chapter1

1. A world-class runner can complete a 100 m dash in about 10 s. Past studies have shown that runners in such a race accelerate uniformly for a time t_u and then run at constant speed for the remainder of the race. A world-class runner is visiting your physics class. You are to develop a procedure that will allow you to determine the uniform acceleration a_u and an approximate value of t_u for the runner in a 100 m dash. By necessity your experiment will be done on a straight track and include your whole class of eleven students.

(a) By checking the line next to each appropriate item in the list below, select the equipment, other than the runner and the track, that your class will need to do the experiment.

____ Stopwatches ____ Tape measures ____ Rulers____ Masking tape

____ Metersticks ____ Starter's pistol ____ String ____ Chalk

(b) Outline the procedure that you would use to determine a_u and t_u, including a labeled diagram of the experimental setup. Use symbols to identify carefully what measurements you would make and include in your procedure how you would use each piece of the equipment you checked in part (a).

(c) Outline the process of data analysis, including how you will identify the portion of the race that has uniform acceleration, and how you would calculate the uniform acceleration.

Part A

1 point is earned: For checking off a distance-measuring device and describing its use in part (b)

1 point is earned: For checking off a stopwatch and describing its use in part (b)

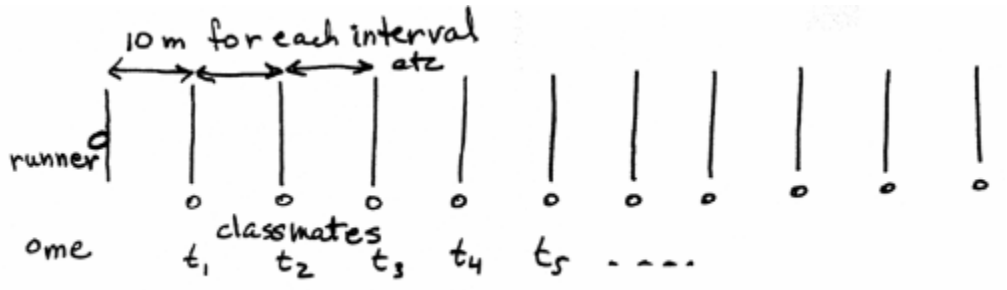
0	1	2

The student response earns all of the following points:

1 point is earned: For checking off a distance-measuring device and describing its use in part (b)

1 point is earned: For checking off a stopwatch and describing its use in part (b)

Part B



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Use the tape measure and chalk to mark off the 100 meters in 10 meter lengths. Set a classmate with a stopwatch at marks as shown. Use the starter's pistol to signal the runner to run and the classmates to start their stopwatches. Each person turns off the stopwatch when the runner reaches his or her mark. You then have measurements of the time to reach each increment of 10 meters.

1 point is earned: For taking distance measurements for 8 to 11 distinct fixed positions per run

1 point is earned: For measuring time for the same 8 to 11 distinct fixed positions, consistent with the description of the experimental setup

2 points are earned: For an experimental technique consistent with being able to determine the requested quantities

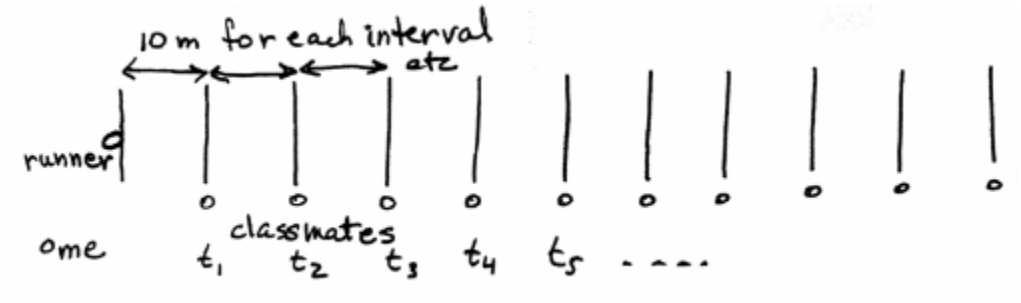
1 point is earned: For a diagram of the experimental setup with clear labels and consistent with the technique described (awarded even if the technique is wrong)

1 point is earned: For a technique that allows data for all positions to be taken in a single run



0	1	2	3	4	5	6

The student response earns all of the following points:



Use the tape measure and chalk to mark off the 100 meters in 10 meter lengths. Set a classmate with a stopwatch at marks as shown. Use the starter's pistol to signal the runner to run and the classmates to start their stopwatches. Each person turns off the stopwatch when the runner reaches his or her mark. You then have measurements of the time to reach each increment of 10 meters.

1 point is earned: For taking distance measurements for 8 to 11 distinct fixed positions per run

1 point is earned: For measuring time for the same 8 to 11 distinct fixed positions, consistent with the description of the experimental setup

2 points are earned: For an experimental technique consistent with being able to determine the requested quantities

1 point is earned: For a diagram of the experimental setup with clear labels and consistent with the technique described (awarded even if the technique is wrong)

1 point is earned: For a technique that allows data for all positions to be taken in a single run

Part C

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Approach A

3 points are earned: For a clear and detailed explanation of the data analysis process

Note: This part of the solution was graded holistically and students could earn between 0 and 3 points depending on the clarity and completeness of their explanation

1 point is earned: For equations or clear prose and use of the data to identify the two distinct regions of motion (constant acceleration and constant velocity)

1 point is earned: For clearly and correctly identifying t_u

1 point is earned: For clearly and correctly identifying a_u

1 point is earned: For having the final answers correct and no incorrect statements or calculations among the correct ones

Approach B

Students needed to clearly indicate which variable was used (acceleration or final velocity) by including the following.

1 point is earned: For a description or diagram that clearly defines all the variables being used

1 point is earned: For a description or diagram showing how the needed variable (acceleration or final velocity) will be determined

2 points are earned: For a successful transformation of the above description into equation form

1 point is earned: For correctly solving the equations obtained

1 point is earned: For work that would determine a correct value of a_u

1 point is earned: For work that would determine a correct value of t_u



0	1	2	3	4	5	6	7

The student response earns all of the following points:

Approach A

3 points are earned: For a clear and detailed explanation of the data analysis process

Note: This part of the solution was graded holistically and students could earn between 0 and 3 points depending on the clarity and completeness of their explanation

1 point is earned: For equations or clear prose and use of the data to identify the two distinct regions of motion (constant acceleration and constant velocity)

1 point is earned: For clearly and correctly identifying t_u

1 point is earned: For clearly and correctly identifying a_u

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1 point is earned: For having the final answers correct and no incorrect statements or calculations among the correct ones

Approach B

Students needed to clearly indicate which variable was used (acceleration or final velocity) by including the following.

1 point is earned: For a description or diagram that clearly defines all the variables being used

1 point is earned: For a description or diagram showing how the needed variable (acceleration or final velocity) will be determined

2 points are earned: For a successful transformation of the above description into equation form

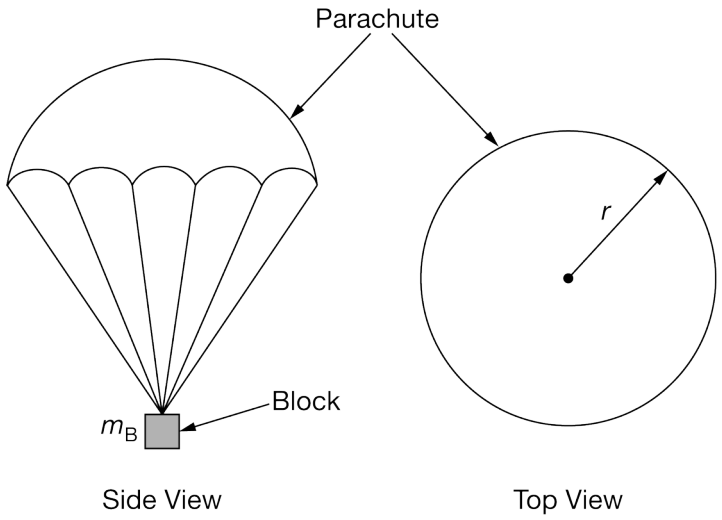
1 point is earned: For correctly solving the equations obtained

1 point is earned: For work that would determine a correct value of a_u

1 point is earned: For work that would determine a correct value of t_u

chapter1

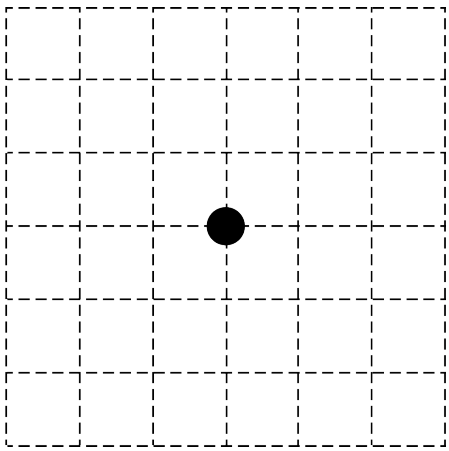
2. This question is a long free-response question. Show your work for each part of the question.



(12 points, suggested time 25 minutes)

A block of mass m_B can be attached to a parachute of negligible mass and radius r, as shown in the two views above. When falling, the parachute experiences an upward force of air resistance. This force increases as the speed of the parachute increases. The falling block and parachute speed up at first but eventually reach a constant speed called terminal speed.

(a) The dot below represents the block-parachute system when it has reached its terminal speed and is falling at this constant speed. On the dot, draw and label the forces (not components) acting on the system. Represent each force by a distinct arrow starting on, and pointing away from, the dot.
Note: Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces.



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A group of students has a collection of parachutes of different radii. They are curious about how the terminal speed depends on the radius of the parachute and find the following equation on the Internet.

$$F_{\text{air}} = Crv^2,$$

where F_{air} is the force exerted by air, r is the parachute's radius, v is the parachute's speed, and C is a constant that has appropriate units.

(b) Determine the units for C.

The units for C are _____ .

(c)

i. Based on the above equation for F_{air}, derive an equation for the system's constant terminal speed v_t in terms of r, C, physical constants, and other relevant quantities that can be measured with equipment usually found in a school physics laboratory, as appropriate. Assume the mass of the parachute is negligible and that the only nonnegligible forces acting on a block-parachute system are the block's weight $m_B g$ and the force of air resistance F_{air}.

ii. Indicate below what quantities could be measured and then graphed to determine whether the equation derived in part (c)(i) correctly relates the parachute's radius and the terminal speed of the block-parachute system.

Vertical axis: _____ Horizontal axis: _____

iii. On the axes below, sketch a graph showing the relationship between the two quantities indicated in part (c)(ii) and based on the equation derived in part (c)(i).

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(d) Design an experimental procedure the students could use to collect the data needed to test whether or not the relationship between the parachute's radius and the terminal speed is described by the graph drawn in part (c)(iii). Assume equipment usually found in a school physics laboratory is available.

In the table below, list the quantities that would be measured in your experiment. Define a symbol to represent each quantity, and also list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

Quantity to be Measured	Symbol for Quantity	Equipment for Measurement

Describe the overall procedure to be used, referring to the table. Provide enough detail so another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table. Include a simple diagram of the setup. As part of your description, be sure to explain how the students would know that they are measuring the constant terminal speed.

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Diagram of Experimental Setup Description of Experimental Procedure

(e) In another experiment, the students collect the following data showing the height of a block above the floor at different times, where the block is dropped from rest at time $t = 0$ with the attached parachute fully open.

Time (s)	Height of Block Above Floor (m)
0.00	1.35
0.10	1.30
0.20	1.18
0.30	1.01
0.40	0.82
0.50	0.63
0.60	0.43
0.70	0.23
0.80	0.03

Use the data shown above to determine the terminal speed of the block.

Part A

chapter1

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

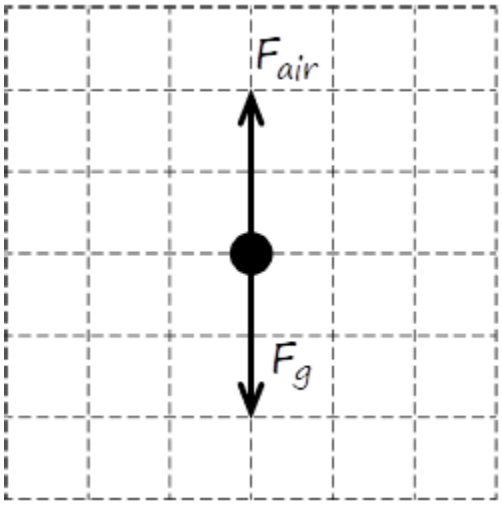
0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a downward force due to gravity, an upward force exerted by the air, and no extraneous forces.
- ☐ 1 point is earned for an upward force and a downward force that are equal in magnitude.

Scoring note: Appropriate labels for the force due to gravity include: F_{G}, F_{g}, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$, $F_{\text{Block,E}}$. The labels G or g are not appropriate labels for force due to gravity. F_{air}, "force of air", "air resistance force", "drag force", or similar labels may be used for the upward force exerted by the air.

Example Response for part (a):



Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for the correct units of kg/m^2 or equivalent.

Part C

chapter1

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

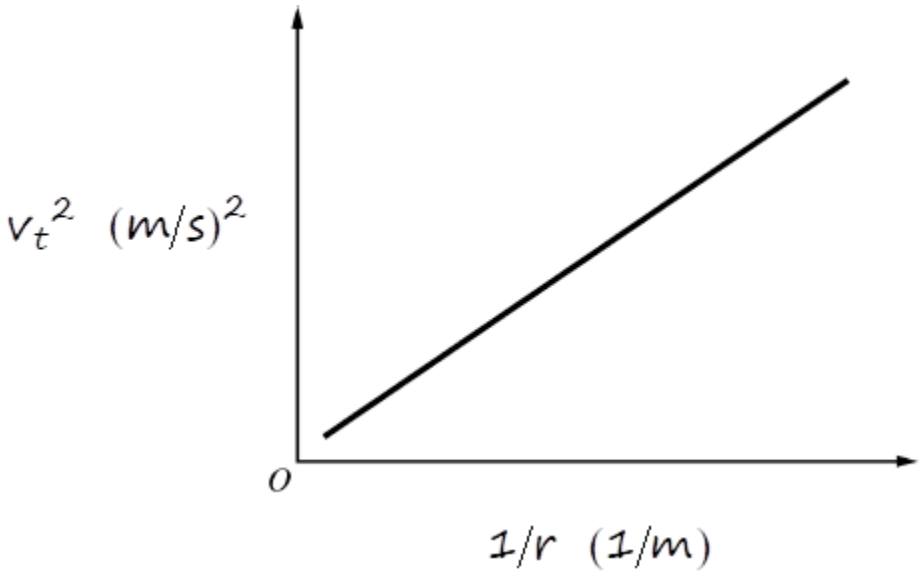
0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for recognizing that at terminal velocity, $F_{\text{air}} = m_B g$
- ☐ 1 point is earned for a correct equation for $v_t = \sqrt{\frac{m_B g}{Cr}}$
- ☐ 1 point is earned for any combination of v_t and r
- ☐ 1 point is earned for a sketch consistent with the variables listed and the equation derived above, where:
  the slope is correctly positive, negative, or zero
  AND
  the concavity is correct (concave-up, concave-down, or a straight line).

Scoring note: Graph does not have to be a straight line to earn this point.

Example Response for 4th point:



Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

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The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for describing how to use appropriate equipment to measure the radii of three different parachutes.
- ☐ 1 point is earned for describing how to use appropriate equipment to measure the speed of a falling parachute.
- ☐ 1 point is earned for describing a way to measure v_T that ensures measurements are taken when terminal speed has been reached.
- ☐ 1 point is earned for attempting to reduce uncertainty by averaging several terminal velocity measurements for the same parachute.

Example Response for part (d):

Quantity to be Measured	Symbol of Quantity	Equipment for Measurement
Radius of parachute	*R*	*Ruler*
Terminal velocity of parachute	v_t	*Motion sensor*

Diagram of Experimental Setup	Description of Experimental Procedure
	*Put the motion sensor on the floor pointed upwards. Measure the radius of a parachute with a ruler. Drop the parachute from a set height over the motion sensor. Record the velocity given by the motion sensor when the velocity becomes constant. Repeat the above measurements several times to reduce error and average the results. Repeat this process for parachutes of at least three different radii.*

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

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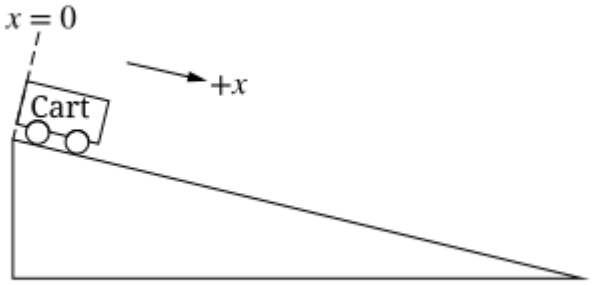
The response accurately includes:

- [ ] 1 point is earned for an answer in the range of $1.9\ \mathrm{m/s}$ to $2.1\ \mathrm{m/s}$

chapter1

3. This question is a long free-response question. Show your work for each part of the question.

This question is a long free-response question. Show your work for each part of the question.



(12 points, suggested time 25 minutes)

(a) Students conduct an experiment to determine the acceleration a of a cart. The cart is released from rest at the top of the ramp at time $t = 0$ and moves down the ramp. The x-axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

	Position x (m)	Time t (s)	
	0.06	0.39	
	0.14	0.59	
	0.24	0.77	
	0.37	0.96	
	0.55	1.20	

i. Indicate which quantities could be graphed to yield a straight line whose slope could be used to determine the acceleration a of the cart. You may use the remaining columns in the table, as needed, to record any quantities (including units) that are not already in the table.

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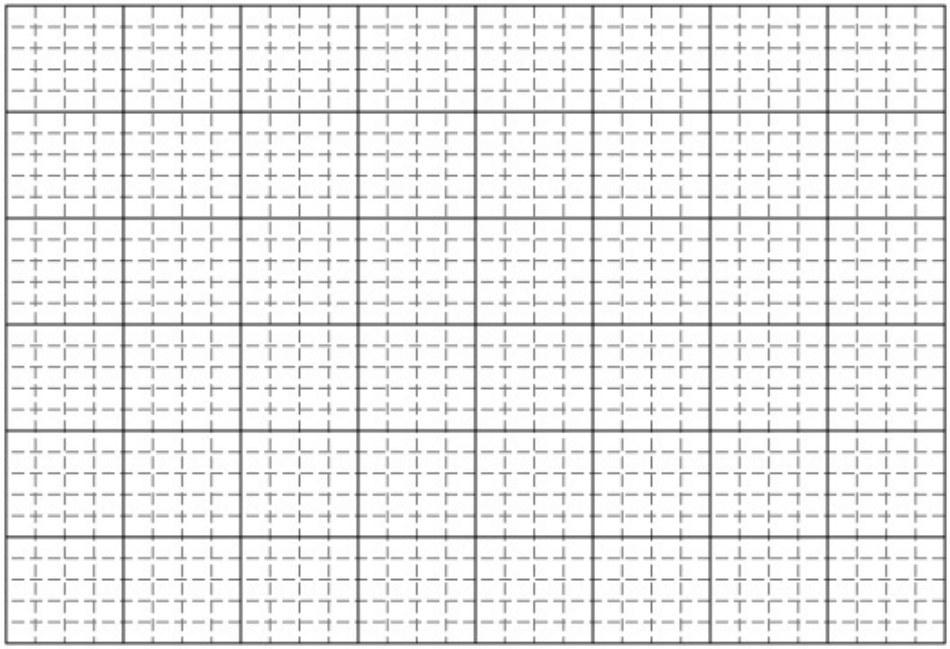
Vertical axis: _____ Horizontal axis: _____

ii. On the following grid, plot the appropriate quantities to create a graph that can be used to determine the acceleration a of the cart as it rolls down the ramp. Clearly scale and label all axes (including units), as appropriate. Draw a straight line that best represents the data.



iii. Using the line you drew in part (a)(ii), calculate an experimental value for the acceleration a of the cart as it rolls down the ramp.

(b) The students are asked to determine an experimental value for the acceleration due to gravity g_{exp} using their data.

i. What additional quantities do the students need to measure in order to calculate g_{exp} from a ?

ii. Write an expression for the value of g_{exp} in terms of a.

(c) The students calculate the value of g_{exp} to be significantly lower than the accepted value of $9.8\ \text{m/s}^2$.

i. What is a physical reason, other than friction or air resistance, that could lead to a significant difference in the experimentally determined value of g_{exp} ?

ii. Briefly explain how the physical reason you identified in part (c)(i) would lead to the decrease in the experimentally determined value of g_{exp} .

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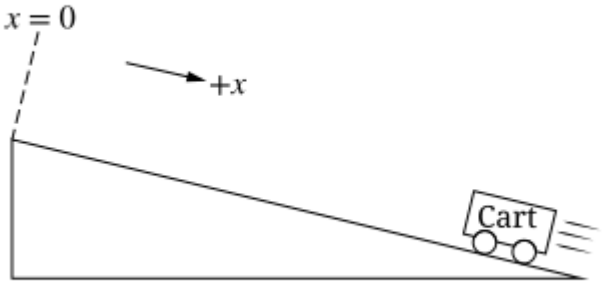
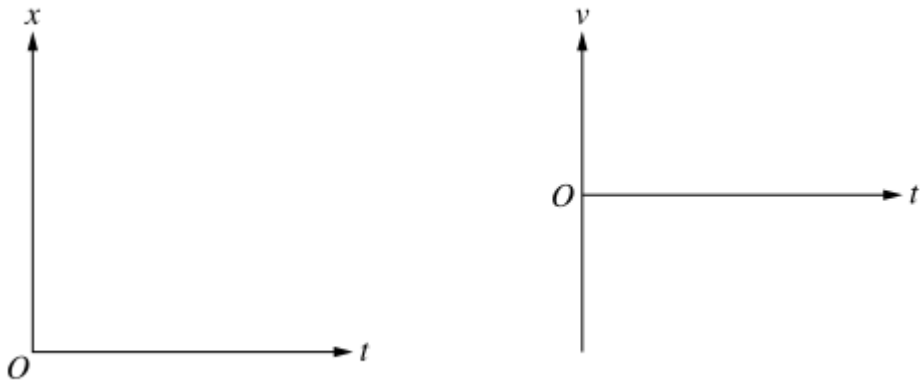


The students want to confirm that the acceleration is the same whether the cart rolls up or down the ramp. The students start the cart at the bottom and give the cart a quick push so that it rolls up the ramp and momentarily comes to rest. The x-axis is still defined to be parallel to the ramp with the origin at the top.

(d) On the following graphs, sketch the position x and velocity v as functions of time t that correspond to the scenario shown while the cart moves up the ramp.



Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

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The response accurately includes **all** the following criteria.

- 1 point is earned for indicating two quantities that, when graphed together, produce a straight line whose slope can be used to determine the acceleration a .
- 1 point is earned for having the axes have a linear scale and identified (labels **OR** units) so that when graphed correctly, the data will spans more than half of the horizontal and vertical axes.
- 1 point is earned for plotting at least 4 of the data points correctly.
- 1 point is earned for drawing a best-fit line that approximates the trend of the data.
- 1 point is earned for attempting to find the slope, $\left(\frac{\text{rise}}{\text{run}}\right)$ or $\left(\frac{\Delta y}{\Delta x}\right)$, of the best-fit line drawn in part (a)(ii).
  Scoring Note: An indication that a calculator was used for linear regression to determine the value of the slope may earn this point.
- 1 point is earned for using the slope in a valid kinematic equation to calculate the acceleration.
  Scoring Note: This point can be earned if evidence of a kinematic equation exists in graphed quantities (e.g., a graph of position as a function of $\frac{1}{2}t^2$).

Example Response

Vertical Axis : *Position*

Horizontal Axis : *Time squared*

Position x (m)	Time t (s)	Time squared t^2 (s^2)
0.06	0.39	0.15
0.14	0.59	0.35
0.24	0.77	0.59
0.37	0.96	0.92
0.55	1.20	1.44

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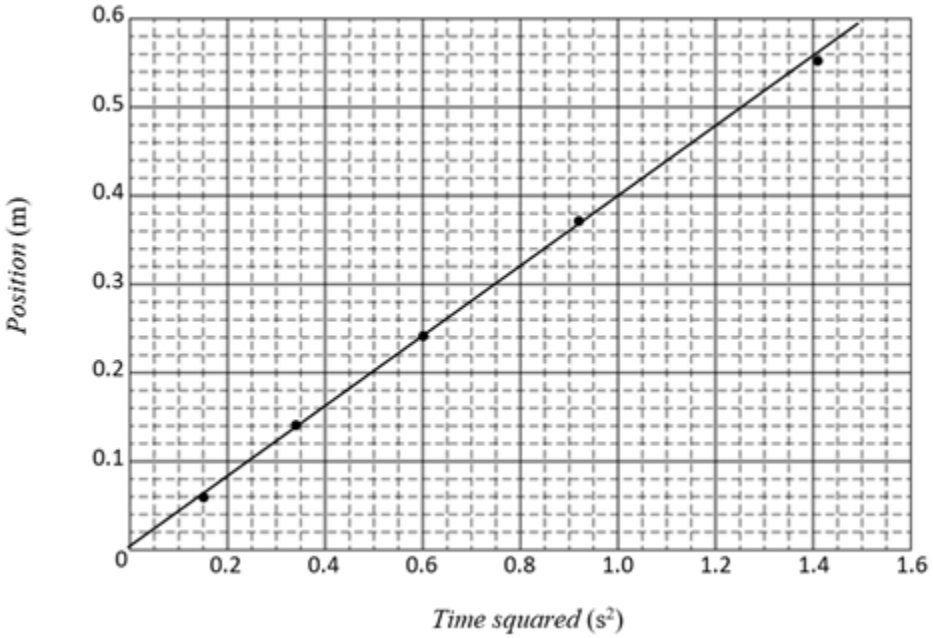
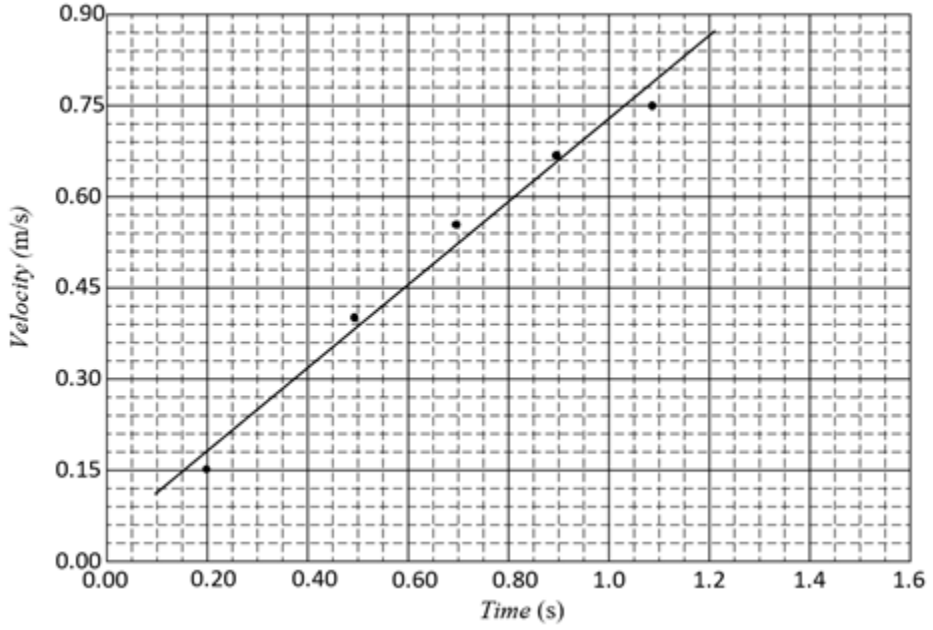


OR



Scoring Note: The following tables represent the most common linearized graphs with the data that were used to determine the acceleration.

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Graph: v vs. t	
$v\left(\frac{\text{m}}{\text{s}}\right)$	t (s)
0.15	0.20
0.40	0.49
0.56	0.68
0.68	0.87
0.75	1.08

Graph: $2x$ vs. t^2	
$2x$ (m)	$t^2\left(\text{s}^2\right)$
0.12	0.15
0.28	0.35
0.48	0.59
0.74	0.92
1.10	1.44

Graph: $2v_{\text{avg}}$ vs. t	
$2v_{\text{avg}}\left(\frac{\text{m}}{\text{s}}\right)$	t (s)
0.31	0.39
0.47	0.59
0.62	0.77
0.77	0.96
0.92	1.20

chapter1

Graph: x vs. $\frac{1}{2}t^2$	
x (m)	$\frac{1}{2}t^2\left(\text{s}^2\right)$
0.06	0.08
0.14	0.17
0.24	0.30
0.37	0.46
0.55	0.72

Graph: v_{avg}^2 vs. x	
$v_{\text{avg}}^2\left(\frac{\text{m}^2}{\text{s}^2}\right)$	x (m)
0.02	0.06
0.06	0.14
0.10	0.24
0.15	0.37
0.21	0.55

Graph: $\sqrt{x}$ vs. t	
$\sqrt{x}\ \left(\sqrt{\text{m}}\right)$	t (s)
0.24	0.39
0.37	0.59
0.49	0.77
0.61	0.96
0.74	1.20

$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta \text{position}}{\Delta \text{time}^2} = \frac{0.48\ \text{m} - 0.18\ \text{m}}{1.2\ \text{s}^2 - 0.4\ \text{s}^2} = 0.375\ \frac{\text{m}}{\text{s}^2}$

$\Delta x = v_0 t + \frac{1}{2}at^2$

$\frac{\Delta x}{t^2} = \frac{1}{2}a$

$\text{slope} \times 2 = a$

$a = 0.75\ \frac{\text{m}}{\text{s}^2}$

Part B

chapter1

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** the following criteria.

- ☐ 1 point is earned for indicating a quantity to be measured.
  Accept **one** of the following:
  - The angle θ with the horizontal
  - The height h and length L of the ramp

  Scoring Note: Stating only the height needs to be measured can earn this point if an energy approach is used.
- ☐ 1 point is earned for providing a correct expression relating the acceleration of gravity to the acceleration measured.
  Scoring Note: If $\cos\theta$ is used, the response must specify that θ was measured from the vertical.

Example Response

$mg_{\text{exp}} \sin\theta = ma$
$g_{\text{exp}} = \frac{a}{\sin\theta}$

OR

$\sin\theta = \frac{h}{L}$
$g_{\text{exp}} = \left(\frac{L}{h}\right)a$

OR

$mg_{\text{exp}}h = \frac{1}{2}mv^2$
$g_{\text{exp}}h = \frac{1}{2}v^2$
$v = \sqrt{2g_{\text{exp}}h}$
$v = at$
$at = \sqrt{2g_{\text{exp}}h}$
$g_{\text{exp}} = \frac{a^2t^2}{2h}$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter1

0	1	2 ✓

The response accurately includes **both** the following criteria.

- ☐ 1 point is earned for identifying a physical factor that could have affected the result.
  Accept **one** of the following:
  - A physical factor in the materials used (*e.g.*, the wheels have nonnegligible rotational inertia, the ramp was bumpy, the wheels were wobbly or not perfectly round, the base of the ramp was not level, the floor was not level).
  - A physical factor in the environment (*e.g.*, the room was being accelerated, elevator, the experiment was performed at high elevation or on a different planet).
  - A physical error in measurement collection (*e.g.*, time, position, or angle was measured incorrectly).

  Scoring Note: A statement of "Human error" does not earn this point.
- ☐ 1 point is earned for correctly indicating the functional dependence between the reason listed in part (c)(i) and g_{exp}
  Accept **one** of the following:
  - Correctly indicating the functional dependence between the physical factor in the materials used and g_{exp} (*e.g.*, if the rotational inertia of the rotating wheels is nonnegligible, the cart will have a smaller acceleration and g_{exp} will be smaller).
  - Correctly indicating the functional dependence between the physical factor in the environment and g_{exp} (*e.g.*, if the experiment was performed at a high elevation, the acceleration will be smaller and g_{exp} will be smaller).
  - Correctly indicating the functional dependence between the physical error in the measurement collection and g_{exp} (*e.g.*, if the angle of the ramp is smaller than the measured value, the cart will have a smaller acceleration and g_{exp} will be smaller).

Example Response

The expression I derived for the value for g_{exp} did not take into consideration that the wheels had any rotational inertia. If the wheels have rotational inertia and are rotating, the acceleration of the cart would be less than $g \sin\theta$, so the value of g_{exp} would be less than $9.8\ \frac{\text{m}}{\text{s}^2}$.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

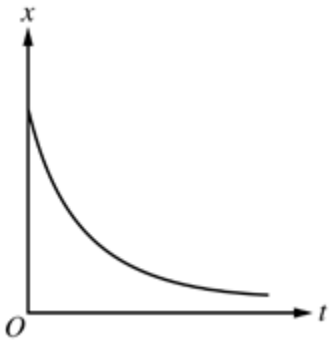
chapter1

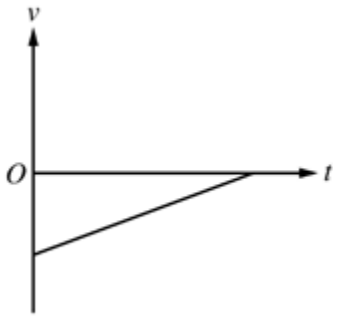
0	1	2 ✓

The response accurately includes **both** the following criteria.

- ☐ 1 point is earned for sketching a concave up curve with an initially negative slope for the graph of position as a function of time.
- ☐ 1 point is earned for **one** of the following:
  - Drawing a line with a positive slope and a negative vertical intercept for the v vs. t graph.
  - Drawing a v vs. t graph that is consistent with the x vs. t graph that shows acceleration.

Example Response





chapter1

4.

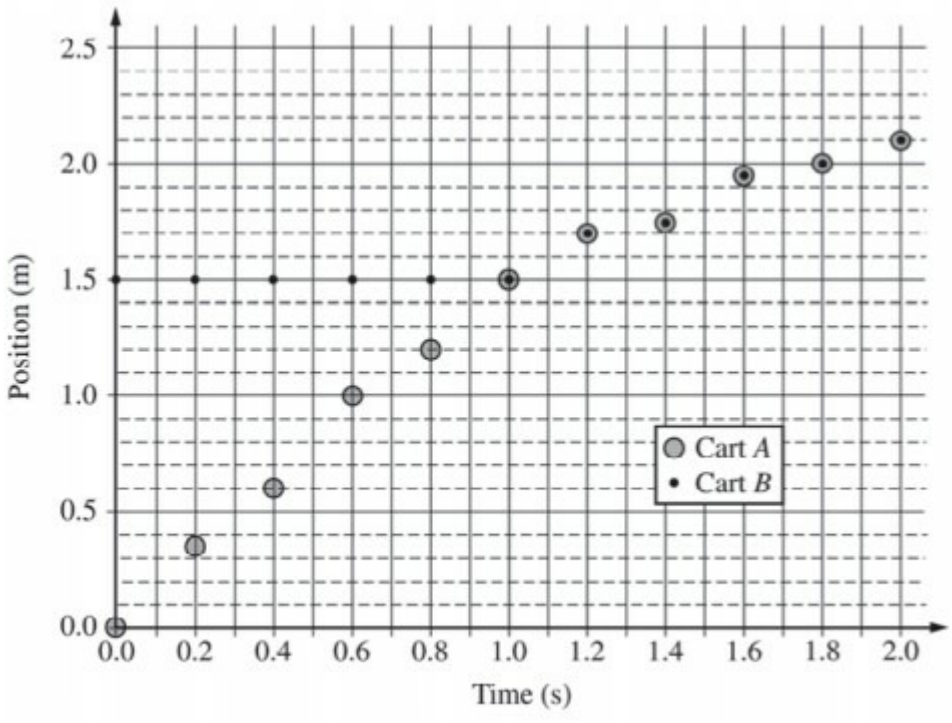


A group of students has two carts, *A* and *B*, with wheels that turn with negligible friction. The carts can travel along a straight horizontal track. Cart *A* has known mass m_A. The students are asked to use a one dimensional collision between the carts to determine the mass of cart *B*. Before the collision, cart *A* travels to the right and cart *B* is initially at rest, as shown above. After the collision, the carts stick together.

(a) Describe an experimental procedure to determine the velocities of the carts before and after a collision, including all the additional equipment you would need. You may include a labeled diagram of your setup to help in your description. Indicate what measurements you would take and how you would take them. Include enough detail so that another student could carry out your procedure.

(b) There will be sources of error in the measurements taken in the experiment, both before and after the collision. For your experimental procedure, will the uncertainty in the calculated value of the mass of cart B be affected more by the error in the measurements taken before the collision or by those taken after the collision, or will it be equally affected by both sets of measurements? Justify your answer.

A group of students took measurements for one collision. A graph of the students' data is shown below.



(c) Given m_A = 0.50 kg, use the graph to calculate the mass of cart *B*. Explicitly indicate the principles used in your calculations.

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(d) The students are now asked to consider the kinetic energy changes in an inelastic collision, specifically whether the initial values of one of the physical quantities affect the fraction of mechanical energy dissipated in the collision. How could you modify the experiment to investigate this question? Be sure to explicitly describe the calculations you would make, specifying all equations you would use (but do not actually do any algebra or arithmetic).

Part A

1 point is earned for a reasonable setup that would allow useful measurements

1 point is earned for indicating all the measurements needed to determine the velocities

1 point is earned for having no obviously extraneous equipment or measurements

Examples:

- Use tape to mark off two distances on the track — one for cart A before the collision and one for the combined carts after the collision. Push cart A to give it an initial speed. Use a stopwatch to measure the time it takes for the cart(s) to cross the marked distances. The speeds are the distances divided by the times.
- Place a motion detector at the left end of the track. Push cart A to give it an initial speed. Record position as a function of time, first for cart A and then for the combined carts A and B.

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned for a reasonable setup that would allow useful measurements

1 point is earned for indicating all the measurements needed to determine the velocities

1 point is earned for having no obviously extraneous equipment or measurements

Examples:

- Use tape to mark off two distances on the track — one for cart A before the collision and one for the combined carts after the collision. Push cart A to give it an initial speed. Use a stopwatch to measure the time it takes for the cart(s) to cross the marked distances. The speeds are the distances divided by the times.
- Place a motion detector at the left end of the track. Push cart A to give it an initial speed. Record position as a function of time, first for cart A and then for the combined carts A and B.

Part B

1 point is earned for indicating a reasonable assumption about the relative size of the measurement errors before and after the collision

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1 point is earned for correctly using the assumption in comparing the effect on the calculated value of the mass of cart B

Example:

> If the measurement errors are of the same magnitude, they will have a greater effect after the collision. The speed of the combined carts will be less than the initial speed of cart A, so errors of the same magnitude will be a greater percentage of the actual value after the collision. So the values after the collision will have a greater effect on the value of the mass of cart B.

0	1	2 ✓

The student response earns all of the following points:

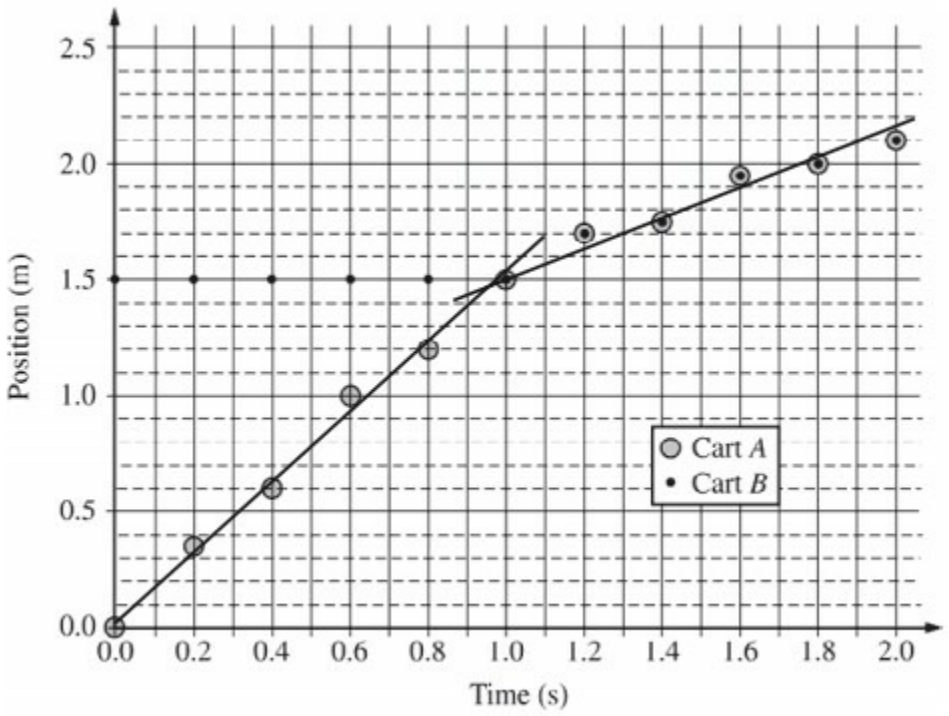
1 point is earned for indicating a reasonable assumption about the relative size of the measurement errors before and after the collision

1 point is earned for correctly using the assumption in comparing the effect on the calculated value of the mass of cart B

Example:

> If the measurement errors are of the same magnitude, they will have a greater effect after the collision. The speed of the combined carts will be less than the initial speed of cart A, so errors of the same magnitude will be a greater percentage of the actual value after the collision. So the values after the collision will have a greater effect on the value of the mass of cart B.

Part C



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1 point is earned for providing sufficient description of the principles used in t he calculation (in either a single explanation or dispersed throughout the calculations)

Conservation of momentum can be used to determine the mass of cart B: *mAvi* = (*mA* + *mB*)*vf*

1 point is earned for correctly recognizing the two regions on the graph corresponding to before and after the collision

1 point is earned for using data from the graph to attempt calculation of speed from slope

1 point is earned for indicating use of the slope of one or two drawn lines to determine one or more speeds (This point cannot be earned if calculations use data points not on the line[s].)

The speed *vi* before the collision is the slope of the best-fit line for the data from 0 to 1 s.

The speed *vf* after the collision is the slope of the best-fit line for the data from 1 s to 2 s.

Using the example lines drawn above:

$v_i = \frac{(1.4-0)m}{(0.9-0)s} = \frac{14}{9}\frac{m}{s}$

$v_f = \frac{(2.1-1.5)m}{(1.9-1.0)s} = \frac{0.6m}{0.9s} = \frac{2}{3}\frac{m}{s}$

Applying conservation of momentum:

$(0.5kg)\left(\frac{14}{9}\frac{m}{s}\right) = (0.5kg + m_B)\left(\frac{2}{3}\frac{m}{s}\right)$

$(0.5kg)\left(\frac{14}{9}\right)\left(\frac{3}{2}\right) = (0.5kg + m_B)$

$(0.5kg)\left(\frac{7}{3}\right) = (0.5kg + m_B)$

$m_B = (0.5kg)\left(\frac{7}{3} - 1\right) = (0.5kg)\frac{4}{3} = \frac{2}{3}kg$

0	1	2	3	4 ✓

The student response earns all of the following points:

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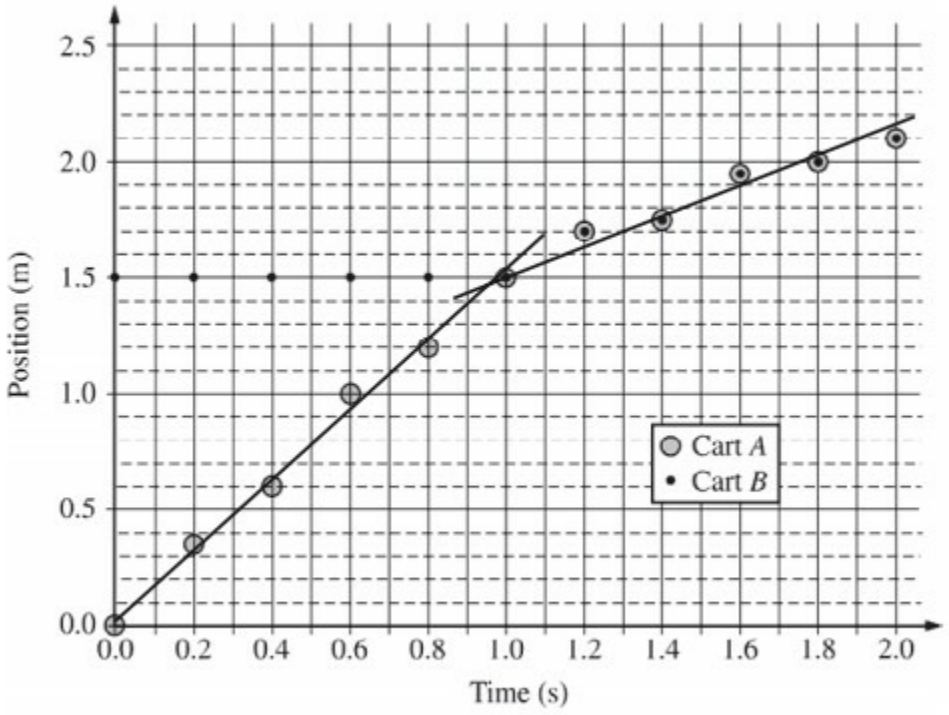

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$m_B = (0.5kg)\left(\frac{7}{3} - 1\right) = (0.5kg)\frac{4}{3} = \frac{2}{3}kg$

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Part D

1 point is earned for an answer consistent with previous responses that indicates a modification of the procedure to accomplish varying the initial speed of cart A or one of the cart masses OR that indicates that the previously described procedure would provide appropriate data, so it does not need modification

1 point is earned for indicating that the data can be used to calculate the kinetic energy *K* before and after the collision

1 point is earned for indicating that the fraction of K lost in the various collisions should be compared

Example:

You could vary the initial speed of cart A. From the data, calculate values of kinetic energy before and after the collision using $K = (1/2)mv2$. Then analyze $((Ki - Kf)/Ki)$ to see if the changes in initial speed give different values.

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned for an answer consistent with previous responses that indicates a modification of the procedure to accomplish varying the initial speed of cart A or one of the cart masses OR that indicates that the previously described procedure would provide appropriate data, so it does not need modification

1 point is earned for indicating that the data can be used to calculate the kinetic energy *K* before and after the collision

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Example:

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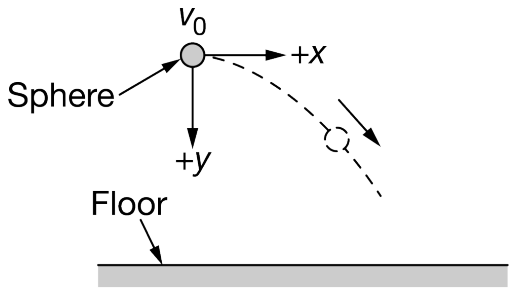
5. This question is a long free-response question. Show your work for each part of the question.

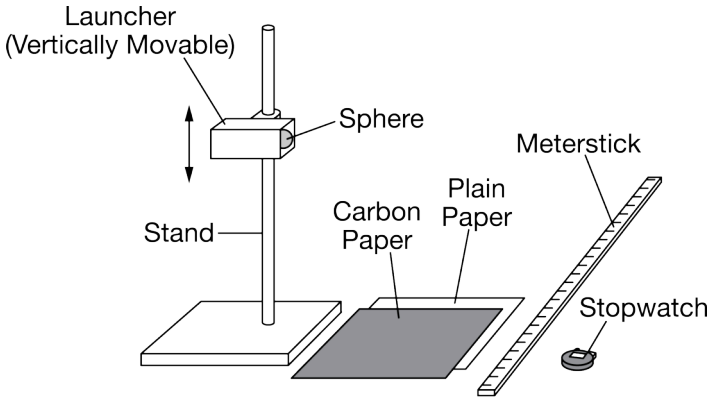


Motion of a Sphere

Students are studying the two-dimensional motion of objects as they move through the air. Specifically, they are examining the behavior of a sphere that is launched horizontally from a location above the floor with an initial velocity v_0 in the $+x$ direction, as shown in the figure. The students assume that the positive directions are along the sphere's initial velocity for horizontal motion and downward for vertical motion.

The horizontal displacement of the object from its starting point is x, and the vertical displacement of the object from its starting point is y. One of the students derives an equation for y in terms of x and other quantities. After examining the equation, the student claims that y is proportional to x^2.

(a) Derive an equation for the vertical coordinate y of the sphere as a function of x, v_0, and physical constants, as appropriate.



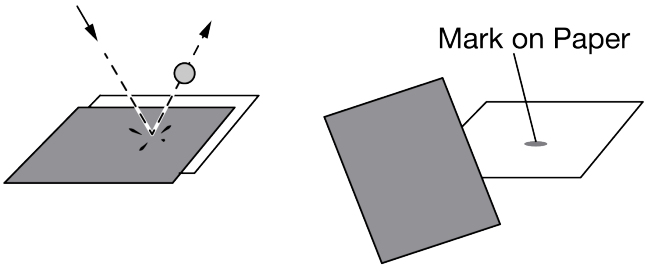
Experimental Apparatus

The students have access to the experimental apparatus shown in the figure. Besides the sphere, the apparatus consists of the following.

A launcher that launches the sphere horizontally, always with the same unknown speed v_0
A stand that can be used to adjust the height of the launcher above the floor

The students also have several sheets of plain paper, several sheets of carbon paper, a meterstick, and a stopwatch. Whenever the sphere bounces off of the carbon paper, the carbon paper produces a visible mark on the sheet of

plain paper that is under the carbon paper, as shown in the figure.



Marking the Paper with the Sphere

(b) Design an experimental procedure that the students could use to test the claim that y is proportional to x^2. In the table below, list the quantities and associated symbols that would be measured in your experiment and the equipment used to measure them. Also list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

Quantity to Be Measured	Symbol for Quantity	Equipment for Measurement

Describe the overall procedure to be used, referring to the table. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table and/or include a simple diagram of the setup.

The students' teacher has a video camera, which is not available for use in the experimental procedure described in part (b). The teacher uses the camera and video analysis software to generate the data shown in the table below. The table shows the sphere's horizontal and vertical displacements x and y, as measured from the location of the sphere's launch point.

Horizontal Displacement $x\ (m)$	Vertical Displacement $y\ (m)$		
0.50	0.065		
1.00	0.245		
1.50	0.560		
2.00	0.990		

(c) The students want to use the teacher's data to determine whether the claim that y is proportional to x^2 is true.

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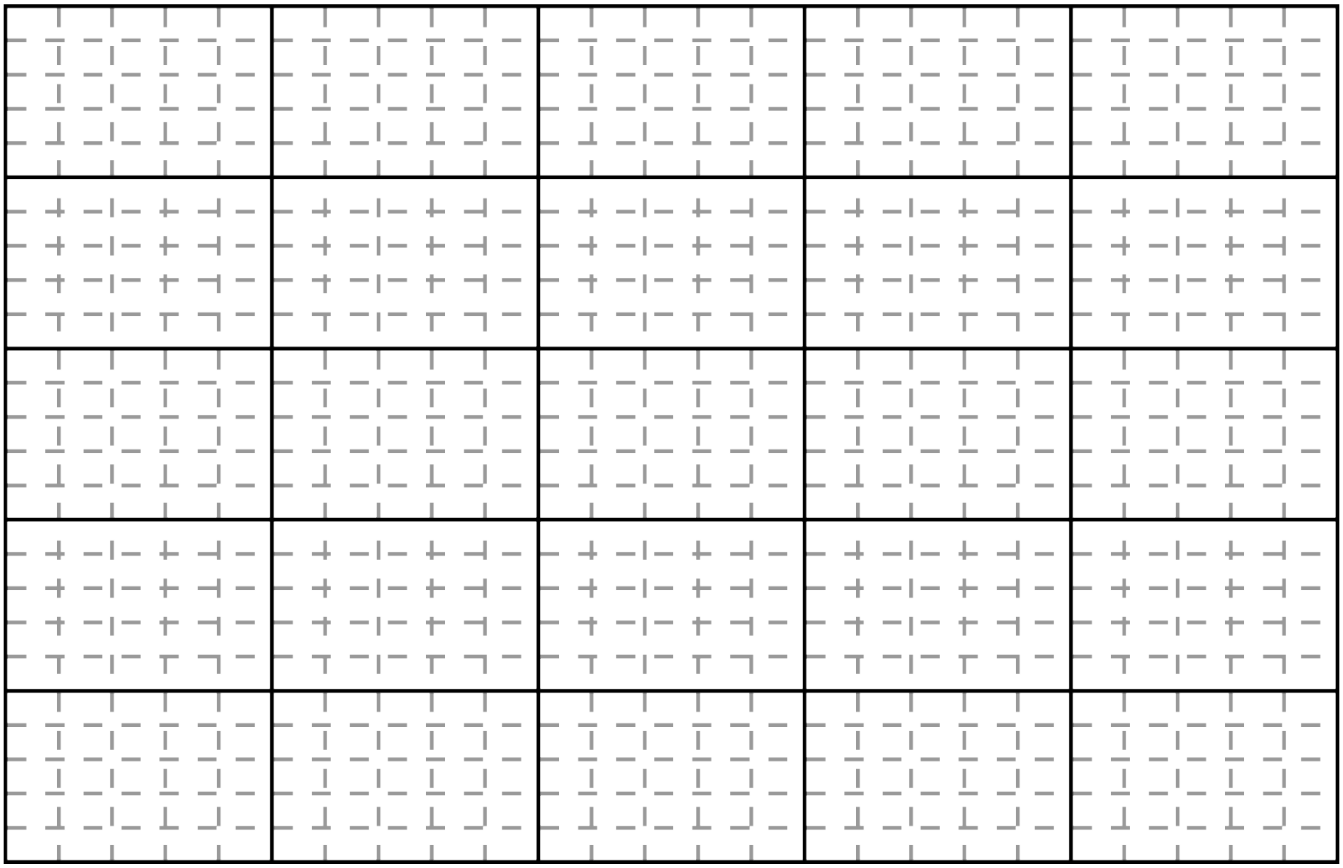
i. Indicate below which quantities could be graphed to determine whether the claim is true. You may use the remaining columns in the table, as needed, to record any quantities (including units) that are not already in the table.

Vertical axis:_____ Horizontal axis: _____

ii. On the grid below, plot the appropriate quantities to determine whether the claim is true. Clearly scale and label all axes, including units as appropriate.



iii. Do the data in the above graph support the claim that y is proportional to x^2 is true?

_____ Yes _____ No

State your reasoning.

(d) The graphs show the sphere's horizontal and vertical velocity components, v_x and v_y, as functions of time t, where $t = 0$ at the instant the sphere is launched. On the same graphs, sketch what v_x and v_y would look like for a different launcher that has a launch speed less than v_0.

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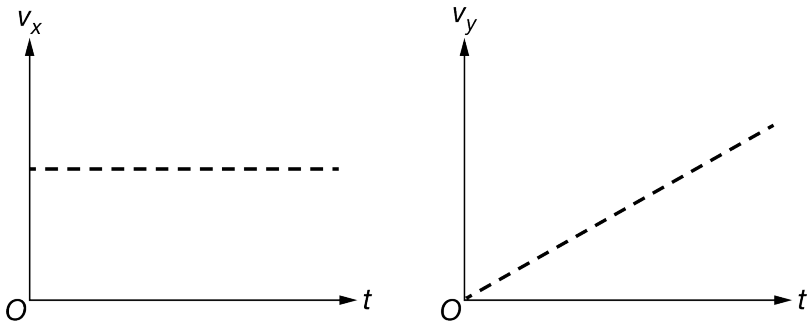

Part (a)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: To earn the first point, it must be possible to use the written equations to arrive at a correct final equation.

0	1	2 ✓

Student response accurately includes two of the following criteria.

- ☐ 1 point is earned for using appropriate kinematics equations for the sphere's horizontal and vertical motions.
- ☐ 1 point is earned for combining the equations to get a correct equation for y in terms of x, v_0 , and g.

Example Responses:

$x = v_{x0}t = v_0t$

$y = y_0 + v_{y0}t + \frac{1}{2}a_yt^2 = \frac{1}{2}gt^2$

$t = x/v_0$

$y = \frac{1}{2}gt^2 = \frac{1}{2}g(x/v_0)^2$ or $\frac{1}{2}\frac{gx^2}{{v_0}^2}$

Part (b)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5 ✓

Student response accurately includes four of the following criteria.

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- [ ] 1 point is earned for listing relevant/appropriate equipment that matches the measured quantities.
- [ ] 1 point is earned for a plausible/practical way to directly or indirectly determine whether the claim is true, using the available equipment.
- [ ] 1 point is earned for measuring x, the horizontal displacement of the sphere.
- [ ] 1 point is earned for measuring *y*, the vertical displacement of the sphere, for at least two different values of *y*.
- [ ] 1 point is earned for attempting to reduce uncertainty using at least three different launch heights, or at least two experimental trials at each of two launch heights.

Example Response:

Quantity to be Measured	Symbol for Quantity	Equipment for Measurement
Initial height above floor for bottom of sphere	*y*	Meter stick
Horizontal distance from launch point to where sphere hits the floor	*x*	Meter stick

1. Move the launcher to the maximum height possible on the stand.

2. With the sphere in the launcher, use the meter stick to measure the height *y* from the floor to the bottom of the sphere. Also, mark the spot on the floor below the center of the sphere.

3. Launch the sphere and note approximately where the sphere hits the floor.

4. Place the two sheets of paper at the location where the sphere hit the floor.

5. Launch the sphere again, so that a mark is made on the plain paper when the sphere hits the sheets of paper. If the sphere misses the paper sheets, reposition the sheets and repeat the launch until the sphere hits the sheets.

6. Put the sphere in the launcher, and use the meter stick to measure the horizontal distance *x* from the mark made in step 1 to the mark on the paper where the sphere hit the floor.

7. Repeat steps 2 through 6 for launch heights of 20%, 40%, 60%, and 80% of the maximum launch height.

Part (c)ii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes both of the following criteria.

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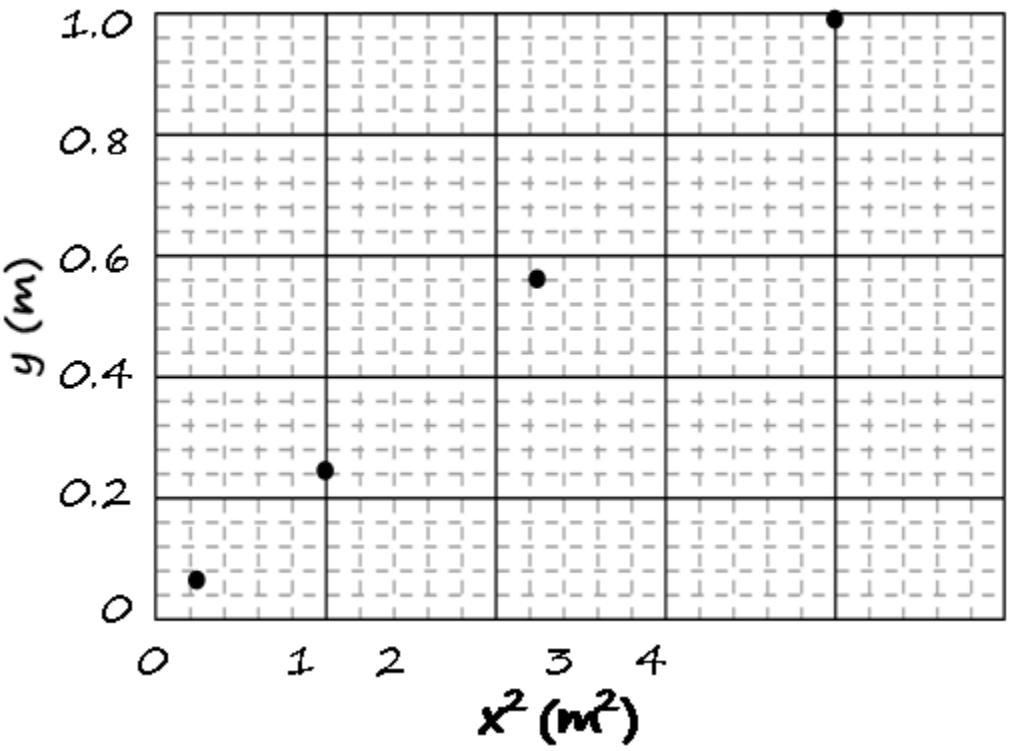
- ☐ 1 point is earned for using a correct scale that uses more than half the grid and for correctly labeling the axes, including units, as appropriate.
- ☐ 1 point is earned for correctly plotting the data or quantities calculated from the data.

Example Response:

Horizontal Displacement x (m)	Vertical Displacement y (m)	x^2 (m^2)	
0.50	0.065	0.25	
1.00	0.245	1.00	
1.50	0.560	2.25	
2.00	0.990	4.00	



Part (c)iii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: if students graph y versus x which results in a parabola, it is not sufficient for a student to simply note that a graph of y versus x is curved, or that the graph "looks like a parabola".) Students must say that y increases by a factor of approximately 4 whenever x is doubled, or similar.

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0	1 ✓

Student response accurately includes the following criteria.

- ☐ 1 point is earned for a reasonable justification that y is proportional to x^2 .

Example Response:

The data points in a graph of y versus x^2 lie along a straight line.

Part (d)

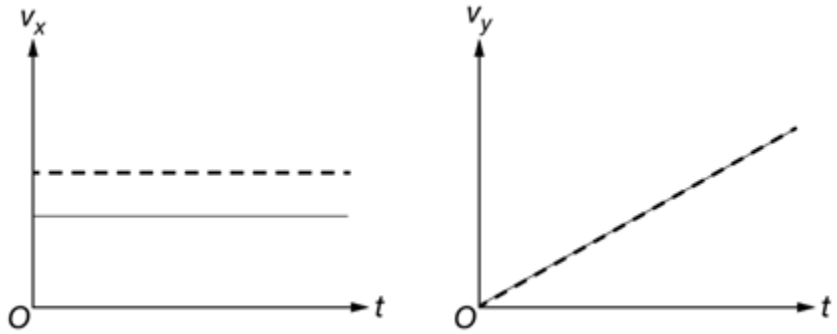
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for a graph of v_x that is a constant and less in magnitude than indicated by the dashed line.
- ☐ 1 point is earned for a graph of v_y that is identical to the graph indicated by the dashed line.

Example Response:



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6. This question is a long free-response question. Show your work for each part of the question.

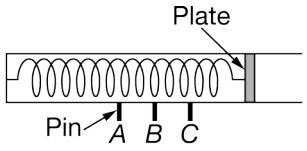


Figure 1. Uncompressed spring

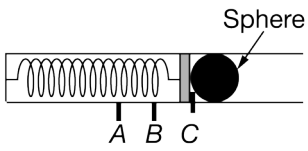


Figure 2. Compressed spring

(12 points, suggested time 25 minutes)

A projectile launcher consists of a spring with an attached plate, as shown in Figure 1. When the spring is compressed, the plate can be held in place by a pin at any of three positions A, B, or C. For example, Figure 2 shows a steel sphere placed against the plate, which is held in place by a pin at position C. The sphere is launched upon release of the pin.

A student hypothesizes that the spring constant of the spring inside the launcher has the same value for different compression distances.

(a) The student plans to test the hypothesis by launching the sphere using the launcher.

i. State a basic physics principle or law the student could use in designing an experiment to test the hypothesis.

ii. Using the principle or law stated in part (a)(i), determine an expression for the spring constant in terms of quantities that can be obtained from measurements made with equipment usually found in a school physics laboratory.

(b) Design an experimental procedure to test the hypothesis in which the student uses the launcher to launch the sphere. Assume equipment usually found in a school physics laboratory is available.

In the table below, list the quantities and associated symbols that would be measured in your experiment. Also list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

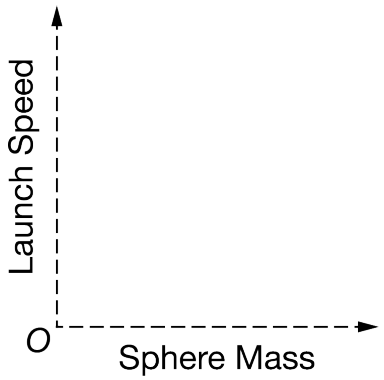
Quantity to be Measured	Symbol for Quantity	Equipment for Measurement

Describe the overall procedure to be used to test the hypothesis that the spring constant of the spring inside the launcher has the same value for different compression distances, referring to the table. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental

uncertainty. As needed, use the symbols defined in the table and/or include a simple diagram of the setup.

(c) Describe how the experimental data could be analyzed to confirm or disconfirm the hypothesis that the spring constant of the spring inside the launcher has the same value for different compression distances.

(d) Another student uses the launcher to consecutively launch several spheres that have the same diameter but different masses, one after another. Each sphere is launched from position A. Consider each sphere's launch speed, which is the speed of the sphere at the instant it loses contact with the plate. On the axes below, sketch a graph of launch speed as a function of sphere mass.



Part A(i) & Part A(ii)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- ☐ An equation that is consistent with a relevant principle or law as written in (a)(i).
- ☐ A valid equation that contains measurable quantities and includes spring constant.
- ☐ A correct and valid algebraic expression for spring constant. The expression must be solved for k .

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: The first, second, and fifth points of part (b) can be earned if the sphere is not launched.

0	1	2	3	4	5 ✓

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The response accurately includes **five** of the following criteria.

- [ ] Listing relevant/appropriate equipment that matches all measured quantities in the experimental procedure.
- [ ] Describing measurements of quantities sufficient to determine the spring constant.
- [ ] A plausible procedure (i.e., can be done in a typical school physics lab) that involves launching the sphere to determine the spring constant.
- [ ] Launching the sphere from at least 2 different initial positions.
- [ ] Attempting to reduce uncertainty (e.g., multiple trials at a pin setting).

Example Response:

The mass of the sphere is measured with a triple beam balance. The launcher is aimed horizontally on a level surface toward a motion sensor. The spring is compressed to pin position A and the spring compression distance is measured. The mass is launched. The motion sensor measures launch speed. The process is repeated three times at position A. The procedure is repeated with the spring compressed to pin positions B and C.

The launcher is aimed horizontally at a height above the ground so that the sphere will follow a projectile path and land on the floor. The spring is compressed to pin position A and the sphere is launched. Measure the mass of the sphere, the initial spring compression, and the vertical and horizontal displacements of the sphere from release to landing position. Repeat three times at pin position A. The procedure is repeated with the spring compressed to pin positions B and C.

Aim the launcher vertically. Compress the spring to pin position A. Launch the sphere vertically. Measure the mass of the sphere, the initial spring compression, and vertical displacement of the sphere above the release position. Repeat three times at pin position A. Repeat the procedure with the spring compressed to pin positions B and C.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: The second point of part (c) is not earned for saying “if the numbers are the same” or similar phrasing that does not address experimental uncertainty.

0	1	2 ✓

The response accurately includes **both** of the following criteria:

- [ ] Comparing the measurements of the spring constant (or a suitable proxy) at all three possible compression distances (A, B, C).
- [ ] Considering uncertainties in confirming the hypothesis (e.g., “If numbers match within experimental uncertainty,” or “If the numbers are about the same”).

Example Response:

For each pin position, take the average $v_{\text{L-avg}}$ of the launch speeds measured at that position. Calculate the spring constant k using the energy conservation relation $\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m_S {v_{\text{L-avg}}}^2$ or $k = \frac{m_S {v_{\text{L-avg}}}^2}{(\Delta x)^2}$. Then compare the k values for

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each spring position. If the values agree within experimental uncertainty, then the hypothesis is confirmed.

For each pin position, take the averages Δx_{avg} and Δy_{avg} of the horizontal and vertical sphere displacements. Calculate the time interval Δt using the kinematics equation $\Delta y_{\text{avg}} = \frac{1}{2} g (\Delta t)^2$, and then calculate the launch speed $v_L = \frac{\Delta x_{\text{avg}}}{\Delta t}$. Calculate the spring constant using the relation $k = \frac{m_S {v_L}^2}{d^2}$. Compare the k values for each spring position. If the values agree within experimental uncertainty, then the hypothesis is confirmed.

For each pin position, take the average Δy_{avg} of the maximum vertical sphere displacement. Use conservation of energy to calculate a value for the spring constant k from the equation

· $\frac{1}{2} k d^2 = mg \Delta y_{\text{avg}}$ (if measuring height from the release (pin) position)

· $\frac{1}{2} k d^2 = mg(\Delta y_{\text{avg}} + d)$ (if measuring height from the spring's uncompressed position)

Compare the k values for each spring position. If the values agree within experimental uncertainty, then the hypothesis is confirmed.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria:

- ☐ A curve where launch speed always decreases with increasing sphere mass.
- ☐ A curve that is entirely concave up AND has the launch speed always decreasing with increasing sphere mass.

Example Response:

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