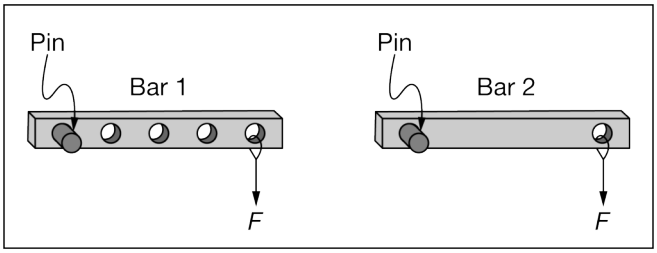
frq8

1. This question is a short free-response question. Show your work for each part of the question.



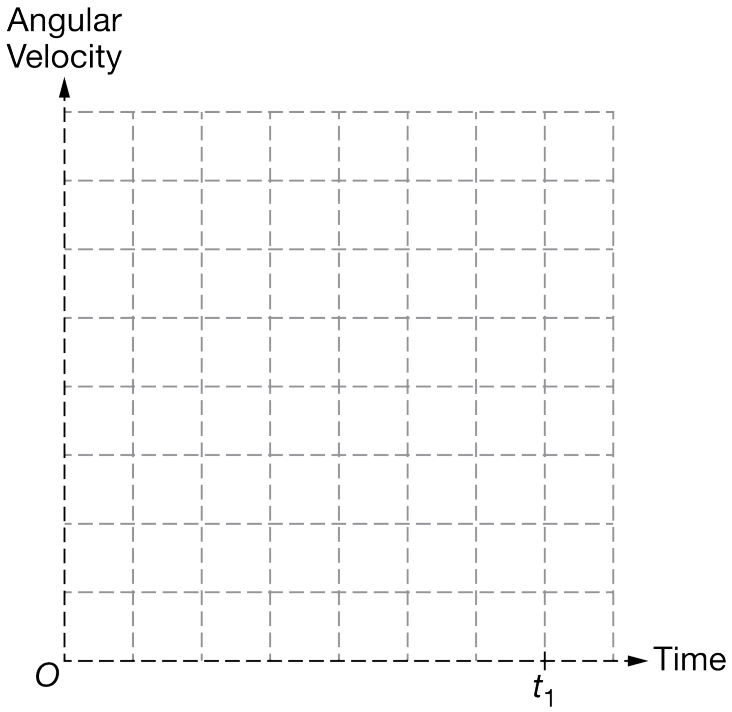
Top View

(7 points, suggested time 13 minutes)

Two bars, bar 1 and bar 2, are at rest on a smooth horizontal table as shown in the top view above. Each bar is connected to the table with a vertical pin. Bar 1 and bar 2 are made from the same material and have the same physical dimensions, but bar 1 has three additional large holes drilled through it. The two bars are each accelerated from rest by forces of equal magnitude F applied at their right end. The forces remain perpendicular to the bars and the forces are exerted for an equal amount of time t_1.

(a) Briefly explain using physics principles why the angular displacement for bar 1 is larger than the angular displacement for bar 2.

(b) On the axes below, sketch the angular velocity of each bar as a function of time. Label the sketch for bar 1 “Bar 1”. Label the sketch for bar 2, “Bar 2”.



frq8

(c) After the force has been applied to both bars for a time t_1, is the angular momentum of bar 1 larger than, smaller than, or the same as the angular momentum of bar 2?

___ Larger ___ Smaller ___ The same

Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Scoring note: A response that includes that the angular displacement is larger because the area under their angular velocity as a function of time graph is larger earns 1 of the 2 points available in part (a).

0	1	2 ✓

The response accurately includes **all** of the following criteria:

- ☐ For a justification that includes that the acceleration is larger for case 1 because the rotational inertia is smaller.
- ☐ For a justification that includes a larger acceleration for the same time means that case 1 has a larger angular displacement.

Example Response:

The rotational inertia for case 1 is smaller than the rotational inertia for case 2. This means that when the two bars have equal torques applied to them, the bar in case 1 has a larger angular acceleration. Since both bars start from rest and are accelerated for the same time, the bar in case 2 has a larger angular displacement.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

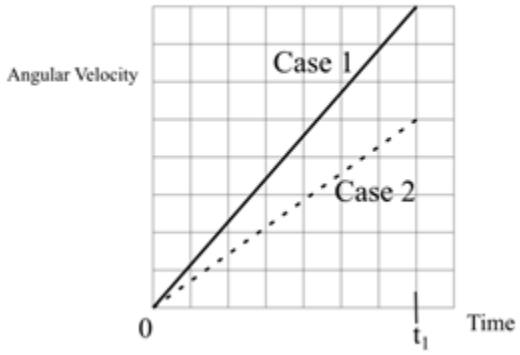
0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

- ☐ For a graph for case 1 and case 2 that both start at the origin
- ☐ For both graphs being linear with a positive slope
- ☐ For a graph for case 1 that has a larger slope than case 2

Example Response:

frq8



Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: Both bars have the same angular momentum.

0	1	2 ✓

The response accurately includes **two** of the following criteria:

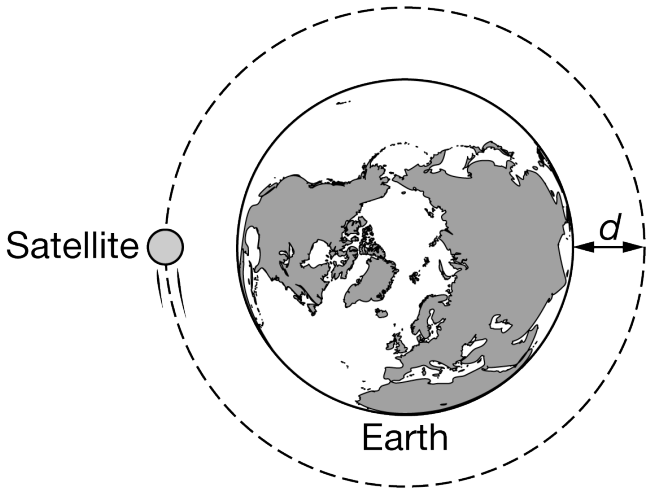
- ☐ For a statement that both bars experience the same torque because they experience the same force at the same distance from the pivot.
- ☐ For a statement that the torque times the time is the same for both cases.

Example Response:

Both bars experience the same torque because they experience the same force at the same distance from the pivot. Change in angular momentum is equal to torque times time, so since the two bars have the same torque exerted for the same time, the two bars have the same change in angular momentum from $t = 0$ to $t = t_1$.

frq8

2. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface, as shown above.

An identical satellite is set into orbit around a new planet. The new planet has the same radius as the radius of Earth but is significantly more massive than Earth. The satellite orbits the same distance d from the surface of the new planet.

(a) Is the speed of the satellite orbiting the new planet greater than, less than, or the same as the speed of the satellite orbiting Earth? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

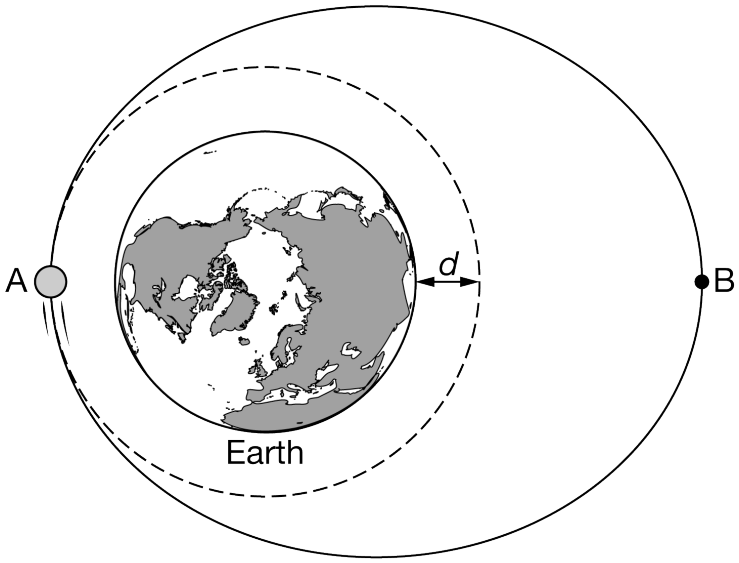
$$\frac{MT^2}{R^3} = C$$

In the equation above

M is the mass of a planet;
T is the orbital period of a satellite in a circular orbit;
R is the distance from the planet's center to the satellite; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

frq8



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point A to point B, and state the following:

Student 1: The satellite is moving faster at point A because the satellite's angular momentum is greater at point A than at point B.

Student 2: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is higher at point B than at point A.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

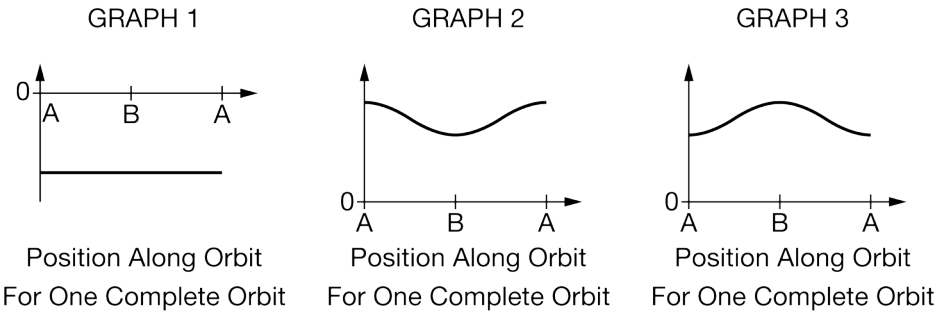
(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite

frq8

as it completes one elliptical orbit, starting and ending at point A.



(h) Which of the above graphs best represents the magnitude of the gravitational field at the location of the satellite as it orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the total mechanical energy of the satellite-Earth system as the satellite orbits? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the satellite around the new planet will orbit with a faster speed.
- ☐ 1 point is earned for a correct explanation in terms of the relationship between gravitational force and centripetal force (or acceleration).

Example Response:

There is greater gravitational force acting on the satellite around the new planet. The increased gravitational force means that the centripetal force $m\frac{v^2}{R}$ *(or acceleration* $\frac{v^2}{R}$*) is greater. Since R is the same in both cases, v must be greater for the satellite orbiting the new planet.*

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

frq8

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a consistent answer AND an attempt at reasoning using functional dependence.
- ☐ 1 point is earned for correctly relating the increased *M* to a decreased *T*.
- ☐ 1 point is earned for correctly explaining how a decreased T increases *v*.

Example Response:

Yes, this is consistent with my reasoning. Since the mass of the new planet is larger, the time it will take to go in an orbit must decrease because M is inversely proportional to T squared. Since the new satellite travels the same circumference orbit, if the second satellite takes less time to go around, it must be going faster.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	✓ 2

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the force was applied in the satellite's direction of motion.
- ☐ 1 point is earned for a correct explanation relating the direction of force with the change in speed.

Example Response:

A force in the direction of the satellite's motion would make the satellite move faster

OR

A force in the direction of the satellite's motion would do positive work on the satellite.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	✓ 1

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

frq8

Student 1 is correct in saying that the satellite is moving faster at A

OR

The first statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is incorrect in saying that the satellite's angular momentum is larger at A than at B

OR

The second statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that the potential energy of the satellite-Earth system is higher at B than at A

OR

The second statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq8

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is incorrect in saying that the satellite is moving faster at B

OR

The first statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 2.
- ☐ 1 point is earned for a correct explanation relating field strength to distance.

Example Response:

Graph 2. The field is larger where the satellite is closer to the planet, so it is larger at A than at B.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 1.

frq8

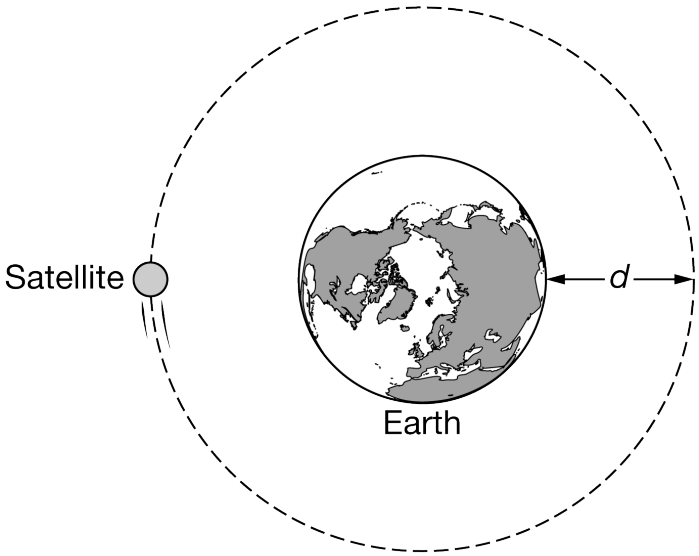
- [ ] 1 point is earned for a correct explanation of why the total mechanical energy is constant in terms of net external force.

Example Response:

Graph 1. The total mechanical energy is constant for the satellite-Earth system because there are no external forces on the system.

frq8

3. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface, as shown above.

An identical satellite is set into orbit around a new planet. The new planet's radius is the same as the radius of Earth but is significantly more massive than Earth. The satellite orbits the new planet with the same orbital period.

(a) Is the distance of the satellite from the surface of the new planet greater than, less than, or equal to d ? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

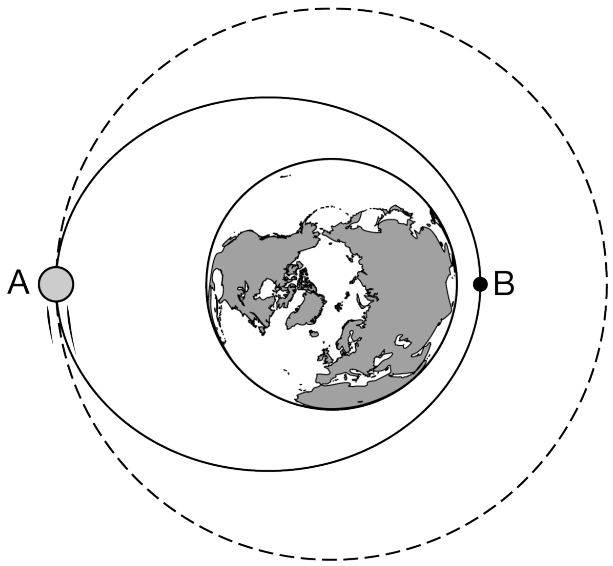
$\frac{Rv^2}{M} = C$

In the equation above

R is the distance from a planet's center to a satellite in a circular orbit;
v is the speed of the satellite;
M is the mass of the planet; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

frq8



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point A to point B, and state the following:

Student 1: The satellite is moving faster at point A because the total mechanical energy of the satellite-Earth system is constant.

Student 2: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is higher at point B than at point A.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

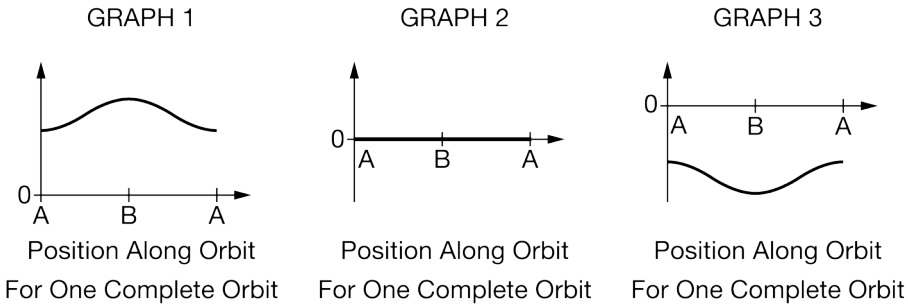
(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite

frq8

as it completes one elliptical orbit, starting and ending at point A.



(h) Which of the above graphs best represents the potential energy of the satellite-Earth system as the satellite orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the torque about the center of Earth exerted on the satellite by Earth's gravitational force as the satellite orbits? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the satellite around the new planet will orbit at a greater distance.
- ☐ 1 point is earned for correctly explaining an increasing distance from the planet to satellite is caused by an increasing speed to maintain a constant period.
  OR
  For correctly explaining an increased distance from the planet to decrease the gravitational force or acceleration on the satellite.

Example Response:

For the satellite to maintain the same period, it must orbit at a greater distance from the new planet because the satellite's orbital speed will be now be greater than before.

OR

"*The distance of the satellite must be further since increasing mass increases the force of gravity keeping the satellite in orbit, to maintain the same period, you would need to be further from the planet to decrease the gravitational force from the larger mass.*

Part B

frq8

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a consistent answer AND an attempt at reasoning using functional dependence.
- ☐ 1 point is earned for using $v = \frac{2\pi R}{T}$ or arguing that *R* must increase if *T* remains the same.
- ☐ 1 point is earned for correctly relating the increase in mass of the planet increases the distance from the planet to satellite.

Example Response:

Yes, this matches with my reasoning in part (a). The mass of the new planet is greater, which means that for C to stay constant the top of the fraction must also be greater. Since v is $\frac{2\pi R}{T}$ *and T is constant, R must be larger for the speed to be larger.*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the force was applied opposite to the satellite's direction of motion.
- ☐ 1 point is earned for a correct explanation relating the direction of force with the changing speed.

Example Response:

The force was applied opposite to the motion of the satellite's direction of motion because this would make the satellite move slower which does negative work on the satellite.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

frq8

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the total mechanical energy of the satellite-Earth system is constant

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is incorrect in saying that the satellite is moving faster at A

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that the satellite is moving faster at B

frq8

OR

The first statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is incorrect in saying that the potential energy of the satellite-Earth system is higher at B than at A

OR

The second statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 3.
- ☐ 1 point is earned for a correct explanation relating potential energy to distance.

Example Response:

Graph 3. The potential energy is higher where the satellite is farther from the planet, so it is higher at A than at B.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq8

0	1	2 ✓

The response accurately includes **both** of the following criteria.

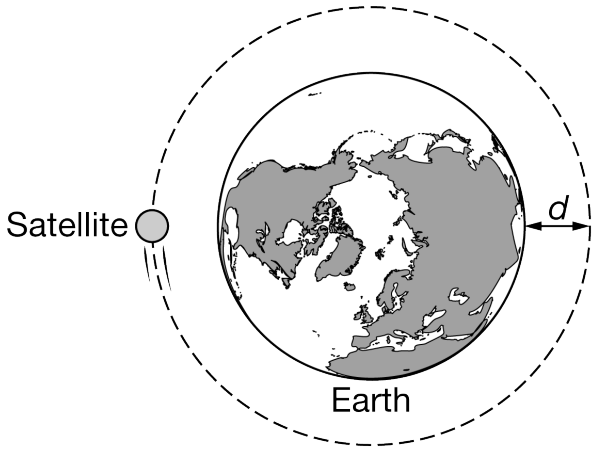
- ☐ 1 point is earned for indicating graph 2.
- ☐ 1 point is earned for a correct explanation why the torque is zero.

Example Response:

Graph 2. The torque is zero since the angle between $\vec{F}$ *and* $\vec{r}$ *is 180 degrees.*

frq8

4. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface.

An identical satellite is set into orbit around a new planet. The new planet's radius is the same as the radius of Earth but the new planet is significantly more massive than Earth. The satellite orbits the new planet with the same speed v.

(a) Is the distance of the satellite from the surface of the new planet greater than, less than, or equal to d ? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

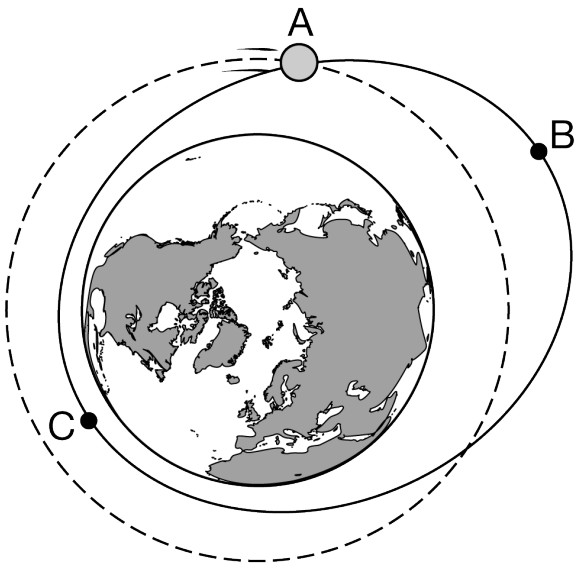
$$\frac{R^3}{MT^2} = C$$

In the equation above

R is the distance from a planet's center to a satellite in a circular orbit;
M is the mass of the planet;
T is the orbital period of the satellite; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

frq8



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit. In the modified orbit, the satellite's speed at point A, after the thrusters were fired, is the same as it was in the original orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point B to point C, and state the following:

Student 1: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is negative.

Student 2: The satellite is moving faster at point C because the total mechanical energy of the satellite-Earth system is greater at point C than at point B.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

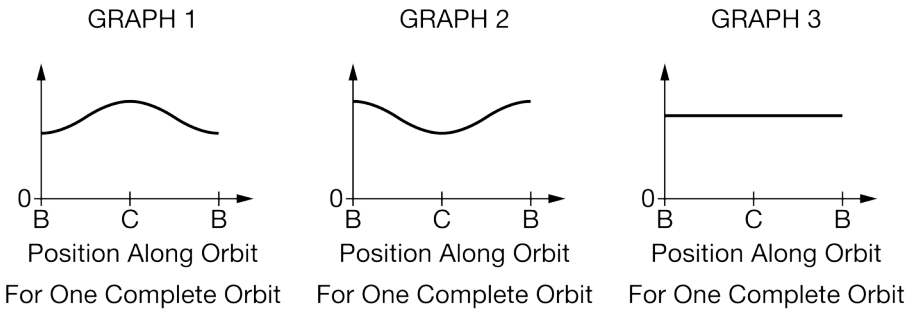
(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is

frq8

incorrect, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite as it completes one elliptical orbit, starting and ending at point B.



(h) Which of the above graphs best represents the magnitude of the acceleration of the satellite as it orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the magnitude of the angular momentum of the satellite with respect to the center of Earth as the satellite orbits? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the satellite around the new planet will orbit at a greater distance.
- ☐ 1 point is earned for a correct explanation in terms of centripetal and gravitational force or acceleration.

Example Response:

Equating the centripetal acceleration v^2/r *with the gravitational acceleration* GM/r^2, *we see that* v^2 *is proportional to* M/r. *So a bigger mass M means that the distance must be greater as well.*

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq8

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for an answer that is consistent with part (a) AND an attempt at reasoning using functional dependence.
- ☐ 1 point is earned for using $v = \frac{2\pi R}{T}$ or for arguing that *R* must increase if *v* remains the same.
- ☐ 1 point is earned for correctly relating the increase in mass of the planet increases the distance from the planet to the satellite.

Example Response:

Yes, it matches my reasoning. Since v is $\frac{2\pi R}{T}$ *and v is the same, if R increases then T must also increase. Since M and T both increase, R must increase to keep C constant.*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the force was applied perpendicular to the satellite's motion.
- ☐ 1 point is earned for a correct explanation relating the direction of force with change in speed.

Example Response:

The force was applied perpendicular to the satellite's motion because this force did not change the speed of the satellite.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

frq8

☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the potential energy of the satellite-Earth system is negative

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is incorrect in saying that the satellite is moving faster at B

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that the satellite is moving faster at C

OR

frq8

The first statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is incorrect in saying that the total mechanical energy of the satellite-Earth system is greater at C than at B.

OR

The second statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 1.
- ☐ 1 point is earned for a correct explanation that the gravitational field or acceleration increases as distance decreases.

Example Response:

Graph 1. The acceleration (or gravitational field) is greater closer to Earth, so it is greater at C than at B.

OR

Graph 1. The centripetal acceleration v^2/r is greater at C because v is greater, and r is smaller than at B.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq8

0	1	2 ✓

The response accurately includes **both** of the following criteria.

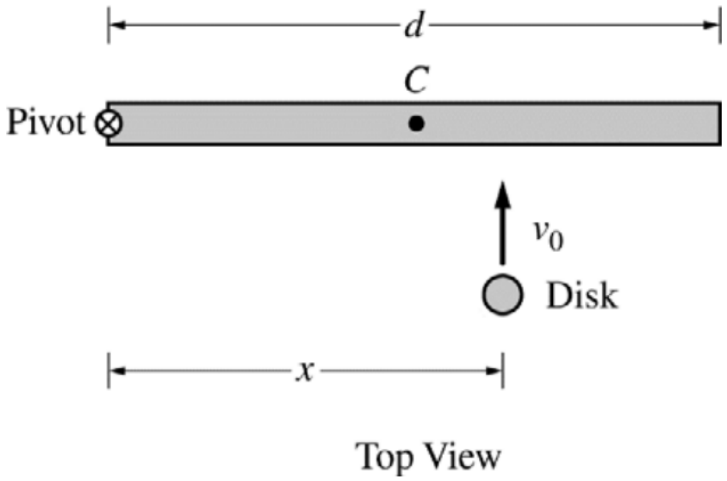
- ☐ 1 point is earned for indicating graph 3.
- ☐ 1 point is earned for a correct explanation why the angular momentum is constant throughout the motion.

Example Response:

Graph 3. The angular momentum is constant because the torque on the satellite is zero.

frq8

5.



The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will stick to the rod a distance x from the pivot. The student wants the roddisk system to end up with as much angular speed as possible.

a. Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point C, at point C, or to the right of point C ?

_____To the left of C _____ At C _____ To the right of C

Briefly explain your reasoning without manipulating equations.

b. On the Internet, a student finds the following equation for the postcollision angular speed ω of the rod in this situation: $\omega = \frac{m_{disk} x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a) ? In other words, does this equation for w have the expected dependence as reasoned in part (a) ?

_____Yes _____No

Briefly explain your reasoning without deriving an equation for ω.

c. Another student deriving an equation for the postcollision angular speed ω of the rod makes a mistake and comes up with $\omega = \frac{I x v_0}{m_{disk} d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

d. Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{disk}x^2$ and its

angular momentum with respect to the pivot is $m_{disk}\ v_0x$. Derive an equation for the postcollision angular speed ω of the rod. Express your answer in terms of d, m_{disk}, I, x, v_0, and physical constants, as appropriate.

e. Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?

_____Greater than _____Less than _____Equal to

Briefly explain your reasoning.

Part A

Correct answer: “To the right of C”

Reasoning cannot earn credit if the incorrect selection is made.

1 point is earned: For an explanation that the torque exerted by the disk or the angular momentum of the disk is greater when farther from the pivot

Example 1: The disk exerts a greater torque on the rod when it pushes the rod farther from the pivot.

Example 2: The disk has greater angular momentum when it's farther from the pivot. The disk loses almost all its speed during the collision and hence gives the rod almost all its angular momentum. So the rod ends up with more angular momentum when the disk hits it farther from the pivot.

0	1 ✓

Student response earns 1 of the following 1 point(s)

Correct answer: “To the right of C”

Reasoning cannot earn credit if the incorrect selection is made.

1 point is earned: For an explanation that the torque exerted by the disk or the angular momentum of the disk is greater when farther from the pivot

Example 1: The disk exerts a greater torque on the rod when it pushes the rod farther from the pivot.

Example 2: The disk has greater angular momentum when it's farther from the pivot. The disk loses almost all its speed during the collision and hence gives the rod almost all its angular momentum. So the rod ends up with more angular momentum when the disk hits it farther from the pivot.

Part B

frq8

Correct answer: “Yes”

If “No” is selected, the explanation may still earn full credit if an incorrect selection was made in part (a).

1 point is earned: For a selection consistent with the selection from part (a)

1 point is earned: For indicating that the equation shows that ω increases with increasing x

Example: According to the equation, ω increases with x; a bigger x produces a bigger angular speed. This agrees with my reasoning from part (a), where I said a bigger x creates a bigger angular speed after the collision.

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

Correct answer: “Yes”

If “No” is selected, the explanation may still earn full credit if an incorrect selection was made in part (a).

1 point is earned: For a selection consistent with the selection from part (a)

1 point is earned: For indicating that the equation shows that ω increases with increasing x

Example: According to the equation, ω increases with x; a bigger x produces a bigger angular speed. This agrees with my reasoning from part (a), where I said a bigger x creates a bigger angular speed after the collision.

Part C

1 point is earned: For focusing on functional dependence (instead of, for example, considering units/dimensions)

1 point is earned: For addressing mdisk, I, or both

1 point is earned: For correctly concluding that the equation is wrong because of the dependence on mdisk, I, or both

Example: If mdisk is large, then more angular momentum will be transferred during the collision. But the equation shows the angular speed decreasing with increasing mdisk, because it is in the denominator.

0	1	2	3 ✓

Student response earns 3 of the following 3 point(s)

1 point is earned: For focusing on functional dependence (instead of, for example, considering units/dimensions)

1 point is earned: For addressing mdisk, I, or both

frq8

1 point is earned: For correctly concluding that the equation is wrong because of the dependence on mdisk, I, or both

> Example: If mdisk is large, then more angular momentum will be transferred during the collision. But the equation shows the angular speed decreasing with increasing mdisk, because it is in the denominator.

Part D

1 point is earned: For using an expression of conservation of angular momentum for the disk and rod

Note: This point is not awarded for equating angular and linear momentum.

1 point is earned: For indicating that the initial angular momentum of the system is equal to mdis v0x

1 point is earned: For a dimensionally correct expression for the post-collision angular momentum that includes Iω

1 point is earned: For indicating the correct rotational inertia of the system after the collision: I+ mdis x2

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

1 point is earned: For using an expression of conservation of angular momentum for the disk and rod

Note: This point is not awarded for equating angular and linear momentum.

1 point is earned: For indicating that the initial angular momentum of the system is equal to mdis v0x

1 point is earned: For a dimensionally correct expression for the post-collision angular momentum that includes Iω

1 point is earned: For indicating the correct rotational inertia of the system after the collision: I+ mdis x2

Part E

Correct answer: “Greater than”

1 point is earned: For indicating, either directly or by analogy to the linear case, that the disk's angular momentum with respect to the pivot changes more in the bouncy scenario than in the original scenario OR for using a similar argument in terms of impulse

Note: This point is for describing what happens to the disk.

1 point is earned: For using conservation of angular momentum or momentum-impulse reasoning to conclude that the rod gains more angular momentum, and hence more angular speed, in the bouncy scenario

Note: This point is for describing what happens to the rod.

> Example: After the bouncy collision, the disk has angular momentum in the clockwise direction. To keep the system angular momentum constant, the magnitude of the rod’s counterclockwise angular momentum must be greater than before.

frq8

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

Correct answer: “Greater than”

1 point is earned: For indicating, either directly or by analogy to the linear case, that the disk's angular momentum with respect to the pivot changes more in the bouncy scenario than in the original scenario OR for using a similar argument in terms of impulse

Note: This point is for describing what happens to the disk.

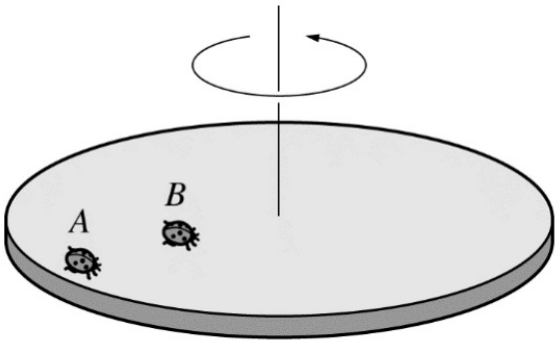
1 point is earned: For using conservation of angular momentum or momentum-impulse reasoning to conclude that the rod gains more angular momentum, and hence more angular speed, in the bouncy scenario

Note: This point is for describing what happens to the rod.

> Example: After the bouncy collision, the disk has angular momentum in the clockwise direction. To keep the system angular momentum constant, the magnitude of the rod’s counterclockwise angular momentum must be greater than before.

frq8

6.



Two ladybugs are standing on a rotating disk that is spinning counterclockwise, as shown in the figure above. Assume that friction in the bearings of the axle is negligible.

a. i. Is the angular speed of ladybug A greater than, less than, or the same as the angular speed of ladybug B ?

_____Greater _____Less _____The same

Briefly justify your answer.

ii. Is the linear speed of ladybug A greater than, less than, or the same as the linear speed of ladybug B ?

_____Greater _____Less _____The same

Briefly justify your answer.

b. Ladybug A begins walking in a circular path in the direction of the disk's rotation. Does the magnitude of the angular momentum of the disk alone (not the ladybugs-disk system) increase, decrease, or stay the same?

_____Increase _____Decrease _____Stay the same

Briefly justify your answer.

c. In a different scenario, a single ladybug is standing near the edge of the disk at a distance of 0.9R from the center, where R is the radius of the disk, as shown in Figure 1 below. The rotational inertia of the ladybugdisk system is I_1 , and the disk completes one rotation in 2.5 s. The ladybug then walks toward the center of the disk to a distance of 0.1R from the center and comes to a stop relative to the disk, as shown in Figure 2. Now the rotational inertia of the system is I_2, and the disk completes one rotation every 2.0 s.

frq8

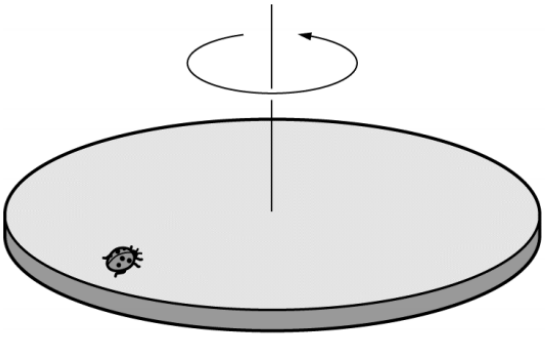


Figure 1

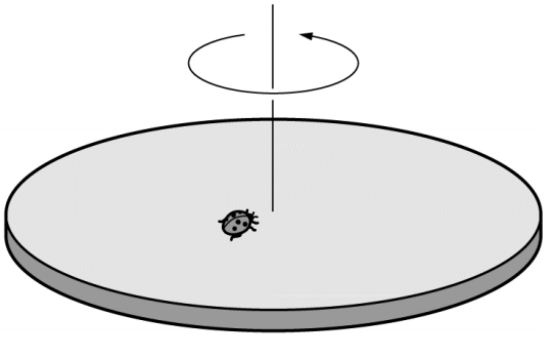


Figure 2

i. Derive an equation for I_2 in terms of I_1

ii. While the ladybug is walking toward the center of the disk, does it exert a torque on the disk?

_____Yes ____No

Briefly explain your reasoning.

Part A

Part(i)

Correct answer: “The same”

1 point is earned: For selecting “The same” and explaining that both ladybugs have the same angular displacement over the same interval of time, or that all points on the disk rotate at the same rate

Notes: No credit is earned if the wrong answer is selected.

No credit is earned for a correct selection with a wrong explanation or no explanation given.

Part(ii)

Correct answer: “Greater”

1 point is earned: For selecting “Greater” and explaining that linear speed is proportional to (or increases with) distance from the center ($v = \omega R$)

Notes: No credit is earned if the wrong answer is selected.

No credit is earned for a correct selection with a wrong explanation or no explanation given.

frq8

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

Part(i)

Correct answer: “The same”

1 point is earned: For selecting “The same” and explaining that both ladybugs have the same angular displacement over the same interval of time, or that all points on the disk rotate at the same rate

Notes: No credit is earned if the wrong answer is selected.

No credit is earned for a correct selection with a wrong explanation or no explanation given.

Part(ii)

Correct answer: “Greater”

1 point is earned: For selecting “Greater” and explaining that linear speed is proportional to (or increases with) distance from the center ($v = \omega R$)

Notes: No credit is earned if the wrong answer is selected.

No credit is earned for a correct selection with a wrong explanation or no explanation given.

Part B

Correct answer: “Decrease”

If the wrong answer is selected, at most one point can be earned for the explanation.

1 point is earned: For stating or implying that the system angular momentum does not change

1 point is earned: For stating that the ladybug gained angular momentum, the disk lost angular momentum

Note: Full credit can be earned for an alternate solution involving the force exerted by the ladybug (while accelerating) causing a torque on the disk.

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

Correct answer: “Decrease”

frq8

If the wrong answer is selected, at most one point can be earned for the explanation.

1 point is earned: For stating or implying that the system angular momentum does not change

1 point is earned: For stating that the ladybug gained angular momentum, the disk lost angular momentum

Note: Full credit can be earned for an alternate solution involving the force exerted by the ladybug (while accelerating) causing a torque on the disk.

Part C

Part(i)

1 point is earned: or using an equation expressing the conservation of angular momentum with I and ω.

I2ω2 = I1ω1

1 point is earned: For correctly substituting the angular speed values and obtaining a correct answer or an algebraic equivalent of the correct answer

$$I_2 = \frac{I_1\omega_1}{\omega_2} = \frac{I_1(1/2.5)}{(1/2)}$$

I2 = 4/5(I1)

Part(ii)

Correct answer: “Yes”

1 point is earned: For selecting “Yes” and for providing evidence that a torque is exerted on the disk (e.g., the disk’s angular momentum/speed changes, or the ladybug exerts a non-radial force on the disk)

Notes: No credit is earned if the wrong answer is selected.

No credit is earned for a correct selection with a wrong explanation or no explanation given

0	1	2	3 ✓

Student response earns 3 of the following 3 point(s)

Part(i)

1 point is earned: or using an equation expressing the conservation of angular momentum with I and ω.

I2ω2 = I1ω1

1 point is earned: For correctly substituting the angular speed values and obtaining a correct answer or an algebraic equivalent of the correct answer

$$I_2 = \frac{I_1\omega_1}{\omega_2} = \frac{I_1(1/2.5)}{(1/2)}$$

frq8

I2 = 4/5(I1)

Part(ii)

Correct answer: “Yes”

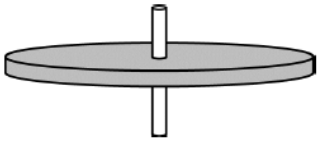
1 point is earned: For selecting “Yes” and for providing evidence that a torque is exerted on the disk (e.g., the disk’s angular momentum/speed changes, or the ladybug exerts a non-radial force on the disk)

Notes: No credit is earned if the wrong answer is selected.

No credit is earned for a correct selection with a wrong explanation or no explanation given

frq8

7.

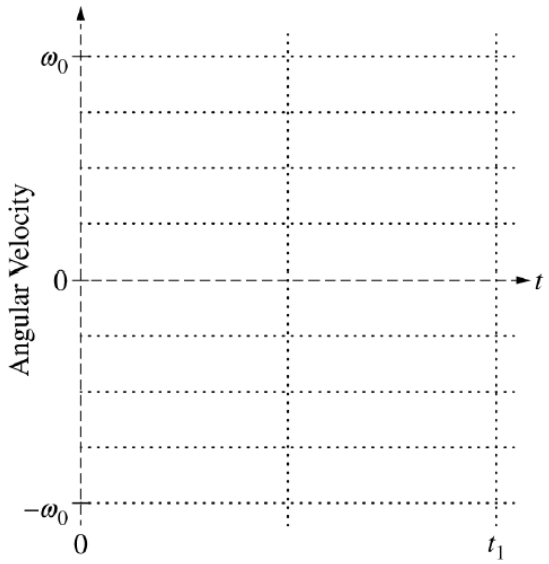


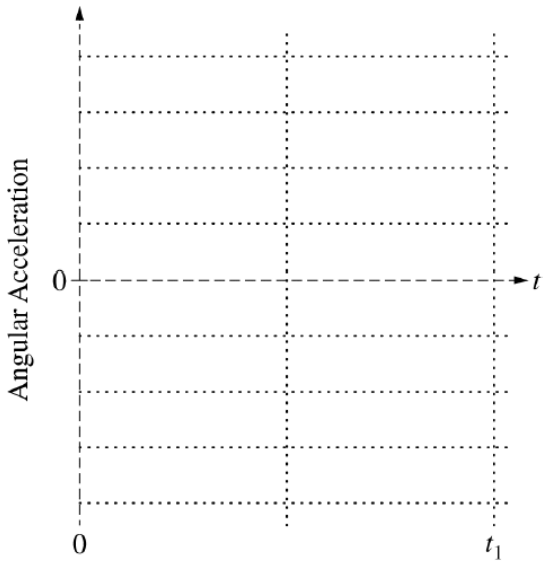
The disk shown above spins about the axle at its center. A student's experiments reveal that, while the disk is spinning, friction between the axle and the disk exerts a constant torque on the disk.

a. At time $t = 0$ the disk has an initial counterclockwise (positive) angular velocity ω_0.The disk later comes to rest at time $t = t_1$.

   i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time t from $t = 0$ until the disk comes to rest at time $t = t_1$.

   ii. On the grid at right below, sketch the disk's angular acceleration as a function of time t from $t = 0$ until the disk comes to rest at time $t = t_1$.





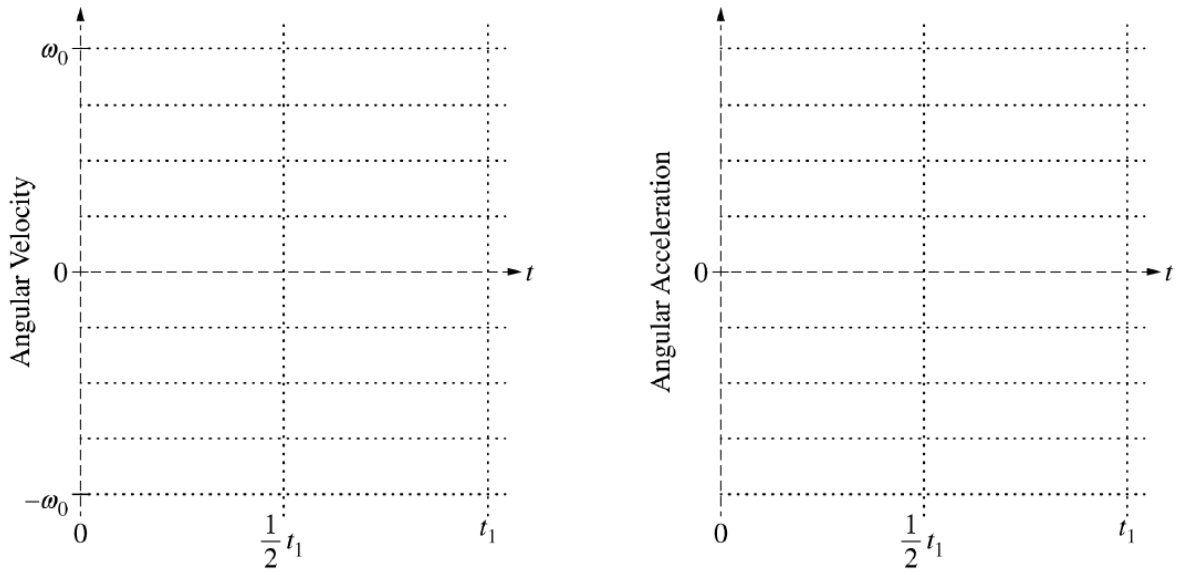
b. The magnitude of the frictional torque exerted on the disk is τ_0. Derive an equation for the rotational inertia I of the disk in terms of τ_0, ω_0, t_1, and physical constants, as appropriate.

c. In another experiment, the disk again has an initial positive angular velocity ω_0 at time $t = 0$. At time $t = \frac{1}{2}t_1$, the student starts dripping oil on the contact surface between the axle and the disk to reduce the friction. As time passes, more and more oil reaches that contact surface, reducing the friction even further.

frq8

i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time from $t = 0$ to $t = t_1$, which is the time at which the disk came to rest in part (a).

ii. On the grid at right below, sketch the disk's angular acceleration as a function of time from $t = 0$ to $t = t_1$.



d. The student is trying to mathematically model the magnitude τ of the torque exerted by the axle on the disk when the oil is present at times $t > \frac{1}{2}t_1$. The student writes down the following two equations, each of which includes a positive constant (C_1 or C_2) with appropriate units.

$$(1)\ \tau = C_1\left(t - \tfrac{1}{2}t_1\right) \text{ (for } t > \tfrac{1}{2}t_1)$$
$$(2)\ \tau = \frac{C_2}{(t+\frac{1}{2}t_1)} \text{ (for } t > \tfrac{1}{2}t_1)$$

Which equation better mathematically models this experiment?

____Equation (1) ____Equation (2)

Briefly explain why the equation you selected is plausible and why the other equation is not plausible.

Part A

PART i

1 point is earned: For a curve that has an angular velocity of +ω0 at time t = 0 and decreases to zero at time t = t1

1 point is earned: For a curve that is a straight line with a negative slope showing the angular velocity approaching zero (can be a positive slope, if the initial angular velocity on the graph is negative)

frq8

PART ii

1 point is earned: For a curve that is negative for the entire time

1 point is earned: For a curve that is a constant nonzero function

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

PART i

1 point is earned: For a curve that has an angular velocity of +ω0 at time t = 0 and decreases to zero at time t = t1

1 point is earned: For a curve that is a straight line with a negative slope showing the angular velocity approaching zero (can be a positive slope, if the initial angular velocity on the graph is negative)

PART ii

1 point is earned: For a curve that is negative for the entire time

1 point is earned: For a curve that is a constant nonzero function

Part B

1 point is earned: For using an equation for the rotational version of Newton's second law

1 point is earned: For using an appropriate rotational kinematics equation α = Δω/Δt

1 point is earned: For a correct answer in terms of the listed quantities, derived from first principles $I = \frac{\tau_0 t_1}{\omega_0}$

Note: This point is still earned if there is a minus sign

Alternate solution using angular momentum and rotational impulse:

1 point is earned: For defining and using angular momentum L = iω

1 point is earned: For using rotational impulse ΔL= τΔt

1 point is earned: For a correct answer in terms of the listed quantities, derived from first principles $I = \frac{\tau_0 t_1}{\omega_0}$

Note: This point is still earned if there is a minus sign

0	1	2	3 ✓

frq8

Student response earns 0 of the following 3 point(s)

1 point is earned: For using an equation for the rotational version of Newton's second law

1 point is earned: For using an appropriate rotational kinematics equation $\alpha = \Delta\omega/\Delta t$

1 point is earned: For a correct answer in terms of the listed quantities, derived from first principles $I = \frac{\tau_0 t_1}{\omega_0}$

Note: This point is still earned if there is a minus sign

Alternate solution using angular momentum and rotational impulse:

1 point is earned: For defining and using angular momentum $L = i\omega$

1 point is earned: For using rotational impulse $\Delta L = \tau\Delta t$

1 point is earned: For a correct answer in terms of the listed quantities, derived from first principles $I = \frac{\tau_0 t_1}{\omega_0}$

Note: This point is still earned if there is a minus sign

Part C

PART i

1 point is earned: For curve with a clear change of slope or curvature at ½t1 and showing a decrease in speed thereafter

1 point is earned: For a curve that indicates slowing at a decreasing rate between times ½ t1 and t1

1 point is earned: For a curve that does not reach zero at or before time t1

PART ii

1 point is earned: For a curve with decreasing magnitude between times ½ t1 and t1

Note: The acceleration may reach zero at or before time t1. If so, it must remain zero for

the remaining time.

0	1	2	3	4	5 ✓

Student response earns 5 of the following 5 point(s)

PART i

1 point is earned: For curve with a clear change of slope or curvature at ½t1 and showing a decrease in speed thereafter

1 point is earned: For a curve that indicates slowing at a decreasing rate between times ½ t1 and t1

1 point is earned: For a curve that does not reach zero at or before time t1

frq8

PART ii

1 point is earned: For a curve with decreasing magnitude between times ½ t1 and t1

Note: The acceleration may reach zero at or before time t1. If so, it must remain zero for

the remaining time.

Part D

Correct answer: “Equation (2)”

Note: If the wrong selection is made, the explanation is not graded.

1 point is earned: For stating that Equation (2) is plausible because the frictional torque decreases with increasing time, whereas in Equation (1) torque increases with increasing time

Examples:

Equation (2) because ⦰ decreases. In Equation (1), it doesn’t.

Equation (2) is plausible because the frictional torque decreases as more oil reaches the contact surface over time. Equation (1) is not plausible because it shows friction increasing as more oil reaches the surface over time.

0	1 ✓

Student response earns 1 of the following 1 point(s)

Correct answer: “Equation (2)”

Note: If the wrong selection is made, the explanation is not graded.

1 point is earned: For stating that Equation (2) is plausible because the frictional torque decreases with increasing time, whereas in Equation (1) torque increases with increasing time

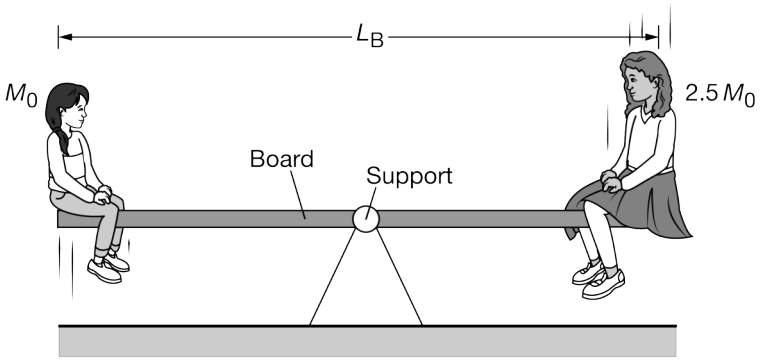
Examples:

Equation (2) because ⦰ decreases. In Equation (1), it doesn’t.

Equation (2) is plausible because the frictional torque decreases as more oil reaches the contact surface over time. Equation (1) is not plausible because it shows friction increasing as more oil reaches the surface over time.

frq8

8. This question is a short free-response question. Show your work for each part of the question.



Note: Figure not drawn to scale.

(7 points, suggested time 13 minutes)

A student of mass M_0 and another student of mass $2.5M_0$ sit at opposite ends of a board of length L_B and negligible mass. The board is free to pivot about a support at its center. At the instant shown above, the board is horizontal.

(a) Derive an expression for the magnitude of the net torque exerted on the students-board system about the support at the instant shown in the figure. Express your answer in terms of M_0, L_B, and physical constants, as appropriate.

(b) The students want to be able to balance the board horizontally without the students touching the ground. They figure out that one student needs to move closer to the support while the other student remains in their original position at the other end of the board.

i. Which student should move closer to the support?

____ The student of mass M_0 ____ The student of mass $2.5M_0$

Briefly explain your reasoning.

ii. Derive an expression for the distance that the student indicated in part (b)(i) should move. Express your answer in terms of M_0, L_B, and physical constants, as appropriate.

(c) Determine the magnitude of the force exerted on the board by the support after the student has moved and the board is balanced horizontally. Express your answer in terms of M_0, L_B, and physical constants, as appropriate.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

frq8

The response accurately includes **both** of the following criteria.

- 1 point is earned for applying the torque equation $\tau = r_{\perp}F$
- 1 point is earned for substituting the correct expressions for the force magnitudes $M_0 g$ and $2.5M_0 g$, both lever arms $L_B/2$, and indicating that the torques are exerted in opposite directions.
  Scoring Note: The following earns both points:
  $\tau_{\text{net}} = (2.5M_0 g)(L_B/2) - (M_0 g)(L_B/2)$
  OR
  $\tau_{\text{net}} = (M_0 g)(L_B/2) - (2.5M_0 g)(L_B/2)$

Example Response for part (a):

$$\begin{aligned}\tau &= r_{\perp}F \\ \tau_{\text{net}} &= (2.5M_0 g)(L_B/2) - (M_0 g)(L_B/2) \\ &= (1.5M_0 g)(L_B/2) \\ &= 0.75M_0 g L_B\end{aligned}$$

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria.

Scoring Note: Correct answer is "The student of mass $2.5M_0$"; if wrong selection, response does not earn credit.

- 1 point is earned for a correct explanation of why the student of mass $2.5M_0$ moves.
  Example Response for 1st point:
  The student of mass M_0 is lighter than the other student, so there must be a longer lever arm (greater distance to pivot) for the student of mass M_0.
  The torques exerted by the two students must balance.
  The center of mass is closer to the more massive student.
- 1 point is earned for writing and using an equation for the rotational version of Newton's second law (i.e., an equation indicating that $\tau_{\text{net}} = 0$ or that the torques balance).
- 1 point is earned for work leading to a correct position of $\frac{L_B}{5}$ from the support.
- 1 point is earned for subtracting the new position from $\frac{L_B}{2}$ to get the distance moved.
  Scoring Notes: answer must be consistent with position calculated.
  An answer of $\frac{L_B}{2} - \frac{L_B}{5}$ is acceptable (or expression consistent with position calculated.)

Example Response for 2nd, 3rd, and 4th points:

Let x be the distance from the pivot to the new location of the student of mass $2.5M_0$.

frq8

Δx *is the distance the student moves*

$\tau_{\text{net}} = I\alpha = 0$
$M_0 g \frac{L_B}{2} = 2.5 M_0 g x$
$\frac{L_B}{2} = 2.5x$
$x = \frac{L_B}{5}$
$\Delta x = \frac{L_B}{2} - \frac{L_B}{5} = (0.5 - 0.2)L_B = 0.3L_B$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

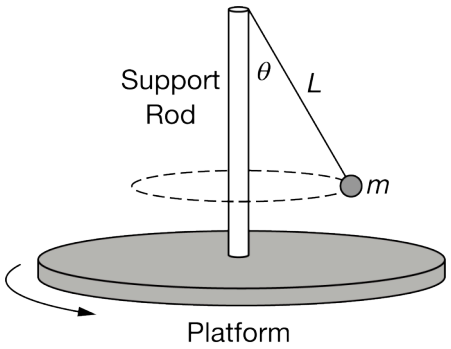
0	1 ✓

The response accurately includes:

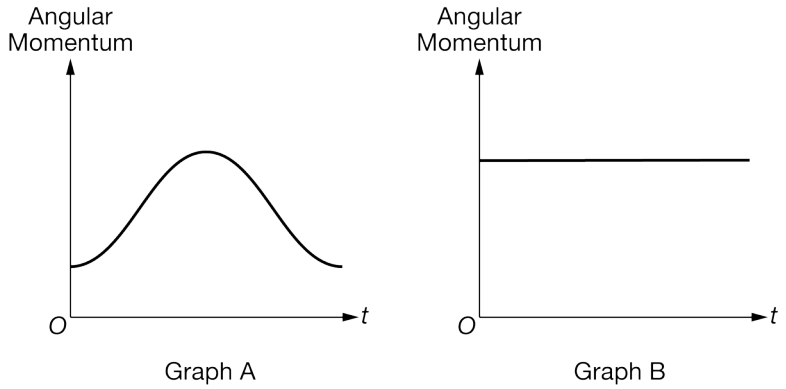
- [ ] 1 point is earned for a correct answer of $3.5 M_0 g$

frq8

9. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A vertical support rod is fixed at the center of a platform. A light string of length L is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass m, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed ω (omega), the string makes an angle θ (theta) with the rod.



(a) The platform rotates with a constant angular speed. Which of the above graphs correctly shows the magnitude of the angular momentum of the platform as a function of time t about a vertical axis at the center of the support rod? Briefly justify your answer.

(b) The platform rotates with a constant angular speed. Is the net torque exerted on the platform about its center clockwise, counterclockwise, or zero? Explain your reasoning.

(c) As the period of rotation becomes very small, what value does θ approach? Without writing any equations or formulas, justify your answer.

(d) A student derives an equation for the square of the period of revolution T of the sphere as a function of θ: $T^2 = \left(4\pi^2\right)\frac{L\cos\theta}{g}$. Indicate whether the equation is consistent with your answer in part (c) and explain why.

(e) In a series of experimental trials, a group of students changes L and adjusts T to keep θ constant. They record L and T for each trial. If the students use the equation given in part (d), what quantities could they graph to yield a

frq8

straight line whose slope could be used to find the acceleration due to gravity, g ?

(f) As the angular speed of the platform increases, does the period of revolution of the sphere increase, decrease, or stay the same? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for identifying the correct graph as graph B.
- ☐ 1 point is earned for a justification that angular momentum is directly proportional to the angular speed which remains constant.

Example Response:

Graph B is correct because the angular speed is constant, and therefore the angular momentum is constant.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that the net torque on the platform is zero.
- ☐ 1 point is earned for a correct statement relating the angular speed to angular acceleration.
- ☐ 1 point is earned for a correct statement relating net torque to angular acceleration.

Example Response:

The net torque on the platform is zero. Because the platform rotates with a constant angular speed, its angular acceleration is zero, and net external torque is equal to the rotational inertia times the angular acceleration, so if the angular acceleration is zero, the net external torque is also zero.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq8

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that correctly relates the period to the angular speed.
- ☐ 1 point is earned for a statement correctly relating the angular speed to the angle theta.

Example Response:

When the period is very small the platform is rotating quickly, making the angular speed very large. When the angular speed is very large, the angle theta is close to 90 degrees.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that is consistent with the reasoning in part (c).
- ☐ 1 point is earned for addressing functional dependence.
- ☐ 1 point is earned for a correct functional dependence argument.

Example Response:

The equation is consistent with part (c) because as theta gets close to 90 degrees, cos theta gets very small, and therefore the period gets very small.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement identifying a T term on one axis.

frq8

- [ ] 1 point is earned for a statement identifying any *L* term on the other axis.
- [ ] 1 point is earned for a correct power relationship for *T* and *L* (i.e., *T* squared and *L*).

Example Response:

They could graph T squared as a function of L.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

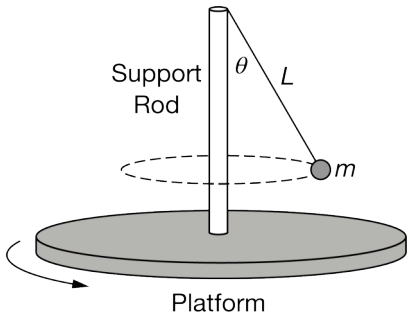
- [ ] 1 point is earned for a statement that the period of rotation decreases.
- [ ] 1 point is earned for a correct explanation of the relationship between the period and the angular speed of the platform.

Example Response:

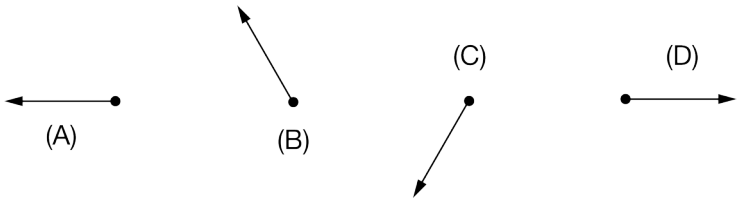
The period will decrease with angular speed because T = 2 pi / omega.

frq8

10. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A vertical support rod is fixed at the center of a platform. A light string of length L is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass m, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed ω (omega), the string makes an angle θ (theta) with the rod.



(a) Identify which of the above vectors represents the net force exerted on the sphere at the moment shown above. Briefly justify your answer.

(b) Is the magnitude of the tension in the string greater than, less than, or equal to the weight of the sphere? Explain your reasoning.

(c) The string can never be completely horizontal no matter how fast the platform is rotating. Without writing any equations or formulas, explain why.

One of the following two equations for the square of the tangential speed v of the sphere as a function of θ is consistent with the statement in part (c) that the string can never be horizontal as the platform is rotating.

Equation 1: $v^2 = \frac{gL(\sin^2\theta)}{(\cos\theta)}$ **Equation 2:** $v^2 = \frac{gL(\cos^2\theta)}{(\sin\theta)}$

(d) Indicate which equation is consistent with the statement in part (c), and explain why. Also explain why the other equation is not consistent with the statement in part (c).

(e) In a series of experimental trials, a group of students changes L and adjusts v to keep θ constant. They record L and v for each trial. Based on the equation in part (d) that you indicated as consistent with the statement in part (c), what quantities could the students graph to yield a straight line whose slope could be used to find the acceleration due to gravity, g ?

frq8

(f) Let I be the rotational inertia of the sphere-rod-platform system about a vertical axis through the center of the support rod. Does I increase, decrease, or stay the same when L increases (but θ stays constant)? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answer: (A), with a correct justification

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement identifying the correct net force vector (A)
- ☐ 1 point is earned for a justification that includes that the net force or acceleration will point toward the center of the circular path

Example Responses:

Figure A because the net force is toward the center of the sphere's circular motion.

OR

The forces acting on the sphere are the force of gravity and the force of tension. The force of gravity is directly toward the ground, and the force of tension is at an angle toward the top of the rod. When the forces are added together, the vertical component of the force of tension and the force of gravity cancel each other out, leaving only the horizontal component of the force of tension, which is demonstrated by vector A.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- ☐ 1 point is earned for a statement that the tension in the string is greater than the weight of the sphere
- ☐ 1 point is earned for an explanation referring to the components of the vectors
- ☐ 1 point is earned for a correct justification that addresses the sum of the vector components

Example Responses:

frq8

The tension is greater than the weight because the vertical component of the tension force must be equal to the weight. The tension also has a horizontal component, so the magnitude will be greater than either of the components.

OR

Explaining that

$\Sigma F_y = F_{Ty} - mg = 0$

$F_T \cos\theta = mg$

$F_T = mg/\cos\theta$

(Since $\cos\theta < 1$*), therefore* $F_T > mg$ *(or > "weight")*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement identifying forces and/or force components in the vertical direction
- ☐ 1 point is earned for a justification that includes that the weight of the sphere must be balanced by the string and so the vertical component of the tension can never be zero

Example Response:

The weight of the sphere is downward and will need to be balanced by the vertical component of the tension. So, there will always be a vertical component of tension.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Notes:

· If students switch sine and cosine (such that they use Equation 2) but then are completely consistent, they can earn 2 of 3 points in part (d).

· If students misinterpret theta as being zero when the string is horizontal, they can earn two of three points in part (d) for using Equation 2 consistently.

0	1	2	3 ✓

frq8

The response accurately includes **three** of the following criteria.

- [ ] 1 point is earned for a statement addressing functional dependence (regardless of the equation chosen)
- [ ] 1 point is earned for indicating that equation 1 shows that as theta goes to 90 degrees, *v* gets very large (infinite/undefined)
- [ ] 1 point is earned for correctly indicating a flaw in equation 2

Example Response:

Equation 1 is consistent with part (c) because as theta gets closer to 90 degrees, the speed gets very large. This shows that no matter how large *v* gets, theta will never get to 90 degrees. Equation 2 is not consistent because it has a finite value for theta equal to 90 degrees.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- [ ] 1 point is earned for a statement identifying any *v* term on one axis
- [ ] 1 point is earned for a statement identifying any *L* term on the other axis
- [ ] 1 point is earned for a statement identifying correct power relationships for *v* and *L* (i.e., *v* squared vs. *L*)

Example Response:

They could graph *v* squared on the *y*-axis as a function of *L* on the *x*-axis.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for a statement identifying that the rotational inertia will increase
- [ ] 1 point is earned for a statement that since the mass is further from the axis, the rotational inertia is bigger

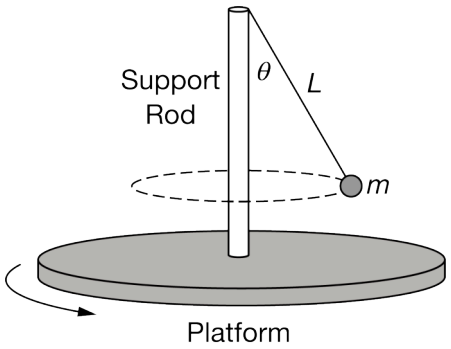
Example Response:

frq8

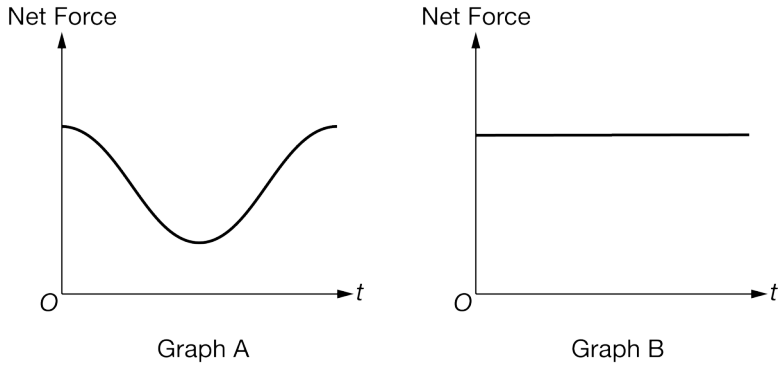
I will increase because for a larger *L*, the distance of the mass from the axis will be larger, and rotational inertia increases with distance.

frq8

11. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A vertical support rod is fixed at the center of a platform. A light string of length L is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass m, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed ω (omega), the string makes an angle θ (theta) with the rod.



(a) The platform rotates with a constant angular speed. Which of the above graphs correctly shows the magnitude of the net force exerted on the sphere as a function of time t ? Briefly justify your answer.

(b) Is the magnitude of the tension in the string greater than, less than, or equal to the centripetal force exerted on the sphere? Explain your reasoning.

(c) The string is nearly vertical when the sphere's tangential speed is very small. Without writing any equations or formulas, explain why.

(d) A student derives an equation for the square of the tangential speed v of the sphere as a function of θ : $v^2 = gL(\tan\theta)(\sin\theta)$. Indicate whether the equation is consistent with the statement in part (c) and explain why.

(e) In a series of experimental trials, a group of students adjusts v and measures θ while keeping all other variables constant. They record v and θ for each trial. If the students use the equation given in part (d), what quantities could they graph to yield a straight line whose slope could be used to find the acceleration due to gravity, g ?

frq8

(f) Let I be the rotational inertia of the sphere-rod-platform system about a vertical axis through the center of the support rod. Does I increase, decrease, or stay the same when θ decreases? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement identifying graph B as correctly describing the magnitude of the net force on the sphere as a function of time.
- ☐ 1 point is earned for a justification that the net force is constant because the speed is constant.

Example Response:

Graph B is correct because the magnitude of the net force on an object in uniform circular motion is constant.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that the magnitude of the tension is greater than the centripetal force exerted on the sphere.
- ☐ 1 point is earned for an explanation referring to the components of the vectors.
- ☐ 1 point is earned for a correct justification that addresses the sum of the vector components.

Example Response:

The tension is greater than the centripetal force because the horizontal component of the tension force must be equal to the centripetal force. The tension also has a vertical component, so the magnitude will be greater than either of the components.

OR

Explaining that

frq8

$\sum F_x = F_{Tx} = mv^2/r$ *where* mv^2/r *is the "centripetal force."*

$F_T \sin\theta = mv^2/r$

$F_T = mv^2/(r\sin\theta)$

(Since $\sin\theta < 1$*), therefore* $F_T > mv^2/r$ *(or >* *"centripetal force")*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that when the speed is very small the centripetal force is very small.
- ☐ 1 point is earned for a justification that includes that if the horizontal force is very small the angle of the tension with the vertical is very small.

Example Response:

A small speed means a small centripetal acceleration. The horizontal component of the tension is equal to the centripetal force, so if it is very small, then the tension will be almost vertical, and the angle will be very small.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that the equation is consistent with the reasoning in part (c).
- ☐ 1 point is earned for addressing the functional dependence.
- ☐ 1 point is earned for a correct functional dependence argument.

Example Response:

It is consistent because as theta goes to zero, both trigonometric terms go to zero, and therefore the speed goes to zero. This matches the statement that the string is nearly vertical when the speed is small.

Part E

frq8

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for identifying any *v* term on one axis.
- ☐ 1 point is earned for identifying any theta term on the other axis.
- ☐ 1 point is earned for a correct power relationship between *v* and trig functions of theta (for example, *v* squared vs. $(\tan\theta)(\sin\theta)$.

Example Response:

They could graph v squared as a function of tan theta sin theta.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the rotational inertia of the sphere-rod-platform-system decreases.
- ☐ 1 point is earned for a justification that includes that the rotational inertia decreases with decreasing distance of the mass from the axis.

Example Response:

I decreases because as theta decreases, the mass gets closer to the axis of rotation and rotational inertia increases with distance from the axis.

frq8

12. This question is a long free-response question. Show your work for each part of the question.

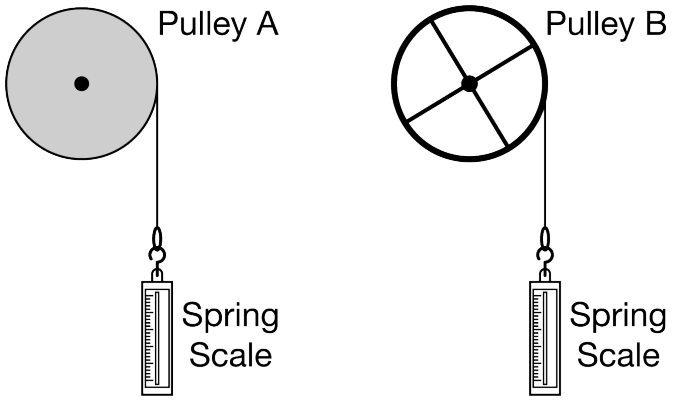


Figure 1

(a) Two identical spring scales are each attached to a cord that is wrapped around a pulley, as shown in Figure 1. The two pulleys have identical mass M_p and radius R_p. However, Pulley A is a solid disk, whereas Pulley B has most of its mass located at its outer rim. Both pulleys can rotate about their centers with negligible friction in their axles.

The pulleys are initially at rest when the two spring scales are pulled with the same constant force F_s, causing both pulleys to rotate. Predict which of the pulleys, if either, will complete one revolution first.

____ Pulley A ____ Pulley B ____ Both pulleys will complete one revolution at the same time.

Without writing or manipulating equations, provide reasoning for your selection.

(b)

i. Derive an equation for the time t_A it takes Pulley A to complete one revolution. Let I_A be the pulley's rotational inertia. Express your answer in terms of F_S, I_A, M_p, R_p, and physical constants, as appropriate.

ii. Is your answer to part (b)(i) consistent with your answer to part (a) ?

____ Yes ____ No

Provide reasoning for whether the two answers are or are not consistent.

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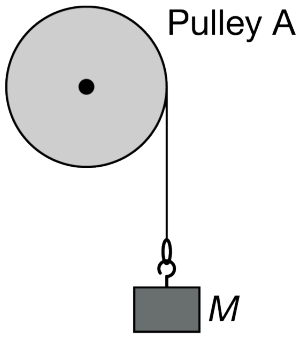


Figure 2

(c) A block of mass M is attached to Pulley A , as shown in Figure 2. The block is released from rest. Describe how the net torque exerted on the pulley and the magnitude of the angular momentum of the pulley are changing, if at all, a short time after the block is released.

Net torque: ____ Increasing ____ Decreasing ____ Not changing

Angular momentum: ____ Increasing ____ Decreasing ____ Not changing

Briefly describe how you arrived at your answers.

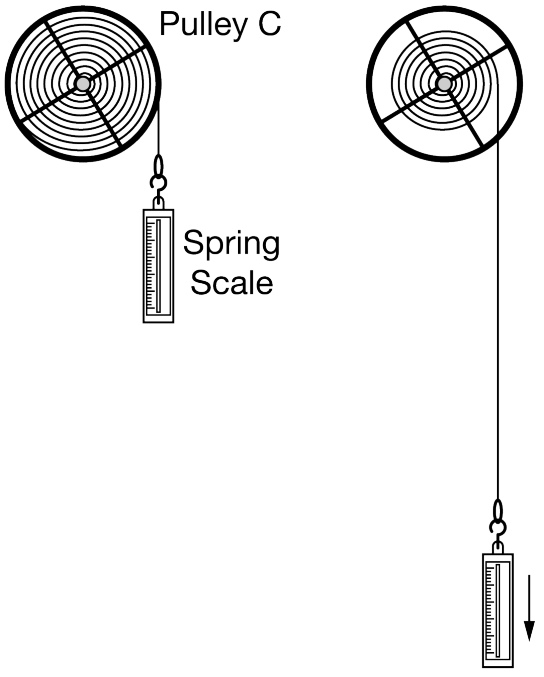


Figure 3

(d) A long cord of negligible mass is wrapped around a third pulley, Pulley C, and attached to a spring scale, as

frq8

shown in Figure 3. The spring scale is pulled downward so that it exerts a constant force of magnitude F_S on the cord. As the spring scale moves down a significant distance, the vertical portion of the cord moves closer and closer to the center of the pulley.

Predict how the net torque exerted on the pulley and the rotational kinetic energy of the pulley will be changing, if at all, as the spring scale moves downward and the vertical portion moves closer to the center of the pulley.

Net torque: ____ Increasing ____ Decreasing ____ Not changing

Rotational kinetic energy: ____ Increasing ____ Decreasing ____ Not changing

Briefly provide reasoning for your predictions.

Part (a)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes both of the following criteria.

Correct answer: Pulley A

Note: If an incorrect selection is made, the response can earn at most one point.

- ☐ 1 point is earned for explaining that pulley B has a higher rotational inertia than pulley A due to its shape or concentration of mass in the outer rim.
- ☐ 1 point is earned for stating that the lower rotational inertia for pulley A means that it will have either a greater acceleration or a faster speed at any given moment in time compared to pulley B.

Example Response:

Since both pulleys have the same force exerted on them at the same distance from their centers, the pulleys experience the same net torque. Since pulley A has a smaller rotational inertia than pulley B, because the mass is distributed closer to the center of rotation, pulley A will have a larger angular acceleration than pulley B. Because pulley A has a larger angular acceleration, it will complete one revolution before pulley B.

Part (b)i

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5 ✓

Student response accurately includes all of the following criteria.

frq8

- 1 point is earned for using the rotational form of Newton's second law.
- 1 point is earned for a correct substitution of $F_S R_P$ for the net torque exerted on the pulley.
- 1 point is earned for using an appropriate kinematics equation or set of equations that can be used to relate time, angular displacement, and angular acceleration.
- 1 point is earned for substituting 2π radians for the angular displacement and a correct or consistent expression for angular acceleration into the kinematics equation.
- 1 point is earned for correctly solving the equation for time and expressing the answer in terms of the listed quantities.

Example Response:

$$\vec{\alpha} = \frac{\vec{\tau}_{net}}{I} \quad \rightarrow \quad \alpha = \frac{F_S R_P}{I_A}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \quad \rightarrow \quad 2\pi = 0 + 0 + \frac{1}{2}\left(\frac{F_S R_P}{I_A}\right){t_A}^2 = \frac{1}{2}\left(\frac{F_S R_P}{I_A}\right){t_A}^2$$

$$4\pi\left(\frac{I_A}{F_S R_P}\right) = {t_A}^2 \quad \rightarrow \quad t_A = \sqrt{4\pi\left(\frac{I_A}{F_S R_P}\right)}$$

Part (b)ii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

Student response accurately includes one of the following criteria.

Note: The selection must be consistent with the answers to (a) and (b)(i) in order to earn credit.

- 1 point is earned for a statement that is consistent with the answers given in parts (a) and (b)(i) and the selection in (b)(ii).

Example Response:

The equation derived in (b)(i) shows that a pulley with a smaller rotational inertia will complete one revolution in less time, consistent with the answer in part (a).

Part (c)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes both of the following criteria.

frq8

Correct answers:

Net torque is not changing.

Angular momentum is increasing.

Note: A correct answer must be selected in order to earn each respective point.

- ☐ 1 point is earned for a correct description of why the net torque of pulley A is not changing.
  Note: This point is not earned for incorrectly stating that the force of tension exerted on pulley A from the rope is equal to the weight of the block attached to the rope. Since the block accelerates and has a nonzero net force, the force of tension exerted by the rope on the pulley cannot be equal to the weight of the block.
- ☐ 1 point is earned for a correct description of why the angular momentum is increasing.
  Example Response:
  The tension force exerted on the pulley by the rope, which provides the torque, is constant.
  The net torque on the pulley is in the same direction that the pulley is rotating, therefore, the pulley's angular speed is increasing.

Part (d)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answers:

Net torque is decreasing.

Rotational kinetic energy is increasing.

Note: A correct answer must be selected in order to earn each respective point.

1 point is earned for correct reasoning that the net torque is decreasing.

Example Response:

The force providing the net torque is moving closer to the pulley's axis of rotation.

1 point is earned for correct reasoning that the rotational kinetic energy is increasing.

Example Response:

The net torque exerted on the pulley is in the same direction that the pulley is rotating.

The pulley's angular speed is increasing.

0	1	2 ✓

Student response accurately includes two of the following criteria.

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Correct answers:

Net torque is decreasing.

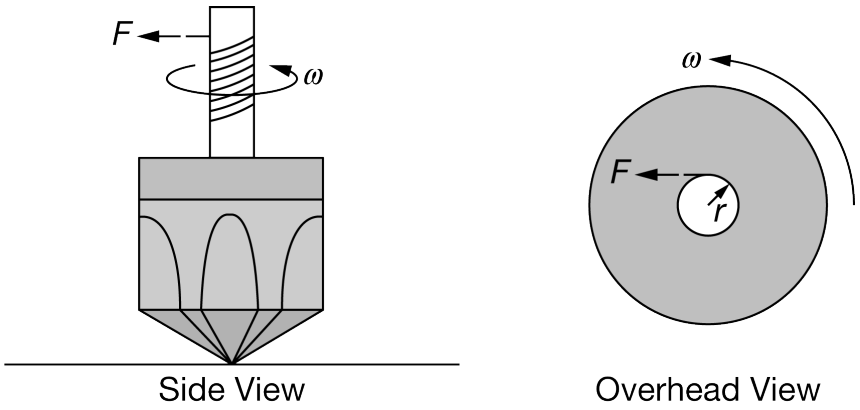
Rotational kinetic energy is increasing.

Note: A correct answer must be selected in order to earn each respective point.

- 1 point is earned for correct reasoning that the net torque is decreasing.
  Example Response:
  The force providing the net torque is moving closer to the pulley's axis of rotation.
- 1 point is earned for correct reasoning that the rotational kinetic energy is increasing.
  Example Response:
  The net torque exerted on the pulley is in the same direction that the pulley is rotating.
  The pulley's angular speed is increasing.

frq8

13. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A light string of total length L is wrapped completely around the axle of a spinning toy top. The radius of the axle is r and the rotational inertia of the top is I. The top is initially at rest and the axle is held vertically with respect to a horizontal table. At time $t = 0$ a force with constant magnitude F pulls the string outwards from the axle of the top, which causes the top to rotate. After a time T the string loses contact with the axle of the top and the top spins on the table with an angular speed ω (omega). Assume the top always remains upright and there is negligible friction between the top and the table.

(a) The original string is replaced with a new string of length $2L$. The string is again pulled with a constant force of magnitude F. When the string loses contact with the top, the angular speed of the top is ω_A. Is ω_A greater than, less than, or equal to ω? Justify your answer without citing or manipulating equations.

(b) The original string of length L is used to spin a second top that also has an axle with radius r, but a larger rotational inertia $I_B = 2I$. The string is pulled with the same constant force of magnitude F. When the string loses contact with the top, the angular speed of the second top is ω_B. Is ω_B greater than, less than, or equal to ω? Justify your answer without citing or manipulating equations.

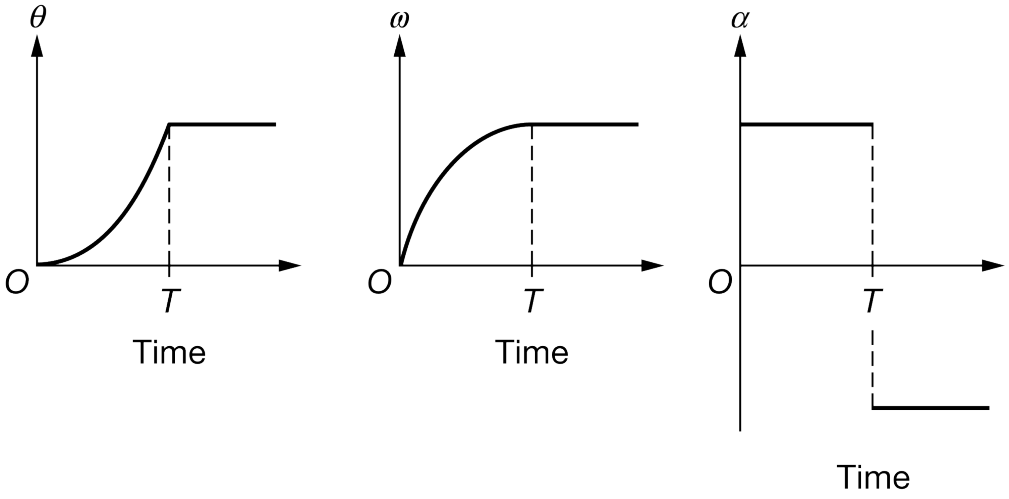
(c) A student derives the following equation to predict the final angular speed of the top: $\omega = \sqrt{\frac{2FL}{I}}$. Whether or not this equation is correct, does it match your reasoning in parts (a) and (b)?

Once the string has been pulled for a certain amount of time T, the string comes off and the top spins on the table.

(d) Describe any changes in the top's angular momentum about a vertical axis through the center of the top after the string loses contact with the top. Explain your answer.

A student makes sketches of the angular displacement θ, angular velocity ω, and angular acceleration α of the top as functions of time. The graphs below represent the time interval from when the string is first pulled until after the string loses contact with the top at time T. Consider the parts of the graph for the time before T and the time after T. Each graph is consistent with the motion of the top for one part and inconsistent for the other part.

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(e) For which part of the graph, either the time before T or the time after T, is the graph of the angular displacement θ inconsistent with the motion of the top as described directly above the graphs? Explain how to correct the part of the graph that is inconsistent.

(f) For which part of the graph, either the time before T or the time after T, is the graph of the angular velocity ω inconsistent with the motion of the top as described directly above the graphs? Explain how to correct the part of the graph that is inconsistent.

(g) For which part of the graph, either the time before T or the time after T, is the graph of the angular acceleration α inconsistent with the motion of the top as described directly above the graphs? Explain how to correct the part of the graph that is inconsistent.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that ω_A is greater than ω
- ☐ 1 point is earned for a correct justification of why ω_A is greater than ω

Example Response:

The angular speed will be larger because a longer string will result in the pulling force acting over a larger distance, which will do more work. The larger work will change the rotational kinetic energy more, which means a larger angular speed.

Part B

frq8

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that ω_B is less than ω
- ☐ 1 point is earned for a correct justification of why ω_B is less than ω

Example Responses:

The same force and string length will result in the same work done to the top, and therefore the same change in rotational kinetic energy. But with a larger rotational inertia, the angular speed will be less.

OR

While the string is being pulled, both tops will experience the same torque and rotate through the same angular displacement. According to the rotational analogue of Newton's second law, the top with greater rotational inertia will experience less angular acceleration. Lower angular acceleration through the same displacement means a lower angular speed.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for an answer that includes any reference to functional relationships between ω and L **OR** ω and I **AND** correctly evaluating whether this matches the reasoning in part (a) or (b)
- ☐ 1 point is earned for correctly identifying that the relationship for length is correct – that ω is proportional to the square root of L, so a larger L results in a larger ω
- ☐ 1 point is earned for correctly identifying that the relationship for I is correct – that ω is proportional to $1/\sqrt{I}$, so if I increases, omega decreases

Example Response:

Yes. In part (a) I said the longer the string the larger the angular speed. Length is in the numerator so a larger L results in a larger angular speed. In part (b) I said the larger the rotational inertia the smaller the angular speed. Rotational inertia is in the denominator so a larger rotational inertia will result in a smaller angular speed.

Part D

frq8

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that includes that the angular momentum will stay the same
- ☐ 1 point is earned for a correct explanation of why the angular momentum remains the same

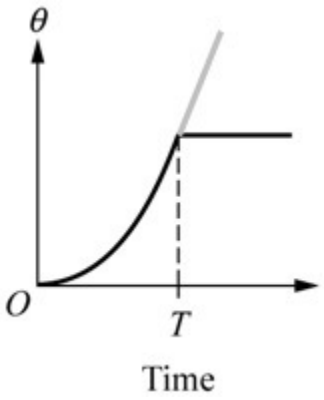
Example Response:

If there are no external forces, like friction, then there are no torques applied to the top so its angular momentum will not change.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Corrected graph, with correction shown in gray:



0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the time after *T* is inconsistent
- ☐ 1 point is earned for a statement that the slope should be positive, not zero, because the top is still spinning

Example Response:

The graph in the segment after the time T is inconsistent with the motion of the top. The top maintains a constant speed

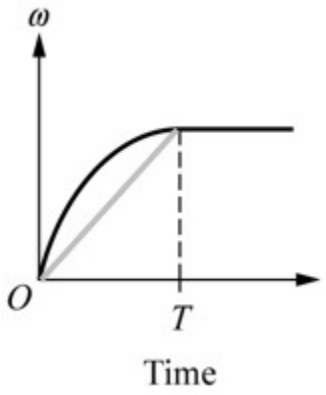
frq8

after the time T, and the horizontal line indicates that it would be at rest. The line should have a positive slope.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Corrected graph, with correction shown in gray:



0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the time before *T* is inconsistent
- ☐ 1 point is earned for a statement that the slope should be constant because the angular acceleration, force, or torque is constant

Example Response:

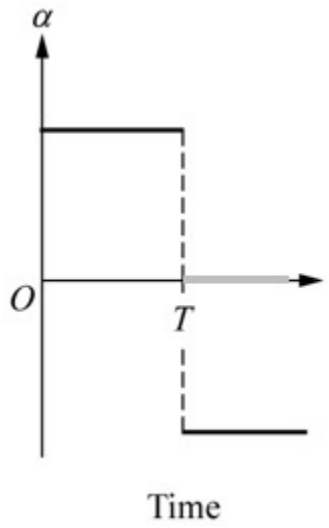
The graph in the segment before the time T is inconsistent with the motion of the top because the rate at which the speed increases is constant since the force is constant. The graph should show a straight line with a positive slope.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Corrected graph, with correction shown in gray:

frq8



0	1	2 ✓

The response accurately includes **both** of the following criteria.

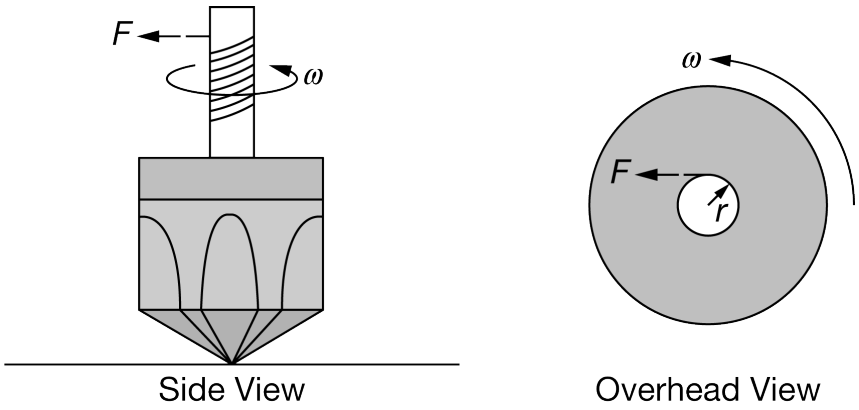
- ☐ 1 point is earned for a statement that the time after *T* is inconsistent
- ☐ 1 point is earned for a statement that the angular acceleration should be zero because there is no net torque on the system

Example Response:

The graph in the segment after the time T is inconsistent with the motion of the top because the velocity is constant as there is no torque exerted on the top and therefore the acceleration should be zero. The graph should show a horizontal line directly on the time axis.

frq8

14. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A light string of total length L is wrapped completely around the axle of a spinning toy top. The radius of the axle is r and the rotational inertia of the top is I. The top is initially at rest and the axle is held vertically with respect to a horizontal table. At time $t = 0$ a force with constant magnitude F pulls the string outwards from the axle of the top, which causes the top to rotate. After a time T the string loses contact with the axle of the top and the top spins on the table with an angular speed ω (omega). Assume the top always remains upright and there is negligible friction between the top and the table.

(a) The string is again wrapped around the axle of the top. The string is pulled with a new constant force $F_A = 2F$, and at time T_A the string loses contact with the top. Is T_A greater than, less than, or equal to T? Justify your answer without citing or manipulating equations.

(b) The string is wrapped around a second top with smaller rotational inertia $I_B = \frac{1}{2}I$. The axle of the second top is the same radius as the radius of the axle of the first top. The same constant force with magnitude F is applied to the string, and at time T_B the string loses contact with the top. Is T_B greater than, less than, or equal to T? Justify your answer without citing or manipulating equations.

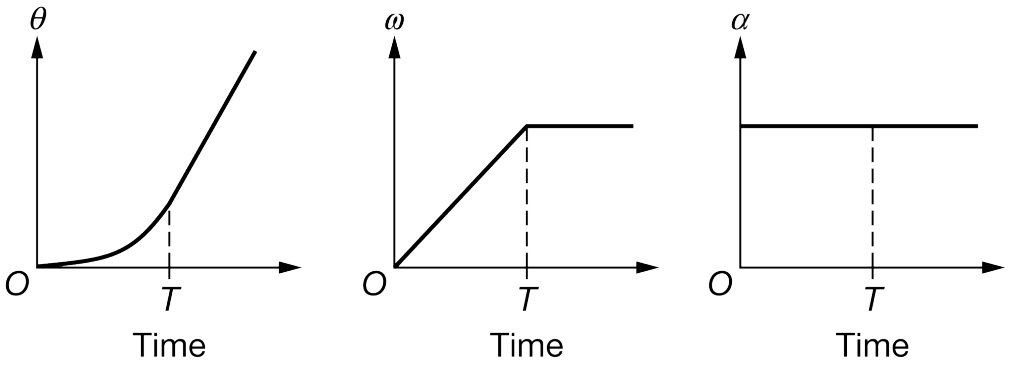
(c) A student derives the following equation to predict the amount of time over which the string is in contact with the top: $T = \sqrt{\frac{2IL}{Fr^2}}$. Whether or not this equation is correct, does it match your reasoning in parts (a) and (b)?

Once the string has been pulled for a certain amount of time T, the string comes off and the top spins on the table. Now consider the case in which friction between the top and the table is not negligible, and the friction exerts a constant torque on the top about a vertical axis through the center of the top.

(d) Describe any changes in the top's angular momentum about a vertical axis through the center of the top after the string loses contact with the top. Explain your answer.

A student makes sketches of the angular displacement θ, angular velocity ω, and angular acceleration α of the top as functions of time. The graphs below represent the time interval from when the string is first pulled until after the string loses contact with the top at time T. Consider the parts of the graph for the time before T and the time after T. Each graph is consistent with the motion of the top for one part and inconsistent for the other part.

frq8



(e) For which part of the graph, either the time before T or the time after T, is the graph of the angular displacement θ inconsistent with the motion of the top in the case in which friction is not negligible? Explain how to correct the part of the graph that is inconsistent.

(f) For which part of the graph, either the time before T or the time after T, is the graph of the angular velocity ω inconsistent with the motion of the top in the case in which friction is not negligible? Explain how to correct the part of the graph that is inconsistent.

(g) For which part of the graph, either the time before T or the time after T, is the graph of the angular acceleration α inconsistent with the motion of the top in the case in which friction is not negligible? Explain how to correct the part of the graph that is inconsistent.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that T_A is less than T.
- ☐ 1 point is earned for a correct justification of why T_A is less than T

Example Response:

A larger force will result in the top spinning faster. If the top spins faster, it will unwind faster which will decrease the time T required for the string to lose contact with the top.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq8

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that T_B is less than T.
- ☐ 1 point is earned for a correct justification of why T_B is less than T

Example Response:

A smaller rotational inertia and the same torque (same force) will result in the top spinning faster. If the top spins faster, it will unwind faster which will decrease the T required for the string to lose contact with the top.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for an answer that includes any reference to functional relationships between T and F **OR** T and I **AND** correctly evaluating whether this matches the reasoning in part (a) or (b).
- ☐ 1 point is earned for correctly identifying that the relationship for force is correct – that T is inversely proportional to the square root of F, so a larger Force means a smaller time.
- ☐ 1 point is earned for correctly identifying that the relationship for I is correct – that time is proportional to the square root of I, so a smaller I means a smaller time.

Example Response:

In part (a) I said a larger force will result in the time to unwind being smaller. Force is in the denominator so a larger force will result in a smaller time. In part (b) I said a smaller rotational inertia will result in a smaller unwind time. Rotational inertia is in the numerator so a smaller rotational inertia will result in a smaller unwind time.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

frq8

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that includes that the angular momentum will decrease.
- ☐ 1 point is earned for a correct explanation of why the angular momentum will decrease.

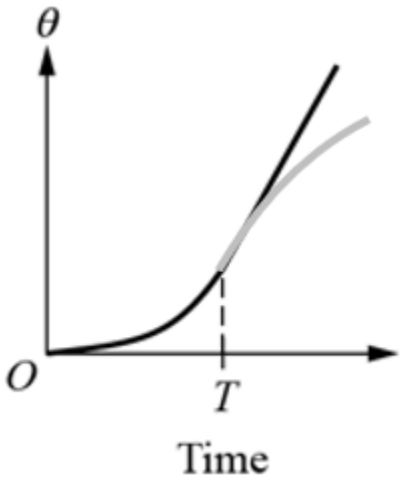
Example Response:

Since there is an external torque due to friction acting on the top then the angular momentum will decrease.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Corrected graph, with correction shown in gray:



✓

0	1	2

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the time after *T* is inconsistent.
- ☐ 1 point is earned for a statement that the slope should decrease because the top is slowing down.

Example Response:

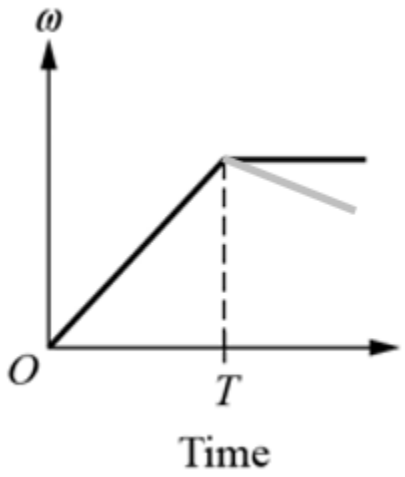
The graph in the segment after the pull time is inconsistent with the motion of the top. The top slows down so the curve should be a concave-down parabola.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

frq8

Corrected graph, with correction shown in gray:



0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the time after *T* is inconsistent.
- ☐ 1 point is earned for a statement that the angular speed should decrease due to the torque from the table.

Example Response:

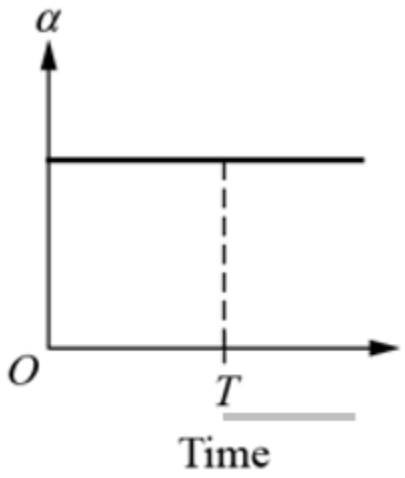
The graph in the segment after the pull time is inconsistent with the motion of the top because the speed is not constant, but it is decreasing. The graph should show a line with a negative slope.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Corrected graph, with correction shown in gray:

frq8



0	1	2 ✓

The response accurately includes **both** of the following criteria.

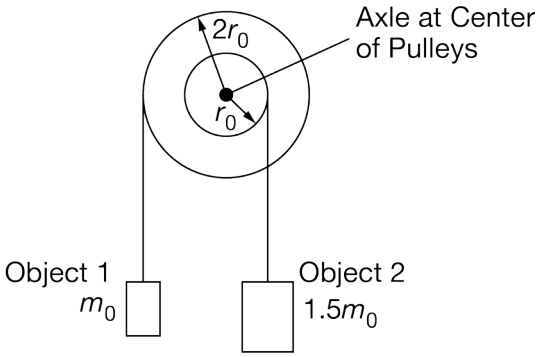
- ☐ 1 point is earned for a statement that the time after *T* is inconsistent.
- ☐ 1 point is earned for a statement that the angular acceleration should be negative because the table exerts a resistive torque on the center of the top which decreases the angular speed.

Example Response:

The graph in the segment after the pull time is inconsistent with the motion of the top because the acceleration is in the opposite direction as the initial period of time. The top is slowing down so the angular acceleration is negative. The graph should show a horizontal line below the time axis.

frq8

15. This question is a short free-response question. Show your work for each part of the question.



(7 points, suggested time 13 minutes)

Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.

m_0 is the mass of object 1.
$1.5m_0$ is the mass of object 2.
r_0 is the radius of the smaller pulley.
$2r_0$ is the radius of the larger pulley.

(a) At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.

i. Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of m_0, r_0, and physical constants, as appropriate.

ii. Object 1 accelerates downward after the pulleys are released. Briefly explain why.

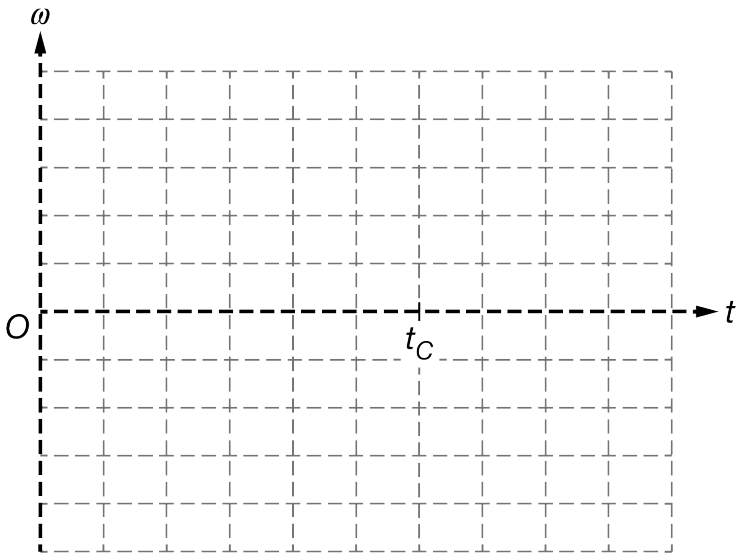
(b) At a later time $t = t_C$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?

___ Same direction ___ Opposite directions

Briefly explain your reasoning.

(c) On the axes below, sketch a graph of the angular velocity ω of the system consisting of the two pulleys as a function of time t. Include the entire time interval shown. The pulleys are released at $t = 0$, and the string is cut at $t = t_C$.

frq8



Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for correct expressions for the torques from the weight of each object. The torques are $m_0g(2r_0)$ for object 1 and $(1.5m_0g)r_0$ for object 2.
- ☐ 1 point is earned for indicating that the two torques are exerted in opposite directions. $\tau_{\text{net}} = m_0g(2r_0) - (1.5m_0g)r_0$
- ☐ 1 point is earned for the derivation of a correct answer of $0.5m_0gr_0$.
- ☐ 1 point is earned for an explanation that object 1 exerts a larger torque than object 2.

Example response for part (a)(i)

$\tau_{\text{net}} = \tau_1 - \tau_2$
$\tau_{\text{net}} = m_0g(2r_0) - (1.5m_0g)r_0$
$\tau_{\text{net}} = 0.5m_0gr_0$

Example response for part (a)(ii)

Object 1 is twice as far from the axle as object 2, while object 2 has only 1.5 *times the weight of object 1. So, object 1 exerts a larger torque.*

frq8

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for a correct answer ‘Opposite directions’ and a correct explanation

Scoring note: If the wrong answer is selected, the response is not graded.

Example Response:

The objects exerted torques in opposite directions, with object 1 exerting a larger torque, so object 1 determines the net torque direction. With the torque from object 1 removed, the net torque and angular acceleration switch direction (becoming clockwise) to the torque from object 2. The angular velocity does not change direction immediately and is still counterclockwise.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

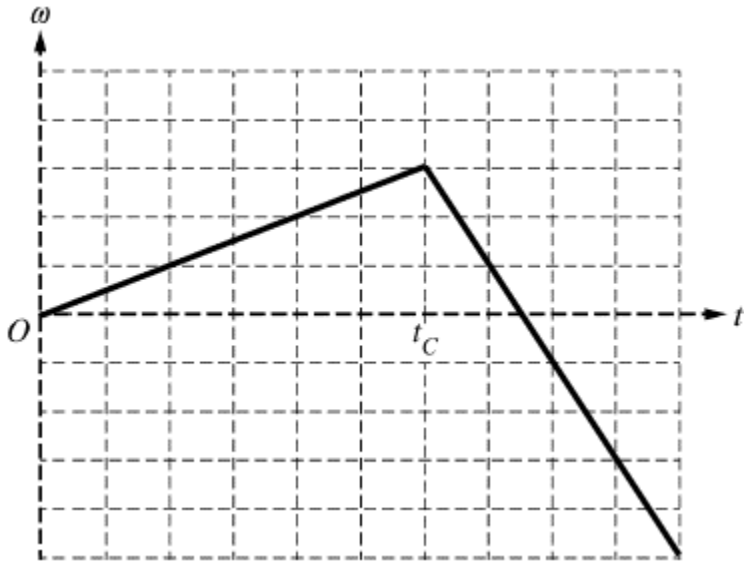
0	1	2 ✓

The response accurately includes **all** of the following criteria.

- [ ] 1 point is earned for a linear graph between 0 and t_C, with an initial angular velocity of zero and nonzero slope.
  Scoring note: The slope can be positive or negative.
- [ ] 1 point is earned for a change in the sign of the slope at $t = t_C$
  AND
  no discontinuity.

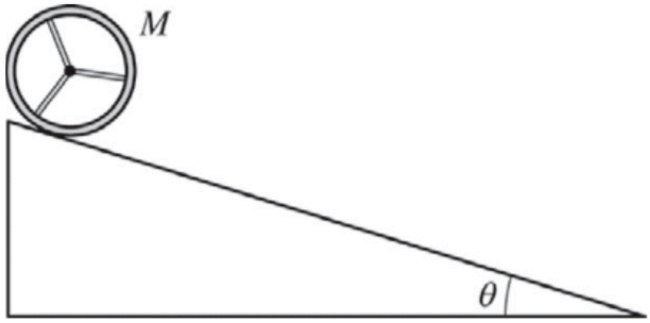
Example Response:

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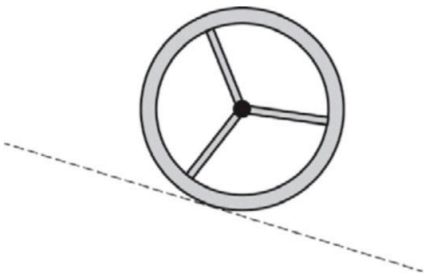
frq8

16. Show your work for each part in the space provided after that part.



A wooden wheel of mass M, consisting of a rim with spokes, rolls down a ramp that makes an angle θ with the horizontal, as shown above. The ramp exerts a force of static friction on the wheel so that the wheel rolls without slipping.

a. i. On the diagram below, draw and label the forces (not components) that act on the wheel as it rolls down the ramp, which is indicated by the dashed line. To clearly indicate at which point on the wheel each force is exerted, draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted. The lengths of the arrows need not indicate the relative magnitudes of the forces



ii. As the wheel rolls down the ramp, which force causes a change in the angular velocity of the wheel with respect to its center of mass?

Briefly explain your reasoning.

b. For this ramp angle, the force of friction exerted on the wheel is less than the maximum possible static friction force. Instead, the magnitude of the force of static friction exerted on the wheel is 40 percent of the magnitude of the force or force component directed opposite to the force of friction. Derive an expression for the linear acceleration of the wheel's center of mass in terms of M, θ, and physical constants, as appropriate.

c. In a second experiment on the same ramp, a block of ice, also with mass M, is released from rest at the same instant the wheel is released from rest, and from the same height. The block slides down the ramp with negligible friction.

i. Which object, if either, reaches the bottom of the ramp with the greatest speed?

frq8

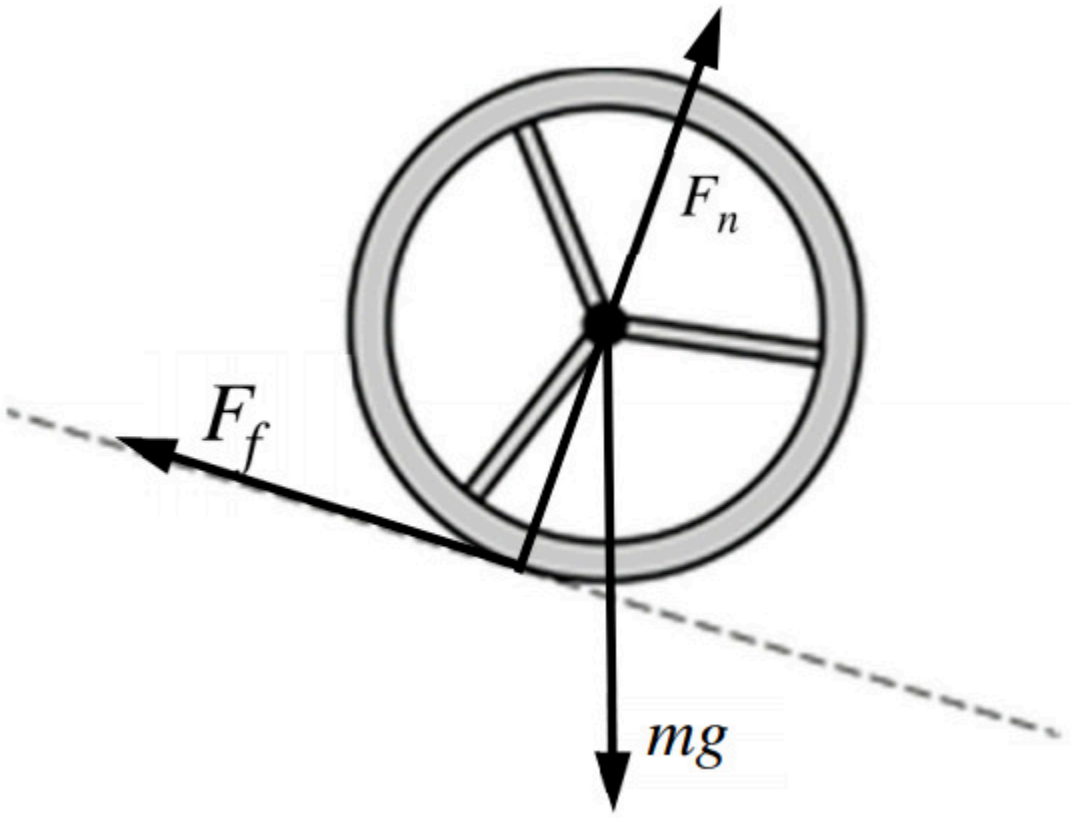
____ Wheel ____ Block ____ Neither; both reach the bottom with the same speed.

Briefly explain your answer, reasoning in terms of forces.

ii. Briefly explain your answer again, now reasoning in terms of energy

Part A

Part (i): 2 points



1 point are earned: For a labeled arrow representing the gravitational force, starting at the wheel's center and directed downward

1 point are earned: For labeled arrows representing the friction and normal forces or a single arrow representing the resultant of the friction and normal forces (i.e., the force exerted on the wheel by the surface), with no extraneous forces

The friction force should start at the wheel-ramp contact and be directed up and left along the ramp.

The normal force should start at the wheel-ramp contact and be perpendicular to the ramp and toward the wheel's center. It does not have to go exactly through the center but must come reasonably close.

Part (ii) - 1 point

Correct answer: The friction force

No points are earned if the wrong force is given.

frq8

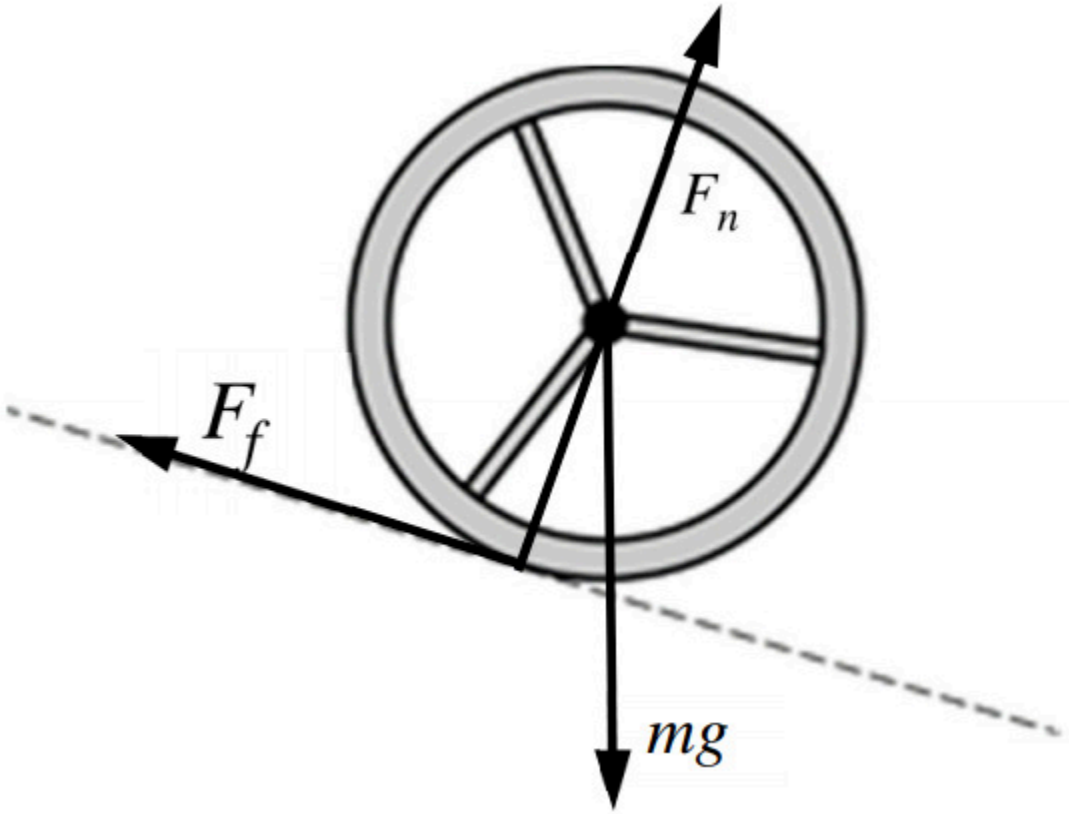
1 point are earned: For correctly explaining that friction is the only force that exerts a torque with respect to the wheel's center of mass

This point is also earned for a causal chain of reasoning about forces: e.g., the gravitational force leads to a normal force (and acceleration down the ramp), which leads to a frictional force, which exerts a torque (or changes the angular velocity).

0	1	2	3 ✓

Student response earns 3 of the following 3 point(s)

Part (i): 2 points



1 point are earned: For a labeled arrow representing the gravitational force, starting at the wheel's center and directed downward

1 point are earned: For labeled arrows representing the friction and normal forces or a single arrow representing the resultant of the friction and normal forces (i.e., the force exerted on the wheel by the surface), with no extraneous forces

The friction force should start at the wheel-ramp contact and be directed up and left along the ramp.

The normal force should start at the wheel-ramp contact and be perpendicular to the ramp and toward the wheel's center. It does not have to go exactly through the center but must come reasonably close.

frq8

Part (ii) - 1 point

Correct answer: The friction force

No points are earned if the wrong force is given.

1 point are earned: For correctly explaining that friction is the only force that exerts a torque with respect to the wheel's center of mass

This point is also earned for a causal chain of reasoning about forces: e.g., the gravitational force leads to a normal force (and acceleration down the ramp), which leads to a frictional force, which exerts a torque (or changes the angular velocity).

Part B

1 point are earned: For an expression for the sum of the force components parallel to the ramp that recognizes that there are two forces with components parallel to the ramp

The expression need not be correct or consistent with the force diagram in part (a).

$$\sum F_{\parallel} = Mg\sin\theta - F_f$$

1 point are earned: For indicating that the frictional force is (0.4) sin Mg q (explicitly or implicitly) and correctly solving for the acceleration in terms of correct variables

$$\sum F_{\parallel} = Mg\sin\theta - (0.4)Mg\sin\theta$$

$$a = \frac{\sum F_{\parallel}}{M} = \frac{Mg\sin\theta - 0.4mg\sin\theta}{M} = \frac{0.6Mg\sin\theta}{M}$$

a = 0.6g sin θ

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

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a = 0.6g sin θ

Part C

Part(i)

Correct answer: Block

No credit for answer without explanation.

1 point are earned: For a correct explanation in terms of forces

Example: The wheel experiences a counteracting frictional force, so the block has a greater net force exerted upon it and therefore has greater acceleration.

Part(ii)

1 point are earned: For a correct explanation in terms of energy conservation

Example: Both object-Earth systems lose the same amount of potential energy and therefore gain the same amount of kinetic energy. With the ice block — but not the wheel — all the kinetic energy is translational, and none is rotational, so the block is faster.

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

Part(i)

Correct answer: Block

No credit for answer without explanation.

1 point are earned: For a correct explanation in terms of forces

Example: The wheel experiences a counteracting frictional force, so the block has a greater net force exerted upon it and therefore has greater acceleration.

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Example: Both object-Earth systems lose the same amount of potential energy and therefore gain the same amount of kinetic energy. With the ice block — but not the wheel — all the kinetic energy is translational, and none is rotational, so the block is faster.