circular curvature, where friction is negligible, as shown above. The maximum height H of the track at each end is much smaller than D.

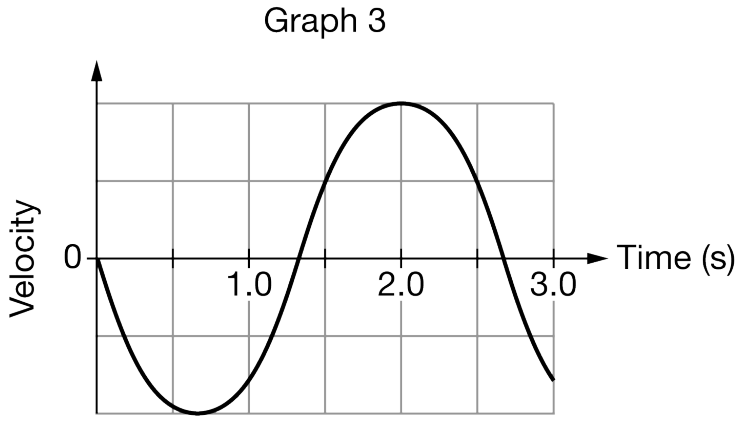
The center of mass of the block oscillates between $x = -D$ and $x = +D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. At time $t = 0$ the block is released from rest at position $x = +D$. The period of the block's oscillation is 2.0 seconds.



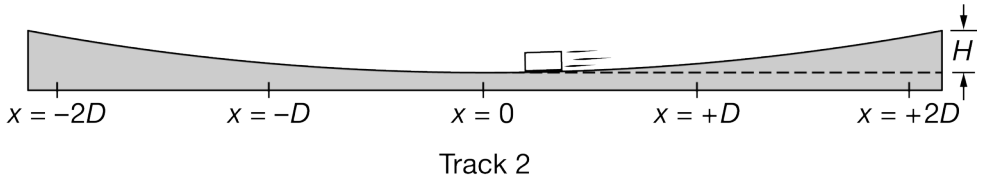
(a) Which of the graphs above, Graph 1 or Graph 2, could represent the block's kinetic energy as a function of its position on the circular track, and which graph could represent the horizontal component of the net force acting on the block as a function of its position on the track? Justify both of your choices with physics principles.

(b) Choose one of the graphs in part (a) above and explain how the graph shows evidence that a restoring force is acting on the block.

frq7



(c) Graph 3 above incorrectly shows the horizontal velocity of the block as a function of time. Identify a feature of the graph that does not correctly represent the motion of the block. Justify your answer.

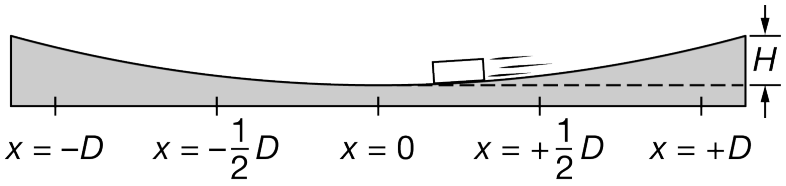


Note: Figure not drawn to scale.

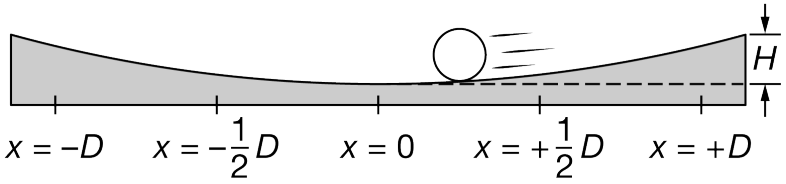
(d) Track 2, shown above, has a horizontal length that is greater than that of Track 1 and the same maximum height H. The block is released from rest at position $x = +2D$ Is the period of the block's oscillation on Track 2 greater than, less than, or equal to the period of oscillation on Track 1? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{h^2}{2dg}}$, where d is the maximum displacement of the block from equilibrium and h is the height of the track at $x = d$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?

frq7



Track 1



Track 3

Note: Figures not drawn to scale.

(f) On Track 1, shown above again for reference, friction is negligible and the center of mass of the block oscillates between a horizontal position of $+D$ and $-D$ with a period of 2.0 seconds. If a sphere with a mass equal to that of the block were to roll without slipping between $+D$ and $-D$ along Track 3 shown above, which is identical to Track 1 but has nonnegligible friction, would the period of the motion of the sphere be greater than, less than, or equal to 2.0 seconds? Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that only Graph 1 represents kinetic energy or that only Graph 2 represents the horizontal component of the net force.
- ☐ 1 point is earned for a valid justification relating the slope of the track to the horizontal force or describing the positive/negative values of the force on each side of the graph.
- ☐ 1 point is earned for a valid justification referring to the speed of the block and its kinetic energy and including a reference to a specific feature of the graph.

Example Response:

Graph 1 represents the kinetic energy of the block and Graph 2 represents the horizontal component of the net force. The kinetic energy is a maximum at $x = 0$ and a minimum at $x = -D$ and $x = +D$. The horizontal component of the net force is zero at $x = 0$ and the magnitude is a maximum at $x = -D$ and $x = +D$. The horizontal component of the net

frq7

force for small oscillations is linear and is proportional to displacement.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for identifying an appropriate feature of one of the graphs consistent with the answer in part (a).
- ☐ 1 point is earned for correctly explaining how a feature of the graph shows evidence of a restoring force.

Example Response:

Graph 2 shows evidence of a restoring force – a restoring force has a magnitude that is proportional to the displacement and opposite in direction to the displacement which is what graph 2 is showing.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for identifying that the period is incorrect,
  OR
  for identifying that the curve should intersect the horizontal axis at 1.0 seconds.
- ☐ 1 point is earned for a claim that the period is too long or that the graph should be compressed horizontally.

Example Response:

The period of the graph shown is incorrect. The problem states that the period is 2 seconds, but the graph shows a period that is longer than 2 seconds.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq7

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that the period of oscillation is longer on Track 2.
- ☐ 1 point is earned for correct reasoning why the average acceleration or average speed is less on Track 2.
- ☐ 1 point is earned for connecting the smaller average acceleration or average speed over the longer distance to a larger time.

Example Response:

The period on track 2 is longer. The block will have the same maximum speed but spread out over a much longer track means that the time to travel the length of the track and back is larger.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence of T on d, with an attempt at a justification.
- ☐ 1 point is earned for correctly indicating whether the dependence of T on d is consistent with the answer to part (d).
  Note: T should increase with d. If part (d) was answered correctly, the equation is inconsistent.

Example Response:

No this does not match my reasoning. In the equation when d is in the denominator, as d increases, T decreases, which means that if the ramp gets longer, the period should decrease, which is not what will happen.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

frq7

The response accurately includes **all** of the following criteria.

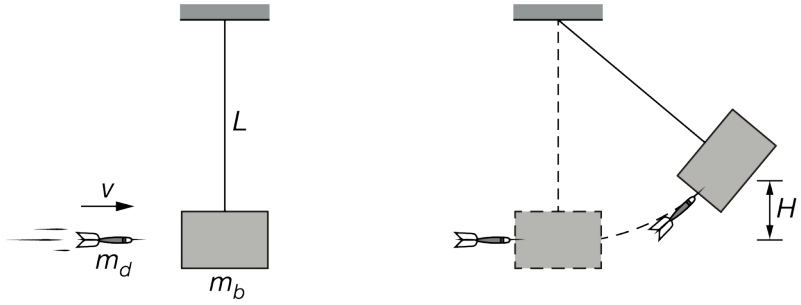
- 1 point is earned for a claim that the period of the sphere would be greater than the period of the block.
- 1 point is earned for a claim that maximum total kinetic energy of the sphere is equal to the maximum kinetic energy of the block,
  OR
  for a claim that the net force on the sphere will be less than that on the block because a frictional force is directed opposite the sphere's motion.
- 1 point is earned for recognizing that some portion of the sphere's kinetic energy is rotational,
  OR
  for recognizing that a smaller net force would result in a smaller average acceleration.

Example Response:

The period of the sphere would be greater. For both the block and the sphere the gravitational potential at the top are equal and so the kinetic energy as each passes through $x = 0$ *are equal. But for the sphere, some of that KE is turned into rotational kinetic energy, therefore it will be traveling, on average, slower than the block and so will take longer to travel the same distance.*

frq7

14. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown at left above. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown at right above. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, if either, experiences a net horizontal force of larger magnitude during the collision. If the net horizontal force is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the experiment was repeated with a dart with the same horizontal speed but with mass smaller than m_d, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the motion of the dart-block system at the instant it first reaches a height of $H/2$, and they make the following statements:

Student 1: When the dart-block system is at a height of $H/2$, the kinetic energy of the system is half of its maximum value, and therefore the speed of the system is half of its maximum speed.

Student 2: When the dart-block system reaches a height of $H/2$, the centripetal force is equal to the tension in the string. Therefore, the acceleration of the system must point toward the fixed end of the string.

(c) Which underlined phrase or phrases, if either, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if either, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if either, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

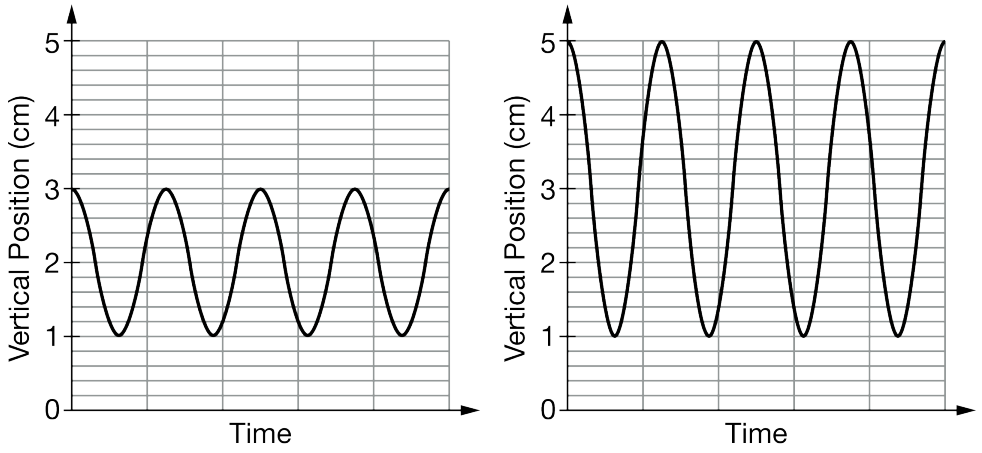
(f) Which underlined phrase or phrases, if either, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

frq7

(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{v}{1+\frac{m_b}{m_d}}\right)^2$. Explain how this equation supports your claim in part (b).

(h) Could the masses of the dart and block both be changed so that the maximum height *H* reached by the dart-block system would remain the same as in the original setup? Justify your answer by referring to the equation in part (g).

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart mass or the string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a statement that both objects experience the same net force because of Newton's third law.

frq7

Example Response:

Both the dart and the block experience the same force because they exert equal and opposite forces on each other because of Newton's third law.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the final height is smaller.
- ☐ 1 point is earned for a statement of conservation of momentum to indicate that the final speed of the dart-block system is smaller **AND** an indication of conservation of energy leading to a smaller height.

Example Response:

Because the momentum of the dart before the collision must be equal to the momentum of the dart-block system after the collision, a smaller dart mass (a smaller dart momentum) will lead to a smaller dart-block speed after the collision (a smaller dart-block momentum). Because the dart-block is traveling slower after the collision, it has less kinetic energy which means that the dart-block will not swing as high.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Responses:

Student 1 is correct in saying that the kinetic energy of the system is half of its maximum at $H/2$.

OR

The first statement

Part D

frq7

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Responses:

Student 1 is incorrect in saying that the speed of the system is half of its maximum speed

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating student 2 has no correct statements.

Example Response:

None

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating both of Student 2’s statements are incorrect.

frq7

Example Responses:

Student 2 is incorrect in saying that centripetal force is equal to the tension in the string, AND incorrect in saying that the acceleration of the system must point toward the center of the circle.

OR

Both statements

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement including functional dependence of the variables.
- ☐ 1 point is earned for correctly asserting that as the mass of the dart decreases in the equation *H* also decreases.

Example Response:

This equation matches my reasoning in part (b)– if the mass of the dart decreases, the height of the swing decreases because a decrease in the mass of the dart means an increase in the ratio $(block\ mass)/(dart\ mass)$*, which is in the denominator of the expression.*

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for “yes”, with an attempt at explanation.
- ☐ 1 point is earned for a statement that both either increase or decrease.
- ☐ 1 point is earned for a statement indicating the factor in increase/decrease (or ratio of masses) is the same.

Example Response:

Yes, if the masses were both doubled, the maximum height would still be H – it works as long as the masses both increase by the same factor .

frq7

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

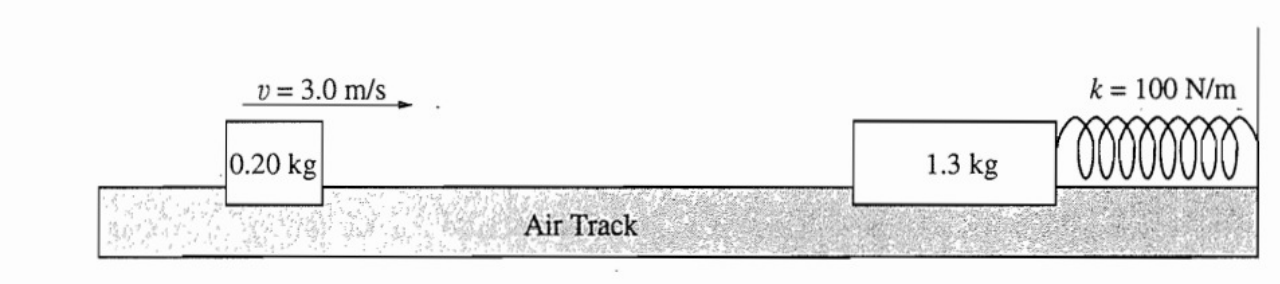
- ☐ 1 point is earned for recognizing that the amplitude of the graph for group 2 is larger.
- ☐ 1 point is earned for a statement discussing the dependence of amplitude on dart mass.
- ☐ 1 point is earned for indicating that the mass of the dart could be larger.

Example Response:

If group 2 used a dart with a larger mass, the dart would have had more momentum, and so the dart-block system would have gone higher (making the amplitude larger), but the string length is the same and so the period would still be the same.

frq7

15.



As shown above, a 0.20-kilogram mass is sliding on a horizontal friction less air track with a speed of 3.0 meters per second when it instantaneously hits and sticks to a 1.3-kilogram mass initially at rest on the track. The 1.3-kilogram mass is connected to one end of a mass less spring, which has a spring constant of 100 newtons per meter. The other end of the spring is fixed.

(a) Determine the following for the 0.20-kilogram mass immediately before the impact.

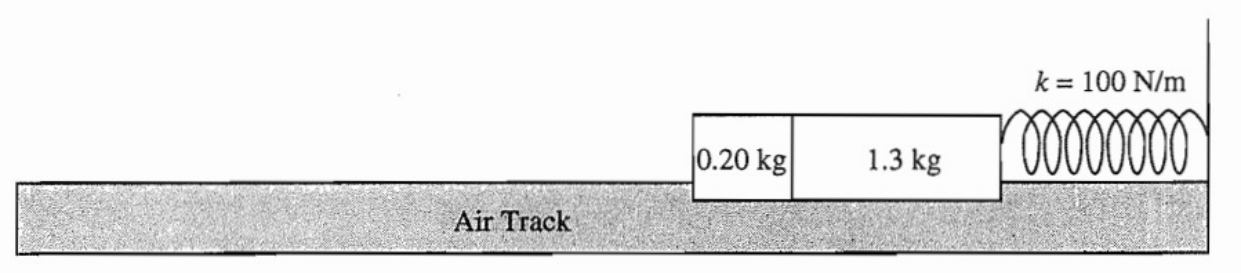
i. Its linear momentum

ii. Its kinetic energy

(b) Determine the following for the combined masses immediately after the impact

i. The linear momentum

ii. The kinetic energy



After the collision, the two masses undergo simple harmonic motion about their position at impact.

(c) Determine the amplitude of the harmonic motion.

(d) Determine the period of the harmonic motion.

Part A

(i)

1 point is earned: For a correct expression for momentum

p = mv

frq7

pi=(0.2kg)(3.0 m/s)

1 point is earned: For the correct answer

pi = 0.6kg * m/s

(ii)

1 point is earned: For a correct expression for kinetic energy

K = ½ mv2

Ki = ½(0.2 kg)(3.0 m/s)2

1 point is earned: For the correct answer

Ki = 0.9J

0	1	2	3	4 ✓

The student response earns four of the following points:

(i)

1 point is earned: For a correct expression for momentum

p = mv

pi=(0.2kg)(3.0 m/s)

1 point is earned: For the correct answer

pi = 0.6kg * m/s

(ii)

1 point is earned: For a correct expression for kinetic energy

K = ½ mv2

Ki = ½(0.2 kg)(3.0 m/s)2

1 point is earned: For the correct answer

Ki = 0.9J

Part B

(i)

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1 point is earned : For any statement of conservation of momentum

p f = p i

1 point is earned : For the correct answer

p f = 0.6kg * m/s

(ii)

Knowing the final momentum, the final velocity can be calculated

p f = m f v f

v f = p f / m f

$= \frac{0.6kg \cdot m/s}{(1.3+0.2)kg}$

1 point is earned : For the correct value of the final velocity

v f = 0.4 m/s

Using the expression for kinetic energy

K f = ½m f v f 2

1 point is earned : For using the sum of the two masses

K f = ½(1.5kg)(0.4 m/s) 2

1 point is earned : For the correct answer

K f = 0.12J

✓

0	1	2	3	4	5

The student response earns five of the following points:

(i)

1 point is earned: For any statement of conservation of momentum

pf = pi

1 point is earned: For the correct answer

pf = 0.6kg * m/s

(ii)

frq7

Knowing the final momentum, the final velocity can be calculated

pf = mfvf

vf = pf / mf

$= \frac{0.6kg \cdot m/s}{(1.3+0.2)kg}$

1 point is earned: For the correct value of the final velocity

vf = 0.4 m/s

Using the expression for kinetic energy

Kf = ½mfvf2

1 point is earned: For using the sum of the two masses

Kf = ½(1.5kg)(0.4 m/s)2

1 point is earned: For the correct answer

Kf = 0.12J

Part C

1 point is earned : For any statement of conservation of energy

The kinetic energy after the impact is equal to the maximum energy stored in the spring.

K f = ½ kA 2

Solving for the amplitude of the motion:

$A = \sqrt{2K_f/k}$

1 point is earned : For substituting the answer to (b)ii for the kinetic energy

$A = \sqrt{2(0.12J)/(100N/m)}$

1 point is earned : For the correct answer

A = 0.05 m

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned : For any statement of conservation of energy

frq7

The kinetic energy after the impact is equal to the maximum energy stored in the spring.

K f = ½ kA 2

Solving for the amplitude of the motion:

$A = \sqrt{2K_f/k}$

1 point is earned : For substituting the answer to (b)ii for the kinetic energy

$A = \sqrt{2(0.12J)/(100N/m)}$

1 point is earned : For the correct answer

A = 0.05 m

Part D

1 point is earned : For the correct expression for the period

$T = 2\pi\sqrt{m/k}$
$T = 2\pi\sqrt{(1.5kg)(100N/m)}$

1 point is earned : For the correct answer

T = 0.77 s

0	1	2 ✓

The student response earns two of the following points:

1 point is earned : For the correct expression for the period

$T = 2\pi\sqrt{m/k}$
$T = 2\pi\sqrt{(1.5kg)(100N/m)}$

1 point is earned : For the correct answer

T = 0.77 s

Units points

1 point is earned: For having both one correct energy unit and one correct momentum unit on answers in parts (a) and (b), and no incorrect units.

frq7

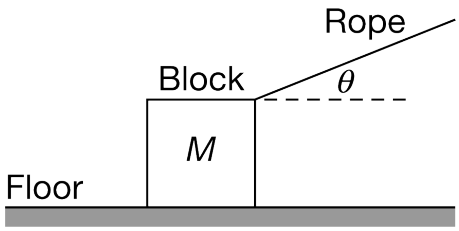
0	1 ✓

The student response earns one of the following points:

1 point is earned: For having both one correct energy unit and one correct momentum unit on answers in parts (a) and (b), and no incorrect units.

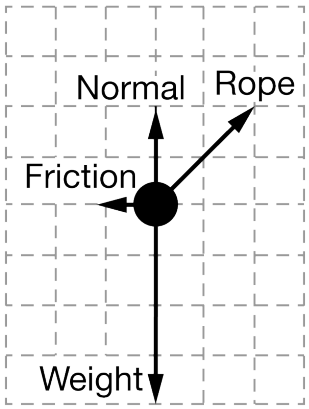
frq7

16. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

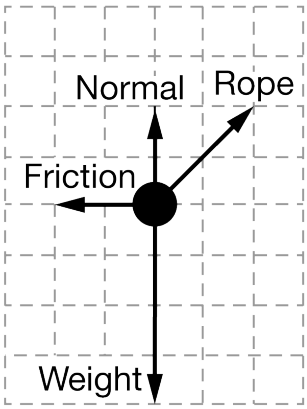


A block of mass M is pulled by a rope that makes an angle θ (theta) above the horizontal, as shown above. The coefficient of kinetic friction between the block and floor is μ (mu).

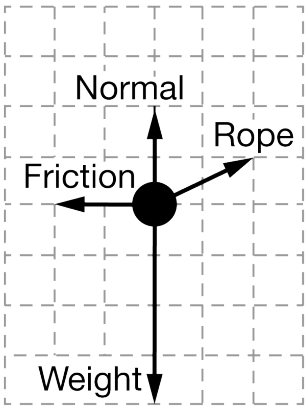
The block is initially at rest and starts accelerating to the right along the floor. Three students each draw a diagram to indicate all of the forces that are exerted on the block as it moves. In the students' diagrams, which are shown below, the lengths of the arrows indicate the relative magnitudes of the forces.



Student 1's Diagram



Student 2's Diagram



Student 3's Diagram

Only one of the students has drawn a diagram that could correctly represent the forces. The other two students' diagrams are incorrect.

(a) Which student has drawn a diagram that could correctly represent the forces exerted on the block?

(b) For the incorrect student diagrams, explain why the diagram could not represent the forces exerted on the block. If there is more than one specific reason a diagram is incorrect, identify each of the reasons.

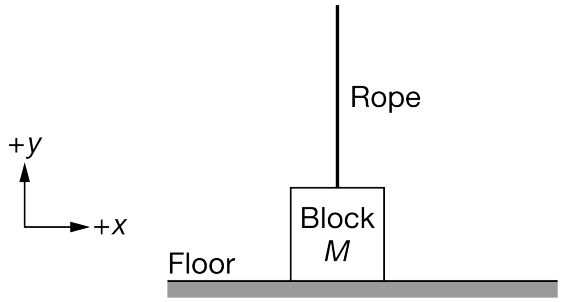
(c) Briefly explain why slightly increasing the angle θ (without changing the magnitude of the force exerted by the rope) could cause a decrease in the block's acceleration.

Suppose the rope is pulled with a force that has a magnitude that is one-third the weight of the block, or $\frac{Mg}{3}$. In this case, when the block accelerates, its acceleration is $a = g\left(\frac{1}{3}\cos\theta + \frac{\mu}{3}\sin\theta - \mu\right)$.

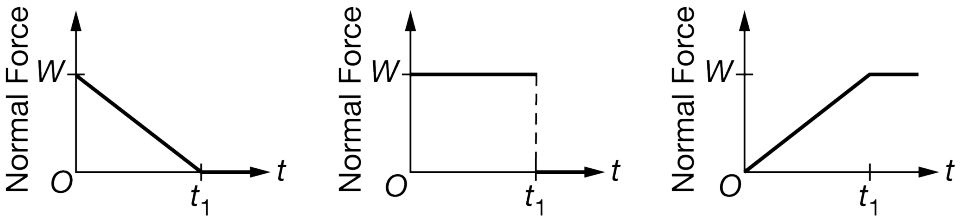
(d) Explain why the above equation is consistent with the claim that an increase in the angle θ could cause a

frq7

decrease in the acceleration.



The rope is now attached to the center of the top of the block, as shown above. The force exerted by the rope is initially zero newtons at time $t = 0$. The force steadily increases, becomes equal to the block's weight W at time t_1, and continues to increase further. Three students sketch the graphs shown below to represent the normal force exerted by the floor on the block as a function of time t. One of the graphs is consistent with this scenario, while the other graphs are not consistent with this scenario.

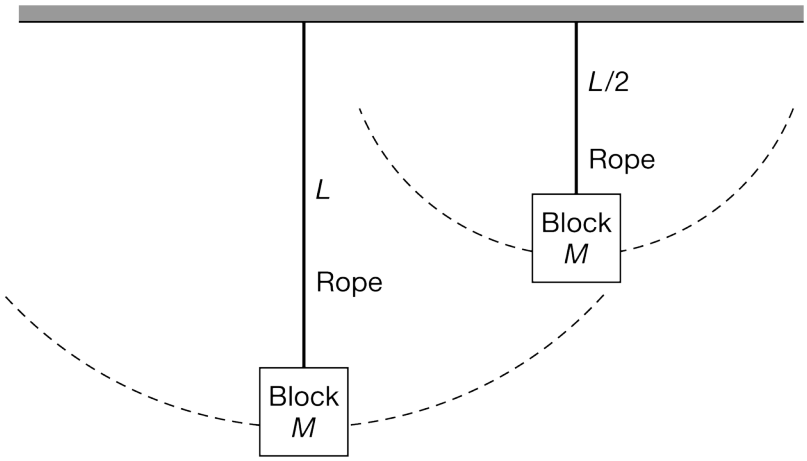


Graph 1 Graph 2 Graph 3

(e) Which graph is consistent with the scenario in which the force exerted by the rope on the block steadily increases?

(f) Identify all features of the other two graphs that make them inconsistent with this scenario. Explain your reasoning.

frq7



Note: Figure not drawn to scale.

With one end of the rope still attached to the block, the other end of the rope is attached to a ceiling to create a pendulum with length L, where L is much greater than the dimensions of the block. A second pendulum is made from an identical block and a rope of length $\frac{L}{2}$, as shown above. Both pendulums are displaced by the same small angle and released from rest.

(g) Does the longer pendulum or the shorter pendulum have a greater period of oscillation? Briefly explain your choice without reference to equations, but by describing the restoring forces exerted on and motion of each block.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating that Student 1 drew a correct diagram.

Example Response:

The diagram for student 1 is correct.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq7

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that Student 2's diagram has equal left and right forces and thus would result in a constant horizontal velocity or no horizontal acceleration.
- ☐ 1 point is earned for indicating that Student 3's diagram has equal left and right forces and thus would result in a constant horizontal velocity or no horizontal acceleration.
- ☐ 1 point is earned for indicating that Student 3's diagram would result in a downward acceleration component since the sum of the upward components is less than the force of gravity.

Example Response:

Student 2's diagram is incorrect because it has equal left and right forces, which would cause the block to move at a constant speed. Student 3's diagram is not correct because it has equal left and right forces as well. Student 3's diagram is also incorrect because the downward components are greater than the upward components. This would cause a downward acceleration, which is not occurring.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the horizontal component of the force exerted by the rope becomes smaller.
- ☐ 1 point is earned for indicating that the net force, therefore, decreases.

Example Response:

If the angle is increased without changing the magnitude, the forward horizontal force would decrease, so the net force on the block could decrease. This would result in a smaller acceleration.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq7

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the cosine term decreases when θ increases.
- ☐ 1 point is earned for indicating that, if the decrease in the cosine term is greater than the increase in the sine term, the acceleration will decrease.

Example Response:

As the angle increases, two terms in the expression are affected. The cosine term will decrease, and the sine of the angle will increase. If the decrease in the cosine term is greater than the increase in the sine term, the acceleration will decrease.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for identifying Graph 1 as consistent with the scenario in which the force exerted by the rope on the block steadily increases.

Example Response:

Graph 1 is consistent with the scenario in which the force exerted by the rope on the block steadily increases

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the normal force is equal to the weight of the block minus the tension force, until the tension equals the weight.

frq7

- [ ] 1 point is earned for indicating that graph 2 is incorrect because the normal force does not remain constant before time t_1.
- [ ] 1 point is earned for indicating that graph 3 is incorrect because the normal force does not increase before time t_1.
- [ ] 1 point is earned for indicating that graph 3 is incorrect because the normal force does not equal the weight after time t_1.

Example Response:

The normal force on the block is equal to the difference between the weight and the tension force, as long as the tension force is less than the weight. If the tension force exceeds the weight, then the normal force becomes zero because the block is lifted off the floor. Thus, graph 2 is incorrect because it indicates the normal force is constant until the tension equals the weight. And graph 3 is incorrect because the normal force is depicted as increasing until it equals the weight rather than decreasing until it is zero.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for indicating that both pendulums have the same maximum restoring force at the same maximum angular displacement from equilibrium.
- [ ] 1 point is earned for indicating that the path the longer pendulum takes is a greater distance so for the same acceleration, it takes a greater time, and thus the longer pendulum has a greater period.

Example Response:

Since both are displaced the same angle, both have the same maximum restoring force, and thus the same maximum acceleration. The longer pendulum travels a longer path, and thus it will take a longer time. Thus, the period of the longer pendulum is greater.

frq7

17. This question is a short free-response question. Show your work for each part of the question.

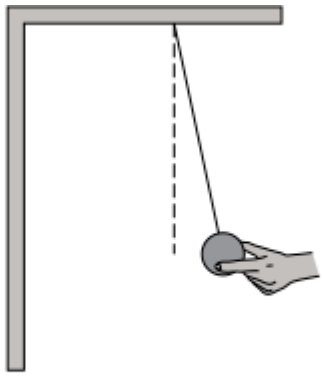


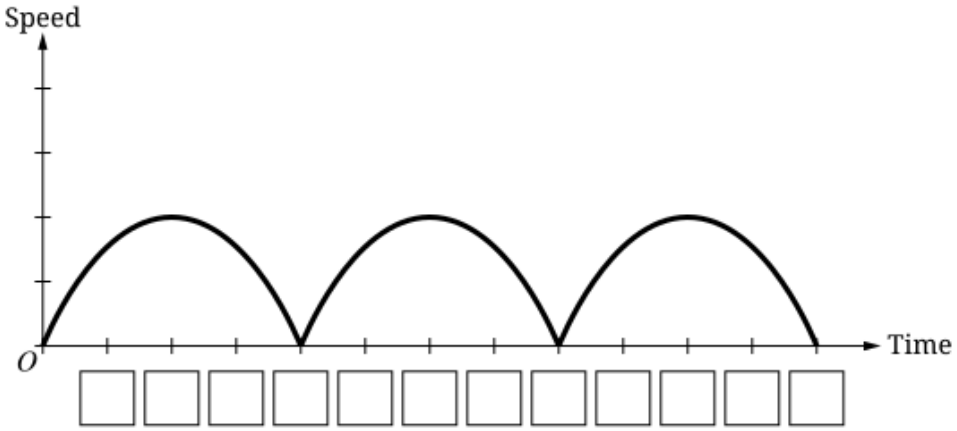
Figure 1

Note: Figure not drawn to scale.

(7 points, suggested time 13 minutes)

Student A and Student B conduct an experiment using a pendulum that consists of a metal sphere attached to a lightweight string that hangs from the ceiling, as shown in Figure 1. The students displace the pendulum a small angle from equilibrium and release it from rest at time $t = 0$.

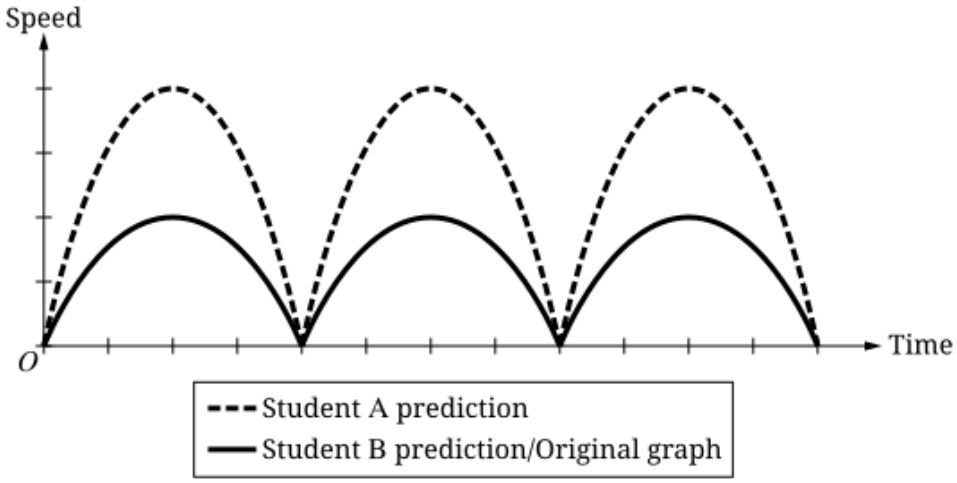
(a) The students create the following graph of the speed of the sphere as a function of time.



i. The sphere first reaches the lowest point of its motion (when the string is vertical) at time $t = t_1$. Indicate time t_1 by writing "t_1" in the appropriate box below the horizontal axis.

ii. The students have a new pendulum, identical to the original except that its sphere has twice the mass. The

students plan to repeat the experiment by releasing the new sphere from rest at the same small angle as before. Student A predicts the graph of speed versus time will look different, and Student B predicts it will look the same as the graph in part (a)(i). The students' predictions are shown in the following graph.



Which student's prediction is correct?

_____Student A _____Student B

Briefly explain your reasoning.

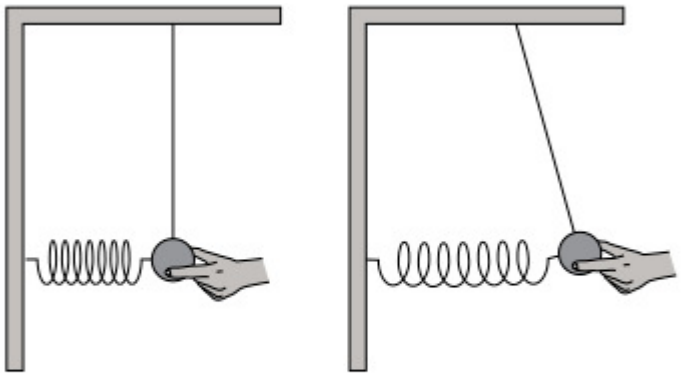


Figure 2 Figure 3

Note: Figures not drawn to scale.

(b) The students attach one end of a light spring to the sphere of the original pendulum and the other end to the wall. Figure 2 shows the new setup when the spring is oriented horizontally at its relaxed length and the string is vertical. Figure 3 shows the pendulum when the students displace the sphere the same small angle as in

frq7

part (a).

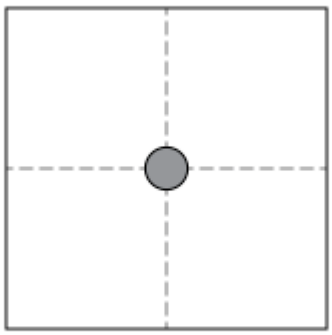
At time $t = 0$, the sphere is then released from rest at this position.

i. The sphere reaches the lowest point of its motion at time $t = t_2$. Compare t_2 to the time at which the sphere reaches its lowest point in part (a)(i), t_1. Is t_2 greater than, less than, or equal to t_1 ?

_____$t_2 > t_1$ _____$t_2 < t_1$ _____$t_2 = t_1$

Briefly justify your reasoning.

ii. The following dot represents the center of mass of the sphere at time t_2 when the sphere reaches the lowest point of its motion. Draw a single arrow representing the direction of the instantaneous acceleration of the sphere. If the acceleration is zero, write "$a = 0$" next to the dot.



iii. After having passed its lowest point, the sphere continues moving toward the wall and momentarily comes to rest. At the moment when the sphere comes to rest, how does the magnitude of the angle the string makes with the vertical compare to the angle from which it was released? Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

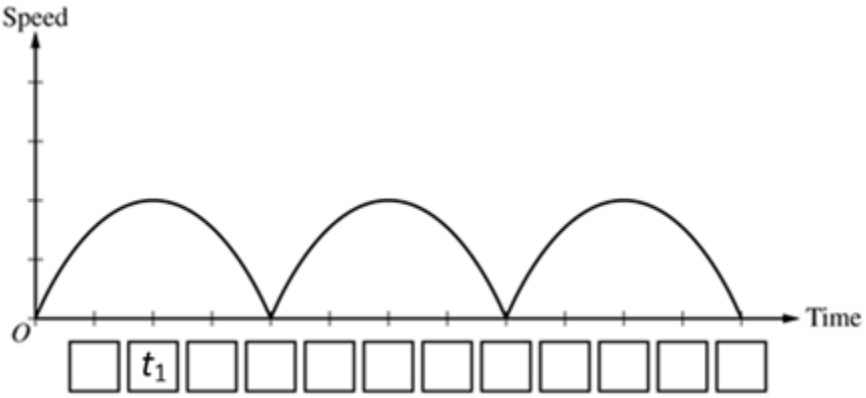
The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for writing t_1 in the correct box below the axis.

frq7

- [ ] 1 point is earned for choosing "Student B" with a relevant justification
  Accept **one** of the following:
  - The equation of the period of the pendulum does not contain mass
  - Mass cancels out when gravitational potential energy is equated to kinetic energy

Example Response



Student B's prediction is correct. The equation for the period of a pendulum does not contain the mass, so the period is unchanged, causing the pendulum to reach the same maximum speed.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5 ✓

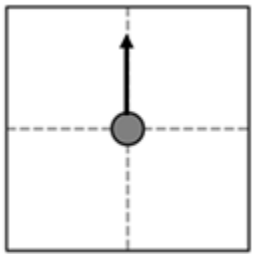
The response accurately includes **all** the following criteria.

- [ ] 1 point is earned for choosing "$t_2 < t_1$" with an attempt at a relevant justification.
- [ ] 1 point is earned for indicating **one** of the following:
  - The greater force results in a greater speed / acceleration
  - The greater spring potential energy results in a greater kinetic energy / speed
- [ ] 1 point is earned for an arrow that points toward the top of the page.
- [ ] 1 point is earned for stating that it is the same angle.
- [ ] 1 point is earned for an explanation that indicates conservation of energy.

Example Response

frq7

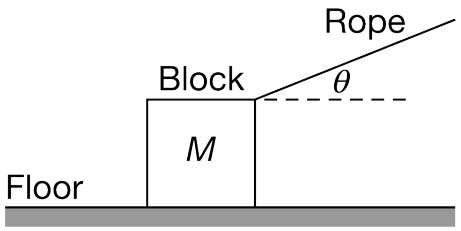
$t_2 < t_1$ *because the net force exerted on the sphere is greater due to the addition of the spring force. A greater net force means that there will also be a greater acceleration.*



The displacement angle will be the same as the original displacement because the total mechanical energy of the sphere-spring-Earth system is constant. The initial elastic and gravitational potential energies are converted to kinetic energy at the equilibrium point and then back to elastic and gravitational potential energies at the turning point.

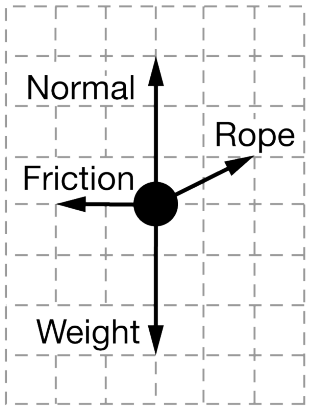
frq7

18. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

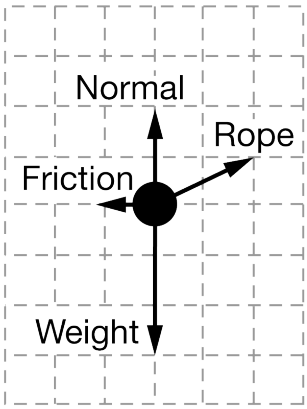


A block of mass M is pulled by a rope that makes an angle θ (theta) above the horizontal, as shown above. The coefficient of kinetic friction between the block and floor is μ (mu).

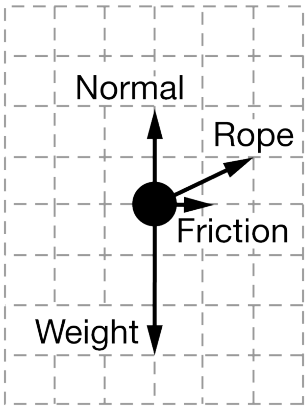
The block is initially at rest and starts accelerating to the right along the floor. Three students each draw a diagram to indicate all of the forces that are exerted on the block as it moves. In the students' diagrams, which are shown below, the lengths of the arrows indicate the relative magnitudes of the forces.



Student 1's Diagram



Student 2's Diagram



Student 3's Diagram

Only one of the students has drawn a diagram that could correctly represent the forces. The other two students' diagrams are incorrect.

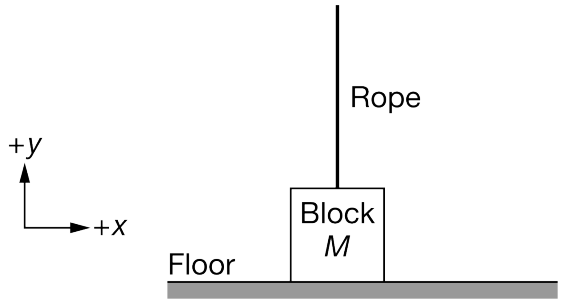
(a) Which student has drawn a diagram that could correctly represent the forces exerted on the block?

(b) For the incorrect student diagrams, explain why the diagram could not represent the forces exerted on the block. If there is more than one specific reason a diagram is incorrect, identify each of the reasons.

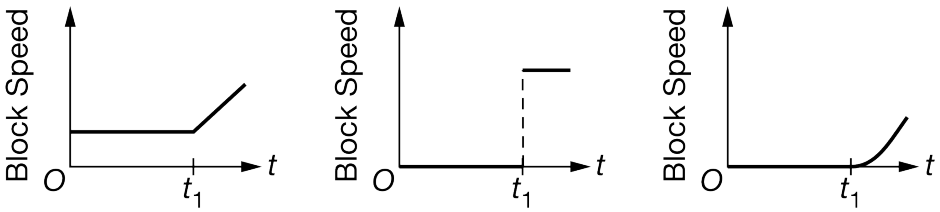
(c) Briefly explain why slightly increasing the force F exerted by the rope on the block (without changing the angle θ) would cause a decrease in the friction force.

(d) The friction force f is $f = \mu(W - F\sin\theta)$, where W is the weight of the block and F is the force exerted by the rope. Explain why this equation is consistent with the claim that an increase in F would cause a decrease in the friction force.

frq7



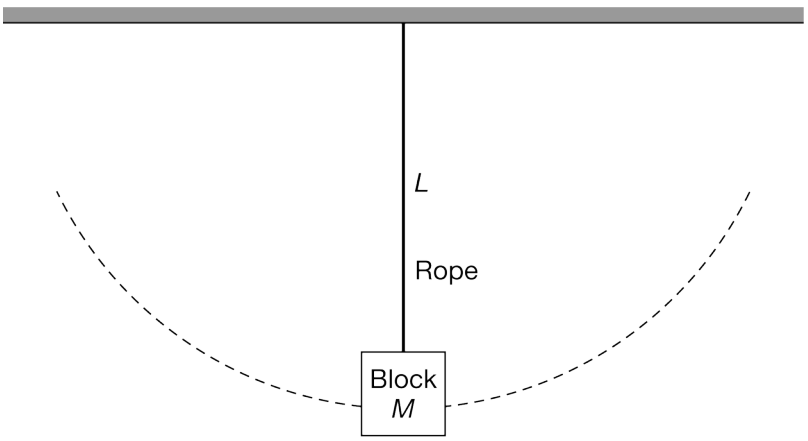
The rope is now attached to the center of the top of the block, as shown above. The force exerted by the rope is initially zero newtons at time $t = 0$. The force steadily increases, becomes equal to the block's weight W at time t_1, and continues to increase further. Three students sketch the graphs shown below to represent the speed of the block as a function of time t. One of the graphs is consistent with this scenario, while the other graphs are not consistent with this scenario.



Graph 1 Graph 2 Graph 3

(e) Which graph is consistent with the scenario in which the force exerted by the rope on the block steadily increases?

(f) Identify all features of the other two graphs that make them inconsistent with this scenario. Explain your reasoning.



Note: Figure not drawn to scale.

frq7

With one end of the rope still attached to the block, the other end of the rope is attached to a ceiling to create a pendulum with length L, where L is much greater than the dimensions of the block. A second identical pendulum is located on the Moon, where the acceleration due to gravity is less than that on Earth. Both pendulums are displaced by the same small angle and released from rest.

(g) Does the pendulum on Earth or the pendulum on the Moon have a greater period of oscillation? Briefly explain your choice without reference to equations, but by describing the restoring forces exerted on and motion of each block.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating that Student 2 drew a correct diagram.

Example Response:

The diagram for student 2 is correct.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that Student 1's diagram has equal left and right forces and thus would result in a constant horizontal velocity or no horizontal acceleration.
- ☐ 1 point is earned for indicating that Student 1's diagram would result in an upward acceleration component since the sum of the upward components is greater than the force of gravity.
- ☐ 1 point is earned for indicating that Student 3's diagram has friction in the incorrect direction.

Example Response:

Student 3 has correctly balanced the vertical forces but has the friction in the same direction as the motion. Student 1 has vertical components in the upward direction that are greater than gravity, so the block would have an upward acceleration. Also, student 1 has equal horizontal components which would not cause an acceleration to the right.

frq7

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that as the pulling force increases, its vertical component increases, decreasing the normal force on the block.
- ☐ 1 point is earned for indicating that since the friction force is proportional to the normal force, the friction force decreases.

Example Response:

As the pulling force increases, its vertical component increases, decreasing the normal force on the block. Since the friction force is proportional to the normal force, the friction force decreases.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for any attempt to use a functional dependence between the friction force and the force *F*.
- ☐ 1 point is earned for indicating that the term with F is subtracted, or that the expression $(W - F\sin\theta)$ decreases when *F* increases.

Example Response:

In the given expression for friction, there is a term that depends on the pulling force F, but that term is subtracted from the weight term. So as F increases, friction decreases.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq7

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for identifying Graph 3 as consistent with the scenario in which the force exerted by the rope on the block steadily increases.

Example Response:

Graph 3 is consistent with the scenario in which the force exerted by the rope on the block steadily increases.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria.

- [ ] 1 point is earned for indicating that graph 1 shows an initial non-zero speed even though the initial speed is zero.
  Note: If it was indicated that there is ONLY a horizontal component of the velocity before the vertical force of the rope acts on the block (from the previous part of the question), this point can be earned for thinking that graph 1 is correct in the first segment but graphs 2 & 3 are not correct in the first segment.
- [ ] 1 point is earned for indicating that the speed after time in graph 1 is incorrect because the upward force is not constant, so the acceleration (slope of speed graph) should not be constant.
- [ ] 1 point is earned for indicating that t_1 graph 2 is incorrect because of the discontinuity in the graph at t_1.
- [ ] 1 point is earned for indicating that graph 2 is incorrect because the speed is not constant after time t_1, it increases since there is a net force.

Example Response:

Graph 1 is incorrect because the speed should be zero before time t_1 and after t_1 it should have an increasing slope since the net force is increasing. Graph 2 is incorrect because, although the speed is initially zero, the speed is not constant after time t_1 because there is an upward net force.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: A response *can* mention the equation for a pendulum, and still get full credit as long as the points described below are earned.

frq7

0	1	2 ✓

The response accurately includes **both** of the following criteria.

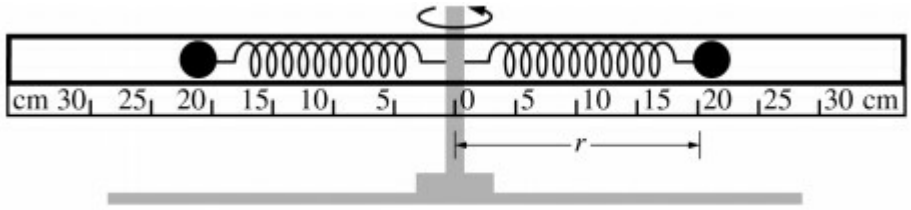
- ☐ 1 point is earned for indicating that the pendulum on the Earth has a greater restoring force at maximum displacement from equilibrium.
- ☐ 1 point is earned for indicating that since the two pendulums have the same length, the amplitude of the motion is the same, and thus the pendulum on the moon, with a smaller acceleration, will take longer to complete once cycle.

Example Response:

Since both pendulums have the same length, they will both have the same amplitude. But the pendulum on the Earth will have a greater restoring force and acceleration, so it will take less time to return to equilibrium. Thus, the pendulum on the moon will have a greater period.

frq7

19.



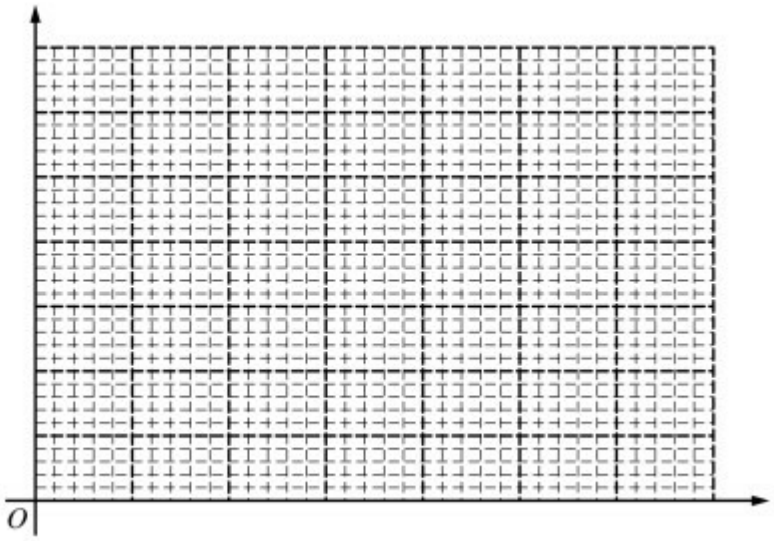
The apparatus shown above consists of two identical springs of negligible mass, each with spring constant k = 20.4 N/m and each attached at one end to a vertical rotating pole. Identical small spheres of mass m are attached to the other ends of the springs. The spheres are constrained to horizontal motion by horizontal guides of negligible friction, each of which has a ruler below it for measuring the radial position r of the sphere. The system can be manually rotated about the pole's axis. In a lab experiment, a student adjusts the rotational speed so that the spheres move to a desired radius r. For each such value of r, the student measures the rotational period T. The student's partially completed data table is shown below. The length of each unstretched spring is L = 0.15 m.

Trial	Radial Position r (m)	Period T (s)	Acceleration a_c $\left(\text{m/s}^2\right)$	Spring Force F_{spring} (N)
1	0.300	0.440	61.2	3.06
2	0.270	0.475	47.2	2.45
3	0.240	0.530		
4	0.210	0.570	25.5	1.22
5	0.180	0.795	11.2	0.61

(a) Calculate the missing values from the table above. Show your work in the space below

(b) On the axes below, plot the data points for the spring force F_{spring} as a function of the acceleration a_c. Label the axes, including the scale. Draw a straight line that best represents the data

frq7

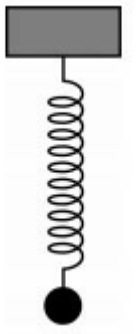


(c)

i. Calculate the slope of your line.

ii. Indicate what the slope calculated in part (c)-i represents.

(d) One sphere and one spring are removed from the rotation apparatus. They are hung vertically so that the sphere is now suspended from the spring, as shown below.



i. Describe a procedure you could use, and the measurements you would make, to verify the value obtained in part (c) using the setup shown above.

ii. Show how you would use the measurements described in part (d)-i to verify the value obtained in part (c).

Part A

frq7

Trial	Radial Position r (m)	Period T (s)	Acceleration a_c $\left(\text{m/s}^2\right)$	Spring Force F_{spring} (N)
1	0.300	0.440	61.2	3.06
2	0.270	0.475	47.2	2.45
3	0.240	0.530	33.7	1.84
4	0.210	0.570	25.5	1.22
5	0.180	0.795	11.2	0.61

1 point is earned for using a correct expression for the centripetal acceleration

$a_c = \frac{v^2}{r}$

1 point is earned for substituting an expression for speed in terms of period

$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{(4\pi^2)(0.240m)}{(0.530s)^2}$

1 point is earned for a correct answer for the centripetal acceleration

ac = 33.7 m/s2

1 point is earned for using a correct expression for the magnitude of the spring force

*F*spring = *kx*

1 point is earned for substituting the correct distance for the stretch of the spring

*F*spring = (20.4 N/m)(0.240 m - 0.15 m)

Correct answer

*F*spring = 1.84 N

0	1	2	3	4	5 ✓

The student response earns all of the following points:

frq7

Trial	Radial Position r (m)	Period T (s)	Acceleration a_c $\left(\text{m/s}^2\right)$	Spring Force F_{spring} (N)
1	0.300	0.440	61.2	3.06
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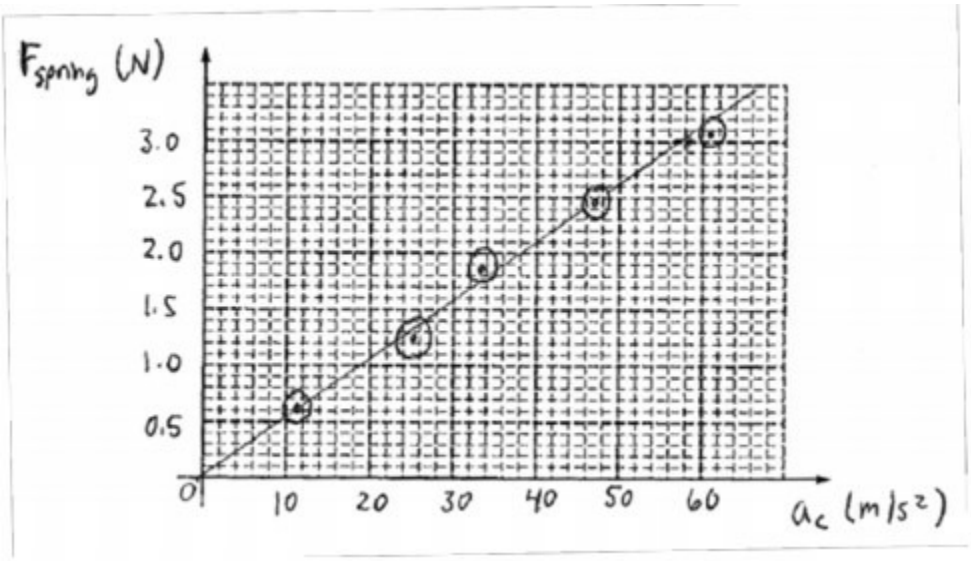
*F*spring = (20.4 N/m)(0.240 m - 0.15 m)

Correct answer

*F*spring = 1.84 N

Part B

frq7



1 point is earned for properly labeling both axes with variables and units

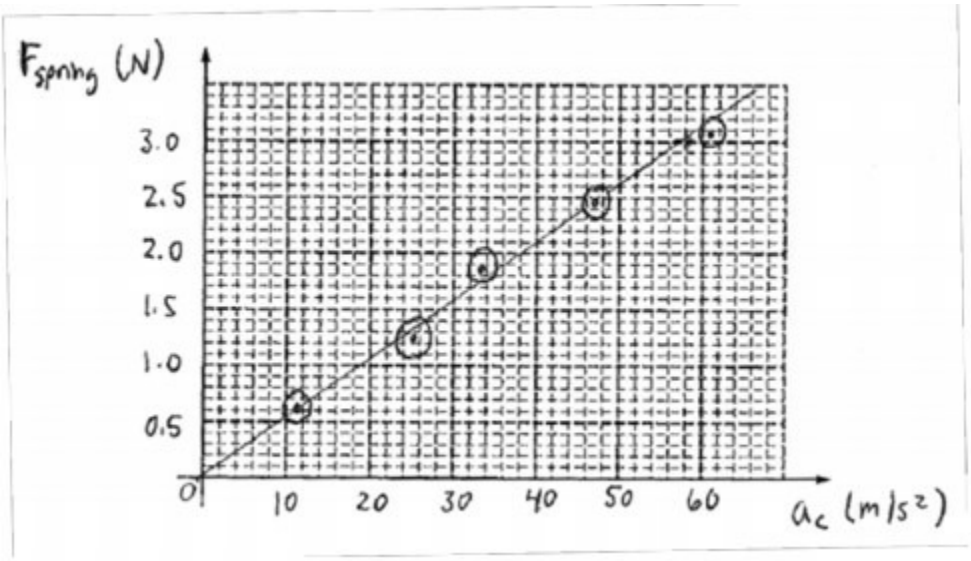
1 point is earned for using and labeling a proper scale on both axes, to use approximately half the grid

1 point is earned for correctly plotting the data points

1 point is earned for drawing a straight line that best represents the data

0	1	2	3	4 ✓

The student response earns all of the following points:



frq7

1 point is earned for properly labeling both axes with variables and units

1 point is earned for using and labeling a proper scale on both axes, to use approximately half the grid

1 point is earned for correctly plotting the data points

1 point is earned for drawing a straight line that best represents the data

Part C

1 point is earned for calculating the slope using points on the straight line in part (b)

$slope = \frac{(3.0N - 0)}{(58m/s^2 - 0)}$

1 point is earned for a correct answer with correct units

slope = 0.052 kg

1 point is earned for indicating the slope is the mass of each sphere

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned for calculating the slope using points on the straight line in part (b)

$slope = \frac{(3.0N - 0)}{(58m/s^2 - 0)}$

1 point is earned for a correct answer with correct units

slope = 0.052 kg

1 point is earned for indicating the slope is the mass of each sphere

Part D

1 point is earned for a procedure that could be used to verify the value found in part (c)

1 point is earned for indicating the measurements that must be made

- Equilibrium Example: Let the sphere hang in equilibrium. Measure how far the spring has stretched from its unstretched length.
- Oscillation Example: You can pull down and release the sphere so that it oscillates and measure the period of oscillation.

frq7

1 point is earned for a correct indication of how the measurements can be used to confirm the value from part (c)

- Equilibrium Example: Once you know the stretch of the spring and you already know the spring constant, you can set the gravitational force equal to the spring force (Fspring = mg = kx) and solve for the mass of the sphere.
- Oscillation Example: With the period of oscillation and the spring constant known, you could use the period of a spring equation $\left(T_s = 2\pi\sqrt{k/m}\right)$ to determine the mass of the sphere.

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned for a procedure that could be used to verify the value found in part (c)

1 point is earned for indicating the measurements that must be made

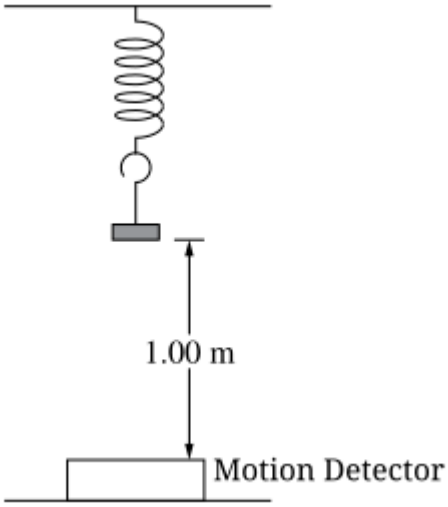
- Equilibrium Example: Let the sphere hang in equilibrium. Measure how far the spring has stretched from its unstretched length.
- Oscillation Example: You can pull down and release the sphere so that it oscillates and measure the period of oscillation.

1 point is earned for a correct indication of how the measurements can be used to confirm the value from part (c)

- Equilibrium Example: Once you know the stretch of the spring and you already know the spring constant, you can set the gravitational force equal to the spring force (Fspring = mg = kx) and solve for the mass of the sphere.
- Oscillation Example: With the period of oscillation and the spring constant known, you could use the period of a spring equation $\left(T_s = 2\pi\sqrt{k/m}\right)$ to determine the mass of the sphere.

frq7

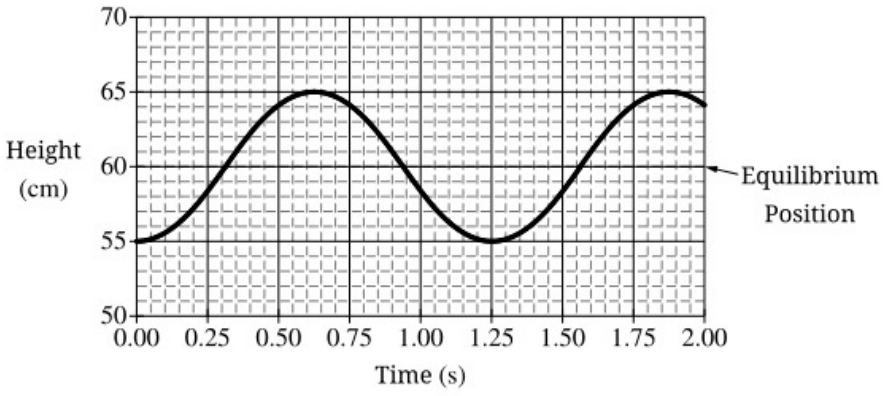
20. This question is a short free-response question. Show your work for each part of the question.



(7 points, suggested time 13 minutes)

A spring of unknown spring constant k_0 is attached to a ceiling. A lightweight hanger is attached to the lower end of the spring, and a motion detector is placed on the floor facing upward directly under the hanger, as shown in the figure above. The bottom of the hanger is $1.00\ \text{m}$ above the motion detector.

A $0.50\ \text{kg}$ object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance d_0 from equilibrium and released at time $t = 0$. The motion detector records the height of the bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.



(a) Using the information given and information taken from the graph, calculate the spring constant.

frq7

(b) At time 0.75 s, the object-spring-Earth system has a total kinetic energy K_0 and a total potential energy U_0. At 1.13 s, the object-spring-Earth system again has a total kinetic energy K_0 and a total potential energy U_0.

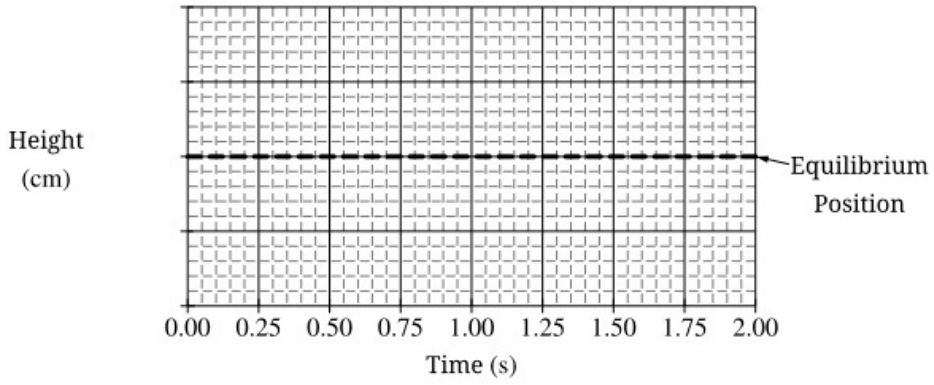
i. Explain how a feature of the graph indicates that the total kinetic energy of the system is the same at these two times.

ii. Briefly explain why the total potential energy of the system is the same at these two times.

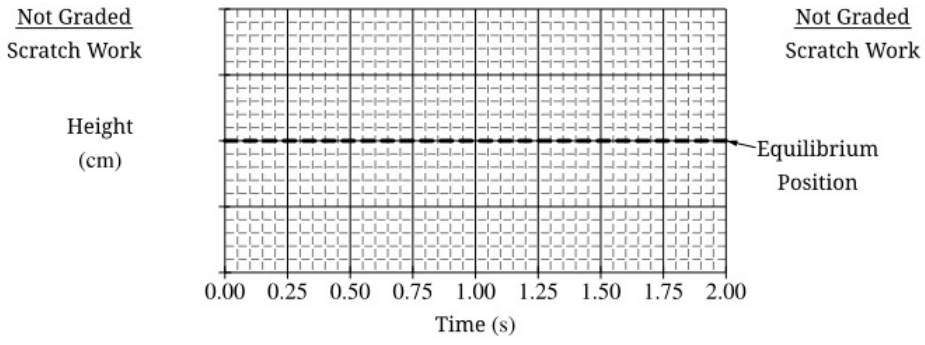
(c) The experiment is repeated with a spring of spring constant $4k_0$ and that has the same length as the original spring. The 0.50 kg object is hung from the new spring and allowed to come to rest at a new equilibrium position.

i. Determine the new equilibrium position above the motion detector.

ii. The object is again pulled down the same distance d_0 from the equilibrium position and released. On the following graph, draw a curve representing the motion of the object after it is released. Label the vertical axis with an appropriate numerical scale. A grid for scratch (practice) work is also provided.



The following graph is provided for scratch work only and will not be graded.



Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq7

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for obtaining a period from the graph of 1.25 seconds.
- ☐ 1 point is earned for substituting the values of period and mass into a valid equation for the spring constant.

  Example Response:

  $T = 1.25 \text{ s}$

  $T = 2\pi\sqrt{\frac{m}{k}}$

  $k_0 = \frac{m}{\left(\frac{T}{2\pi}\right)^2} = \frac{0.50 \text{ kg}}{\left(\frac{1.25 \text{ s}}{2\pi}\right)^2} = 12.6 \text{ N/m}$

- ☐ **Alternate Solution to part (a)**

  1 point is earned for stating that the spring is stretched 0.40 m to its equilibrium position (since the equilibrium height is 60 cm and the original height was 100 cm).
- ☐ 1 point is earned for substituting the amount of spring stretch and mass into a valid equation for the spring constant.

  Example Response (Alternate Solution):

  The string is stretched 0.40 m *under a force of* $mg = 5 \text{ N}$. *Since* $F = -kx$, *we have* $k = \frac{mg}{x} = \frac{5}{0.4} \text{ N/m} = 12.5 \text{ N/m}$.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a valid reason why the kinetic energy is the same at both times.

  Example Responses:

  The magnitude of the slope of the graph is the same at both times, this means the speed and therefore the kinetic energy is the same at both times

  OR

  The object is the same distance from equilibrium at both times, so the kinetic energy must be the same.
- ☐ 1 point is earned for a valid reason why the total potential energy is the same at both times.

  Example Responses:

  The total energy of the system is constant, so if K is the same at both times, U must be also.

  OR

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The total energy of the system is constant, and equal energy is transferred from gravitational potential to spring potential.

Part C

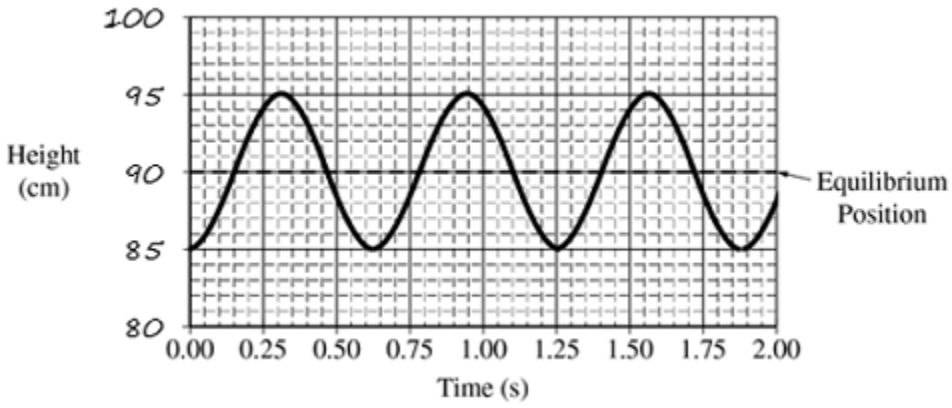
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** the following criteria.

- ☐ 1 point is earned for writing $90\ \text{cm}$ or $0.90\ \text{m}$.
- ☐ 1 point is earned for **both** of the following:
  · A graph that has the same amplitude as the original graph.
  · A graph that is centered on the new equilibrium value consistent with first section of c.
- ☐ 1 point is earned for a graph with half the period as the original graph

Example Response:



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21. This question is a short free-response question. Show your work for each part of the question.

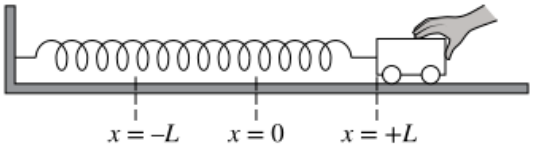


Figure 1

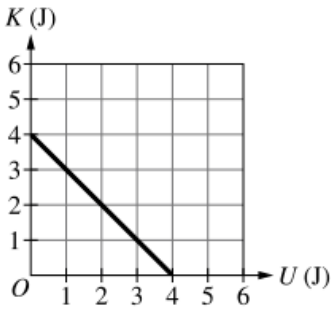


Figure 2

(7 points, suggested time 13 minutes)

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions $x = +L$ and $x = -L$. Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.

(a) On the graph of kinetic energy K versus potential energy U shown in Figure 2, the values for the x-intercept and y-intercept are the same. Briefly explain why this is true, using physics principles.

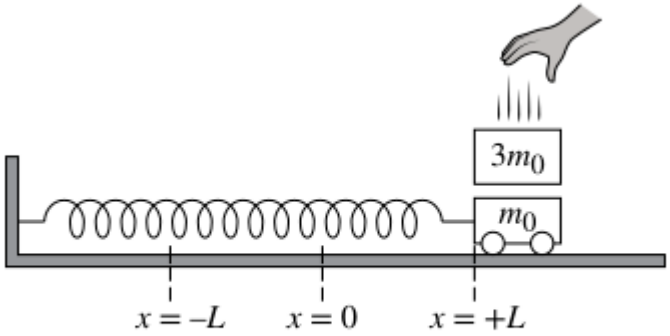


Figure 3

When the cart is at $+L$ and momentarily at rest, a block is dropped onto the cart, as shown in Figure 3. The block sticks to the cart, and the block-cart-spring system continues to oscillate between $-L$ and $+L$. The masses of the cart and the block are m_0 and $3m_0$, respectively.

(b) The frequency of oscillation before the block is dropped onto the cart is f_1. The frequency of oscillation after

frq7

the block is dropped onto the cart is f_2. Calculate the numerical value of the ratio $\frac{f_2}{f_1}$.

(c) The dashed line in Figure 4 shows the kinetic energy K versus potential energy U of the block-cart-spring system after the block is dropped onto the cart. This graph is identical to the graph shown in Figure 2 for the cart-spring system before the block is dropped onto the cart.

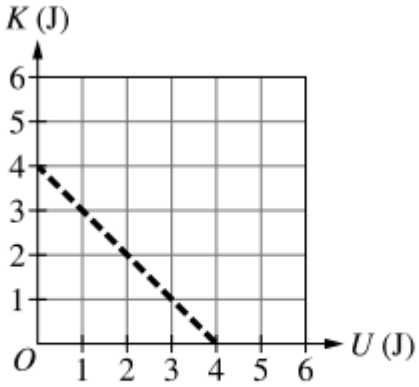


Figure 4

i. Briefly explain why the two graphs must be the same, using physics principles.

ii. After the block is dropped onto the cart, consider a system that consists only of the cart and the spring. On Figure 4, sketch a solid line that shows the kinetic energy of the system that consists of the cart and the spring but not the block after the block is dropped onto the cart.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes the following criterion.

- ☐ 1 point is earned for an explanation that indicates that the maximum kinetic energy and maximum potential energy are the same due to energy conservation.
  Scoring Note: This point may be earned for only stating “conservation of energy.”

Example Response

The maximum kinetic energy and maximum potential energy of the car-spring system are both $4\ \text{J}$*, because energy is conserved in this system.*

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Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** the following criteria.

- ☐ 1 point is earned for using the equation for frequency or period in a ratio.
- ☐ 1 point is earned for substituting the total mass $4m_0$ into the correct ratio: $\frac{f_2}{f_1}$ or $\frac{T_1}{T_2}$.

Example Response

$\frac{\frac{1}{2\pi}\sqrt{\frac{k}{m_2}}}{\frac{1}{2\pi}\sqrt{\frac{k}{m_1}}}$ OR $\frac{\frac{1}{2\pi}\sqrt{\frac{k}{m_1}}}{\frac{1}{2\pi}\sqrt{\frac{k}{m_2}}}$ OR $\frac{2\pi\sqrt{\frac{m_2}{k}}}{2\pi\sqrt{\frac{m_1}{k}}}$ OR $\frac{2\pi\sqrt{\frac{m_1}{k}}}{2\pi\sqrt{\frac{m_2}{k}}}$

Scoring Note: Simplified versions of the above ratios also earn this point.

$T = 2\pi\sqrt{\frac{m}{k}}$

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\frac{f_2}{f_1} = \frac{\frac{1}{2\pi}\sqrt{\frac{k}{4m_0}}}{\frac{1}{2\pi}\sqrt{\frac{k}{m_0}}}$

$\frac{f_2}{f_1} = \frac{1}{2}$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** the following criteria.

- ☐ 1 point is earned for a valid explanation in terms of work or energy for why the systems' energies should be the same.
  Accept **one** of the following:
  - No work is done on the system

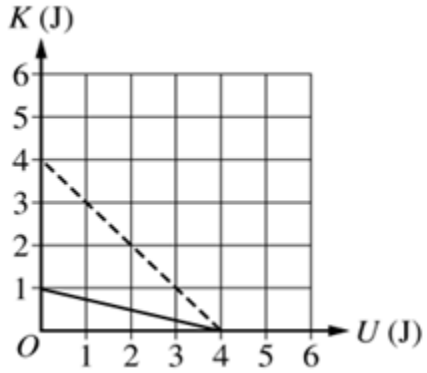
frq7

- The maximum spring potential energy is the same
- The force exerted on the system is perpendicular to the direction of motion

- [ ] 1 point is earned for drawing a single straight line with a horizontal intercept that is the same as the horizontal intercept of the original graph of $4\,\mathrm{J}$.
- [ ] 1 point is earned for drawing a line with a vertical intercept that is less than the vertical intercept in the original graph.
- [ ] 1 point is earned for drawing a line with the correct vertical intercept of $1\mathrm{J}$.

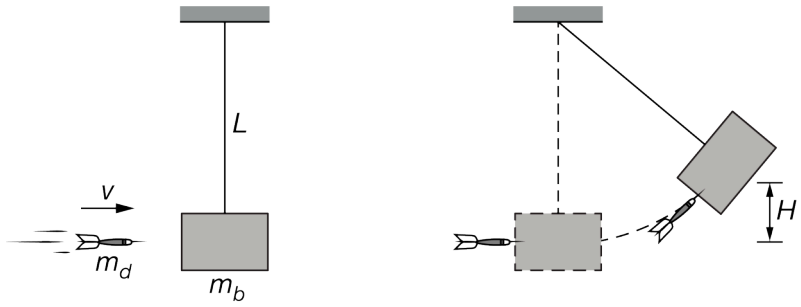
Example Response

The maximum potential energy of the system does not depend upon the mass of the system, therefore there will be no change when the block is added.



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22. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown above at left. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown above at right. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, if either, experiences an acceleration of larger magnitude during the collision. If the acceleration is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the experiment were repeated with a block with mass greater than m_b, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the tension in the string at the moment immediately after the collision and state the following:

Student 1: The tension in the string immediately after the collision is greater than the weight of the dart-block system because the acceleration is upward at that instant.

Student 2: The tension in the string is equal to the weight of the block-dart system because there is no vertical acceleration immediately after the collision.

(c) Which underlined phrase or phrases, if any, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if any, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if any, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

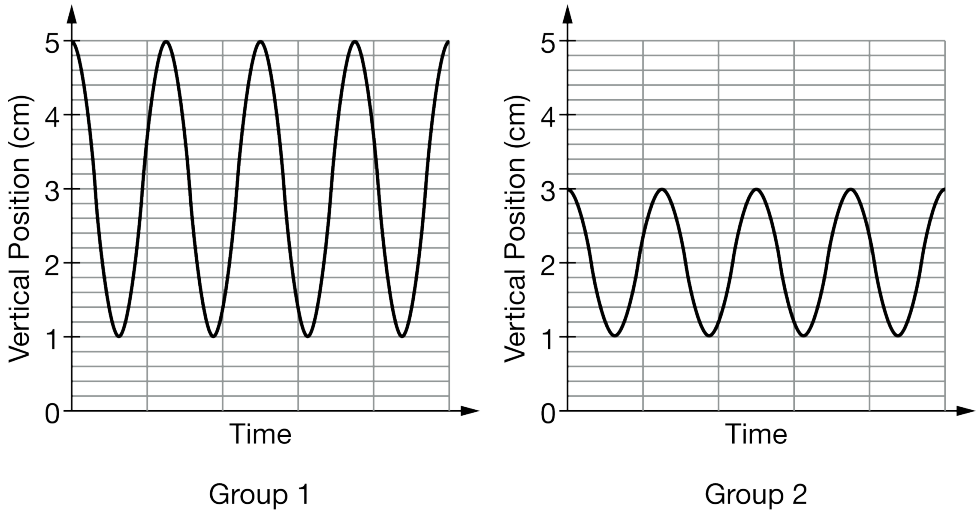
(f) Which underlined phrase or phrases, if any, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{m_d v}{m_d + m_b}\right)^2$. Explain how this equation supports your claim in part (b).

frq7

(h) Consider the case in which the collision between the dart and the block is perfectly elastic (the dart bounces off the block to the left). In this case, is the speed of the block immediately after the collision greater than, less than, or the same as the speed of the block in the original case? Justify your answer.

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart speed or the string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a statement that the dart experiences the larger acceleration because it has the smaller mass with equal force exerted on it because of Newton's third law.

Example Response:

Both the dart and the block experience the same force because they exert equal and opposite forces on each other because of Newton's third law, however the mass of the dart is smaller and so experiences a larger acceleration.

frq7

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the final height is smaller.
- ☐ 1 point is earned for a statement of conservation of momentum to indicate that the final speed of the dart-block system is smaller **AND** an indication of conservation of energy leading to a smaller height.

Example Response:

Because the momentum of the dart before the collision must be equal to the momentum of the dart-block system after the collision, a larger block mass will lead to a smaller dart-block speed after the collision because of conservation of momentum. Because the dart-block system is traveling slower after the collision, it has less kinetic energy which means that the dart-block system will not swing as high.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating both of Student 1's statements are correct.

Example Responses:

Student 1's entire statement is correct

OR

Both statements

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq7

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating student 1 has no incorrect statements.

Example Response:

None

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating student 2 has no correct statements.

Example Response:

None

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating both of Student 2's statements are incorrect.

Example Responses:

Student 2's entire statement is incorrect.

OR

frq7

Both statements

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement including functional dependence of the variables.
- ☐ 1 point is earned for correctly asserting that as the mass of the block increases in the equation H decreases.

Example Response:

This equation matches my reasoning in part b – if the mass of the block increases, the height of the swing decreases because the mass of the block is in the denominator.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for "greater", with an attempt at an explanation.
- ☐ 1 point is earned for a statement indicating a larger change in momentum of the dart.
- ☐ 1 point is earned for a statement using conservation of momentum to show that the block momentum/ speed is greater.

Example Response:

When the dart bounces off the block, it has a larger change in momentum (going from v to $-v$ instead of v to a smaller v), which means that the block also has a larger change in momentum (since both objects receive the same impulse and experience the same change in momentum). Because the block has a greater change in momentum, it has a larger speed after the collision than in the original case.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq7

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for recognizing that the amplitude of the graph for group 2 is smaller.
- ☐ 1 point is earned for a statement discussing the dependence of amplitude on dart momentum.
- ☐ 1 point is earned for indicating that the speed of the dart could be smaller.

Example Response:

If group 2 used a dart with a smaller initial speed, the dart would have had less momentum, and so the dart-block system would not have gone as high (making the amplitude smaller), but the string length is the same and so the period would still be the same.