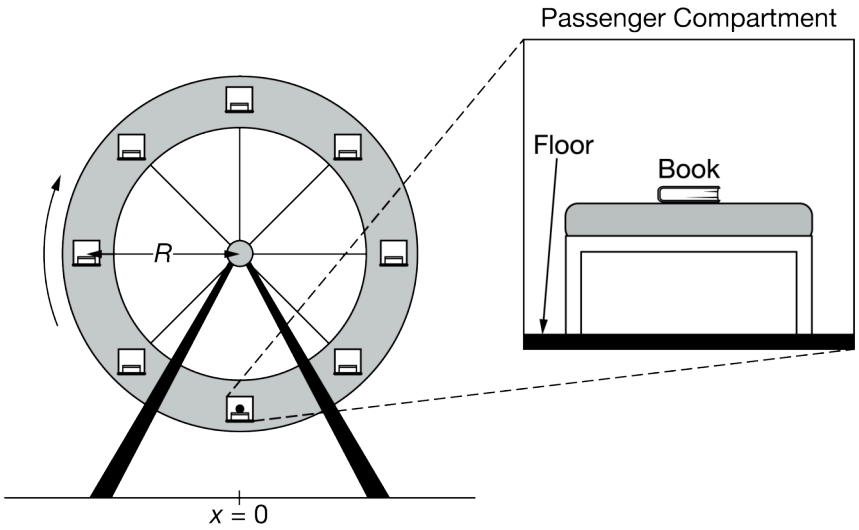
chapter2parta

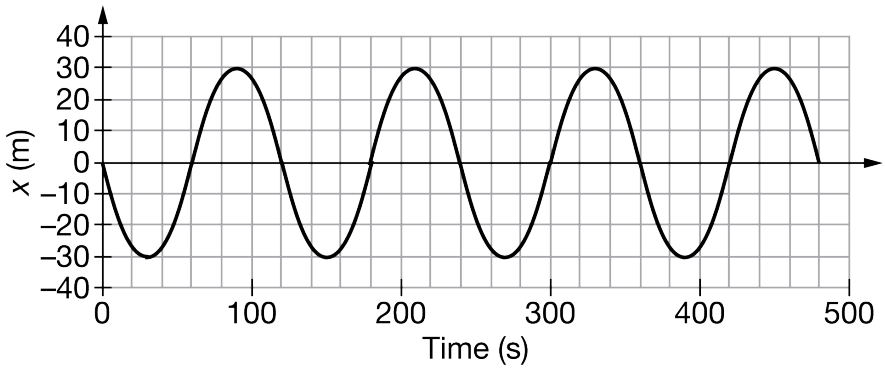
1. This question is a long free-response question. Show your work for each part of the question.

(12 points, suggested time 25 minutes)



A passenger compartment of a rotating amusement park ride contains a bench on which a book of mass m_b is placed, as indicated by the dot in the left figure above. The compartment moves with a constant angular speed about the center of the ride along a circular path of radius R. The bench remains horizontal throughout the compartment's motion. The right figure above shows a magnified view of the compartment.

The graph below shows the horizontal (x) component of the book's position as a function of time, where the $+x$ -direction is to the right.



(a)

i. Determine the period of revolution of the book.

ii. Calculate the tangential speed v_b (not the angular speed) of the book.

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(b)

i. On the dot below, which represents the book, draw and label the forces (not components) that act on the book at the lowest point of its circular path. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



ii. At the lowest point of the circular path, the book is moving only in the horizontal direction. In what direction, if any, is the net vertical force on the book?

_____ Up _____ Down _____ No direction, since the net vertical force is equal to zero

Without deriving any equations, briefly explain your reasoning in terms of the book's motion.

(c) Derive an algebraic equation for the vertical force that the bench exerts on the book at the lowest point of the circular path in terms of the book's mass m_b, tangential speed v_b, radius R of the path, and physical constants, as appropriate. Do not substitute any numerical values for variables or physical constants.

(d) At the lowest point of the circular path, is the force that the bench exerts on the book greater than, less than, or equal to the weight of the book?

_____ Greater Than _____ Less Than _____ Equal to

Briefly explain how your answers in (b)ii and (c) support your selection.

Part A(i)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ An indication of the correct period of 120 seconds, 2 minutes, OR the equivalent.

Part A(ii)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

$(\pi/2)\ \mathrm{m/s}$ is acceptable.

$\pi R/(60\,\mathrm{s})$ or $(\pi R/60)\ \mathrm{m/s}$ can only earn 1 of the two points.

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0	1	2 ✓

The response accurately includes both of the following criteria:

- ☐ For using the correct radius of $30\ \text{m}$ as found from the graph
- ☐ For substitution of the period, consistent with the answer in (a)(i), into a correct expression

Example Response:

$\text{Speed} = d/T = 2\pi R/T = 2\pi(30\ \text{m})/(120\ \text{s}) = 1.6\ \text{m/s}$

Part B(i)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes three of the following criteria.

- ☐ For a correctly drawn and labeled normal force
- ☐ For a correctly drawn and labeled gravitational force
- ☐ For drawing no extraneous forces, given that at least one force is drawn

Example Response:



Part B(ii)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: “Up”

Response cannot earn credit if incorrect selection is made.

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0	1 ✓

The response accurately includes:

- [ ] An explicit or implicit assertion based on forces.

Example Response:

Because there is a centripetal force

Claim: The direction of the net vertical force on the book is up.

Evidence: The book is at the lowest point of its circular path. For an object in uniform circular motion, the centripetal force is always directed toward the center of the circular path.

Reasoning Statement: The net force is the centripetal force on the book, which must be upward toward the center of the book's circular path.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes three of the following criteria.

- [ ] For using Newton's 2nd law correctly or consistent with the diagram in (b)(i)
- [ ] For a correct expression for the centripetal acceleration
- [ ] For a correct final expression with no numbers substituted for values

Example Response:

$\Sigma \vec{F} = m\vec{a}$ (for the rotating system)

$F_{net} = ma_c = mv^2/r$

$a_c = mv^2/r = m_b v_b^2/R$

$F_{\text{net}} = m_b v_b^2/R = F_N - F_G$

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$$F_N = m_b v_b^2/R + F_G$$
$$= m_b\left(v_b^2/R + g\right)$$

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answer: “Greater than”

A response with an incorrect selection consistent with parts (b)(ii) and (c) can earn 2 points.

0	1	2 ✓

The response accurately includes both of the following criteria:

- ☐ For correct reasoning with a selection consistent with the answer from (b)(ii) and (c)
- ☐ For explaining why the force from the bench is greater than the weight of the book.

Example Response:

Claim: At the lowest point of the circular path, the force that the bench exerts on the book is greater than the weight of the book.

Evidence: The direction of the net vertical force on the book is up. (Part (b)(ii) answer)

The book exerts a vertical force on the bench of magnitude $m_b\left(v_b^2/R + g\right)$. (Part (c) answer)

Reasoning Statements:

The upward force exerted by the bench must be greater than the downward gravitational force, in order for the net force to be upward as stated in the answer to (b)(ii). The magnitude $m_b\left(v_b^2/R + g\right)$ of the force exerted by the bench derived in part (c) is greater than the book’s weight $m_b g$.

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2. A student wants to determine the coefficient of static friction between a long, flat wood board and a small wood block.

a. Describe an experiment for determining the coefficient of static friction between the wood board and the wood block. Assume equipment usually found in a school physics laboratory is available.

i. Draw a diagram of the experimental setup of the board and block. In your diagram, indicate each quantity that would be measured and draw or state what equipment would be used to measure each quantity.

ii. Describe the overall procedure to be used, including any steps necessary to reduce experimental uncertainty. Give enough detail so that another student could replicate the experiment.

b. Derive an equation for the coefficient of static friction in terms of quantities measured in the procedure from part (a).

A physics class consisting of six lab groups wants to test the hypothesis that the coefficient of static friction between the board and the block equals the coefficient of kinetic friction between the board and the block. Each group determines the coefficients of kinetic and static friction between the board and the block. The groups' results are shown below, with the class averages indicated in the bottom row.

Lab Group Number	Coefficient of Kinetic Friction	Coefficient of Static Friction
1	0.45	0.54
2	0.46	0.52
3	0.42	0.56
4	0.43	0.55
5	0.74	0.23
6	0.44	0.54
Average	0.49	0.49

c. Based on these data, what conclusion should the students make about the hypothesis that the coefficients of static and kinetic friction are equal?

_____ The static and kinetic coefficients are equal.

_____ The static and kinetic coefficients are not equal.

Briefly justify your reasoning.

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d. A metal disk is glued to the top of the wood block. The mass of the block-disk system is twice the mass of the original block. Does the coefficient of static friction between the bottom of the block and the board increase, decrease, or remain the same when the disk is added to the block?

_____ Increase _____Decrease _____Remain he same

Briefly state your reasoning

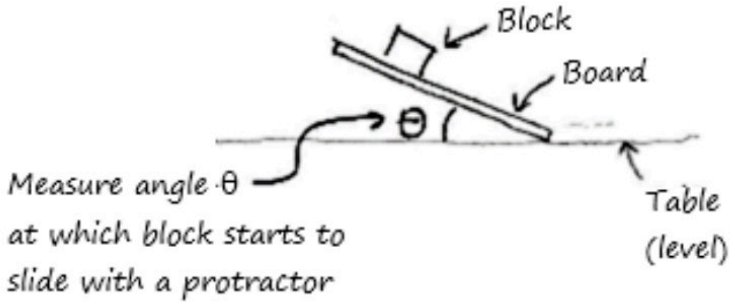
Part A

Part(i)

1 point is earned: For drawing a diagram of an experimental setup to measure the coefficient of friction that is feasible in a school physics lab

1 point is earned: For indicating the measurements necessary for calculating the coefficient of friction

1 point is earned: For indicating equipment necessary for measuring the quantities required to calculate the coefficient of friction



Part(ii)

1 point is earned: For a description that is consistent with the diagram in part (a)(i), in enough detail that another student could replicate the experiment

1 point is earned: For a description that is a conceptually valid method to find quantities that would allow a calculation of a friction coefficient

1 point is earned: For including a valid method for reducing experimental error

Example:

> With the block at rest on the board, slowly lift one end of board until the block just begins to slide. Measure the angle between board and table and repeat several times with block at different locations on the board (with multiple trials at each location).

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0	1	2	3	4	5	6 ✓

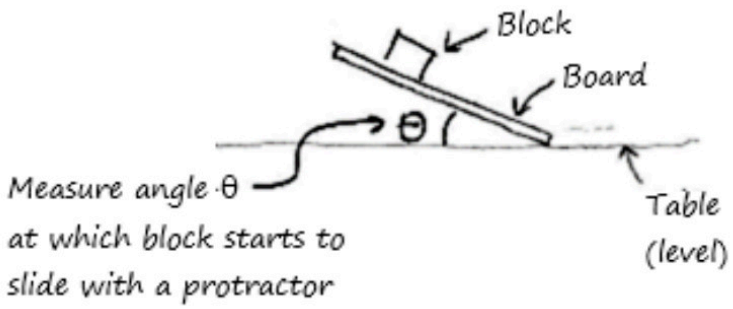
Student response earns 6 of the following 6 point(s)

Part(i)

1 point is earned: For drawing a diagram of an experimental setup to measure the coefficient of friction that is feasible in a school physics lab

1 point is earned: For indicating the measurements necessary for calculating the coefficient of friction

1 point is earned: For indicating equipment necessary for measuring the quantities required to calculate the coefficient of friction



Part(ii)

1 point is earned: For a description that is consistent with the diagram in part (a)(i), in enough detail that another student could replicate the experiment

1 point is earned: For a description that is a conceptually valid method to find quantities that would allow a calculation of a friction coefficient

1 point is earned: For including a valid method for reducing experimental error

Example:

> With the block at rest on the board, slowly lift one end of board until the block just begins to slide. Measure the angle between board and table and repeat several times with block at different locations on the board (with multiple trials at each location).

Part B

Note: In order to earn full credit for part (b), all terms (variables) must be indicated in the diagram and/or procedure of part (a).

1 point is earned: For using Newton's second law (or reasoning in terms of zero net force) in one dimension, parallel to the board's surface, either explicitly or implicitly

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1 point is earned: For using Newton's second law (or reasoning in terms of zero net force) in one dimension, perpendicular to the board's surface, either explicitly or implicitly

Note: Replacing the normal force with mg is "implicit" use of Newton's second law in the perpendicular direction for a horizontal surface. For a tilted surface, the appropriate trigonometric term should be included.

1 point is earned: For a correct derived expression of the coefficient of static friction in terms of quantities indicated in part (a)

0	1	2	3 ✓

Student response earns 3 of the following 3 point(s)

Note: In order to earn full credit for part (b), all terms (variables) must be indicated in the diagram and/or procedure of part (a).

1 point is earned: For using Newton's second law (or reasoning in terms of zero net force) in one dimension, parallel to the board's surface, either explicitly or implicitly

1 point is earned: For using Newton's second law (or reasoning in terms of zero net force) in one dimension, perpendicular to the board's surface, either explicitly or implicitly

Note: Replacing the normal force with mg is "implicit" use of Newton's second law in the perpendicular direction for a horizontal surface. For a tilted surface, the appropriate trigonometric term should be included.

1 point is earned: For a correct derived expression of the coefficient of static friction in terms of quantities indicated in part (a)

Part C

Correct answer: "The static and kinetic coefficients are not equal."

Reasoning cannot earn credit if the incorrect selection is made.

1 point is earned: For identifying group 5's results as outliers, or indicating the presence of an outlier

1 point is earned: For a conclusion that coefficients are not the same, justified by either removing the outlier or noting the coefficients are different for each group

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

Correct answer: "The static and kinetic coefficients are not equal."

Reasoning cannot earn credit if the incorrect selection is made.

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1 point is earned: For identifying group 5's results as outliers, or indicating the presence of an outlier

1 point is earned: For a conclusion that coefficients are not the same, justified by either removing the outlier or noting the coefficients are different for each group

Part D

1 point is earned: For a valid argument that indicates the coefficient of static friction is a property of the two surfaces, and is consistent with the selected answer

Example 1: Selects "Remain the same"

Referring to the equation in part (b), coefficient does not depend on mass.

Example 2: Selects "Remain the same"

The coefficient depends on the nature of the surfaces involved, not the masses or normal force of the objects involved.

Example 3: Selects "Decrease"

The increased normal force will cause smoothing of the surfaces, decreasing the coefficient of friction.

Example 4: Selects "Increase"

The increased normal force will cause the surfaces to become gouged, increasing the coefficient of friction.

0	1 ✓

Student response earns 1 of the following 1 point(s)

1 point is earned: For a valid argument that indicates the coefficient of static friction is a property of the two surfaces, and is consistent with the selected answer

Example 1: Selects "Remain the same"

Referring to the equation in part (b), coefficient does not depend on mass.

Example 2: Selects "Remain the same"

The coefficient depends on the nature of the surfaces involved, not the masses or normal force of the objects involved.

Example 3: Selects "Decrease"

The increased normal force will cause smoothing of the surfaces, decreasing the coefficient of friction.

Example 4: Selects "Increase"

The increased normal force will cause the surfaces to become gouged, increasing the coefficient of friction.

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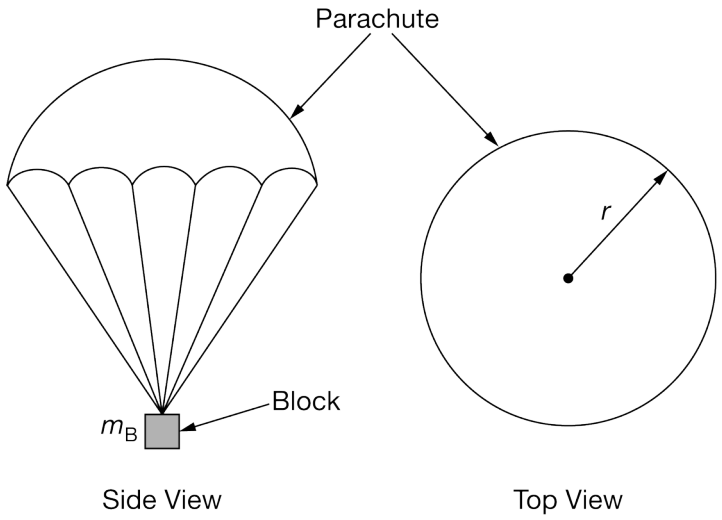
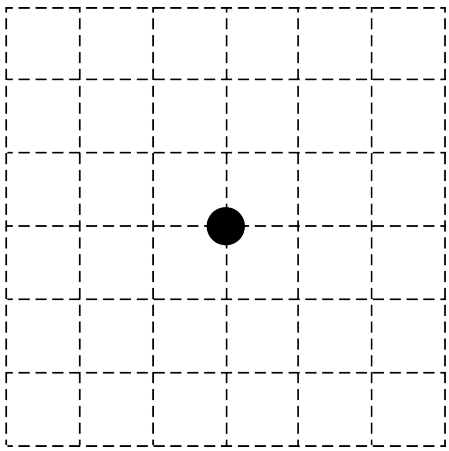
3. This question is a long free-response question. Show your work for each part of the question.



(12 points, suggested time 25 minutes)

A block of mass m_B can be attached to a parachute of negligible mass and radius r, as shown in the two views above. When falling, the parachute experiences an upward force of air resistance. This force increases as the speed of the parachute increases. The falling block and parachute speed up at first but eventually reach a constant speed called terminal speed.

(a) The dot below represents the block-parachute system when it has reached its terminal speed and is falling at this constant speed. On the dot, draw and label the forces (not components) acting on the system. Represent each force by a distinct arrow starting on, and pointing away from, the dot.
Note: Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces.



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A group of students has a collection of parachutes of different radii. They are curious about how the terminal speed depends on the radius of the parachute and find the following equation on the Internet.

$$F_{\text{air}} = Crv^2,$$

where F_{air} is the force exerted by air, r is the parachute's radius, v is the parachute's speed, and C is a constant that has appropriate units.

(b) Determine the units for C.

The units for C are _____ .

(c)

i. Based on the above equation for F_{air}, derive an equation for the system's constant terminal speed v_t in terms of r, C, physical constants, and other relevant quantities that can be measured with equipment usually found in a school physics laboratory, as appropriate. Assume the mass of the parachute is negligible and that the only nonnegligible forces acting on a block-parachute system are the block's weight $m_B g$ and the force of air resistance F_{air}.

ii. Indicate below what quantities could be measured and then graphed to determine whether the equation derived in part (c)(i) correctly relates the parachute's radius and the terminal speed of the block-parachute system.

Vertical axis: _____ Horizontal axis: _____

iii. On the axes below, sketch a graph showing the relationship between the two quantities indicated in part (c)(ii) and based on the equation derived in part (c)(i).

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(d) Design an experimental procedure the students could use to collect the data needed to test whether or not the relationship between the parachute's radius and the terminal speed is described by the graph drawn in part (c)(iii). Assume equipment usually found in a school physics laboratory is available.

In the table below, list the quantities that would be measured in your experiment. Define a symbol to represent each quantity, and also list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

Quantity to be Measured	Symbol for Quantity	Equipment for Measurement

Describe the overall procedure to be used, referring to the table. Provide enough detail so another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table. Include a simple diagram of the setup. As part of your description, be sure to explain how the students would know that they are measuring the constant terminal speed.

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Diagram of Experimental Setup Description of Experimental Procedure

(e) In another experiment, the students collect the following data showing the height of a block above the floor at different times, where the block is dropped from rest at time $t = 0$ with the attached parachute fully open.

Time (s)	Height of Block Above Floor (m)
0.00	1.35
0.10	1.30
0.20	1.18
0.30	1.01
0.40	0.82
0.50	0.63
0.60	0.43
0.70	0.23
0.80	0.03

Use the data shown above to determine the terminal speed of the block.

Part A

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

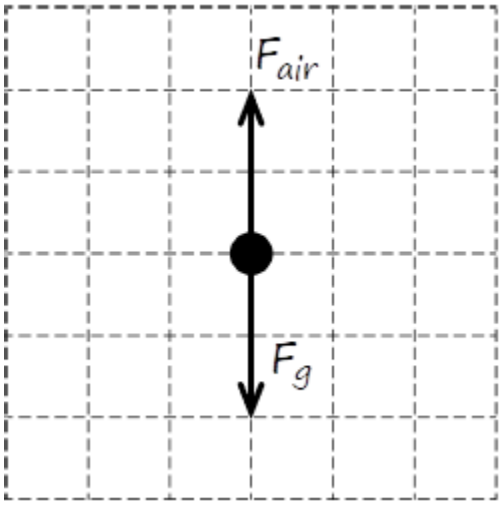
0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a downward force due to gravity, an upward force exerted by the air, and no extraneous forces.
- ☐ 1 point is earned for an upward force and a downward force that are equal in magnitude.

Scoring note: Appropriate labels for the force due to gravity include: F_{G}, F_{g}, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$, $F_{\text{Block,E}}$. The labels G or g are not appropriate labels for force due to gravity. F_{air}, "force of air", "air resistance force", "drag force", or similar labels may be used for the upward force exerted by the air.

Example Response for part (a):



Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for the correct units of kg/m^2 or equivalent.

Part C

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

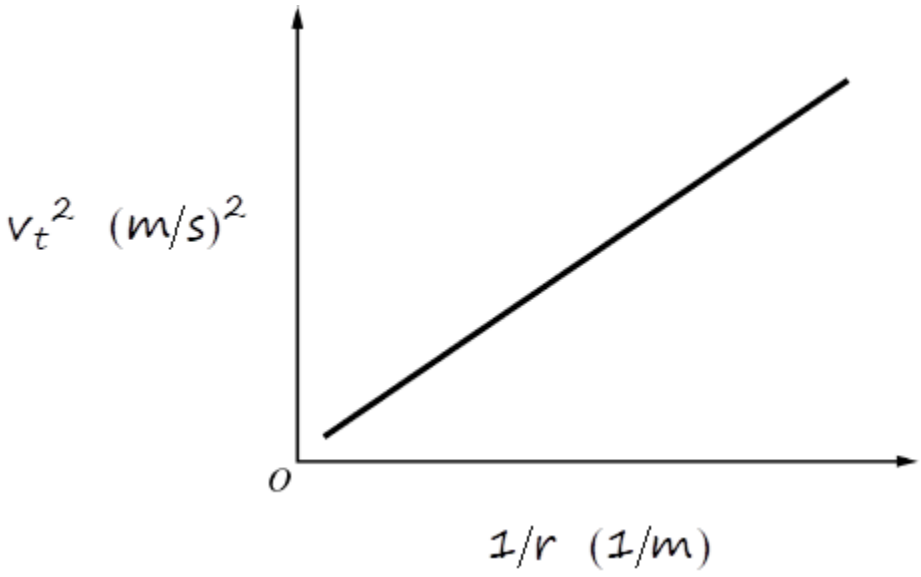
0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for recognizing that at terminal velocity, $F_{\text{air}} = m_B g$
- ☐ 1 point is earned for a correct equation for $v_t = \sqrt{\frac{m_B g}{Cr}}$
- ☐ 1 point is earned for any combination of v_t and r
- ☐ 1 point is earned for a sketch consistent with the variables listed and the equation derived above, where:
  the slope is correctly positive, negative, or zero
  AND
  the concavity is correct (concave-up, concave-down, or a straight line).

Scoring note: Graph does not have to be a straight line to earn this point.

Example Response for 4th point:



Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

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The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for describing how to use appropriate equipment to measure the radii of three different parachutes.
- ☐ 1 point is earned for describing how to use appropriate equipment to measure the speed of a falling parachute.
- ☐ 1 point is earned for describing a way to measure v_T that ensures measurements are taken when terminal speed has been reached.
- ☐ 1 point is earned for attempting to reduce uncertainty by averaging several terminal velocity measurements for the same parachute.

Example Response for part (d):

Quantity to be Measured	Symbol of Quantity	Equipment for Measurement
Radius of parachute	*R*	*Ruler*
Terminal velocity of parachute	v_t	*Motion sensor*

Diagram of Experimental Setup	Description of Experimental Procedure
	*Put the motion sensor on the floor pointed upwards. Measure the radius of a parachute with a ruler. Drop the parachute from a set height over the motion sensor. Record the velocity given by the motion sensor when the velocity becomes constant. Repeat the above measurements several times to reduce error and average the results. Repeat this process for parachutes of at least three different radii.*

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

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The response accurately includes:

- [ ] 1 point is earned for an answer in the range of $1.9\ \mathrm{m/s}$ to $2.1\ \mathrm{m/s}$

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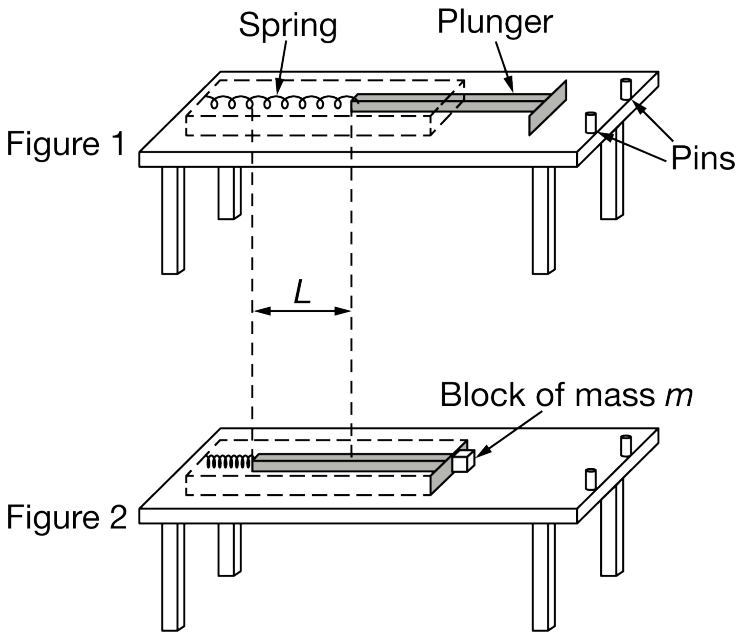
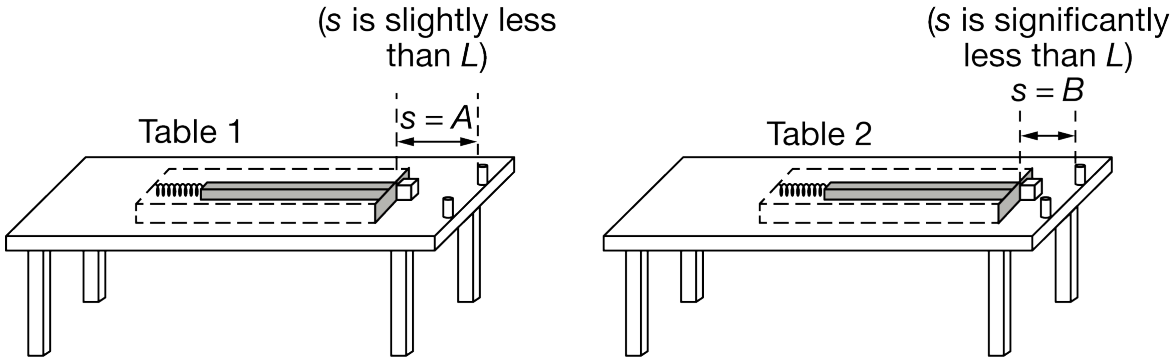
4. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A spring-loaded launcher can be securely attached to a tabletop. When the spring is at its uncompressed length, the plunger is in the position shown above in Figure 1. A small wood block can be pressed against the end of the plunger, compressing the spring a distance L, as shown in Figure 2. When the block is released, the plunger pushes the block along the tabletop. Two pins are attached to the edge of the table to prevent the plunger from extending beyond the table. When the plunger hits the pins, the block then leaves the table. There is nonnegligible friction between the block and the tabletop.



The launcher can be moved closer to or farther from the edge of the tabletop. Students set up tables 1 and 2, as shown above, with tabletops that are made of the same material. On table 1, the launcher is positioned so that the distance $s = A$ between the launcher and the edge of the tabletop is slightly less than L. On table 2, the launcher is positioned so that the distance $s = B$ between the launcher and the edge of the tabletop is significantly less than L. On both tables, the block is released from rest and loses contact with the plunger at the moment the plunger reaches the pins. Tables 1 and 2 are the same height.

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k is the spring constant of the spring.
m is the mass of the block.
L is the distance the spring is compressed.
μ (mu) is the coefficient of kinetic friction between the block and the tabletop.
s is the distance between the launcher and the edge of the tabletop.

Assume the spring is ideal, the plunger has negligible mass, the block loses contact with the plunger immediately after it reaches the end of the table, and there is non-negligible friction between the tabletops and the blocks.

(a) Without manipulating equations, explain why the block launched from table 1 could land farther from the table than the block launched from table 2 does.

(b) Does the block launched from table 1 spend more, less, or the same amount of time in the air than the block launched from table 2 does? Explain your reasoning.

(c) Consider the Earth-block system. Determine the change, if any, in the total mechanical energy of the system from the instant the block leaves the table to the instant immediately before it reaches the ground.

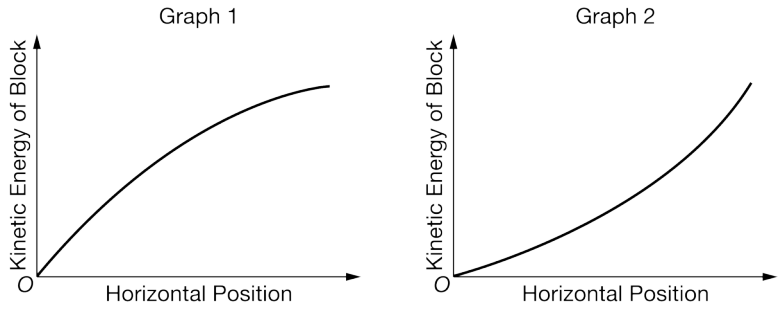
The students correctly derive an equation for the horizontal distance D from the edge of the tabletop that the block lands, in terms of s (the distance between the launcher and the edge of the tabletop):

$$D = t\sqrt{\frac{k}{m}(2Ls - s^2) - 2\mu g s},$$

where t is the time interval between the instant the block leaves the table and the instant immediately before it reaches the ground.

(d) Does this equation for D support your argument from part (a) that the block launched from table 1 could land farther from the table than the block launched from table 2? Briefly explain why or why not.

(e) The students now use the same setups to launch blocks that are identical to each other but more massive than the original blocks. The blocks are made of the same material as the original blocks. Does this change in the mass of the blocks make it more likely or less likely that the block launched from table 1 goes farther than the block launched from table 2 does? Briefly explain your reasoning.



(f) Two students each try to sketch a graph of the kinetic energy of the block as a function of its position from launch to the time the block reaches the edge of the table. Their graphs are shown above. Which graph best represents the kinetic energy of the block? Justify your selection.

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Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for applying work/energy or momentum/impulse to explain why launch speed could be greater for Table 1.
- ☐ 1 point is earned for indicating that a larger *s* enables the spring to exert a force on the block for a longer time or distance, or that a larger *s* corresponds to more elastic potential energy being transferred to the block.
- ☐ 1 point is earned for indicating that the block could gain more speed and/or kinetic energy and/or momentum on Table 1.

Example Response:

The distance s is greater for table 1 than table 2 which means that the force that the spring exerts on the block is exerted over a longer distance, allowing the force to do more work on the block. Since the spring force does more work on the block, the change in kinetic energy could be greater, meaning the block could be launched at a greater speed from table 1. The block from table 1 will travel a larger horizontal distance because it was launched with a greater horizontal speed.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating the same time.
- ☐ 1 point is earned for indicating that both blocks will have the same initial vertical speed or zero initial vertical speed.
- ☐ 1 point is earned for indicating that either:
  only the vertical motion determines the fall time,
  OR
  The vertical displacement is the same in both cases
  OR
  The vertical acceleration is the same in both cases.

Example Response:

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The blocks launched from table 1 and table 2 will spend the same amount of time in the air because they have an initial vertical velocity of zero and start from the same height above the floor.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: just “zero” earns this point.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for indicating there is no change in the total mechanical energy.

Example Response:

After the block leaves the table the total mechanical energy does not change.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for addressing the functional dependence of *D* on *s*.
- ☐ 1 point is earned for correctly indicating how a specific term in the expression changes with increasing *s*.
- ☐ 1 point is earned for indicating that an increase in s could result in an increase in *D*, or in the right-hand-side of the equation
  Note: If a student answers “no” the third point is awarded for indicating that an increase in s results in a decrease in *D*, or in the right-hand side of the equation.

Example Response:

Yes. The closer s is to L, the greater the $(2Ls - L^\wedge 2)$ *part of the equation would be. If that term of the equation is greater, D will be greater.*

OR

Yes. Because s is always less than L, Ls *is greater than* $s^\wedge 2$*. So as s increases, the equation increases.*

Part E

chapter2parta

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating "Less likely", with an attempt at an explanation.
- ☐ 1 point is earned for noting that the larger mass increases the force of friction.
- ☐ 1 point is earned for indicating that a larger frictional force will dissipate more energy (or result in a smaller net force on the block/acceleration of the block), which results in a lower launch speed, and therefore smaller distance.
  Note: Part (e) can be answered quantitatively. See alternate solution below.

It is less likely that the block in 1 will go farther. The larger mass means that the force of friction is larger, so the net force on the block is less, resulting in a lower launch speed. On table 1, the frictional force is exerted over a longer distance, and so has a bigger effect on reducing the speed compared to table 2.

OR

Less likely. The first term is multiplied by k/m; so this term decreases with bigger m. The second term doesn't change with m. So, the difference between the two terms is smaller, reducing the effect of increasing s on the distance traveled.

Part E – Alternate Solution

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

*The response accurately includes **all** of the following criteria.*

- ☐ *1 point is earned for indicating "less likely", with an attempt at an explanation.*
- ☐ *1 point is earned for addressing the functional dependence of D on m in the given equation.*
- ☐ *1 point is earned for explaining how the right side of the equation becomes smaller with reference to both terms in the substitution, therefore, D becomes smaller.*

Example Response:

Less likely. The first term is multiplied by k/m; so this term decreases with bigger m. The second term doesn't change with m. So, the difference between the two terms is smaller, reducing the effect of increasing s on the distance traveled."

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	✓ 2

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that Graph 1 is the correct graph, with an attempt at an explanation.
- ☐ 1 point is earned for correctly addressing the decreasing slope of the curve for large values of horizontal position.

Example Response:

Graph 1 is the correct graph. The slope decreases with increasing position. This makes sense because, as the spring expands, it exerts a smaller force on the block. The smaller force means that the kinetic energy increases by a smaller amount for a given change in position.

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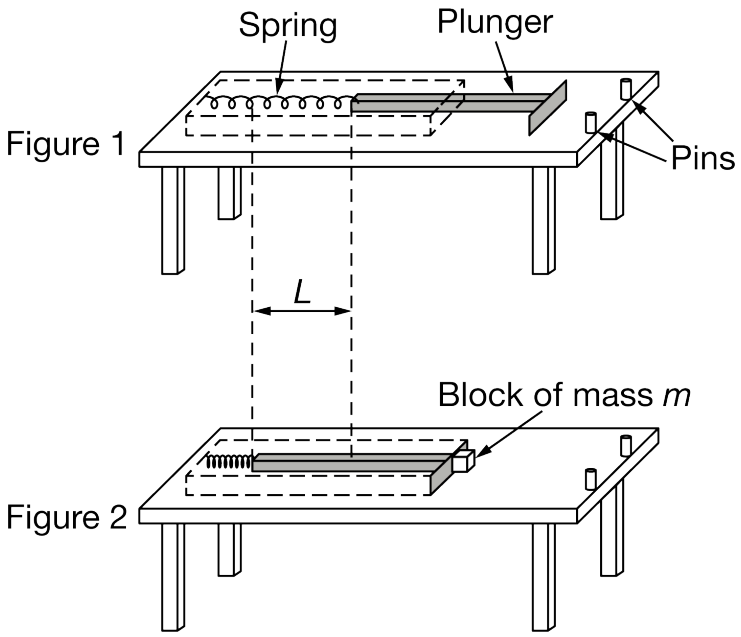
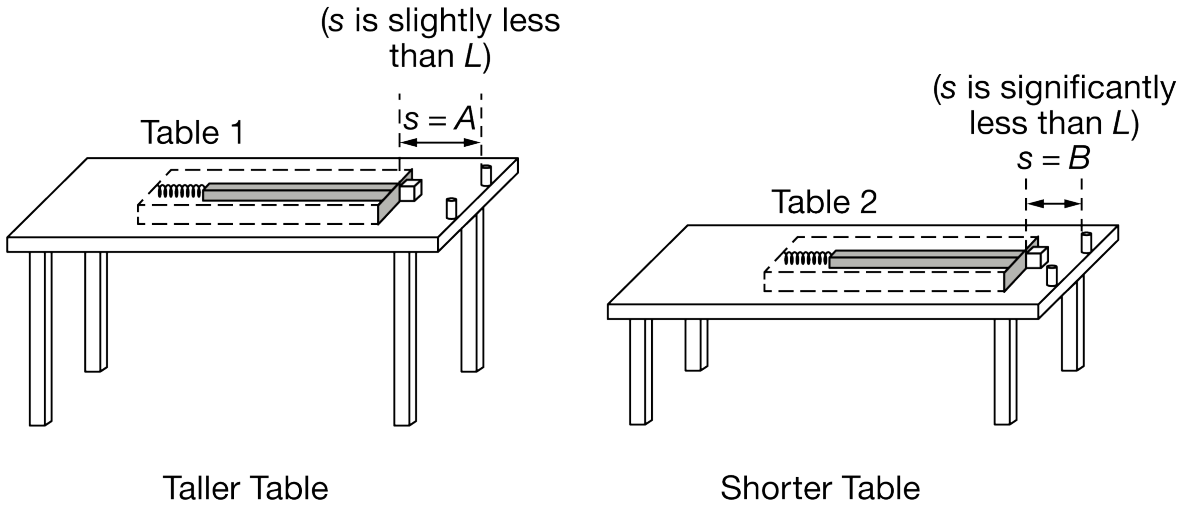
5. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A spring-loaded launcher can be securely attached to a tabletop. When the spring is at its uncompressed length, the plunger is in the position shown above in Figure 1. A small wood block can be pressed against the end of the plunger, compressing the spring a distance L as shown in Figure 2. When the block is released, the plunger pushes the block along the tabletop. Two pins are attached to the edge of the table to prevent the plunger from extending beyond the table. When the plunger hits the pins, the block then leaves the table. There is nonnegligible friction between the block and the tabletop.



The launcher can be moved closer to or farther from the edge of the tabletop. Students set up tables 1 and 2, as shown above, with tabletops that are made of the same material. On table 1, which is taller than table 2, the

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launcher is positioned so that the distance $s = A$ between the launcher and the edge of the tabletop is slightly less than L. On table 2, the launcher is positioned so that the distance $s = B$ between the launcher and the edge of the tabletop is significantly less than L. On both tables, the block is released from rest and loses contact with the plunger at the moment the plunger reaches the pins.

k is the spring constant of the spring.
m is the mass of the block.
L is the distance the spring is compressed.
μ (mu) is the coefficient of kinetic friction between the block and the tabletop.
s is the distance between the launcher and the edge of the tabletop.

Assume the spring is ideal, the plunger has negligible mass, the block loses contact with the plunger immediately after it reaches the end of the table, and there is non-negligible friction between the tabletops and the blocks.

(a) Without manipulating equations, explain why the block launched from table 2 could have a greater speed when it leaves the table than the block launched from table 1 does.

(b) Does the block launched from table 2 spend more, less, or the same amount of time in the air than the block launched from table 1 does? Explain your reasoning.

(c) Consider the system consisting of only the block. Determine the change, if any, in the total mechanical energy of the system from the instant the block leaves the table to the instant immediately before it reaches the ground.

The students correctly derive an equation for the speed v of the block when it is at the edge of the tabletop, in terms of s (the distance between the launcher and the edge of the tabletop):

$v = \sqrt{\frac{k}{m}(2Ls - s^2) - 2\mu g s}$.

(d) Does this equation for v support your argument from part (a) that the block launched from table 2 could have a larger speed when it leaves the table than the block on table 1 does? Briefly explain why or why not.

(e) The students now repeat the scenario with identical setups except that the coefficient of kinetic friction between the blocks and the table is larger. Does this change in the coefficient of friction make it more likely or less likely that the block launched from table 1 has a greater speed when it leaves the table than the block on table 2 does? Briefly explain your reasoning.

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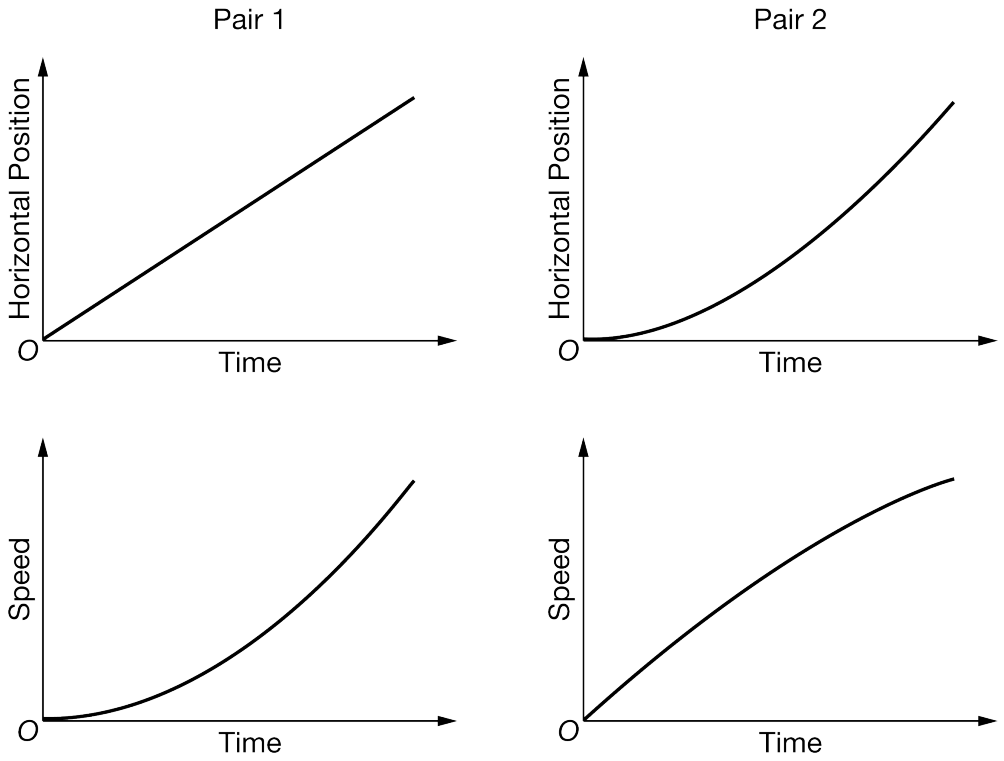

(f) Two students each try to sketch a graph of the horizontal position and speed of the block as a function of time while the block is being pushed by the plunger. Their graphs are shown above. In which pair of graphs are the two graphs consistent with each other? Justify your selection.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for attempting to apply work/energy or momentum/impulse to explain why launch speed could be greater for Table 2.
- ☐ 1 point is earned for indicating that a larger *s* for the block on Table 1 enables friction to exert a force on the block for a longer time or distance, or that a larger *s* corresponds to more energy being dissipated.
- ☐ 1 point is earned for correctly applying work/energy or momentum/impulse to explain why launch speed could be greater for Table 2.

Example Response:

The distance s is greater for table 1 than table 2 which means that the force of friction from the table can be exerted for a

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longer distance, allowing the friction force to dissipate more energy. Since the friction force dissipates more energy, the reduction in kinetic energy due to friction is greater, meaning the block will be launched at a smaller speed from table 1.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the block launched from Table 2 will spend less time in the air.
- ☐ 1 point is earned for indicating that both blocks will have the same initial vertical speed or zero initial vertical speed.
- ☐ 1 point is earned for indicating that either:
  only the vertical motion determines the fall time,
  OR
  the vertical displacement is different for the two cases.

Example Response:

The block launched from table 2 will spend less time in the air because both blocks have an initial vertical velocity of zero, but the block on table 2 starts from a lower height above the floor.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for indicating that the change is mgh.

Example Response:

mgh

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for addressing the functional dependence of *v* on *s*.
- ☐ 1 point is earned for correctly indicating how a specific term in the expression changes with increasing *s*.
- ☐ 1 point is earned for indicating that an increase in *s* could result in a decrease in *v,* or in the right-hand-side of the equation.
  Scoring note: If a student answers "no" the third point is awarded for indicating that an increase in *s* results in an increase in *v*, or in the right-hand side of the equation.

Example Response:

Yes, because the equation indicates that v could increase as s decreases like my argument in part (a). In the equation, the expression under the radical could increase when s decreases because there are two "s" terms with negative signs. If the number under the radical increases, v will increase.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating "Less likely", with an attempt at an explanation.
- ☐ 1 point is earned for noting that the larger coefficient of friction results in a larger frictional force.
- ☐ 1 point is earned for indicating that a larger frictional force will dissipate more energy (or result in a smaller net force on the block/acceleration of the block), which results in a lower launch speed.

Example Response:

It is less likely that the block in 1 will have a greater launch speed. The larger coefficient of friction means that the force of friction is larger, so the net force on the block is less, resulting in a lower launch speed. On table 1, the frictional force is exerted over a longer distance, and so has a bigger effect on reducing the speed compared to table 2.

Part E-Alternate Solution

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2	3 ✓

*The response accurately includes **all** of the following criteria.*

- ☐ *1 point is earned for indicating "less likely" with an attempt at an explanation.*
- ☐ *1 point is earned for addressing the functional dependence of v on the coefficient of friction (μ) in the given "correct" equation.*
- ☐ *1 point is earned for explaining how the right side of the equation becomes smaller with reference to both terms in the substitution, therefore v becomes smaller.*

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating pair 2, with an attempt at an explanation.
- ☐ 1 point is earned for either correctly ruling out the inconsistent position and time graphs in pair 1,
  OR
  For indicating that the speed graph in pair 2 is consistent with the position graph (the speed graph corresponds to the slope of the position graph).

Example Response:

Pair 2 has the two consistent graphs. The slope of the position graph is initially zero, and then increases, which is the behavior of the speed graph.

OR

Pair 2 has the two consistent graphs. In pair 1, the position graph is a straight line, so the speed graph should be constant.

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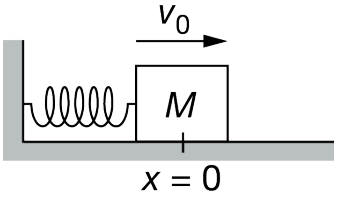
6. This question is a long free-response question. Show your work for each part of the question.

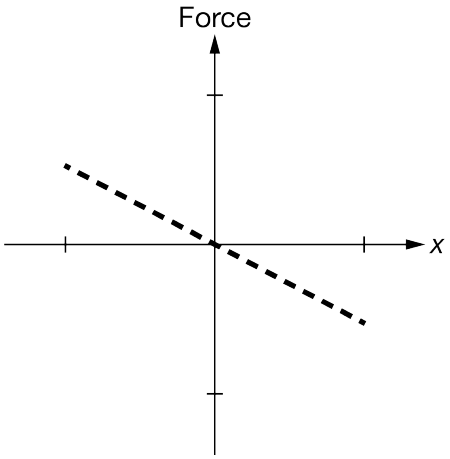


A block of mass M is attached to a spring of negligible mass and can slide on a horizontal surface along the x -direction, as shown above. There is friction present between the block and the surface. The spring exerts no force on the block when the center of the block is located at $x = 0$. At the instant shown above, the block is located at $x = 0$ and moving toward the right with speed $v = v_0$.

(a) For the instant shown above, with the block located at $x = 0$ and moving to the right, predict the direction of the net force on the block. If the net force is zero, select "The net force is zero."

____ To the left ____ The net force is zero ____ To the right

Briefly justify your prediction.



(b) The above graph indicates the force exerted on the block by the spring as a function of position x, with the positive direction toward the right. On the same graph, draw a graph of the net force on the block, as a function of position x, for situations where the block is moving toward the right.

(c) Is the graph you drew in part (b) consistent with your prediction in part (a) ?

____ Yes ____ No

Provide claims for why the answers to parts (a) and (b) are or are not consistent.

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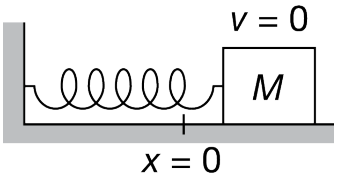

(d) In the figure above, the block of mass M has come to rest at a location to the right of the spring's equilibrium position. The spring has a spring constant k, and the coefficients of kinetic and static friction between the block and the surface are μ_k and μ_s, respectively.

i. On the dot below, which represents the block at rest and to the right of $x = 0$, draw and label the forces (not components) that act on the block. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

ii. Derive an expression for the maximum displacement possible for the block that will allow the block to still remain at rest. Express your answer in terms of M, k, μ_k, μ_s, and physical constants, as appropriate.

Part (a)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

Student response accurately includes the following criteria.

- ☐ 1 point is earned for statingthat the friction force is opposite the direction of motion.

Example Response:

["To the left" is selected.]

Because in this case the block is at x = 0, the spring force on the block is 0. The normal force and the gravitational force add to 0. Because the block is moving to the right, there is a friction force to the left. Therefore, the net force on the block is equal to the frictional force and is pointed to the left..

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Part (b)

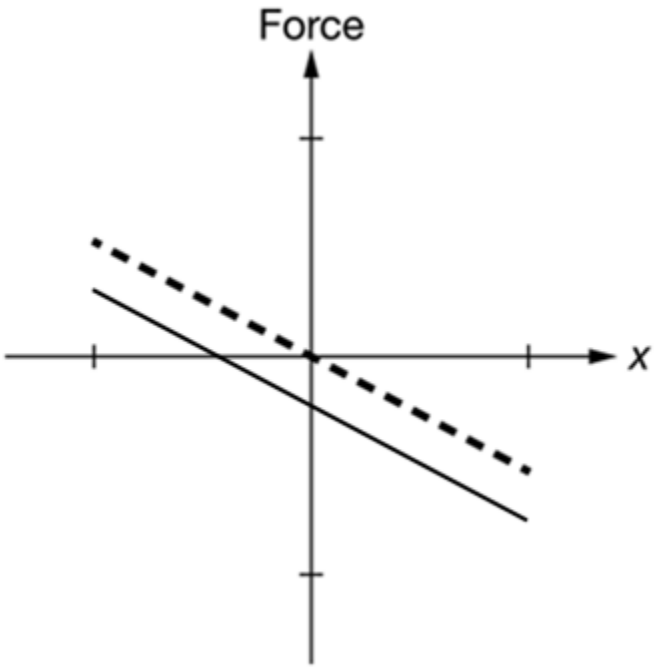
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

Student response accurately includes all of the following criteria.

- ☐ 1 point is earned for indicating a negative net force at zero displacement.
- ☐ 1 point is earned for creating a graph that is either above the spring force everywhere or below the spring force everywhere (so friction force is always in the same direction).
- ☐ 1 point is earned for creating a graph that is a straight line and parallel to the spring force graph.

Example Response:



Part (c)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: For this part, the selection of “Yes” or “No” is not graded alone. It must be consistent with the explanation in order to receive the second point.

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0	1	2 ✓

Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for providing a claim that addresses the graph at an x-value of zero.
- ☐ 1 point is earned for providing a claim that is consistent with the answers to parts (a) and (b) AND with the selection of "Yes" or "No" in part (c).

Example Response:

The graph indicates a negative net force at zero displacement, consistent with the leftward net force predicted in part (a).

Part (d)i

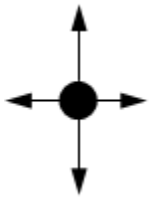
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

Student response accurately includes all of the following criteria.

- ☐ 1 point is earned for indicating a downward gravitational force and an upward normal force.
- ☐ 1 point is earned for indicating a spring force to the left.
- ☐ 1 point is earned for indicating a friction force to the right.

Example Response:



Part (d)ii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

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Student response accurately includes all of the following criteria.

- [ ] 1 point is earned for equating the spring force and the friction force: $F_{\text{spring}} = F_f$
- [ ] 1 point is earned for providing correct expressions representing the spring force and the friction force: $kx_{\max} = \mu_S F_n = \mu_S Mg$
- [ ] 1 point is earned for correctly solving for the maximum displacement in terms of the listed quantities: $x_{\max} = \mu_S Mg/k$

Example Response:

$\Sigma \overrightarrow{F} = m\overrightarrow{a}$
$\Sigma F_x = ma_x$
$F_{fs} - F_s = ma_x$
$F_{fs} - F_s = M(0)$
$F_{fs} - F_s = 0$
$F_s = F_{fs}$
$kx = \mu_s F_n$
$kx_{\max} = \mu_s F_n$
$x_{\max} = \frac{\mu_s F_n}{k}$

$\Sigma \overrightarrow{F} = m\overrightarrow{a}$
$\Sigma F_y = ma_y$
$F_n - F_g = ma_y$
$F_n - F_g = M(0)$
$F_n - F_g = 0$
$F_n = F_g$
$F_n = mg$
$F_n = Mg$

$x_{\max} = \frac{\mu_s Mg}{k}$

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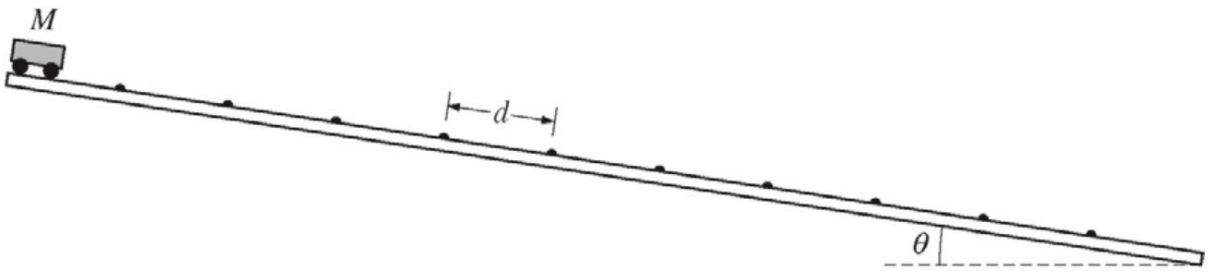
7.



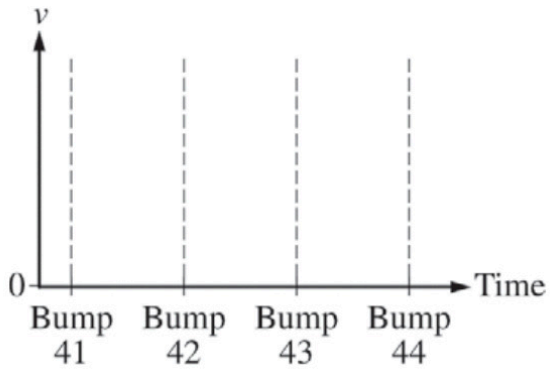
Note: Figure not drawn to scale.

A long track, inclined at an angle θ to the horizontal, has small speed bumps on it. The bumps are evenly spaced a distance d apart, as shown in the figure above. The track is actually much longer than shown, with over 100 bumps. A cart of mass M is released from rest at the top of the track. A student notices that after reaching the 40th bump the cart's average speed between successive bumps no longer increases, reaching a maximum value v_{avg}. This means the time interval taken to move from one bump to the next bump becomes constant.

a. Consider the cart's motion between bump 41 and bump 44.

   i. In the figure below, sketch a graph of the cart's velocity v as a function of time from the moment it reaches bump 41 until the moment it reaches bump 44.

   ii. Over the same time interval, draw a dashed horizontal line at $v = v_{avg}$. Label the line"v_{avg}"



b. Suppose the distance between the bumps is increased but everything else stays the same. Is the maximum speed of the cart now greater than, less than, or the same as it was with the bumps closer together?

   ____Greater than ____Less than ____The same as

   Briefly explain your reasoning.

c. With the bumps returned to the original spacing, the track is tilted to a greater ramp angle θ. Is the maximum speed of the cart greater than, less than, or the same as it was when the ramp angle was smaller?

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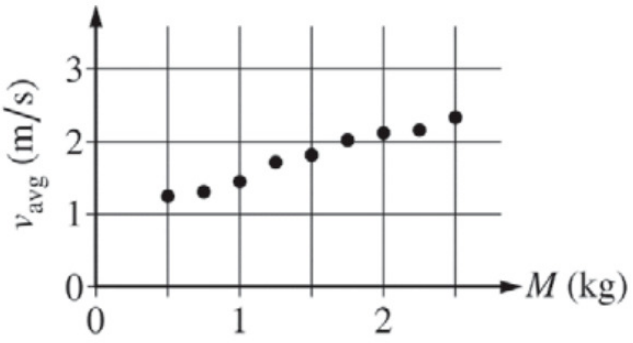
____Greater than ____Less than ____The same as

Briefly explain your reasoning.

d. Before deriving an equation for a quantity such as v_{avg} , it can be useful to come up with an equation that is intuitively expected to be true. That way, the derivation can be checked later to see if it makes sense physically. A student comes up with the following equation for the cart's maximum average speed: $v_{avg} = C\frac{Mg\sin\theta}{d}$, where C is a positive constant.

i. To test the equation, the student rolls a cart down the long track with speed bumps many times in front of a motion detector. The student varies the mass M of the cart with each trial but keeps everything else the same. The graph shown below is the student's plot of the data for v_{avg} as a function of M.



Are these data consistent with the student's equation?

____Yes ____No

Briefly explain your reasoning.

ii. Another student suggests that whether or not the data above are consistent with the equation, the equation could be incorrect for other reasons. Does the equation make physical sense?

____Yes ____No

Briefly explain your reasoning.

Part A

Part(i)

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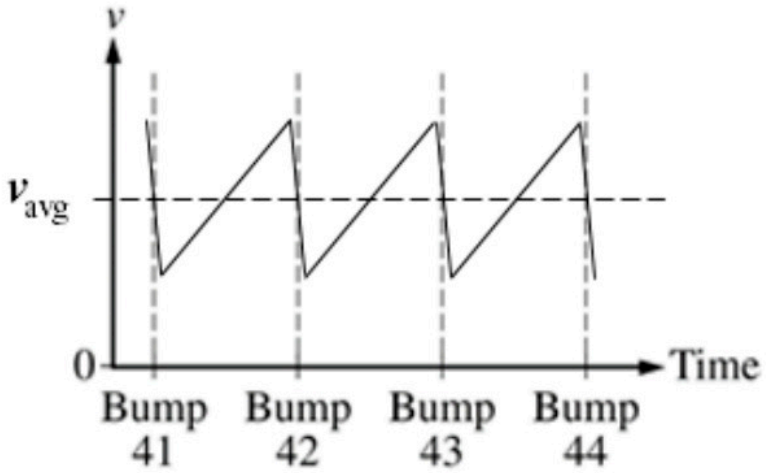

1 point is earned: For having a constant upward slope in each segment between bumps

1 point is earned: For having the velocity “reset” abruptly at each bump (i.e., as a sawtooth function, not a sinusoidal curve) to a minimum positive value that is the same for each bump

1 point is earned: For having the same maximum value in each cycle that occurs near the bump times (This point can be earned for a sinusoidal curve with peaks at the bump times.)

Part(ii)

1 point is earned: For drawing a vavg line that is horizontal and consistent with the graph drawn, even if that graph is wrong

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Part(i)

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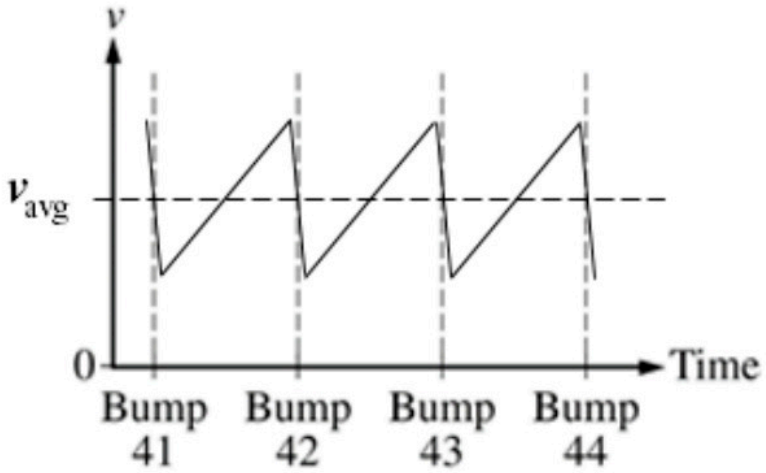

1 point is earned: For having a constant upward slope in each segment between bumps

1 point is earned: For having the velocity "reset" abruptly at each bump (i.e., as a sawtooth function, not a sinusoidal curve) to a minimum positive value that is the same for each bump

1 point is earned: For having the same maximum value in each cycle that occurs near the bump times (This point can be earned for a sinusoidal curve with peaks at the bump times.)

Part(ii)

1 point is earned: For drawing a vavg line that is horizontal and consistent with the graph drawn, even if that graph is wrong

Part B

Correct answer: Greater than

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

No points are earned if the wrong answer is selected.

1 point is earned: For indicating that there is more time between bumps

1 point is earned: For connecting that the cart has more time or more distance to accelerate between bumps

Example: The maximum speed is greater because the cart has more space (or time) to accelerate (build up speed) between bumps.

For indicating that the potential energy difference increases due to increased height between successive bumps

For relating the increase in potential energy to an increase in kinetic energy

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0	1	2 ✓

Student response earns 2 of the following 2 point(s)

Correct answer: Greater than

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

No points are earned if the wrong answer is selected.

1 point is earned: For indicating that there is more time between bumps

1 point is earned: For connecting that the cart has more time or more distance to accelerate between bumps

Example: The maximum speed is greater because the cart has more space (or time) to accelerate (build up speed) between bumps.

For indicating that the potential energy difference increases due to increased height between successive bumps

For relating the increase in potential energy to an increase in kinetic energy

Part C

Correct answer: Greater than

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

No points are earned if the wrong answer is selected.

1 point is earned: For indicating that the acceleration is greater

1 point is earned: For indicating that the component of the gravitational force increases

Example: The maximum speed is greater because the cart has more space (or time) to accelerate (build up speed) between bumps.

For indicating that the potential energy difference increases due to increased height between successive bumps

For relating the increase in potential energy to an increase in kinetic energy

0	1	2 ✓

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Student response earns 2 of the following 2 point(s)

Correct answer: Greater than

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

No points are earned if the wrong answer is selected.

1 point is earned: For indicating that the acceleration is greater

1 point is earned: For indicating that the component of the gravitational force increases

Example: The maximum speed is greater because the cart has more space (or time) to accelerate (build up speed) between bumps.

For indicating that the potential energy difference increases due to increased height between successive bumps

For relating the increase in potential energy to an increase in kinetic energy

Part D

Part (i)

Correct answer: No

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

1 point is earned: For indicating that vavg is not proportional to M

1 point is earned: For connecting the equation to the data

Examples:

> The y-intercept of the graph is not zero, but the equation indicates that it should be zero.
>
> Doubling the mass from the graph does not double vavg, but the equation indicates that it should double.
>
> If “yes” is selected, one point may be earned for explaining that vavg increases with M or that vavg looks (approximately) proportional to M for a limited portion of the data (e.g., the points at M = 1.0 kg and M = 2.0 kg).

Part (ii)

Correct answer: No

1 point is earned: For indicating that the distance dependency is incorrect

1 point is earned: For indicating that, according to the equation, greater d leads to a smaller vavg,

OR for stating or implying the contradiction between this inverse relation and the reasoning of part (b)

Credit is earned for any answer that is consistent with reasoning in part (b).

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Example:

> According to the equation, a larger d corresponds to a smaller vavg, because they are inversely proportional. But according to the reasoning of part (b), a bigger distance d between the bumps leads to a larger maximum average speed, showing that the equation is implausible.

If “yes” is selected, one point can be earned for indicating that an increase in the angle increases vavg.

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Part (i)

Correct answer: No

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

1 point is earned: For indicating that vavg is not proportional to M

1 point is earned: For connecting the equation to the data

Examples:

> The y-intercept of the graph is not zero, but the equation indicates that it should be zero.
>
> Doubling the mass from the graph does not double vavg, but the equation indicates that it should double.
>
> If “yes” is selected, one point may be earned for explaining that vavg increases with M or that vavg looks (approximately) proportional to M for a limited portion of the data (e.g., the points at M = 1.0 kg and M = 2.0 kg).

Part (ii)

Correct answer: No

1 point is earned: For indicating that the distance dependency is incorrect

1 point is earned: For indicating that, according to the equation, greater d leads to a smaller vavg,

OR for stating or implying the contradiction between this inverse relation and the reasoning of part (b)

Credit is earned for any answer that is consistent with reasoning in part (b).

Example:

> According to the equation, a larger d corresponds to a smaller vavg, because they are inversely proportional. But according to the reasoning of part (b), a bigger distance d between the bumps leads to a larger maximum average speed, showing that the equation is implausible.

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If “yes” is selected, one point can be earned for indicating that an increase in the angle increases vavg.

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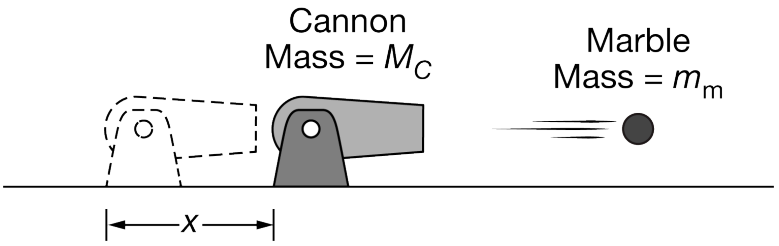
8. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch marbles of mass m_{m} horizontally from a toy cannon of mass M_C, where M_C is always greater than m_{m}, as shown above. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students use a new marble with a different mass launched at the same speed v relative to the ground. The students record the distance x the cannon slides backward for each launch.

(a) Without manipulating equations, briefly describe what happens to the distance x that the cannon slides backward as students increase the mass of the marble. Explain your reasoning.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\mathrm{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) Without manipulating or stating equations, briefly describe whether it would take more work to stop the marble or to stop the cannon after the marble has been launched. Justify your answer.

(d) Students correctly derive the following equation relating the work W done on an object initially at rest, the momentum p of the object, and the mass m of the object: $W = \frac{p^2}{2m}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble-cannon system during the short time interval in which the marble is accelerated inside the cannon and make the following claims:

Student 1: The cannon is pushing the marble forward, and the marble is pushing back with equal and opposite force. Since the work done by the cannon cancels out the work done by the marble, the mechanical energy of the system is constant.

Student 2: The mechanical energy of the system increases while the marble is being accelerated. The marble gains kinetic energy to the right, and the cannon gains an equal amount of kinetic energy to the left.

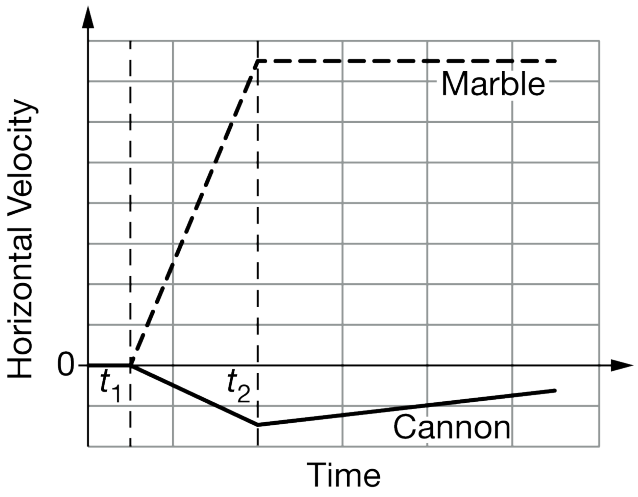
(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is

incorrect, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity of the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Provide three pieces of evidence that together explain why the magnitude of the rate of change of the cannon's velocity is less than the magnitude of the rate of change of the marble's velocity during the time interval from t_1 to t_2.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to indicate that a larger marble mass means a larger cannon speed
  OR
  For using a combination of Newton's 3rd law leading to Newton's 2nd law to indicate that a larger marble mass means a larger cannon speed.
- ☐ 1 point is earned for explaining why the larger cannon speed results in sliding a greater distance.

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Example Response:

The initial momentum of the marble-cannon system is zero, so it will be zero after the marble is launched as well. A greater marble mass then means that the backward cannon speed is greater. Since the acceleration due to friction is the same regardless of cannon speed, a greater cannon speed means that the cannon will slide a greater distance.

OR

While the marble is in the cannon, the force on the marble has the same magnitude as the force on the cannon (according to Newton's 3rd law). If the final speed of the marble is the same each time but the mass of the marble increases, the net force on the marble must increase (according to Newton's 2nd law) and, therefore, the net force on the cannon must increase as well (the force of friction remains constant) giving the cannon a larger final speed resulting in a greater distance.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence between the mass m_m and x.
- ☐ 1 point is earned for a claim that as m_m in the equation increases, x in the equation also increases.

Example Response:

Yes, the equation matches that as the mass of the marble increases the slide distance of the cannon also increases. The mass of the marble is in the numerator of the equation so if the numerator increases the slide distance will also increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim that the marble requires more work to stop, with attempt at a justification.
- ☐ 1 point is earned for a valid justification.

Example Response:

The marble requires more work to stop because the marble has a greater kinetic energy thus a greater change in kinetic

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energy in order to stop.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim consistent with the answer to (c).
- ☐ 1 point is earned for a claim that both the marble and cannon have equal momentum when the marble is launched and for recognizing that the marble's smaller mass in the denominator means it has more work done on it.

Example Response:

Yes. By conservation of momentum, the cannon and marble have the same magnitude of momentum immediately after the launch. Since p is the same, but m is smaller for the marble, the kinetic energy is greater for the marble since m is in the denominator, which means that the marble had more work done on it.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1's statement that is correct is the marble is pushing back with equal and opposite force

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1 ✓

The response accurately includes

- [ ] 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 1's statement that is incorrect is work done by the cannon cancels out the work done by the marble and mechanical energy is constant.

OR

The second statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If a student considered the cannon to include a spring launching mechanism, then the first statement would be incorrect. A student who states "none" in part (g) and "both" in part (h) earns full credit for both parts.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2's statement that is correct is mechanical energy increases while the marble is being accelerated.

OR

The first statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student's 2 statement that is incorrect is the cannon gains an equal amount of kinetic energy to the left.

OR

The second statement

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Though the slope change in the graph occurs at t_2 (rather than "in the interval from t_1 to t_2 "), a student who indicates the effect of force of friction by discussing the slope change at t_2 is awarded this point.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for recognizing that the same force is exerted on both the marble and cannon.
- ☐ 1 point is earned for indicating that the larger mass of the cannon results in its acceleration being less.
- ☐ 1 point is earned for indicating the force of friction acting opposite the cannon's direction of motion will further reduce the acceleration of the cannon.

Example Response:

Even the though the force on the cannon and the marble is the same due to Newton's third law, the cannon's acceleration is smaller. So, its velocity increases less than the marble because the cannon's mass is larger. The cannon also has a frictional force opposing its motion, which decreases the net force and therefore the acceleration.

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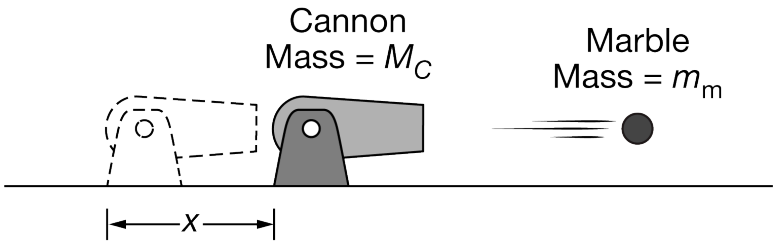
9. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch identical marbles of mass m_{m} horizontally from a toy cannon of varying mass M_C, where M_C is always greater than m_{m}, as shown above. The students can attach weights to the cannon to change M_C. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students vary the mass of the cannon and launch a marble with the same speed v relative to the ground. The students record the distance x that the cannon slides backward.

(a) Without manipulating equations, explain how the distance x that the cannon slides backward changes, if at all, when marbles are launched from more massive cannons.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\mathrm{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) If constant horizontal forces were exerted over the same distance on the cannon and on the marble after the launch, which object would require a force with a greater magnitude to completely stop it? Without manipulating or referencing equations, briefly explain your reasoning.

(d) Students correctly derive the following equation relating the force F exerted on an object initially at rest over a distance d, the mass m of the object, and the final momentum p of the object: $F = \frac{p^2}{2md}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble after it has been launched by the cannon but before it reaches the ground. Both students agree that air resistance is negligible and make the following claims.

Student 1: The mechanical energy of the marble is constant. There are no external forces exerted on the marble, so its energy does not change.

Student 2: The mechanical energy decreases as the marble falls. This happens because the gravitational force is doing work on the marble.

(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

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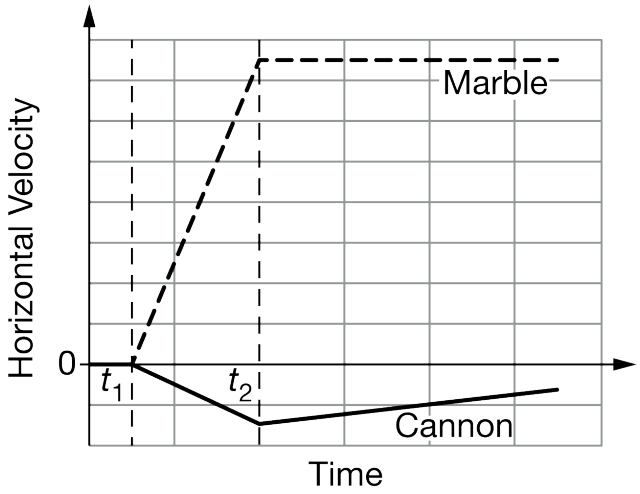
(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity of the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Describe the motion of the center of mass of the marble-cannon system during the time interval from t_1 to t_2. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Students may earn the points for a correct discussion regarding the force of friction. A more massive cannon leads to a greater normal force increasing the force of friction. This would cause the net force on the cannon to the left to be smaller while the marble was in the cannon (giving it an initially smaller speed once the marble has left the cannon) resulting in a smaller distance traveled.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to indicate that a larger cannon mass means a smaller cannon speed

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OR

For using a combination of Newton's 3rd law leading to Newton's 2nd law to indicate that a larger cannon mass means a smaller cannon speed.

☐ 1 point is earned for explaining why the smaller cannon speed results in sliding a shorter distance.

Example Response:

The initial momentum of the marble-cannon system is zero, so it will be zero after the marble is launched as well. A more massive cannon then means that the backward cannon speed is less. A smaller cannon speed means that the cannon will slide a shorter distance.

OR

While the marble is in the cannon, the force on the marble has the same magnitude as the force on the cannon (according to Newton's 3rd law). If the mass of the marble stays constant but the mass of the cannon increases, the cannon will have smaller acceleration and smaller speed (according to Newton's 2nd law) resulting in a smaller distance.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

☐ 1 point is earned for using functional dependence between the mass and the distance x.

☐ 1 point is earned for a claim that as M_c in the equation increases, x in the equation decreases.

Example Response:

Yes, the equation matches the prediction that as the mass of the cannon increases the distance the cannon slides backwards decreases. From the equation, the mass of the cannon and the distance it slides backwards are inversely proportional. The mass of the cannon is in the denominator of the equations so as the mass increases the distance the cannon slides backwards will decreases.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

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- [ ] 1 point is earned for a claim that it would take more force to stop the marble, with an attempt at a justification.
- [ ] 1 point is earned for a valid justification using either work/energy or momentum.

Example Response:

It would take more force to stop the marble because the change in kinetic energy to stop the marble is greater so the work on the marble needs to be greater.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for a claim consistent with the answer to (c).
- [ ] 1 point is earned for a claim that both the marble and cannon have equal momentum when the marble is launched and for recognizing that the marble's smaller mass in the denominator means it requires more force to stop.

Example Response:

Yes. By conservation of momentum, the cannon and marble have the same magnitude of momentum immediately after the launch. Since p is the same, but m is smaller for the marble, the force is greater for the marble since m is in the denominator.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for a statement of "None".

Example Response:

None

Part F

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating both statements are incorrect.

Example Response:

Both parts of Student 1's statement are incorrect.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

The gravitational force is doing work on the marble

OR

The second statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

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Example Response:

The mechanical energy decreases as the marble falls.

OR

The first statement

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating the center of mass moves to the right.
- ☐ 1 point is earned for indicating that there is an external force on the system due to friction.
- ☐ 1 point is earned for indicating that the net force on the system is needed to move the center on mass.

Example Response:

During the launch, the only horizontal external force exerted on the marble-cannon system is the frictional force exerted on the cannon. The friction force is directed to the right, so the net force on the system is to the right. This means the center of mass of the system is accelerating to the right.

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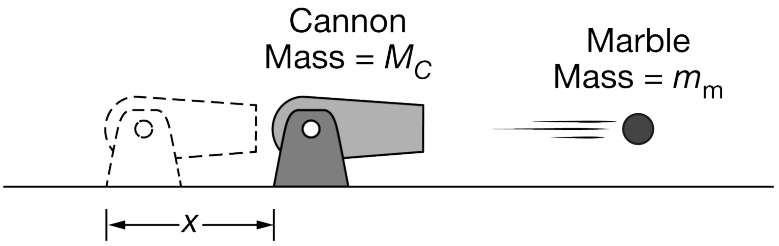
10. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch identical marbles of mass m_{m} horizontally from a toy cannon of mass M_C, where $M_C > m_{\mathrm{m}}$, as shown above. The cannon can be adjusted to change the launch speed v of the marble relative to the ground. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students vary the launch speed v and record the distance x the cannon slides backward for each launch speed.

(a) Without manipulating equations, briefly describe what happens to the distance x that the cannon slides backward as students increase the launch speed. Explain your reasoning.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\mathrm{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) Without manipulating or stating equations, briefly describe whether the cannon or the marble has more kinetic energy immediately after the marble leaves the cannon. Justify your answer.

(d) Students correctly derive an equation relating the kinetic energy K, the momentum p, and the mass m of an object: $K = \frac{p^2}{2m}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble after it has been launched by the cannon but before it reaches the ground. Both students agree that air resistance is negligible and make the following claims.

Student 1: There are two external forces exerted on the marble while it is in the air, the gravitational force and the force exerted by the cannon. Therefore, the mechanical energy of the marble changes while it is in the air.

Student 2: The cannon does not exert a force on the marble after the launch. Therefore, the mechanical energy of the marble is constant.

(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

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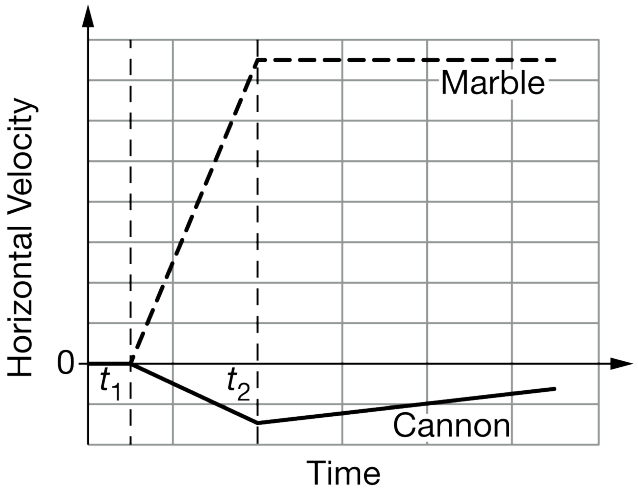
(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity for the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Describe how to find the change in the distance between the center of mass of the cannon and the center of mass of the marble from time t_1 to time t_2. Assume that before time t_1 the center of mass of the marble is at the same position as the center of mass of the cannon.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to indicate that a larger marble speed means a larger cannon speed
  OR
  For using a combination of Newton's 3rd law leading to Newton's 2nd law to indicate that a larger marble speed means a larger cannon speed.
- ☐ 1 point is earned for correctly explaining why the larger cannon speed results in sliding a greater distance.

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Example Response:

The initial momentum of the marble-cannon system is zero, so it will be zero after the marble is launched as well. A greater marble speed then means that the backward cannon speed is greater. Since the acceleration due to friction is the same regardless of cannon speed, a greater cannon speed means that the cannon will slide a greater distance.

OR

While the marble is in the cannon, the force on the marble has the same magnitude as the force on the cannon (according to Newton's 3rd law). If the mass of the marble stays constant but the final speed of the marble increases, the net force on the marble must increase (according to Newton's 2nd law) and, therefore, the net force on the cannon must increase as well (the force of friction remains constant) giving the cannon a larger final speed resulting in a greater distance.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence.
- ☐ 1 point is earned for a claim that as v in the equation increases, x in the equation also increases.

Example Response:

Yes, the equation matches the prediction that as the velocity of the marble increases the distance the cannon moves backward also increases. In the equation as velocity increases the distance would increase. Since the velocity is in the numerator of the equation, as the numerator of a fraction increases the distance x would increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim that the marble has more kinetic energy, with an attempt at a justification.
- ☐ 1 point is earned for a valid justification.

Example Response:

By conservation of momentum, mass and velocity are inversely proportional. Since velocity is squared when determining

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kinetic energy, a greater speed increases the kinetic energy by a larger factor than a smaller mass decreases the energy.

OR

The cannon and marble exert forces of equal magnitude on each other. However, the marble will move a greater distance than the cannon while these forces are exerted. So, the marble has more work done on it, and therefore a greater kinetic energy.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim consistent with the answer to (c).
- ☐ 1 point is earned for a claim that both the marble and cannon have equal momentum when the marble is launched and for recognizing that the marble's smaller mass in the denominator means it has more kinetic energy.

Example Response:

Yes. By conservation of momentum, the cannon and marble have the same magnitude of momentum immediately after the launch. Since p is the same, but m is smaller for the marble, the kinetic energy is greater for the marble since m is in the denominator.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

The correct statement from student 1 is that mechanical energy of the marble changes while it is in the air

OR

The second statement

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Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for an incorrect statement and no correct statements.

Example Response:

The incorrect statement for student 1 is that the cannon exerts a force on the marble while it is in the air

OR

The first statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

The correct statement for student 2 is that the cannon no longer exerts a force on the marble

OR

The first statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

The incorrect statement for student 2 is that the mechanical energy of the marble is constant

OR

The second statement

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for recognizing that displacement can be calculated by finding the area under a *v* vs. *t* graph.
- ☐ 1 point is earned for recognizing that the total displacement will be represented by combining the areas under each *v* vs. *t* graph.
- ☐ 1 point is earned for indicating that, for the cannon, either the area (which is negative) is to be subtracted
  OR
  the absolute value of the area is to be added, to the area under the marble graph.

Example Response:

The area under the velocity vs. time graph represents the displacement of an object. To find the change in the distance, subtract the areas of the two graphs between t_1 *and* t_2.

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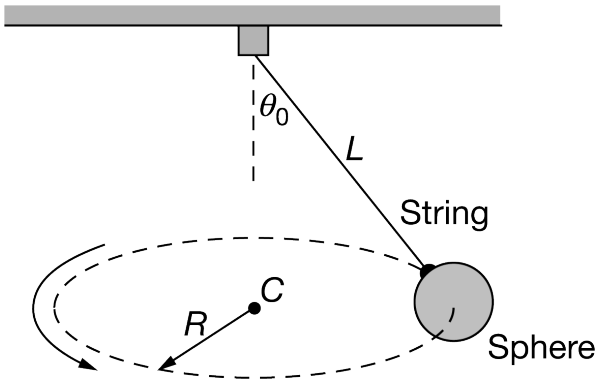
11. This question is a long free-response question. Show your work for each part of the question.



(12 points, suggested time 25 minutes)

A small sphere of mass M is suspended by a string of length L. The sphere is made to move in a horizontal circle of radius R at a constant speed, as shown above. The center of the circle is labeled point C, and the string makes an θ_0 with the vertical.

(a) Two students are discussing the motion of the sphere and make the following statements.

Student 1: None of the forces exerted on the sphere are in the direction of point C, the center of the circular path. Therefore, I don't see how there can be a centripetal force exerted on the sphere to make it move in a circle.

Student 2: I see another problem. The tension force exerted by the string is at an angle from the vertical. Therefore, its vertical component must be less than the weight Mg of the sphere. That means the net force on the sphere has a downward vertical component, and the sphere should move downward as well as moving around in a circle.

i. What is one aspect of Student 1's reasoning that is incorrect?

ii. What is one aspect of Student 2's reasoning that is incorrect?

(b)

i. Derive an equation for the magnitude of the net force exerted on the sphere. Express your answer in terms of M, θ_0, and physical constants, as appropriate.

ii. Describe one aspect or step in your derivation of part (b)(i) that can be correctly linked to your answer to either part (a)(i) or part (a)(ii).

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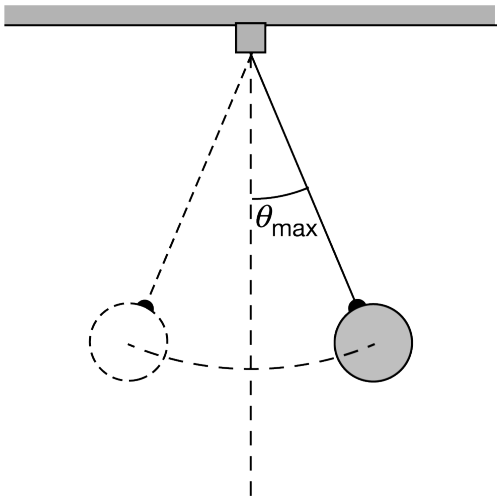
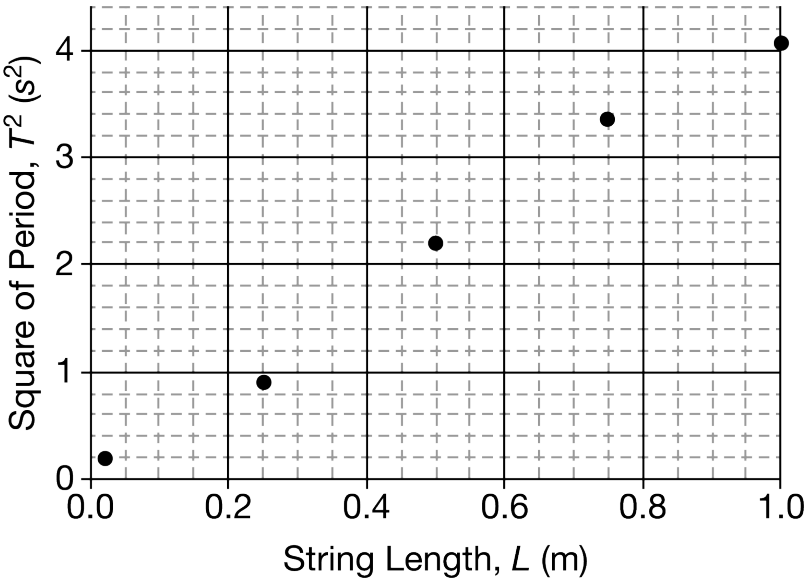


Instead of moving in a horizontal circle, the sphere now moves in a vertical plane so that it is a simple pendulum, as shown above. The maximum angle θ_{max} that the string makes from the vertical can be assumed to be small. The graph below shows data for the square of the pendulum period T as a function of string length L.



(c) On the graph above, draw a best-fit line for the data. Then use the line to calculate a numerical value for the gravitational acceleration g.

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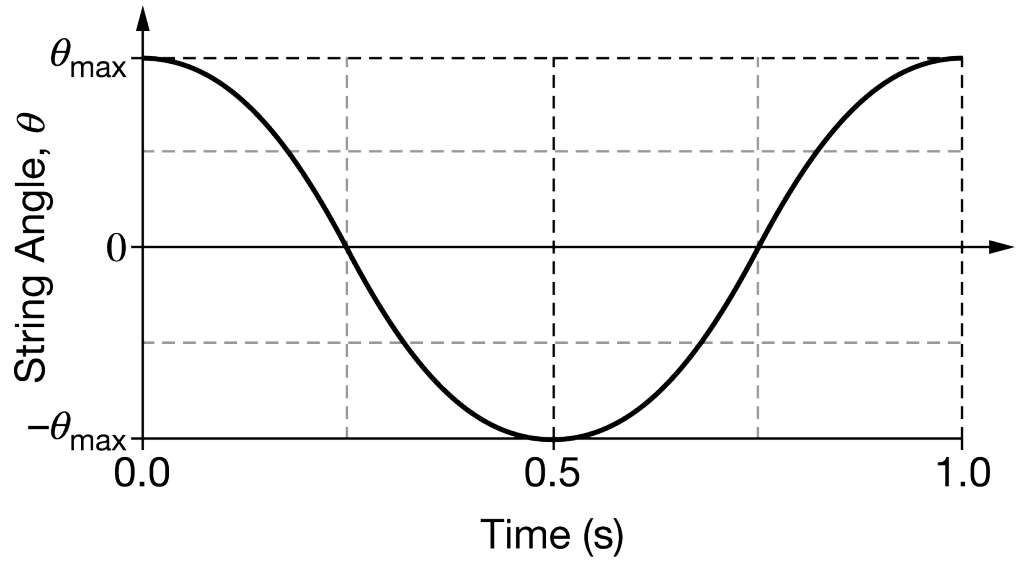
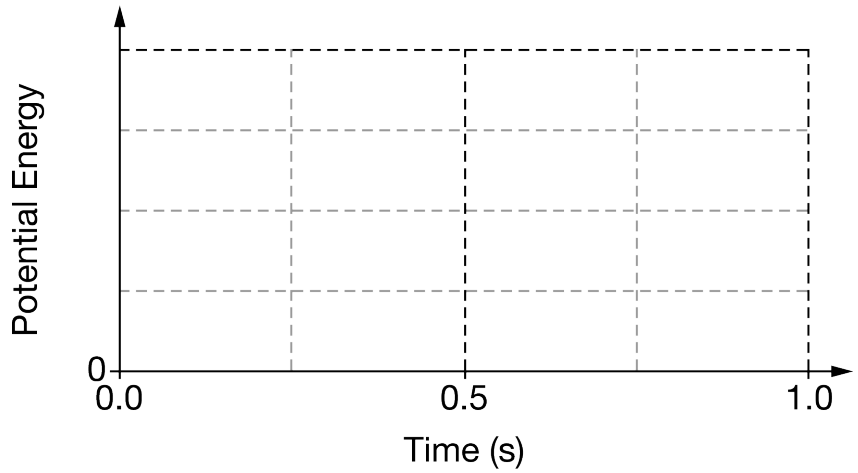


(d) The graph above shows the angle θ from the vertical as a function of time for the pendulum. On the axes below, sketch a graph of the gravitational potential energy of the sphere-Earth system for the same time interval. Take the zero of potential energy to be when the potential energy has its minimum value.



(e) As the sphere swings back and forth, it must also rotate a small amount during each swing. The figures below indicate the direction that the sphere rotates as it is swinging in each direction.

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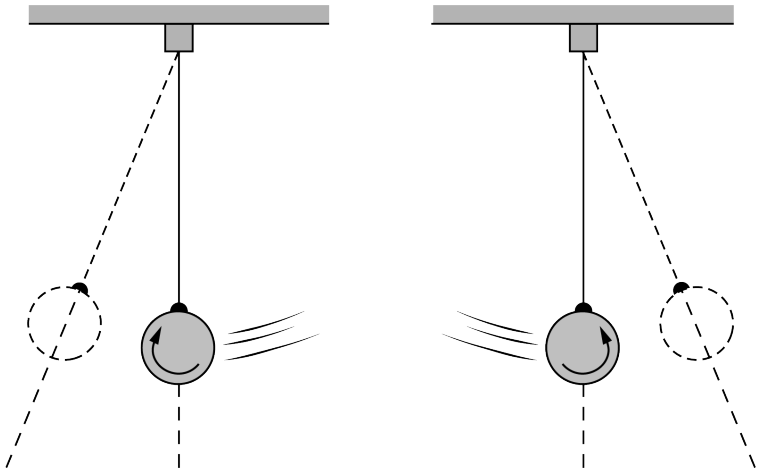

In order for the sphere's rotation to change direction, a torque must be exerted on the sphere. When the sphere is at its maximum rightward displacement, what is the direction of the torque exerted on the sphere with respect to the point of attachment between the sphere and string?

_____ Clockwise _____ Counterclockwise

Briefly state why the torque is in the direction you indicated.

Part A(i)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes a claim (explicit or implied) that the sphere can have a centripetal force without any individual force pointing toward the center of its circular path.

Example Response:

The centripetal force is provided by the net force on an object.

The tension force from the string has a horizontal component, which provides the centripetal force.

Part A(ii)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

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The response accurately includes a claim (explicit or implied) that the tension force is or can be larger than Mg

Example Response:

The tension force is larger than Mg.

The tension force is larger than Mg, allowing the vertical component to be equal to Mg and the net force to have zero vertical component, as it must.

Part B(i)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes all three of the following criteria:

- ☐ Using Newton's second law for vertical force components.
- ☐ Writing the horizontal component of string tension in terms of angle.
- ☐ Substituting a correct tension force and writing an answer in terms of the stated quantities.

Example Response:

$\sum F_y = 0$
$F_T \cos\theta - Mg = 0$ or $F_T \cos\theta = Mg$

$F_{\text{net}} = F_x = F_T \sin\theta$

$F_T \cos\theta = Mg,\ \text{so}\ \ F_T = Mg/\cos\theta$
$F_{\text{net}} = F_T \sin\theta = \left(\frac{Mg}{\cos\theta}\right)\sin\theta\ \ \text{or}\ \ Mg\tan\theta$

Part B(ii)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes a correct statement linking the derivation to the answer in either (a)(i) or (a)(ii).

Example Response:

The net force F_{net} is in the horizontal direction and equals the horizontal component $F_T \sin\theta$, as indicated by the

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statement $F_{\text{net}} = F_T \sin\theta$ in the derivation.

The tension force was shown to be $F_T = Mg/\cos\theta$, which is greater than Mg as stated in the answer to (a)(ii).

Part C

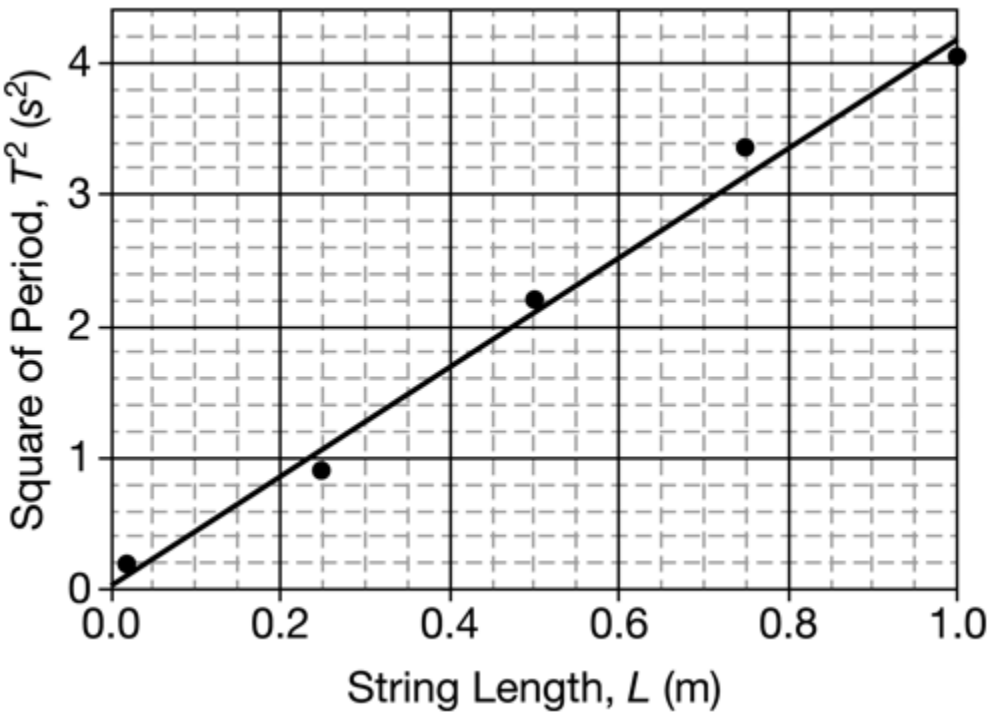
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes all three of the following criteria:

- ☐ Drawing a reasonable straight line of best fit.
- ☐ Correctly calculating the slope or its inverse, using points on the line drawn.
- ☐ A calculation of *g* consistent with the calculated slope or slope inverse.

Example Response:



$\text{Slope} = \frac{\Delta(T^2)}{\Delta L} = \frac{4.15-1.3}{1.0-0.3} = 4.07\ \text{s}^2/\text{m}$

$T = 2\pi\sqrt{L/g}$ so $T^2 = 4\pi^2 L/g$

$\text{Slope} = 4\pi^2/g$

$g = 4\pi^2/\text{Slope} = 4\pi^2/\left(4.07\ \text{s}^2/\text{m}\right) = 9.7\ \text{m/s}^2$

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Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

The maxima should be equal in energy but may have any energy value.

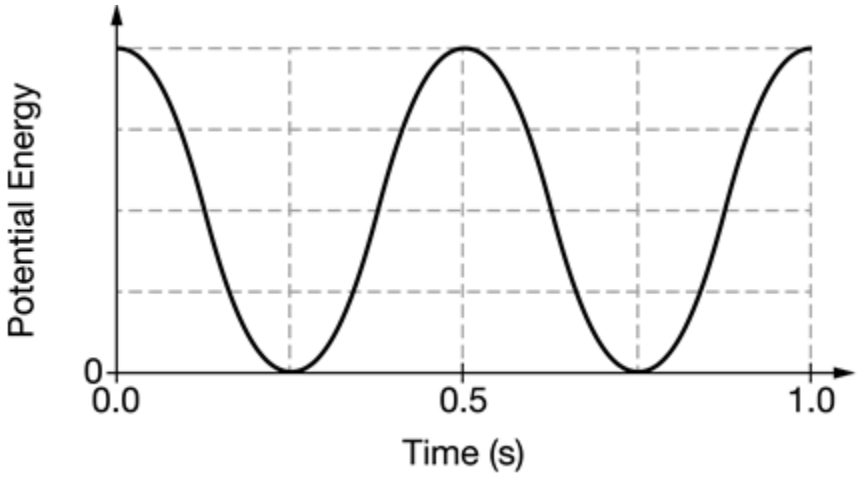
A graph with maxima at 0 and 1 s, and a minimum at 0.5 s, can earn the second point only.

0	1	2 ✓

The response accurately includes both of the following criteria:

- ☐ A graph with equal maxima at 0, 0.5 s, and 1 s, and minima of zero at 0.25 s and 0.75 s.
- ☐ A graph that has even symmetry (mirror symmetry) about time $t = 0.5$ s.

Example Response:



Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If the incorrect selection is made, the response is not graded.

0	1 ✓

The response accurately includes a correct selection AND correct reasoning in terms of the changing direction of the sphere's rotation.

Example Response:

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The correct answer is clockwise.

At the sphere’s maximum rightward displacement, the sphere’s rotation is changing from counterclockwise (swinging to the right) to clockwise (swinging to the left). So the torque must be clockwise.
At the sphere’s maximum rightward displacement, the gravitational force (taken to act at the center of the sphere) exerts a clockwise torque about the point of attachment to the string.

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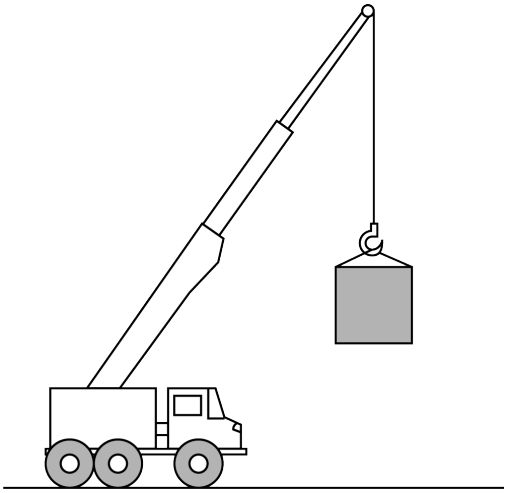
12. This question is a long free-response question. Show your work for each part of the question.

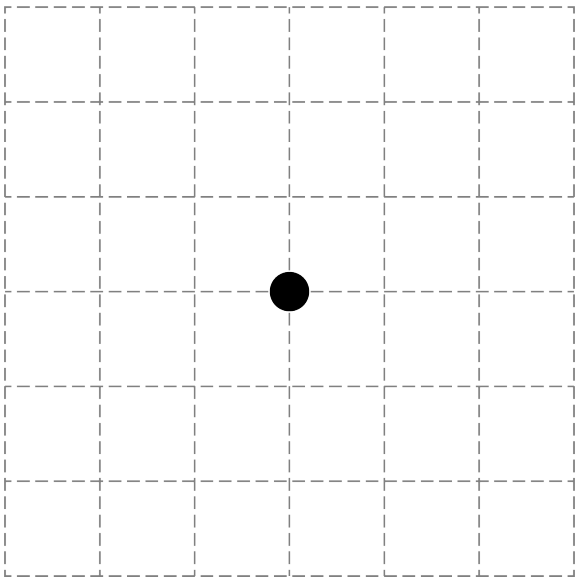


(12 points, suggested time 25 minutes)

A crane and box are both initially at rest. At time $t = 0\ \text{s}$, the crane begins to drive forward at a constant speed of $0.5\ \frac{\text{m}}{\text{s}}$, while also lifting the box with an upward acceleration of $1\ \frac{\text{m}}{\text{s}^2}$. The box does not swing while being lifted by the crane.

(a) On the dot below, which represents the box, draw and label the forces (not components) that are exerted on the box during the scenario described above. Represent each force by a distinct arrow starting on, and pointing away from, the dot.

Note: Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces.



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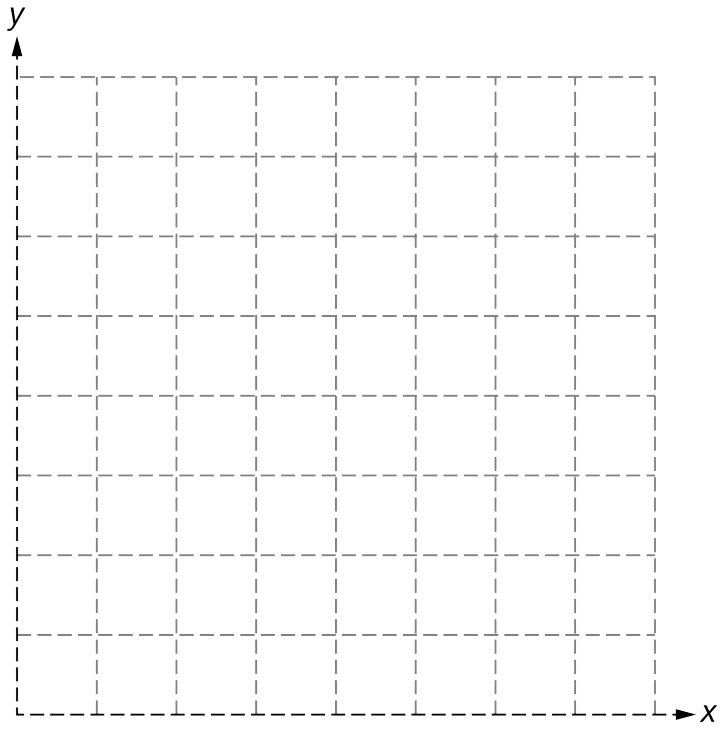
(b) On the grid below, sketch the shape of the path taken by the box as it is lifted by the crane as viewed by a stationary observer.



Justify the shape of the path you drew above.

(c) Assume that, at $t = 0$ s, the horizontal position of the box is $x = 0$ and it has a vertical position of $y = 0$. Derive an equation that describes the vertical position y of the box as a function of the horizontal position x of the box.

(d) Does your equation from part (c) agree with your sketch in part (b)? Justify your response.

At time $t = t_1$, the crane begins to accelerate forward, reaching a new constant velocity at time $t = t_2$. The box accelerates upward during the entire time interval $t_1 < t < t_2$.

(e) The dot below represents the box during the time interval $t_1 < t < t_2$. On the dot, draw an arrow indicating the direction of the net force exerted on the block during this interval. The arrow should start on, and point away from, the dot.

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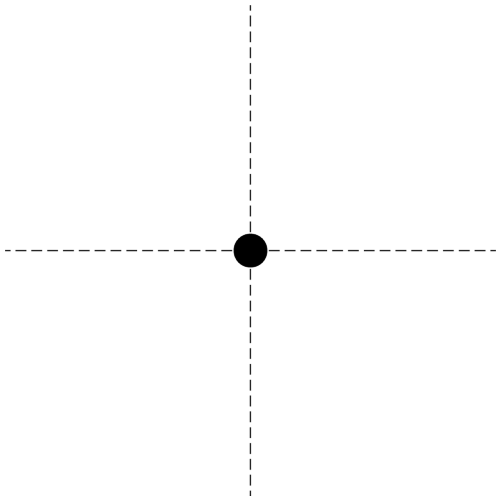

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Scoring notes:

· G or g are not acceptable labels for the force of gravity

· A maximum of two points can be earned if there are any extraneous forces.

0	1	2	3 ✓

The response accurately includes **one** of the following criteria:

- ☐ For an arrow directed upwards labeled Tension (or F_T)
- ☐ For an arrow directed downwards labeled Force of Gravity (or mg, or F_g).
- ☐ For an arrow labeled tension that is larger than the arrow labeled Force of Gravity.

Example Response:

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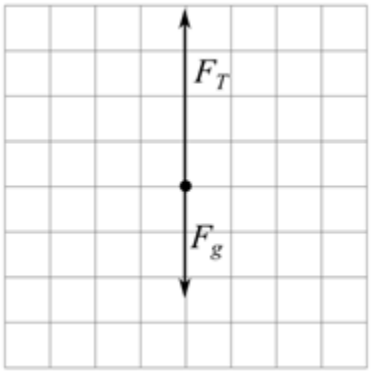


Part B

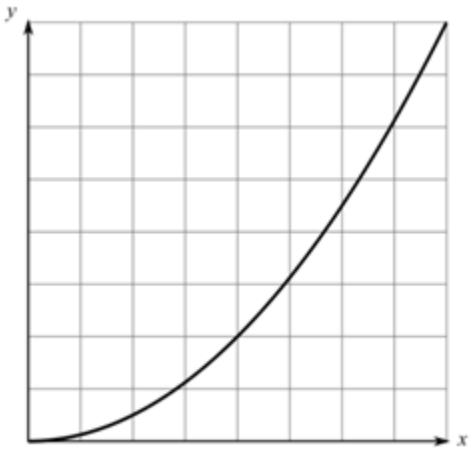
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

- ☐ For a sketch showing a non-linear path
- ☐ For a sketch that is concave up
- ☐ For a justification that addresses that the box is moving at a constant speed in the horizontal direction and accelerating in the vertical direction which leads to a concave up parabola.

Example Response:



Since the box is moving at a constant speed in the horizontal direction and accelerating in the vertical direction, the path that the box takes should look like a concave up parabola

Part C

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

- ☐ For a kinematic equation correctly relating the horizontal position of the box to the horizontal component of the speed of the box and the time
  $v_x = \frac{x}{t}$
  $0.5\ \text{m/s} = \frac{x}{t}$
  $t = \frac{x}{0.5\ \text{m/s}}$
- ☐ For a kinematic equation correctly relating the vertical position of the box as a function of the vertical acceleration of the box and the time
  $y = \frac{1}{2}a_y t^2$
  $y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)t^2$
- ☐ For correctly combining the vertical and horizontal motions of the box into a single equation.
  $y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)\left(\frac{x}{0.5\ \text{m/s}}\right)^2$
  $y = \left(2\ \text{m}^{-1}\right)x^2$

Example Response:

$v_x = \frac{x}{t}$
$0.5\ \text{m/s} = \frac{x}{t}$
$t = \frac{x}{0.5\ \text{m/s}}$

$y = \frac{1}{2}a_y t^2$
$y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)t^2$

$y = \frac{1}{2}\left(1.0\ \text{m/s}^2\right)\left(\frac{x}{0.5\ \text{m/s}}\right)^2$
$y = \left(2\ \text{m}^{-1}\right)x^2$

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

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The response accurately includes **both** of the following criteria.

- ☐ For an attempt to use functional dependence to describe how the equation and the sketch in part B relate
- ☐ For correctly stating that the vertical position is proportional to the horizontal position squared and so the shape of the path traveled by the box should be a parabola.

Example Response:

Yes. My equation in part (c) describes a parabola with a minimum at the origin, which matches my sketch in part (b).

Part E

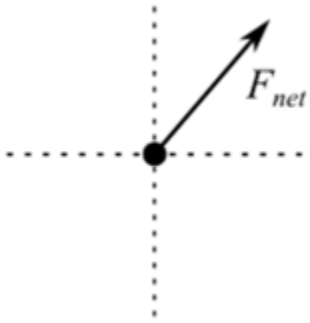
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes the following criteria:

- ☐ One point for an arrow labeled ΣF or net force, pointing upwards and to the right.

Example Response:



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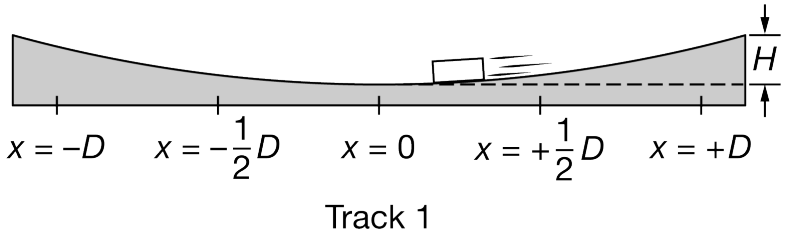
13. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

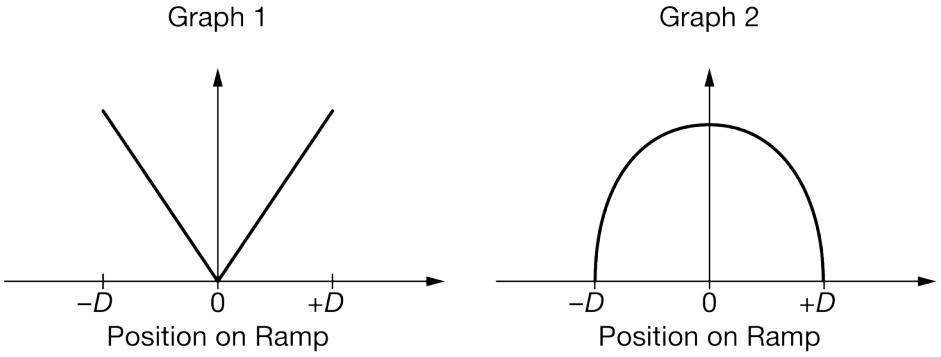


Track 1

Note: Figure not drawn to scale.

A block slides back and forth without any appreciable loss of energy on a track with a slight circular curvature, as shown above. The maximum height H of the track at each end is much smaller than D.

The center of mass of the block oscillates between $x = -D$ and $x = +D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. The block undergoes two full oscillations in 8.0 seconds.



(a) Which of the graphs above, Graph 1 or Graph 2, could represent the magnitude of the horizontal component of the block's acceleration as a function of its position on the ramp, and which graph could represent the magnitude of the block's momentum as a function of its position on the ramp? Justify both your choices with physics principles.

(b) For the system consisting of only the block, describe how the mechanical energy of the system changes, if at all, as the block moves from $x = 0$ to $x = -D$. Justify your answer.

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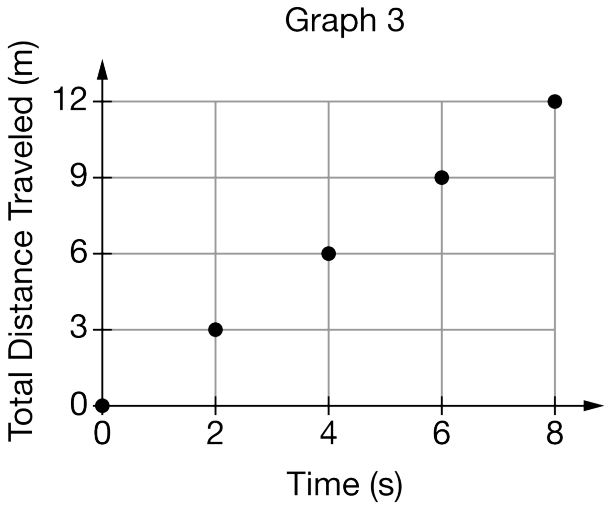
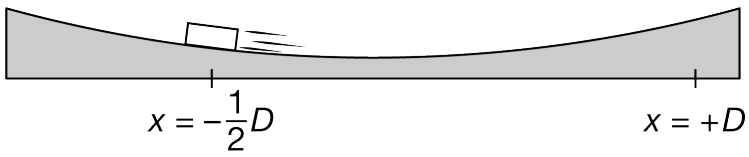


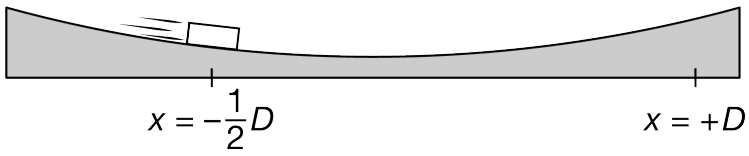
(c) Graph 3 above shows the total horizontal distance traveled by the block as a function of time as it completes two oscillations. Using the graph, determine the distance D between the midpoint of the ramp and the maximum position of the block on the ramp. Explain how you arrived at your answer.

(d) Suppose the entire setup was moved to the Moon, where the gravitational field strength is less than that on Earth. The center of mass of the block again oscillates between $x = -D$ and $x = +D$. Would the period of the block's oscillation on the Moon be greater than, less than, or equal to the period of oscillation on Earth? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{D^2}{2Hg}}$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?



Block on Track 2 at Time $t = T_1$



Block on Track 2 at Time $t = T_2$

Note: Figures not drawn to scale.

(f) Track 2 is identical to Track 1 except there is nonnegligible friction between the block and Track 2. The block

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is released from rest on Track 2 at position $x = +D$. At time $t = T_1$, the block moves to the left, passing through position $x = -\frac{1}{2}D$, as shown above. After the block reverses direction near $x = -D$, the block moves to the right, passing through position $x = -\frac{1}{2}D$ at time $t = T_2$. At which time, T_1 or T_2, if either, is the magnitude of the tangential acceleration of the block greater? Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that only Graph 1 represents acceleration or that only Graph 2 represents momentum.
- ☐ 1 point is earned for a valid justification correctly relating force to acceleration and including a reference to a specific feature of the graph.
- ☐ 1 point is earned for a valid justification relating velocity to momentum and including a reference to a specific feature of the graph.

Example Response:

Graph 1 represents the magnitude of the cart's horizontal acceleration, and Graph 2 represents the block's momentum. The horizontal component of the acceleration will look like the net force graph, which should be a restoring force, and so should be proportional to D. The momentum of the cart graph will look like the velocity graph which has a maximum at $x = 0$ *and is zero at D and* $-D$.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for any statement that between $x = 0$ and $x = -D$, the kinetic or mechanical energy decreases.
- ☐ 1 point is earned for a clear indication that the mechanical energy of the system consists of only the kinetic energy of the block.

Example Response:

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The mechanical energy of the block is only kinetic energy, so between $x = 0$ and $x = -D$, the mechanical energy (kinetic energy), decreases.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an attempt at relating the distance traveled to the amplitude.
- ☐ 1 point is earned for a correct answer: $D = 1.5\ \text{m}$.

Example Response:

D is equal to 1.5 meters. In two complete oscillations the block travels $8D$, in 8 seconds. From the graph we can see that the total distance traveled in 8 seconds is 12 meters, so the distance D is $(12\ \text{m})/8 = 1.5\ \text{m}$.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that the period of oscillation is longer on the moon.
- ☐ 1 point is earned for correct physical reasoning why the average speed or average acceleration is less on the moon.
- ☐ 1 point is earned for connecting smaller average speed or smaller average acceleration over the same distance to a longer time.

Example Response:

The period of oscillation on the moon would be greater. The average speed of the box on the moon would be smaller because the acceleration due to gravity is smaller, so for a smaller average speed, over the same distance, the time to oscillate would be larger.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence of *T* on *g*, with an attempt at justification.
- ☐ 1 point is earned for correctly indicating whether the dependence of *T* on *g* is consistent with the answer to part (d).
  Note: *T* should increase when *g* decreases. If part (d) was answered correctly, the equation is consistent.

Example Response:

Yes, this makes sense with my reasoning. Since the acceleration due to gravity, g, is in the denominator, when g is smaller, (like it is on the Moon), the period T is larger.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

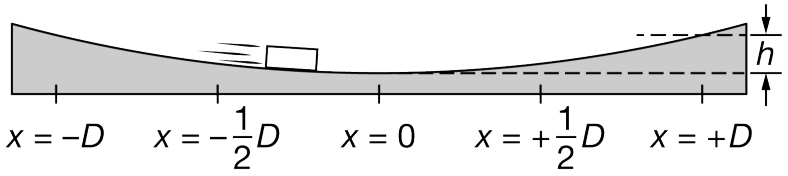
- ☐ 1 point is earned for a claim that the acceleration is greater at T_1.
- ☐ 1 point is earned for stating that both forces (component of gravity and friction) act on the block.
- ☐ 1 point is earned for correctly relating the sum of the forces on the block to the acceleration of the block at both T_1 OR T_2.

Example Response:

The tangential acceleration is greater at time T_1. At time T_1 both the component of gravity along the track and the friction force are in the same direction, while at T_2, the force of friction is in the opposite direction, so the force along the track is less at T_2, or greater at T_1. Since the force along the track is greater at T_1, the tangential acceleration of the block is greater at T_1.

chapter2parta

14. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

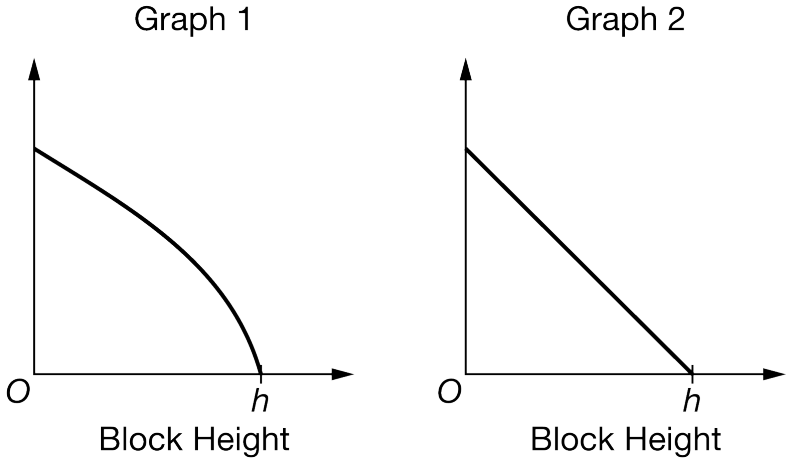


Track 1

Note: Figure not drawn to scale.

A block slides back and forth on a track with a slight circular curvature, where friction is negligible, as shown above. The maximum height h of the track at each end is much smaller than D.

The center of mass of the block oscillates between $x = -D$ and $x = +D$, reaching a maximum height h every time the block is at $x = +D$ or $x = -D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. At time $t = 0$, the speed of the block is zero. The next time that the block's speed is zero is at time $t = 2.0$ s.



(a) Which of the graphs above, Graph 1 or Graph 2, could represent the kinetic energy of the block as a function of its height on the track, and which graph could represent the speed as a function of the block's height on the track? Justify both of your choices with physics principles.

(b) For the system consisting of the block and Earth, describe how the mechanical energy of the system changes, if at all, as the block moves from $x = +D$ to $x = 0$. Justify your answer.

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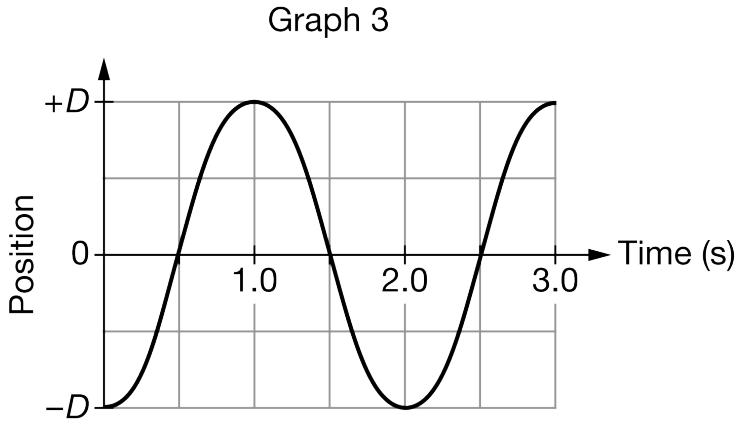
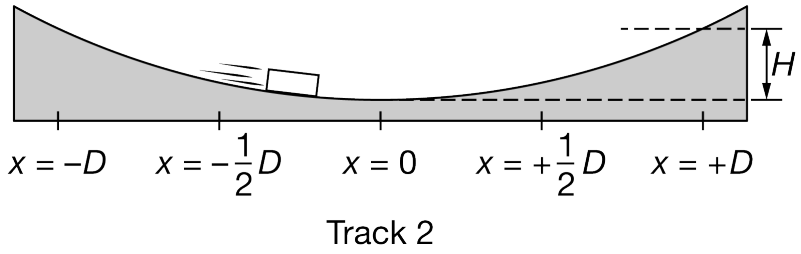


(c) Graph 3 above incorrectly represents the horizontal position of the block as a function of time. Identify a feature of the graph that does not correctly represent the motion of the block. Justify your answer.



Note: Figure not drawn to scale.

(d) Track 2 is similar to Track 1, but the maximum height H on Track 2 is greater than h and is still much smaller than D, as shown above. The center of mass of the block oscillates between $x = -D$ and $x = +D$, the same as on Track 1. How does the period of the block's oscillation on Track 2 compare to the period of the block's oscillation on Track 1? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{H^2}{2Dg}}$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?

(f) When the block reaches $x = 0$ at the bottom of the circular track, is the force exerted by the track on the block greater than, less than, or equal to the weight of the block? Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

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The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that only Graph 1 represents the speed as a function of height OR that only Graph 2 represents the kinetic energy as a function of height.
- ☐ 1 point is earned for a valid justification correctly relating speed to height and including a reference to a specific feature of the graph.
  Note: It acceptable to explain that speed is proportional to the square root of the kinetic energy.
- ☐ 1 point is earned for a valid justification that kinetic energy decreases linearly with increasing height and including a reference to a specific feature of the graph.

Example Response:

Graph 1 is the speed as a function of height and Graph 2 is the kinetic energy of the block as a function of height. Speed is equal to zero at h, but does not decrease linearly since the acceleration changes as the slope of the track changes. The kinetic energy however, changes linearly with height, since it is related to gravitational potential energy which changes linearly with height.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: The mechanical energy stays the same.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the mechanical energy of the system consists of gravitational potential energy and kinetic energy of the block.
- ☐ 1 point is earned for a statement that no work is done on the system, so the mechanical energy is constant.

Example Response:

For this system, the total mechanical energy is both the kinetic energy of the block and the gravitational potential energy of the block-Earth system. Because there are no net external forces that do work on the system, the total mechanical energy of the system remains constant.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

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The response accurately includes **both** of the following criteria.

- 1 point is earned for identifying that a maxima or minima on the graph indicates zero velocity,
  OR
  for identifying that the period on the graph is incorrect (2.0 seconds),
  OR
  for identifying that the graph should not have a zero slope at 1.0 seconds.
- 1 point is earned for a claim that the period should be longer (4.0 seconds) with no incorrect statements,
  OR
  for a claim that the next maxima should occur at 2.0 seconds with no incorrect statements,
  OR
  for a claim that the velocity should be zero (or flat slope) at only 0 and 2.0 seconds with no incorrect statements,
  OR
  for a claim that the graph should be "stretched" to twice its width along the *x*-axis (or similar) with no incorrect statements.

Example Response:

The period in this graph is incorrect – the period should be 4 seconds, since it takes 2 seconds just to go from $-D$ *to D, which is only half a period. Also, the velocity (the slope of the graph) should be zero at zero seconds and 2 seconds, not at 1 second.*

The problem states that the speed of the block is zero at $t = 0$ s *and* $t = 2$ s*, so the graph should have a maximum at* $t = 2$ s *and not* $t = 1$ s.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- 1 point is earned for a claim that the period of oscillation is shorter on Track 2.
- 1 point is earned for correct reasoning why the average speed or average acceleration is greater on Track 2 (connecting increased total mechanical energy to a greater average speed or connecting greater average acceleration to a steeper track).
- 1 point is earned for connecting greater average speed or greater average acceleration over the same distance to a shorter time.

Example Response:

The period is shorter on track 2 because the average speed is greater on track 2 because the total mechanical energy is larger since H is larger. Because the average speed is larger, over the same distance the block takes less time to travel from D to $-D$ *and back.*

Part E

chapter2parta

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence of *T* on *H,* with an attempt at a justification.
- ☐ 1 point is earned for correctly indicating whether the dependence of *T* on *H* is consistent with the answer to part (d).

Example Response:

No this does not match my claim above. Because H is in the numerator, an increase in H should lead to an increase in T according to the equation. However, in part (d) I found that an increase in H leads to a shorter period (T).

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that the force exerted on the block by the track is greater than the weight.
- ☐ 1 point is earned for describing the net force in terms of the force of track on the block (or normal force) and weight only.
- ☐ 1 point is earned for describing that the net force is directed upward because the block is in circular motion.

Example Response:

The force exerted on the box by the track is greater than the weight at the lowest point, since the force from the track minus the weight of the block has to be equal to the centripetal force up.

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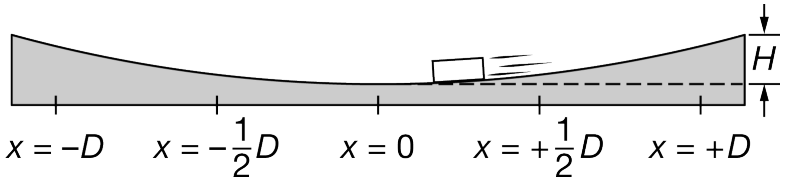
15. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

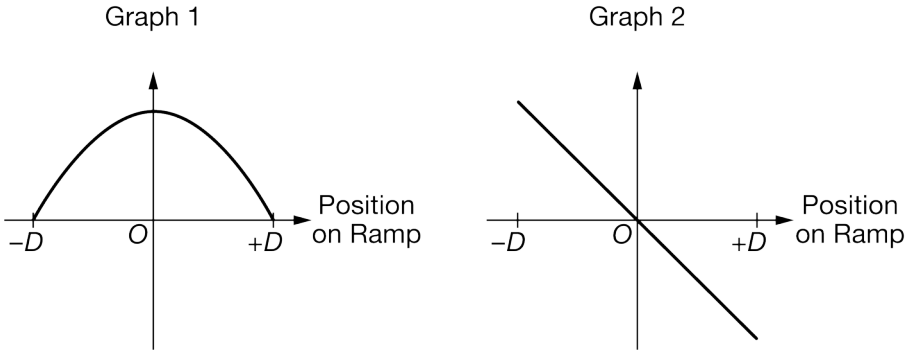


Track 1

Note: Figure not drawn to scale.

A block slides back and forth on a track with a slight circular curvature, where friction is negligible, as shown above. The maximum height H of the track at each end is much smaller than D.

The center of mass of the block oscillates between $x = -D$ and $x = +D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. At time $t = 0$ the block is released from rest at position $x = +D$. The period of the block's oscillation is 2.0 seconds.



(a) Which of the graphs above, Graph 1 or Graph 2, could represent the block's kinetic energy as a function of its position on the circular track, and which graph could represent the horizontal component of the net force acting on the block as a function of its position on the track? Justify both of your choices with physics principles.

(b) Choose one of the graphs in part (a) above and explain how the graph shows evidence that a restoring force is acting on the block.

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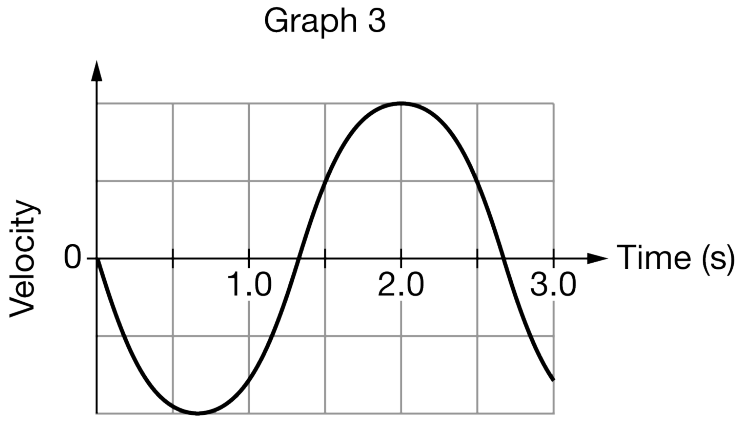
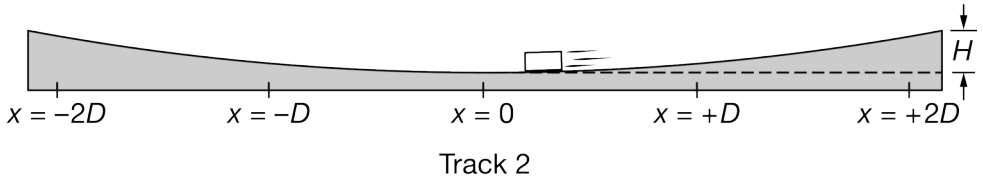


(c) Graph 3 above incorrectly shows the horizontal velocity of the block as a function of time. Identify a feature of the graph that does not correctly represent the motion of the block. Justify your answer.



Note: Figure not drawn to scale.

(d) Track 2, shown above, has a horizontal length that is greater than that of Track 1 and the same maximum height H. The block is released from rest at position $x = +2D$ Is the period of the block's oscillation on Track 2 greater than, less than, or equal to the period of oscillation on Track 1? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{h^2}{2dg}}$, where d is the maximum displacement of the block from equilibrium and h is the height of the track at $x = d$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?

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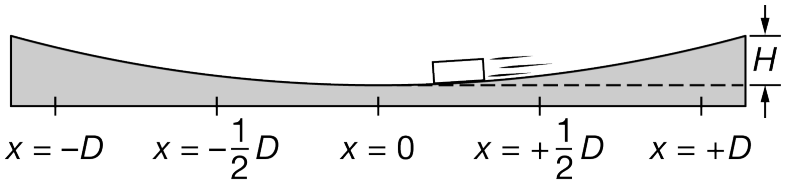
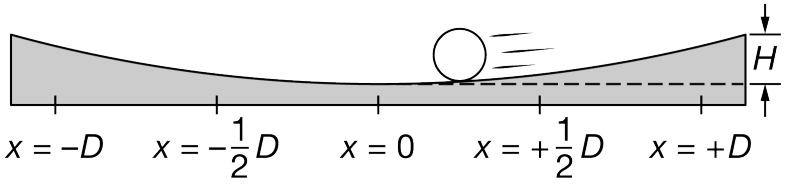


Track 1



Track 3

Note: Figures not drawn to scale.

(f) On Track 1, shown above again for reference, friction is negligible and the center of mass of the block oscillates between a horizontal position of $+D$ and $-D$ with a period of 2.0 seconds. If a sphere with a mass equal to that of the block were to roll without slipping between $+D$ and $-D$ along Track 3 shown above, which is identical to Track 1 but has nonnegligible friction, would the period of the motion of the sphere be greater than, less than, or equal to 2.0 seconds? Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that only Graph 1 represents kinetic energy or that only Graph 2 represents the horizontal component of the net force.
- ☐ 1 point is earned for a valid justification relating the slope of the track to the horizontal force or describing the positive/negative values of the force on each side of the graph.
- ☐ 1 point is earned for a valid justification referring to the speed of the block and its kinetic energy and including a reference to a specific feature of the graph.

Example Response:

Graph 1 represents the kinetic energy of the block and Graph 2 represents the horizontal component of the net force. The kinetic energy is a maximum at $x = 0$ and a minimum at $x = -D$ and $x = +D$. The horizontal component of the net force is zero at $x = 0$ and the magnitude is a maximum at $x = -D$ and $x = +D$. The horizontal component of the net

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force for small oscillations is linear and is proportional to displacement.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for identifying an appropriate feature of one of the graphs consistent with the answer in part (a).
- ☐ 1 point is earned for correctly explaining how a feature of the graph shows evidence of a restoring force.

Example Response:

Graph 2 shows evidence of a restoring force – a restoring force has a magnitude that is proportional to the displacement and opposite in direction to the displacement which is what graph 2 is showing.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for identifying that the period is incorrect,
  OR
  for identifying that the curve should intersect the horizontal axis at 1.0 seconds.
- ☐ 1 point is earned for a claim that the period is too long or that the graph should be compressed horizontally.

Example Response:

The period of the graph shown is incorrect. The problem states that the period is 2 seconds, but the graph shows a period that is longer than 2 seconds.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that the period of oscillation is longer on Track 2.
- ☐ 1 point is earned for correct reasoning why the average acceleration or average speed is less on Track 2.
- ☐ 1 point is earned for connecting the smaller average acceleration or average speed over the longer distance to a larger time.

Example Response:

The period on track 2 is longer. The block will have the same maximum speed but spread out over a much longer track means that the time to travel the length of the track and back is larger.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence of T on d, with an attempt at a justification.
- ☐ 1 point is earned for correctly indicating whether the dependence of T on d is consistent with the answer to part (d).
  Note: T should increase with d. If part (d) was answered correctly, the equation is inconsistent.

Example Response:

No this does not match my reasoning. In the equation when d is in the denominator, as d increases, T decreases, which means that if the ramp gets longer, the period should decrease, which is not what will happen.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

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The response accurately includes **all** of the following criteria.

- 1 point is earned for a claim that the period of the sphere would be greater than the period of the block.
- 1 point is earned for a claim that maximum total kinetic energy of the sphere is equal to the maximum kinetic energy of the block,
  OR
  for a claim that the net force on the sphere will be less than that on the block because a frictional force is directed opposite the sphere's motion.
- 1 point is earned for recognizing that some portion of the sphere's kinetic energy is rotational,
  OR
  for recognizing that a smaller net force would result in a smaller average acceleration.

Example Response:

The period of the sphere would be greater. For both the block and the sphere the gravitational potential at the top are equal and so the kinetic energy as each passes through $x = 0$ *are equal. But for the sphere, some of that KE is turned into rotational kinetic energy, therefore it will be traveling, on average, slower than the block and so will take longer to travel the same distance.*

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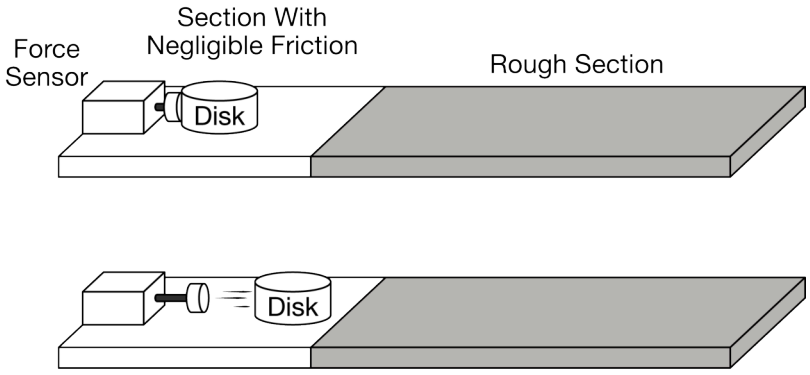
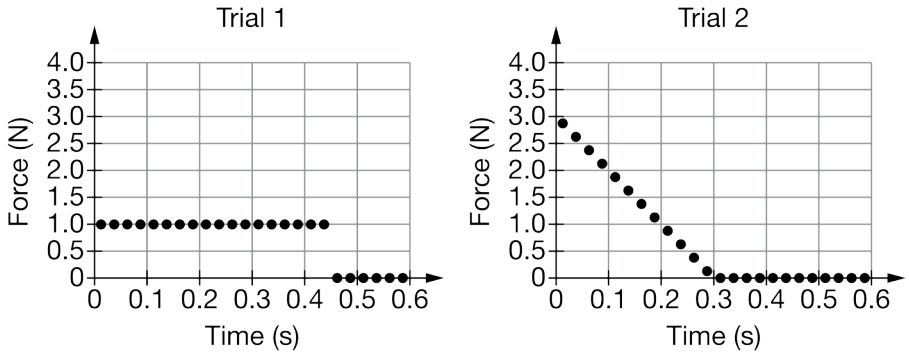
16. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Students use a force sensor to push a disk of mass m across a horizontal table. The force sensor pushes the disk over a short time interval. The magnitude of the force and the duration of the time interval can be set by the students. After being pushed by the sensor, the disk first slides along a surface with negligible friction and then enters a rough section where friction is not negligible, as shown above.

The students perform two trials in which the magnitude of the force exerted by the force sensor and the time interval are different. They create the graphs of force as a function of time shown below. The disk enters the rough section in each trial after $t = 0.5$ s.



(a) In which of the two trials, if either, is the change in momentum of the disk between 0 seconds and 0.5 seconds greater? Explain your reasoning.

(b) In both trials, the disk comes to rest in the rough section before reaching the edge of the table. In which trial, if either, does the disk travel a greater distance in the rough section? Explain your reasoning.

(c) The students repeat Trial 1 with a disk that is made from the same material as the original disk and has the same mass as the original disk but that has a larger radius (and therefore more surface area is in contact with the surface). The same force is exerted on the new disk over the same time interval as in Trial 1. Would the distance traveled in the rough section by the new disk be greater than, less than, or the same as the distance traveled by the original disk in Trial 1? Explain your reasoning.

Students derive the following equation, which may or may not be correct, to describe the distance D traveled by

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the original disk on the rough section: $D = \frac{(F_{\text{avg}}{}^2)t^2}{2\mu mg}$, where F_{avg} is the average force exerted by the sensor, t is the time that the disk is in contact with the sensor, μ (mu) is the coefficient of kinetic friction, and m is the mass of the disk.

(d) Without algebraic manipulation of equations or attempting to derive this equation, indicate whether the above equation is consistent with your answer in part (b). Explain your reasoning.

A student measures the distance x_0 that the disk travels while in contact with the force sensor in Trial 1. The disk then travels a distance D along the rough section. The student claims that if a force with twice the magnitude of the original force was exerted on the disk for a distance x_0, the disk would travel a distance $2D$ in the rough section.

(e) Is the above claim correct or incorrect? Justify your answer without citing or manipulating equations.

The student finds the following equation on the Internet describing the relationship between the magnitude F of the force exerted on the disk, the distance x the disk is pushed, and the distance D it slides in the rough section: $D = C\frac{Fx}{\mu mg}$, where C is a constant with appropriate units.

(f) Indicate whether or not the equation above is consistent with your statement in part (e) relating D and x. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the impulse or $\mathrm{F(t)}$ or area under the curve is the impulse or change in momentum
  OR
  For recognizing that the acceleration can be determined from the force.
- ☐ 1 point is earned for correctly indicating the area is equal to the $\mathrm{F_{ave}(t)}$ or $\mathrm{F_t/2}$ or $0.45\ \mathrm{Ns}$
  OR
  For addressing that the mass value is the same or need not be known.
- ☐ 1 point is earned for indicating that both disks have the same change in momentum
  OR
  For correct application of kinematics including the use of the time.

Example Response:

The area under each curve is the change in momentum, and since both graphs have the same area under the curve, $0.45\ Ns$*, both have the same change in momentum.*

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OR

The change in momentum is the same for both pucks because the same average force was applied to the same mass for the same time frame causing the same launch velocity.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks.
- ☐ 1 point is earned for a correct relationship between the speed OR momentum OR kinetic energy and the distance traveled in the rough section consistent with their answer to part (a).

Example Response:

Both disks experience the same frictional force in both trials, so the acceleration will be the same for both disks, and since both disks start with the same initial speed, they will slide the same distance.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating both disks would have the same launch speed with the same force over the same time interval.
  OR
  For indicating that both disks will have the same kinetic energy OR momentum when a force is applied over the same time.
- ☐ 1 point is earned for indicating that the surface area of the disk does not affect the frictional force.
- ☐ 1 point is earned for indicating that the new block will travel the same distance before stopping.

Example Response:

Both disks will have the same launch speed because they experience the same force and have the same mass. The surface area of the disks does not affect the frictional force, so the new block will travel the same distance before stopping.

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Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an answer consistent with part (b), with an attempt to use the functional dependence of D on F_{avg} and/or t.
- ☐ 1 point is earned for correct reasoning (i.e., that $F_{\text{avg}}t$ is the same in both trials, so D is the same).

Example Response:

The equation is consistent with my reasoning in part (b). In the equation D is proportional to the square of the product of F_{avg} times t, which is the same for both trials, so D is the same.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: “Correct”.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a disk that is pushed with twice the force will have twice the original energy.
- ☐ 1 point is earned for a correct application of conservation of energy or work.
- ☐ 1 point is earned for indicating that since the work to stop the disk is also proportional to the distance to stop it, the disk will travel twice the distance before stopping.

Example Response:

A disk that is pushed from rest with twice the force for the same distance will have twice the work done one it, and so will have twice the original energy. The work done to stop the disk is equal to the energy that the disk has after being pushed. Work is also proportional to the distance to stop it; the new disk will travel twice the distance before stopping.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the equation is consistent with their statement in part (e) AND an attempt at a justification using functional dependence.
- ☐ 1 point is earned for correctly indicating D is proportional to F.

Example Response:

The given equation is consistent with my reasoning in part (e). Because D in the equation is directly proportional to F, pushing with twice the force will result in a distance D that is twice as large.

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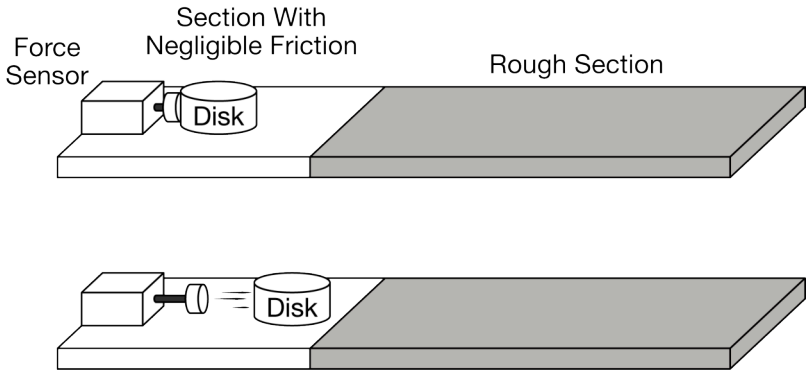
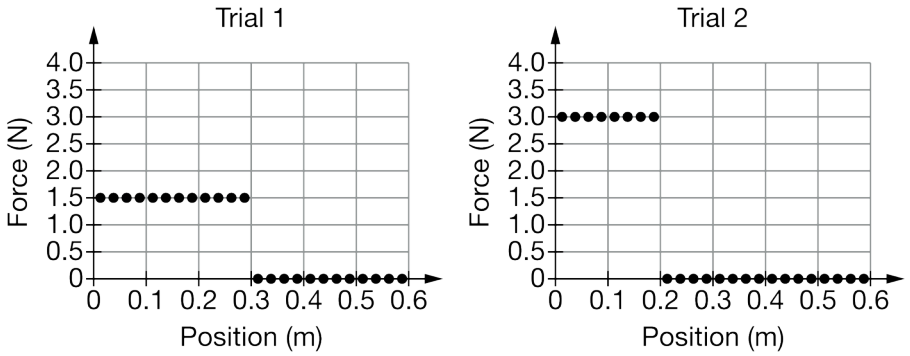
17. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Students use a force sensor to push a disk of mass m across a horizontal table. The force sensor pushes the disk over a short distance. The magnitude of the force and the distance the disk is pushed can be set by the students. After being pushed by the sensor, the disk first slides along a surface with negligible friction and then enters a rough section where friction is not negligible, as shown above.

The students perform two trials in which the magnitude of the force exerted by the force sensor and the distance the disk is pushed are different. They create the graphs of force as a function of position shown below. The disk enters the rough surface in each trial after moving 0.6 meters.



(a) In which of the two trials, if either, is the change in energy of the disk the greatest during the interval shown in the graphs? Explain your reasoning.

(b) In both trials, the disk comes to rest in the rough section before reaching the edge of the table. In which trial, if either, does the disk travel a shorter distance in the rough section? Explain your reasoning.

(c) The students repeat Trial 2 with a disk that is made from the same material as the original disk but that is less massive. The same constant force is exerted on the less massive disk over the same distance as in Trial 2. Would the distance traveled in the rough section by the less massive disk be greater than, less than, or the same as the distance traveled in the rough section by the original disk in Trial 2? Explain your reasoning.

Students derive the following equation, which may or may not be correct, to describe the distance D traveled by

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the original disk in Trial 2 on the rough section: $D = \frac{Fx}{\mu mg}$, where F is the force exerted by the sensor, x is the distance over which the sensor exerts a force on the disk, μ (mu) is the coefficient of kinetic friction, and m is the mass of the disk.

(d) Without algebraic manipulation of equations or attempting to derive this equation, indicate whether the equation is consistent with your answer in part (b). Explain your reasoning.

A student measures the time t that the disk is in contact with the force sensor in Trial 2. The disk then travels a distance D along the rough section. The student claims that if the same force is exerted on the disk for a time $2t$, the disk would travel a distance $2D$ in the rough section.

(e) Is the above claim correct or incorrect? Justify your answer without citing or manipulating equations.

The student finds the following equation on the Internet describing the relationship between the time t the disk is pushed and the distance D it slides in the rough section: $D = C\frac{F^2t^2}{2\mu m^2 g}$, where C is a constant with appropriate units.

(f) Indicate whether or not the above equation is consistent with your statement in part (e) relating D and t. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that trial 2 has the graph with the greatest area under the curve
  OR
  For recognizing that the acceleration can be determined from the force.
- ☐ 1 point is earned for indicating the area represents the work done
  OR
  For addressing that the mass value is the same OR need not be known.
- ☐ 1 point is earned for relating the greatest work done to the greatest change in energy
  OR
  For correct application of kinematics including the use of the distance traveled.

Example Response:

Trial 2. The work done on the disk is represented by the area under the force vs position graph, so the graph with the largest area under the curve has the greatest work done on it. Graph 2 has the greatest area, and so the disk in trial 2 had the greatest amount of work done on it.

Part B

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks.
- ☐ 1 point is earned for indicating that the disk with the smaller launch speed will travel a shorter distance in the rough section.

Example Response:

Trial 1. The disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks. This means that the disk with the smaller speed after the push will travel a smaller distance.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a less massive disk would have a greater launch speed with the same force over the same distance.
- ☐ 1 point is earned for indicating that the distance traveled in the rough section, for a given speed, does not depend on the mass of the disk.
- ☐ 1 point is earned for indicating that the less massive disk will travel farther before stopping.

Example Response:

Farther than the original disk in trial 2. When pushed by the same force over the same distance, a less massive disk would have a larger launch speed. Since the mass of the disk doesn't affect the distance traveled through the rough section, the less massive disk will travel farther before stopping.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an answer consistent with part (b), with an attempt to use the functional dependence of *D* on *F* and/or *x*.
- ☐ 1 point is earned for correct reasoning (i.e., that Fx is greater in Trial 2, so *D* is greater).

Example Response:

This equation matches my reasoning in part (b). Since D is inversely proportional mass, if the mass increases, the distance D decreases, since m is in the numerator.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: “Incorrect”.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a disk that is pushed for twice the time will have twice the speed.
- ☐ 1 point is earned for a correct application of conservation of energy or work for the disk in the rough section.
- ☐ 1 point is earned for indicating that the stopping distance is proportional to kinetic energy, not speed.

Example Response:

A disk that is pushed from rest with the same force for twice the time will have twice the momentum. Since the mass of the disk doesn’t change, the second disk will have twice the speed. However, the stopping distance is proportional to the kinetic energy which is proportional to the velocity squared.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the equation is consistent (or inconsistent) with the statement in part (e) AND an attempt at a justification using functional dependence.
- ☐ 1 point is earned for indicating that the distance is proportional to *t* squared.

Example Response:

It is consistent. If t is doubled, the distance traveled should quadruple, because D is proportional to t squared.

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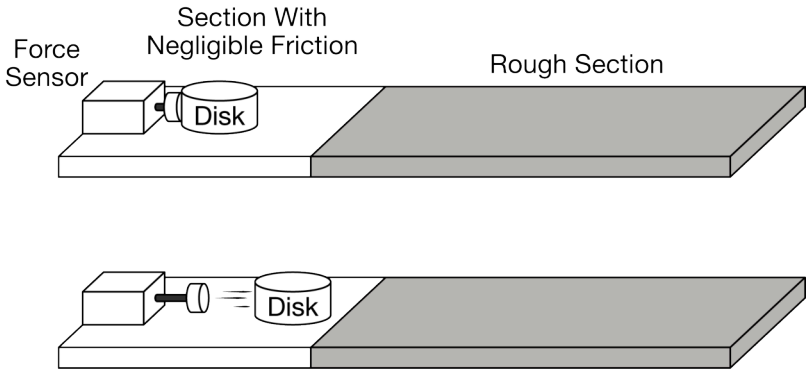
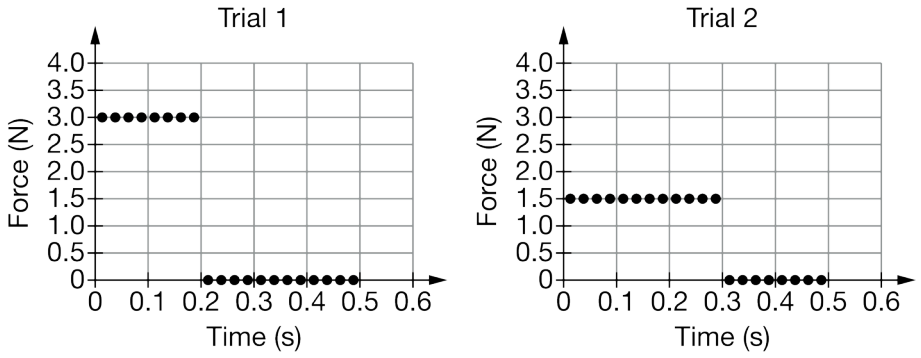
18. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Students use a force sensor to push a disk of mass m across a horizontal table. The force sensor pushes the disk with a force of constant magnitude over a short time interval. The magnitude of the force and the duration of the time interval can be set by the students. After being pushed by the sensor, the disk first slides along a surface with negligible friction and then enters a rough section where friction is not negligible, as shown above.

The students perform two trials in which the magnitude of the force exerted by the force sensor and the time interval are different. They create the graphs of force as a function of time shown below. The disk enters the rough surface in each trial after $t = 0.5$ s.



(a) In which of the two trials, if either, is the speed of the disk the greatest at time 0.5 seconds? Explain your reasoning.

(b) In both trials, the disk comes to rest in the rough section before reaching the edge of the table. In which trial, if either, does the disk travel a greater distance in the rough section? Explain your reasoning.

(c) The students repeat Trial 1 with a disk that is made from the same material as the original disk but that is more massive. The same force is exerted on the more massive disk over the same time interval as in Trial 1. Would the distance traveled in the rough section by the more massive disk be greater than, less than, or the same as the distance traveled by the original disk in Trial 1? Explain your reasoning.

Students derive the following equation, which may or may not be correct, to describe the distance D traveled by

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the original disk on the rough section: $D = \frac{F^2t^2}{2\mu mg}$, where F is the force exerted by the sensor, t is the time that the disk is in contact with the sensor, μ (mu) is the coefficient of kinetic friction, and m is the mass of the disk.

(d) Without algebraic manipulation of equations or attempting to derive this equation, indicate whether the above equation is consistent with your answer in part (b). Explain your reasoning.

A student measures the distance x_0 that the disk travels while in contact with the force sensor in Trial 1. The disk then travels a distance D along the rough section. The student claims that if the same force is exerted on the disk for a distance $2x_0$, the disk would travel a distance $2D$ in the rough section.

(e) Is the above claim correct or incorrect? Justify your answer without citing or manipulating equations.

The student finds the following equation on the Internet describing the relationship between the distance x the disk is pushed and the distance D it slides in the rough section: $D = C\frac{Fx^2}{\mu mg}$, where C is a constant with appropriate units.

(f) Indicate whether or not the equation above is consistent with your statement in part (e) relating D and x. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that trial 1 has the graph with the greatest area under the curve OR impulse OR $F(t)$ OR addressing the numerical values of 0.45 Ns vs 0.6 Ns
  OR
  For recognizing that the acceleration can be determined from the force.
- ☐ 1 point is earned for indicating the area represents the change in momentum
  OR
  For addressing that the mass value is the same OR need not be known
- ☐ 1 point is earned for relating the greatest change in momentum to the greatest change in speed
  OR
  For correct application of kinematics including the use of the time.

Example Response:

Trial 1 has the biggest area under the curve, and since the area under a force vs time graph is impulse or change in momentum, the trial with the biggest area will have the biggest change in momentum. Both disks start at rest so the disk with the biggest change in momentum has the biggest change in speed.

OR

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Because the mass of the puck in both trials is the same, the trial with the largest force has the largest acceleration. Since both pucks start at rest, the puck with the largest acceleration will have the largest speed when pushed for the same amount of time.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks.
- ☐ 1 point is earned for a correct relationship between the speed OR momentum OR kinetic energy and the distance traveled in the rough section consistent with their answer to part (a).

Example Response:

In both trials the disk experiences the same frictional force, so the acceleration is the same in both trials. The puck with the greatest launch speed travels the furthest before coming to rest because it will take a longer time to stop with the same acceleration.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a more massive disk would have a smaller launch speed OR momentum with the same force over the same time interval
  OR
  For indicating that a more massive disk will have less kinetic energy when a force is applied over the same time.
- ☐ 1 point is earned for indicating that the mass of the disk does not affect the distance traveled in the rough section.
- ☐ 1 point is earned for indicating that the more massive block will travel less far before stopping.

Example Response:

The more massive disk will have a smaller launch speed after being pushed with the same force for the same time. The pucks will have the same acceleration in the rough section of the track, regardless of the mass, so the more massive block

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will not travel as far before stopping.

OR

The disk would have less kinetic energy over the same time frame, because the force would be applied over a smaller distance. Since the masses are equal, the F_f on the disks would be the same in the rough patch, for a disc with less kinetic energy entering the rough region, and experiencing the same F_f, the disk would not go as far.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an answer consistent with part (b), with an attempt to use the functional dependence of *D* on *F* and/or *t*.
- ☐ 1 point is earned for correct reasoning (i.e., that F_t is greater in trial 1, so *D* is greater).

Example Response:

The equation is consistent with my answer to part (b). D increases with the product F_t, so if this product increases, the distance D will also increase.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a disk that is pushed for twice the original distance will have twice the original energy.
- ☐ 1 point is earned for a correct application of conservation of energy or work.
- ☐ 1 point is earned for indicating that since the work to stop the disk is also proportional to the distance to stop it, the disk will travel twice the distance before stopping.

Example Response:

A disk that is pushed from rest for twice the original distance will have twice the work done on it and so will have twice the energy. The work done to stop the disk is equal to the energy that the disk has after being pushed. Work is also proportional to the distance to stop it; the disk will travel twice the distance before stopping.

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Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the equation is not consistent (or consistent) with their statement in part (e) AND an attempt at justification using functional dependence.
- ☐ 1 point is earned for correctly indicating that D is proportional to x.

Example Response:

The given equation is not consistent with my statement in (e). In the equation D is directly proportional to x squared. But in part (e) I stated that if the distance x doubles, D will also double.

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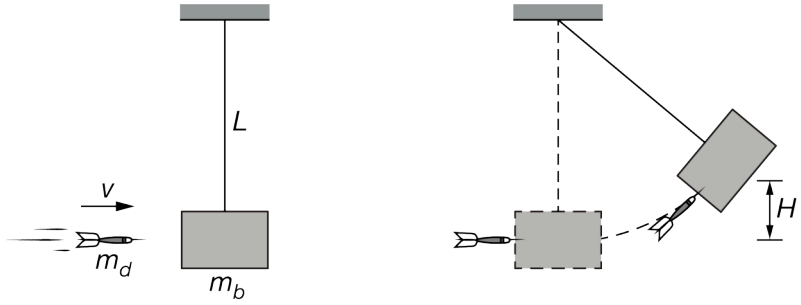
19. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown at left above. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown at right above. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, if either, experiences a net horizontal force of larger magnitude during the collision. If the net horizontal force is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the experiment was repeated with a dart with the same horizontal speed but with mass smaller than m_d, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the motion of the dart-block system at the instant it first reaches a height of $H/2$, and they make the following statements:

Student 1: When the dart-block system is at a height of $H/2$, the kinetic energy of the system is half of its maximum value, and therefore the speed of the system is half of its maximum speed.

Student 2: When the dart-block system reaches a height of $H/2$, the centripetal force is equal to the tension in the string. Therefore, the acceleration of the system must point toward the fixed end of the string.

(c) Which underlined phrase or phrases, if either, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if either, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if either, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases, if either, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

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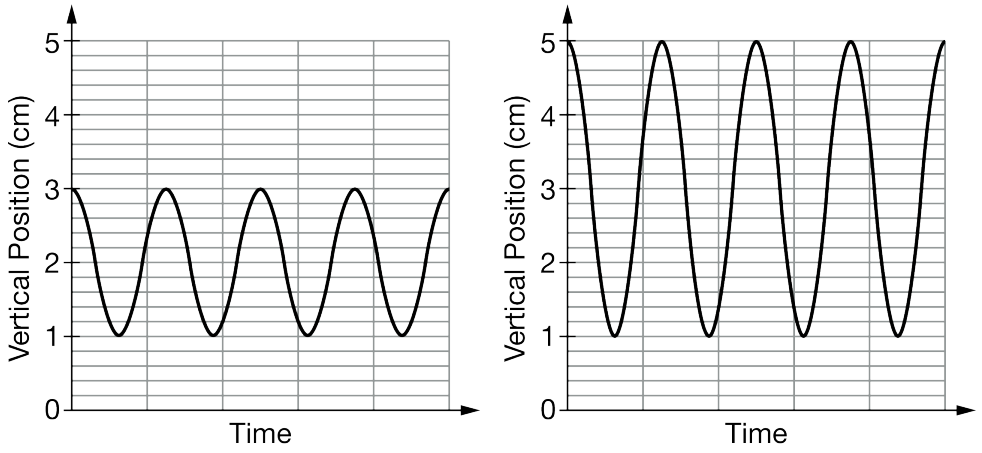
(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{v}{1+\frac{m_b}{m_d}}\right)^2$. Explain how this equation supports your claim in part (b).

(h) Could the masses of the dart and block both be changed so that the maximum height *H* reached by the dart-block system would remain the same as in the original setup? Justify your answer by referring to the equation in part (g).

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart mass or the string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a statement that both objects experience the same net force because of Newton's third law.

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Example Response:

Both the dart and the block experience the same force because they exert equal and opposite forces on each other because of Newton's third law.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the final height is smaller.
- ☐ 1 point is earned for a statement of conservation of momentum to indicate that the final speed of the dart-block system is smaller **AND** an indication of conservation of energy leading to a smaller height.

Example Response:

Because the momentum of the dart before the collision must be equal to the momentum of the dart-block system after the collision, a smaller dart mass (a smaller dart momentum) will lead to a smaller dart-block speed after the collision (a smaller dart-block momentum). Because the dart-block is traveling slower after the collision, it has less kinetic energy which means that the dart-block will not swing as high.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Responses:

Student 1 is correct in saying that the kinetic energy of the system is half of its maximum at $H/2$.

OR

The first statement

Part D

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for an incorrect statement and no correct statements.

Example Responses:

Student 1 is incorrect in saying that the speed of the system is half of its maximum speed

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for indicating student 2 has no correct statements.

Example Response:

None

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for indicating both of Student 2's statements are incorrect.

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Example Responses:

Student 2 is incorrect in saying that centripetal force is equal to the tension in the string, AND incorrect in saying that the acceleration of the system must point toward the center of the circle.

OR

Both statements

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement including functional dependence of the variables.
- ☐ 1 point is earned for correctly asserting that as the mass of the dart decreases in the equation *H* also decreases.

Example Response:

This equation matches my reasoning in part (b)– if the mass of the dart decreases, the height of the swing decreases because a decrease in the mass of the dart means an increase in the ratio $(block\ mass)/(dart\ mass)$*, which is in the denominator of the expression.*

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for “yes”, with an attempt at explanation.
- ☐ 1 point is earned for a statement that both either increase or decrease.
- ☐ 1 point is earned for a statement indicating the factor in increase/decrease (or ratio of masses) is the same.

Example Response:

Yes, if the masses were both doubled, the maximum height would still be H – it works as long as the masses both increase by the same factor .

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Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for recognizing that the amplitude of the graph for group 2 is larger.
- ☐ 1 point is earned for a statement discussing the dependence of amplitude on dart mass.
- ☐ 1 point is earned for indicating that the mass of the dart could be larger.

Example Response:

If group 2 used a dart with a larger mass, the dart would have had more momentum, and so the dart-block system would have gone higher (making the amplitude larger), but the string length is the same and so the period would still be the same.

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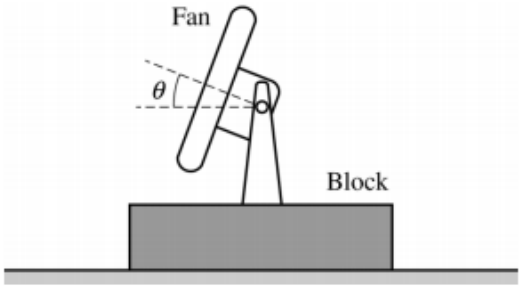
20.



A student in a physics lab has a block with a fan attached to it, as shown in the figure above. The fan has a pivot so that the angle θ it makes with the horizontal can be adjusted between 0° and 90°. When the fan is pointed horizontally so that θ = 0°, the block accelerates from rest along a track, even though there is friction between the block and the track. For the following questions, assume that the student has access to equipment that would usually be found in a school physics laboratory.

a. Describe an experimental procedure that the student could use to measure the force F that the air exerts on the fan-block system when the fan is turned on. Assume that the magnitude of this force is the same for all angles θ.

    i. What quantities would be measured?

    ii. What equipment would be used for the measurements, and how would that equipment be used? Include a labeled diagram of the experimental setup.

    iii. Describe the overall procedure to be used. Give enough detail so that another student could replicate the experiment.

    iv. Describe how the force can be determined from the measurements described in parts (a)i–iii.

b. Describe an experimental procedure that would allow the student to use the fan-block system, with the **fan turned on**, to find the coefficient of kinetic friction between the block and the track.

    i. What quantities would be measured?

    ii. What equipment would be used for the measurements, and how would that equipment be used? Include a labeled diagram of the experimental setup.

    iii. Describe the overall procedure to be used. Give enough detail so that another student could replicate the experiment.

    iv. On the dot to the right, which represents the fan-block system, draw and label the forces (not components) that are exerted on the system during the experiment described in parts (b)i–iii. Represent each force by a distinct arrow starting on, and pointing away from, the dot.

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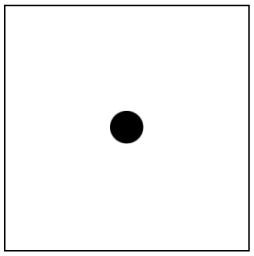
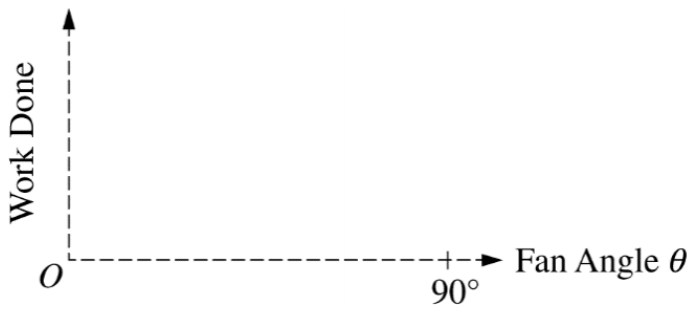


c. Describe how to analyze the data from the experiment described in parts (b)i–iii in order to determine the coefficient of kinetic friction between the block and the track. Your analysis may include the force F exerted by the air on the fan-block system. Do not add to the free-body diagram in part (b)iv.

d. In a different experiment, the student attaches the block to the top of a low-friction cart. On the axes below, sketch a graph of the work done by air on the fan-block-cart system as a function of q when the system travels a fixed distance D



Part A

Part(i, ii, and iii)

Note: Parts i, ii, and iii are read together

1 point is earned: For measuring the mass or weight of the relevant system

1 point is earned: For making a measurement of force (or something from which a force can be calculated) with the fan turned on

Note: fan angles of either 0° or 90° need not be measured directly, since these are given as the minimum and maximum angles possible. If the procedure involves intermediate angles, then these angles must be measured.

Example 1 (variant - Example 1a in parentheses):

(a)i The downward force exerted by the fan-block system (or “mass”, as read on a balance, of the system)

(a)ii A spring scale (or mechanical balance)

(a)iii Set the fan angle to 90°. With the fan off, hang the system from a spring scale to find its weight (or place the system on a mechanical balance to find its mass). Turn the fan on, and record the force from the spring scale

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(or the mass reading from the mechanical balance).

Example 2:

(a)i Mass of the fan-block system and low-friction cart. Velocity of the cart as a function of time.

(a)ii Scale to measure mass. Motion detector to measure velocity as a function of time.

(a)iii Place the system and a low-friction cart on a scale to find the total mass of the fan-block and cart. Next, on a horizontal surface, place the fan-block on the cart, set the fan angle to zero, turn on the fan, and release the cart from rest. Use the motion detector to graph the cart velocity as a function of time.

Part(iv)

1 point is earned: For correctly indicating a method to determine the force exerted by air on the fan, using the measurements and procedure from parts (a) i, ii, and iii

Example 1 (and 1a), continued from above: Take the difference in the force readings with the fan off and the fan on. (1a: Multiply the difference in the mass readings by g). This difference is the force exerted by air on the fan.

Example 2, continued from above: Estimate the average slope of the graph, which is the acceleration. Multiply acceleration by the total mass to find the force exerted by air on the fan.

0	1	2	3 ✓

Student response earns 3 of the following 3 point(s)

Part(i, ii, and iii)

Note: Parts i, ii, and iii are read together

1 point is earned: For measuring the mass or weight of the relevant system

1 point is earned: For making a measurement of force (or something from which a force can be calculated) with the fan turned on

Note: fan angles of either 0° or 90° need not be measured directly, since these are given as the minimum and maximum angles possible. If the procedure involves intermediate angles, then these angles must be measured.

Example 1 (variant - Example 1a in parentheses):

(a)i The downward force exerted by the fan-block system (or “mass”, as read on a balance, of the system)

(a)ii A spring scale (or mechanical balance)

(a)iii Set the fan angle to 90°. With the fan off, hang the system from a spring scale to find its weight (or place the system on a mechanical balance to find its mass). Turn the fan on, and record the force from the spring scale (or the mass reading from the mechanical balance).

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Example 2:

(a)i Mass of the fan-block system and low-friction cart. Velocity of the cart as a function of time.

(a)ii Scale to measure mass. Motion detector to measure velocity as a function of time.

(a)iii Place the system and a low-friction cart on a scale to find the total mass of the fan-block and cart. Next, on a horizontal surface, place the fan-block on the cart, set the fan angle to zero, turn on the fan, and release the cart from rest. Use the motion detector to graph the cart velocity as a function of time.

Part(iv)

1 point is earned: For correctly indicating a method to determine the force exerted by air on the fan, using the measurements and procedure from parts (a) i, ii, and iii

Example 1 (and 1a), continued from above: Take the difference in the force readings with the fan off and the fan on. (1a: Multiply the difference in the mass readings by g). This difference is the force exerted by air on the fan.

Example 2, continued from above: Estimate the average slope of the graph, which is the acceleration. Multiply acceleration by the total mass to find the force exerted by air on the fan.

Part B

Part(i, ii, and iii)

Note: Parts i, ii, and iii are read together

Note: In parts (b) i, ii, and iii, it is acceptable to say "the same as in part (a), with the following changes:", with changes to the procedure in part (a) then listed.

1 point is earned: For describing measurements that would allow determination of the acceleration

1 point is earned: For describing a procedure that is practical/feasible in a school laboratory

Example 3:

Place the fan-block on the track, with angle = 0. Place a motion detector behind it, set to graph velocity vs. time. Turn fan on, and release from rest. Repeat the procedure several times.

Example 4:

Place the fan-block on the track, with angle = 0. Place a photogate just in front of the block, and the second photogate a meter or two in front of the block. Measure distance from front of block to the front of that second photogate. Turn on fan and release the block from rest. The photogates measure the time it takes the block to travel the measured distance to the second photogate.

Example 5:

(b)i Angle of the fan. Velocity of fan-block system as a function of time. (b)ii Protractor to measure the angle. Motion detector to measure the velocity and time. (b)iii Adjust the fan angle until the fan-block slides with a constant speed after being given a push. Use the motion detector to determine whether the speed is constant (zero slope on a graph of velocity as a function of time).

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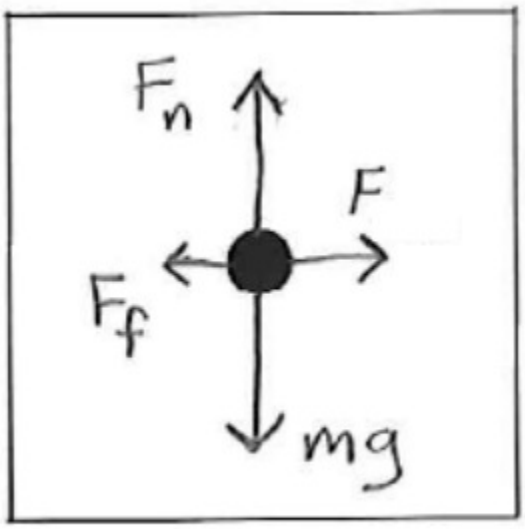
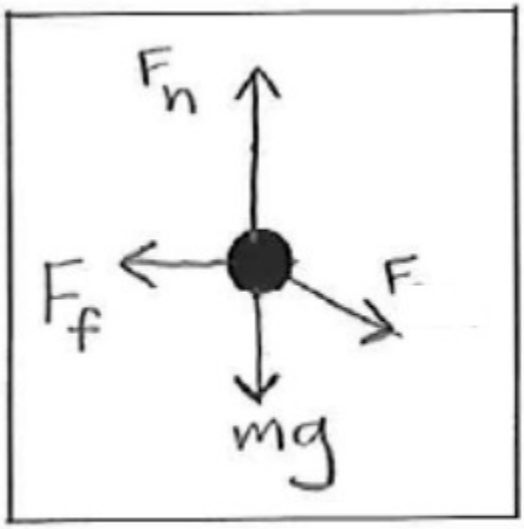
Part(iv)

1 point is earned: For correctly drawn and labeled vectors for the force exerted by air on fan and for the frictional force exerted on the block by the track, corresponding to the experiment described in part (b)iii. Other forces correctly drawn and labeled AND no incorrect forces, corresponding to the experiment described in part (b)iii.



Examples 3 & 4



Example 5

0	1	2	3 ✓

Student response earns 3 of the following 3 point(s)

Part(i, ii, and iii)

Note: Parts i, ii, and iii are read together

Note: In parts (b) i, ii, and iii, it is acceptable to say "the same as in part (a), with the following changes:", with changes to the procedure in part (a) then listed.

1 point is earned: For describing measurements that would allow determination of the acceleration

1 point is earned: For describing a procedure that is practical/feasible in a school laboratory

Example 3:

Place the fan-block on the track, with angle = 0. Place a motion detector behind it, set to graph velocity vs. time. Turn fan on, and release from rest. Repeat the procedure several times.

Example 4:

Place the fan-block on the track, with angle = 0. Place a photogate just in front of the block, and the second photogate a meter or two in front of the block. Measure distance from front of block to the front of that second

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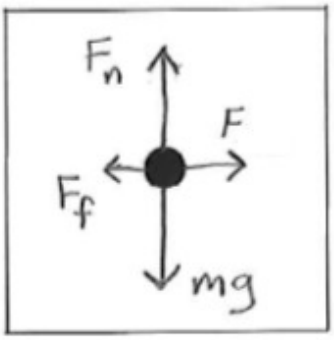
photogate. Turn on fan and release the block from rest. The photogates measure the time it takes the block to travel the measured distance to the second photogate.

Example 5:

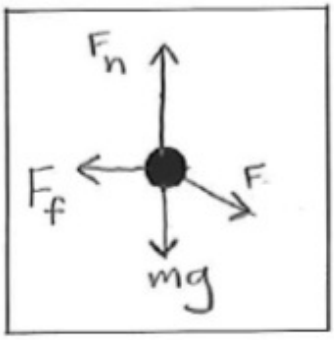
(b)i Angle of the fan. Velocity of fan-block system as a function of time. (b)ii Protractor to measure the angle. Motion detector to measure the velocity and time. (b)iii Adjust the fan angle until the fan-block slides with a constant speed after being given a push. Use the motion detector to determine whether the speed is constant (zero slope on a graph of velocity as a function of time).

Part(iv)

1 point is earned: For correctly drawn and labeled vectors for the force exerted by air on fan and for the frictional force exerted on the block by the track, corresponding to the experiment described in part (b)iii. Other forces correctly drawn and labeled AND no incorrect forces, corresponding to the experiment described in part (b)iii.



Examples 3 & 4



Example 5

Part C

1 point is earned: For using kinematic data to obtain acceleration, a, or showing that a = 0

Alternatively the point can be earned using the Work-Energy solution with kinetic energy K = (1/2)mv2

1 point is earned: For relating frictional force and coefficient of kinetic friction as Ff = μKFN

1 point is earned: For a correct representation of the normal force, correctly referring back to a measurement of the fan-block system

FN = mg if θ = 0 or FN = mg + Fsin θ if θ ≠ 0

1 point is earned: For using Newton's second law, consistently with normal force above, for the appropriate forces or force components in the direction of the fan's motion that would lead to the determination of the coefficient of kinetic friction

Fnet = ma

F - Ff = ma

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$\mu_k = \frac{F-ma}{mg}$ if $\theta = 0$ or $\mu_k = \frac{F\cos\theta - ma}{mg + F\sin\theta}$ if $\theta \neq 0$

Alternatively the point can be earned using the Work-Energy solution that would lead to the determination of the coefficient of kinetic friction.

Example 3 (continued from above): θ = 0 Determine the slope of each v vs. t graph, to get the acceleration for each trial, and average the accelerations. Call this average acceleration a. The kinetic frictional force exerted by track on block is Ff = μFn = μmg, where m is the mass of the fan-block as found in part (a), since the acceleration is purely horizontal and hence Fn and mg must balance. So, by Newton's second law, letting F denote the force exerted by the air on the fan found in part (a), we get

Fnet = ma

F - Ff = ma

F - μmg = ma

Solve for μ to get μ = (F-ma) / mg

Example 4 (continued from above):

θ = 0 Let d denote distance from block to second photogate and t denote the corresponding time. Since v0 = 0, d = 1/2 at2 and hence a = 2d/r2. For each trial, calculate that acceleration, and take the average, which becomes our best estimate of a. [Rest of solution is as in example 3]

Example 5 (continued from above): θ = 0 Since the acceleration is zero, the net horizontal force is zero. From Newton's second law,

Ff = μmg = Fcos θ

Solving for μk:

μk = (Fcos θ - ma)/(mg + Fsin θ) if θ ≠ 0



0	1	2	3	4

Student response earns 4 of the following 4 point(s)

1 point is earned: For using kinematic data to obtain acceleration, a, or showing that a = 0

Alternatively the point can be earned using the Work-Energy solution with kinetic energy K = (1/2)mv2

1 point is earned: For relating frictional force and coefficient of kinetic friction as Ff = μKFN

1 point is earned: For a correct representation of the normal force, correctly referring back to a measurement of the fan-block system

FN = mg if θ = 0 or FN = mg + Fsin θ if θ ≠ 0

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1 point is earned: For using Newton's second law, consistently with normal force above, for the appropriate forces or force components in the direction of the fan's motion that would lead to the determination of the coefficient of kinetic friction

Fnet = ma

F - Ff = ma

$\mu_k = \frac{F-ma}{mg}$ if $\theta = 0$ or $\mu_k = \frac{F\cos\theta - ma}{mg + F\sin\theta}$ if $\theta \neq 0$

Alternatively the point can be earned using the Work-Energy solution that would lead to the determination of the coefficient of kinetic friction.

> Example 3 (continued from above): θ = 0 Determine the slope of each v vs. t graph, to get the acceleration for each trial, and average the accelerations. Call this average acceleration a. The kinetic frictional force exerted by track on block is Ff = μFn = μmg, where m is the mass of the fan-block as found in part (a), since the acceleration is purely horizontal and hence Fn and mg must balance. So, by Newton's second law, letting F denote the force exerted by the air on the fan found in part (a), we get
>
> > Fnet = ma
> >
> > F - Ff = ma
> >
> > F - μmg = ma
>
> Solve for μ to get μ = (F-ma) / mg
>
> Example 4 (continued from above):
>
> θ = 0 Let d denote distance from block to second photogate and t denote the corresponding time. Since v0 = 0, d = 1/2 at2 and hence a = 2d/r2. For each trial, calculate that acceleration, and take the average, which becomes our best estimate of a. [Rest of solution is as in example 3]
>
> Example 5 (continued from above): θ = 0 Since the acceleration is zero, the net horizontal force is zero. From Newton's second law,
>
> Ff = μmg = Fcos θ
>
> Solving for μk:
>
> μk = (Fcos θ - ma)/(mg + Fsin θ) if θ ≠ 0

Part D

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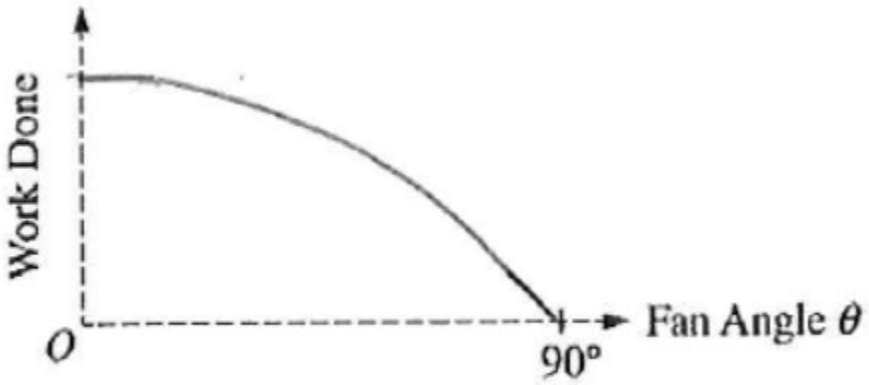


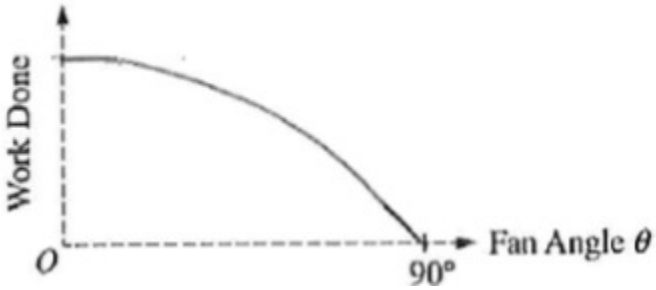
A correct graph shows the work done as proportional to cosθ

1 point is earned: For showing a curve that continually decreases from a maximum value at 0° to a minimum value of zero at 90°

1 point is earned: For showing a curve that is concave-downward (i.e. that has a negative second derivative)

0	1	2 ✓

Student response earns 2 of the following 2 point(s)



A correct graph shows the work done as proportional to cosθ

1 point is earned: For showing a curve that continually decreases from a maximum value at 0° to a minimum value of zero at 90°

1 point is earned: For showing a curve that is concave-downward (i.e. that has a negative second derivative)

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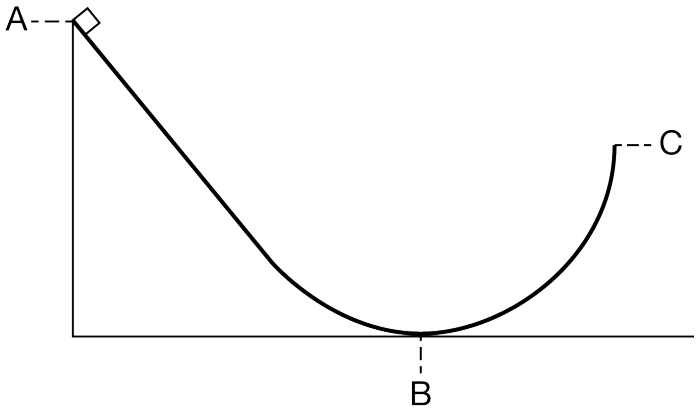
21. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)



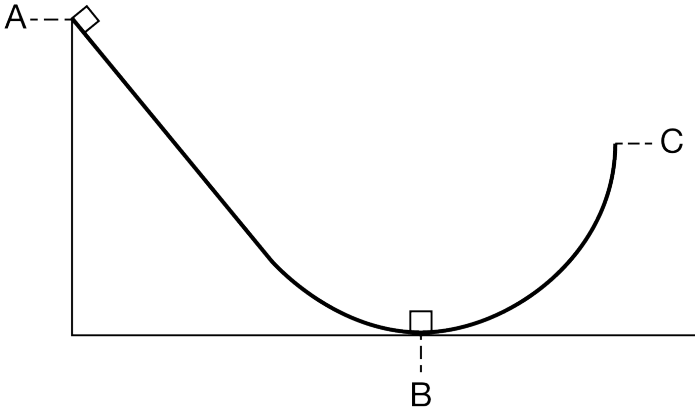
The figure above shows a track consisting of a straight section that connects to a circular section. The bottommost point of the track is at location B, and the track ends at location C, where the track is vertical. A block is released from rest at location A. There is negligible friction between the track and the box.

(a) Is the magnitude of the normal force exerted on the block by the track when the block is at location B greater than, less than, or equal to the normal force immediately before the block reaches location C? Justify your answer.

(b) In a coherent, paragraph-length response, describe the relative angle between the direction of the normal force exerted on the block by the track and the direction of the block's acceleration at the following times.

Immediately after the block is released at location A
When the block is at location B
Immediately before the block reaches location C

Specifically, indicate whether the normal force is parallel to the acceleration, perpendicular to the acceleration, or neither, at each location. Use physical principles to justify your responses.



(c) The scenario is repeated, but an identical block is at rest at location B, as shown above. When the block at

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location A is released from rest, it slides down the track and collides with and sticks to the identical block. The two blocks then continue sliding up the ramp. Is the maximum height reached by the two blocks greater than, less than, or equal to the maximum height reached by the block in the original scenario? Briefly explain your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the normal force at B is greater than it is immediately before C.
- ☐ 1 point is earned for a correct explanation that correctly uses either or both of the following effects:
  · At B, the speed *v* is greater so the net force $F_{\text{net}} = mv^2/r$ is greater
  · At B, the normal force must "overcome" the downward weight force, so the normal force $F_n = F_{\text{net}} + mg$ is greater.

Example Response:

At B, the speed v is greater, so the net force mv^2/r is greater, resulting in a greater normal force than at just before location C.

OR

At B, the normal force must "overcome" the downward weight force, so the normal force $(= F_{\text{net}} + mg)$ is greater than at just before location C.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the normal force is always perpendicular to the surface of the track.
- ☐ 1 point is earned for relating the acceleration to the net force.
- ☐ 1 point is earned for indicating the acceleration at location A is along the ramp and thus perpendicular to the normal force.
- ☐ 1 point is earned for indicating that at location B the acceleration is parallel to the normal force.

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- [ ] 1 point is earned for indicating that at position just before location C the acceleration is neither perpendicular nor parallel to the normal force.
- [ ] 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

The normal force is always perpendicular to the track surface. At location A, the block is accelerating from rest in a straight line down the ramp, so the acceleration is along the ramp and thus perpendicular to the normal force.

At B, the acceleration is upward, so it is in the same direction as the normal force.

Just before C, the forces of gravity (downward) and the normal force (perpendicular to the track) by vector summation result in the net force, which is in the direction of the acceleration. –The acceleration therefore is at an angle to the normal force, that is neither parallel nor perpendicular.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for indicating the height of the two blocks is less than that of the original block.
- [ ] 1 point is earned for a correct explanation of why the height is less than the original block.

Example Response:

In an inelastic collision linear momentum is constant and kinetic energy is not constant. The two-block system has less energy than the first block had at the same position. All of the kinetic energy at that point can now be converted to gravitational potential energy. Which means that the two-block system will not go as high as the first block.

OR

By conservation of momentum, the speed of the two blocks after the collision is half the speed of the original block. Since the blocks are going slower, they will not go as high as the original block did.

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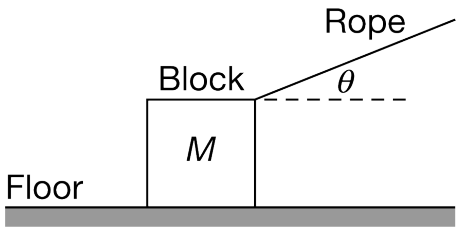
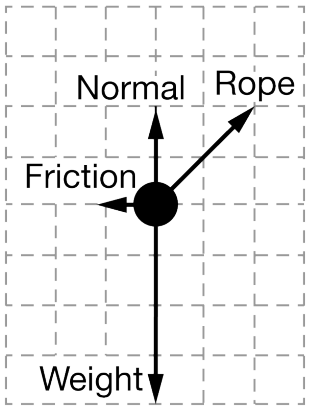
22. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

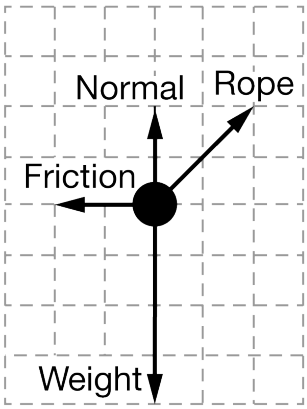


A block of mass M is pulled by a rope that makes an angle θ (theta) above the horizontal, as shown above. The coefficient of kinetic friction between the block and floor is μ (mu).

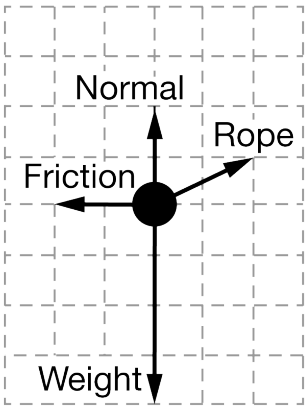
The block is initially at rest and starts accelerating to the right along the floor. Three students each draw a diagram to indicate all of the forces that are exerted on the block as it moves. In the students' diagrams, which are shown below, the lengths of the arrows indicate the relative magnitudes of the forces.



Student 1's Diagram



Student 2's Diagram



Student 3's Diagram

Only one of the students has drawn a diagram that could correctly represent the forces. The other two students' diagrams are incorrect.

(a) Which student has drawn a diagram that could correctly represent the forces exerted on the block?

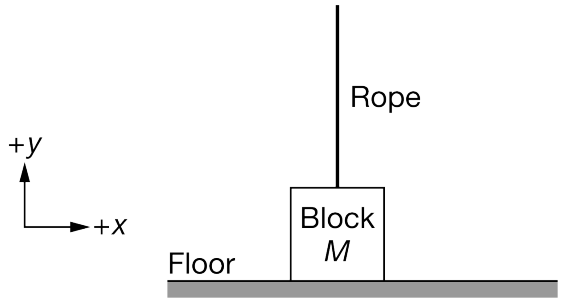
(b) For the incorrect student diagrams, explain why the diagram could not represent the forces exerted on the block. If there is more than one specific reason a diagram is incorrect, identify each of the reasons.

(c) Briefly explain why slightly increasing the angle θ (without changing the magnitude of the force exerted by the rope) could cause a decrease in the block's acceleration.

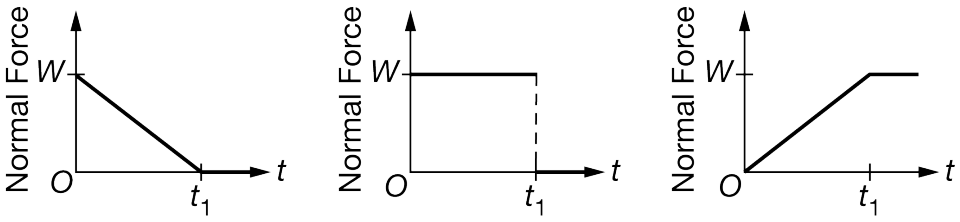
Suppose the rope is pulled with a force that has a magnitude that is one-third the weight of the block, or $\frac{Mg}{3}$. In this case, when the block accelerates, its acceleration is $a = g\left(\frac{1}{3}\cos\theta + \frac{\mu}{3}\sin\theta - \mu\right)$.

(d) Explain why the above equation is consistent with the claim that an increase in the angle θ could cause a

decrease in the acceleration.



The rope is now attached to the center of the top of the block, as shown above. The force exerted by the rope is initially zero newtons at time $t = 0$. The force steadily increases, becomes equal to the block's weight W at time t_1, and continues to increase further. Three students sketch the graphs shown below to represent the normal force exerted by the floor on the block as a function of time t. One of the graphs is consistent with this scenario, while the other graphs are not consistent with this scenario.



Graph 1 Graph 2 Graph 3

(e) Which graph is consistent with the scenario in which the force exerted by the rope on the block steadily increases?

(f) Identify all features of the other two graphs that make them inconsistent with this scenario. Explain your reasoning.

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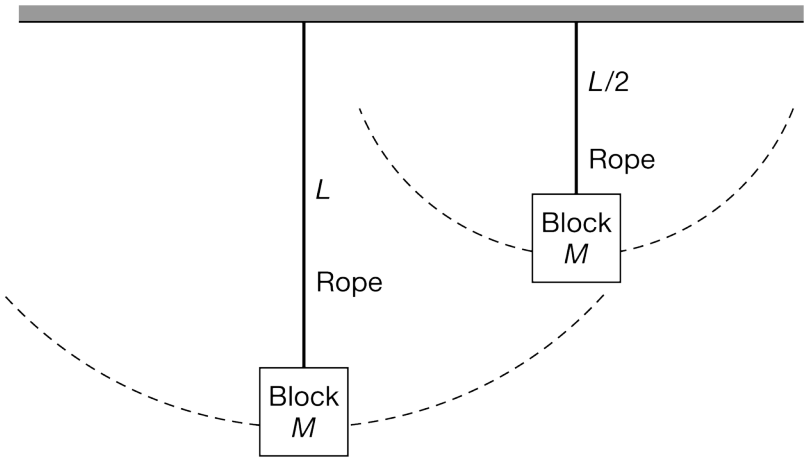

Note: Figure not drawn to scale.

With one end of the rope still attached to the block, the other end of the rope is attached to a ceiling to create a pendulum with length L, where L is much greater than the dimensions of the block. A second pendulum is made from an identical block and a rope of length $\frac{L}{2}$, as shown above. Both pendulums are displaced by the same small angle and released from rest.

(g) Does the longer pendulum or the shorter pendulum have a greater period of oscillation? Briefly explain your choice without reference to equations, but by describing the restoring forces exerted on and motion of each block.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating that Student 1 drew a correct diagram.

Example Response:

The diagram for student 1 is correct.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that Student 2's diagram has equal left and right forces and thus would result in a constant horizontal velocity or no horizontal acceleration.
- ☐ 1 point is earned for indicating that Student 3's diagram has equal left and right forces and thus would result in a constant horizontal velocity or no horizontal acceleration.
- ☐ 1 point is earned for indicating that Student 3's diagram would result in a downward acceleration component since the sum of the upward components is less than the force of gravity.

Example Response:

Student 2's diagram is incorrect because it has equal left and right forces, which would cause the block to move at a constant speed. Student 3's diagram is not correct because it has equal left and right forces as well. Student 3's diagram is also incorrect because the downward components are greater than the upward components. This would cause a downward acceleration, which is not occurring.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the horizontal component of the force exerted by the rope becomes smaller.
- ☐ 1 point is earned for indicating that the net force, therefore, decreases.

Example Response:

If the angle is increased without changing the magnitude, the forward horizontal force would decrease, so the net force on the block could decrease. This would result in a smaller acceleration.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the cosine term decreases when θ increases.
- ☐ 1 point is earned for indicating that, if the decrease in the cosine term is greater than the increase in the sine term, the acceleration will decrease.

Example Response:

As the angle increases, two terms in the expression are affected. The cosine term will decrease, and the sine of the angle will increase. If the decrease in the cosine term is greater than the increase in the sine term, the acceleration will decrease.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for identifying Graph 1 as consistent with the scenario in which the force exerted by the rope on the block steadily increases.

Example Response:

Graph 1 is consistent with the scenario in which the force exerted by the rope on the block steadily increases

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the normal force is equal to the weight of the block minus the tension force, until the tension equals the weight.

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- [ ] 1 point is earned for indicating that graph 2 is incorrect because the normal force does not remain constant before time t_1.
- [ ] 1 point is earned for indicating that graph 3 is incorrect because the normal force does not increase before time t_1.
- [ ] 1 point is earned for indicating that graph 3 is incorrect because the normal force does not equal the weight after time t_1.

Example Response:

The normal force on the block is equal to the difference between the weight and the tension force, as long as the tension force is less than the weight. If the tension force exceeds the weight, then the normal force becomes zero because the block is lifted off the floor. Thus, graph 2 is incorrect because it indicates the normal force is constant until the tension equals the weight. And graph 3 is incorrect because the normal force is depicted as increasing until it equals the weight rather than decreasing until it is zero.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for indicating that both pendulums have the same maximum restoring force at the same maximum angular displacement from equilibrium.
- [ ] 1 point is earned for indicating that the path the longer pendulum takes is a greater distance so for the same acceleration, it takes a greater time, and thus the longer pendulum has a greater period.

Example Response:

Since both are displaced the same angle, both have the same maximum restoring force, and thus the same maximum acceleration. The longer pendulum travels a longer path, and thus it will take a longer time. Thus, the period of the longer pendulum is greater.

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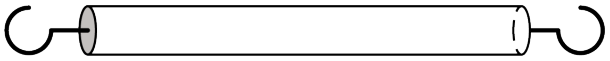
23. This question is a long free-response question. Show your work for each part of the question.



(12 points, suggested time 25 minutes)

A group of students is investigating how the thickness of a plastic rod affects the maximum force F_{max} with which the rod can be pulled without breaking. Two students are discussing models to represent how F_{max} depends on rod thickness.
Student A claims that F_{max} is directly proportional to the radius of the rod.
Student B claims that F_{max} is directly proportional to the cross-sectional area of the rod—the area of the base of the cylinder, shaded gray in the figure above.

(a) The students have a collection of many rods of the same material. The rods are all the same length but come in a range of six different thicknesses. Design an experimental procedure to determine which student's model, if either, correctly represents how F_{max} depends on rod thickness.

In the table below, list the quantities that would be measured in your experiment. Define a symbol to represent each quantity, and also list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

Quantity to be Measured	Symbol for Quantity	Equipment for Measurement

Describe the overall procedure to be used, referring to the table. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table and/or include a simple diagram of the setup.

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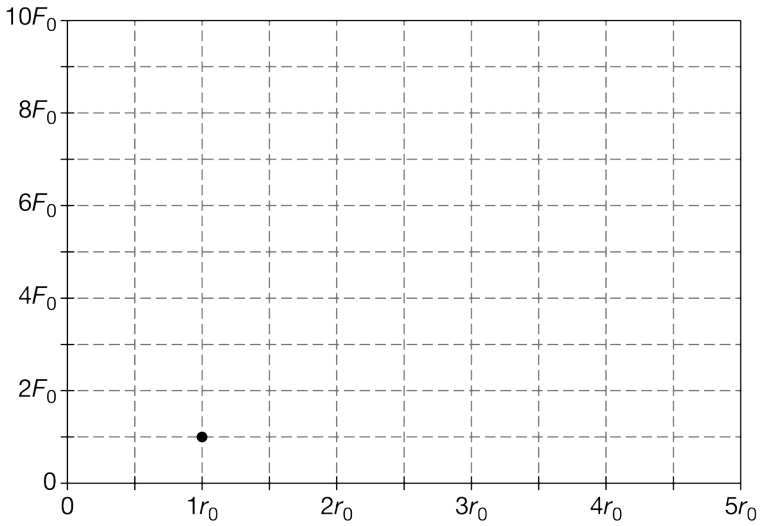
(b) For a rod of radius r_0, it is determined that $F_{\max}$ is F_0, as indicated by the dot on the grid below. On the grid, draw and label graphs corresponding to the two students' models of the dependence of $F_{\max}$ on rod radius. Clearly label each graph "A" or "B," corresponding to the appropriate model.



The table below shows results of measurements taken by another group of students for rods of different thicknesses.

Rod radius (mm)	0.5	1.0	1.5	2.0	2.5
$F_{\max}$ (N)	40	120	320	520	900

(c) On the grid below, plot the data points from the table. Clearly scale and label all axes, including units. Draw either a straight line or a curve that best represents the data.

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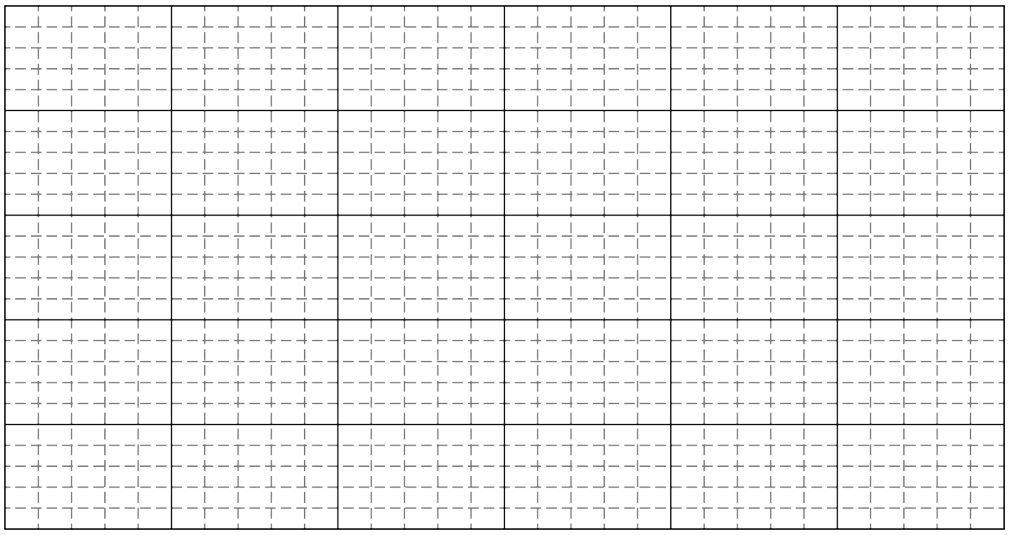

(d) Which student's model is more closely represented by the evidence shown in the graph you drew in part (c) ?

____ Student A's model: F_{max} is directly proportional to the radius of the rod.

____ Student B's model: F_{max} is directly proportional to the cross-sectional area of the rod.

Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for measuring the radius or diameter of rods with different radii using an appropriate tool.
- ☐ 1 point is earned for measuring force using an appropriate tool.
- ☐ 1 point is earned for a plausible/practical way to directly or indirectly determine F_{max} for a given rod.
- ☐ 1 point is earned for attempting to reduce experimental uncertainty in an experiment that involves breaking the rods.

Example Response:

Measure the diameter D of each rod with a ruler.

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Students should pull on the rod with the force probe until the rod breaks.

Record the force $F_{\max}$ just before breaking.

Repeat each trial several times to reduce error.

Then trade for a new set of rods with different radii.

Repeat this experiment for several different radii rods.

Part B

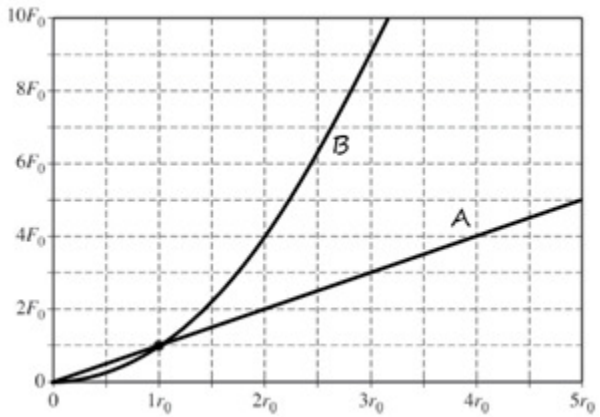
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a straight-line graph marked "A" with a slope of $\frac{F_0}{r_0}$
- ☐ 1 point is earned for a graph marked "B" that is concave up
- ☐ 1 point is earned for a graph marked "B" that shows a quadratic relationship at the correct points
- ☐ 1 point is earned for two graphs that both contain the point (r_0, F_0)

Example Response:



Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

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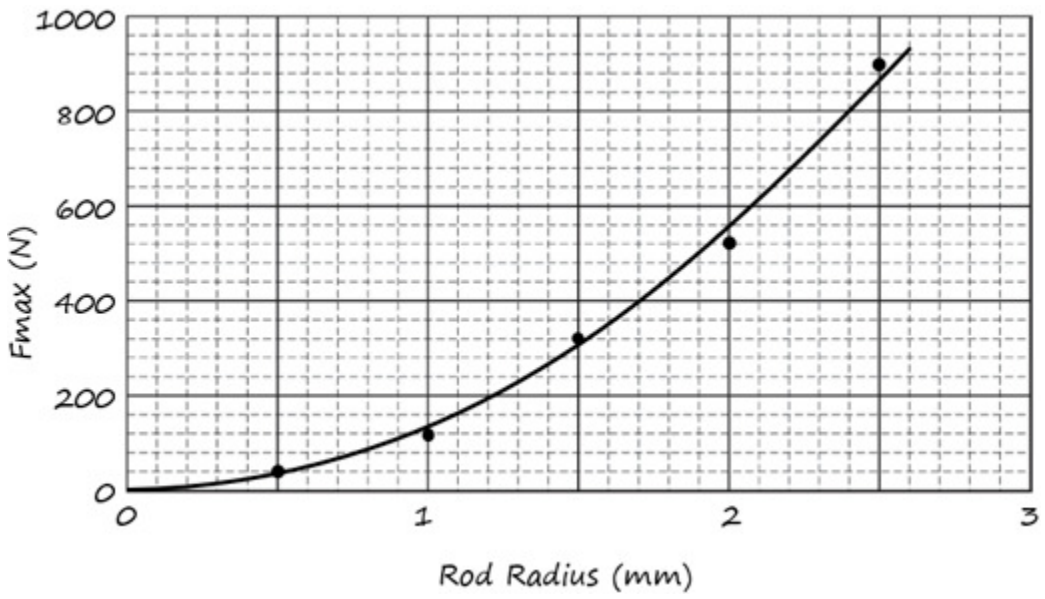
The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for linear scales with appropriate labels and units
  AND
  for a graph where the plotted points cover at least half of the grid's width and height.
- ☐ 1 point is earned for plotting the points correctly.
- ☐ 1 point is earned for drawing a reasonable best-fit curve.

Example Response:



Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for identifying Model B and for indicating that $F_{\max}$ increases as the square of the radius increases

Example Response:

In this graph, $F_{\max}$ seems to be proportional to R^2, so that if we graph $F_{\max}$ on the vertical axis and R^2 on the horizontal axis, it should show a linear graph.

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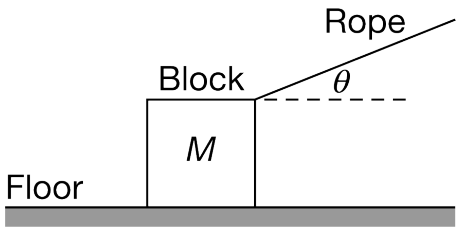
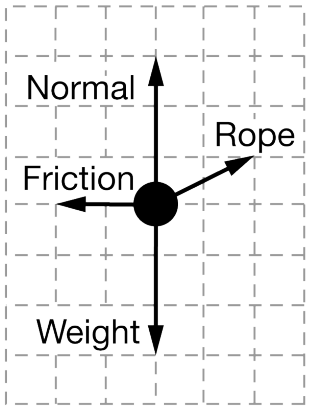
24. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

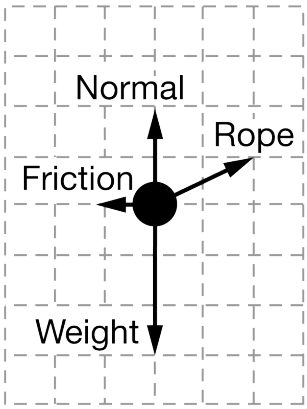


A block of mass M is pulled by a rope that makes an angle θ (theta) above the horizontal, as shown above. The coefficient of kinetic friction between the block and floor is μ (mu).

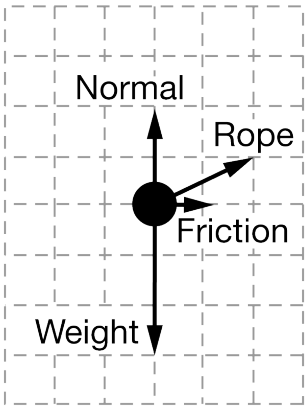
The block is initially at rest and starts accelerating to the right along the floor. Three students each draw a diagram to indicate all of the forces that are exerted on the block as it moves. In the students' diagrams, which are shown below, the lengths of the arrows indicate the relative magnitudes of the forces.



Student 1's Diagram



Student 2's Diagram



Student 3's Diagram

Only one of the students has drawn a diagram that could correctly represent the forces. The other two students' diagrams are incorrect.

(a) Which student has drawn a diagram that could correctly represent the forces exerted on the block?

(b) For the incorrect student diagrams, explain why the diagram could not represent the forces exerted on the block. If there is more than one specific reason a diagram is incorrect, identify each of the reasons.

(c) Briefly explain why slightly increasing the force F exerted by the rope on the block (without changing the angle θ) would cause a decrease in the friction force.

(d) The friction force f is $f = \mu(W - F\sin\theta)$, where W is the weight of the block and F is the force exerted by the rope. Explain why this equation is consistent with the claim that an increase in F would cause a decrease in the friction force.

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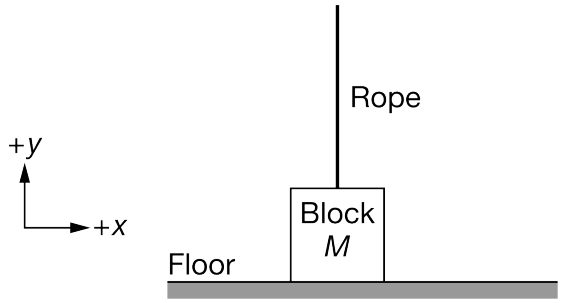
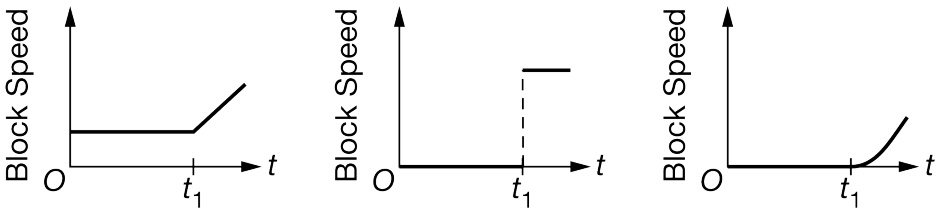


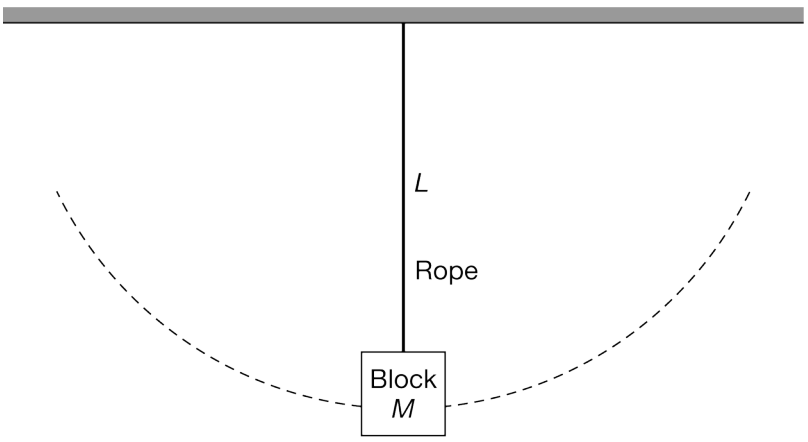
The rope is now attached to the center of the top of the block, as shown above. The force exerted by the rope is initially zero newtons at time $t = 0$. The force steadily increases, becomes equal to the block's weight W at time t_1, and continues to increase further. Three students sketch the graphs shown below to represent the speed of the block as a function of time t. One of the graphs is consistent with this scenario, while the other graphs are not consistent with this scenario.



Graph 1 Graph 2 Graph 3

(e) Which graph is consistent with the scenario in which the force exerted by the rope on the block steadily increases?

(f) Identify all features of the other two graphs that make them inconsistent with this scenario. Explain your reasoning.



Note: Figure not drawn to scale.

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With one end of the rope still attached to the block, the other end of the rope is attached to a ceiling to create a pendulum with length L, where L is much greater than the dimensions of the block. A second identical pendulum is located on the Moon, where the acceleration due to gravity is less than that on Earth. Both pendulums are displaced by the same small angle and released from rest.

(g) Does the pendulum on Earth or the pendulum on the Moon have a greater period of oscillation? Briefly explain your choice without reference to equations, but by describing the restoring forces exerted on and motion of each block.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating that Student 2 drew a correct diagram.

Example Response:

The diagram for student 2 is correct.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that Student 1's diagram has equal left and right forces and thus would result in a constant horizontal velocity or no horizontal acceleration.
- ☐ 1 point is earned for indicating that Student 1's diagram would result in an upward acceleration component since the sum of the upward components is greater than the force of gravity.
- ☐ 1 point is earned for indicating that Student 3's diagram has friction in the incorrect direction.

Example Response:

Student 3 has correctly balanced the vertical forces but has the friction in the same direction as the motion. Student 1 has vertical components in the upward direction that are greater than gravity, so the block would have an upward acceleration. Also, student 1 has equal horizontal components which would not cause an acceleration to the right.

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Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that as the pulling force increases, its vertical component increases, decreasing the normal force on the block.
- ☐ 1 point is earned for indicating that since the friction force is proportional to the normal force, the friction force decreases.

Example Response:

As the pulling force increases, its vertical component increases, decreasing the normal force on the block. Since the friction force is proportional to the normal force, the friction force decreases.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for any attempt to use a functional dependence between the friction force and the force *F*.
- ☐ 1 point is earned for indicating that the term with F is subtracted, or that the expression $(W - F\sin\theta)$ decreases when *F* increases.

Example Response:

In the given expression for friction, there is a term that depends on the pulling force F, but that term is subtracted from the weight term. So as F increases, friction decreases.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1 ✓

The response accurately includes

- ☐ 1 point is earned for identifying Graph 3 as consistent with the scenario in which the force exerted by the rope on the block steadily increases.

Example Response:

Graph 3 is consistent with the scenario in which the force exerted by the rope on the block steadily increases.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that graph 1 shows an initial non-zero speed even though the initial speed is zero.
  Note: If it was indicated that there is ONLY a horizontal component of the velocity before the vertical force of the rope acts on the block (from the previous part of the question), this point can be earned for thinking that graph 1 is correct in the first segment but graphs 2 & 3 are not correct in the first segment.
- ☐ 1 point is earned for indicating that the speed after time in graph 1 is incorrect because the upward force is not constant, so the acceleration (slope of speed graph) should not be constant.
- ☐ 1 point is earned for indicating that t_1 graph 2 is incorrect because of the discontinuity in the graph at t_1.
- ☐ 1 point is earned for indicating that graph 2 is incorrect because the speed is not constant after time t_1, it increases since there is a net force.

Example Response:

Graph 1 is incorrect because the speed should be zero before time t_1 and after t_1 it should have an increasing slope since the net force is increasing. Graph 2 is incorrect because, although the speed is initially zero, the speed is not constant after time t_1 because there is an upward net force.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: A response *can* mention the equation for a pendulum, and still get full credit as long as the points described below are earned.

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0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the pendulum on the Earth has a greater restoring force at maximum displacement from equilibrium.
- ☐ 1 point is earned for indicating that since the two pendulums have the same length, the amplitude of the motion is the same, and thus the pendulum on the moon, with a smaller acceleration, will take longer to complete once cycle.

Example Response:

Since both pendulums have the same length, they will both have the same amplitude. But the pendulum on the Earth will have a greater restoring force and acceleration, so it will take less time to return to equilibrium. Thus, the pendulum on the moon will have a greater period.

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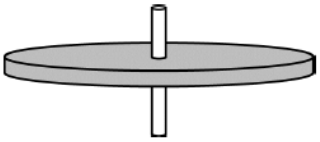
25.

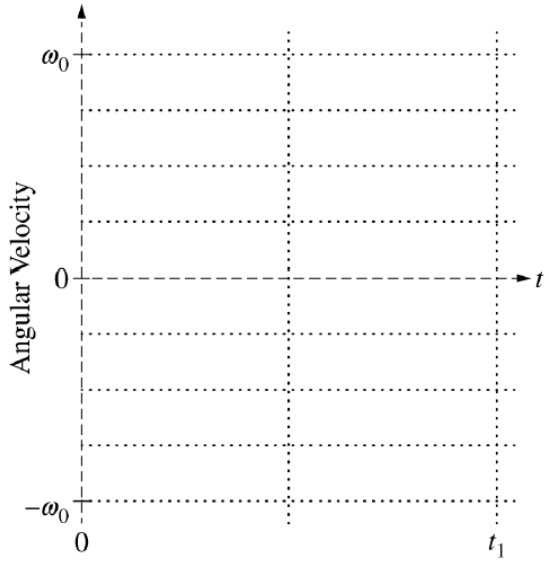


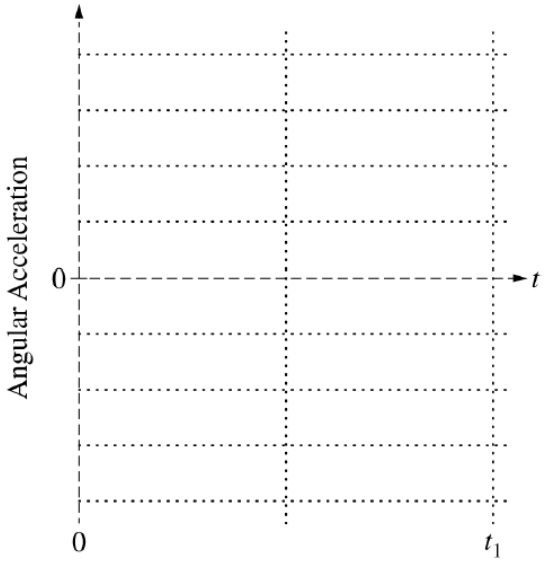
The disk shown above spins about the axle at its center. A student's experiments reveal that, while the disk is spinning, friction between the axle and the disk exerts a constant torque on the disk.

a. At time $t = 0$ the disk has an initial counterclockwise (positive) angular velocity ω_0.The disk later comes to rest at time $t = t_1$.

   i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time t from $t = 0$ until the disk comes to rest at time $t = t_1$.

   ii. On the grid at right below, sketch the disk's angular acceleration as a function of time t from $t = 0$ until the disk comes to rest at time $t = t_1$.





b. The magnitude of the frictional torque exerted on the disk is τ_0. Derive an equation for the rotational inertia I of the disk in terms of τ_0, ω_0, t_1, and physical constants, as appropriate.

c. In another experiment, the disk again has an initial positive angular velocity ω_0 at time $t = 0$. At time $t = \frac{1}{2}t_1$, the student starts dripping oil on the contact surface between the axle and the disk to reduce the friction. As time passes, more and more oil reaches that contact surface, reducing the friction even further.

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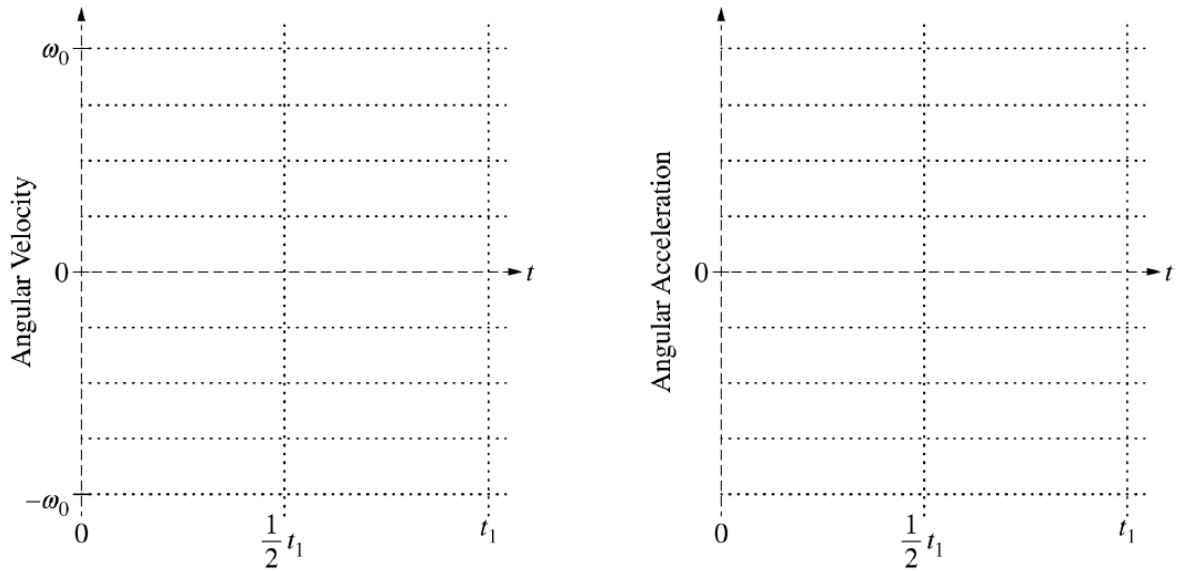
i. On the grid at left below, sketch a graph that could represent the disk's angular velocity as a function of time from $t = 0$ to $t = t_1$, which is the time at which the disk came to rest in part (a).

ii. On the grid at right below, sketch the disk's angular acceleration as a function of time from $t = 0$ to $t = t_1$.



d. The student is trying to mathematically model the magnitude τ of the torque exerted by the axle on the disk when the oil is present at times $t > \frac{1}{2}t_1$. The student writes down the following two equations, each of which includes a positive constant (C_1 or C_2) with appropriate units.

$$(1)\ \tau = C_1\left(t - \tfrac{1}{2}t_1\right) \text{ (for } t > \tfrac{1}{2}t_1\text{)}$$
$$(2)\ \tau = \frac{C_2}{(t+\frac{1}{2}t_1)} \text{ (for } t > \tfrac{1}{2}t_1\text{)}$$

Which equation better mathematically models this experiment?

____Equation (1) ____Equation (2)

Briefly explain why the equation you selected is plausible and why the other equation is not plausible.

Part A

PART i

1 point is earned: For a curve that has an angular velocity of +ω0 at time t = 0 and decreases to zero at time t = t1

1 point is earned: For a curve that is a straight line with a negative slope showing the angular velocity approaching zero (can be a positive slope, if the initial angular velocity on the graph is negative)

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PART ii

1 point is earned: For a curve that is negative for the entire time

1 point is earned: For a curve that is a constant nonzero function

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

PART i

1 point is earned: For a curve that has an angular velocity of +ω0 at time t = 0 and decreases to zero at time t = t1

1 point is earned: For a curve that is a straight line with a negative slope showing the angular velocity approaching zero (can be a positive slope, if the initial angular velocity on the graph is negative)

PART ii

1 point is earned: For a curve that is negative for the entire time

1 point is earned: For a curve that is a constant nonzero function

Part B

1 point is earned: For using an equation for the rotational version of Newton's second law

1 point is earned: For using an appropriate rotational kinematics equation α = Δω/Δt

1 point is earned: For a correct answer in terms of the listed quantities, derived from first principles $I = \frac{\tau_0 t_1}{\omega_0}$

Note: This point is still earned if there is a minus sign

Alternate solution using angular momentum and rotational impulse:

1 point is earned: For defining and using angular momentum L = iω

1 point is earned: For using rotational impulse ΔL= τΔt

1 point is earned: For a correct answer in terms of the listed quantities, derived from first principles $I = \frac{\tau_0 t_1}{\omega_0}$

Note: This point is still earned if there is a minus sign

0	1	2	3 ✓

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Student response earns 0 of the following 3 point(s)

1 point is earned: For using an equation for the rotational version of Newton's second law

1 point is earned: For using an appropriate rotational kinematics equation $\alpha = \Delta\omega/\Delta t$

1 point is earned: For a correct answer in terms of the listed quantities, derived from first principles $I = \frac{\tau_0 t_1}{\omega_0}$

Note: This point is still earned if there is a minus sign

Alternate solution using angular momentum and rotational impulse:

1 point is earned: For defining and using angular momentum $L = i\omega$

1 point is earned: For using rotational impulse $\Delta L = \tau\Delta t$

1 point is earned: For a correct answer in terms of the listed quantities, derived from first principles $I = \frac{\tau_0 t_1}{\omega_0}$

Note: This point is still earned if there is a minus sign

Part C

PART i

1 point is earned: For curve with a clear change of slope or curvature at ½t1 and showing a decrease in speed thereafter

1 point is earned: For a curve that indicates slowing at a decreasing rate between times ½ t1 and t1

1 point is earned: For a curve that does not reach zero at or before time t1

PART ii

1 point is earned: For a curve with decreasing magnitude between times ½ t1 and t1

Note: The acceleration may reach zero at or before time t1. If so, it must remain zero for

the remaining time.

0	1	2	3	4	5 ✓

Student response earns 5 of the following 5 point(s)

PART i

1 point is earned: For curve with a clear change of slope or curvature at ½t1 and showing a decrease in speed thereafter

1 point is earned: For a curve that indicates slowing at a decreasing rate between times ½ t1 and t1

1 point is earned: For a curve that does not reach zero at or before time t1

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PART ii

1 point is earned: For a curve with decreasing magnitude between times ½ t1 and t1

Note: The acceleration may reach zero at or before time t1. If so, it must remain zero for

the remaining time.

Part D

Correct answer: “Equation (2)”

Note: If the wrong selection is made, the explanation is not graded.

1 point is earned: For stating that Equation (2) is plausible because the frictional torque decreases with increasing time, whereas in Equation (1) torque increases with increasing time

Examples:

Equation (2) because ◈ decreases. In Equation (1), it doesn’t.

Equation (2) is plausible because the frictional torque decreases as more oil reaches the contact surface over time. Equation (1) is not plausible because it shows friction increasing as more oil reaches the surface over time.

0	1 ✓

Student response earns 1 of the following 1 point(s)

Correct answer: “Equation (2)”

Note: If the wrong selection is made, the explanation is not graded.

1 point is earned: For stating that Equation (2) is plausible because the frictional torque decreases with increasing time, whereas in Equation (1) torque increases with increasing time

Examples:

Equation (2) because ◈ decreases. In Equation (1), it doesn’t.

Equation (2) is plausible because the frictional torque decreases as more oil reaches the contact surface over time. Equation (1) is not plausible because it shows friction increasing as more oil reaches the surface over time.

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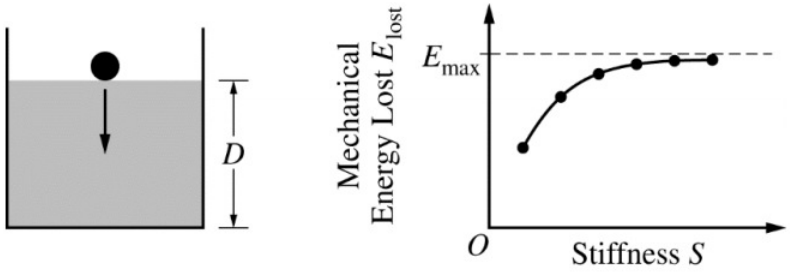
26.



Food scientists have created a new oil. At room temperature, the oil is a liquid. As the oil gets colder, however, it stiffens (thickens) into a sticky gel. To explore the properties of the oil, the scientists fill a container with the oil to a height D, as shown in the figure above on the left. They drop a small steel ball of mass M from rest at the top of the oil. Using video to capture the ball's motion, the scientists calculate E_{lost} , the mechanical energy lost by the ball-Earth system from the time the ball enters the oil to the time just before the ball strikes the bottom of the container. The scientists also define the "stiffness" S of the oil as a quantity proportional to the force required to move a rod through the oil at a standard constant speed. The scientists calculate E_{lost} and S at several different temperatures, ranging from room temperature to the lowest temperature at which the ball still falls through the oil. The graph above on the right shows E_{lost} as a function of S for the calculated data points and a best-fit curve.

a. Give a physical reason why the curve in the graph would not reach the vertical (E_{lost}) axis even if the scientists had taken data over a broader range of temperatures.

b. As S increases, E_{lost} approaches a maximum value labeled E_{max} on the graph above. Write an equation for E_{max} in terms of *M, D*, and physical constants, as appropriate.

   Justify your answer.

c. One of the scientists, in trying to represent the relationship between the oil stiffness and the mechanical energy lost, writes down the equation $2\ E_{lost} = CS^2$, where *C* is a constant with appropriate units. Another scientist points out that this equation cannot be correct. Give two reasons why the equation cannot be correct.

d. Further attempting to model the ball's motion, the scientists write the following equation for the time t the ball takes to fall through the oil: $t = Z/S$, where Z is a constant with appropriate units. Is this equation plausible—in other words, does it make physical sense?

   ____ Plausible ____ Not plausible

   Briefly explain your reasoning.

e. ΔK is the change in kinetic energy of the ball between the time it is released from rest and the time just before the ball strikes the bottom of the container. On the axes below, sketch ΔK as a function of S, the oil stiffness.

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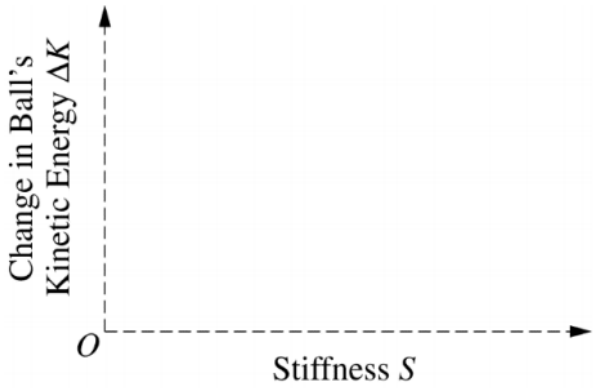

Part A

1 point is earned: For indicating (explicitly or implicitly) that intersecting the vertical axis corresponds to zero stiffness

1 point is earned: For correctly indicating that no measurement of stiffness could be zero

Example: The vertical axis is where S = 0 , indicating a substance puts up no resistance to getting stirred. But even at room temperature the oil has some stiffness. Hence, the lowest measured stiffness is positive, not zero, and the graph reflects this.

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

1 point is earned: For indicating (explicitly or implicitly) that intersecting the vertical axis corresponds to zero stiffness

1 point is earned: For correctly indicating that no measurement of stiffness could be zero

Example: The vertical axis is where S = 0 , indicating a substance puts up no resistance to getting stirred. But even at room temperature the oil has some stiffness. Hence, the lowest measured stiffness is positive, not zero, and the graph reflects this.

Part B

1 point is earned: For a correct equation: E_{max} = MgD OR E_{max} = -MgD

1 point is earned: For stating that the maximum energy that can be lost is equal to the initial energy of the system

1 point is earned: For stating that the initial energy of the system is the initial gravitational potential energy

Example: The maximum energy that can be lost is equal to the initial energy, which is all gravitational potential energy: MgD

Alternate solution for the last 2 points

chapter2parta

1 point is earned: For a statement or the application of mechanical energy as the sum of kinetic and potential energies

1 point is earned: For combining the change in the gravitational potential energy with a small change in kinetic energy as the ball falls to make a valid argument for E_{max}

> Example: In a very stiff oil that the ball can barely fall through, it gains essentially no kinetic energy. In this case the overall change in mechanical energy, $\Delta K + \Delta U$, is approximately equal to ΔU because ΔK is essentially zero.
>
> So, $E_{max} = |\Delta U| = MgD$.

0	1	2	3 ✓

Student response earns 3 of the following 3 point(s)

1 point is earned: For a correct equation: $E_{max} = MgD$ OR $E_{max} = -MgD$

1 point is earned: For stating that the maximum energy that can be lost is equal to the initial energy of the system

1 point is earned: For stating that the initial energy of the system is the initial gravitational potential energy

Example: The maximum energy that can be lost is equal to the initial energy, which is all gravitational potential energy: MgD

Alternate solution for the last 2 points

1 point is earned: For a statement or the application of mechanical energy as the sum of kinetic and potential energies

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> Example: In a very stiff oil that the ball can barely fall through, it gains essentially no kinetic energy. In this case the overall change in mechanical energy, $\Delta K + \Delta U$, is approximately equal to ΔU because ΔK is essentially zero.
>
> So, $E_{max} = |\Delta U| = MgD$.

Part C

2 points are earned: For each of the following, up to a maximum of 2 points

1 point for each valid, independent statement that the equation does not match the graph (maximum of 2 points).

1 point for a statement that demonstrates that E_{lost} cannot increase indefinitely.

> Example statements (1 point each, any two could be combined for 2 points): The equation indicates that E_{lost} increases indefinitely with S, but the graph has an upper limit for E_{lost}. The equation represents a parabola (or is concave-up), which does not match the graph. It doesn't make physical sense that E_{lost} could be more than the available potential energy MgD.

chapter2parta

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

2 points are earned: For each of the following, up to a maximum of 2 points

1 point for each valid, independent statement that the equation does not match the graph (maximum of 2 points).

1 point for a statement that demonstrates that E_{lost} cannot increase indefinitely.

> Example statements (1 point each, any two could be combined for 2 points): The equation indicates that E_{lost} increases indefinitely with S, but the graph has an upper limit for E_{lost}. The equation represents a parabola (or is concave-up), which does not match the graph. It doesn't make physical sense that E_{lost} could be more than the available potential energy MgD.

Part D

Correct answer: "Not plausible"

Note: The explanation can earn the first point if the incorrect selection is made.

1 point is earned: For a statement or implication that the equation indicates that t increases as S decreases

1 point is earned: For a statement or implication that t should be larger as S increases

Note: If only one of the above statements is written explicitly, and "Not plausible" is selected, the second point is earned by implication.

> Example 1 (2 points): "Not plausible". According to the equation, a less stiff oil makes the sphere take more time to reach the bottom.
>
> Example 2 (2 points): "Not plausible". A greater stiffness means that the ball should take a longer time to fall through the oil.
>
> Example 3 (1 point): "Plausible". The equation says that bigger S means bigger time, which agrees with the idea that a thicker oil resists the ball's motion more.

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

Correct answer: "Not plausible"

Note: The explanation can earn the first point if the incorrect selection is made.

chapter2parta

1 point is earned: For a statement or implication that the equation indicates that t increases as S decreases

1 point is earned: For a statement or implication that t should be larger as S increases

Note: If only one of the above statements is written explicitly, and “Not plausible” is selected, the second point is earned by implication.

> Example 1 (2 points): “Not plausible”. According to the equation, a less stiff oil makes the sphere take more time to reach the bottom.
>
> Example 2 (2 points): “Not plausible”. A greater stiffness means that the ball should take a longer time to fall through the oil.
>
> Example 3 (1 point): “Plausible”. The equation says that bigger S means bigger time, which agrees with the idea that a thicker oil resists the ball’s motion more.

Part E

1 point is earned: For drawing a concave-upward curve that approaches the horizontal at higher S

1 point is earned: For having the maximum value of ΔK at the lowest graphed value of S, which can be S = 0 or a positive value

Note: This point is not earned if the graph indicates that ΔK approaches infinity as S approaches zero.

1 point is earned: For high values of S, the graph approaches and does not cross the horizontal axis

Note: This point is earned if the graph touches the horizontal axis without crossing it.



0	1	2	3

Student response earns 3 of the following 3 point(s)

1 point is earned: For drawing a concave-upward curve that approaches the horizontal at higher S

1 point is earned: For having the maximum value of ΔK at the lowest graphed value of S, which can be S = 0 or a positive value

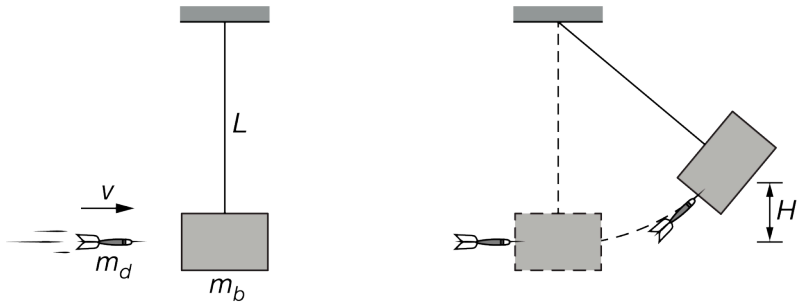
Note: This point is not earned if the graph indicates that ΔK approaches infinity as S approaches zero.

1 point is earned: For high values of S, the graph approaches and does not cross the horizontal axis

Note: This point is earned if the graph touches the horizontal axis without crossing it.

chapter2parta

27. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown above at left. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown above at right. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, if either, experiences an acceleration of larger magnitude during the collision. If the acceleration is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the experiment were repeated with a block with mass greater than m_b, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the tension in the string at the moment immediately after the collision and state the following:

Student 1: The tension in the string immediately after the collision is greater than the weight of the dart-block system because the acceleration is upward at that instant.

Student 2: The tension in the string is equal to the weight of the block-dart system because there is no vertical acceleration immediately after the collision.

(c) Which underlined phrase or phrases, if any, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if any, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if any, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases, if any, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{m_d v}{m_d + m_b}\right)^2$. Explain how this equation supports your claim in part (b).

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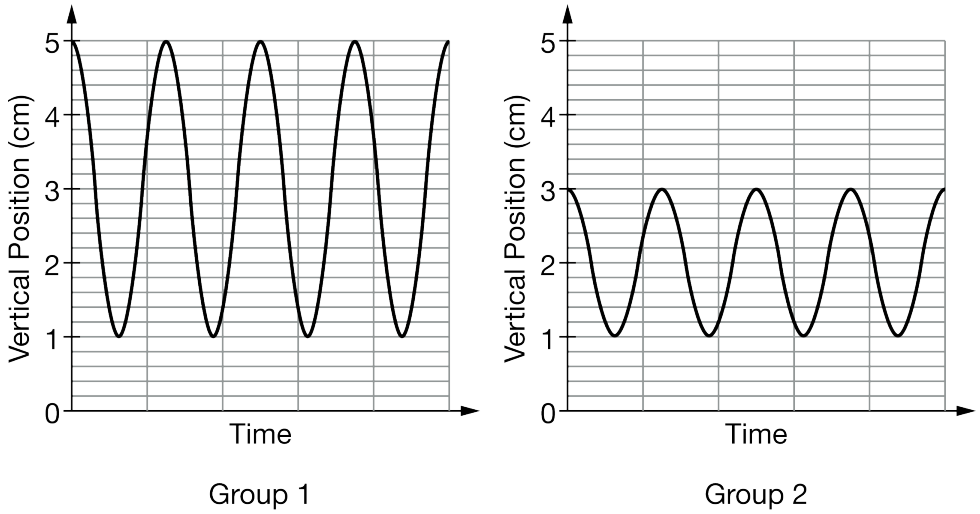
(h) Consider the case in which the collision between the dart and the block is perfectly elastic (the dart bounces off the block to the left). In this case, is the speed of the block immediately after the collision greater than, less than, or the same as the speed of the block in the original case? Justify your answer.

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart speed or the string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a statement that the dart experiences the larger acceleration because it has the smaller mass with equal force exerted on it because of Newton's third law.

Example Response:

Both the dart and the block experience the same force because they exert equal and opposite forces on each other because of Newton's third law, however the mass of the dart is smaller and so experiences a larger acceleration.

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Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the final height is smaller.
- ☐ 1 point is earned for a statement of conservation of momentum to indicate that the final speed of the dart-block system is smaller **AND** an indication of conservation of energy leading to a smaller height.

Example Response:

Because the momentum of the dart before the collision must be equal to the momentum of the dart-block system after the collision, a larger block mass will lead to a smaller dart-block speed after the collision because of conservation of momentum. Because the dart-block system is traveling slower after the collision, it has less kinetic energy which means that the dart-block system will not swing as high.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating both of Student 1's statements are correct.

Example Responses:

Student 1's entire statement is correct

OR

Both statements

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating student 1 has no incorrect statements.

Example Response:

None

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating student 2 has no correct statements.

Example Response:

None

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating both of Student 2's statements are incorrect.

Example Responses:

Student 2's entire statement is incorrect.

OR

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Both statements

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement including functional dependence of the variables.
- ☐ 1 point is earned for correctly asserting that as the mass of the block increases in the equation H decreases.

Example Response:

This equation matches my reasoning in part b – if the mass of the block increases, the height of the swing decreases because the mass of the block is in the denominator.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for "greater", with an attempt at an explanation.
- ☐ 1 point is earned for a statement indicating a larger change in momentum of the dart.
- ☐ 1 point is earned for a statement using conservation of momentum to show that the block momentum/ speed is greater.

Example Response:

When the dart bounces off the block, it has a larger change in momentum (going from v to $-v$ instead of v to a smaller v), which means that the block also has a larger change in momentum (since both objects receive the same impulse and experience the same change in momentum). Because the block has a greater change in momentum, it has a larger speed after the collision than in the original case.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

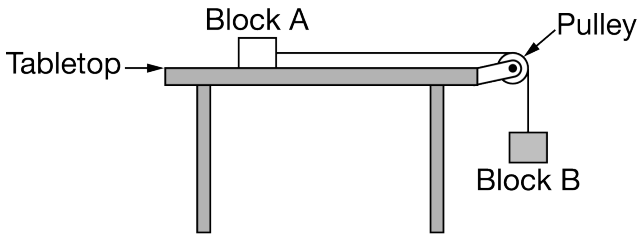
- ☐ 1 point is earned for recognizing that the amplitude of the graph for group 2 is smaller.
- ☐ 1 point is earned for a statement discussing the dependence of amplitude on dart momentum.
- ☐ 1 point is earned for indicating that the speed of the dart could be smaller.

Example Response:

If group 2 used a dart with a smaller initial speed, the dart would have had less momentum, and so the dart-block system would not have gone as high (making the amplitude smaller), but the string length is the same and so the period would still be the same.

chapter2parta

28. This question is a long free-response question. Show your work for each part of the question.



(12 points, suggested time 25 minutes)

This problem explores how the relative masses of two blocks affect the acceleration of the blocks. Block A, of mass m_A, rests on a horizontal tabletop. There is negligible friction between block A and the tabletop. Block B, of mass m_B, hangs from a light string that runs over a pulley and attaches to block A, as shown above. The pulley has negligible mass and spins with negligible friction about its axle. The blocks are released from rest.

(a)

i. Suppose the mass of block A is much greater than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

ii. Now suppose the mass of block A is much less than the mass of block B. Estimate the magnitude of the acceleration of the blocks after release.

Briefly explain your reasoning without deriving or using equations.

(b) Now suppose neither block's mass is much greater than the other, but that they are not necessarily equal. The dots below represent block A and block B, as indicated by the labels. On each dot, draw and label the forces (not components) exerted on that block after release. Represent each force by a distinct arrow starting on, and pointing away from, the dot.



(c) Derive an equation for the acceleration of the blocks after release in terms of m_A, m_B, and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

(d) Consider the scenario from part (a)(ii), where the mass of block A is much less than the mass of block B. Does your equation for the acceleration of the blocks from part (c) agree with your reasoning in part (a)(ii)?

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_____ Yes _____ No

Briefly explain your reasoning by addressing why, according to your equation, the acceleration becomes (or approaches) a certain value when m_A is much less than m_B.

(e) While the blocks are accelerating, the tension in the vertical portion of the string is T_1. Next, the pulley of negligible mass is replaced with a second pulley whose mass is not negligible. When the blocks are accelerating in this scenario, the tension in the vertical portion of the string is T_2. How do the two tensions compare to each other?

_____ $T_2 > T_1$ _____ $T_2 = T_1$ _____ $T_2 < T_1$

Briefly explain your reasoning.

Part A(i)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answers: “Zero”, “small”, “negligible”, “much less than “g”, or “$<< g$”.

0	1	2 ✓

The response accurately includes **both** of the following criteria:

- ☐ For a correct answer and attempt at a consistent justification.
- ☐ For correct reasoning.

Example 1 Point Response:

Nearly zero. Because block A is much heavier than block B .

Example 2 Point Responses:

Very small. Because block A has a large inertia, it won’t speed up much.

Close to zero because block B is so light that it can hardly budge block A .

Claim: The acceleration of the blocks is zero/small/negligible/ “$<< g$” .

Evidence: The mass of block A is much greater than the mass of block B .

Reasoning Statements: Very small. Because block A has a large inertia, it won’t speed up much or close to zero because block B is so light that it can hardly budge block A .

Part A(ii)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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Correct Answer: g , $9.8\ \mathrm{m/s^2}$, or $10\ \mathrm{m/s^2}$ (or just 9.8 or 10)

0	1 ✓

The response does not accurately include:

- [ ] An indication that the acceleration is nearly equal to g and a correct justification.

Example Response:

Nearly equal to g . Because block B is almost in free fall.

$10\ \mathrm{m/s^2}$, because block A has negligible mass and the tension in the string is nearly zero.

Claim: The acceleration of the blocks is close to g .

Evidence: The mass of block A is much less than the mass of block B .There is negligible friction between block A and the tabletop. The pulley has negligible mass and spins with negligible friction about its axle.

Reasoning Statements: Nearly equal to g . Because block B is almost in free fall or $10\ \mathrm{m/s^2}$, because block A has negligible mass and the tension in the string is nearly zero.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: N , F_{N} , "normal force," F_{Table} , "table force," or any other label can be used to indicate the force is "normal" or comes from the table.

Note: F_{g} , F_{grav} , W , mg , $m_A g$, "gravity," "grav force," may be used.

Note: G or g are not acceptable.

Note: "tension," "string force," F_T, F_{tension}, F_{string}, F_S, T, or some other label indicating that the force comes from the string or from tension may be used.

Note: $m_B g$, F_{mB}, "force from block B " or other indications that the force is "created" by block B are NOT acceptable.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- [ ] A correct normal force on block A with acceptable label.
- [ ] Correct gravitational forces with acceptable label on both diagrams, and no extraneous forces on either

diagram.

- [ ] Correct tension forces with acceptable label on both diagrams.

Example Response:



Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Writing a "whole system" equation is also sufficient to earn the first two points.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- [ ] For using separate Newton's second law equations for each block.
- [ ] For combining the equations with correct notation, including correctly using m_A and m_B , indicating that the same tension force acts on both blocks, *and* that they share the same acceleration.
- [ ] For a correct equation for a with supporting work.

Example Response:

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$$\sum F_B = m_B a$$
$$m_B g - F_T = m_B a$$
$$\sum F_A = m_A a$$
$$F_T = m_A a$$
$$m_B g - m_A a = m_B a$$
$$m_B g = (m_B + m_A) a$$
$$a = \frac{m_B g}{(m_B + m_A)}$$

Part C - Alternate Solution

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: *Writing the correct whole-system equation is sufficient to earn the first two points of part (c).*

0	1	2	3 ✓

*The response accurately includes **three** of the following criteria.*

- ☐ *For writing a "whole-system" equation for the total mass that does not contain internal forces.*
- ☐ *For substituting the net force and system mass with correct quantities.*
- ☐ *For a correct equation for a with supporting work.*

Example Response:

$F_{\text{net}} = m_{\text{total}} a$

$m_B g = (m_A + m_B) a$

$a = \frac{m_B}{m_A + m_B} g$

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answer: "Yes"

chapter2parta

Note: “No” is acceptable if the equation is inconsistent with the answer in (a)(ii).

0	1 ✓

The response accurately includes

- ☐ A valid reasoning that addresses the result in part (c) and the reasoning in part (a)(ii).

Example Response:

Claim:
Yes, the equation for the acceleration of the blocks from part (c) agrees with the reasoning in part (a)(ii).

OR
No, the equation for the acceleration of the blocks from part (c) does not agree with the reasoning in part (a)(ii).

Evidence:

The mass of block A is much less than the mass of block B .
$a = \frac{m_B}{m_A+m_B} g$ (derived as part (c) answer)

Reasoning Statements for “Yes” claim:
When m_A is much less than m_B , it can be neglected in the equation derived in part (c), giving an acceleration close to g as stated in (a)(ii).

Reasoning Statements for “No” claim, if the answer in part (a)(ii) is wrong:
When m_A is much less than m_B , it can be neglected in the equation derived in part (c), giving an acceleration close to g . This disagrees with the value of ___ stated in (a)(ii).

Reasoning Statements for “No” claim, if the answer in part (c) is wrong:

When m_A is much less than m_B , it can be neglected in the equation derived in part (c), giving an acceleration of ___. This disagrees with the value of g stated in (a)(ii).

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answer: $T_2 > T_1$.

Note: A maximum of 1 point can be earned for part (e) if an incorrect selection is made.

0	1	2 ✓

The response accurately includes **both** of the following criteria:

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- For reasoning that the acceleration of both blocks is smaller.
- For doing any one of the following, consistent with the answer selection and Newton's second law for block B .
  - Concluding that a smaller acceleration implies that T_2 is greater than T_1
  - Concluding that an unchanged acceleration implies that T_2 is the same as T_1
  - Concluding that a larger acceleration implies that T_2 is less than T_1

Example Response:

Claim: $T_2 > T_1$

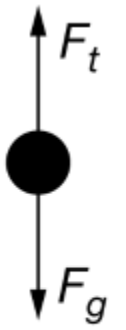
Evidence:

The pulleys spin with negligible friction about the axle.

The original pulley has negligible mass.

The second pulley's mass is not negligible.

$$\vec{a} = \frac{\sum \vec{F}}{m}$$



Block B

Reasoning Statements:

The rotational inertia of the second pulley results in a smaller acceleration for the blocks. Block B must have a smaller net force to have a smaller acceleration, so the rope tension must be larger than before (closer in magnitude to the gravitational force on block B).

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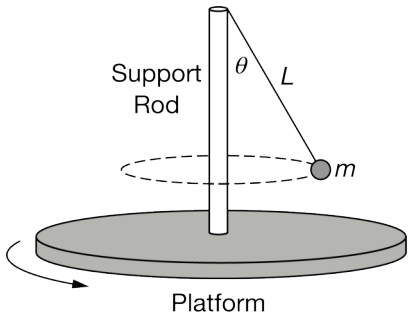
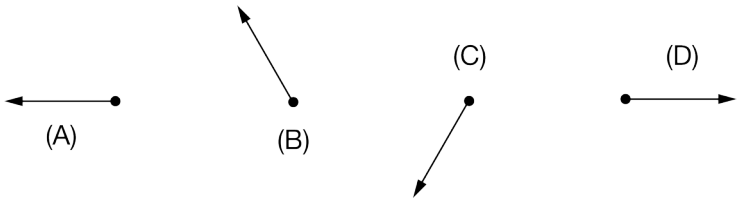
29. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A vertical support rod is fixed at the center of a platform. A light string of length L is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass m, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed ω (omega), the string makes an angle θ (theta) with the rod.



(a) Identify which of the above vectors represents the net force exerted on the sphere at the moment shown above. Briefly justify your answer.

(b) Is the magnitude of the tension in the string greater than, less than, or equal to the weight of the sphere? Explain your reasoning.

(c) The string can never be completely horizontal no matter how fast the platform is rotating. Without writing any equations or formulas, explain why.

One of the following two equations for the square of the tangential speed v of the sphere as a function of θ is consistent with the statement in part (c) that the string can never be horizontal as the platform is rotating.

Equation 1: $v^2 = \frac{gL(\sin^2\theta)}{(\cos\theta)}$ **Equation 2:** $v^2 = \frac{gL(\cos^2\theta)}{(\sin\theta)}$

(d) Indicate which equation is consistent with the statement in part (c), and explain why. Also explain why the other equation is not consistent with the statement in part (c).

(e) In a series of experimental trials, a group of students changes L and adjusts v to keep θ constant. They record L and v for each trial. Based on the equation in part (d) that you indicated as consistent with the statement in part (c), what quantities could the students graph to yield a straight line whose slope could be used to find the acceleration due to gravity, g ?

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(f) Let I be the rotational inertia of the sphere-rod-platform system about a vertical axis through the center of the support rod. Does I increase, decrease, or stay the same when L increases (but θ stays constant)? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answer: (A), with a correct justification

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement identifying the correct net force vector (A)
- ☐ 1 point is earned for a justification that includes that the net force or acceleration will point toward the center of the circular path

Example Responses:

Figure A because the net force is toward the center of the sphere's circular motion.

OR

The forces acting on the sphere are the force of gravity and the force of tension. The force of gravity is directly toward the ground, and the force of tension is at an angle toward the top of the rod. When the forces are added together, the vertical component of the force of tension and the force of gravity cancel each other out, leaving only the horizontal component of the force of tension, which is demonstrated by vector A.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- ☐ 1 point is earned for a statement that the tension in the string is greater than the weight of the sphere
- ☐ 1 point is earned for an explanation referring to the components of the vectors
- ☐ 1 point is earned for a correct justification that addresses the sum of the vector components

Example Responses:

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The tension is greater than the weight because the vertical component of the tension force must be equal to the weight. The tension also has a horizontal component, so the magnitude will be greater than either of the components.

OR

Explaining that

$\Sigma F_y = F_{Ty} - mg = 0$

$F_T \cos\theta = mg$

$F_T = mg/\cos\theta$

(Since $\cos\theta < 1$*), therefore* $F_T > mg$ *(or > "weight")*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement identifying forces and/or force components in the vertical direction
- ☐ 1 point is earned for a justification that includes that the weight of the sphere must be balanced by the string and so the vertical component of the tension can never be zero

Example Response:

The weight of the sphere is downward and will need to be balanced by the vertical component of the tension. So, there will always be a vertical component of tension.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Notes:

· If students switch sine and cosine (such that they use Equation 2) but then are completely consistent, they can earn 2 of 3 points in part (d).

· If students misinterpret theta as being zero when the string is horizontal, they can earn two of three points in part (d) for using Equation 2 consistently.

0	1	2	3 ✓

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The response accurately includes **three** of the following criteria.

- [ ] 1 point is earned for a statement addressing functional dependence (regardless of the equation chosen)
- [ ] 1 point is earned for indicating that equation 1 shows that as theta goes to 90 degrees, *v* gets very large (infinite/undefined)
- [ ] 1 point is earned for correctly indicating a flaw in equation 2

Example Response:

Equation 1 is consistent with part (c) because as theta gets closer to 90 degrees, the speed gets very large. This shows that no matter how large *v* gets, theta will never get to 90 degrees. Equation 2 is not consistent because it has a finite value for theta equal to 90 degrees.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- [ ] 1 point is earned for a statement identifying any *v* term on one axis
- [ ] 1 point is earned for a statement identifying any *L* term on the other axis
- [ ] 1 point is earned for a statement identifying correct power relationships for *v* and *L* (i.e., *v* squared vs. *L*)

Example Response:

They could graph *v* squared on the *y*-axis as a function of *L* on the *x*-axis.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for a statement identifying that the rotational inertia will increase
- [ ] 1 point is earned for a statement that since the mass is further from the axis, the rotational inertia is bigger

Example Response:

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I will increase because for a larger *L*, the distance of the mass from the axis will be larger, and rotational inertia increases with distance.

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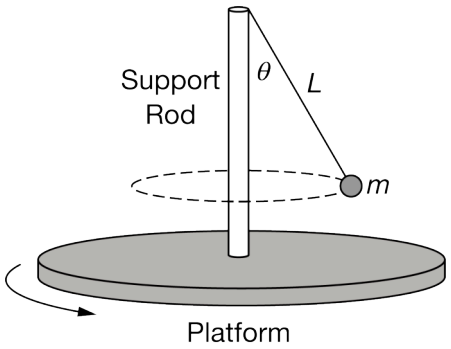
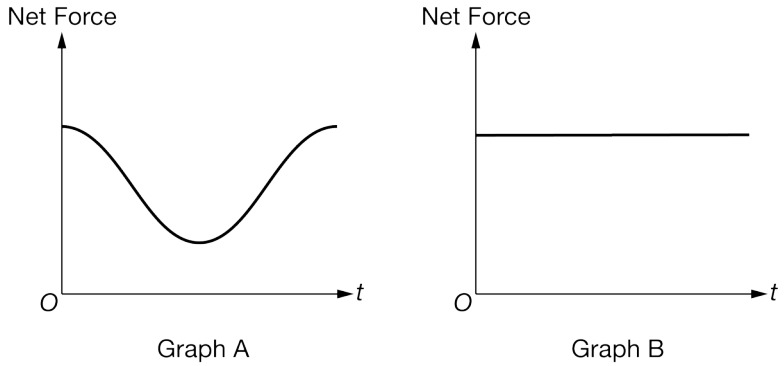
30. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A vertical support rod is fixed at the center of a platform. A light string of length L is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass m, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed ω (omega), the string makes an angle θ (theta) with the rod.



(a) The platform rotates with a constant angular speed. Which of the above graphs correctly shows the magnitude of the net force exerted on the sphere as a function of time t ? Briefly justify your answer.

(b) Is the magnitude of the tension in the string greater than, less than, or equal to the centripetal force exerted on the sphere? Explain your reasoning.

(c) The string is nearly vertical when the sphere's tangential speed is very small. Without writing any equations or formulas, explain why.

(d) A student derives an equation for the square of the tangential speed v of the sphere as a function of θ : $v^2 = gL(\tan\theta)(\sin\theta)$. Indicate whether the equation is consistent with the statement in part (c) and explain why.

(e) In a series of experimental trials, a group of students adjusts v and measures θ while keeping all other variables constant. They record v and θ for each trial. If the students use the equation given in part (d), what quantities could they graph to yield a straight line whose slope could be used to find the acceleration due to gravity, g ?

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(f) Let I be the rotational inertia of the sphere-rod-platform system about a vertical axis through the center of the support rod. Does I increase, decrease, or stay the same when θ decreases? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement identifying graph B as correctly describing the magnitude of the net force on the sphere as a function of time.
- ☐ 1 point is earned for a justification that the net force is constant because the speed is constant.

Example Response:

Graph B is correct because the magnitude of the net force on an object in uniform circular motion is constant.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that the magnitude of the tension is greater than the centripetal force exerted on the sphere.
- ☐ 1 point is earned for an explanation referring to the components of the vectors.
- ☐ 1 point is earned for a correct justification that addresses the sum of the vector components.

Example Response:

The tension is greater than the centripetal force because the horizontal component of the tension force must be equal to the centripetal force. The tension also has a vertical component, so the magnitude will be greater than either of the components.

OR

Explaining that

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$\sum F_x = F_{Tx} = mv^2/r$ *where* mv^2/r *is the "centripetal force."*

$F_T \sin\theta = mv^2/r$

$F_T = mv^2/(r\sin\theta)$

(Since $\sin\theta < 1$*), therefore* $F_T > mv^2/r$ *(or >* *"centripetal force")*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that when the speed is very small the centripetal force is very small.
- ☐ 1 point is earned for a justification that includes that if the horizontal force is very small the angle of the tension with the vertical is very small.

Example Response:

A small speed means a small centripetal acceleration. The horizontal component of the tension is equal to the centripetal force, so if it is very small, then the tension will be almost vertical, and the angle will be very small.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that the equation is consistent with the reasoning in part (c).
- ☐ 1 point is earned for addressing the functional dependence.
- ☐ 1 point is earned for a correct functional dependence argument.

Example Response:

It is consistent because as theta goes to zero, both trigonometric terms go to zero, and therefore the speed goes to zero. This matches the statement that the string is nearly vertical when the speed is small.

Part E

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for identifying any *v* term on one axis.
- ☐ 1 point is earned for identifying any theta term on the other axis.
- ☐ 1 point is earned for a correct power relationship between *v* and trig functions of theta (for example, *v* squared vs. $(\tan\theta)(\sin\theta)$.

Example Response:

They could graph v squared as a function of tan theta sin theta.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the rotational inertia of the sphere-rod-platform-system decreases.
- ☐ 1 point is earned for a justification that includes that the rotational inertia decreases with decreasing distance of the mass from the axis.

Example Response:

I decreases because as theta decreases, the mass gets closer to the axis of rotation and rotational inertia increases with distance from the axis.

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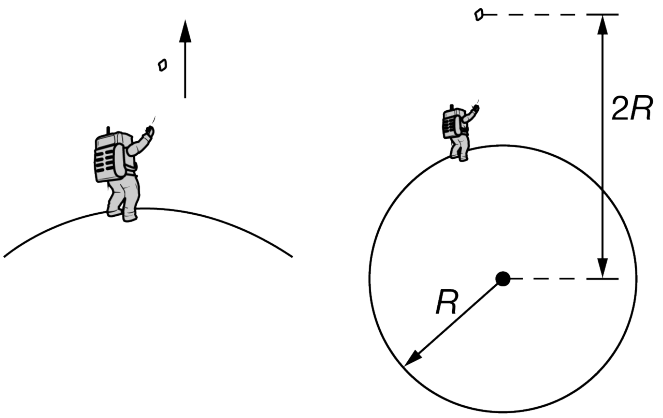
31. This question is a short free-response question. Show your work for each part of the question.

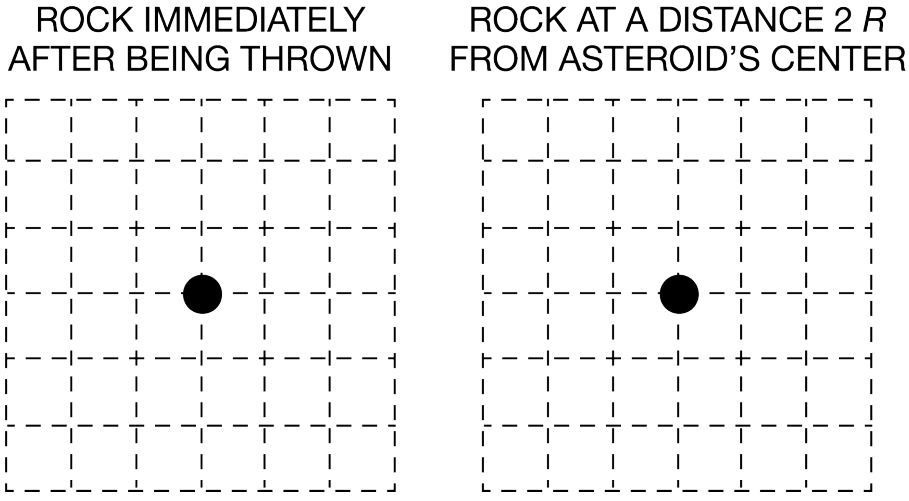


Note: Figures not drawn to scale.

An astronaut stands on the surface of a spherical asteroid that has a weak gravitational field but no atmosphere. The astronaut throws a small rock straight upward, as shown at left above. The rock travels upward until it reaches a distance of $2R$ from the center of the asteroid, where R is the asteroid's radius, and then falls back down to the asteroid's surface.

(a) The dots below represent the rock immediately after being thrown and when it is a distance $2R$ from the asteroid's center. On each dot, draw and label the forces (not components) exerted on the rock at the indicated time. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Note: Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces at both times.



(b) The graph at left below shows the rock's velocity as a function of time after being thrown upward with an initial velocity v_{01}. The dashed line shows what the velocity would be if the rock experienced a constant acceleration, and the solid curve shows the rock's actual velocity.

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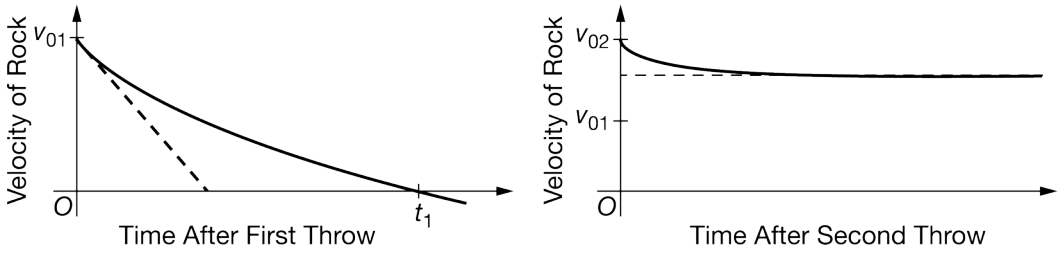
After the rock lands back on the surface, the astronaut then throws the rock straight upward a second time but with a greater force so that the rock's initial speed v_{02} is greater than initial speed v_{01} for the first throw. The graph at right below shows the rock's velocity as a function of time after being thrown upward the second time.



In a clear, coherent, paragraph-length response that may also contain equations and/or drawings, make claims for the following aspects of the rock's motion.

For the first throw, state a claim for what is happening at time t_1.
State a claim about the strength of the gravitational force exerted on the rock as the rock gets farther from the asteroid.
For the first throw, state a claim about the relationship about the strength of the gravitational force exerted on the rock as the rock gets farther from the asteroid and the behavior indicated between time 0 and t_1 in the velocity-time graph.
For the second throw, state a claim about why there is a horizontal asymptote for the velocity.

Part (a)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes both of the following criteria.

- ☐ 1 point is earned for a downward gravitational force on each dot.
- ☐ 1 point is earned for a smaller magnitude force when the rock is at 2*R* from the asteroid's center, and no extraneous forces.

Example Response:

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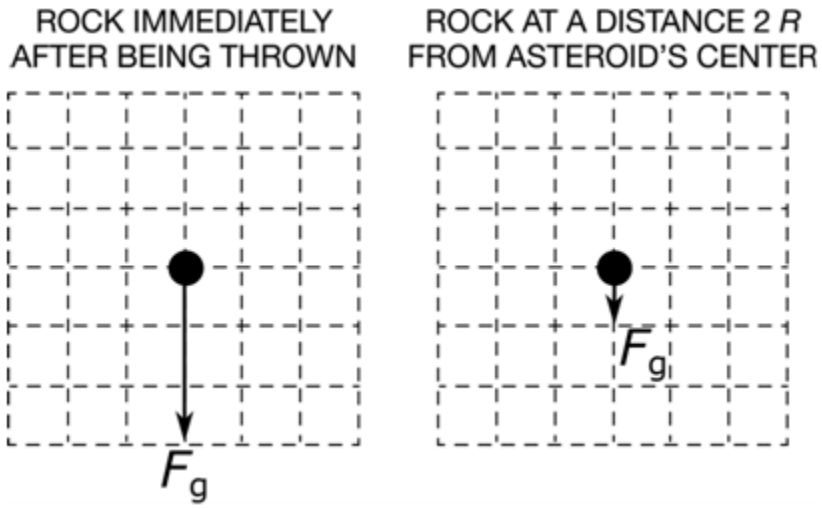

Part (b)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5 ✓

Student response accurately includes all of the following criteria.

- ☐ 1 point is earned for stating that time t_1 is when the rock reaches its maximum height or distance from the center of the asteroid on the first throw.
- ☐ 1 point is earned for a statement that the gravitational force gets weaker farther away from the asteroid.
- ☐ 1 point is earned for stating or implying that the weaker gravitational force means that the acceleration (or slope of velocity-time graph) gets smaller in magnitude OR stating that the change (reduction) in speed is less (or speed is greater) compared to the constant-acceleration graph.
- ☐ 1 point is earned for one of the following regarding the second throw.
  - stating that, if the rock gets far enough away from the asteroid, the gravitation force can become negligible
  - stating that the rock has enough speed to escape the gravitational field
  - stating that the rock is far enough away from the asteroid that it moves at a (nearly) constant speed
- ☐ 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response

Example Response:

On the first throw, since the gravitational force exerted on the rock gets weaker as the rock gets farther from the asteroid, the rock's acceleration gets less in magnitude. This can be seen by the slope of the graph (on the left) getting shallower from time 0 to t_1 . At t_1 , the rock's velocity is zero and it has reached its maximum height. On the second throw, the rock is thrown farther, and reaches a height where the gravitational force (or acceleration) is negligible, and the rock's velocity approaches a constant value.

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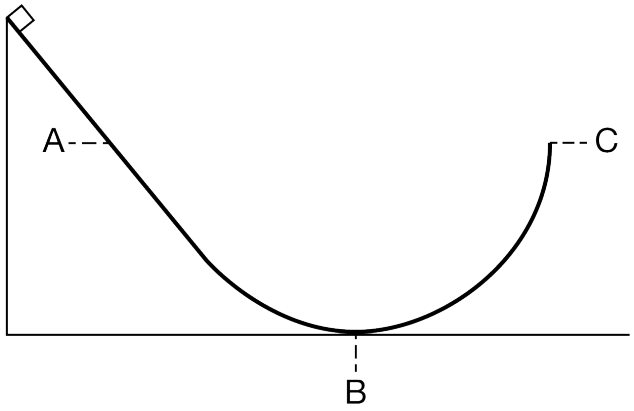
32. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)



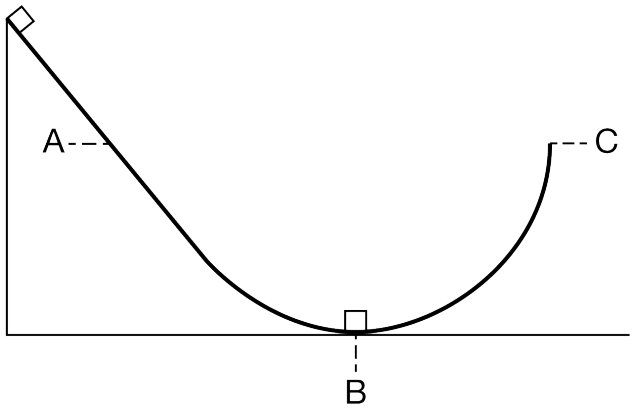
The figure above shows a track consisting of a straight section that connects to a circular section. Three locations are indicated: location A on the straight section of the track, location B at the bottommost point of the track, and location C at the point where the track is vertical. Locations A and C are at the same height. A block is released from rest at the top of the straight section of the track. There is negligible friction between the track and the block.

(a) Is the magnitude of the normal force exerted on the block by the track at location A greater than, less than, or equal to the normal force at location B? Justify your answer.

(b) In a coherent, paragraph-length response, describe the relative angle between the direction of the block's velocity and the direction of the block's acceleration at the following times.

When the block is at location A
When the block is at location B
Immediately before the block reaches location C

Specifically, indicate whether the velocity is parallel to the acceleration, perpendicular to the acceleration, or neither, at each location. Use physical principles to justify your responses.



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(c) The scenario is repeated, but an identical block is at rest at location B, as shown above. When the block at the top of the straight section of the track is released from rest, it slides down the track and collides elastically with the identical block and comes to rest. The identical block then begins sliding up the ramp. Is the maximum height reached by the identical block greater than, less than, or equal to the maximum height reached by the block in the original scenario? Briefly explain your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the normal force at A is less than at B.
- ☐ 1 point is earned for a correct explanation including both locations A and B (e.g., at A the normal force is less than the weight, but at B it is greater than the weight).

Example Response:

At A, the block's acceleration is along the straight track, so the normal force must cancel the component of the gravitational force normal to the track and is therefore less in magnitude than the block's weight. At B, the block's acceleration and net force are upward, so the upward normal force must be greater than the block's weight.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the velocity is always parallel to the track.
- ☐ 1 point is earned for relating the acceleration to the net force.
- ☐ 1 point is earned for indicating the acceleration at location A is along the track and thus parallel to velocity.
- ☐ 1 point is earned for indicating that at location B the velocity is perpendicular to the acceleration.
- ☐ 1 point is earned for indicating that just before location C the acceleration is neither perpendicular nor parallel to the normal force.
- ☐ 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the

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paragraph-length response.

Example Response:

The velocity of the box is along the track.

At location A, the box is accelerating down the ramp because there is a net force exerted on the box down the ramp. The net force and acceleration always have the same direction. If the net force is down the track then the acceleration is down the track. So that makes the velocity and acceleration in the same direction at location A, therefore parallel.

At location B, the bottommost point, the box is moving in a circle. Here the velocity is still along the track. Since location B is the bottommost point of a circle, the net force and acceleration would be directed toward the center of that circle. So that makes the velocity and acceleration perpendicular at location B.

Just before location C, the box is both moving in a circle and slowing down as it rises above the ground. The velocity is in the direction of motion, which is upward. Since the box is slowing down, the acceleration must have a downward component. The acceleration would also have a component toward the center of the block's circular path. So, the acceleration is neither parallel nor perpendicular to the upward velocity at just before location C.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the maximum height reached by the second block is the same as that of the original block.
- ☐ 1 point is earned for a correct explanation of why the second block reaches the same height.

Example Response:

Since the collision is elastic and the first block comes to rest, the second block has the same kinetic energy that the first block had just before the collision, which is the same as the first block's kinetic energy at the bottom of the ramp in the original scenario.