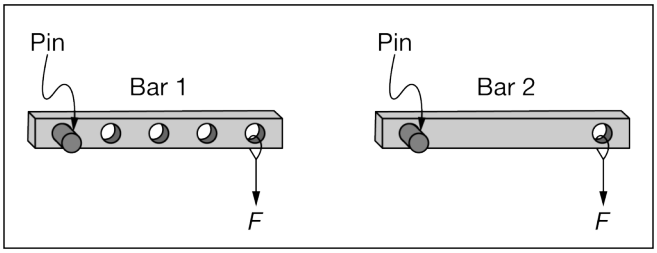
frq8

1. This question is a short free-response question. Show your work for each part of the question.



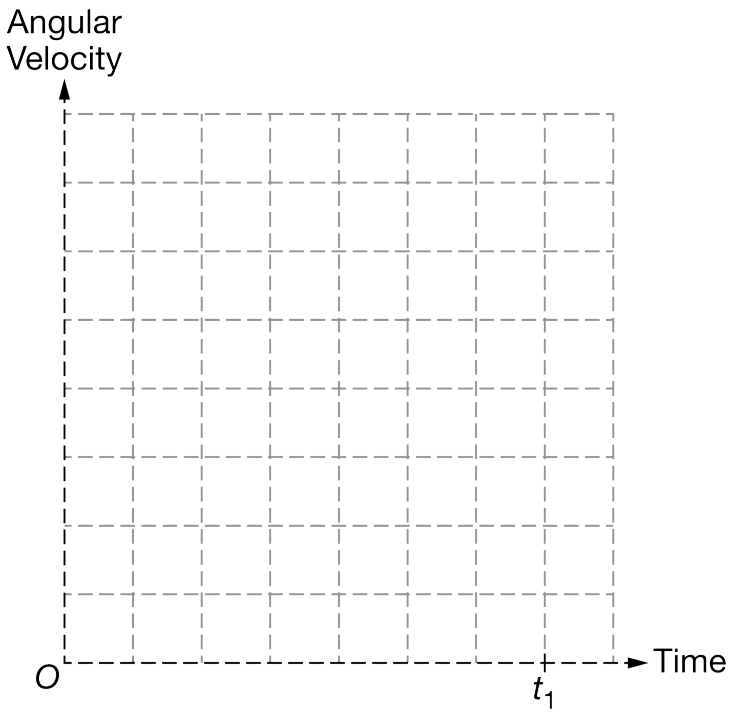
Top View

(7 points, suggested time 13 minutes)

Two bars, bar 1 and bar 2, are at rest on a smooth horizontal table as shown in the top view above. Each bar is connected to the table with a vertical pin. Bar 1 and bar 2 are made from the same material and have the same physical dimensions, but bar 1 has three additional large holes drilled through it. The two bars are each accelerated from rest by forces of equal magnitude F applied at their right end. The forces remain perpendicular to the bars and the forces are exerted for an equal amount of time t_1.

(a) Briefly explain using physics principles why the angular displacement for bar 1 is larger than the angular displacement for bar 2.

(b) On the axes below, sketch the angular velocity of each bar as a function of time. Label the sketch for bar 1 “Bar 1”. Label the sketch for bar 2, “Bar 2”.



frq8

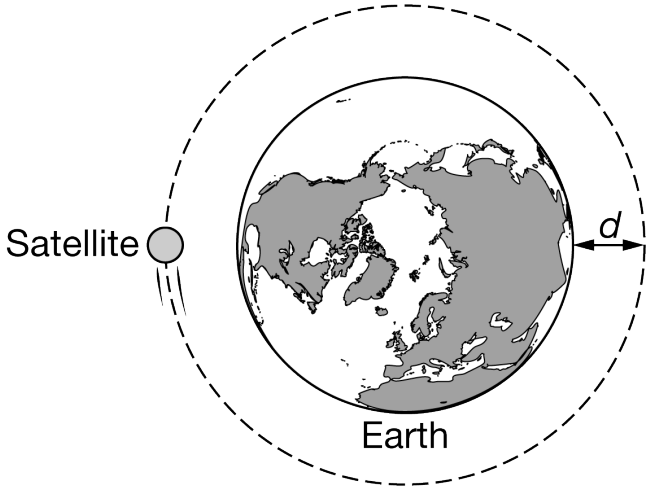
(c) After the force has been applied to both bars for a time t_1, is the angular momentum of bar 1 larger than, smaller than, or the same as the angular momentum of bar 2?

___ Larger ___ Smaller ___ The same

Justify your answer.

frq8

2. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface, as shown above.

An identical satellite is set into orbit around a new planet. The new planet has the same radius as the radius of Earth but is significantly more massive than Earth. The satellite orbits the same distance d from the surface of the new planet.

(a) Is the speed of the satellite orbiting the new planet greater than, less than, or the same as the speed of the satellite orbiting Earth? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

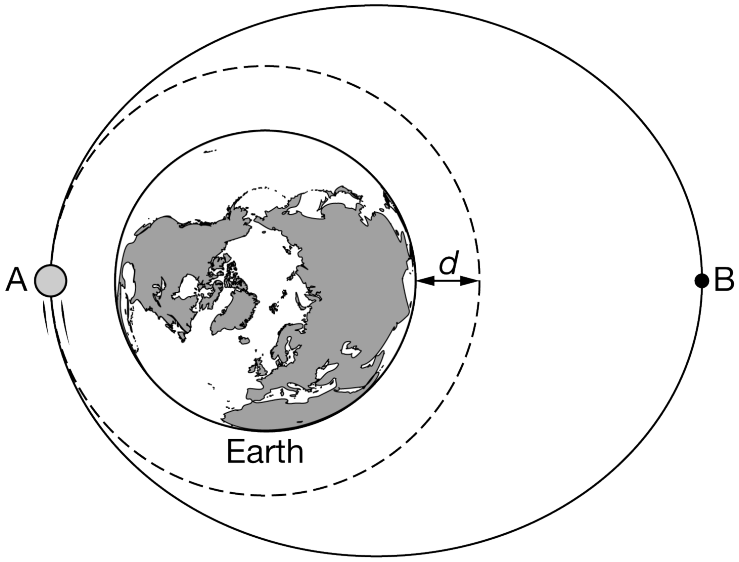
$$\frac{MT^2}{R^3} = C$$

In the equation above

M is the mass of a planet;
T is the orbital period of a satellite in a circular orbit;
R is the distance from the planet's center to the satellite; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

frq8



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point A to point B, and state the following:

Student 1: The satellite is moving faster at point A because the satellite's angular momentum is greater at point A than at point B.

Student 2: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is higher at point B than at point A.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

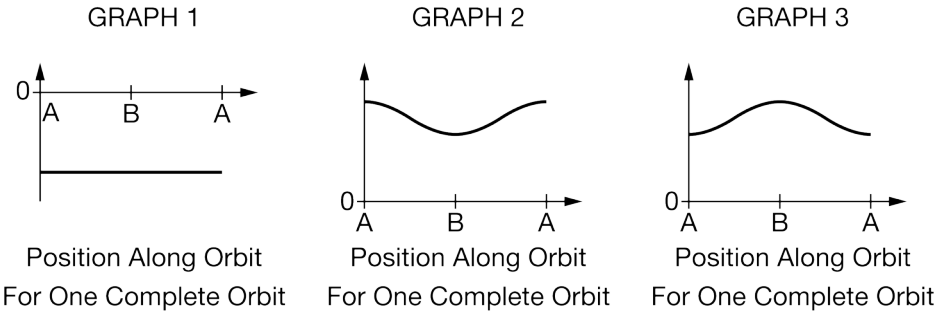
(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite as it completes one elliptical orbit, starting and ending at point A.

frq8

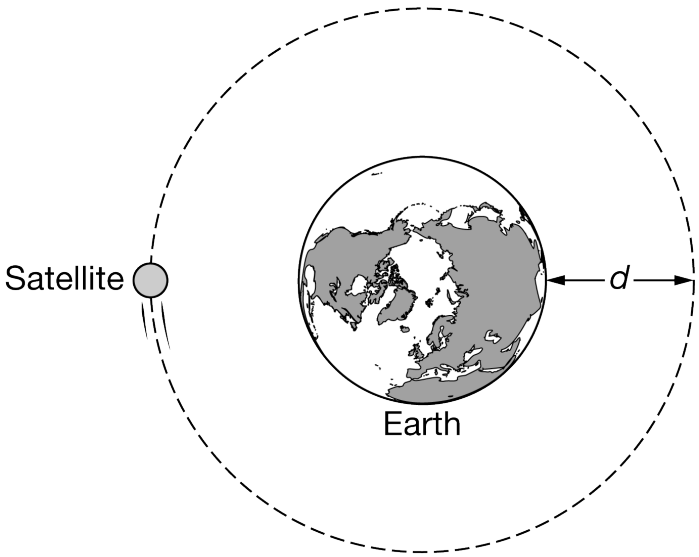


(h) Which of the above graphs best represents the magnitude of the gravitational field at the location of the satellite as it orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the total mechanical energy of the satellite-Earth system as the satellite orbits? Briefly explain your reasoning.

frq8

3. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface, as shown above.

An identical satellite is set into orbit around a new planet. The new planet's radius is the same as the radius of Earth but is significantly more massive than Earth. The satellite orbits the new planet with the same orbital period.

(a) Is the distance of the satellite from the surface of the new planet greater than, less than, or equal to d ? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

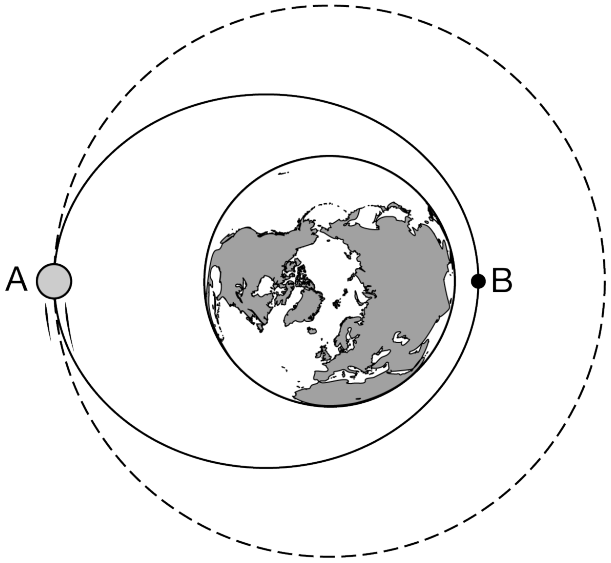
$$\frac{Rv^2}{M} = C$$

In the equation above

R is the distance from a planet's center to a satellite in a circular orbit;
v is the speed of the satellite;
M is the mass of the planet; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

frq8



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point A to point B, and state the following:

Student 1: The satellite is moving faster at point A because the total mechanical energy of the satellite-Earth system is constant.

Student 2: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is higher at point B than at point A.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

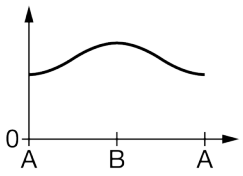
(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite

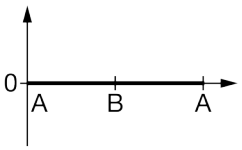
as it completes one elliptical orbit, starting and ending at point A.

GRAPH 1



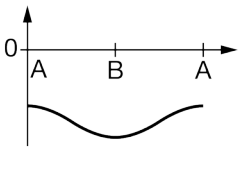
Position Along Orbit
For One Complete Orbit

GRAPH 2



Position Along Orbit
For One Complete Orbit

GRAPH 3



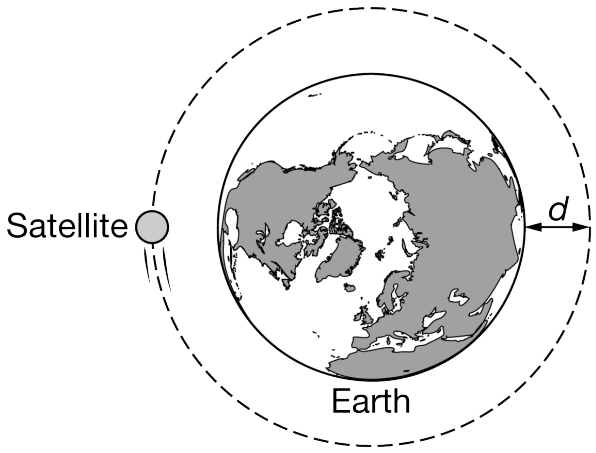
Position Along Orbit
For One Complete Orbit

(h) Which of the above graphs best represents the potential energy of the satellite-Earth system as the satellite orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the torque about the center of Earth exerted on the satellite by Earth's gravitational force as the satellite orbits? Briefly explain your reasoning.

frq8

4. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface.

An identical satellite is set into orbit around a new planet. The new planet's radius is the same as the radius of Earth but the new planet is significantly more massive than Earth. The satellite orbits the new planet with the same speed v.

(a) Is the distance of the satellite from the surface of the new planet greater than, less than, or equal to d ? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

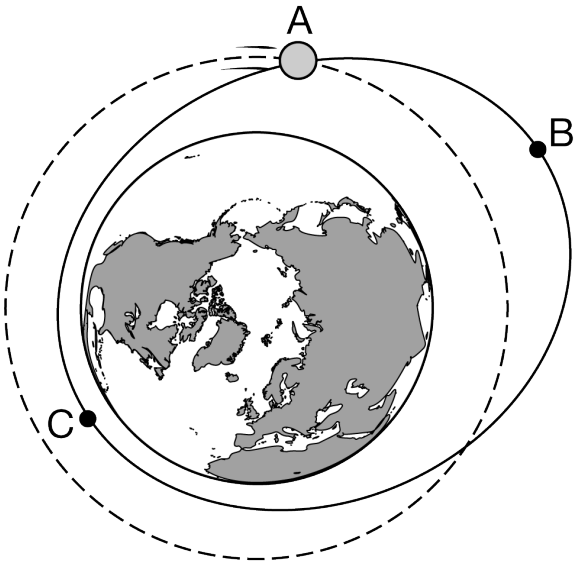
$$\frac{R^3}{MT^2} = C$$

In the equation above

R is the distance from a planet's center to a satellite in a circular orbit;
M is the mass of the planet;
T is the orbital period of the satellite; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

frq8



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit. In the modified orbit, the satellite's speed at point A, after the thrusters were fired, is the same as it was in the original orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point B to point C, and state the following:

Student 1: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is negative.

Student 2: The satellite is moving faster at point C because the total mechanical energy of the satellite-Earth system is greater at point C than at point B.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

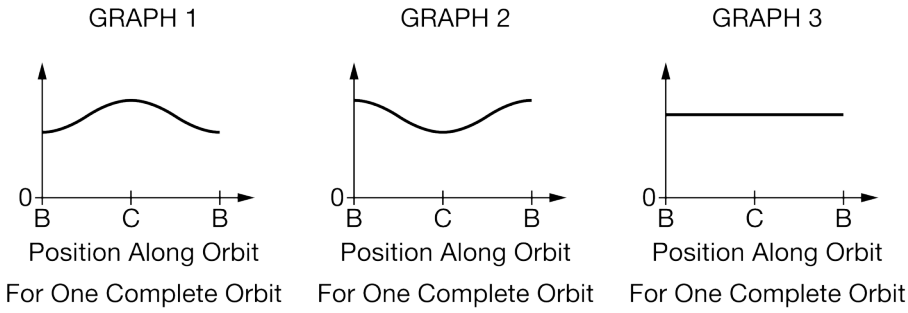
(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

frq8

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite as it completes one elliptical orbit, starting and ending at point B.

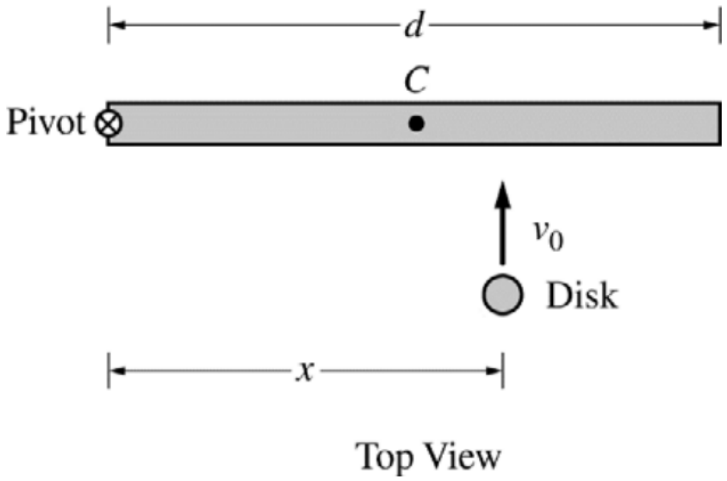


(h) Which of the above graphs best represents the magnitude of the acceleration of the satellite as it orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the magnitude of the angular momentum of the satellite with respect to the center of Earth as the satellite orbits? Briefly explain your reasoning.

frq8

5.



The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will stick to the rod a distance x from the pivot. The student wants the roddisk system to end up with as much angular speed as possible.

a. Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point C, at point C, or to the right of point C ?

_____To the left of C _____ At C _____ To the right of C

Briefly explain your reasoning without manipulating equations.

b. On the Internet, a student finds the following equation for the postcollision angular speed ω of the rod in this situation: $\omega = \frac{m_{disk} x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a) ? In other words, does this equation for w have the expected dependence as reasoned in part (a) ?

_____Yes _____No

Briefly explain your reasoning without deriving an equation for ω.

c. Another student deriving an equation for the postcollision angular speed ω of the rod makes a mistake and comes up with $\omega = \frac{I x v_0}{m_{disk} d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

d. Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{disk}x^2$ and its

angular momentum with respect to the pivot is $m_{disk}\ v_0x$. Derive an equation for the postcollision angular speed ω of the rod. Express your answer in terms of d, m_{disk}, I, x, v_0, and physical constants, as appropriate.

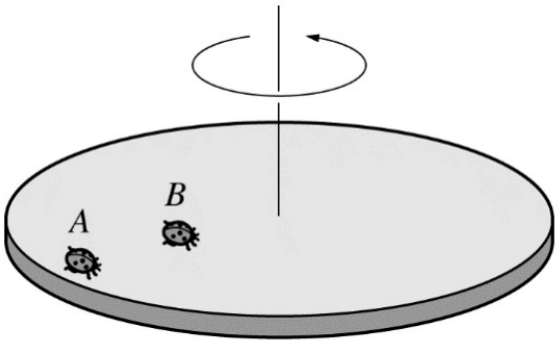
e. Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?

_____Greater than _____Less than _____Equal to

Briefly explain your reasoning.

frq8

6.



Two ladybugs are standing on a rotating disk that is spinning counterclockwise, as shown in the figure above. Assume that friction in the bearings of the axle is negligible.

a. i. Is the angular speed of ladybug A greater than, less than, or the same as the angular speed of ladybug B ?

_____Greater _____Less _____The same

Briefly justify your answer.

ii. Is the linear speed of ladybug A greater than, less than, or the same as the linear speed of ladybug B ?

_____Greater _____Less _____The same

Briefly justify your answer.

b. Ladybug A begins walking in a circular path in the direction of the disk's rotation. Does the magnitude of the angular momentum of the disk alone (not the ladybugs-disk system) increase, decrease, or stay the same?

_____Increase _____Decrease _____Stay the same

Briefly justify your answer.

c. In a different scenario, a single ladybug is standing near the edge of the disk at a distance of 0.9R from the center, where R is the radius of the disk, as shown in Figure 1 below. The rotational inertia of the ladybugdisk system is I_1 , and the disk completes one rotation in 2.5 s. The ladybug then walks toward the center of the disk to a distance of 0.1R from the center and comes to a stop relative to the disk, as shown in Figure 2. Now the rotational inertia of the system is I_2, and the disk completes one rotation every 2.0 s.

frq8

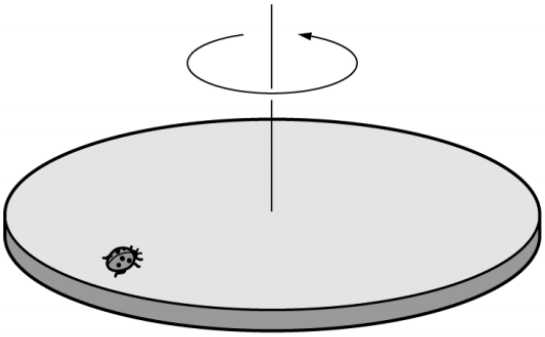


Figure 1

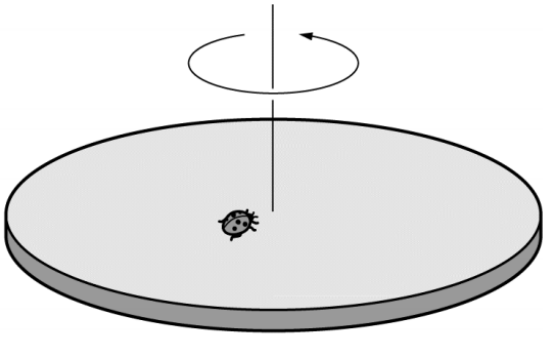


Figure 2

i. Derive an equation for I_2 in terms of I_1

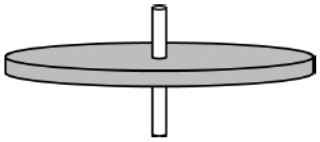
ii. While the ladybug is walking toward the center of the disk, does it exert a torque on the disk?

_____Yes ____No

Briefly explain your reasoning.

frq8

7.

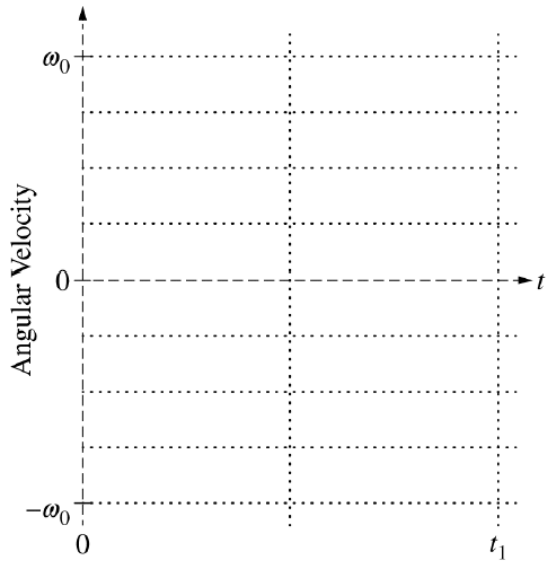


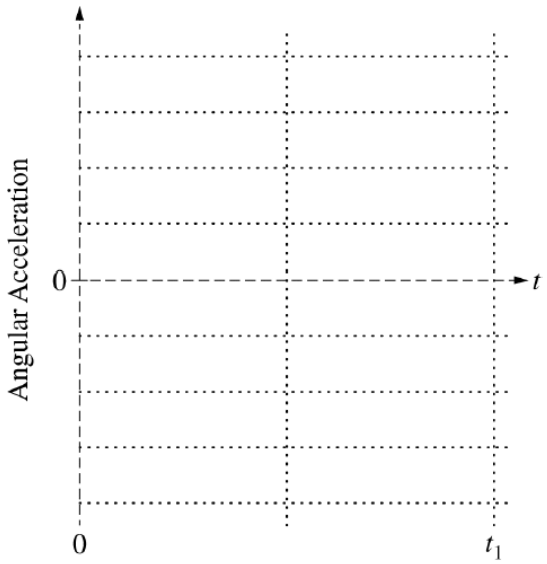
The disk shown above spins about the axle at its center. A student's experiments reveal that, while the disk is spinning, friction between the axle and the disk exerts a constant torque on the disk.

a. At time $t = 0$ the disk has an initial counterclockwise (positive) angular velocity ω_0.The disk later comes to rest at time $t = t_1$.

   i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time t from $t = 0$ until the disk comes to rest at time $t = t_1$.

   ii. On the grid at right below, sketch the disk's angular acceleration as a function of time t from $t = 0$ until the disk comes to rest at time $t = t_1$.





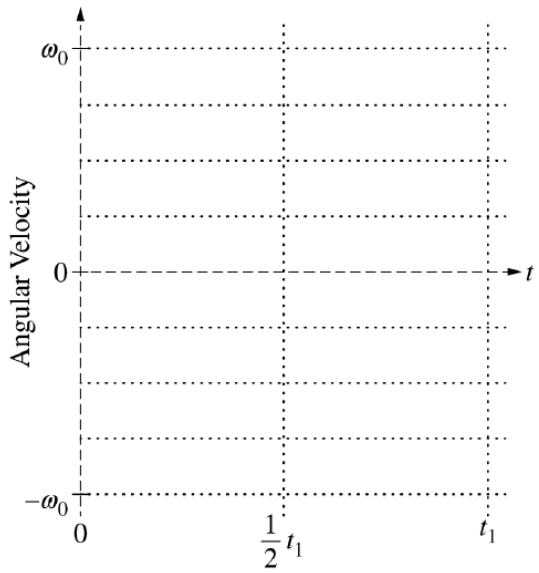
b. The magnitude of the frictional torque exerted on the disk is τ_0. Derive an equation for the rotational inertia I of the disk in terms of τ_0, ω_0, t_1, and physical constants, as appropriate.

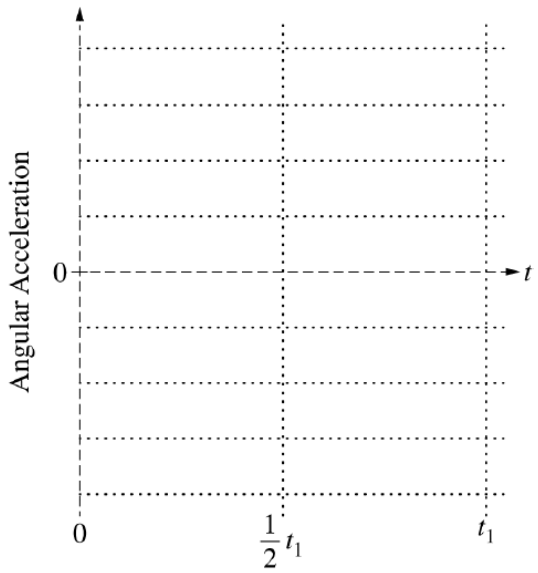
c. In another experiment, the disk again has an initial positive angular velocity ω_0 at time $t = 0$. At time $t = \frac{1}{2}t_1$, the student starts dripping oil on the contact surface between the axle and the disk to reduce the friction. As time passes, more and more oil reaches that contact surface, reducing the friction even further.

frq8

i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time from $t = 0$ to $t = t_1$, which is the time at which the disk came to rest in part (a).

ii. On the grid at right below, sketch the disk's angular acceleration as a function of time from $t = 0$ to $t = t_1$.





d. The student is trying to mathematically model the magnitude τ of the torque exerted by the axle on the disk when the oil is present at times $t > \frac{1}{2}t_1$. The student writes down the following two equations, each of which includes a positive constant (C_1 or C_2) with appropriate units.

$$(1)\ \tau = C_1\left(t - \tfrac{1}{2}t_1\right) \text{ (for } t > \tfrac{1}{2}t_1\text{)}$$
$$(2)\ \tau = \frac{C_2}{\left(t + \frac{1}{2}t_1\right)} \text{ (for } t > \tfrac{1}{2}t_1\text{)}$$

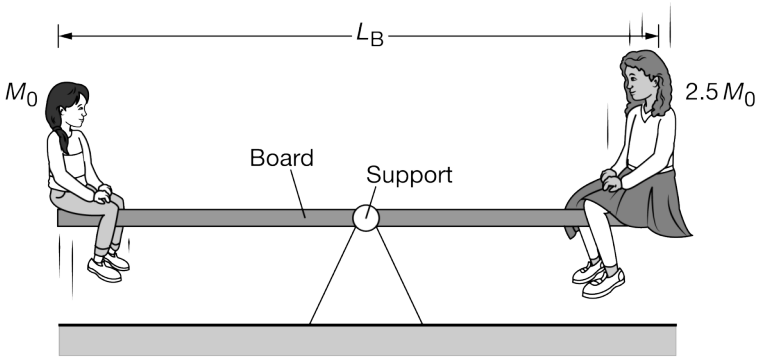
Which equation better mathematically models this experiment?

____Equation (1) ____Equation (2)

Briefly explain why the equation you selected is plausible and why the other equation is not plausible.

frq8

8. This question is a short free-response question. Show your work for each part of the question.



Note: Figure not drawn to scale.

(7 points, suggested time 13 minutes)

A student of mass M_0 and another student of mass $2.5M_0$ sit at opposite ends of a board of length L_B and negligible mass. The board is free to pivot about a support at its center. At the instant shown above, the board is horizontal.

(a) Derive an expression for the magnitude of the net torque exerted on the students-board system about the support at the instant shown in the figure. Express your answer in terms of M_0, L_B, and physical constants, as appropriate.

(b) The students want to be able to balance the board horizontally without the students touching the ground. They figure out that one student needs to move closer to the support while the other student remains in their original position at the other end of the board.

i. Which student should move closer to the support?

____ The student of mass M_0 ____ The student of mass $2.5M_0$

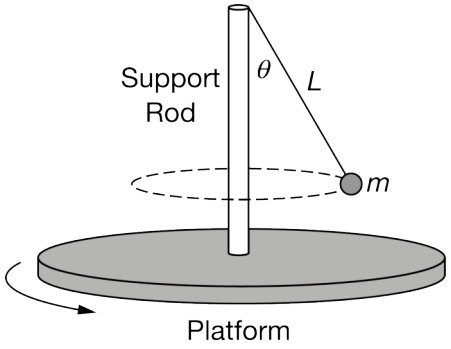
Briefly explain your reasoning.

ii. Derive an expression for the distance that the student indicated in part (b)(i) should move. Express your answer in terms of M_0, L_B, and physical constants, as appropriate.

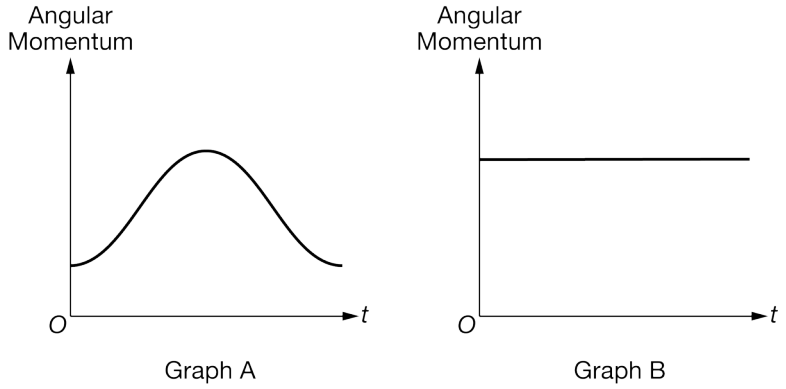
(c) Determine the magnitude of the force exerted on the board by the support after the student has moved and the board is balanced horizontally. Express your answer in terms of M_0, L_B, and physical constants, as appropriate.

frq8

9. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A vertical support rod is fixed at the center of a platform. A light string of length L is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass m, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed ω (omega), the string makes an angle θ (theta) with the rod.



(a) The platform rotates with a constant angular speed. Which of the above graphs correctly shows the magnitude of the angular momentum of the platform as a function of time t about a vertical axis at the center of the support rod? Briefly justify your answer.

(b) The platform rotates with a constant angular speed. Is the net torque exerted on the platform about its center clockwise, counterclockwise, or zero? Explain your reasoning.

(c) As the period of rotation becomes very small, what value does θ approach? Without writing any equations or formulas, justify your answer.

(d) A student derives an equation for the square of the period of revolution T of the sphere as a function of θ: $T^2 = \left(4\pi^2\right)\frac{L\cos\theta}{g}$. Indicate whether the equation is consistent with your answer in part (c) and explain why.

(e) In a series of experimental trials, a group of students changes L and adjusts T to keep θ constant. They record L and T for each trial. If the students use the equation given in part (d), what quantities could they graph to yield a straight line whose slope could be used to find the acceleration due to gravity, g ?

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(f) As the angular speed of the platform increases, does the period of revolution of the sphere increase, decrease, or stay the same? Briefly explain your reasoning.

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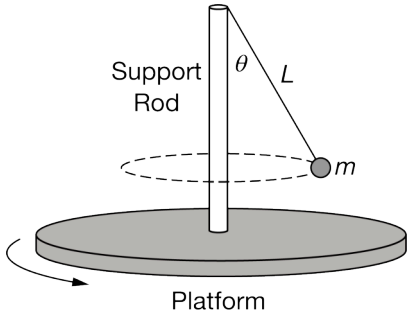
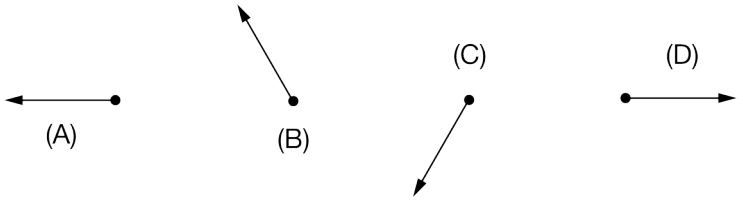
10. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A vertical support rod is fixed at the center of a platform. A light string of length L is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass m, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed ω (omega), the string makes an angle θ (theta) with the rod.



(a) Identify which of the above vectors represents the net force exerted on the sphere at the moment shown above. Briefly justify your answer.

(b) Is the magnitude of the tension in the string greater than, less than, or equal to the weight of the sphere? Explain your reasoning.

(c) The string can never be completely horizontal no matter how fast the platform is rotating. Without writing any equations or formulas, explain why.

One of the following two equations for the square of the tangential speed v of the sphere as a function of θ is consistent with the statement in part (c) that the string can never be horizontal as the platform is rotating.

Equation 1: $v^2 = \frac{gL(\sin^2\theta)}{(\cos\theta)}$ **Equation 2:** $v^2 = \frac{gL(\cos^2\theta)}{(\sin\theta)}$

(d) Indicate which equation is consistent with the statement in part (c), and explain why. Also explain why the other equation is not consistent with the statement in part (c).

(e) In a series of experimental trials, a group of students changes L and adjusts v to keep θ constant. They record L and v for each trial. Based on the equation in part (d) that you indicated as consistent with the statement in part (c), what quantities could the students graph to yield a straight line whose slope could be used to find the acceleration due to gravity, g ?

(f) Let I be the rotational inertia of the sphere-rod-platform system about a vertical axis through the center of the

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support rod. Does I increase, decrease, or stay the same when L increases (but θ stays constant)? Briefly explain your reasoning.

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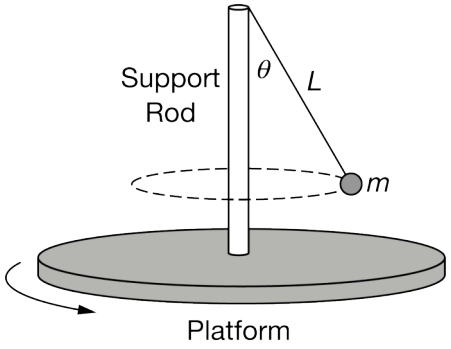
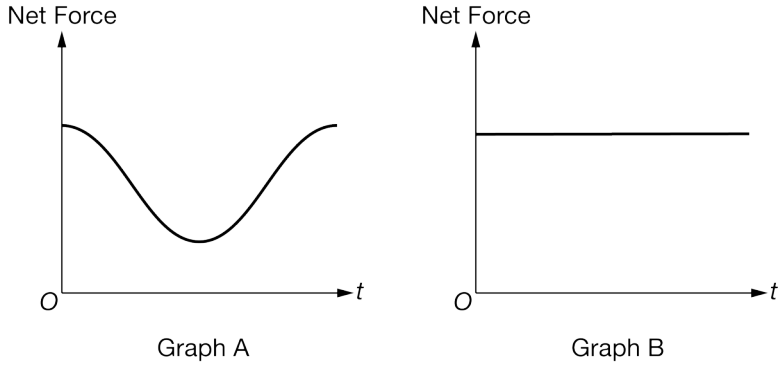
11. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A vertical support rod is fixed at the center of a platform. A light string of length L is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass m, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed ω (omega), the string makes an angle θ (theta) with the rod.



(a) The platform rotates with a constant angular speed. Which of the above graphs correctly shows the magnitude of the net force exerted on the sphere as a function of time t ? Briefly justify your answer.

(b) Is the magnitude of the tension in the string greater than, less than, or equal to the centripetal force exerted on the sphere? Explain your reasoning.

(c) The string is nearly vertical when the sphere's tangential speed is very small. Without writing any equations or formulas, explain why.

(d) A student derives an equation for the square of the tangential speed v of the sphere as a function of θ : $v^2 = gL(\tan\theta)(\sin\theta)$. Indicate whether the equation is consistent with the statement in part (c) and explain why.

(e) In a series of experimental trials, a group of students adjusts v and measures θ while keeping all other variables constant. They record v and θ for each trial. If the students use the equation given in part (d), what quantities could they graph to yield a straight line whose slope could be used to find the acceleration due to gravity, g ?

(f) Let I be the rotational inertia of the sphere-rod-platform system about a vertical axis through the center of the

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support rod. Does I increase, decrease, or stay the same when θ decreases? Briefly explain your reasoning.

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12. This question is a long free-response question. Show your work for each part of the question.

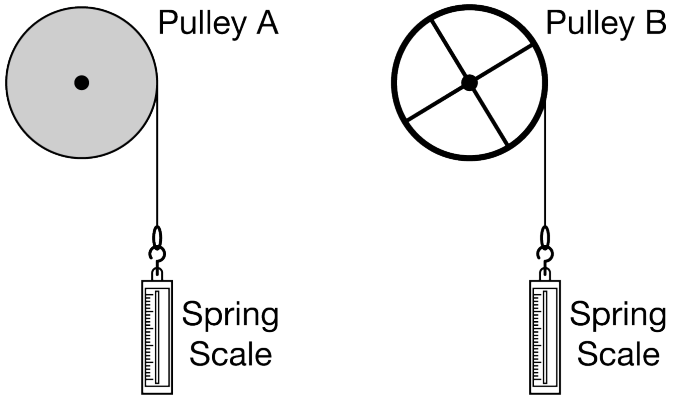

Figure 1

(a) Two identical spring scales are each attached to a cord that is wrapped around a pulley, as shown in Figure 1. The two pulleys have identical mass M_p and radius R_p. However, Pulley A is a solid disk, whereas Pulley B has most of its mass located at its outer rim. Both pulleys can rotate about their centers with negligible friction in their axles.

The pulleys are initially at rest when the two spring scales are pulled with the same constant force F_s, causing both pulleys to rotate. Predict which of the pulleys, if either, will complete one revolution first.

____ Pulley A ____ Pulley B ____ Both pulleys will complete one revolution at the same time.

Without writing or manipulating equations, provide reasoning for your selection.

(b)

i. Derive an equation for the time t_A it takes Pulley A to complete one revolution. Let I_A be the pulley's rotational inertia. Express your answer in terms of F_S, I_A, M_p, R_p, and physical constants, as appropriate.

ii. Is your answer to part (b)(i) consistent with your answer to part (a) ?

____ Yes ____ No

Provide reasoning for whether the two answers are or are not consistent.

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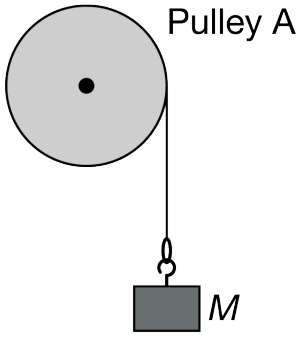

Figure 2

(c) A block of mass M is attached to Pulley A , as shown in Figure 2. The block is released from rest. Describe how the net torque exerted on the pulley and the magnitude of the angular momentum of the pulley are changing, if at all, a short time after the block is released.

Net torque: ____ Increasing ____ Decreasing ____ Not changing

Angular momentum: ____ Increasing ____ Decreasing ____ Not changing

Briefly describe how you arrived at your answers.

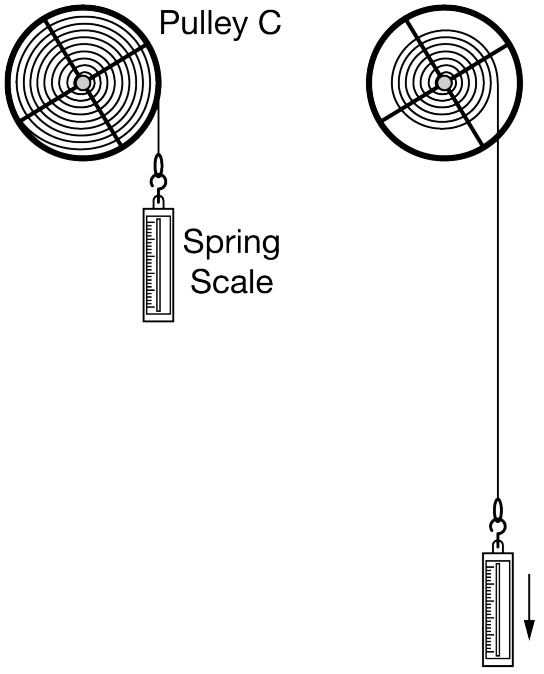

Figure 3

(d) A long cord of negligible mass is wrapped around a third pulley, Pulley C, and attached to a spring scale, as shown in Figure 3. The spring scale is pulled downward so that it exerts a constant force of magnitude F_S on the

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cord. As the spring scale moves down a significant distance, the vertical portion of the cord moves closer and closer to the center of the pulley.

Predict how the net torque exerted on the pulley and the rotational kinetic energy of the pulley will be changing, if at all, as the spring scale moves downward and the vertical portion moves closer to the center of the pulley.

Net torque: ____ Increasing ____ Decreasing ____ Not changing

Rotational kinetic energy: ____ Increasing ____ Decreasing ____ Not changing

Briefly provide reasoning for your predictions.

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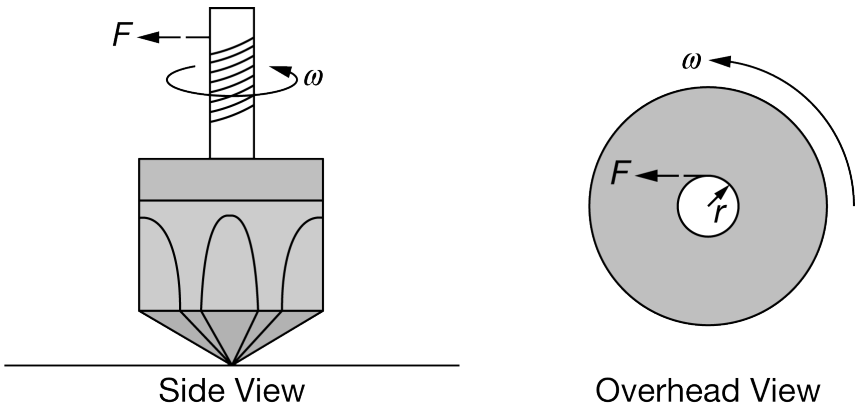
13. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A light string of total length L is wrapped completely around the axle of a spinning toy top. The radius of the axle is r and the rotational inertia of the top is I. The top is initially at rest and the axle is held vertically with respect to a horizontal table. At time $t = 0$ a force with constant magnitude F pulls the string outwards from the axle of the top, which causes the top to rotate. After a time T the string loses contact with the axle of the top and the top spins on the table with an angular speed ω (omega). Assume the top always remains upright and there is negligible friction between the top and the table.

(a) The original string is replaced with a new string of length $2L$. The string is again pulled with a constant force of magnitude F. When the string loses contact with the top, the angular speed of the top is ω_A. Is ω_A greater than, less than, or equal to ω? Justify your answer without citing or manipulating equations.

(b) The original string of length L is used to spin a second top that also has an axle with radius r, but a larger rotational inertia $I_B = 2I$. The string is pulled with the same constant force of magnitude F. When the string loses contact with the top, the angular speed of the second top is ω_B. Is ω_B greater than, less than, or equal to ω? Justify your answer without citing or manipulating equations.

(c) A student derives the following equation to predict the final angular speed of the top: $\omega = \sqrt{\frac{2FL}{I}}$. Whether or not this equation is correct, does it match your reasoning in parts (a) and (b)?

Once the string has been pulled for a certain amount of time T, the string comes off and the top spins on the table.

(d) Describe any changes in the top's angular momentum about a vertical axis through the center of the top after the string loses contact with the top. Explain your answer.

A student makes sketches of the angular displacement θ, angular velocity ω, and angular acceleration α of the top as functions of time. The graphs below represent the time interval from when the string is first pulled until after the string loses contact with the top at time T. Consider the parts of the graph for the time before T and the time after T. Each graph is consistent with the motion of the top for one part and inconsistent for the other part.

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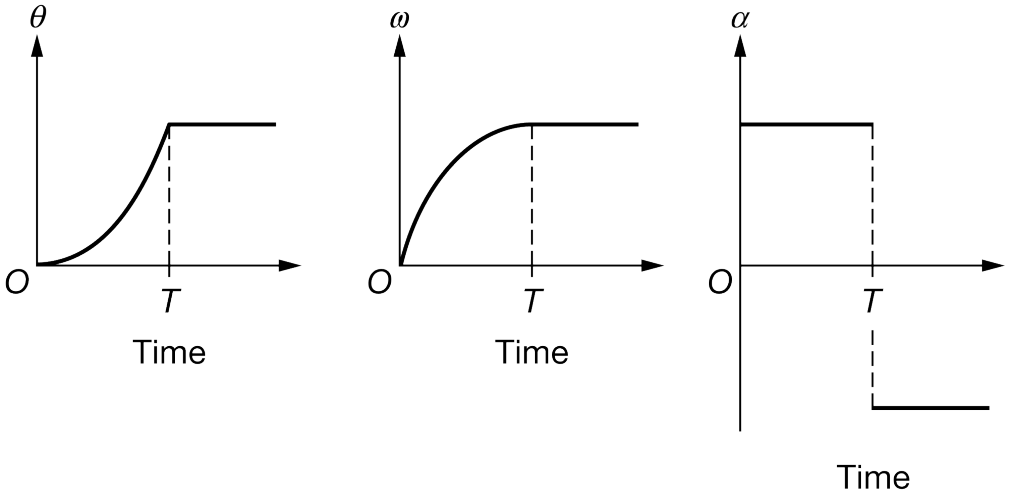

(e) For which part of the graph, either the time before T or the time after T, is the graph of the angular displacement θ inconsistent with the motion of the top as described directly above the graphs? Explain how to correct the part of the graph that is inconsistent.

(f) For which part of the graph, either the time before T or the time after T, is the graph of the angular velocity ω inconsistent with the motion of the top as described directly above the graphs? Explain how to correct the part of the graph that is inconsistent.

(g) For which part of the graph, either the time before T or the time after T, is the graph of the angular acceleration α inconsistent with the motion of the top as described directly above the graphs? Explain how to correct the part of the graph that is inconsistent.

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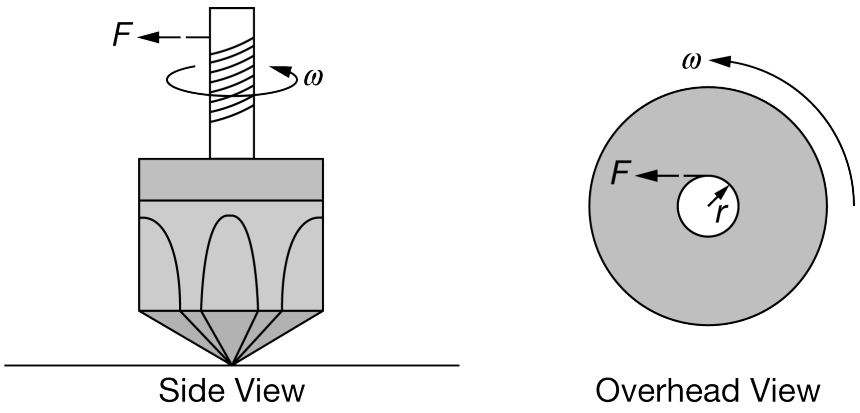
14. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A light string of total length L is wrapped completely around the axle of a spinning toy top. The radius of the axle is r and the rotational inertia of the top is I. The top is initially at rest and the axle is held vertically with respect to a horizontal table. At time $t = 0$ a force with constant magnitude F pulls the string outwards from the axle of the top, which causes the top to rotate. After a time T the string loses contact with the axle of the top and the top spins on the table with an angular speed ω (omega). Assume the top always remains upright and there is negligible friction between the top and the table.

(a) The string is again wrapped around the axle of the top. The string is pulled with a new constant force $F_A = 2F$, and at time T_A the string loses contact with the top. Is T_A greater than, less than, or equal to T? Justify your answer without citing or manipulating equations.

(b) The string is wrapped around a second top with smaller rotational inertia $I_B = \frac{1}{2}I$. The axle of the second top is the same radius as the radius of the axle of the first top. The same constant force with magnitude F is applied to the string, and at time T_B the string loses contact with the top. Is T_B greater than, less than, or equal to T? Justify your answer without citing or manipulating equations.

(c) A student derives the following equation to predict the amount of time over which the string is in contact with the top: $T = \sqrt{\frac{2IL}{Fr^2}}$. Whether or not this equation is correct, does it match your reasoning in parts (a) and (b)?

Once the string has been pulled for a certain amount of time T, the string comes off and the top spins on the table. Now consider the case in which friction between the top and the table is not negligible, and the friction exerts a constant torque on the top about a vertical axis through the center of the top.

(d) Describe any changes in the top's angular momentum about a vertical axis through the center of the top after the string loses contact with the top. Explain your answer.

A student makes sketches of the angular displacement θ, angular velocity ω, and angular acceleration α of the top as functions of time. The graphs below represent the time interval from when the string is first pulled until after the string loses contact with the top at time T. Consider the parts of the graph for the time before T and the time after T. Each graph is consistent with the motion of the top for one part and inconsistent for the other part.

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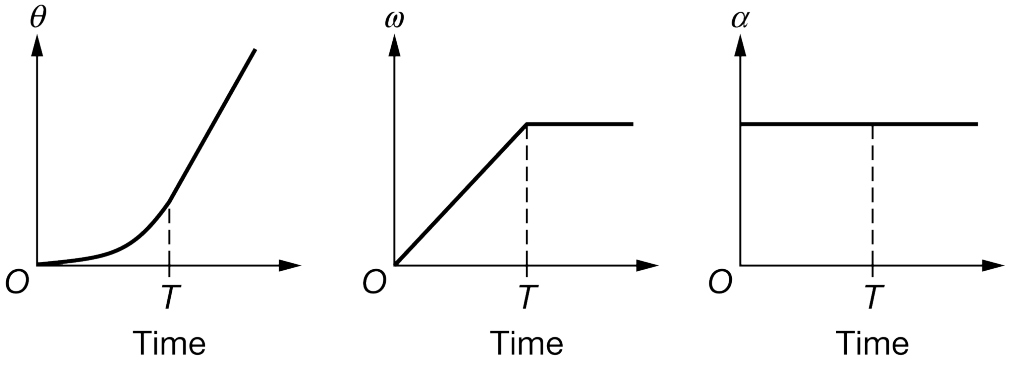

(e) For which part of the graph, either the time before T or the time after T, is the graph of the angular displacement θ inconsistent with the motion of the top in the case in which friction is not negligible? Explain how to correct the part of the graph that is inconsistent.

(f) For which part of the graph, either the time before T or the time after T, is the graph of the angular velocity ω inconsistent with the motion of the top in the case in which friction is not negligible? Explain how to correct the part of the graph that is inconsistent.

(g) For which part of the graph, either the time before T or the time after T, is the graph of the angular acceleration α inconsistent with the motion of the top in the case in which friction is not negligible? Explain how to correct the part of the graph that is inconsistent.

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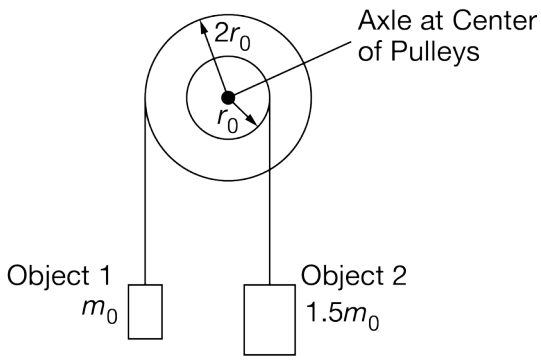
15. This question is a short free-response question. Show your work for each part of the question.



(7 points, suggested time 13 minutes)

Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.

m_0 is the mass of object 1.
$1.5m_0$ is the mass of object 2.
r_0 is the radius of the smaller pulley.
$2r_0$ is the radius of the larger pulley.

(a) At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.

i. Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of m_0, r_0, and physical constants, as appropriate.

ii. Object 1 accelerates downward after the pulleys are released. Briefly explain why.

(b) At a later time $t = t_C$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?

___ Same direction ___ Opposite directions

Briefly explain your reasoning.

(c) On the axes below, sketch a graph of the angular velocity ω of the system consisting of the two pulleys as a function of time t. Include the entire time interval shown. The pulleys are released at $t = 0$, and the string is cut at $t = t_C$.

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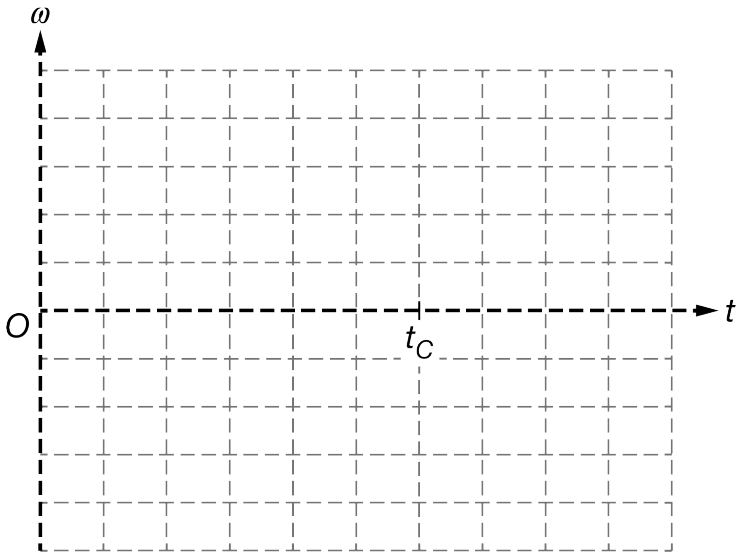

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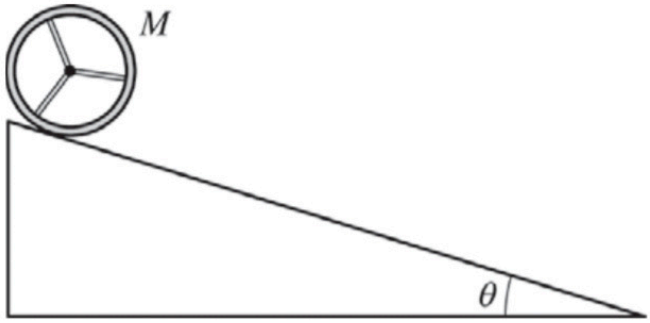
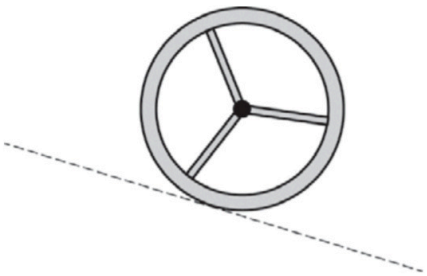
16. Show your work for each part in the space provided after that part.



A wooden wheel of mass M, consisting of a rim with spokes, rolls down a ramp that makes an angle θ with the horizontal, as shown above. The ramp exerts a force of static friction on the wheel so that the wheel rolls without slipping.

a. i. On the diagram below, draw and label the forces (not components) that act on the wheel as it rolls down the ramp, which is indicated by the dashed line. To clearly indicate at which point on the wheel each force is exerted, draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted. The lengths of the arrows need not indicate the relative magnitudes of the forces



ii. As the wheel rolls down the ramp, which force causes a change in the angular velocity of the wheel with respect to its center of mass?

Briefly explain your reasoning.

b. For this ramp angle, the force of friction exerted on the wheel is less than the maximum possible static friction force. Instead, the magnitude of the force of static friction exerted on the wheel is 40 percent of the magnitude of the force or force component directed opposite to the force of friction. Derive an expression for the linear acceleration of the wheel's center of mass in terms of M, θ, and physical constants, as appropriate.

c. In a second experiment on the same ramp, a block of ice, also with mass M, is released from rest at the same instant the wheel is released from rest, and from the same height. The block slides down the ramp with negligible friction.

i. Which object, if either, reaches the bottom of the ramp with the greatest speed?

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____ Wheel ____ Block ____ Neither; both reach the bottom with the same speed.

Briefly explain your answer, reasoning in terms of forces.

ii. Briefly explain your answer again, now reasoning in terms of energy