frq7

1. A 20 kg box on a horizontal frictionless surface is moving to the right at a speed of 4.0 m/s. The box hits and remains attached to one end of a spring of negligible mass whose other end is attached to a wall. As a result, the spring compresses a maximum distance of 0.50 m, and the box then oscillates back and forth.

(a)

i. The spring does work on the box from the moment the box first hits the spring to the moment the spring first reaches its maximum compression. Indicate whether the work done by the spring is positive, negative, or zero.

____ Positive ____ Negative ____ Zero

Justify your answer.

ii. Calculate the magnitude of the work described in part i.

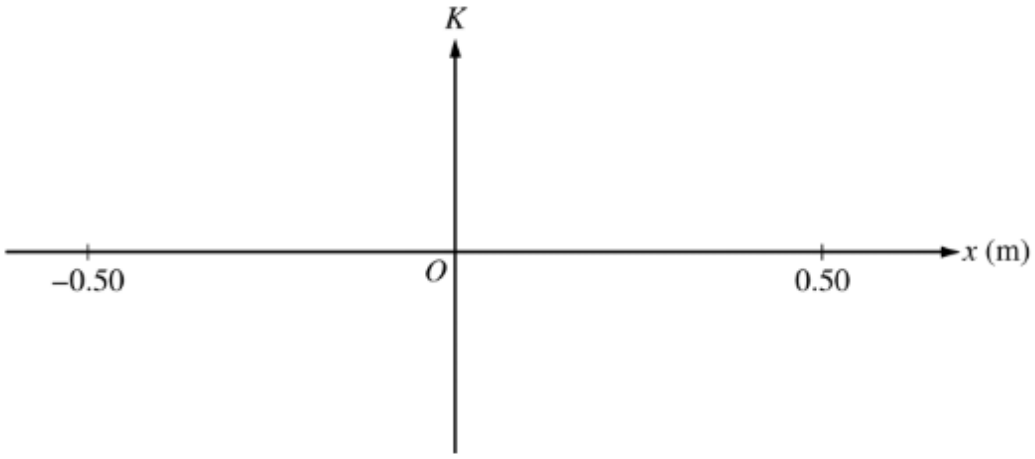
(b) Calculate the spring constant of the spring.

(c) Calculate the magnitude of the maximum acceleration of the box.

(d) Calculate the frequency of the oscillation of the box.

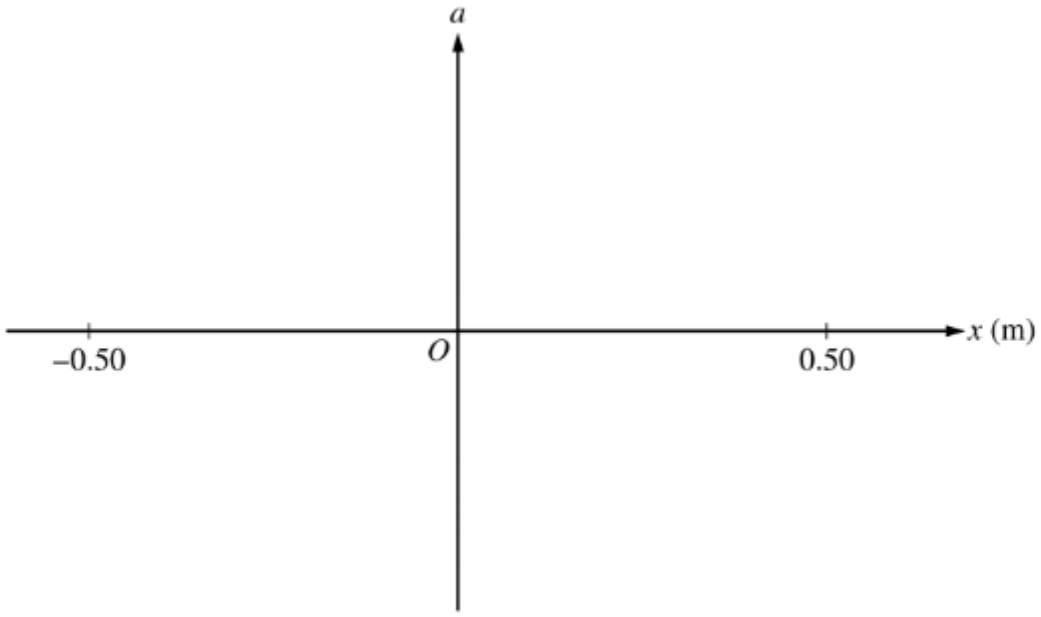
(e) Let x = 0 be the point where the box makes contact with the spring, with positive x directed toward the right.

i. On the axes below, sketch the kinetic energy *K* of the oscillating box as a function of *position* x for the range x = -0.50 m to x = +0.50 m.



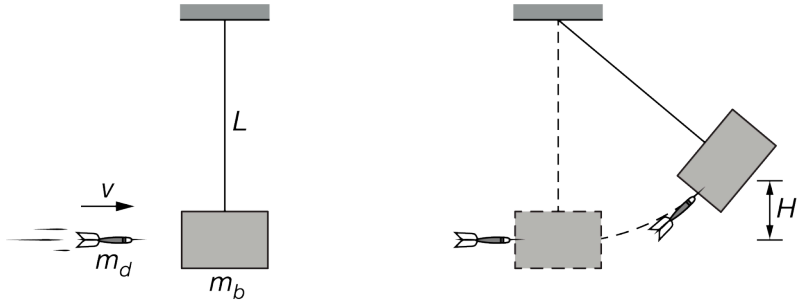
ii. On the axes below, sketch the acceleration a of the oscillating box as a function of *position* x for the range x = -0.50 m to x = +0.50 m.

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2. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown at left above. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown at right above. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, the dart or the block, if either, experiences an impulse of larger magnitude during the collision. If the impulse is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the speed of the dart as it embeds itself into the block is greater than *v*, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the motion of the dart-block system at the instant it reaches height *H*. The students make the following statements:

Student 1: The speed of the system at height *H* is zero. Since it is moving in circular motion and centripetal acceleration is given by $a_c = \frac{v^2}{r}$, the acceleration must also be zero.

Student 2: The acceleration of the system at height *H* can't be zero, or the system would not swing back down. The acceleration must be equal to *g* at height *H*.

(c) Which underlined phrase or phrases, if either, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if either, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if either, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases, if either, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

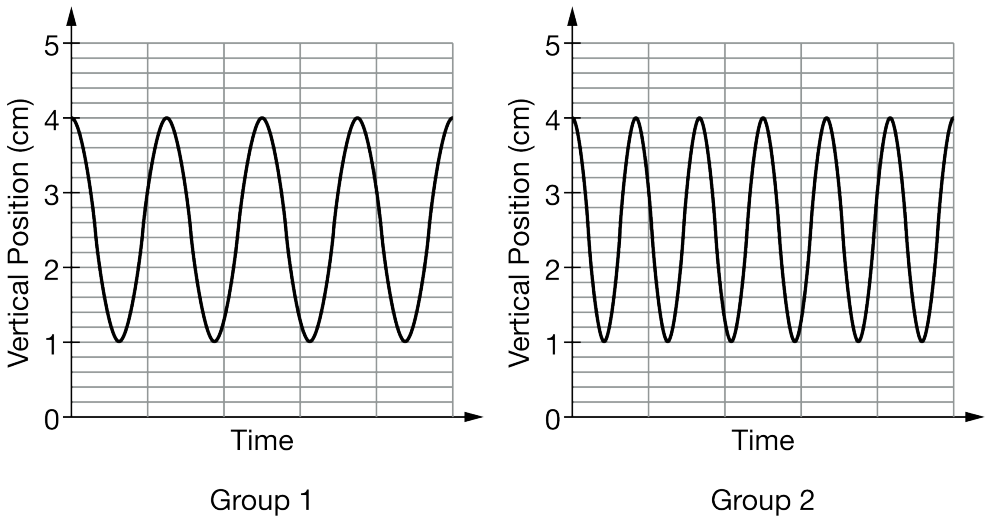
(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{m_d v}{m_d + m_b}\right)^2$. Explain how this equation supports your claim in part (b).

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The students perform several experimental trials. In each trial, a different speed v is used, and the resulting maximum height reached by the center of mass of the dart-block system is measured.

(h) Which quantities could the students graph to produce a straight line whose slope can be used to find the acceleration due to gravity, g ?

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.

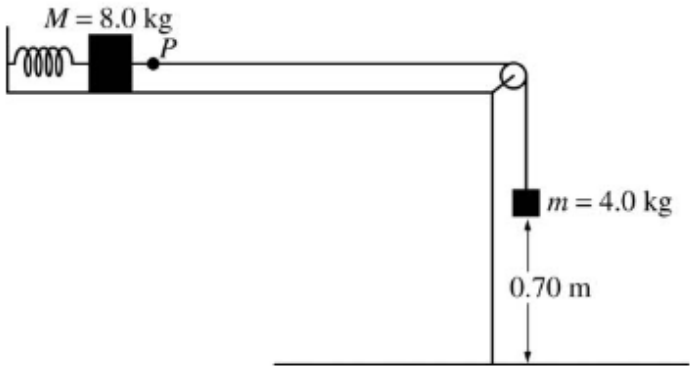


Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart mass, initial dart speed, block mass, or string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

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3.



An ideal spring of unstretched length 0.20 m is placed horizontally on a frictionless table as shown above. One end of the spring is fixed and the other end is attached to a block of mass M = 8.0 kg. The 8.0 kg block is also attached to a massless string that passes over a small frictionless pulley. A block of mass m = 4.0 kg hangs from the other end of the string. When this spring-and-blocks system is in equilibrium, the length of the spring is 0.25 m and the 4.0 kg block is 0.70 m above the floor.

(a) On the figures below, draw free-body diagrams showing and labeling the forces on each block when the system is in equilibrium.

M = 8.0 kg



m = 4.0 kg



(b) Calculate the tension in the string.

(c) Calculate the force constant of the spring.

The string is now cut at point P.

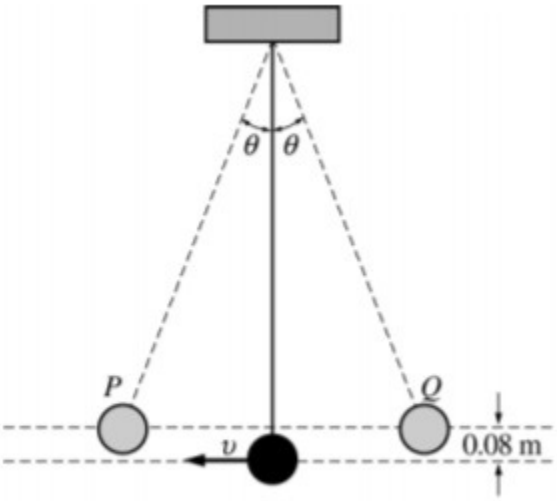
(d) Calculate the time taken by the 4.0 kg block to hit the floor.

(e) Calculate the frequency of oscillation of the 8.0 kg block.

(f) Calculate the maximum speed attained by the 8.0 kg block.

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4. A simple pendulum consists of a bob of mass 0.085 kg attached to a string of length 1.5 m. The pendulum is raised to point Q, which is 0.08 m above its lowest position, and released so that it oscillates with small amplitude θ between the points P and Q as shown below.



Note: Figure not drawn to scale.

a. On the figures below, draw free-body diagrams showing and labeling the forces acting on the bob in each of the situations described.

i. When it is at point P



ii. When it is in motion at its lowest position



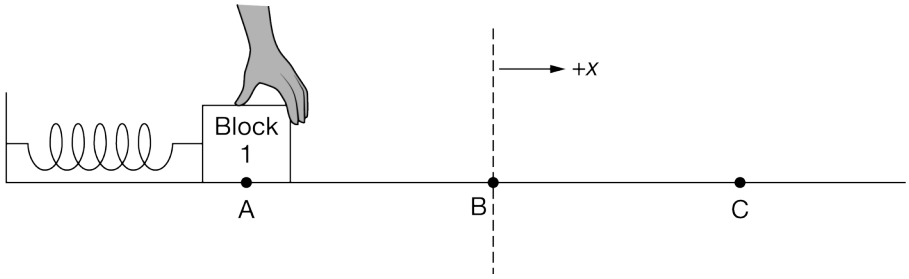
b. Calculate the speed u of the bob at its lowest position.

c. Calculate the tension in the string when the bob is passing through its lowest position.

d. Describe one modification that could be made to double the period of oscillation.

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5. This question is a short free-response question. Show your work for each part of the question.



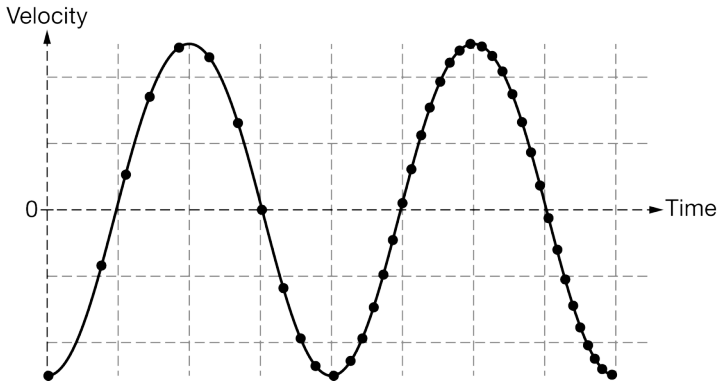
(7 points, suggested time 13 minutes)

Block 1 is at rest on a horizontal surface and is connected to a wall by an ideal spring. Friction between block 1 and the surface is negligible. Block 1 is held at rest at point A, to the left of point B which is the equilibrium position of the spring-block system, as shown in the figure. Block 1 is then released and allowed to oscillate. Some time later, block 1 is momentarily at rest at point C. Consider the positive horizontal direction to be toward the right.

(a) The dot below represents block 1 just after it is released. On the dot, draw an arrow indicating the direction of the net force exerted on the block just after it is released. The arrow should start on, and point away from, the dot.



Net Force on Block 1

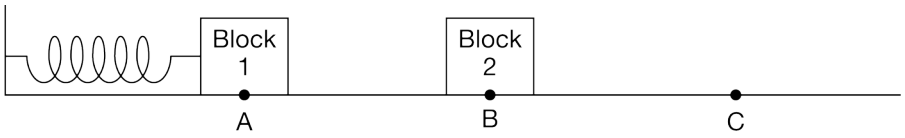


After the block has been oscillating on the table, a motion detector is turned on and the velocity of block 1 as a function of time is measured and plotted in a graph. The students correctly draw a best-fit curve that represents the

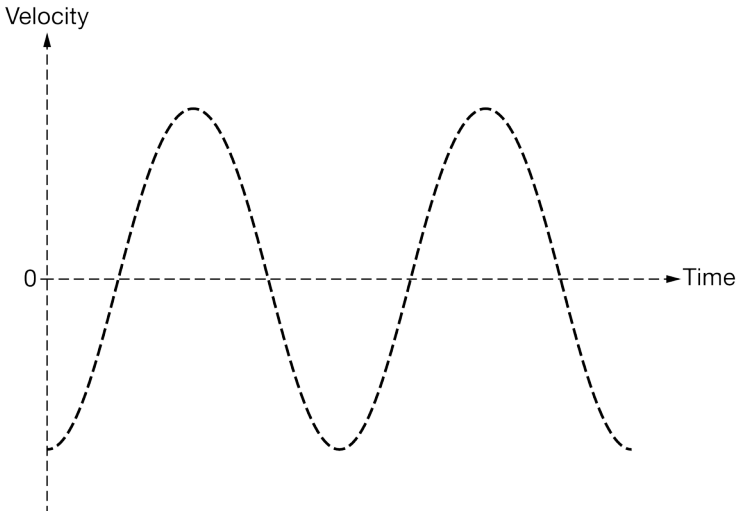
collected data, as shown above.

(b) A student looking at the graph of velocity as a function of time makes the following claim.
"*The motion sensor was turned on when the block was at point A and was moving to the left*."

Does the data in this graph support the student's claim about the location AND the direction of the block's motion when the motion sensor was turned on? For any incorrect claims, if any, state the correct position and/or direction of motion. Justify your answer.

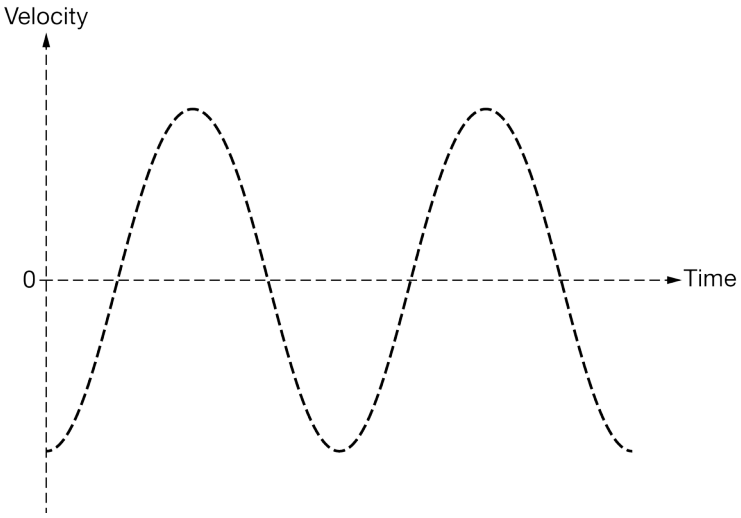


(c) Block 1 is stopped, and again block 1 is held at rest at point A. A second identical block, block 2, is placed at point B. Block 1 is released from rest, and then collides with and sticks to Block 2. After the collision, the two blocks oscillate together on the spring. On the graph below, sketch a curve to indicate the new velocity as a function of time of the two blocks after the collision. For reference, the dashed curve represents the students' best-fit curve of the velocity as a function of time described in part (b).



Note: Do any scratch (practice) work on the graph below. Sketches made below will NOT be graded.

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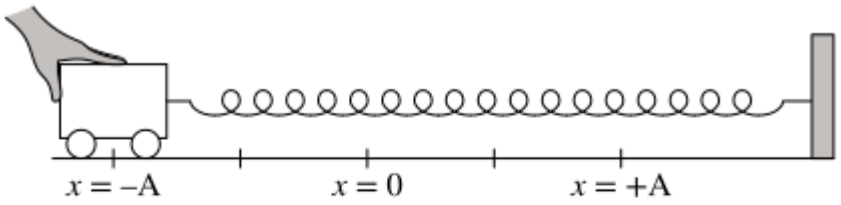


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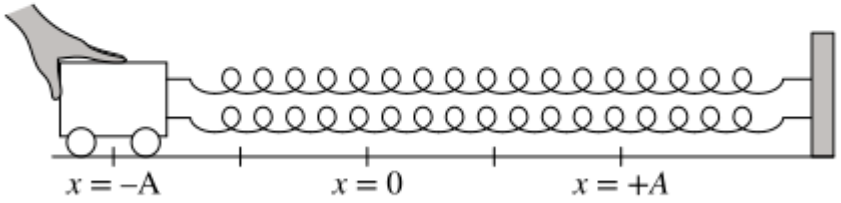
6. This question is a short free-response question. Show your work for each part of the question.

(7 points, suggested time 13 minutes)

The following two scenarios describe a cart attached to either one or two identical springs which are at their relaxed length when the cart is at position $x = 0$. In each scenario, the cart is pulled to position $x = -A$, is released from rest, and oscillates with negligible friction between $x = +A$ and $x = -A$.



Scenario 1



Scenario 2

In Scenario 1, the cart is attached to the wall with one spring as shown. The total mechanical energy of the cart-spring system in Scenario 1 is E_1.

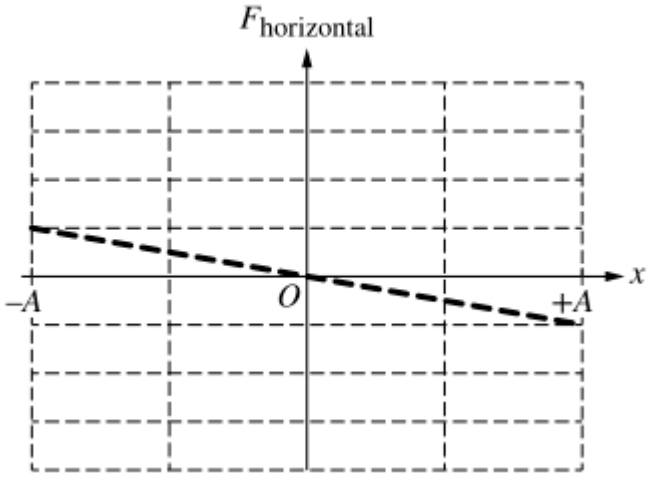
(a) For Scenario 1, derive an expression for the kinetic energy of the cart in terms of E_1 when the cart is at position $x = +\frac{A}{2}$.

(b) As shown in the figure for Scenario 2, a second identical spring is attached between the same cart and the wall before the cart is pulled to position $x = -A$.

i. Determine the total mechanical energy of the cart-springs system in Scenario 2 in terms of E_1.

ii. The maximum speed of the cart in Scenario 1 is v_1 and the maximum speed of the cart in Scenario 2 is v_2. Calculate the numerical value of the ratio $\frac{v_2}{v_1}$.

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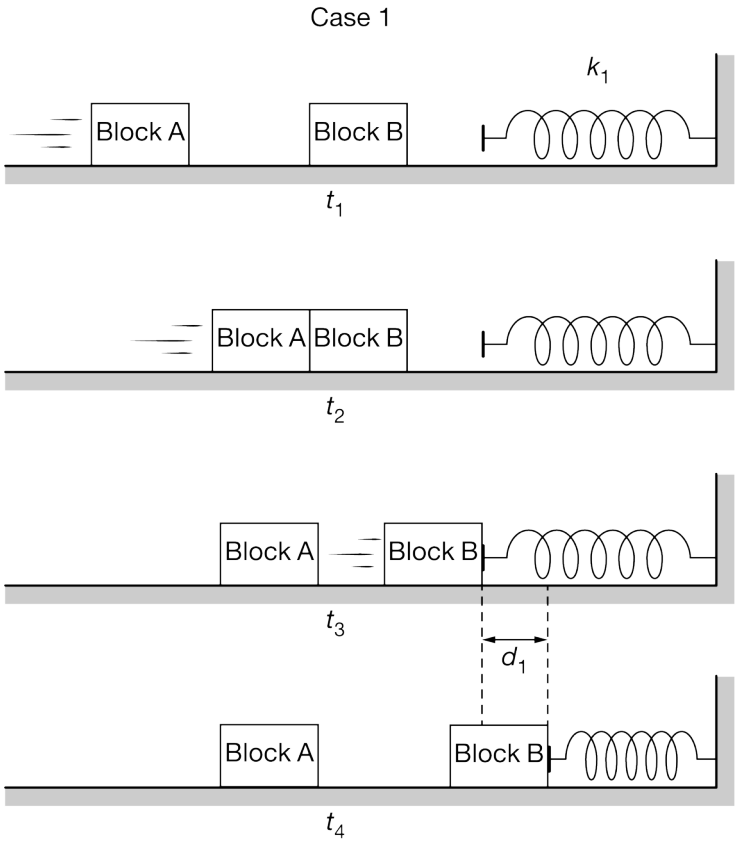


A student correctly sketches a graph of the horizontal component of the net force exerted on the cart by the spring in Scenario 1 as a function of position, as shown above with a dashed line.

(c) On the same graph above, using a solid line, sketch the horizontal component of the net force exerted on the cart in Scenario 2 with two springs attached.

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7. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



The figure above shows Case 1 of an experiment, in which the following events occur.

- At time t_1, block A slides to the right along a horizontal surface toward identical block B, which is at rest.
- At time t_2, block A collides elastically with block B and block A comes to rest.
- At time t_3, the right side of block B reaches an ideal, horizontal spring with negligible mass and spring constant k_1. The spring is initially at its relaxed (uncompressed) length and fixed to a wall.
- At time t_4, the spring is compressed a maximum distance d_1 and block B is instantaneously at rest.

Assume there is negligible friction between each block and the surface. For parts (a)-(d), consider the system consisting of block A, block B, and the spring.

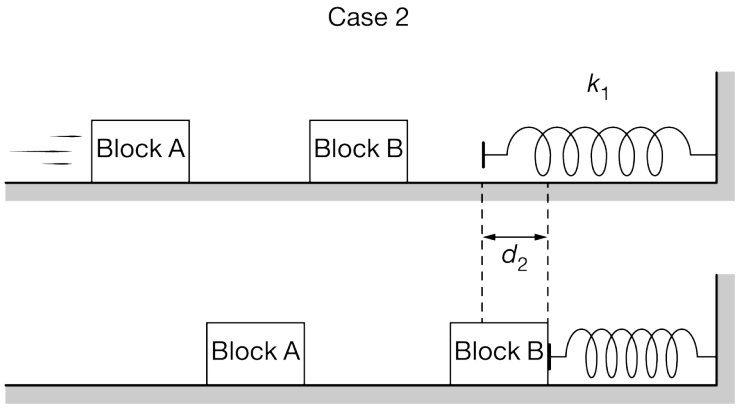
(a) Indicate whether the linear momentum of the two-block–spring system is either constant or not constant during the time interval from t_1 to immediately before t_3, and explain why.

(b) Indicate whether the mechanical energy of the two-block–spring system is either constant or not constant during the time interval from t_1 to immediately before t_3, and explain why.

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(c) Indicate whether the linear momentum of the two-block–spring system is either constant or not constant during the time interval t_3 from to t_4, and explain why.

(d) Indicate whether the mechanical energy of the two-block–spring system is either constant or not constant during the time interval from t_3 to t_4, and explain why.



In Case 2 shown above, the setup is identical to Case 1 except that the speed of block A before colliding with block B is greater than in Case 1. Block A again comes to rest immediately after colliding with block B. The spring compresses a maximum distance d_2.

(e) How does the new maximum spring compression d_2 compare to d_1, the spring compression in Case 1? (Is d_2 greater than d_1, the same as d_1, or less than d_1 ?)

(f) Briefly explain your reasoning for part (e).

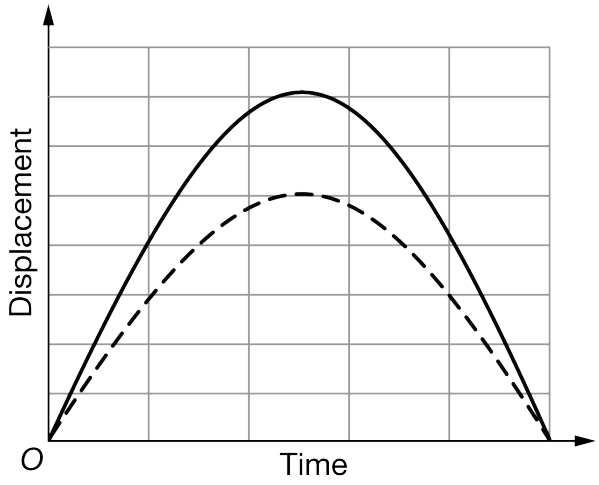
Students attempt to determine a general equation for the maximum compression distance d of a spring with spring constant k. Blocks A and B each have mass m and block A has initial speed v. The students come up with the following two equations.

Equation 1: $d = C_1\sqrt{\frac{m}{vk}}$ **Equation 2:** $d = C_2\sqrt{\frac{m}{k}}v$

In the two equations, C_1 and C_2 are positive constants with appropriate units.

(g) Without algebraic manipulation of equations or deriving the correct equation, explain which equation better supports your explanation for the compression distance in part (f).

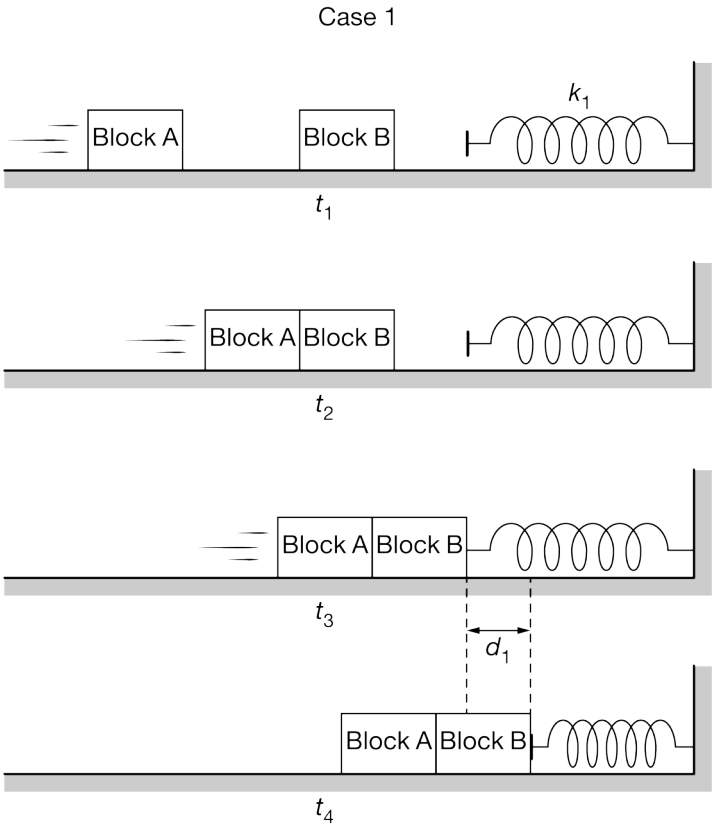
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(h) The graph above shows the displacement of the blocks while block B is in contact with the spring. The solid line corresponds to either Case 1 or Case 2, while the dashed line corresponds to the other case. Does the dashed-line curve correspond to Case 1 (slower block A) or Case 2 (faster block A)? Explain your reasoning.

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8. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



The figure above shows Case 1 of an experiment, in which the following events occur.

- At time t_1, block A slides to the right along a horizontal surface toward identical block B, which is at rest.
- At time t_2, block A collides with and sticks to block B.
- At time t_3, block A and block B are still stuck together and the right side of block B reaches an ideal, horizontal spring with negligible mass and spring constant k_1. The spring is initially at its relaxed (uncompressed) length and fixed to a wall.
- At time t_4, the spring is compressed a maximum distance d_1 and the blocks are instantaneously at rest.

Assume there is negligible friction between each block and the surface.

A student observes the experiment and makes the following claim.

"*Since some mechanical energy is dissipated when the blocks stick together, the momentum of the two-block system decreases.*"

(a) Indicate whether the claim is ***correct*** or ***incorrect*** and explain why.

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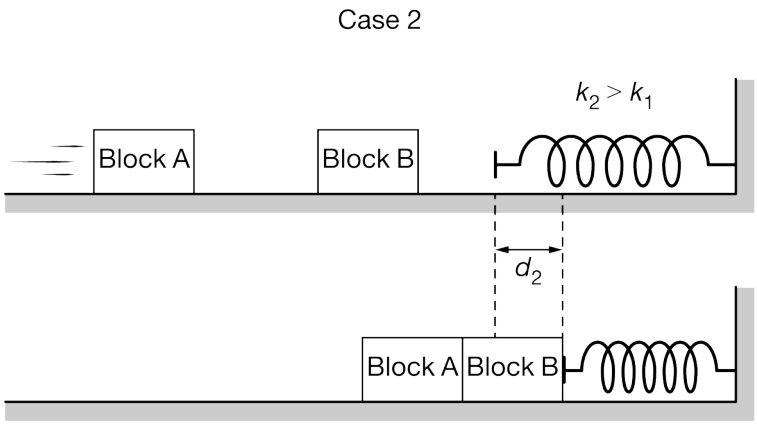
Another student makes the following claim about the collision between block A and block B.

"*When the blocks collide the total momentum of the two-block system doesn't change, so the kinetic energy of the system must also be constant.*"

(b) Indicate whether the claim is ***correct*** or ***incorrect*** and explain why.

(c) During the time interval from t_3 to t_4, is the magnitude of the acceleration of the two-block system increasing, decreasing, or constant? Briefly explain why.

(d) During the time interval from t_3 to t_4, does the spring do positive, negative, or zero work on the two-block system? Briefly explain your answer.



In Case 2 shown above, the setup is identical to Case 1 except that a stiffer spring is used. The spring in Case 2 has a spring constant k_2 that is greater than k_1. Block A slides with the same speed as in Case 1 and the spring compresses a maximum distance d_2.

(e) How does the new maximum spring compression distance d_2 in Case 2 compare to d_1 in Case 1? (Is d_2 greater than d_1, the same as d_1, or less than d_1 ?)

(f) Briefly explain your reasoning for part (e).

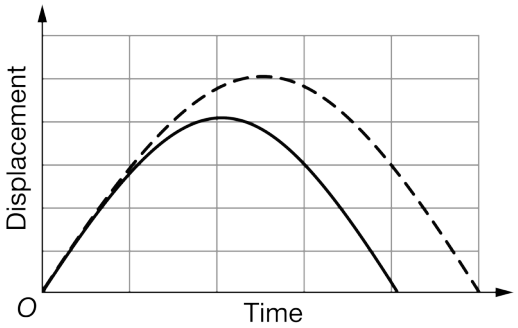
Students attempt to determine a general equation for the maximum compression distance d of a spring with spring constant k. Block A has mass m_A and initial speed v, and block B has mass m_B. The students come up with the following two equations.

Equation 1: $d = \frac{C_1 m_A v}{\sqrt{(k)(m_A+m_B)}}$ **Equation 2:** $d = C_2\left(k\sqrt{m_A + m_B}\right)v$

In the two equations, C_1 and C_2 are positive constants with appropriate units.

(g) Without algebraic manipulation of equations or deriving the correct equation, explain which equation better supports your explanation for the compression distance in part (f).

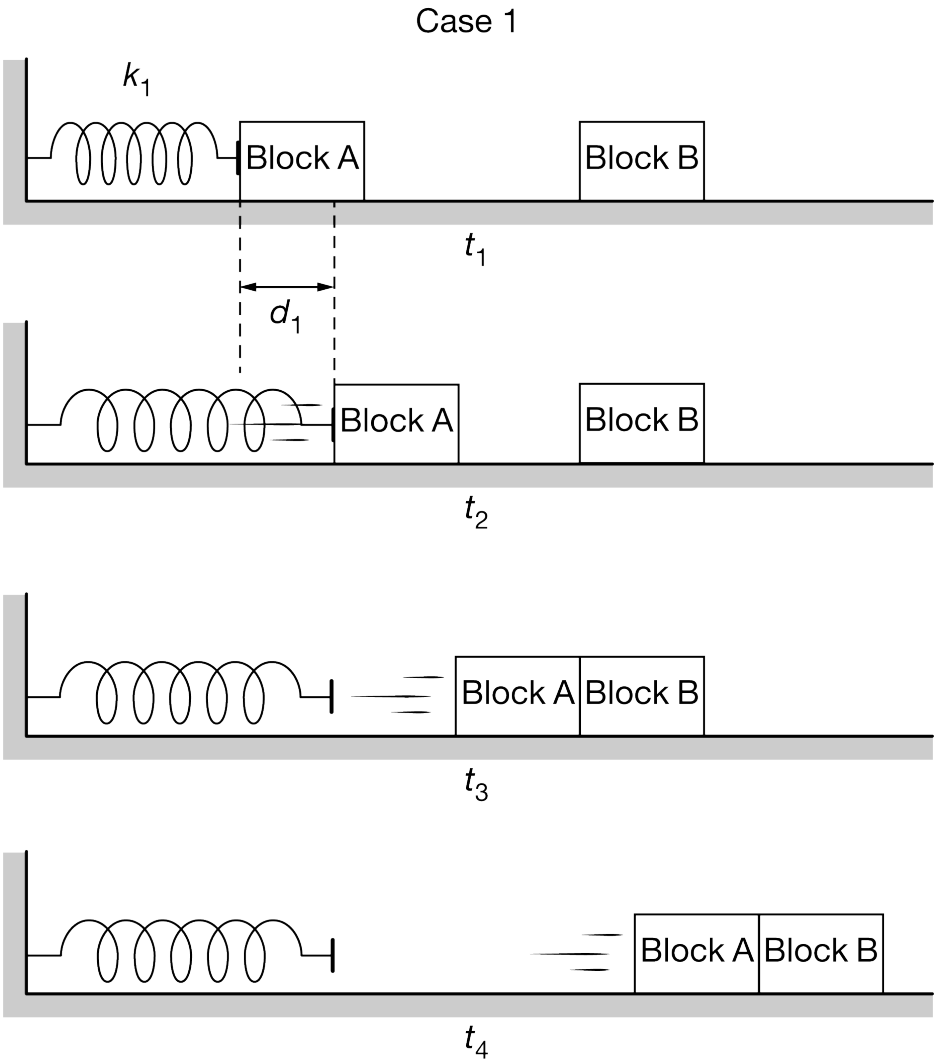
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(h) The graph above shows the displacement of the blocks while block B is in contact with the spring. The solid line corresponds to either Case 1 or Case 2, while the dashed line corresponds to the other case. Does the solid-line curve correspond to Case 1 (with a smaller spring constant) or Case 2 (with a larger spring constant)? Explain your reasoning.

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9. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



The figure above shows Case 1 of an experiment. Initially, block A is held in place while in contact with a horizontal spring with spring constant k_1 that is compressed a distance d_1 and fixed to a wall. The following events occur.

- At time t_1, block A is released from rest.
- At time t_2, the ideal spring reaches its uncompressed length, and block A leaves the spring.
- At time t_3, block A collides with and sticks to identical block B, which is initially at rest.
- At time t_4, block A and block B are sliding together along the surface.

Assume there is negligible friction between each block and the surface.

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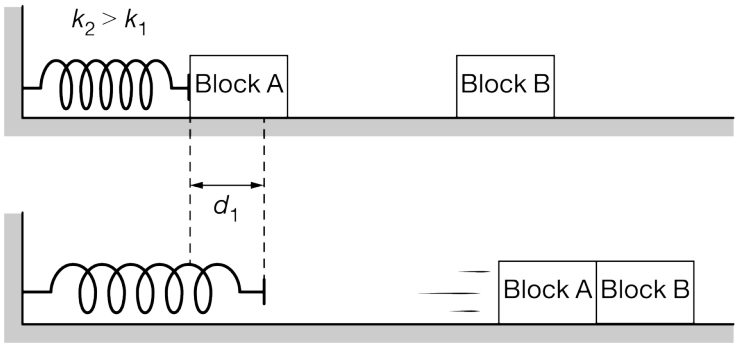
(a) Indicate whether the speed of the center of mass of the two-block system is either constant or not constant during the time interval from t_1 to t_2, and explain why.

(b) Indicate whether the mechanical energy of the two-block system is either constant or not constant during the time interval from t_1 to t_2, and explain why.

(c) Indicate whether the speed of the center of mass of the two-block system is either constant or not constant during the time interval from t_2 to t_4, and explain why.

(d) Indicate whether the mechanical energy of the two-block system is either constant or not constant during the time interval from t_2 to t_4, and explain why.

Case 2



In Case 2 shown above, the setup is identical to Case 1 except that a stiffer spring is used. The spring in Case 2 has a spring constant k_2 that is greater than k_1. The spring is compressed the same distance, d_1, before block A is released. As in Case 1, the blocks stick together when they collide.

(e) Is the speed of the blocks after the collision in Case 2 greater than, less than, or the same as the speed of the blocks after the collision in Case 1?

(f) Briefly explain your reasoning for part (e).

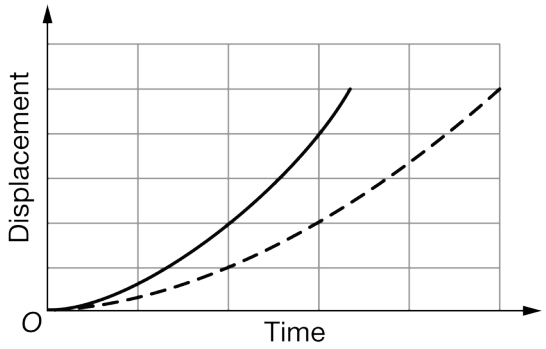
Students attempt to determine a general equation for the speed v of the blocks after the collision. A spring with spring constant k is initially compressed a distance d, block A has mass m_A, and block B has mass m_B. The students come up with the following two equations.

Equation 1: $v = C_1\left(\frac{m_B+m_A}{\sqrt{km_A}}\right)d$ **Equation 2:** $v = C_2\left(\frac{\sqrt{km_A}}{m_A+m_B}\right)d$

In the two equations, C_1 and C_2 are positive constants with appropriate units.

(g) Without algebraic manipulation of equations or deriving the correct equation, explain which equation better supports your explanation for the speed of the two blocks in part (f).

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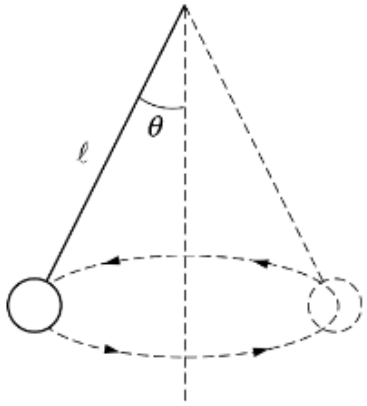


The graph above shows the displacement of block A while it is in contact with the spring for Case 1 and Case 2. The solid line corresponds to either Case 1 or Case 2, while the dashed line corresponds to the other case.

(h) Does the solid-line curve best correspond to Case 1 (with a smaller spring constant) or to Case 2 (with a larger spring constant)? Explain your reasoning.

frq7

10.



A ball attached to a string of length l swings in a horizontal circle, as shown above, with a constant speed. The string makes an angle θ with the vertical, and T is the magnitude of the tension in the string. Express your answers to the following in terms of the given quantities and fundamental constants.

a. On the figure below, draw and label vectors to represent all the forces acting on the ball when it is at the position shown in the diagram. The lengths of the vectors should be consistent with the relative magnitudes of the forces.



b. Determine the mass of the ball.

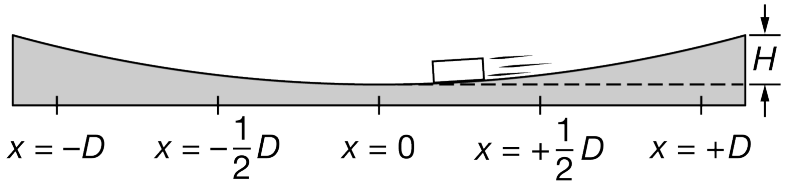
c. Determine the speed of the ball.

d. Determine the frequency of revolution of the ball.

e. Suppose that the string breaks as the ball swings in its circular path. Qualitatively describe the trajectory of the ball after the string breaks but before it hits the ground.

frq7

11. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

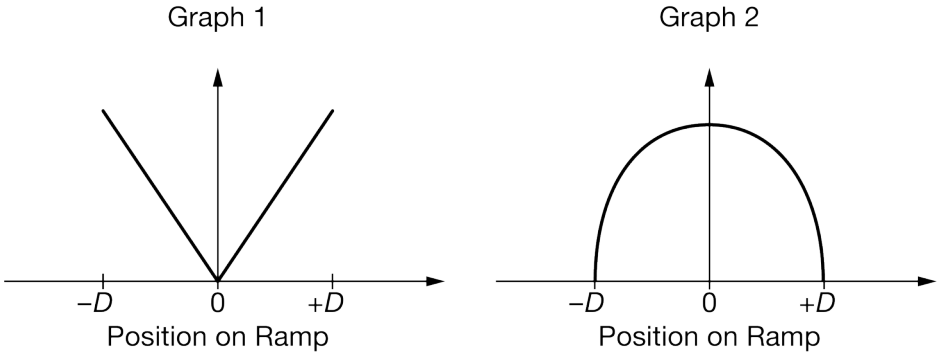


Track 1

Note: Figure not drawn to scale.

A block slides back and forth without any appreciable loss of energy on a track with a slight circular curvature, as shown above. The maximum height H of the track at each end is much smaller than D.

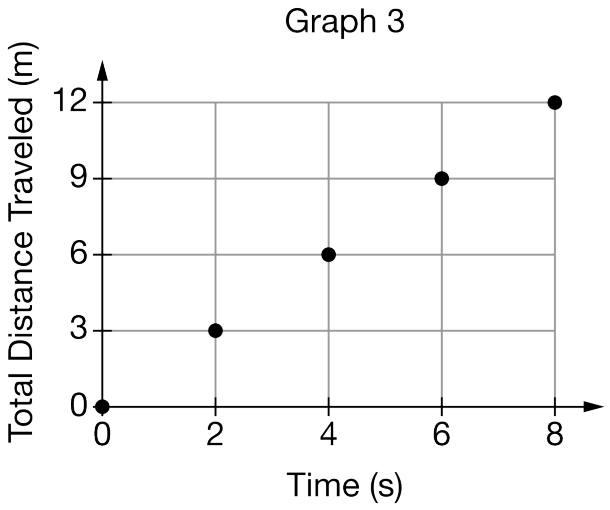
The center of mass of the block oscillates between $x = -D$ and $x = +D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. The block undergoes two full oscillations in 8.0 seconds.



(a) Which of the graphs above, Graph 1 or Graph 2, could represent the magnitude of the horizontal component of the block's acceleration as a function of its position on the ramp, and which graph could represent the magnitude of the block's momentum as a function of its position on the ramp? Justify both your choices with physics principles.

(b) For the system consisting of only the block, describe how the mechanical energy of the system changes, if at all, as the block moves from $x = 0$ to $x = -D$. Justify your answer.

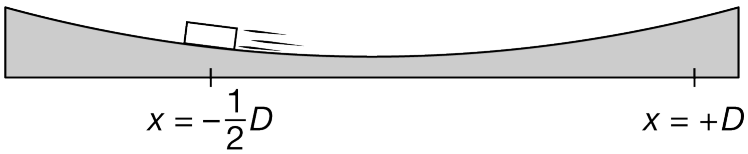
frq7



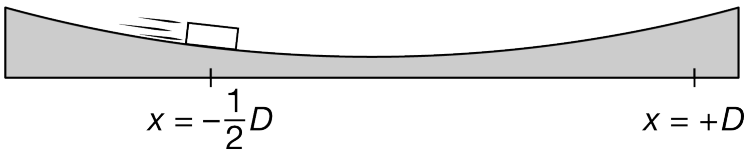
(c) Graph 3 above shows the total horizontal distance traveled by the block as a function of time as it completes two oscillations. Using the graph, determine the distance D between the midpoint of the ramp and the maximum position of the block on the ramp. Explain how you arrived at your answer.

(d) Suppose the entire setup was moved to the Moon, where the gravitational field strength is less than that on Earth. The center of mass of the block again oscillates between $x = -D$ and $x = +D$. Would the period of the block's oscillation on the Moon be greater than, less than, or equal to the period of oscillation on Earth? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{D^2}{2Hg}}$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?



Block on Track 2 at Time $t = T_1$



Block on Track 2 at Time $t = T_2$

Note: Figures not drawn to scale.

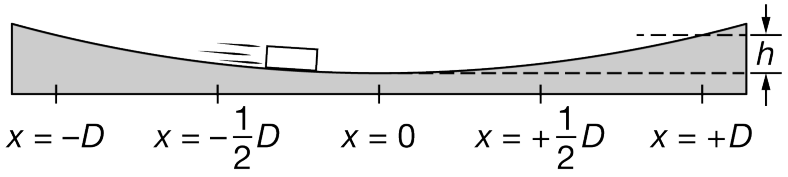
(f) Track 2 is identical to Track 1 except there is nonnegligible friction between the block and Track 2. The block

frq7

is released from rest on Track 2 at position $x = +D$. At time $t = T_1$, the block moves to the left, passing through position $x = -\frac{1}{2}D$, as shown above. After the block reverses direction near $x = -D$, the block moves to the right, passing through position $x = -\frac{1}{2}D$ at time $t = T_2$. At which time, T_1 or T_2, if either, is the magnitude of the tangential acceleration of the block greater? Justify your answer.

frq7

12. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

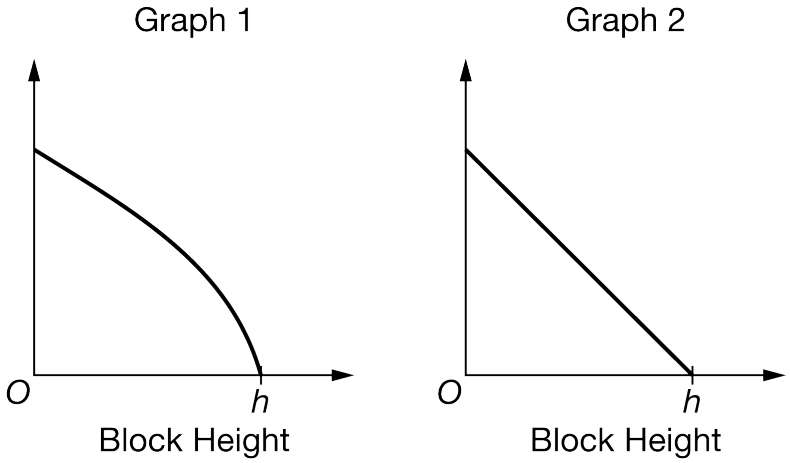


Track 1

Note: Figure not drawn to scale.

A block slides back and forth on a track with a slight circular curvature, where friction is negligible, as shown above. The maximum height h of the track at each end is much smaller than D.

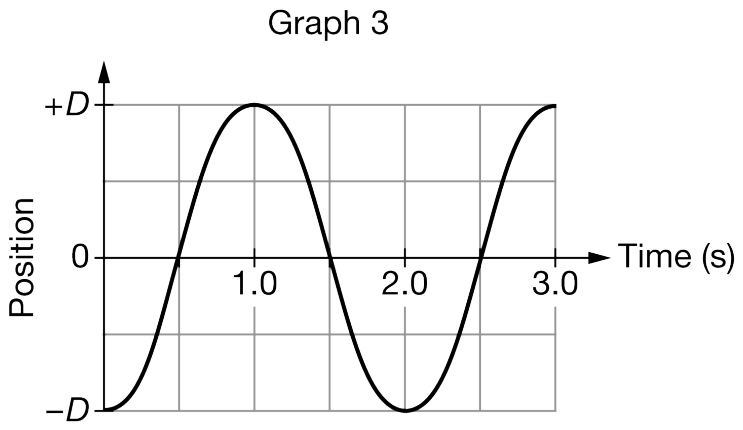
The center of mass of the block oscillates between $x = -D$ and $x = +D$, reaching a maximum height h every time the block is at $x = +D$ or $x = -D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. At time $t = 0$, the speed of the block is zero. The next time that the block's speed is zero is at time $t = 2.0$ s.



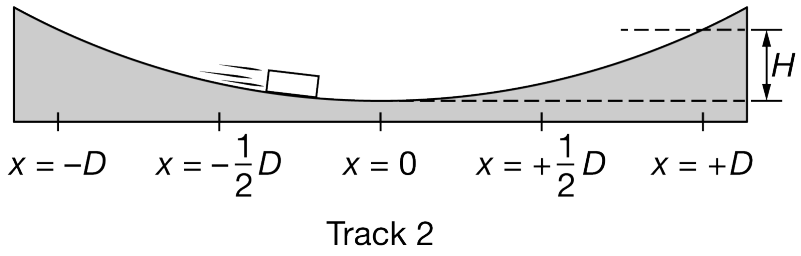
(a) Which of the graphs above, Graph 1 or Graph 2, could represent the kinetic energy of the block as a function of its height on the track, and which graph could represent the speed as a function of the block's height on the track? Justify both of your choices with physics principles.

(b) For the system consisting of the block and Earth, describe how the mechanical energy of the system changes, if at all, as the block moves from $x = +D$ to $x = 0$. Justify your answer.

frq7



(c) Graph 3 above incorrectly represents the horizontal position of the block as a function of time. Identify a feature of the graph that does not correctly represent the motion of the block. Justify your answer.



Note: Figure not drawn to scale.

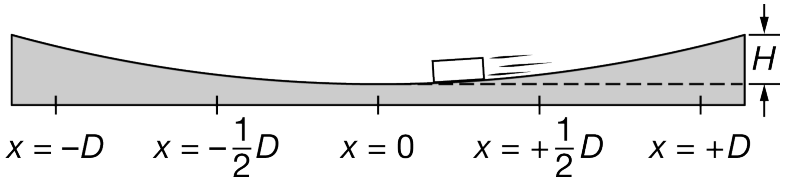
(d) Track 2 is similar to Track 1, but the maximum height H on Track 2 is greater than h and is still much smaller than D, as shown above. The center of mass of the block oscillates between $x = -D$ and $x = +D$, the same as on Track 1. How does the period of the block's oscillation on Track 2 compare to the period of the block's oscillation on Track 1? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{H^2}{2Dg}}$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?

(f) When the block reaches $x = 0$ at the bottom of the circular track, is the force exerted by the track on the block greater than, less than, or equal to the weight of the block? Justify your answer.

frq7

13. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



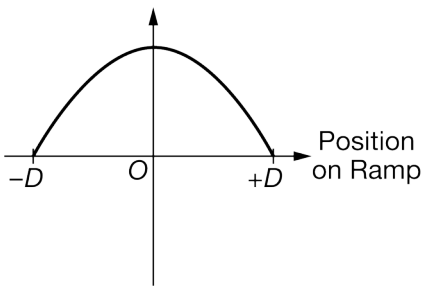
Track 1

Note: Figure not drawn to scale.

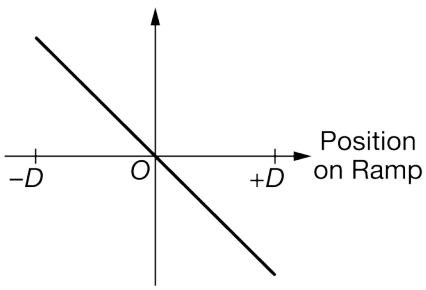
A block slides back and forth on a track with a slight circular curvature, where friction is negligible, as shown above. The maximum height H of the track at each end is much smaller than D.

The center of mass of the block oscillates between $x = -D$ and $x = +D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. At time $t = 0$ the block is released from rest at position $x = +D$. The period of the block's oscillation is 2.0 seconds.

Graph 1



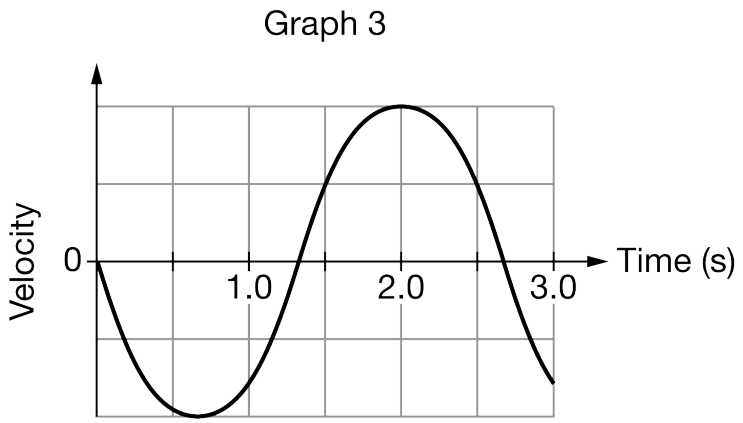
Graph 2



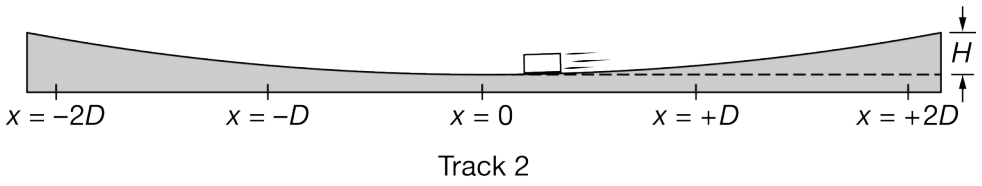
(a) Which of the graphs above, Graph 1 or Graph 2, could represent the block's kinetic energy as a function of its position on the circular track, and which graph could represent the horizontal component of the net force acting on the block as a function of its position on the track? Justify both of your choices with physics principles.

(b) Choose one of the graphs in part (a) above and explain how the graph shows evidence that a restoring force is acting on the block.

frq7



(c) Graph 3 above incorrectly shows the horizontal velocity of the block as a function of time. Identify a feature of the graph that does not correctly represent the motion of the block. Justify your answer.



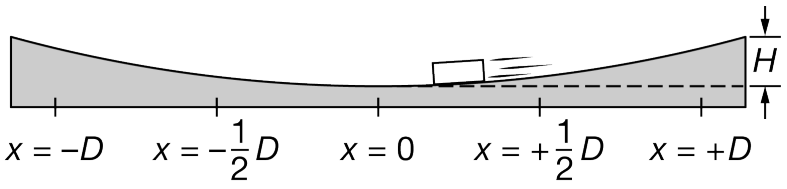
Track 2

Note: Figure not drawn to scale.

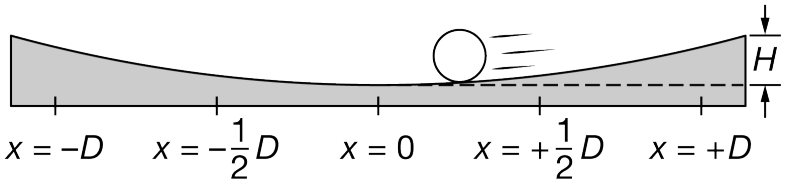
(d) Track 2, shown above, has a horizontal length that is greater than that of Track 1 and the same maximum height H. The block is released from rest at position $x = +2D$ Is the period of the block's oscillation on Track 2 greater than, less than, or equal to the period of oscillation on Track 1? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{h^2}{2dg}}$, where d is the maximum displacement of the block from equilibrium and h is the height of the track at $x = d$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?

frq7



Track 1



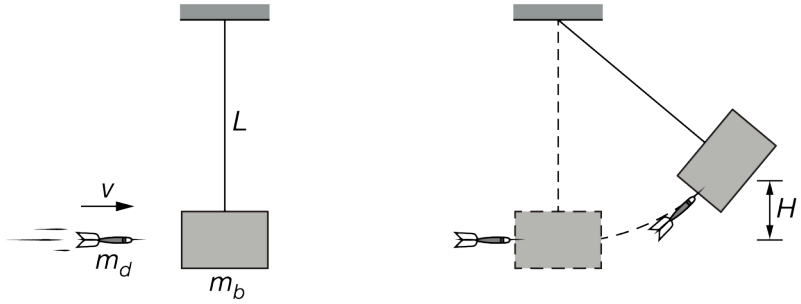
Track 3

Note: Figures not drawn to scale.

(f) On Track 1, shown above again for reference, friction is negligible and the center of mass of the block oscillates between a horizontal position of $+D$ and $-D$ with a period of 2.0 seconds. If a sphere with a mass equal to that of the block were to roll without slipping between $+D$ and $-D$ along Track 3 shown above, which is identical to Track 1 but has nonnegligible friction, would the period of the motion of the sphere be greater than, less than, or equal to 2.0 seconds? Justify your answer.

frq7

14. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown at left above. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown at right above. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, if either, experiences a net horizontal force of larger magnitude during the collision. If the net horizontal force is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the experiment was repeated with a dart with the same horizontal speed but with mass smaller than m_d, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the motion of the dart-block system at the instant it first reaches a height of $H/2$, and they make the following statements:

Student 1: When the dart-block system is at a height of $H/2$, the kinetic energy of the system is half of its maximum value, and therefore the speed of the system is half of its maximum speed.

Student 2: When the dart-block system reaches a height of $H/2$, the centripetal force is equal to the tension in the string. Therefore, the acceleration of the system must point toward the fixed end of the string.

(c) Which underlined phrase or phrases, if either, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if either, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if either, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

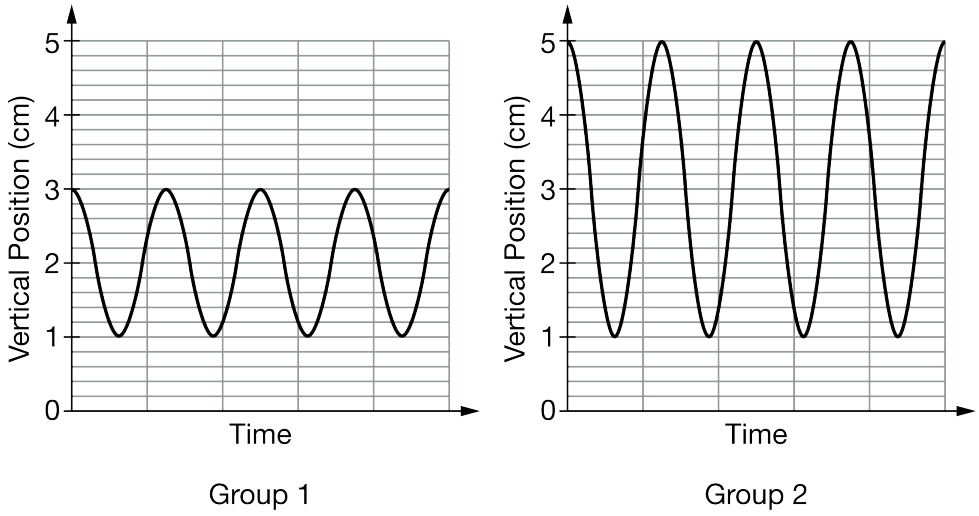
(f) Which underlined phrase or phrases, if either, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{v}{1+\frac{m_b}{m_d}}\right)^2$. Explain how this

equation supports your claim in part (b).

(h) Could the masses of the dart and block both be changed so that the maximum height *H* reached by the dart-block system would remain the same as in the original setup? Justify your answer by referring to the equation in part (g).

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.

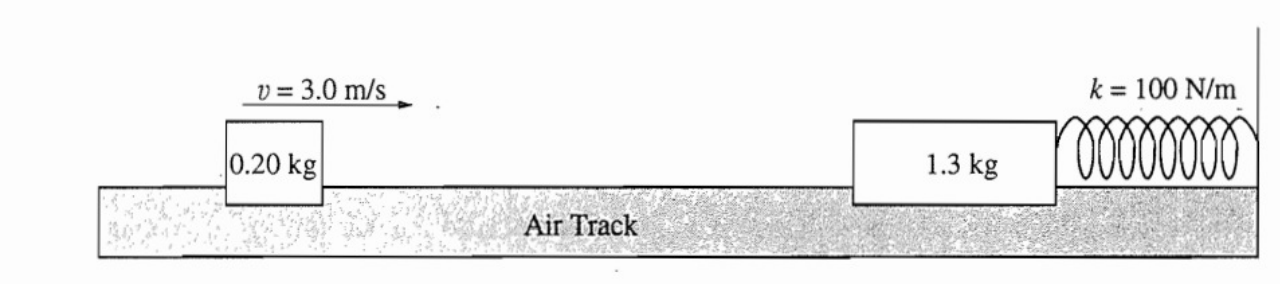


Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart mass or the string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

frq7

15.



As shown above, a 0.20-kilogram mass is sliding on a horizontal friction less air track with a speed of 3.0 meters per second when it instantaneously hits and sticks to a 1.3-kilogram mass initially at rest on the track. The 1.3-kilogram mass is connected to one end of a mass less spring, which has a spring constant of 100 newtons per meter. The other end of the spring is fixed.

(a) Determine the following for the 0.20-kilogram mass immediately before the impact.

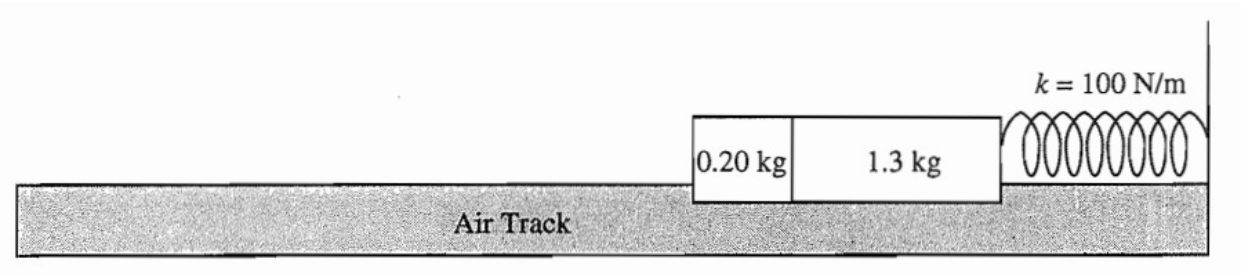
i. Its linear momentum

ii. Its kinetic energy

(b) Determine the following for the combined masses immediately after the impact

i. The linear momentum

ii. The kinetic energy



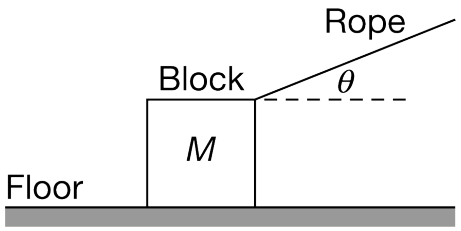
After the collision, the two masses undergo simple harmonic motion about their position at impact.

(c) Determine the amplitude of the harmonic motion.

(d) Determine the period of the harmonic motion.

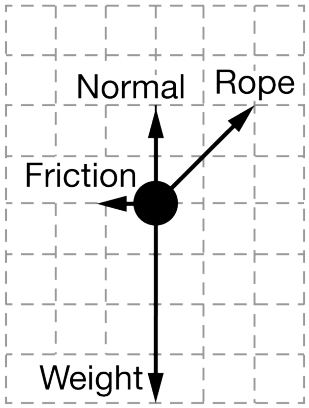
frq7

16. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

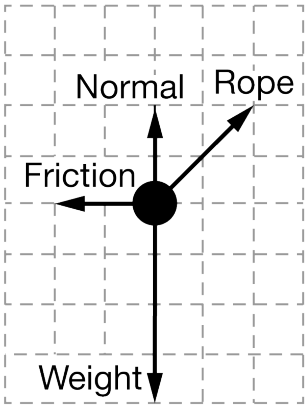


A block of mass M is pulled by a rope that makes an angle θ (theta) above the horizontal, as shown above. The coefficient of kinetic friction between the block and floor is μ (mu).

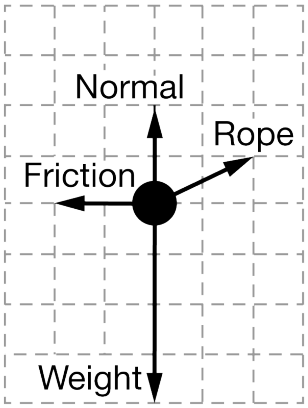
The block is initially at rest and starts accelerating to the right along the floor. Three students each draw a diagram to indicate all of the forces that are exerted on the block as it moves. In the students' diagrams, which are shown below, the lengths of the arrows indicate the relative magnitudes of the forces.



Student 1's Diagram



Student 2's Diagram



Student 3's Diagram

Only one of the students has drawn a diagram that could correctly represent the forces. The other two students' diagrams are incorrect.

(a) Which student has drawn a diagram that could correctly represent the forces exerted on the block?

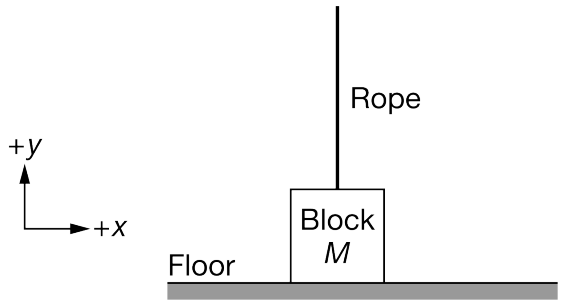
(b) For the incorrect student diagrams, explain why the diagram could not represent the forces exerted on the block. If there is more than one specific reason a diagram is incorrect, identify each of the reasons.

(c) Briefly explain why slightly increasing the angle θ (without changing the magnitude of the force exerted by the rope) could cause a decrease in the block's acceleration.

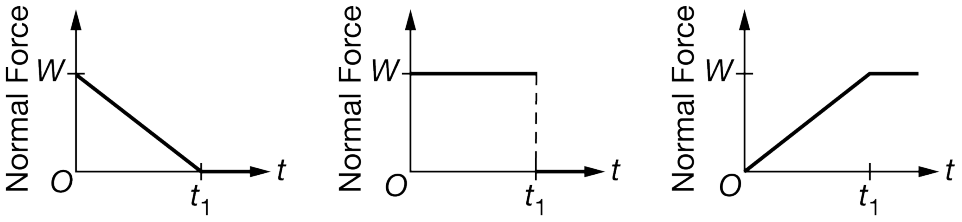
Suppose the rope is pulled with a force that has a magnitude that is one-third the weight of the block, or $\frac{Mg}{3}$. In this case, when the block accelerates, its acceleration is $a = g\left(\frac{1}{3}\cos\theta + \frac{\mu}{3}\sin\theta - \mu\right)$.

(d) Explain why the above equation is consistent with the claim that an increase in the angle θ could cause a decrease in the acceleration.

frq7



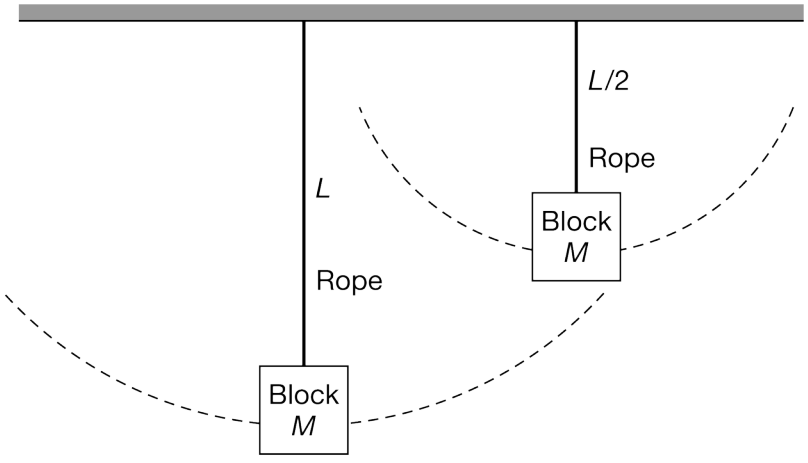
The rope is now attached to the center of the top of the block, as shown above. The force exerted by the rope is initially zero newtons at time $t = 0$. The force steadily increases, becomes equal to the block's weight W at time t_1, and continues to increase further. Three students sketch the graphs shown below to represent the normal force exerted by the floor on the block as a function of time t. One of the graphs is consistent with this scenario, while the other graphs are not consistent with this scenario.



Graph 1 Graph 2 Graph 3

(e) Which graph is consistent with the scenario in which the force exerted by the rope on the block steadily increases?

(f) Identify all features of the other two graphs that make them inconsistent with this scenario. Explain your reasoning.



Note: Figure not drawn to scale.

frq7

With one end of the rope still attached to the block, the other end of the rope is attached to a ceiling to create a pendulum with length L, where L is much greater than the dimensions of the block. A second pendulum is made from an identical block and a rope of length $\frac{L}{2}$, as shown above. Both pendulums are displaced by the same small angle and released from rest.

(g) Does the longer pendulum or the shorter pendulum have a greater period of oscillation? Briefly explain your choice without reference to equations, but by describing the restoring forces exerted on and motion of each block.

frq7

17. This question is a short free-response question. Show your work for each part of the question.

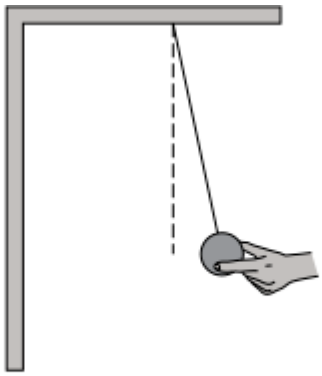


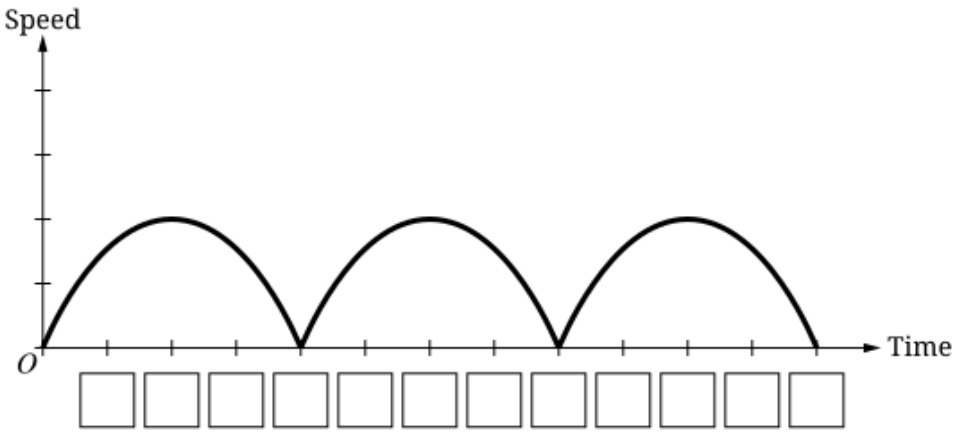
Figure 1

Note: Figure not drawn to scale.

(7 points, suggested time 13 minutes)

Student A and Student B conduct an experiment using a pendulum that consists of a metal sphere attached to a lightweight string that hangs from the ceiling, as shown in Figure 1. The students displace the pendulum a small angle from equilibrium and release it from rest at time $t = 0$.

(a) The students create the following graph of the speed of the sphere as a function of time.

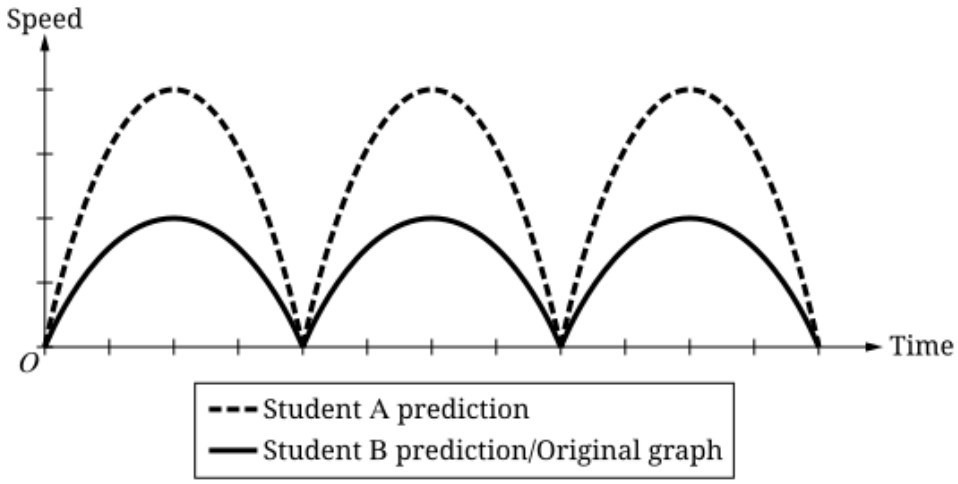


i. The sphere first reaches the lowest point of its motion (when the string is vertical) at time $t = t_1$. Indicate time t_1 by writing "t_1" in the appropriate box below the horizontal axis.

ii. The students have a new pendulum, identical to the original except that its sphere has twice the mass. The students plan to repeat the experiment by releasing the new sphere from rest at the same small angle as before.

frq7

Student A predicts the graph of speed versus time will look different, and Student B predicts it will look the same as the graph in part (a)(i). The students' predictions are shown in the following graph.



Which student's prediction is correct?

_____Student A _____Student B

Briefly explain your reasoning.

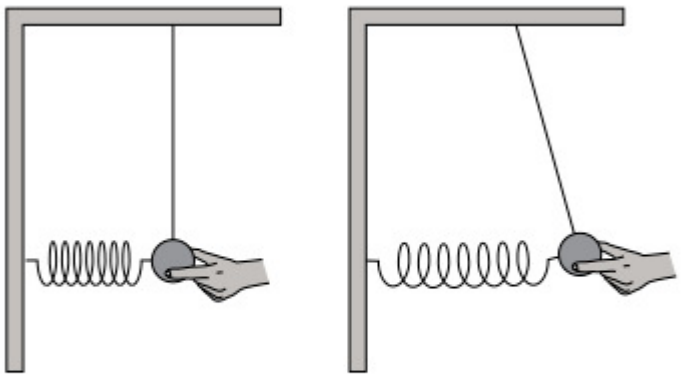


Figure 2 Figure 3

Note: Figures not drawn to scale.

(b) The students attach one end of a light spring to the sphere of the original pendulum and the other end to the wall. Figure 2 shows the new setup when the spring is oriented horizontally at its relaxed length and the string is vertical. Figure 3 shows the pendulum when the students displace the sphere the same small angle as in part (a).

At time $t = 0$, the sphere is then released from rest at this position.

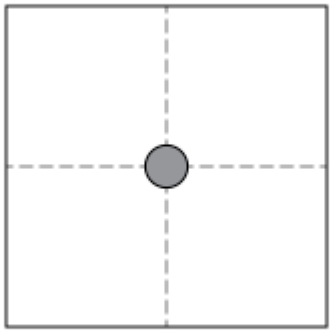
frq7

i. The sphere reaches the lowest point of its motion at time $t = t_2$. Compare t_2 to the time at which the sphere reaches its lowest point in part (a)(i), t_1. Is t_2 greater than, less than, or equal to t_1 ?

_____$t_2 > t_1$ _____$t_2 < t_1$ _____$t_2 = t_1$

Briefly justify your reasoning.

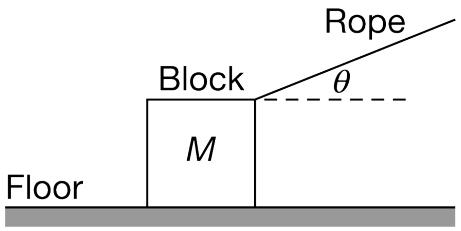
ii. The following dot represents the center of mass of the sphere at time t_2 when the sphere reaches the lowest point of its motion. Draw a single arrow representing the direction of the instantaneous acceleration of the sphere. If the acceleration is zero, write "$a = 0$" next to the dot.



iii. After having passed its lowest point, the sphere continues moving toward the wall and momentarily comes to rest. At the moment when the sphere comes to rest, how does the magnitude of the angle the string makes with the vertical compare to the angle from which it was released? Explain your reasoning.

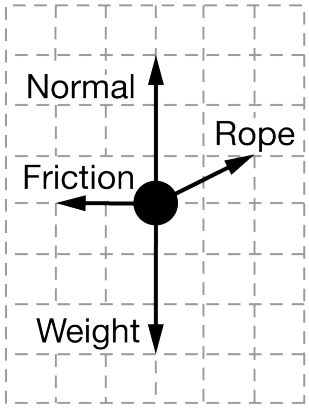
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18. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

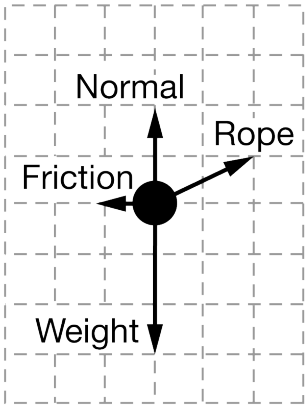


A block of mass M is pulled by a rope that makes an angle θ (theta) above the horizontal, as shown above. The coefficient of kinetic friction between the block and floor is μ (mu).

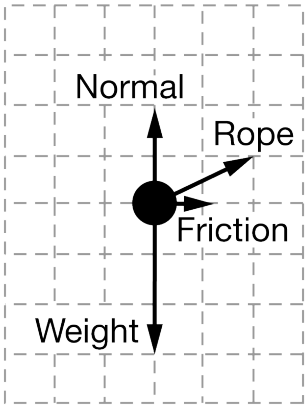
The block is initially at rest and starts accelerating to the right along the floor. Three students each draw a diagram to indicate all of the forces that are exerted on the block as it moves. In the students' diagrams, which are shown below, the lengths of the arrows indicate the relative magnitudes of the forces.



Student 1's Diagram



Student 2's Diagram



Student 3's Diagram

Only one of the students has drawn a diagram that could correctly represent the forces. The other two students' diagrams are incorrect.

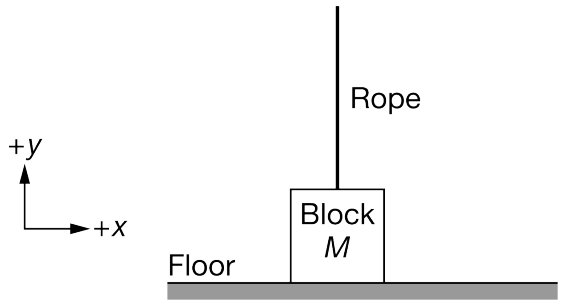
(a) Which student has drawn a diagram that could correctly represent the forces exerted on the block?

(b) For the incorrect student diagrams, explain why the diagram could not represent the forces exerted on the block. If there is more than one specific reason a diagram is incorrect, identify each of the reasons.

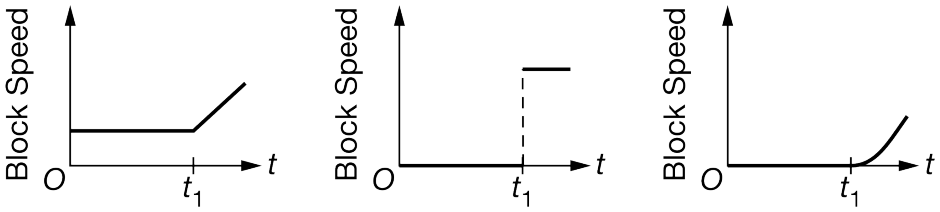
(c) Briefly explain why slightly increasing the force F exerted by the rope on the block (without changing the angle θ) would cause a decrease in the friction force.

(d) The friction force f is $f = \mu(W - F\sin\theta)$, where W is the weight of the block and F is the force exerted by the rope. Explain why this equation is consistent with the claim that an increase in F would cause a decrease in the friction force.

frq7



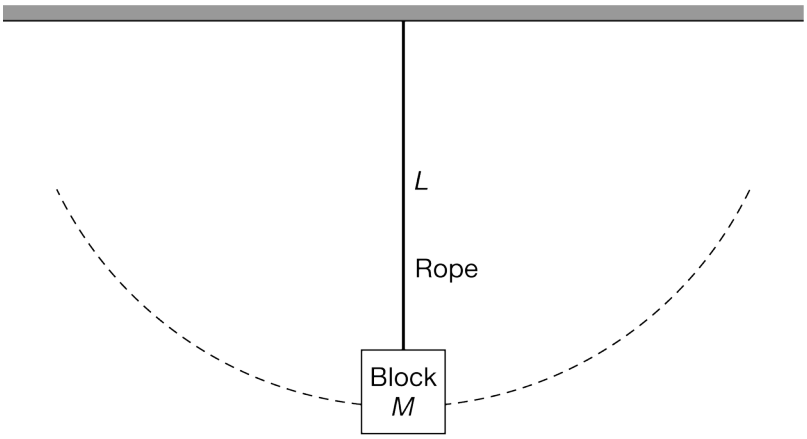
The rope is now attached to the center of the top of the block, as shown above. The force exerted by the rope is initially zero newtons at time $t = 0$. The force steadily increases, becomes equal to the block's weight W at time t_1, and continues to increase further. Three students sketch the graphs shown below to represent the speed of the block as a function of time t. One of the graphs is consistent with this scenario, while the other graphs are not consistent with this scenario.



Graph 1 Graph 2 Graph 3

(e) Which graph is consistent with the scenario in which the force exerted by the rope on the block steadily increases?

(f) Identify all features of the other two graphs that make them inconsistent with this scenario. Explain your reasoning.



Note: Figure not drawn to scale.

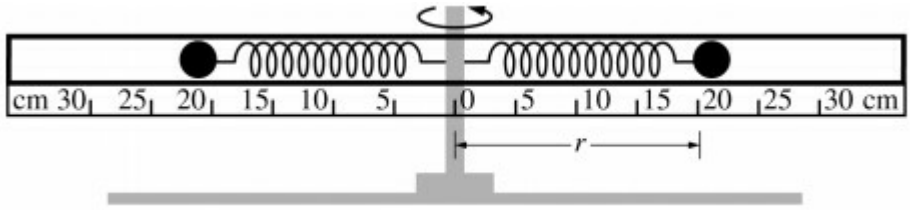
With one end of the rope still attached to the block, the other end of the rope is attached to a ceiling to create a

pendulum with length L, where L is much greater than the dimensions of the block. A second identical pendulum is located on the Moon, where the acceleration due to gravity is less than that on Earth. Both pendulums are displaced by the same small angle and released from rest.

(g) Does the pendulum on Earth or the pendulum on the Moon have a greater period of oscillation? Briefly explain your choice without reference to equations, but by describing the restoring forces exerted on and motion of each block.

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19.



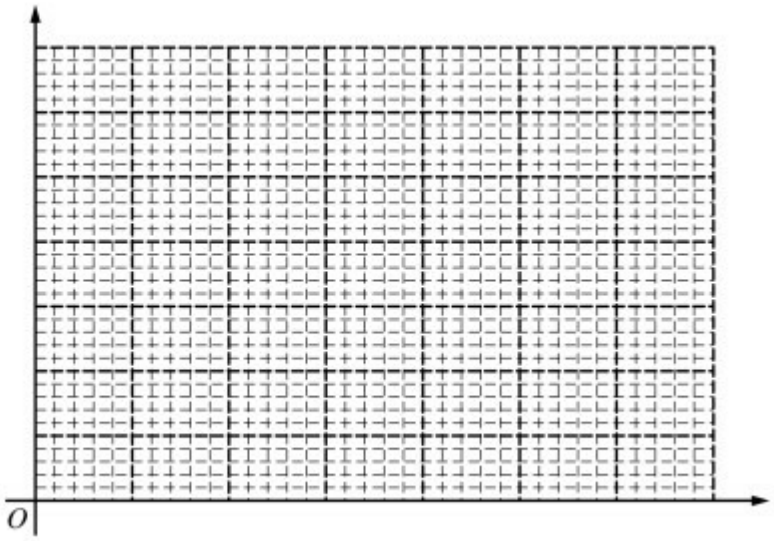
The apparatus shown above consists of two identical springs of negligible mass, each with spring constant k = 20.4 N/m and each attached at one end to a vertical rotating pole. Identical small spheres of mass m are attached to the other ends of the springs. The spheres are constrained to horizontal motion by horizontal guides of negligible friction, each of which has a ruler below it for measuring the radial position r of the sphere. The system can be manually rotated about the pole's axis. In a lab experiment, a student adjusts the rotational speed so that the spheres move to a desired radius r. For each such value of r, the student measures the rotational period T. The student's partially completed data table is shown below. The length of each unstretched spring is L = 0.15 m.

Trial	Radial Position r (m)	Period T (s)	Acceleration a_c $\left(\text{m/s}^2\right)$	Spring Force F_{spring} (N)
1	0.300	0.440	61.2	3.06
2	0.270	0.475	47.2	2.45
3	0.240	0.530		
4	0.210	0.570	25.5	1.22
5	0.180	0.795	11.2	0.61

(a) Calculate the missing values from the table above. Show your work in the space below

(b) On the axes below, plot the data points for the spring force F_{spring} as a function of the acceleration a_c. Label the axes, including the scale. Draw a straight line that best represents the data

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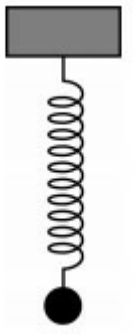


(c)

i. Calculate the slope of your line.

ii. Indicate what the slope calculated in part (c)-i represents.

(d) One sphere and one spring are removed from the rotation apparatus. They are hung vertically so that the sphere is now suspended from the spring, as shown below.

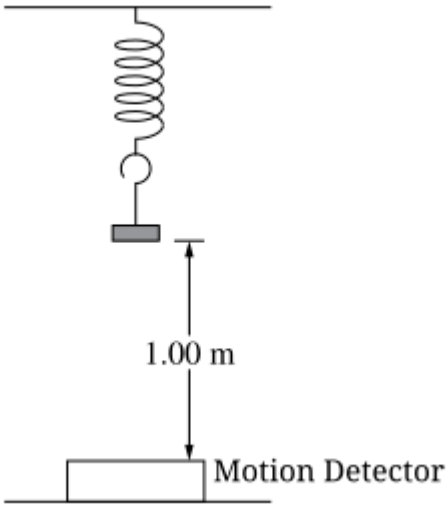


i. Describe a procedure you could use, and the measurements you would make, to verify the value obtained in part (c) using the setup shown above.

ii. Show how you would use the measurements described in part (d)-i to verify the value obtained in part (c).

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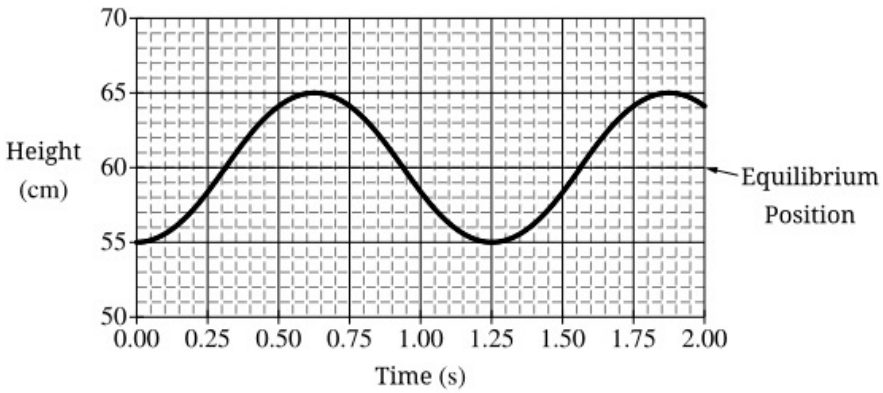
20. This question is a short free-response question. Show your work for each part of the question.



(7 points, suggested time 13 minutes)

A spring of unknown spring constant k_0 is attached to a ceiling. A lightweight hanger is attached to the lower end of the spring, and a motion detector is placed on the floor facing upward directly under the hanger, as shown in the figure above. The bottom of the hanger is $1.00\ \text{m}$ above the motion detector.

A $0.50\ \text{kg}$ object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance d_0 from equilibrium and released at time $t = 0$. The motion detector records the height of the bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.



(a) Using the information given and information taken from the graph, calculate the spring constant.

(b) At time $0.75\ \text{s}$, the object-spring-Earth system has a total kinetic energy K_0 and a total potential energy U_0.

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At 1.13 s, the object-spring-Earth system again has a total kinetic energy K_0 and a total potential energy U_0.

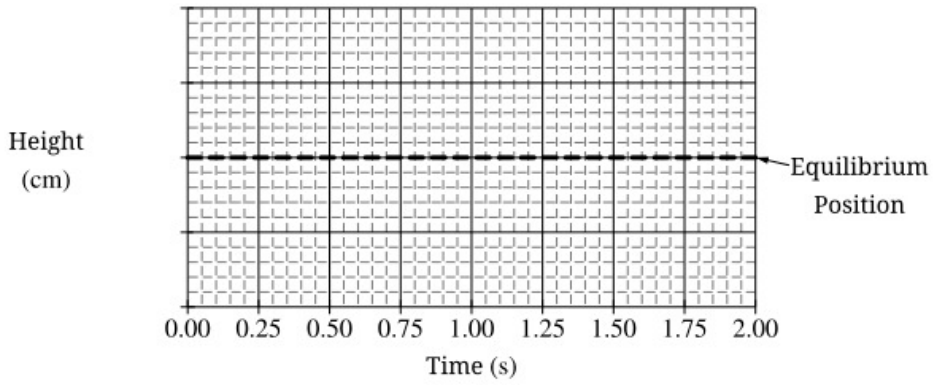
i. Explain how a feature of the graph indicates that the total kinetic energy of the system is the same at these two times.

ii. Briefly explain why the total potential energy of the system is the same at these two times.

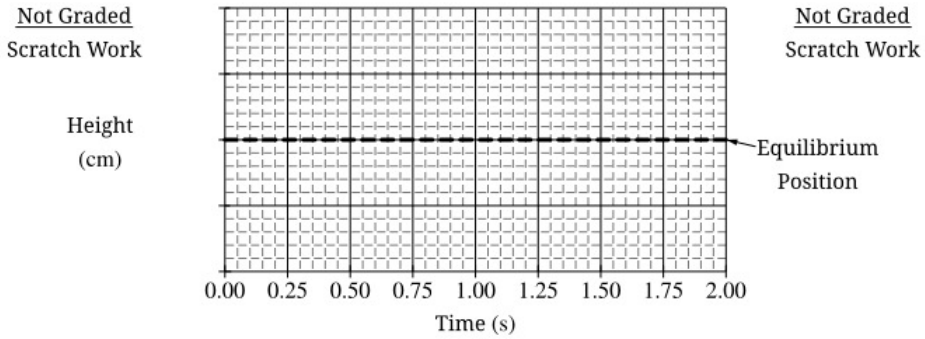
(c) The experiment is repeated with a spring of spring constant $4k_0$ and that has the same length as the original spring. The 0.50 kg object is hung from the new spring and allowed to come to rest at a new equilibrium position.

i. Determine the new equilibrium position above the motion detector.

ii. The object is again pulled down the same distance d_0 from the equilibrium position and released. On the following graph, draw a curve representing the motion of the object after it is released. Label the vertical axis with an appropriate numerical scale. A grid for scratch (practice) work is also provided.



The following graph is provided for scratch work only and will not be graded.



frq7

21. This question is a short free-response question. Show your work for each part of the question.

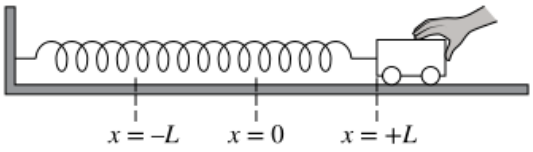


Figure 1

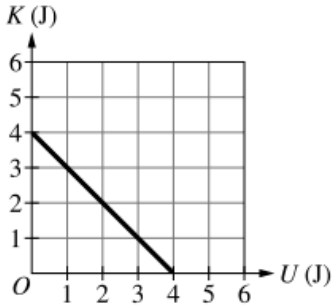


Figure 2

(7 points, suggested time 13 minutes)

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions $x = +L$ and $x = -L$. Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.

(a) On the graph of kinetic energy K versus potential energy U shown in Figure 2, the values for the x-intercept and y-intercept are the same. Briefly explain why this is true, using physics principles.

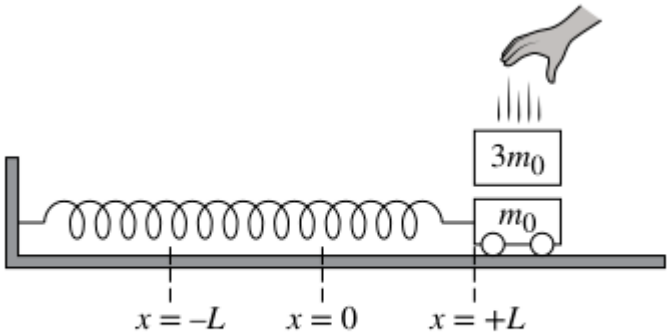


Figure 3

When the cart is at $+L$ and momentarily at rest, a block is dropped onto the cart, as shown in Figure 3. The block sticks to the cart, and the block-cart-spring system continues to oscillate between $-L$ and $+L$. The masses of the cart and the block are m_0 and $3m_0$, respectively.

(b) The frequency of oscillation before the block is dropped onto the cart is f_1. The frequency of oscillation after

the block is dropped onto the cart is f_2. Calculate the numerical value of the ratio $\frac{f_2}{f_1}$.

(c) The dashed line in Figure 4 shows the kinetic energy K versus potential energy U of the block-cart-spring system after the block is dropped onto the cart. This graph is identical to the graph shown in Figure 2 for the cart-spring system before the block is dropped onto the cart.

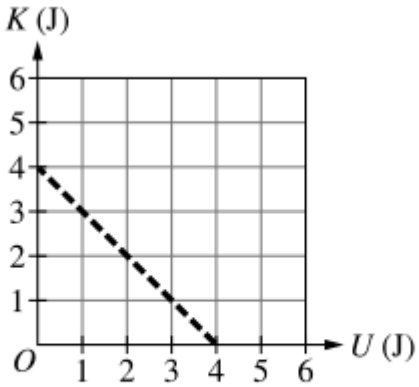


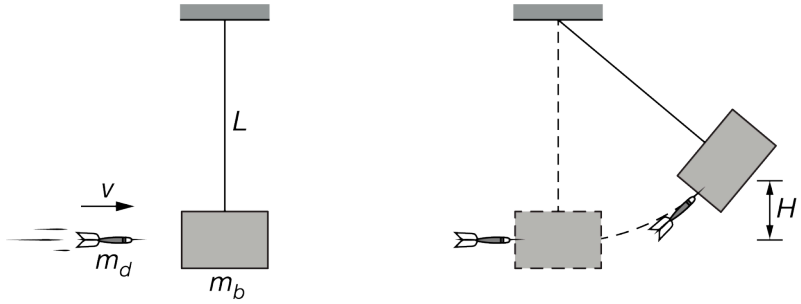
Figure 4

i. Briefly explain why the two graphs must be the same, using physics principles.

ii. After the block is dropped onto the cart, consider a system that consists only of the cart and the spring. On Figure 4, sketch a solid line that shows the kinetic energy of the system that consists of the cart and the spring but not the block after the block is dropped onto the cart.

frq7

22. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown above at left. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown above at right. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, if either, experiences an acceleration of larger magnitude during the collision. If the acceleration is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the experiment were repeated with a block with mass greater than m_b, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the tension in the string at the moment immediately after the collision and state the following:

Student 1: The tension in the string immediately after the collision is greater than the weight of the dart-block system because the acceleration is upward at that instant.

Student 2: The tension in the string is equal to the weight of the block-dart system because there is no vertical acceleration immediately after the collision.

(c) Which underlined phrase or phrases, if any, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if any, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

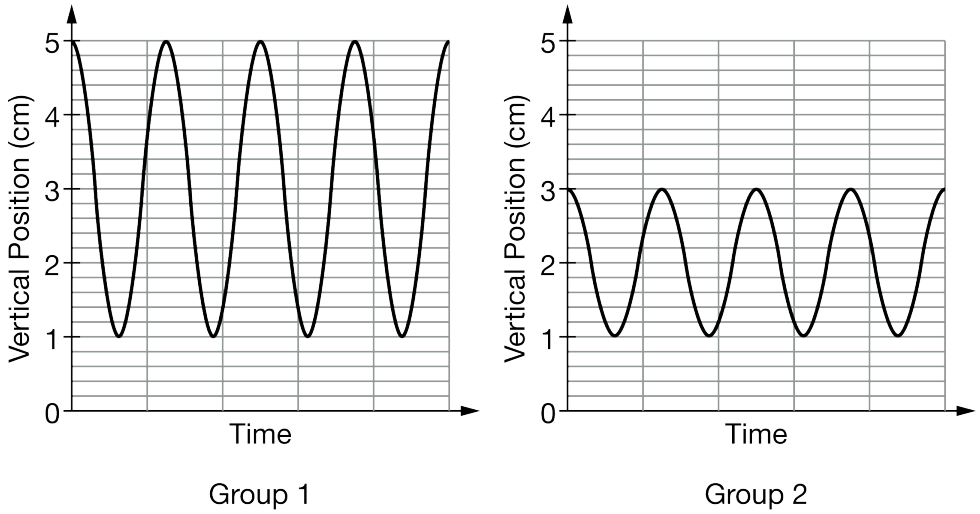
(e) Which underlined phrase or phrases, if any, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases, if any, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{m_d v}{m_d + m_b}\right)^2$. Explain how this equation supports your claim in part (b).

(h) Consider the case in which the collision between the dart and the block is perfectly elastic (the dart bounces off the block to the left). In this case, is the speed of the block immediately after the collision greater than, less than, or the same as the speed of the block in the original case? Justify your answer.

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart speed or the string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.