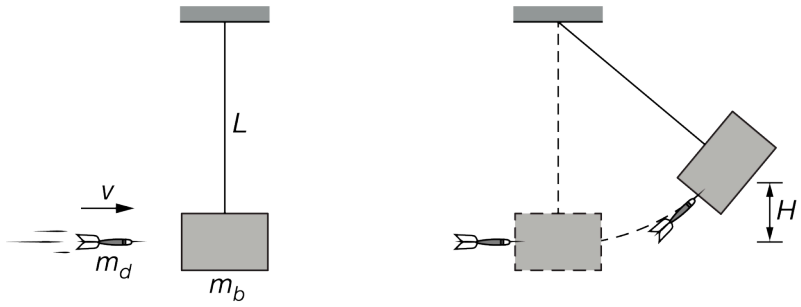
chapter4

1. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown at left above. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown at right above. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, the dart or the block, if either, experiences an impulse of larger magnitude during the collision. If the impulse is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the speed of the dart as it embeds itself into the block is greater than *v*, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the motion of the dart-block system at the instant it reaches height *H*. The students make the following statements:

Student 1: The speed of the system at height *H* is zero. Since it is moving in circular motion and centripetal acceleration is given by $a_c = \frac{v^2}{r}$, the acceleration must also be zero.

Student 2: The acceleration of the system at height *H* can't be zero, or the system would not swing back down. The acceleration must be equal to *g* at height *H*.

(c) Which underlined phrase or phrases, if either, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if either, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if either, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases, if either, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

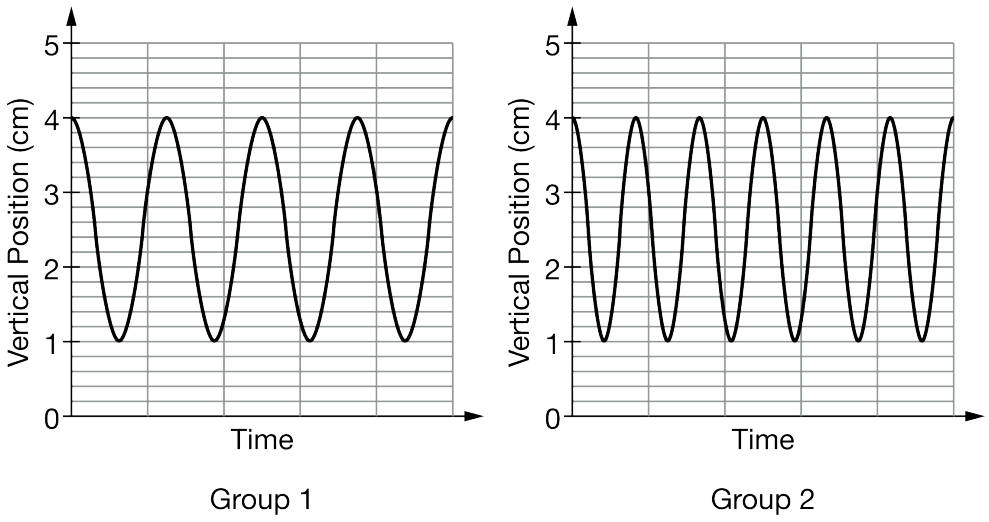
(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{m_d v}{m_d + m_b}\right)^2$. Explain how this

equation supports your claim in part (b).

The students perform several experimental trials. In each trial, a different speed v is used, and the resulting maximum height reached by the center of mass of the dart-block system is measured.

(h) Which quantities could the students graph to produce a straight line whose slope can be used to find the acceleration due to gravity, g ?

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart mass, initial dart speed, block mass, or string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct explanation, in terms of force & time (using Newton's third law)
  OR
  a correct explanation in terms of change in momentum (using conservation of momentum).

chapter4

Example Response:

Both the dart and the block experience the same impulse because they exert equal and opposite forces on each other because of Newton's third law, and the forces are exerted for equal times.

OR

By conservation of momentum, the decrease in the momentum of the dart must be equal and opposite to the increase in the momentum of the block, and therefore the impulse is the same in magnitude for both objects.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the final height is greater.
- ☐ 1 point is earned for a statement of conservation of momentum to indicate that the final speed of the dart-block system is greater **AND** an indication of conservation of energy leading to a greater height.

Example Response:

Because the momentum of the dart before the collision must be equal to the momentum of the dart-block system after the collision, a higher dart speed (a higher dart momentum) will lead to a higher dart-block speed after the collision (a higher dart-block momentum). Because the dart-block is traveling faster after the collision, it has more kinetic energy which means that the dart-block will swing higher.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Responses:

Student 1 is correct in saying that the speed at height H is zero

OR

chapter4

The first statement.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Responses:

Student 1 is incorrect in saying that the acceleration is zero at the height H

OR

The second statement.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Responses:

Student 2 is correct in saying the acceleration of the system at height H can't be zero or the system would not swing back down

OR

The first statement.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Responses:

Student 2 is incorrect in saying that the acceleration is g at height H

OR

The second statement.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement including functional dependence of the variables.
- ☐ 1 point is earned for correctly asserting that as the speed increases in the equation *H* also increases .

Example Response:

This equation matches my reasoning in part (b) – if the speed of the dart increases, the height of the swing increases because v is in the numerator.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for plotting a variant of *v* on one axis.

chapter4

- [ ] 1 point is earned for plotting a variant of *H* on the other axis.
- [ ] 1 point is earned for a correct power relationship between the two variables, i.e., *H* vs. *v* squared.

Example Response:

Students could graph H as a function of v squared.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- [ ] 1 point is earned for recognizing that the period in the second graph is shorter.
- [ ] 1 point is earned for a statement about the dependence of the period on the length of the pendulum.
- [ ] 1 point is earned for indicating that the length of the pendulum must be shorter for a shorter period.

Example Response:

Since the period is shorter for group 2, and the period is related to the length, if the period became shorter with no other changes, the length of the string must have been shortened.

chapter4

2. A new kind of toy ball is advertised to "bounce perfectly elastically" off hard surfaces. A student suspects, however, that no collision can be perfectly elastic. The student hypothesizes that the collisions are very close to being perfectly elastic for low-speed collisions but that they deviate more and more from being perfectly elastic as the collision speed increases.

a. Design an experiment to test the student's hypothesis about collisions of the ball with a hard surface. The student has equipment that would usually be found in a school physics laboratory.

i. What quantities would be measured?

ii. What equipment would be used for the measurements, and how would that equipment be used?

iii. Describe the procedure to be used to test the student's hypothesis. Give enough detail so that another student could replicate the experiment.

b. Describe how you would represent the data in a graph or table. Explain how that representation would be used to determine whether the data are consistent with the student's hypothesis.

c. A student carries out the experiment and analysis described in parts (a) and (b). The student immediately concludes that something went wrong in the experiment because the graph or table shows behavior that is elastic for low-speed collisions but appears to violate a basic physics principle for high-speed collisions.

i. Give an example of a graph or table that indicates nearly elastic behavior for low-speed collisions but appears to violate a basic physics principle for high-speed collisions.

ii. State one physics principle that appears to be violated in the graph or table given in part (c)i. Several physics principles might appear to be violated, but you only need to identify one.

Briefly explain what aspect of the graph or table indicates that the physics principle is violated, and why.

Part A

4 points total

Parts i, ii, and iii are scored as a unit

1 point is earned: For an overall plan in which quantities are measured that could be used to compare mechanical energy before and after a collision with a hard surface

1 point is earned: For a conceptually plausible plan to measure pre- and post-collision positions and/or speeds that could be used to compare pre- and post-collision mechanical energies, without extraneous equipment and/or measurements

1 point is earned: For having lab equipment and measurement procedures well specified

1 point is earned: For a procedure that includes trials of different pre-collision speeds, ranging from low speed to high speed (as is needed to test the student's hypothesis)

Example 1:

chapter4

i. The drop height of the ball and the bounce height.

ii. A meterstick to measure the heights and a video camera to record the ball's motion.

iii. Place the meterstick upright against the wall. Drop the ball from 10 different drop heights, using the video camera to record the bounce heights.

Example 2:

i. The speed of the ball immediately before and immediately after it bounces.

ii. A photogate near the floor, at a height just above the diameter of the ball, to measure the ball's speed.

iii. Drop the ball through the photogate. Record the speeds measured by the photogate before and after the bounce. Change the drop height of the ball at least five times, covering a range of heights from "low" to "high."

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Parts i, ii, and iii are scored as a unit

1 point is earned: For an overall plan in which quantities are measured that could be used to compare mechanical energy before and after a collision with a hard surface

1 point is earned: For a conceptually plausible plan to measure pre- and post-collision positions and/or speeds that could be used to compare pre- and post-collision mechanical energies, without extraneous equipment and/or measurements

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Example 1:

1. The drop height of the ball and the bounce height.

2. A meterstick to measure the heights and a video camera to record the ball's motion.

3. Place the meterstick upright against the wall. Drop the ball from 10 different drop heights, using the video camera to record the bounce heights.

Example 2:

1. The speed of the ball immediately before and immediately after it bounces.

2. A photogate near the floor, at a height just above the diameter of the ball, to measure the ball's speed.

3. Drop the ball through the photogate. Record the speeds measured by the photogate before and after the bounce. Change the drop height of the ball at least five times, covering a range of heights from "low" to "high."

chapter4

Part B

4 points total

1 point is earned: For describing how to plot or otherwise represent the data in a way that could be used to test the hypothesis

1 point is earned: For describing how to compare the post-collision to pre-collision mechanical energy (or a plausible alternative) to quantify the elasticity of the collision

1 point is earned: For comparing the low-speed versus high-speed results

1 point is earned: For addressing the hypothesis with an analysis such as a slope, ratio, or difference

Example 1:

> Make a graph of the bounce height h_f as a function of the drop height h_i. If the data are consistent with the hypothesis, then the data will (1) lie close to the line $h_f = h_i$ for low drop heights, and (2) lie below this line for high drop heights

Example 2:

> Make a graph of v_f^2-v_i^2 as a function of v_i, where v_f and v_i are the ball's speed just after and just before the bounce, respectively. If the data are consistent with the hypothesis, then v_f^2-v_i^2 will (1) be close to zero for low speeds, and (2) be negative for high speeds.

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

1 point is earned: For describing how to plot or otherwise represent the data in a way that could be used to test the hypothesis

1 point is earned: For describing how to compare the post-collision to pre-collision mechanical energy (or a plausible alternative) to quantify the elasticity of the collision

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> Make a graph of v_f^2-v_i^2 as a function of v_i, where v_f and v_i are the ball's speed just after and just before the

chapter4

bounce, respectively. If the data are consistent with the hypothesis, then v_f^2-v_i^2 will (1) be close to zero for low speeds, and (2) be negative for high speeds.

Part C

4 points total

Part(i)

1 point is earned: For drawing a graph or table that shows that the low-speed collisions are nearly perfectly elastic

1 point is earned: For drawing a graph or table that shows a violation of a physics principle for higher-speed collisions

Example for energy conservation:

A graph of the ratio (Post-collision mechanical energy / Pre-collision mechanical energy) as a function of precollision speed, in which the graph stays near 1.0 for low initial speeds but becomes greater than 1.0 for high-speed collisions.

Part(ii)

1 point is earned: For a correct description of the aspect of the graph or table that shows a violation of the physical principle indicated

1 point is earned: For a correct explanation of why the representation shows a violation of the physical principle indicated

Example for energy conservation using the graph described above:

The value of the energy ratio shows a violation of conservation of energy when it becomes greater than 1.0 because the final energy cannot be greater than the initial energy.

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Part(i)

1 point is earned: For drawing a graph or table that shows that the low-speed collisions are nearly perfectly elastic

1 point is earned: For drawing a graph or table that shows a violation of a physics principle for higher-speed collisions

Example for energy conservation:

A graph of the ratio (Post-collision mechanical energy / Pre-collision mechanical energy) as a function of precollision speed, in which the graph stays near 1.0 for low initial speeds but becomes greater than 1.0 for high-speed collisions.

Part(ii)

chapter4

1 point is earned: For a correct description of the aspect of the graph or table that shows a violation of the physical principle indicated

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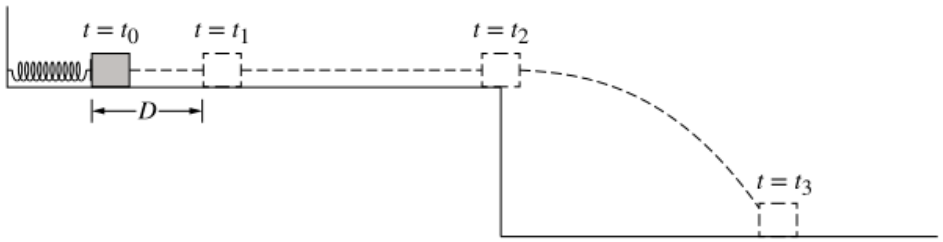
Example for energy conservation using the graph described above:

> The value of the energy ratio shows a violation of conservation of energy when it becomes greater than 1.0 because the final energy cannot be greater than the initial energy.

chapter4

3. This question is a long free-response question. Show your work for each part of the question.



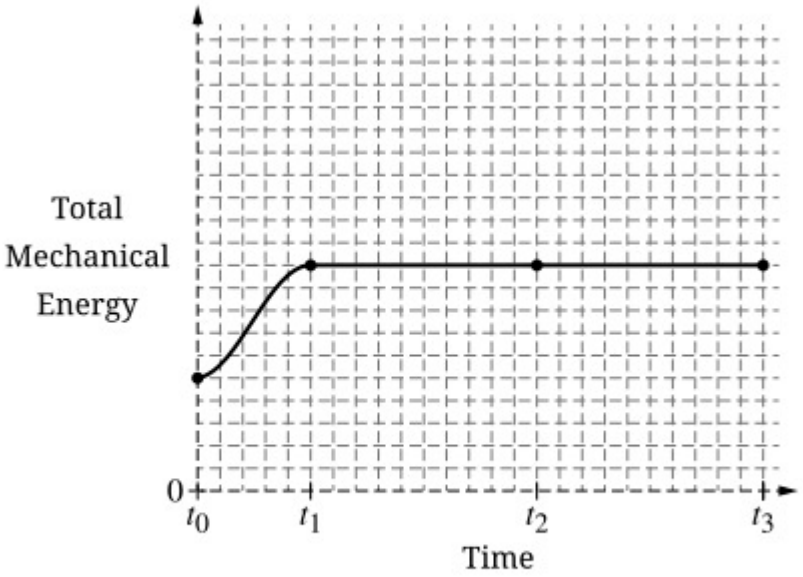
(12 points, suggested time 25 minutes)

A block is launched from a table by a compressed spring that is attached to a wall. The spring is initially compressed a distance D before the block is launched, as shown above. There is negligible friction between the block and the table.

At time $t = t_0$ the block is released from rest.
At $t = t_1$ the block loses contact with the spring.
At $t = t_2$ the block leaves the edge of the table.
At $t = t_3$ the block lands on the floor.

The graph below shows the total mechanical energy of the system containing the block and Earth from $t = t_0$ to immediately before $t = t_3$. The gravitational potential energy is defined to be zero when the block is on the floor.

chapter4



Note: Time axis not drawn to scale.

(a) On the same axes, sketch a graph of the total mechanical energy of just the block from time t_0 to time t_3.

The procedure is repeated with a second block, Block 2, which is more massive than the original block, Block 1. Block 2 is placed on the table, and the spring is compressed the same distance D as before.

(b) For Block 2, is the horizontal distance traveled from the edge of the table to where it hits the ground greater than, less than, or equal to the corresponding distance for Block 1?

___ Greater than ___ Less than ___ Equal to

Justify your answer.

(c) Derive a general expression for X, the horizontal distance traveled by a block of mass m_B, from the edge of the table to where it hits the floor. Let k_0 represent the spring constant and h_0 represent the height of the tabletop above the floor. Express your answer in terms of D, m_B, k_0, h_0, and physical constants, as appropriate.

(d) Does your expression derived in part (c) support your reasoning in part (b)?

___ Yes ___ No

Justify your answer.

(e) Is the kinetic energy of Block 2 immediately after it leaves the spring the same as or different than the kinetic energy of Block 1 immediately after it leaves the spring?

___ Same ___ Different

Briefly justify your answer.

Part A

chapter4

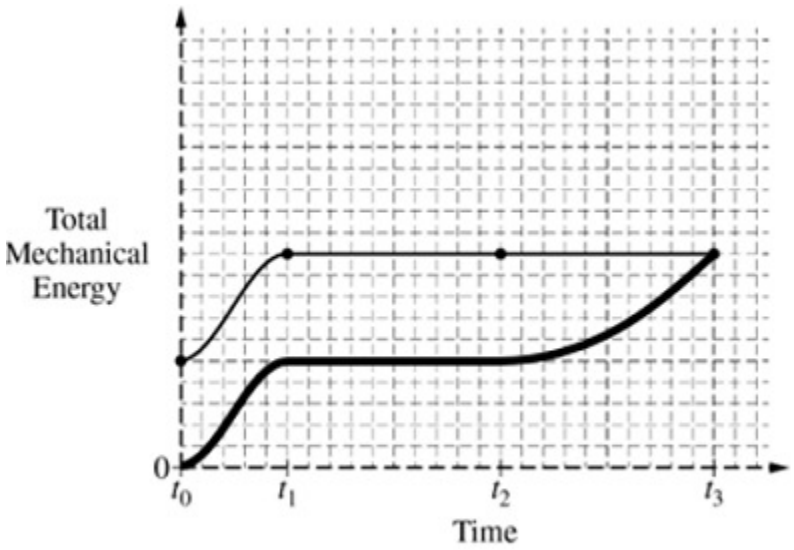
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for curve that **both**:
  · starts at 0 at t_0
  · is reasonably similar in shape to the original curve between t_0 and t_1
- ☐ 1 point is earned for **both** of the following:
  · a non-zero horizontal line from $t_1 - t_2$
  · a curve or function that is continuous at t_1
- ☐ 1 point is earned for **all** of the following:
  · a line or curve with a positive slope from $t_2 - t_3$
  · a curve or function that is continuous at t_2
  · the same energy as the original curve at t_3

Example Response:



Note: Time axis not drawn to scale.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

chapter4

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for **both** of the following:
  · Selecting “Less”
  · An attempt at a relevant justification.
- ☐ 1 point is earned for indicating that Block 2 has a smaller horizontal speed
- ☐ 1 point is earned for indicating that the fall time is the same in both scenarios

Example Response:

In both situations there is the same spring potential energy at the beginning which means both blocks are launched with the same kinetic energy. However, the more massive block will have a smaller launch speed. Because both blocks are launched from the same table height, they fall for the same amount of time. This means that the more massive block which is launched with a smaller speed travels a smaller horizontal distance before landing.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for using conservation of energy to find the speed of a block after leaving the spring.
  $v = \sqrt{\frac{k_0 D^2}{m_B}}$
- ☐ 1 point is earned for correctly finding the fall time to be $\sqrt{\frac{2h_0}{g}}$
- ☐ 1 point is earned for multiplying the speed and fall time to find the horizontal distance in terms of the given variables and physical constants.

Example Response:

$\frac{1}{2}k_0 D^2 = \frac{1}{2}m_B {v_h}^2$

${v_h}^2 = k_0 D^2 / m_B \quad \rightarrow \quad v_h = D\sqrt{k_0/m_B}$

$h_0 = \frac{1}{2}gt^2 \quad \rightarrow \quad t = \sqrt{2h_0/g}$

$X = v_h t = \left(D\sqrt{k_0/m_B}\right)\left(\sqrt{2h_0/g}\right)$

$X = D\sqrt{\frac{2k_0 h_0}{m_B g}}$

chapter4

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for addressing the functional dependence between X and m_B in the equation derived in part (c)
  Scoring Note: It is not necessary to use the functional dependence correctly to earn this point.
- ☐ 1 point is earned for **both** of the following:
  · Correctly identifying the relationship between X and m_B for the expression derived in part (c)
  · Comparing the relationship to the reasoning in part (b)

Example Response:

m_B is in the denominator of the expression for X, so the larger mass of block 2 *means that X is less for block* 2*, consistent with my answer in part b.*

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

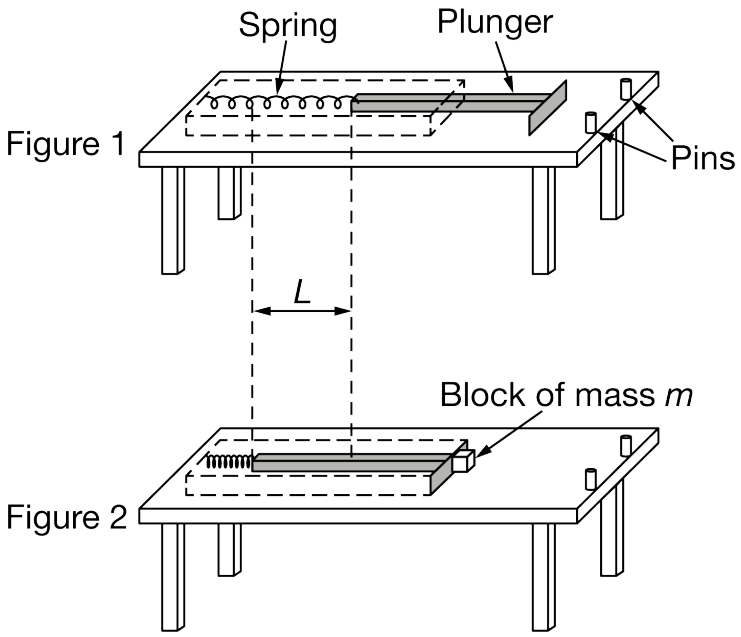
- ☐ 1 point is earned for **both** of the following:
  · Selecting “Same”
  · Providing a valid justification

Example Response:

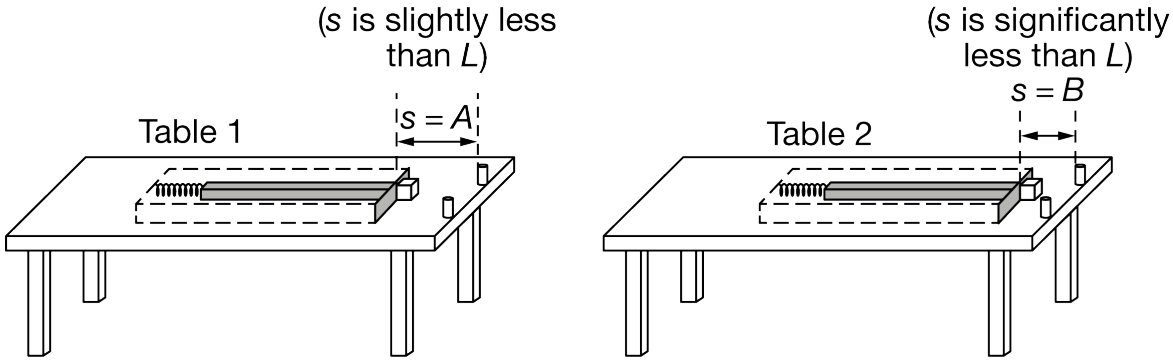
The spring stores the same amount of potential energy because it is compressed the same distance D. So the same amount of energy is transferred to kinetic energy of each block.

chapter4

4. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A spring-loaded launcher can be securely attached to a tabletop. When the spring is at its uncompressed length, the plunger is in the position shown above in Figure 1. A small wood block can be pressed against the end of the plunger, compressing the spring a distance L, as shown in Figure 2. When the block is released, the plunger pushes the block along the tabletop. Two pins are attached to the edge of the table to prevent the plunger from extending beyond the table. When the plunger hits the pins, the block then leaves the table. There is nonnegligible friction between the block and the tabletop.



The launcher can be moved closer to or farther from the edge of the tabletop. Students set up tables 1 and 2, as shown above, with tabletops that are made of the same material. On table 1, the launcher is positioned so that the distance $s = A$ between the launcher and the edge of the tabletop is slightly less than L. On table 2, the launcher is positioned so that the distance $s = B$ between the launcher and the edge of the tabletop is significantly less than L. On both tables, the block is released from rest and loses contact with the plunger at the moment the plunger reaches the pins. Tables 1 and 2 are the same height.

k is the spring constant of the spring.
m is the mass of the block.
L is the distance the spring is compressed.
μ (mu) is the coefficient of kinetic friction between the block and the tabletop.
s is the distance between the launcher and the edge of the tabletop.

Assume the spring is ideal, the plunger has negligible mass, the block loses contact with the plunger immediately after it reaches the end of the table, and there is non-negligible friction between the tabletops and the blocks.

(a) Without manipulating equations, explain why the block launched from table 1 could land farther from the table than the block launched from table 2 does.

(b) Does the block launched from table 1 spend more, less, or the same amount of time in the air than the block launched from table 2 does? Explain your reasoning.

(c) Consider the Earth-block system. Determine the change, if any, in the total mechanical energy of the system from the instant the block leaves the table to the instant immediately before it reaches the ground.

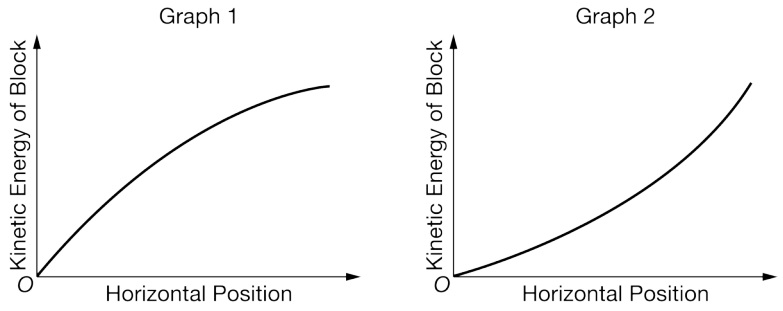
The students correctly derive an equation for the horizontal distance D from the edge of the tabletop that the block lands, in terms of s (the distance between the launcher and the edge of the tabletop):

$$D = t\sqrt{\frac{k}{m}(2Ls - s^2) - 2\mu g s},$$

where t is the time interval between the instant the block leaves the table and the instant immediately before it reaches the ground.

(d) Does this equation for D support your argument from part (a) that the block launched from table 1 could land farther from the table than the block launched from table 2? Briefly explain why or why not.

(e) The students now use the same setups to launch blocks that are identical to each other but more massive than the original blocks. The blocks are made of the same material as the original blocks. Does this change in the mass of the blocks make it more likely or less likely that the block launched from table 1 goes farther than the block launched from table 2 does? Briefly explain your reasoning.



(f) Two students each try to sketch a graph of the kinetic energy of the block as a function of its position from launch to the time the block reaches the edge of the table. Their graphs are shown above. Which graph best represents the kinetic energy of the block? Justify your selection.

chapter4

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for applying work/energy or momentum/impulse to explain why launch speed could be greater for Table 1.
- ☐ 1 point is earned for indicating that a larger *s* enables the spring to exert a force on the block for a longer time or distance, or that a larger *s* corresponds to more elastic potential energy being transferred to the block.
- ☐ 1 point is earned for indicating that the block could gain more speed and/or kinetic energy and/or momentum on Table 1.

Example Response:

The distance s is greater for table 1 than table 2 which means that the force that the spring exerts on the block is exerted over a longer distance, allowing the force to do more work on the block. Since the spring force does more work on the block, the change in kinetic energy could be greater, meaning the block could be launched at a greater speed from table 1. The block from table 1 will travel a larger horizontal distance because it was launched with a greater horizontal speed.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating the same time.
- ☐ 1 point is earned for indicating that both blocks will have the same initial vertical speed or zero initial vertical speed.
- ☐ 1 point is earned for indicating that either:
  only the vertical motion determines the fall time,
  OR
  The vertical displacement is the same in both cases
  OR
  The vertical acceleration is the same in both cases.

Example Response:

chapter4

The blocks launched from table 1 and table 2 will spend the same amount of time in the air because they have an initial vertical velocity of zero and start from the same height above the floor.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: just “zero” earns this point.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for indicating there is no change in the total mechanical energy.

Example Response:

After the block leaves the table the total mechanical energy does not change.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for addressing the functional dependence of *D* on *s*.
- ☐ 1 point is earned for correctly indicating how a specific term in the expression changes with increasing *s*.
- ☐ 1 point is earned for indicating that an increase in s could result in an increase in *D*, or in the right-hand-side of the equation
  Note: If a student answers “no” the third point is awarded for indicating that an increase in s results in a decrease in *D*, or in the right-hand side of the equation.

Example Response:

Yes. The closer s is to L, the greater the $(2Ls - L^\wedge 2)$ *part of the equation would be. If that term of the equation is greater, D will be greater.*

OR

Yes. Because s is always less than L, Ls *is greater than* $s^\wedge 2$*. So as s increases, the equation increases.*

Part E

chapter4

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating "Less likely", with an attempt at an explanation.
- ☐ 1 point is earned for noting that the larger mass increases the force of friction.
- ☐ 1 point is earned for indicating that a larger frictional force will dissipate more energy (or result in a smaller net force on the block/acceleration of the block), which results in a lower launch speed, and therefore smaller distance.
  Note: Part (e) can be answered quantitatively. See alternate solution below.

It is less likely that the block in 1 will go farther. The larger mass means that the force of friction is larger, so the net force on the block is less, resulting in a lower launch speed. On table 1, the frictional force is exerted over a longer distance, and so has a bigger effect on reducing the speed compared to table 2.

OR

Less likely. The first term is multiplied by k/m; so this term decreases with bigger m. The second term doesn't change with m. So, the difference between the two terms is smaller, reducing the effect of increasing s on the distance traveled.

Part E – Alternate Solution

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

*The response accurately includes **all** of the following criteria.*

- ☐ *1 point is earned for indicating "less likely", with an attempt at an explanation.*
- ☐ *1 point is earned for addressing the functional dependence of D on m in the given equation.*
- ☐ *1 point is earned for explaining how the right side of the equation becomes smaller with reference to both terms in the substitution, therefore, D becomes smaller.*

Example Response:

Less likely. The first term is multiplied by k/m; so this term decreases with bigger m. The second term doesn't change with m. So, the difference between the two terms is smaller, reducing the effect of increasing s on the distance traveled."

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1	2 ✓

The response accurately includes **both** of the following criteria.

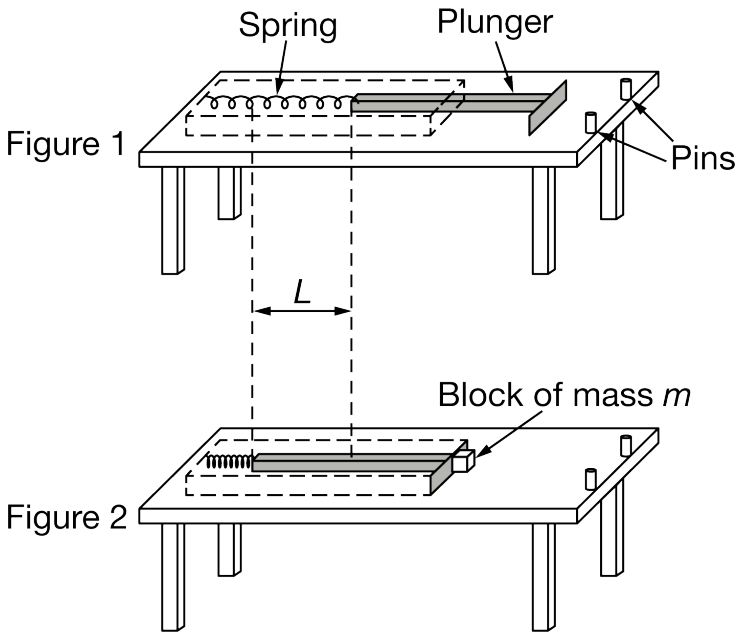
- ☐ 1 point is earned for indicating that Graph 1 is the correct graph, with an attempt at an explanation.
- ☐ 1 point is earned for correctly addressing the decreasing slope of the curve for large values of horizontal position.

Example Response:

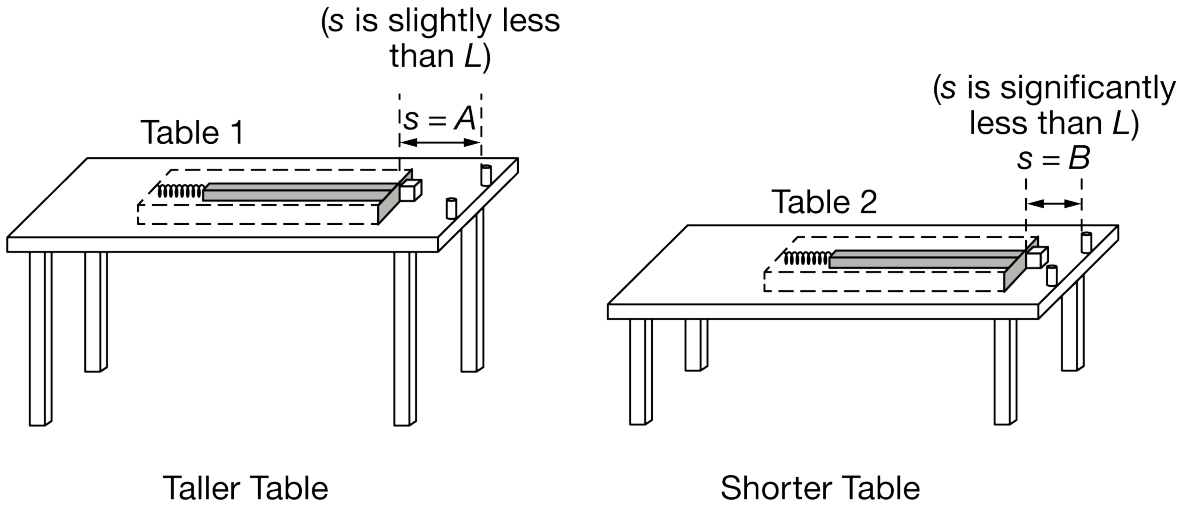
Graph 1 is the correct graph. The slope decreases with increasing position. This makes sense because, as the spring expands, it exerts a smaller force on the block. The smaller force means that the kinetic energy increases by a smaller amount for a given change in position.

chapter4

5. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A spring-loaded launcher can be securely attached to a tabletop. When the spring is at its uncompressed length, the plunger is in the position shown above in Figure 1. A small wood block can be pressed against the end of the plunger, compressing the spring a distance L as shown in Figure 2. When the block is released, the plunger pushes the block along the tabletop. Two pins are attached to the edge of the table to prevent the plunger from extending beyond the table. When the plunger hits the pins, the block then leaves the table. There is nonnegligible friction between the block and the tabletop.



The launcher can be moved closer to or farther from the edge of the tabletop. Students set up tables 1 and 2, as shown above, with tabletops that are made of the same material. On table 1, which is taller than table 2, the

launcher is positioned so that the distance $s = A$ between the launcher and the edge of the tabletop is slightly less than L. On table 2, the launcher is positioned so that the distance $s = B$ between the launcher and the edge of the tabletop is significantly less than L. On both tables, the block is released from rest and loses contact with the plunger at the moment the plunger reaches the pins.

k is the spring constant of the spring.
m is the mass of the block.
L is the distance the spring is compressed.
μ (mu) is the coefficient of kinetic friction between the block and the tabletop.
s is the distance between the launcher and the edge of the tabletop.

Assume the spring is ideal, the plunger has negligible mass, the block loses contact with the plunger immediately after it reaches the end of the table, and there is non-negligible friction between the tabletops and the blocks.

(a) Without manipulating equations, explain why the block launched from table 2 could have a greater speed when it leaves the table than the block launched from table 1 does.

(b) Does the block launched from table 2 spend more, less, or the same amount of time in the air than the block launched from table 1 does? Explain your reasoning.

(c) Consider the system consisting of only the block. Determine the change, if any, in the total mechanical energy of the system from the instant the block leaves the table to the instant immediately before it reaches the ground.

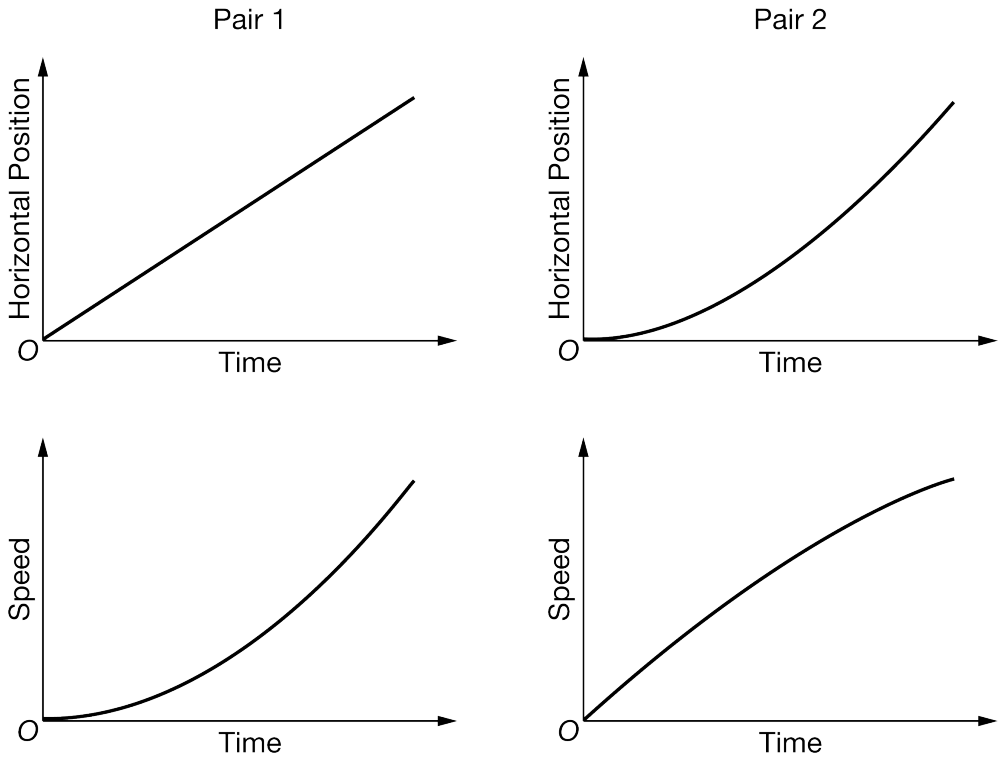
The students correctly derive an equation for the speed v of the block when it is at the edge of the tabletop, in terms of s (the distance between the launcher and the edge of the tabletop):

$v = \sqrt{\frac{k}{m}(2Ls - s^2) - 2\mu g s}$.

(d) Does this equation for v support your argument from part (a) that the block launched from table 2 could have a larger speed when it leaves the table than the block on table 1 does? Briefly explain why or why not.

(e) The students now repeat the scenario with identical setups except that the coefficient of kinetic friction between the blocks and the table is larger. Does this change in the coefficient of friction make it more likely or less likely that the block launched from table 1 has a greater speed when it leaves the table than the block on table 2 does? Briefly explain your reasoning.

chapter4



(f) Two students each try to sketch a graph of the horizontal position and speed of the block as a function of time while the block is being pushed by the plunger. Their graphs are shown above. In which pair of graphs are the two graphs consistent with each other? Justify your selection.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for attempting to apply work/energy or momentum/impulse to explain why launch speed could be greater for Table 2.
- ☐ 1 point is earned for indicating that a larger *s* for the block on Table 1 enables friction to exert a force on the block for a longer time or distance, or that a larger *s* corresponds to more energy being dissipated.
- ☐ 1 point is earned for correctly applying work/energy or momentum/impulse to explain why launch speed could be greater for Table 2.

Example Response:

The distance s is greater for table 1 than table 2 which means that the force of friction from the table can be exerted for a

chapter4

longer distance, allowing the friction force to dissipate more energy. Since the friction force dissipates more energy, the reduction in kinetic energy due to friction is greater, meaning the block will be launched at a smaller speed from table 1.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the block launched from Table 2 will spend less time in the air.
- ☐ 1 point is earned for indicating that both blocks will have the same initial vertical speed or zero initial vertical speed.
- ☐ 1 point is earned for indicating that either:
  only the vertical motion determines the fall time,
  OR
  the vertical displacement is different for the two cases.

Example Response:

The block launched from table 2 will spend less time in the air because both blocks have an initial vertical velocity of zero, but the block on table 2 starts from a lower height above the floor.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for indicating that the change is mgh.

Example Response:

mgh

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for addressing the functional dependence of v on s.
- ☐ 1 point is earned for correctly indicating how a specific term in the expression changes with increasing s.
- ☐ 1 point is earned for indicating that an increase in s could result in a decrease in v, or in the right-hand-side of the equation.
  Scoring note: If a student answers "no" the third point is awarded for indicating that an increase in s results in an increase in v, or in the right-hand side of the equation.

Example Response:

Yes, because the equation indicates that v could increase as s decreases like my argument in part (a). In the equation, the expression under the radical could increase when s decreases because there are two "s" terms with negative signs. If the number under the radical increases, v will increase.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating "Less likely", with an attempt at an explanation.
- ☐ 1 point is earned for noting that the larger coefficient of friction results in a larger frictional force.
- ☐ 1 point is earned for indicating that a larger frictional force will dissipate more energy (or result in a smaller net force on the block/acceleration of the block), which results in a lower launch speed.

Example Response:

It is less likely that the block in 1 will have a greater launch speed. The larger coefficient of friction means that the force of friction is larger, so the net force on the block is less, resulting in a lower launch speed. On table 1, the frictional force is exerted over a longer distance, and so has a bigger effect on reducing the speed compared to table 2.

Part E-Alternate Solution

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1	2	3 ✓

*The response accurately includes **all** of the following criteria.*

- ☐ *1 point is earned for indicating "less likely" with an attempt at an explanation.*
- ☐ *1 point is earned for addressing the functional dependence of v on the coefficient of friction (μ) in the given "correct" equation.*
- ☐ *1 point is earned for explaining how the right side of the equation becomes smaller with reference to both terms in the substitution, therefore v becomes smaller.*

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating pair 2, with an attempt at an explanation.
- ☐ 1 point is earned for either correctly ruling out the inconsistent position and time graphs in pair 1,
  OR
  For indicating that the speed graph in pair 2 is consistent with the position graph (the speed graph corresponds to the slope of the position graph).

Example Response:

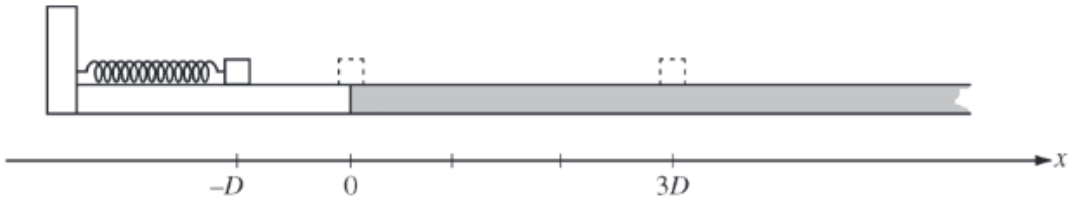
Pair 2 has the two consistent graphs. The slope of the position graph is initially zero, and then increases, which is the behavior of the speed graph.

OR

Pair 2 has the two consistent graphs. In pair 1, the position graph is a straight line, so the speed graph should be constant.

chapter4

6.

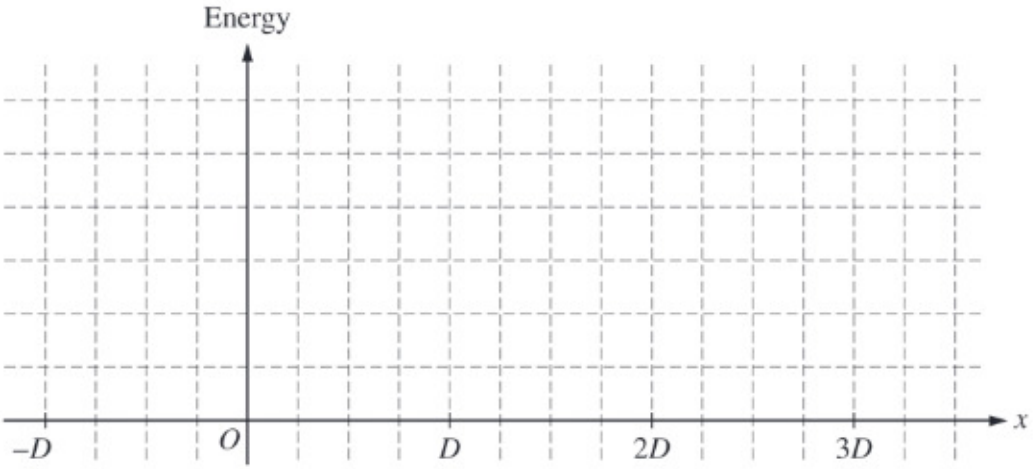


A block is initially at position x = 0 and in contact with an uncompressed spring of negligible mass. The block is pushed back along a frictionless surface from position *x = 0 to x = -D*, as shown above, compressing the spring by an amount $\Delta x = D$. The block is then released. At *x = 0* the block enters a rough part of the track and eventually comes to rest at position *x = D*. The coefficient of kinetic friction between the block and the rough track is μ.

(a) On the axes below, sketch and label graphs of the following two quantities as a function of the position of the block between x = -D and x = 3D. You do not need to calculate values for the vertical axis, but the same vertical scale should be used for both quantities.

i. The kinetic energy *K* of the block

ii. The potential energy *U* of the block-spring system



The spring is now compressed twice as much, to $\Delta x = 2D$. A student is asked to predict whether the final position of the block will be twice as far at *x = 6D*. The student reasons that since the spring will be compressed twice as much as before, the block will have more energy when it leaves the spring, so it will slide farther along the track before stopping at position *x = 6D*.

b)

i. Which aspects of the student's reasoning, if any, are correct? Explain how you arrived at your answer.

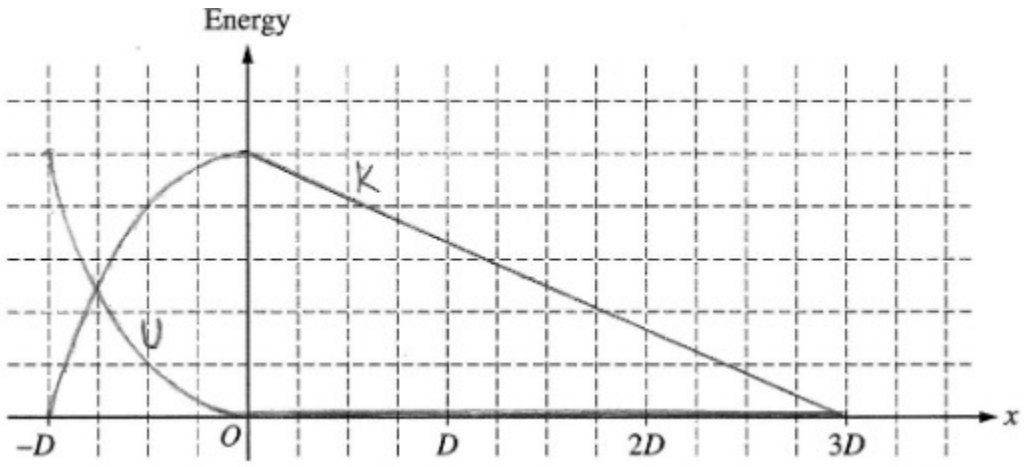
chapter4

ii. Which aspects of the student's reasoning, if any, are incorrect? Explain how you arrived at your answer.

(c) Use quantitative reasoning, including equations as needed, to develop an expression for the new final position of the block. Express your answer in terms of D.

(d) Explain how any correct aspects of the student's reasoning identified in part (b) are expressed by your mathematical relationships in part (c). Explain how your relationships in part (c) correct any incorrect aspects of the student's reasoning identified in part (b). Refer to the relationships you wrote in part (c), not just the final answer you obtained by manipulating those relationships.

Part A



1 point is earned: For sketching either energy curve with a reasonably correct shape between $x = -D$ and $x = 0$, with zero and maximum values at the correct locations

1 point is earned: For sketching two curves from $x = -D$ to $x = 0$ with shapes and values such that the total energy is constant (even if the curves are incorrect)

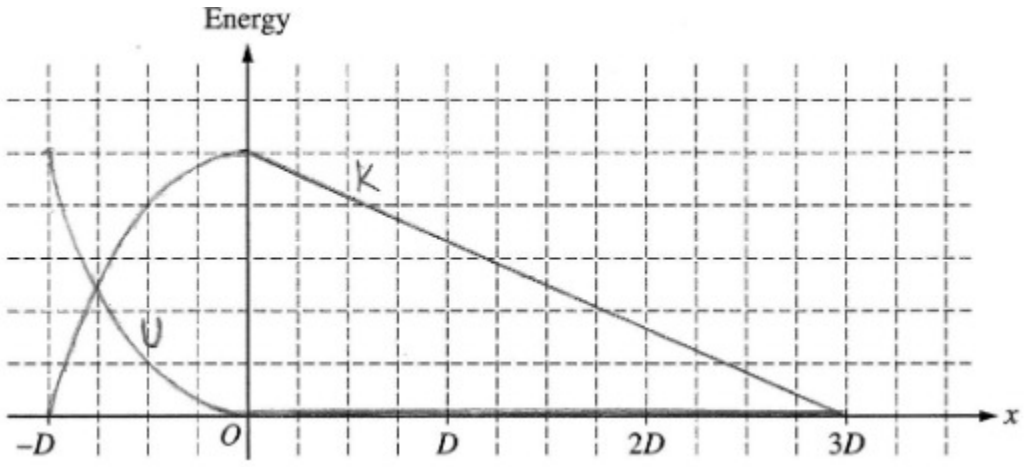
1 point is earned: For sketching potential energy equal to zero from $x = 0$ to $x = 3D$

1 point is earned: For sketching kinetic energy as a linear function from its maximum value at $x = 0$ to zero at $x = 3D$

0	1	2	3	4 ✓

The student response earns all of the following points:

chapter4



1 point is earned: For sketching either energy curve with a reasonably correct shape between $x = -D$ and $x = 0$, with zero and maximum values at the correct locations

1 point is earned: For sketching two curves from $x = -D$ to $x = 0$ with shapes and values such that the total energy is constant (even if the curves are incorrect)

1 point is earned: For sketching potential energy equal to zero from $x = 0$ to $x = 3D$

1 point is earned: For sketching kinetic energy as a linear function from its maximum value at $x = 0$ to zero at $x = 3D$

Part B

(i)

1 point is earned: For identifying that the student is correct that the block will have more energy when it leaves the spring

(ii)

1 point is earned: For identifying that the student is incorrect about the new final position of the block because the spring's energy does not scale linearly with its compression

✓

0	1	2

The student response earns all of the following points:

(i)

1 point is earned: For identifying that the student is correct that the block will have more energy when it leaves the spring

(ii)

1 point is earned: For identifying that the student is incorrect about the new final position of the block because the

chapter4

spring’s energy does not scale linearly with its compression

Part C

1 point is earned: For indicating that the final energy in the spring (which becomes the mechanical energy of the block as it reaches the rough track) is four times the original energy in the spring

1 point is earned: For indicating that the frictional force remains the same

1 point is earned: For equating the initial energy in the spring to an expression that shows that the energy dissipated by friction is proportional to the distance the block slides down the rough track

Example:

$U_1 = \frac{1}{2}kD^2 and, U_2 = \frac{1}{2}k(2D)^2, so, U_2 = 4U_1$
$W_1 = \mu mg(3D) and, W_2 = \mu mg\Delta x_2$
$W_1 = U_1 and, W_2 = U_2 = 4U_1 = 4W_1$
$\mu mg\Delta x_2 = 4(\mu mg(3D))$
$\Delta x_2 = 4(3D) = 12D$

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For indicating that the final energy in the spring (which becomes the mechanical energy of the block as it reaches the rough track) is four times the original energy in the spring

1 point is earned: For indicating that the frictional force remains the same

1 point is earned: For equating the initial energy in the spring to an expression that shows that the energy dissipated by friction is proportional to the distance the block slides down the rough track

Example:

$U_1 = \frac{1}{2}kD^2 and, U_2 = \frac{1}{2}k(2D)^2, so, U_2 = 4U_1$
$W_1 = \mu mg(3D) and, W_2 = \mu mg\Delta x_2$
$W_1 = U_1 and, W_2 = U_2 = 4U_1 = 4W_1$
$\mu mg\Delta x_2 = 4(\mu mg(3D))$
$\Delta x_2 = 4(3D) = 12D$

Part D

1 point is earned: For indicating that the student’s correct reasoning that the block has more energy in the second situation is expressed by the calculations comparing the initial energy in the spring

1 point is earned: For indicating that the student’s correct reasoning that the block will slide farther is expressed by an

chapter4

equation that indicates that the work done by friction to stop the block in the second situation is some factor greater than the work done in the first situation

1 point is earned: For indicating that the student's incorrect reasoning that energy scales linearly with the spring's compression is corrected by the expression for the initial energy of the spring

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For indicating that the student's correct reasoning that the block has more energy in the second situation is expressed by the calculations comparing the initial energy in the spring

1 point is earned: For indicating that the student's correct reasoning that the block will slide farther is expressed by an equation that indicates that the work done by friction to stop the block in the second situation is some factor greater than the work done in the first situation

1 point is earned: For indicating that the student's incorrect reasoning that energy scales linearly with the spring's compression is corrected by the expression for the initial energy of the spring

chapter4

7. This question is a long free-response question. Show your work for each part of the question.

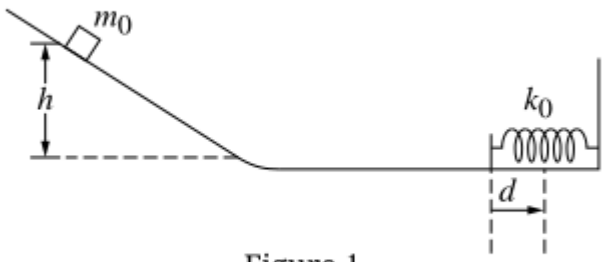


Figure 1

(12 points, suggested time 25 minutes)

In an experiment, a block is released from rest on a ramp, as shown in Figure 1. The block slides down an incline and across a horizontal surface, where it compresses a spring and then momentarily comes to rest. Frictional forces between the block and surfaces are negligible.

(a) The experiment is repeated, but the block is released from a greater height on the ramp. Will the new compression distance of the spring be greater than, less than, or equal to the original compression distance?

_____ Greater than _____ Less than _____ Equal to

Briefly justify your selection without deriving or using equations.

(b) Derive an expression for the distance d that the spring compresses. Express your answer in terms of the mass of the block m_0, the height h from which the block is released, the spring constant k_0, and physical constants, as appropriate.

(c) Does the equation you derived in part (b) agree with your reasoning in part (a)? Justify your answer.

(d) After the block compresses the spring, the block remains attached to the spring and oscillates. A student finds an incorrect equation for the period of oscillation T for the block-spring system on the Internet, $T = C\sqrt{\frac{h}{m_0 g}}$, where C is a constant with appropriate units. Explain why this equation does not make physical sense.

chapter4

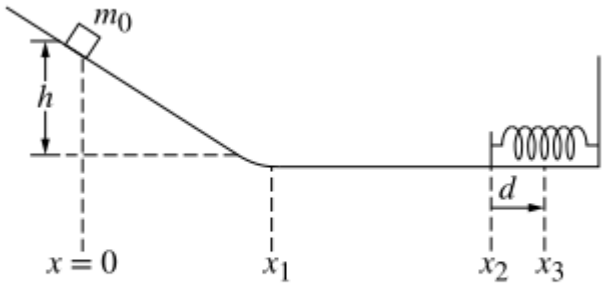


Figure 2

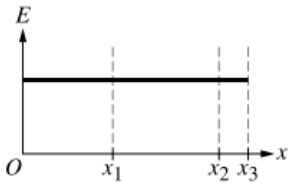
Figure 2 shows the same experiment with four horizontal positions indicated.

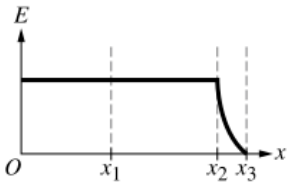
(e) Consider the following systems:

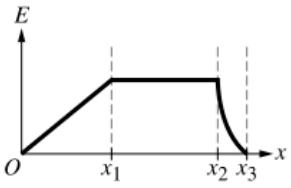
System A	Block only
System B	Block and Earth only
System C	Block, Earth, and spring

Each of the following graphs represents the mechanical energy E as a function of the horizontal position x of the block for one of the systems.

i. Under each graph, write the letter "A," "B," or "C" to indicate the system that is represented by the graph.







System_____ System_____ System_____

chapter4

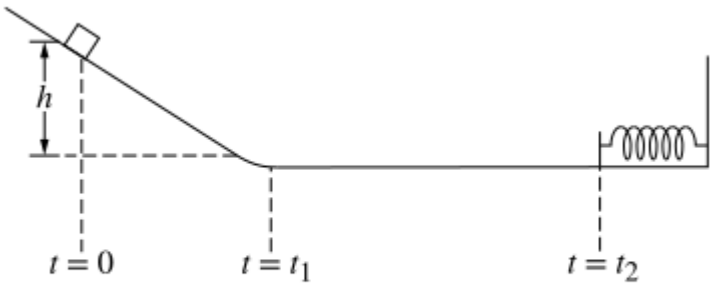
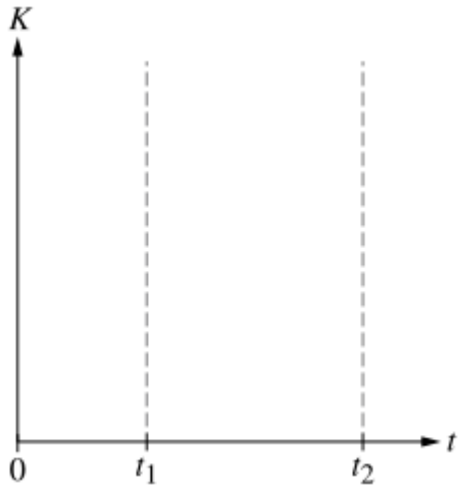


Figure 3

Figure 3 indicates the times the block is at three specific positions during the same experiment. The block is released from rest at $t = 0$. At time t_1, the block reaches the horizontal surface. Time t_2 indicates the time immediately before the block makes contact with the spring.

ii. On the following axes, sketch a graph of the kinetic energy K of the block as a function of time t from time $t = 0$ to time $t = t_2$.



Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

chapter4

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for selecting “Greater than” with an attempt at any relevant justification.
- ☐ 1 point is earned for stating **one** of the following correct justifications:
  - The initial gravitational potential energy of the block-Earth system immediately before the block is released from rest is converted into elastic potential energy in the spring when the spring is fully compressed
  - The kinetic energy of the block immediately before the block collides with the spring is converted into elastic potential energy in the spring when the spring is fully compressed

Scoring Note: If an incorrect selection is made, a maximum of one point may be earned.

Example Response

Greater than because the block-Earth system will have more mechanical energy as height increases. Therefore, more energy is available to transfer into the spring after the block fully compresses the spring.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a multistep derivation that begins with an application of conservation of energy.
- ☐ 1 point is earned for a correct equation: $d = \sqrt{\frac{2m_0gh}{k_0}}$.

Example Response

$E_0 = E_f$
$U_{g0} + U_{s0} + K_0 + W_{\text{External}} = U_{gf} + U_{sf} + K_f$
$U_{g0} = U_{sf}$
$m_0gh = \frac{1}{2}k_0d^2$
$d = \sqrt{\frac{2m_0gh}{k_0}}$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the equation from part (b) is in agreement or is not in agreement with the reasoning provided in part (a) that is consistent with the equation derived in part (b).
- ☐ 1 point is earned for a statement that correctly applies the functional dependence of d and h.

Example Response

Yes. My equation agrees with my reasoning. My equation has h in the numerator so as h increases, d also increases. This agrees with my reasoning that a greater initial gravitational potential energy will result in a greater spring compression distance.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes the following criterion.

- ☐ 1 point is earned for indicating **one** of the following:
  - The period only depends on the internal properties of the spring and the mass of the block
  - Height h does not impact the period
  - Gravitational acceleration g does not impact the period
  - The only factors that affect the period of a mass spring oscillator is the mass and spring constant
  - The mass should be in the numerator rather than in the denominator

Example Response

The period of oscillation for a mass and spring should only depend on the mass of the block and the properties of the spring (spring constant). This equation has g and h in it, which does not make physical sense because these variables will not impact the properties of the spring or the mass of the block, so they will not affect the period of oscillation.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

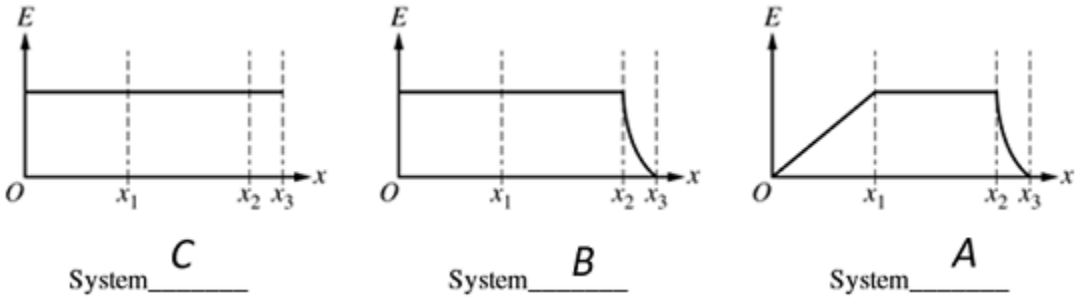
0	1	2	3	4	5 ✓

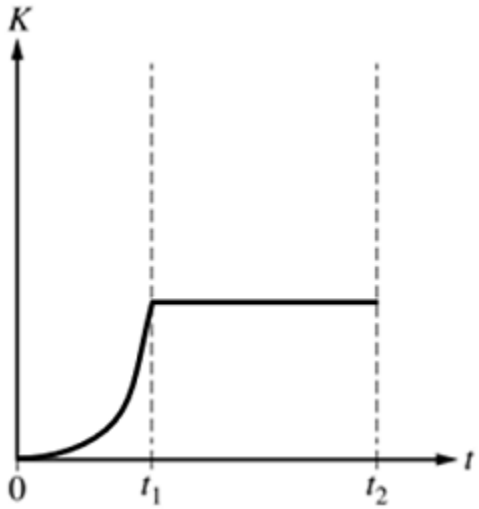
The response accurately includes **all** of the following criteria.

- 1 point is earned for indicating that the left graph corresponds to System C.
- 1 point is earned for indicating **either** of the following:
  - Middle graph corresponds to System B
  - Right graph corresponds to System A

  Scoring Note: If the same letter is written under multiple graphs, the point for the correct corresponding graph is not earned.
- 1 point is earned for drawing an increasing function between $t = 0$ and $t = t_1$.
- 1 point is earned for drawing a concave up curve between $t = 0$ and $t = t_1$.
- 1 point is earned for drawing a horizontal line between $t = t_1$ and $t = t_2$.

Example Response



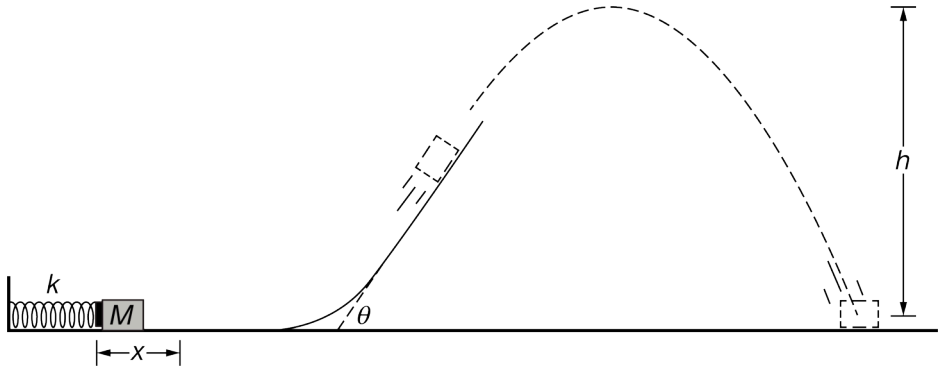


chapter4

Scoring Note: The graph does not have to be continuous at $t = t_1$ to earn full credit.

chapter4

8. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A box is pressed against a horizontal spring, compressing the spring from its relaxed length. The box is then released, and the spring launches the box horizontally along a track that ends in a ramp, as shown above. The box has enough speed to leave the ramp, and the box reaches a maximum vertical height above the floor. Assume there is negligible friction between the box and the track and air resistance is negligible.

k is the spring constant of the spring.
x is the distance the spring is compressed.
M is the mass of the box.
θ (theta) is the angle of the ramp from the horizontal.
h is the maximum height reached by the box.

The scenario is repeated using a stiffer spring with a spring constant that has a greater value for k than that of the original spring. The stiffer spring is compressed the same distance x, and the same box is used.

(a) Indicate how using a stiffer spring would affect the maximum height reached by the box, and explain why without mathematically deriving a relation for h.

(b) Students derive an equation for the maximum height, $h = \frac{(\sin^2\theta)kx^2}{2Mg}$, which may or may not be correct. Is this equation consistent with your claim in part (a)? Explain why or why not.

Two students are discussing the motion of the box when it is at its maximum height and make the following statements:

Student 1: The speed of the block at its maximum height is zero because the kinetic energy is a minimum at the top of the trajectory.

Student 2: Only the vertical component of the velocity is zero at the maximum height, because the vertical component of the acceleration is zero at the top of the trajectory.

(c) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

chapter4

(d) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

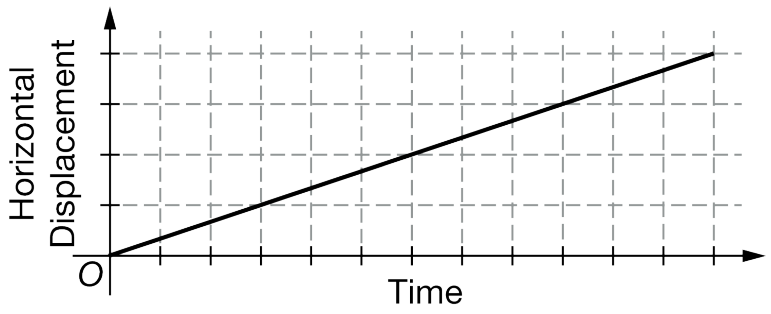
(e) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(g) Consider the system consisting of the box and the spring, but not Earth. How does the energy of the system when the spring is fully compressed compare to the energy of the system at the moment immediately before the box hits the ground? Justify your answer.

(h) If the friction between the track and the box were not negligible, state one way in which the setup could be changed such that the maximum height of the box was the same as in the original setup.

The graph below shows the horizontal component of the displacement of the box as a function of time. The graph represents the entire time the box is in the air after the box is launched from the ramp at an angle θ to the horizontal.



(i) How would the slope of the graph be different, if at all, if the ramp angle were larger than θ, but less than 90 degrees, and the height of the end of the ramp were the same? Justify your response.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that with a stiffer spring the spring constant would increase (or the average force would increase), so the speed of the box as it leaves the spring will be larger.
- ☐ 1 point is earned for a statement that because the speed of the box as it leaves the spring will be greater, and everything else remains the same, the maximum height of the block will increase.

chapter4

Example Response:

A stiffer spring means that the spring constant is larger, which means that there is more initial energy stored in the spring (or that the average net force on the box is greater), which means that the launch speed is greater, so with everything else the same the box will reach a larger maximum height.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating "yes", with an attempt at a justification using functional dependence.
- ☐ 1 point is earned for indicating that, since k is in the numerator if k increases, h will increase.

Example Response:

Yes, it is consistent with my reasoning – since k is in the numerator, if k increases, h will also increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the kinetic energy is a minimum at the top of the trajectory.

OR

The second statement

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 1 is incorrect in saying that the speed of the block is zero at the max height

OR

The first statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that only the vertical component of the speed is zero at the max height

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

chapter4

- [ ] 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 2 is incorrect in saying that the vertical component of the acceleration is zero at the top of the trajectory.

OR

The second statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- [ ] 1 point is earned for indicating that the energy of the box/spring system remains the same.
- [ ] 1 point is earned for indicating that there is mechanical energy in the spring
  OR
  For indicating that the box returns to the same height.
- [ ] 1 point is earned for indicating that just before the box lands all the energy is kinetic energy, and none is lost
  OR
  For indicating that the net work done by gravity is zero.

Example Response:

The energy of the spring-box system initially is spring potential energy, and just before the box lands, all the energy is kinetic energy, and because there is no net work done by outside forces, the energy of the spring box system is the same.

OR

The energy of the box/spring system remains the same because the box returns to the same height, and so the net work done by gravity is zero.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Valid changes also include making the ramp steeper, trading out the spring for a stiffer (larger k) spring, or using a box of less mass.

chapter4

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for indicating a valid change that could be made so that the maximum height would be the same.

Example Response:

Nonnegligible friction would cause mechanical energy to be dissipated. Therefore, the compression distance could be increased so that the spring transfers more energy to the box, offsetting the loss in energy from friction.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

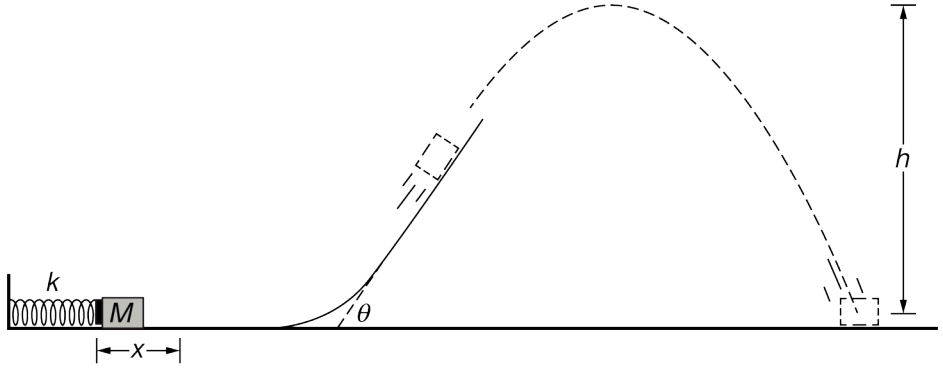
- [ ] 1 point is earned for indicating that the slope of the line would be smaller.
- [ ] 1 point is earned for indicating that with a larger angle, the horizontal component of the initial velocity is smaller.
- [ ] 1 point is earned for indicating that the slope of the horizontal position as a function of time graph is equal to the horizontal velocity of the box.

Example Response:

Since the graph is of horizontal position as a function of time, the slope of the line represents the horizontal velocity as a function of time. If the box were launched at a larger angle, the horizontal component of the velocity would be smaller and so this line would start at zero still but would have a smaller, but still positive, slope.

chapter4

9. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A box is pressed against a horizontal spring, compressing the spring from its relaxed length. The box is then released and the spring launches the box horizontally along a track that ends in a ramp, as shown above. The box has enough speed to leave the ramp, and the box reaches a maximum vertical height above the floor. Assume there is negligible friction between the box and the track and air resistance is negligible.

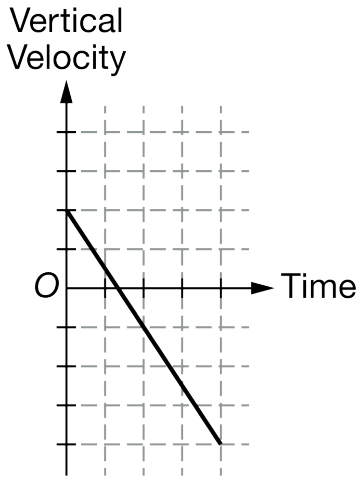
k is the spring constant of the spring.
x is the distance the spring is compressed.
M is the mass of the box.
θ (theta) is the angle of the ramp from the horizontal.
h is the maximum height reached by the box.

The scenario is repeated with the box compressing the spring a distance that is larger than the original distance x.

(a) Indicate how h in this second scenario compares to h in the original scenario, and explain why without mathematically deriving a relation for h.

(b) Students derive an equation for the maximum height h in the original scenario, $h = \frac{(\sin^2\theta)kx^2}{2Mg}$, which may or may not be correct. Is this equation consistent with your claim in part (a)? Explain why or why not.

chapter4



Two students, student 1 and student 2, are discussing the graph shown above, which shows the vertical component of the velocity of the box from when it leaves the ramp to immediately before it hits the floor. The students make the following statements.

Student 1: This graph shows that the vertical acceleration of the box is constant, and that the box's speed decreases for the entire time shown.

Student 2: The area between the graph and the time axis is the box's vertical displacement, but the graph is incorrect because the positive displacement should be equal to the negative displacement.

(c) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(g) Consider the system consisting of only the box, but not the spring or Earth. How does the energy of the system when the spring is fully compressed compare to the energy of the system at the moment immediately before the box hits the ground? Justify your answer.

The angle of the ramp is changed such that it is larger than θ but less than 90 degrees, while the height of the end of the ramp remains the same.

(h) How would the maximum height of the box compare to the original scenario in which the ramp angle was equal to θ ? Justify your answer.

(i) Would the slope of the graph of vertical velocity as a function of time shown above part (c) be different for this scenario in which the box is launched from the larger angle? Justify your response.

Part A

chapter4

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that with a larger compression distance the energy stored in the spring is greater (or the average force would increase or the spring acts on the box for a longer time/distance), so the speed of the box as it leaves the spring will be larger.
- ☐ 1 point is earned for a statement that because the speed of the box as it leaves the spring will be greater, and everything else remains the same, the maximum height of the block will increase.

Example Response:

A larger compression distance means that more energy is stored initially in the spring, which means that the launch speed is greater, so with everything else the same the box will reach a larger maximum height.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for the answer yes, and an attempt at a justification using functional dependence.
- ☐ 1 point is earned for indicating that, since x is in the numerator if x increases, h will increase.

Example Response:

Yes, it is consistent with my reasoning – since x is in the numerator, if x increases, h will also increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

chapter4

☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the vertical acceleration of the box is constant

OR

The first statement

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 1 is incorrect in saying that the box's speed decreases for the entire time shown.

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that the area between the graph and the time axis is the box's vertical displacement

OR

chapter4

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 2 is incorrect in saying that the positive displacement should be equal to the negative displacement.

OR

The second statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the energy of the box system increases
  OR
  For indicating that energy is greater from the spring doing work on box.
- ☐ 1 point is earned for indicating that there is no initial mechanical energy in the box system
  OR
  For indicating that the box returns to the same height.
- ☐ 1 point is earned for indicating that just before the box lands all the energy is kinetic energy
  OR
  For indicating that the net work done by gravity is zero.

Example Response:

The energy of the box system initially is zero because the spring is not in the system. Just before the box lands, all the mechanical energy in the system is kinetic energy, and because there is no net work done by gravity because the box returns to the same height, the energy of the box system increases.

chapter4

OR

The energy of the system increases, because the system does not include the spring, and the spring does work on the box. The box eventually returns to the same height, so the net work done on the system from gravity is zero.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating that increasing the angle would increase the vertical component of the velocity which would increase the maximum height.

Example Response:

Increasing the angle of the ramp would increase the vertical component of the velocity so the box would be able to go higher.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

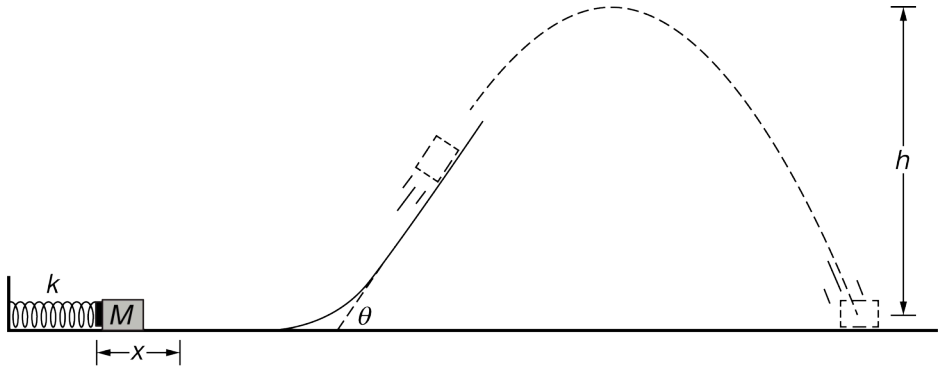
- ☐ 1 point is earned for indicating that the slope of the line would be the same.
- ☐ 1 point is earned for indicating that the slope of a velocity as a function of time graph is equal to the (vertical) acceleration.
- ☐ 1 point is earned for indicating that the vertical acceleration is constant (g).

Example Response:

Since the slope of the vertical velocity vs. time is equal to the acceleration, and the acceleration is constant throughout this problem (g), the slope of the line would remain the same even if the angle of the ramp were increased.

chapter4

10. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A box is pressed against a horizontal spring, compressing the spring from its relaxed length. The box is then released and the spring launches the box horizontally along a track that ends in a ramp, as shown above. The box has enough speed to leave the ramp, and the box reaches a maximum vertical height above the floor. Assume there is negligible friction between the box and the track and air resistance is negligible.

k is the spring constant of the spring.
x is the distance the spring is compressed.
M is the mass of the box.
θ (theta) is the angle of the ramp from the horizontal.
h is the maximum height reached by the box.

The scenario is repeated using a different box with a greater mass. The spring is compressed the same distance x.

(a) Indicate how h in this second scenario compares to h in the original scenario, and explain why without mathematically deriving a relation for h.

(b) Students derive an equation for h in the original scenario, $h = \frac{(\sin^2\theta)kx^2}{2Mg}$, which may or may not be correct. Is this equation consistent with your claim in part (a)? Explain why or why not.

Two students are discussing the work done on the system consisting of only the box between the instant just before the spring begins accelerating the box and the instant the box reaches the end of the ramp. The students make the following statements.

Student 1: At first, the mechanical energy of the system increases because the spring does positive work on the system. Then, the mechanical energy of the system increases again as the box travels up the ramp because the box's potential energy increases.

Student 2: The mechanical energy of the system remains constant while the spring pushes the box because the work done on the box by the spring cancels the work done by the spring on the box. Then, when the block is on the ramp, the mechanical energy of the system decreases because the box slows down.

(c) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***,

write “none.”

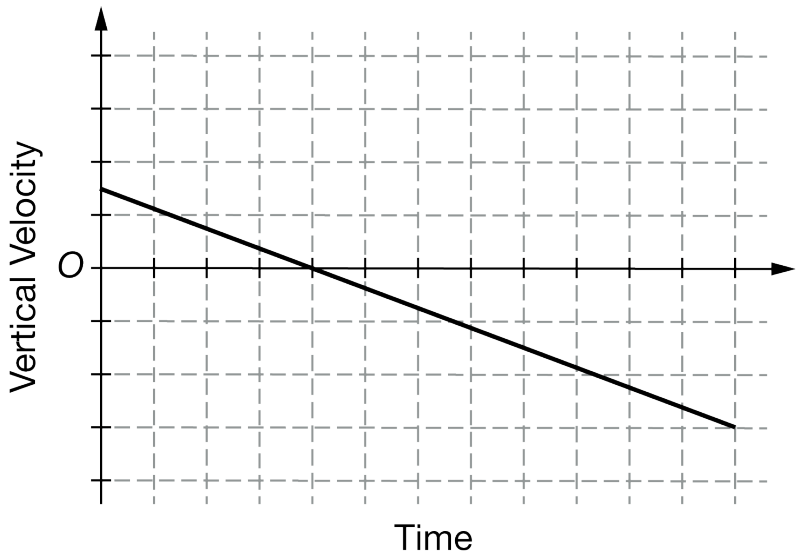
(d) Which underlined phrase or phrases in student 1’s statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write “none.”

(e) Which underlined phrase or phrases in student 2’s statement are ***correct***, if either? If neither phrase is ***correct***, write “none.”

(f) Which underlined phrase or phrases in student 2’s statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write “none.”

(g) Does the mechanical energy of the box-spring-Earth system increase, decrease, or remain the same from the moment the spring is released until just before the box hits the ground? Justify your answer.

(h) If air resistance were not negligible, how would that affect your answer to part (g), if at all? Justify your answer.



(i) The graph above shows the vertical component of the velocity of the box as a function of time. The graph represents the entire time the box is in the air when the box is launched from the ramp at an angle θ to the horizontal. How would the total area under the curve be different, if at all, if the ramp angle θ were increased but still less than 90 degrees, without changing the height of the end of the ramp?

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

chapter4

- [ ] 1 point is earned for a statement that with a larger mass (in the second scenario), the energy stored in the spring is the same, but the speed of the box on the table is smaller
  OR
  For a larger mass and an equivalent average force, the average acceleration will be smaller so the speed of the box on the table will be smaller.
- [ ] 1 point is earned for a statement that because the speed of the box as it leaves the spring will be less, and everything else remains the same. The maximum height of the block will decrease.

Example Response:

A larger mass means that although there is the same spring potential energy, the box will have a smaller speed as it approaches the bottom of the ramp. This means that the launch speed is less, so with everything else the same the box will reach a smaller maximum height.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	✓ 2

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for the answer yes, and an attempt at a justification using functional dependence.
- [ ] 1 point is earned for Since M is in the denominator, if M increases, h will decrease.

Example Response:

Yes, it is consistent with my reasoning – since M is in the denominator, if M increases, h will decrease.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	✓ 1

The response accurately includes

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the mechanical energy of the system increases because the spring does positive work on the system

chapter4

OR

The first statement

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that when the block is on the ramp, the mechanical energy of the system decreases because the box slows down

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 2 is incorrect in saying that the mechanical energy of the system remains constant while the spring pushes the box because the work done on the box by the spring cancels the work done by the spring on the box

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the energy of the box-spring-Earth system remains constant.
- ☐ 1 point is earned for indicating that initially there is spring potential energy.
- ☐ 1 point is earned for indicating that just before the box lands all the energy is kinetic energy.

Example Response:

The energy of the box-spring-Earth system initially is elastic potential energy because all the energy is stored in the spring. Just before the box lands, all the mechanical energy in the system is kinetic energy, and because there is no net work done by gravity because the box returns to the same height, the energy of the box-spring-Earth system is constant.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a statement that air resistance would decrease the mechanical energy of the system.

Example Response:

If there was air resistance it would remove energy from the box-spring-Earth system, decreasing the mechanical energy of the system.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a statement that the total area under the curve would be the same.

chapter4

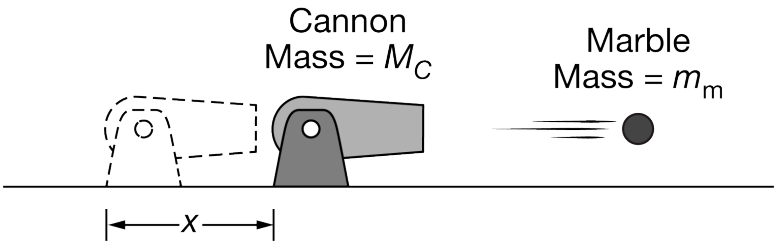
- [ ] 1 point is earned for a statement that the area under the curve of a vertical velocity vs. time graph is the vertical displacement.
- [ ] 1 point is earned for a statement that the box starts and ends at the same height as in the original scenario, so the vertical displacement is the same as in the original.

Example Response:

Since the area under the vertical velocity vs. time graph is the vertical displacement, and the box starts and ends at the same height, the vertical displacement will still be the same in both cases, and so the total area under the curve will be the same for both as well.

chapter4

11. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch marbles of mass m_{m} horizontally from a toy cannon of mass M_C, where M_C is always greater than m_{m}, as shown above. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students use a new marble with a different mass launched at the same speed v relative to the ground. The students record the distance x the cannon slides backward for each launch.

(a) Without manipulating equations, briefly describe what happens to the distance x that the cannon slides backward as students increase the mass of the marble. Explain your reasoning.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\mathrm{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) Without manipulating or stating equations, briefly describe whether it would take more work to stop the marble or to stop the cannon after the marble has been launched. Justify your answer.

(d) Students correctly derive the following equation relating the work W done on an object initially at rest, the momentum p of the object, and the mass m of the object: $W = \frac{p^2}{2m}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble-cannon system during the short time interval in which the marble is accelerated inside the cannon and make the following claims:

Student 1: The cannon is pushing the marble forward, and the marble is pushing back with equal and opposite force. Since the work done by the cannon cancels out the work done by the marble, the mechanical energy of the system is constant.

Student 2: The mechanical energy of the system increases while the marble is being accelerated. The marble gains kinetic energy to the right, and the cannon gains an equal amount of kinetic energy to the left.

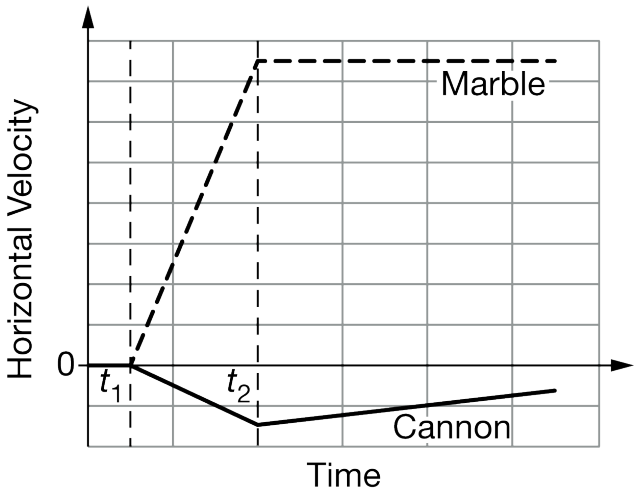
(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is

incorrect, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity of the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Provide three pieces of evidence that together explain why the magnitude of the rate of change of the cannon's velocity is less than the magnitude of the rate of change of the marble's velocity during the time interval from t_1 to t_2.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to indicate that a larger marble mass means a larger cannon speed
  OR
  For using a combination of Newton's 3rd law leading to Newton's 2nd law to indicate that a larger marble mass means a larger cannon speed.
- ☐ 1 point is earned for explaining why the larger cannon speed results in sliding a greater distance.

chapter4

Example Response:

The initial momentum of the marble-cannon system is zero, so it will be zero after the marble is launched as well. A greater marble mass then means that the backward cannon speed is greater. Since the acceleration due to friction is the same regardless of cannon speed, a greater cannon speed means that the cannon will slide a greater distance.

OR

While the marble is in the cannon, the force on the marble has the same magnitude as the force on the cannon (according to Newton's 3rd law). If the final speed of the marble is the same each time but the mass of the marble increases, the net force on the marble must increase (according to Newton's 2nd law) and, therefore, the net force on the cannon must increase as well (the force of friction remains constant) giving the cannon a larger final speed resulting in a greater distance.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence between the mass m_m and x.
- ☐ 1 point is earned for a claim that as m_m in the equation increases, x in the equation also increases.

Example Response:

Yes, the equation matches that as the mass of the marble increases the slide distance of the cannon also increases. The mass of the marble is in the numerator of the equation so if the numerator increases the slide distance will also increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim that the marble requires more work to stop, with attempt at a justification.
- ☐ 1 point is earned for a valid justification.

Example Response:

The marble requires more work to stop because the marble has a greater kinetic energy thus a greater change in kinetic

chapter4

energy in order to stop.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim consistent with the answer to (c).
- ☐ 1 point is earned for a claim that both the marble and cannon have equal momentum when the marble is launched and for recognizing that the marble's smaller mass in the denominator means it has more work done on it.

Example Response:

Yes. By conservation of momentum, the cannon and marble have the same magnitude of momentum immediately after the launch. Since p is the same, but m is smaller for the marble, the kinetic energy is greater for the marble since m is in the denominator, which means that the marble had more work done on it.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1's statement that is correct is the marble is pushing back with equal and opposite force

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 1's statement that is incorrect is work done by the cannon cancels out the work done by the marble and mechanical energy is constant.

OR

The second statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If a student considered the cannon to include a spring launching mechanism, then the first statement would be incorrect. A student who states "none" in part (g) and "both" in part (h) earns full credit for both parts.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2's statement that is correct is mechanical energy increases while the marble is being accelerated.

OR

The first statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student's 2 statement that is incorrect is the cannon gains an equal amount of kinetic energy to the left.

OR

The second statement

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Though the slope change in the graph occurs at t_2 (rather than "in the interval from t_1 to t_2 "), a student who indicates the effect of force of friction by discussing the slope change at t_2 is awarded this point.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

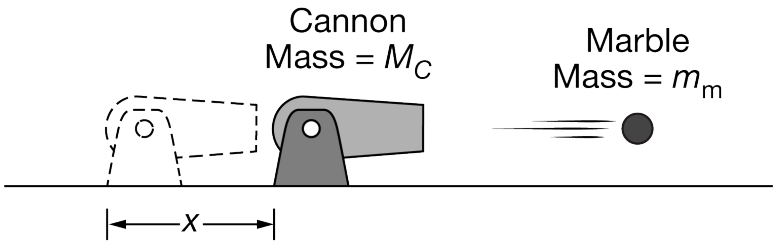
- ☐ 1 point is earned for recognizing that the same force is exerted on both the marble and cannon.
- ☐ 1 point is earned for indicating that the larger mass of the cannon results in its acceleration being less.
- ☐ 1 point is earned for indicating the force of friction acting opposite the cannon's direction of motion will further reduce the acceleration of the cannon.

Example Response:

Even the though the force on the cannon and the marble is the same due to Newton's third law, the cannon's acceleration is smaller. So, its velocity increases less than the marble because the cannon's mass is larger. The cannon also has a frictional force opposing its motion, which decreases the net force and therefore the acceleration.

chapter4

12. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch identical marbles of mass m_{m} horizontally from a toy cannon of varying mass M_C, where M_C is always greater than m_{m}, as shown above. The students can attach weights to the cannon to change M_C. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students vary the mass of the cannon and launch a marble with the same speed v relative to the ground. The students record the distance x that the cannon slides backward.

(a) Without manipulating equations, explain how the distance x that the cannon slides backward changes, if at all, when marbles are launched from more massive cannons.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\text{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) If constant horizontal forces were exerted over the same distance on the cannon and on the marble after the launch, which object would require a force with a greater magnitude to completely stop it? Without manipulating or referencing equations, briefly explain your reasoning.

(d) Students correctly derive the following equation relating the force F exerted on an object initially at rest over a distance d, the mass m of the object, and the final momentum p of the object: $F = \frac{p^2}{2md}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble after it has been launched by the cannon but before it reaches the ground. Both students agree that air resistance is negligible and make the following claims.

Student 1: The mechanical energy of the marble is constant. There are no external forces exerted on the marble, so its energy does not change.

Student 2: The mechanical energy decreases as the marble falls. This happens because the gravitational force is doing work on the marble.

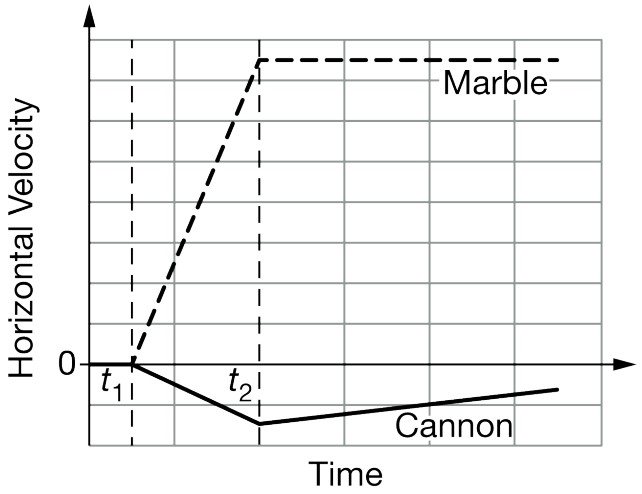
(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

chapter4

(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity of the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Describe the motion of the center of mass of the marble-cannon system during the time interval from t_1 to t_2. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Students may earn the points for a correct discussion regarding the force of friction. A more massive cannon leads to a greater normal force increasing the force of friction. This would cause the net force on the cannon to the left to be smaller while the marble was in the cannon (giving it an initially smaller speed once the marble has left the cannon) resulting in a smaller distance traveled.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to indicate that a larger cannon mass means a smaller cannon speed

chapter4

OR

For using a combination of Newton's 3rd law leading to Newton's 2nd law to indicate that a larger cannon mass means a smaller cannon speed.

☐ 1 point is earned for explaining why the smaller cannon speed results in sliding a shorter distance.

Example Response:

The initial momentum of the marble-cannon system is zero, so it will be zero after the marble is launched as well. A more massive cannon then means that the backward cannon speed is less. A smaller cannon speed means that the cannon will slide a shorter distance.

OR

While the marble is in the cannon, the force on the marble has the same magnitude as the force on the cannon (according to Newton's 3rd law). If the mass of the marble stays constant but the mass of the cannon increases, the cannon will have smaller acceleration and smaller speed (according to Newton's 2nd law) resulting in a smaller distance.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

☐ 1 point is earned for using functional dependence between the mass and the distance x.

☐ 1 point is earned for a claim that as M_c in the equation increases, x in the equation decreases.

Example Response:

Yes, the equation matches the prediction that as the mass of the cannon increases the distance the cannon slides backwards decreases. From the equation, the mass of the cannon and the distance it slides backwards are inversely proportional. The mass of the cannon is in the denominator of the equations so as the mass increases the distance the cannon slides backwards will decreases.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

chapter4

- ☐ 1 point is earned for a claim that it would take more force to stop the marble, with an attempt at a justification.
- ☐ 1 point is earned for a valid justification using either work/energy or momentum.

Example Response:

It would take more force to stop the marble because the change in kinetic energy to stop the marble is greater so the work on the marble needs to be greater.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim consistent with the answer to (c).
- ☐ 1 point is earned for a claim that both the marble and cannon have equal momentum when the marble is launched and for recognizing that the marble's smaller mass in the denominator means it requires more force to stop.

Example Response:

Yes. By conservation of momentum, the cannon and marble have the same magnitude of momentum immediately after the launch. Since p is the same, but m is smaller for the marble, the force is greater for the marble since m is in the denominator.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a statement of "None".

Example Response:

None

Part F

chapter4

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating both statements are incorrect.

Example Response:

Both parts of Student 1's statement are incorrect.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

The gravitational force is doing work on the marble

OR

The second statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

chapter4

Example Response:

The mechanical energy decreases as the marble falls.

OR

The first statement

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

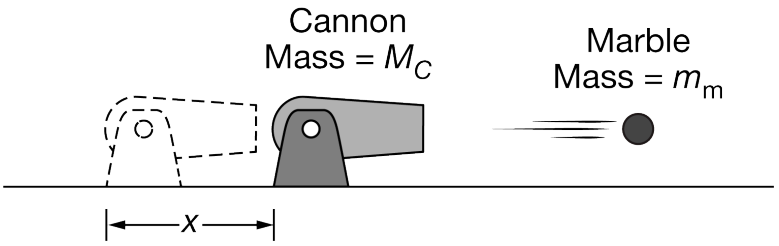
- ☐ 1 point is earned for indicating the center of mass moves to the right.
- ☐ 1 point is earned for indicating that there is an external force on the system due to friction.
- ☐ 1 point is earned for indicating that the net force on the system is needed to move the center on mass.

Example Response:

During the launch, the only horizontal external force exerted on the marble-cannon system is the frictional force exerted on the cannon. The friction force is directed to the right, so the net force on the system is to the right. This means the center of mass of the system is accelerating to the right.

chapter4

13. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch identical marbles of mass m_{m} horizontally from a toy cannon of mass M_C, where $M_C > m_{\text{m}}$, as shown above. The cannon can be adjusted to change the launch speed v of the marble relative to the ground. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students vary the launch speed v and record the distance x the cannon slides backward for each launch speed.

(a) Without manipulating equations, briefly describe what happens to the distance x that the cannon slides backward as students increase the launch speed. Explain your reasoning.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\text{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) Without manipulating or stating equations, briefly describe whether the cannon or the marble has more kinetic energy immediately after the marble leaves the cannon. Justify your answer.

(d) Students correctly derive an equation relating the kinetic energy K, the momentum p, and the mass m of an object: $K = \frac{p^2}{2m}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble after it has been launched by the cannon but before it reaches the ground. Both students agree that air resistance is negligible and make the following claims.

Student 1: There are two external forces exerted on the marble while it is in the air, the gravitational force and the force exerted by the cannon. Therefore, the mechanical energy of the marble changes while it is in the air.

Student 2: The cannon does not exert a force on the marble after the launch. Therefore, the mechanical energy of the marble is constant.

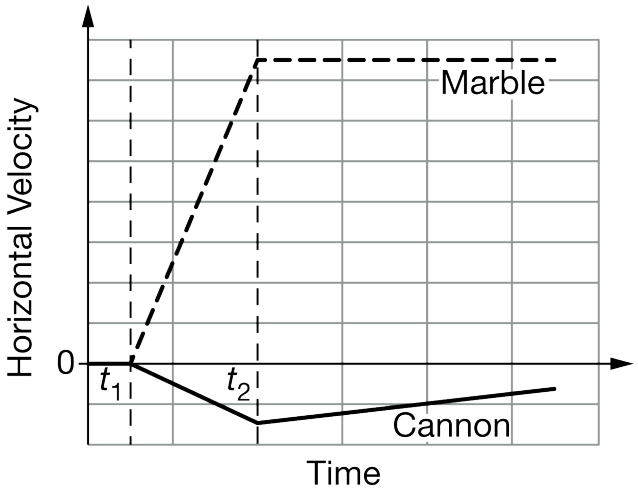
(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

chapter4

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity for the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Describe how to find the change in the distance between the center of mass of the cannon and the center of mass of the marble from time t_1 to time t_2. Assume that before time t_1 the center of mass of the marble is at the same position as the center of mass of the cannon.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to indicate that a larger marble speed means a larger cannon speed
  OR
  For using a combination of Newton's 3rd law leading to Newton's 2nd law to indicate that a larger marble speed means a larger cannon speed.
- ☐ 1 point is earned for correctly explaining why the larger cannon speed results in sliding a greater distance.

chapter4

Example Response:

The initial momentum of the marble-cannon system is zero, so it will be zero after the marble is launched as well. A greater marble speed then means that the backward cannon speed is greater. Since the acceleration due to friction is the same regardless of cannon speed, a greater cannon speed means that the cannon will slide a greater distance.

OR

While the marble is in the cannon, the force on the marble has the same magnitude as the force on the cannon (according to Newton's 3rd law). If the mass of the marble stays constant but the final speed of the marble increases, the net force on the marble must increase (according to Newton's 2nd law) and, therefore, the net force on the cannon must increase as well (the force of friction remains constant) giving the cannon a larger final speed resulting in a greater distance.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence.
- ☐ 1 point is earned for a claim that as v in the equation increases, x in the equation also increases.

Example Response:

Yes, the equation matches the prediction that as the velocity of the marble increases the distance the cannon moves backward also increases. In the equation as velocity increases the distance would increase. Since the velocity is in the numerator of the equation, as the numerator of a fraction increases the distance x would increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim that the marble has more kinetic energy, with an attempt at a justification.
- ☐ 1 point is earned for a valid justification.

Example Response:

By conservation of momentum, mass and velocity are inversely proportional. Since velocity is squared when determining

chapter4

kinetic energy, a greater speed increases the kinetic energy by a larger factor than a smaller mass decreases the energy.

OR

The cannon and marble exert forces of equal magnitude on each other. However, the marble will move a greater distance than the cannon while these forces are exerted. So, the marble has more work done on it, and therefore a greater kinetic energy.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim consistent with the answer to (c).
- ☐ 1 point is earned for a claim that both the marble and cannon have equal momentum when the marble is launched and for recognizing that the marble's smaller mass in the denominator means it has more kinetic energy.

Example Response:

Yes. By conservation of momentum, the cannon and marble have the same magnitude of momentum immediately after the launch. Since p is the same, but m is smaller for the marble, the kinetic energy is greater for the marble since m is in the denominator.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

The correct statement from student 1 is that mechanical energy of the marble changes while it is in the air

OR

The second statement

chapter4

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for an incorrect statement and no correct statements.

Example Response:

The incorrect statement for student 1 is that the cannon exerts a force on the marble while it is in the air

OR

The first statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

The correct statement for student 2 is that the cannon no longer exerts a force on the marble

OR

The first statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

The incorrect statement for student 2 is that the mechanical energy of the marble is constant

OR

The second statement

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for recognizing that displacement can be calculated by finding the area under a *v* vs. *t* graph.
- ☐ 1 point is earned for recognizing that the total displacement will be represented by combining the areas under each *v* vs. *t* graph.
- ☐ 1 point is earned for indicating that, for the cannon, either the area (which is negative) is to be subtracted
  OR
  the absolute value of the area is to be added, to the area under the marble graph.

Example Response:

The area under the velocity vs. time graph represents the displacement of an object. To find the change in the distance, subtract the areas of the two graphs between t_1 *and* t_2.

chapter4

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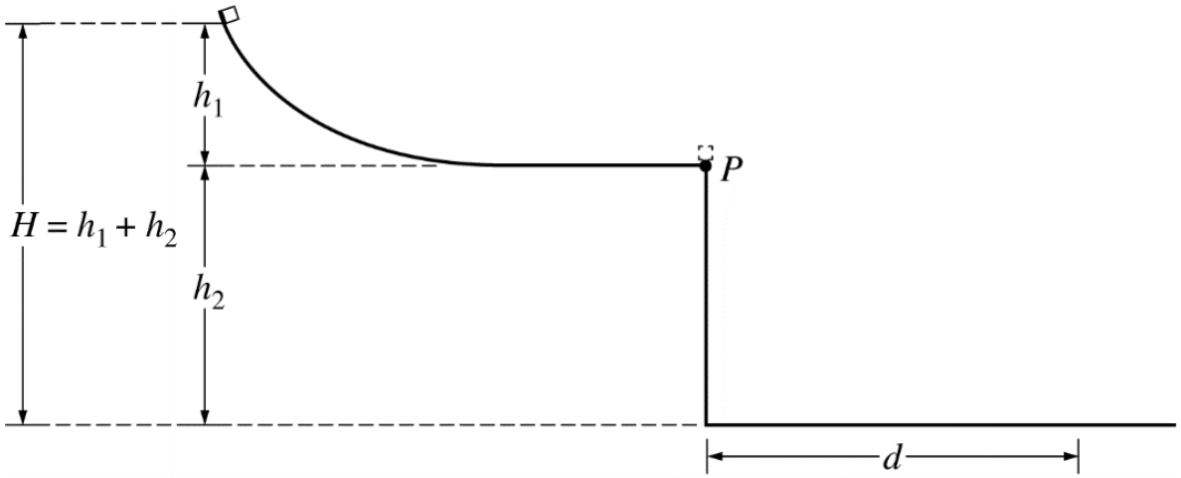


A student releases a block of mass m from rest at the top of a slide of height h_1. The block moves down the slide and off the end of a table of height h_2 , landing on the floor a horizontal distance d from the edge of the table. Friction and air resistance are negligible. The overall height H of the setup is determined by the height of the room. Therefore, if h_1 is increased, h_2 must decrease by the same amount so that the sum $h_1 + h_2$ remains equal to H. The student wants to adjust h_1 and h_2 to make d as large as possible.

a. i. Without using equations, explain why making h_1 very small would cause d to be small, even though h_2 would be large.

ii. Without using equations, explain why making h_2 very small would cause d to be small, even though h_1 would be large

b. Derive an equation for d in terms of h_1, h_2, m, and physical constants, as appropriate.

c. i. Write the equation or step in your derivation in part (b) (not your final answer) that supports your reasoning in part (a)i.

Briefly explain your choice.

ii. Write the equation or step in your derivation in part (b) (not your final answer) that supports your reasoning in part (a)ii.

Briefly explain your choice.

d. If the experiment is repeated on the Moon without changing h_1 or h_2 , will the new landing distance d be greater than, less than, or the same as the landing distance when the experiment is performed on Earth?

_____Greater than _____Less than _____The same as

Briefly explain your choice.

chapter4

Part A

Part(i)

1 point is earned: For recognizing a small horizontal velocity for the block at point P

1 point is earned: For stating or implying that the block doesn't go very far even though there is a longer fall time

Example: Even though the large h2 allows the block to stay in the air for a long time, the small h1 means the system loses little gravitational potential energy and hence gains little kinetic energy so that it leaves the table with too little speed to cover much distance while aloft.

Part(ii)

1 point is earned: For recognizing a large horizontal velocity for the block at point P

1 point is earned: For mentioning a short time of flight for the falling block

Example: Now the block gains a lot of kinetic energy on the ramp so that it leaves the table with a large horizontal speed. But the small h2 means the block spends so little time in the air that it lands before covering much horizontal distance.

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Part(i)

1 point is earned: For recognizing a small horizontal velocity for the block at point P

1 point is earned: For stating or implying that the block doesn't go very far even though there is a longer fall time

Example: Even though the large h2 allows the block to stay in the air for a long time, the small h1 means the system loses little gravitational potential energy and hence gains little kinetic energy so that it leaves the table with too little speed to cover much distance while aloft.

Part(ii)

1 point is earned: For recognizing a large horizontal velocity for the block at point P

1 point is earned: For mentioning a short time of flight for the falling block

Example: Now the block gains a lot of kinetic energy on the ramp so that it leaves the table with a large horizontal speed. But the small h2 means the block spends so little time in the air that it lands before covering much horizontal distance.

Part B

1 point is earned: For using an energy conservation statement (corresponds to step 1 in example below)

1 point is earned: For setting up a time of flight calculation using vertical acceleration (corresponds to step 2a in example

chapter4

below)

Note: derivation of t must be shown to earn credit.

1 point is earned: For using vt = d to determine distance traveled (corresponds to step 2b in example below)

Example:

Step 1: Use energy conservation to find the block's speed at the bottom of ramp, which equals the launch speed v1 at point P:

mgh1 = 1/2mv12 ⇒ v1 = √gh1

Step 2a: Find the time of flight, using the independence of the horizontal and vertical motion. Since the block leaves the table with zero vertical speed, the vertical kinematic equation y = y0 + vy0t + 1/2ayt2 reduces h2 = 1/2 gt2 (taking downward as positive). Solve for time to get t = √(2h2/g).

Step 2b. Find the horizontal distance d covered while in the air after point P. Since there are no horizontal forces exerted on the block, its horizontal velocity stays constant at v1. So, the block travels a horizontal distance

$$d = v_i t = \sqrt{2gh_1}\sqrt{\frac{2h_2}{g}} = 2\sqrt{h_1 h_2}$$

✓

0	1	2	3

Student response earns 3 of the following 3 point(s)

1 point is earned: For using an energy conservation statement (corresponds to step 1 in example below)

1 point is earned: For setting up a time of flight calculation using vertical acceleration (corresponds to step 2a in example below)

Note: derivation of t must be shown to earn credit.

1 point is earned: For using vt = d to determine distance traveled (corresponds to step 2b in example below)

Example:

Step 1: Use energy conservation to find the block's speed at the bottom of ramp, which equals the launch speed v1 at point P:

mgh1 = 1/2mv12 ⇒ v1 = √gh1

Step 2a: Find the time of flight, using the independence of the horizontal and vertical motion. Since the block leaves the table with zero vertical speed, the vertical kinematic equation y = y0 + vy0t + 1/2ayt2 reduces h2 = 1/2 gt2 (taking downward as positive). Solve for time to get t = √(2h2/g).

Step 2b. Find the horizontal distance d covered while in the air after point P. Since there are no horizontal forces exerted on the block, its horizontal velocity stays constant at v1. So, the block travels a horizontal distance

chapter4

$$d = v_i t = \sqrt{2gh_1}\sqrt{\frac{2h_2}{g}} = 2\sqrt{h_1 h_2}$$

Part C

Part(i)

1 point is earned: For mentioning a correct equation or step of reasoning (energy conservation on ramp)

1 point is earned: For correctly explaining why this equation or step mirrors the reasoning of (a)i

Example: My energy conservation reasoning (part (b) step 1) mirrors my reasoning that

a small ramp height corresponds to a low potential energy (mgh1), which is converted into low kinetic energy and hence a low speed.

Part(ii)

1 point is earned: For mentioning a correct equation or step of reasoning (time of flight)

1 point is earned: For correctly explaining why that equation or step mirrors the reasoning of (a)ii

Example: My time-of-flight calculation (part (b) step 2a) shows that t is proportional to √(h2), so a smaller falling distance h2 corresponds to a smaller flight time.

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Part(i)

1 point is earned: For mentioning a correct equation or step of reasoning (energy conservation on ramp)

1 point is earned: For correctly explaining why this equation or step mirrors the reasoning of (a)i

Example: My energy conservation reasoning (part (b) step 1) mirrors my reasoning that

a small ramp height corresponds to a low potential energy (mgh1), which is converted into low kinetic energy and hence a low speed.

Part(ii)

1 point is earned: For mentioning a correct equation or step of reasoning (time of flight)

1 point is earned: For correctly explaining why that equation or step mirrors the reasoning of (a)ii

Example: My time-of-flight calculation (part (b) step 2a) shows that t is proportional to √(h2), so a smaller falling distance h2 corresponds to a smaller flight time.

Part D

chapter4

Correct answer: “The same as”

Note: explanation is scored regardless of checkbox.

Point can be earned if response is consistent with answer in part (b)

1 point is earned: For reasoning that is consistent with the functional dependence of the part (b) answer

Example:

As found in part (b), the distance d does not depend on g. So, doing the experiment on the Moon instead of Earth makes no difference. The increased time of flight (since weaker gravity makes the block take longer to land) compensates for the small speed gained by the block on the ramp.

Checked “Greater than” then g must be in the denominator in the incorrect equation in part (b)

Checked “Less than” then g must be in the numerator in the incorrect equation in part (b)

0	1 ✓

Student response earns 1 of the following 1 point(s)

Correct answer: “The same as”

Note: explanation is scored regardless of checkbox.

Point can be earned if response is consistent with answer in part (b)

1 point is earned: For reasoning that is consistent with the functional dependence of the part (b) answer

Example:

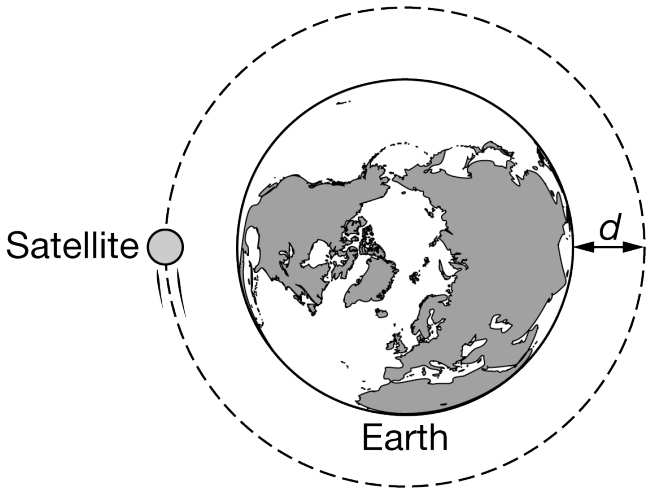
As found in part (b), the distance d does not depend on g. So, doing the experiment on the Moon instead of Earth makes no difference. The increased time of flight (since weaker gravity makes the block take longer to land) compensates for the small speed gained by the block on the ramp.

Checked “Greater than” then g must be in the denominator in the incorrect equation in part (b)

Checked “Less than” then g must be in the numerator in the incorrect equation in part (b)

chapter4

15. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface, as shown above.

An identical satellite is set into orbit around a new planet. The new planet has the same radius as the radius of Earth but is significantly more massive than Earth. The satellite orbits the same distance d from the surface of the new planet.

(a) Is the speed of the satellite orbiting the new planet greater than, less than, or the same as the speed of the satellite orbiting Earth? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

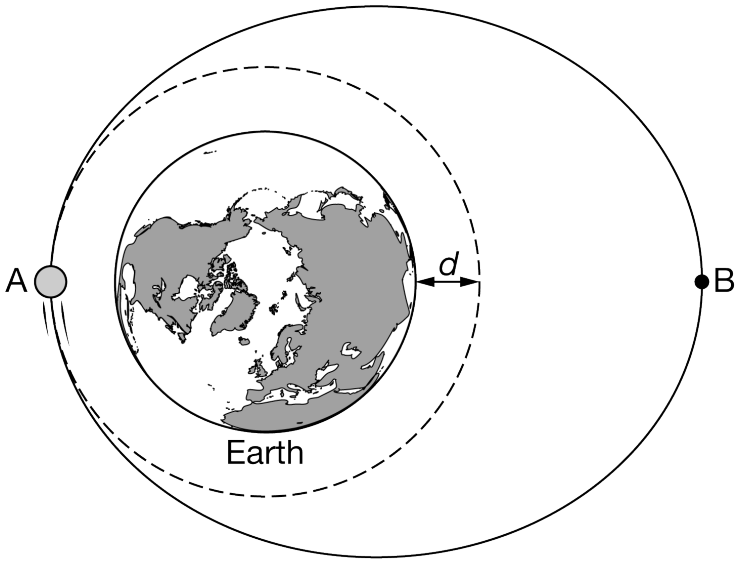
$$\frac{MT^2}{R^3} = C$$

In the equation above

M is the mass of a planet;
T is the orbital period of a satellite in a circular orbit;
R is the distance from the planet's center to the satellite; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

chapter4



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point A to point B, and state the following:

Student 1: The satellite is moving faster at point A because the satellite's angular momentum is greater at point A than at point B.

Student 2: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is higher at point B than at point A.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

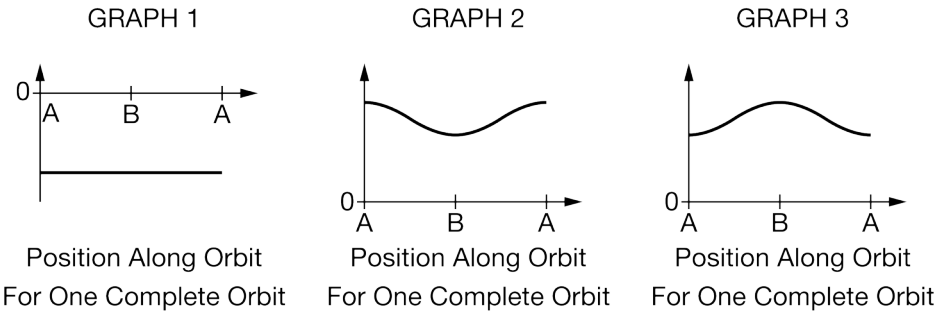
(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite

as it completes one elliptical orbit, starting and ending at point A.



(h) Which of the above graphs best represents the magnitude of the gravitational field at the location of the satellite as it orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the total mechanical energy of the satellite-Earth system as the satellite orbits? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the satellite around the new planet will orbit with a faster speed.
- ☐ 1 point is earned for a correct explanation in terms of the relationship between gravitational force and centripetal force (or acceleration).

Example Response:

There is greater gravitational force acting on the satellite around the new planet. The increased gravitational force means that the centripetal force $m\frac{v^2}{R}$ *(or acceleration* $\frac{v^2}{R}$*) is greater. Since R is the same in both cases, v must be greater for the satellite orbiting the new planet.*

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

chapter4

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a consistent answer AND an attempt at reasoning using functional dependence.
- ☐ 1 point is earned for correctly relating the increased *M* to a decreased *T*.
- ☐ 1 point is earned for correctly explaining how a decreased T increases *v*.

Example Response:

Yes, this is consistent with my reasoning. Since the mass of the new planet is larger, the time it will take to go in an orbit must decrease because M is inversely proportional to T squared. Since the new satellite travels the same circumference orbit, if the second satellite takes less time to go around, it must be going faster.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	✓ 2

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the force was applied in the satellite's direction of motion.
- ☐ 1 point is earned for a correct explanation relating the direction of force with the change in speed.

Example Response:

A force in the direction of the satellite's motion would make the satellite move faster

OR

A force in the direction of the satellite's motion would do positive work on the satellite.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	✓ 1

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

chapter4

Student 1 is correct in saying that the satellite is moving faster at A

OR

The first statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is incorrect in saying that the satellite's angular momentum is larger at A than at B

OR

The second statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that the potential energy of the satellite-Earth system is higher at B than at A

OR

The second statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is incorrect in saying that the satellite is moving faster at B

OR

The first statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 2.
- ☐ 1 point is earned for a correct explanation relating field strength to distance.

Example Response:

Graph 2. The field is larger where the satellite is closer to the planet, so it is larger at A than at B.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 1.

chapter4

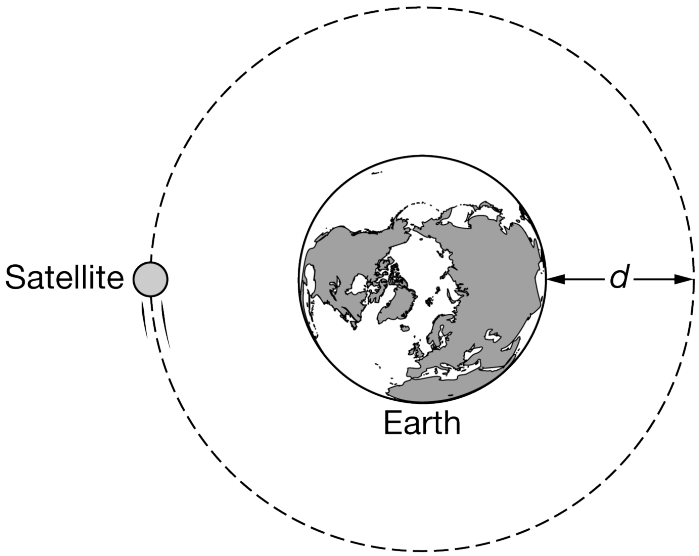
- [ ] 1 point is earned for a correct explanation of why the total mechanical energy is constant in terms of net external force.

Example Response:

Graph 1. The total mechanical energy is constant for the satellite-Earth system because there are no external forces on the system.

chapter4

16. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface, as shown above.

An identical satellite is set into orbit around a new planet. The new planet's radius is the same as the radius of Earth but is significantly more massive than Earth. The satellite orbits the new planet with the same orbital period.

(a) Is the distance of the satellite from the surface of the new planet greater than, less than, or equal to d ? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

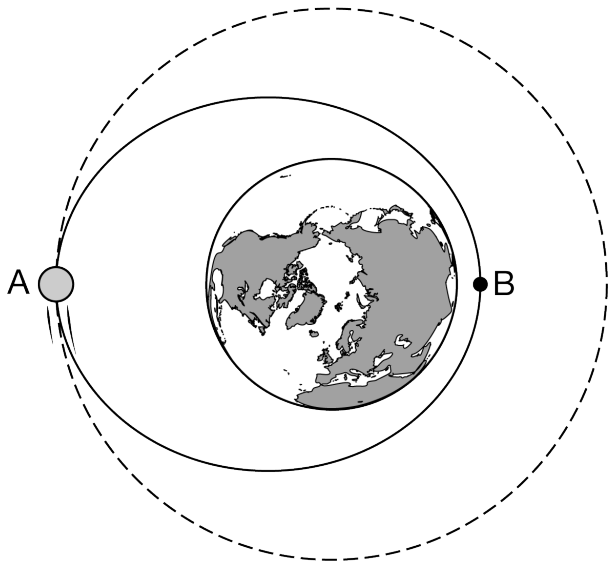
$\frac{Rv^2}{M} = C$

In the equation above

R is the distance from a planet's center to a satellite in a circular orbit;
v is the speed of the satellite;
M is the mass of the planet; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

chapter4



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point A to point B, and state the following:

Student 1: The satellite is moving faster at point A because the total mechanical energy of the satellite-Earth system is constant.

Student 2: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is higher at point B than at point A.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

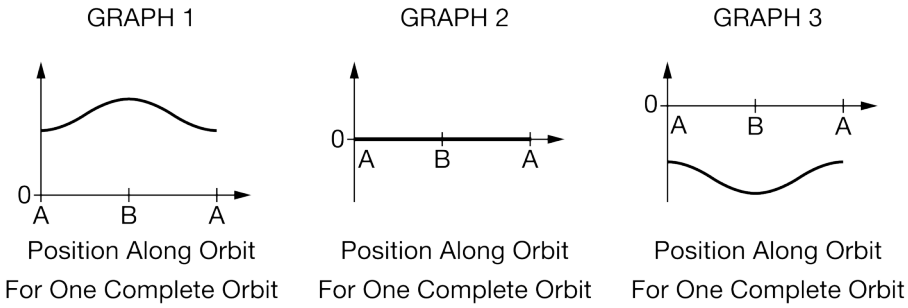
(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite

as it completes one elliptical orbit, starting and ending at point A.



(h) Which of the above graphs best represents the potential energy of the satellite-Earth system as the satellite orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the torque about the center of Earth exerted on the satellite by Earth's gravitational force as the satellite orbits? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the satellite around the new planet will orbit at a greater distance.
- ☐ 1 point is earned for correctly explaining an increasing distance from the planet to satellite is caused by an increasing speed to maintain a constant period.

  OR

  For correctly explaining an increased distance from the planet to decrease the gravitational force or acceleration on the satellite.

Example Response:

For the satellite to maintain the same period, it must orbit at a greater distance from the new planet because the satellite's orbital speed will be now be greater than before.

OR

"*The distance of the satellite must be further since increasing mass increases the force of gravity keeping the satellite in orbit, to maintain the same period, you would need to be further from the planet to decrease the gravitational force from the larger mass.*

Part B

chapter4

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a consistent answer AND an attempt at reasoning using functional dependence.
- ☐ 1 point is earned for using $v = \frac{2\pi R}{T}$ or arguing that *R* must increase if *T* remains the same.
- ☐ 1 point is earned for correctly relating the increase in mass of the planet increases the distance from the planet to satellite.

Example Response:

Yes, this matches with my reasoning in part (a). The mass of the new planet is greater, which means that for C to stay constant the top of the fraction must also be greater. Since v is $\frac{2\pi R}{T}$ *and T is constant, R must be larger for the speed to be larger.*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the force was applied opposite to the satellite's direction of motion.
- ☐ 1 point is earned for a correct explanation relating the direction of force with the changing speed.

Example Response:

The force was applied opposite to the motion of the satellite's direction of motion because this would make the satellite move slower which does negative work on the satellite.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

chapter4

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the total mechanical energy of the satellite-Earth system is constant

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is incorrect in saying that the satellite is moving faster at A

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that the satellite is moving faster at B

chapter4

OR

The first statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is incorrect in saying that the potential energy of the satellite-Earth system is higher at B than at A

OR

The second statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 3.
- ☐ 1 point is earned for a correct explanation relating potential energy to distance.

Example Response:

Graph 3. The potential energy is higher where the satellite is farther from the planet, so it is higher at A than at B.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1	2 ✓

The response accurately includes **both** of the following criteria.

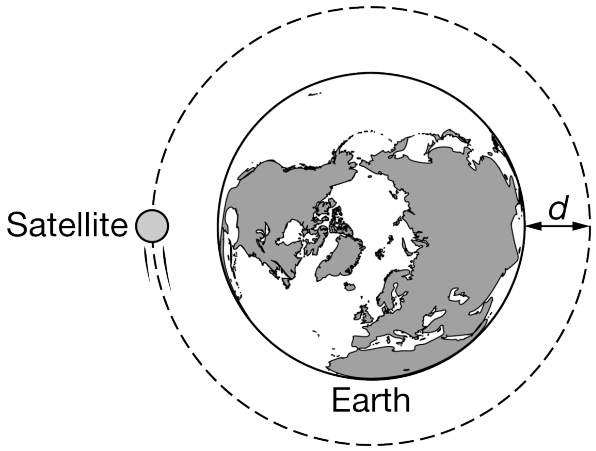
- ☐ 1 point is earned for indicating graph 2.
- ☐ 1 point is earned for a correct explanation why the torque is zero.

Example Response:

Graph 2. The torque is zero since the angle between $\vec{F}$ *and* $\vec{r}$ *is 180 degrees.*

chapter4

17. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface.

An identical satellite is set into orbit around a new planet. The new planet's radius is the same as the radius of Earth but the new planet is significantly more massive than Earth. The satellite orbits the new planet with the same speed v.

(a) Is the distance of the satellite from the surface of the new planet greater than, less than, or equal to d ? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

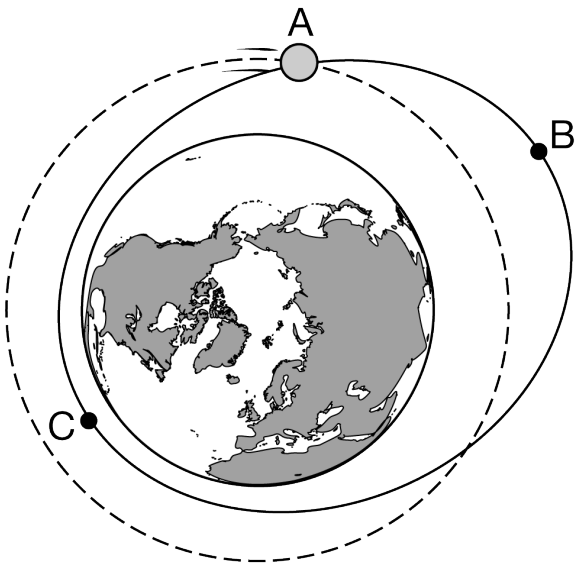
$$\frac{R^3}{MT^2} = C$$

In the equation above

R is the distance from a planet's center to a satellite in a circular orbit;
M is the mass of the planet;
T is the orbital period of the satellite; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

chapter4



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit. In the modified orbit, the satellite's speed at point A, after the thrusters were fired, is the same as it was in the original orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point B to point C, and state the following:

Student 1: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is negative.

Student 2: The satellite is moving faster at point C because the total mechanical energy of the satellite-Earth system is greater at point C than at point B.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

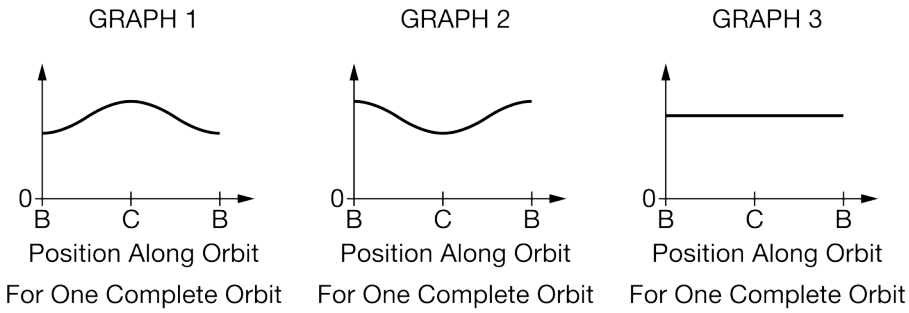
(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is

incorrect, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite as it completes one elliptical orbit, starting and ending at point B.



(h) Which of the above graphs best represents the magnitude of the acceleration of the satellite as it orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the magnitude of the angular momentum of the satellite with respect to the center of Earth as the satellite orbits? Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the satellite around the new planet will orbit at a greater distance.
- ☐ 1 point is earned for a correct explanation in terms of centripetal and gravitational force or acceleration.

Example Response:

Equating the centripetal acceleration v^2/r *with the gravitational acceleration* GM/r^2, *we see that* v^2 *is proportional to* M/r. *So a bigger mass M means that the distance must be greater as well.*

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for an answer that is consistent with part (a) AND an attempt at reasoning using functional dependence.
- ☐ 1 point is earned for using $v = \frac{2\pi R}{T}$ or for arguing that *R* must increase if *v* remains the same.
- ☐ 1 point is earned for correctly relating the increase in mass of the planet increases the distance from the planet to the satellite.

Example Response:

Yes, it matches my reasoning. Since v is $\frac{2\pi R}{T}$ *and v is the same, if R increases then T must also increase. Since M and T both increase, R must increase to keep C constant.*

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the force was applied perpendicular to the satellite's motion.
- ☐ 1 point is earned for a correct explanation relating the direction of force with change in speed.

Example Response:

The force was applied perpendicular to the satellite's motion because this force did not change the speed of the satellite.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

chapter4

☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is correct in saying that the potential energy of the satellite-Earth system is negative

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1 is incorrect in saying that the satellite is moving faster at B

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is correct in saying that the satellite is moving faster at C

OR

chapter4

The first statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes:

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2 is incorrect in saying that the total mechanical energy of the satellite-Earth system is greater at C than at B.

OR

The second statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating graph 1.
- ☐ 1 point is earned for a correct explanation that the gravitational field or acceleration increases as distance decreases.

Example Response:

Graph 1. The acceleration (or gravitational field) is greater closer to Earth, so it is greater at C than at B.

OR

Graph 1. The centripetal acceleration v^2/r is greater at C because v is greater, and r is smaller than at B.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1	2 ✓

The response accurately includes **both** of the following criteria.

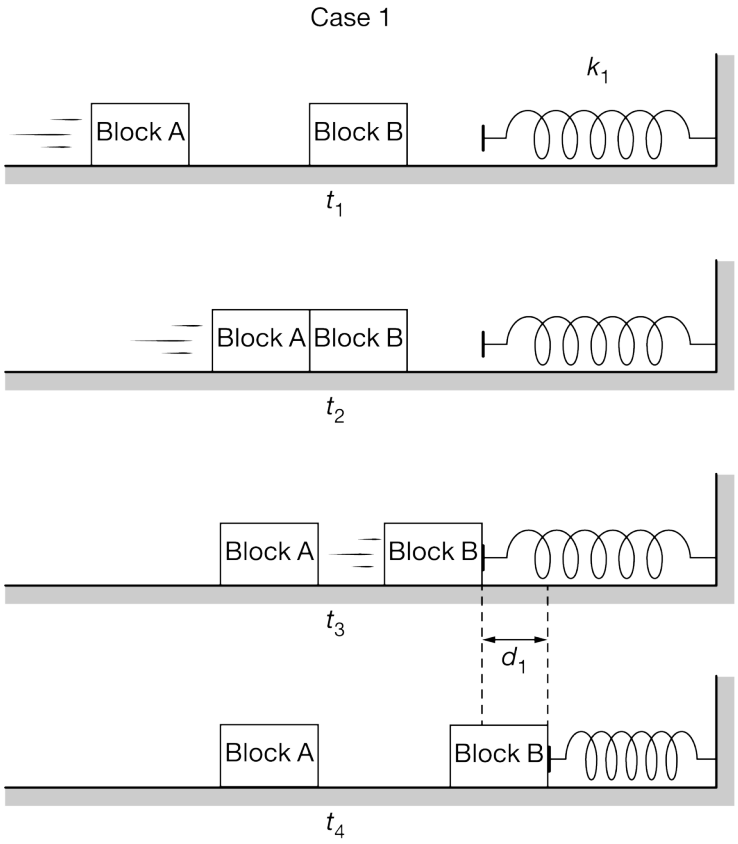
- ☐ 1 point is earned for indicating graph 3.
- ☐ 1 point is earned for a correct explanation why the angular momentum is constant throughout the motion.

Example Response:

Graph 3. The angular momentum is constant because the torque on the satellite is zero.

chapter4

18. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



The figure above shows Case 1 of an experiment, in which the following events occur.

- At time t_1, block A slides to the right along a horizontal surface toward identical block B, which is at rest.
- At time t_2, block A collides elastically with block B and block A comes to rest.
- At time t_3, the right side of block B reaches an ideal, horizontal spring with negligible mass and spring constant k_1. The spring is initially at its relaxed (uncompressed) length and fixed to a wall.
- At time t_4, the spring is compressed a maximum distance d_1 and block B is instantaneously at rest.

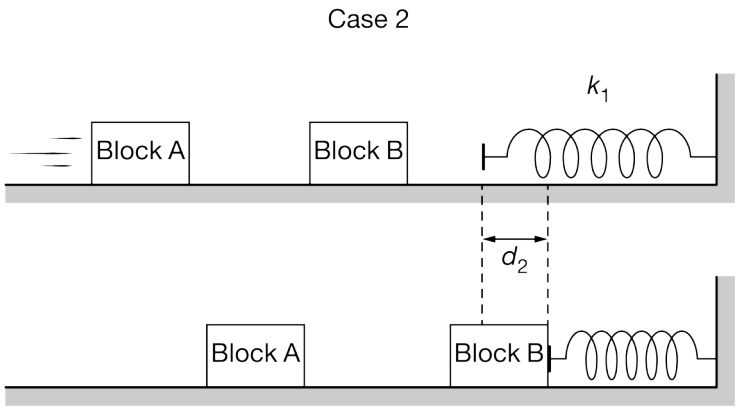
Assume there is negligible friction between each block and the surface. For parts (a)-(d), consider the system consisting of block A, block B, and the spring.

(a) Indicate whether the linear momentum of the two-block–spring system is either constant or not constant during the time interval from t_1 to immediately before t_3, and explain why.

(b) Indicate whether the mechanical energy of the two-block–spring system is either constant or not constant during the time interval from t_1 to immediately before t_3, and explain why.

(c) Indicate whether the linear momentum of the two-block–spring system is either constant or not constant during the time interval t_3 from to t_4, and explain why.

(d) Indicate whether the mechanical energy of the two-block–spring system is either constant or not constant during the time interval from t_3 to t_4, and explain why.



In Case 2 shown above, the setup is identical to Case 1 except that the speed of block A before colliding with block B is greater than in Case 1. Block A again comes to rest immediately after colliding with block B. The spring compresses a maximum distance d_2.

(e) How does the new maximum spring compression d_2 compare to d_1, the spring compression in Case 1? (Is d_2 greater than d_1, the same as d_1, or less than d_1 ?)

(f) Briefly explain your reasoning for part (e).

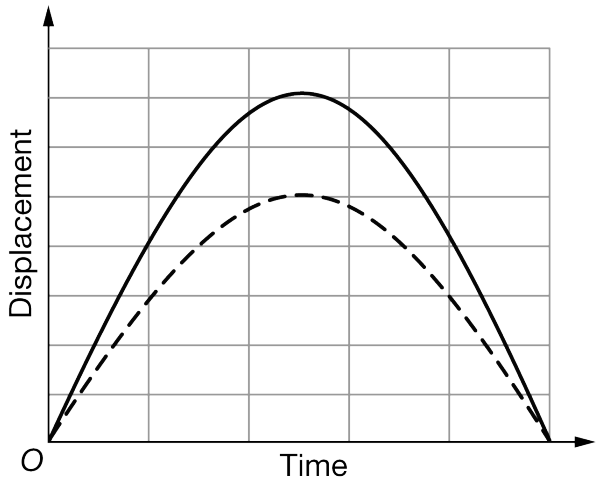
Students attempt to determine a general equation for the maximum compression distance d of a spring with spring constant k. Blocks A and B each have mass m and block A has initial speed v. The students come up with the following two equations.

Equation 1: $d = C_1\sqrt{\frac{m}{vk}}$ **Equation 2:** $d = C_2\sqrt{\frac{m}{k}}v$

In the two equations, C_1 and C_2 are positive constants with appropriate units.

(g) Without algebraic manipulation of equations or deriving the correct equation, explain which equation better supports your explanation for the compression distance in part (f).

chapter4



(h) The graph above shows the displacement of the blocks while block B is in contact with the spring. The solid line corresponds to either Case 1 or Case 2, while the dashed line corresponds to the other case. Does the dashed-line curve correspond to Case 1 (slower block A) or Case 2 (faster block A)? Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that linear momentum is constant, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the linear momentum is constant.

Example Response:

No net external forces are exerted on the two-block–spring system, so the momentum of the system is constant.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

chapter4

- ☐ 1 point is earned for indicating that mechanical energy is constant, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the mechanical energy is constant.

Example Response:

The mechanical energy of the system is constant because the collision is elastic.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that linear momentum is not constant, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the linear momentum is not constant.

Example Response:

The linear momentum is not constant, because a net external force is exerted on the two-block–spring system.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that mechanical energy is constant, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the mechanical energy is constant.

Example Response:

The mechanical energy is constant because no work is being done on the system by an external force.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for stating the compression distance is larger.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that block A transfers all of its kinetic energy to block B (i.e., block B has more kinetic energy when it reaches the spring than in Case 1).
- ☐ 1 point is earned for indicating that the greater kinetic energy means a larger compression because the elastic potential energy is equal to the initial kinetic energy.

Example Response:

Block A transfers all of its kinetic energy to block B in the elastic collision, so block B has more kinetic energy than in case 1. This means that there is more spring potential energy when the spring is compressed, so it is compressed a larger amount.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If part (g) is answered with a correct interpretation of Equation 1 consistent with an incorrect response in part (f), both points can be earned in part (g).

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that Equation 2 best models the compression with an attempt at an explanation that addresses functional dependence.
- ☐ 1 point is earned for a correct explanation that addresses the dependence of d on v.

chapter4

Example Response:

Since v is proportional to d in equation 2, as v increases d will increase, as predicted in part (f).

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

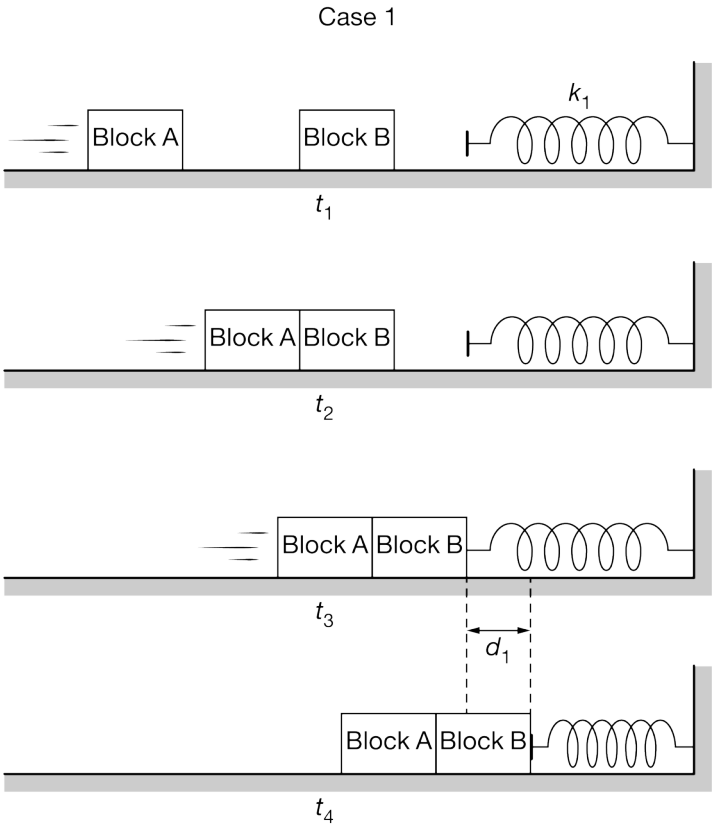
- ☐ 1 point is earned for indicating the dashed line corresponds to Case 1, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation regarding the slope of the displacement vs. time graph
  OR
  For a correct explanation addressing the amplitude (energy).

Example Response:

The dashed-line has an initially smaller slope representing a smaller initial speed of block B. Both graphs have zero slope where the block B momentarily stops at the spring's maximum displacement. The maximum displacement is smaller for the initially slower Block B. Therefore, the dashed-line graph corresponds to Case 1.
OR
The smaller maximum displacement for the dashed line is consistent with less energy being stored in the spring. This would indicate that block B had less initial kinetic energy when it collided with the spring corresponding to Case 1.

chapter4

19. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



The figure above shows Case 1 of an experiment, in which the following events occur.

- At time t_1, block A slides to the right along a horizontal surface toward identical block B, which is at rest.
- At time t_2, block A collides with and sticks to block B.
- At time t_3, block A and block B are still stuck together and the right side of block B reaches an ideal, horizontal spring with negligible mass and spring constant k_1. The spring is initially at its relaxed (uncompressed) length and fixed to a wall.
- At time t_4, the spring is compressed a maximum distance d_1 and the blocks are instantaneously at rest.

Assume there is negligible friction between each block and the surface.

A student observes the experiment and makes the following claim.

"*Since some mechanical energy is dissipated when the blocks stick together, the momentum of the two-block system decreases.*"

(a) Indicate whether the claim is ***correct*** or ***incorrect*** and explain why.

chapter4

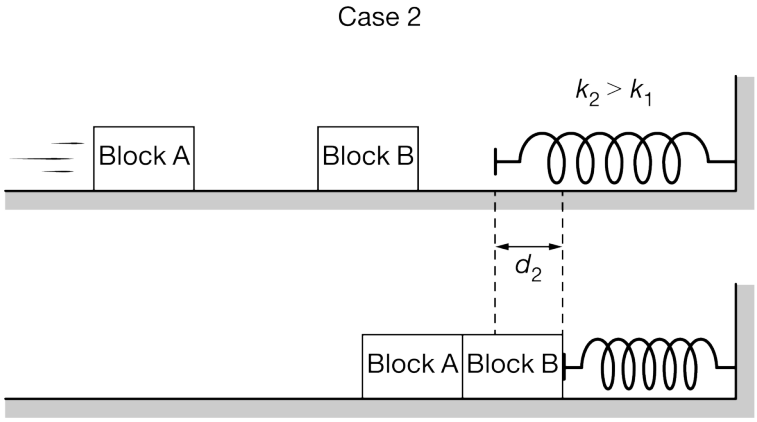
Another student makes the following claim about the collision between block A and block B.

"*When the blocks collide the total momentum of the two-block system doesn't change, so the kinetic energy of the system must also be constant.*"

(b) Indicate whether the claim is ***correct*** or ***incorrect*** and explain why.

(c) During the time interval from t_3 to t_4, is the magnitude of the acceleration of the two-block system increasing, decreasing, or constant? Briefly explain why.

(d) During the time interval from t_3 to t_4, does the spring do positive, negative, or zero work on the two-block system? Briefly explain your answer.



In Case 2 shown above, the setup is identical to Case 1 except that a stiffer spring is used. The spring in Case 2 has a spring constant k_2 that is greater than k_1. Block A slides with the same speed as in Case 1 and the spring compresses a maximum distance d_2.

(e) How does the new maximum spring compression distance d_2 in Case 2 compare to d_1 in Case 1? (Is d_2 greater than d_1, the same as d_1, or less than d_1 ?)

(f) Briefly explain your reasoning for part (e).

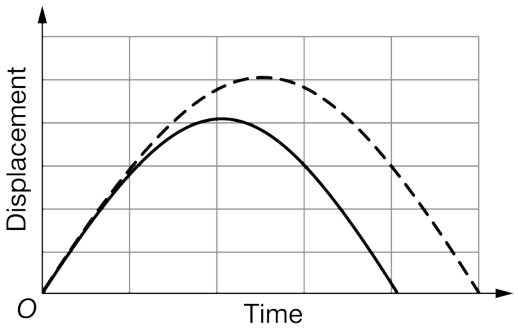
Students attempt to determine a general equation for the maximum compression distance d of a spring with spring constant k. Block A has mass m_A and initial speed v, and block B has mass m_B. The students come up with the following two equations.

Equation 1: $d = \frac{C_1 m_A v}{\sqrt{(k)(m_A+m_B)}}$ **Equation 2:** $d = C_2\left(k\sqrt{m_A + m_B}\right)v$

In the two equations, C_1 and C_2 are positive constants with appropriate units.

(g) Without algebraic manipulation of equations or deriving the correct equation, explain which equation better supports your explanation for the compression distance in part (f).

chapter4



(h) The graph above shows the displacement of the blocks while block B is in contact with the spring. The solid line corresponds to either Case 1 or Case 2, while the dashed line corresponds to the other case. Does the solid-line curve correspond to Case 1 (with a smaller spring constant) or Case 2 (with a larger spring constant)? Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the claim is incorrect, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the momentum does not change.

Example Response:

The claim is incorrect. The momentum does not change because no net external force is exerted on the two-block system.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the claim is incorrect, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation.

chapter4

Example Response:

The claim is incorrect. Energy is dissipated during the collision and therefore the kinetic energy decreases.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the magnitude of the acceleration is increasing, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation.

Example Response:

The force exerted by the spring increases with increasing compression.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the spring does negative work on the two-block system, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why negative work is done.

Example Response:

The spring does negative work on the two-block system because the spring exerts a net external force opposite the direction of motion of the system.

OR

The spring force does negative work on the two-block system because the force exerted from the spring decreases the kinetic energy of the system.

Part E

chapter4

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for stating the compression distance is smaller.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the two-block system has the same kinetic energy before reaching the spring as in Case 1 (or that the maximum elastic potential energy in the spring is the same as in Case 1).
- ☐ 1 point is earned for indicating that the elastic potential energy is directly related to both spring constant and compression distance, or that a greater spring constant means a smaller compression distance for the same potential energy.

Example Response:

The two-block system has the same kinetic energy when it reaches the spring. All the kinetic energy is converted to spring potential energy. Since the spring constant is now greater, the maximum compression distance must be less in order to have the same amount of potential energy at maximum compression.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If part (g) is answered with a correct interpretation of Equation 2 consistent with an incorrect response in part (f), both points can be earned in part (g).

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that Equation 1 best models the compression with an attempt at an

chapter4

explanation that addresses functional dependence.

- [ ] 1 point is earned for a correct explanation that addresses the dependence of d on k.

Example Response:

Since k is in the denominator of Equation 1, as k increases d will decrease, as predicted in part (f).

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

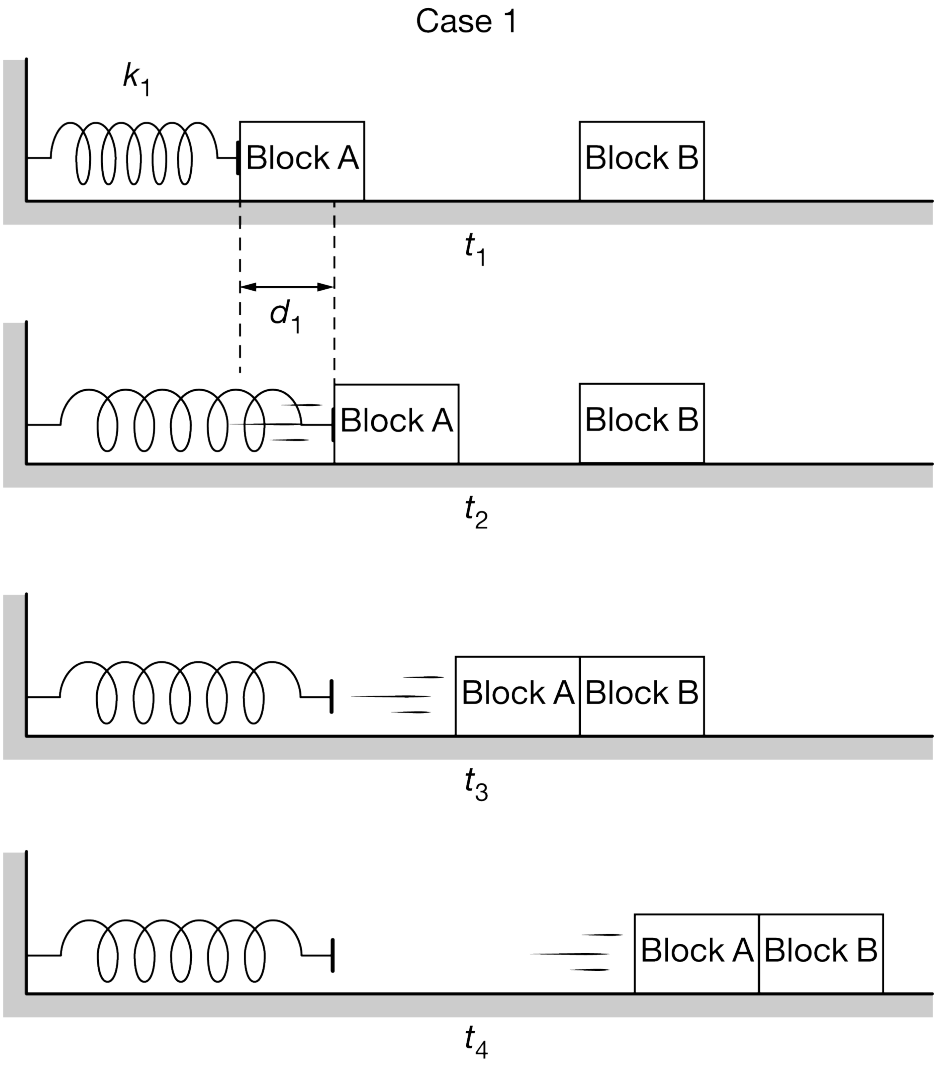
- [ ] 1 point is earned for indicating the solid line corresponds to Case 2, with an attempt at an explanation.
- [ ] 1 point is earned for a correct explanation addressing either the amplitude or period (or both).

Example Responses:

The smaller maximum displacement for the solid line is consistent with the larger spring constant in case 2.
OR
The period of an object oscillating on a spring is shorter when the spring constant is larger, so the solid line represents case 2.

chapter4

20. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



The figure above shows Case 1 of an experiment. Initially, block A is held in place while in contact with a horizontal spring with spring constant k_1 that is compressed a distance d_1 and fixed to a wall. The following events occur.

- At time t_1, block A is released from rest.
- At time t_2, the ideal spring reaches its uncompressed length, and block A leaves the spring.
- At time t_3, block A collides with and sticks to identical block B, which is initially at rest.
- At time t_4, block A and block B are sliding together along the surface.

Assume there is negligible friction between each block and the surface.

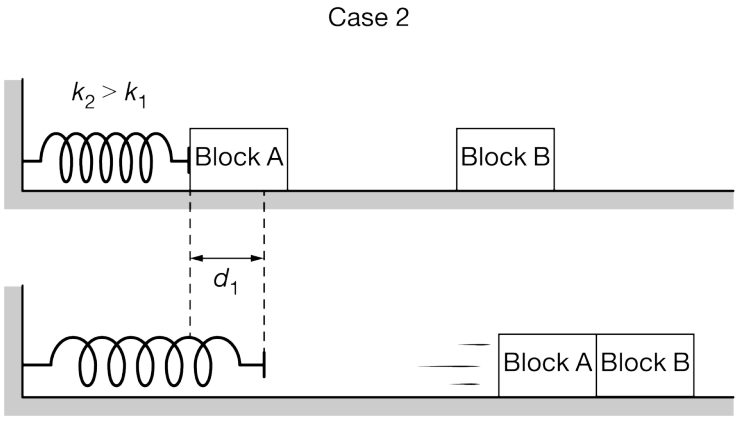
chapter4

(a) Indicate whether the speed of the center of mass of the two-block system is either constant or not constant during the time interval from t_1 to t_2, and explain why.

(b) Indicate whether the mechanical energy of the two-block system is either constant or not constant during the time interval from t_1 to t_2, and explain why.

(c) Indicate whether the speed of the center of mass of the two-block system is either constant or not constant during the time interval from t_2 to t_4, and explain why.

(d) Indicate whether the mechanical energy of the two-block system is either constant or not constant during the time interval from t_2 to t_4, and explain why.



In Case 2 shown above, the setup is identical to Case 1 except that a stiffer spring is used. The spring in Case 2 has a spring constant k_2 that is greater than k_1. The spring is compressed the same distance, d_1, before block A is released. As in Case 1, the blocks stick together when they collide.

(e) Is the speed of the blocks after the collision in Case 2 greater than, less than, or the same as the speed of the blocks after the collision in Case 1?

(f) Briefly explain your reasoning for part (e).

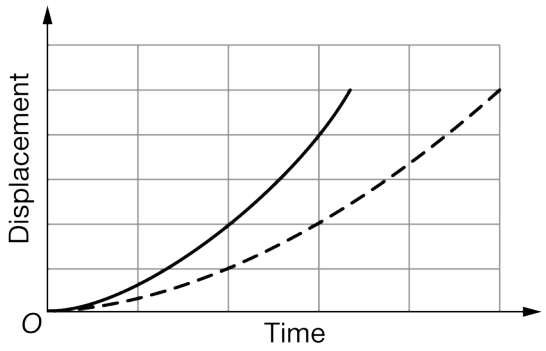
Students attempt to determine a general equation for the speed v of the blocks after the collision. A spring with spring constant k is initially compressed a distance d, block A has mass m_A, and block B has mass m_B. The students come up with the following two equations.

Equation 1: $v = C_1\left(\frac{m_B+m_A}{\sqrt{km_A}}\right)d$ **Equation 2:** $v = C_2\left(\frac{\sqrt{km_A}}{m_A+m_B}\right)d$

In the two equations, C_1 and C_2 are positive constants with appropriate units.

(g) Without algebraic manipulation of equations or deriving the correct equation, explain which equation better supports your explanation for the speed of the two blocks in part (f).

chapter4



The graph above shows the displacement of block A while it is in contact with the spring for Case 1 and Case 2. The solid line corresponds to either Case 1 or Case 2, while the dashed line corresponds to the other case.

(h) Does the solid-line curve best correspond to Case 1 (with a smaller spring constant) or to Case 2 (with a larger spring constant)? Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the speed of the center of mass is not constant, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the center of mass speed is not constant.

Example Response:

The speed of the center of mass is not constant, because a net external force is exerted on the two-block system.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

chapter4

- ☐ 1 point is earned for indicating that mechanical energy is not constant, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the mechanical energy is not constant.

Example Response:

The mechanical energy is not constant, because the spring is doing work on the two-block system

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the speed of the center of mass of the system is constant, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the speed of the center of mass of the system is constant.

Example Response:

The speed of the center of mass of the system is constant, because no net external force is exerted on the two-block system.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that mechanical energy is not constant, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the mechanical energy is not constant.

Example Response:

The mechanical energy is not constant, because the collision is inelastic and therefore the kinetic energy decreases.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for stating the speed in Case 2 is greater than that in Case 1.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that more energy is transferred to block A by the spring.
- ☐ 1 point is earned for indicating that a larger kinetic energy means larger initial speed for block A before the collision with block B and, therefore, a larger final speed for the two blocks after the collision.

Example Response:

Because the spring constant is greater in case 2, more energy is transferred to block A by the spring and block A has more kinetic energy than in case 1. The larger kinetic energy means a larger initial speed for block A before the collision with block B, so there is more momentum and a greater final speed.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If (g) is answered with a correct interpretation of Equation 1 consistent with an incorrect response in part (f), both points can be earned in part (g)

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that Equation 2 best models the compression with an attempt at an explanation that addresses functional dependence.
- ☐ 1 point is earned for a correct explanation that addresses the dependence of v on k.

chapter4

Example Response:

Since v and k are directly related (k is in the numerator of the expression), a larger k means that v will be larger, as predicted in part (f).

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

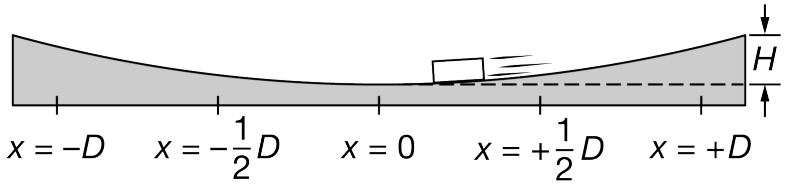
- ☐ 1 point is earned for indicating the solid line corresponds to Case 2, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation that the block will spend less time in contact with the spring with a stiffer spring constant (or that the speed/slope will be larger).

Example Response:

The solid line has a greater slope at the end, which means a greater speed, consistent with the larger spring constant in case 2.
OR
The solid line corresponds to a shorter period of a block attached to a spring, consistent with the larger spring constant in case 2.

chapter4

21. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

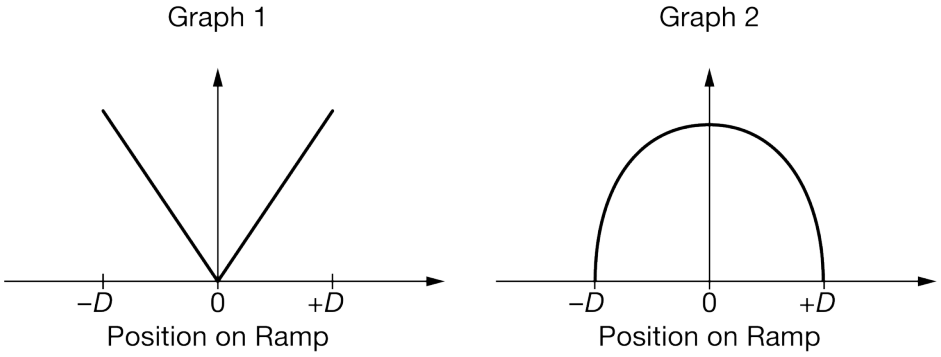


Track 1

Note: Figure not drawn to scale.

A block slides back and forth without any appreciable loss of energy on a track with a slight circular curvature, as shown above. The maximum height H of the track at each end is much smaller than D.

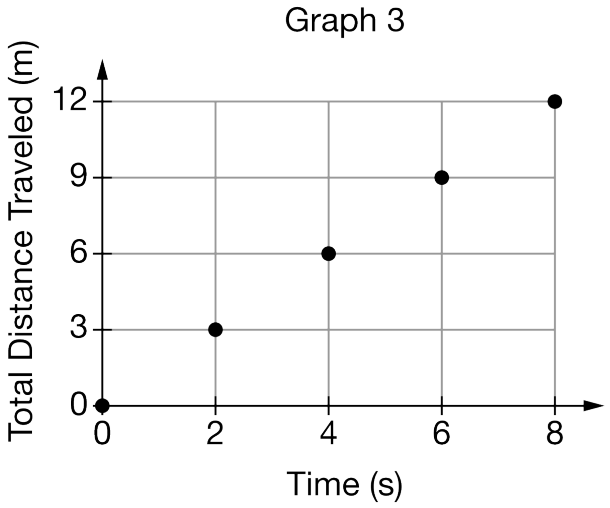
The center of mass of the block oscillates between $x = -D$ and $x = +D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. The block undergoes two full oscillations in 8.0 seconds.



(a) Which of the graphs above, Graph 1 or Graph 2, could represent the magnitude of the horizontal component of the block's acceleration as a function of its position on the ramp, and which graph could represent the magnitude of the block's momentum as a function of its position on the ramp? Justify both your choices with physics principles.

(b) For the system consisting of only the block, describe how the mechanical energy of the system changes, if at all, as the block moves from $x = 0$ to $x = -D$. Justify your answer.

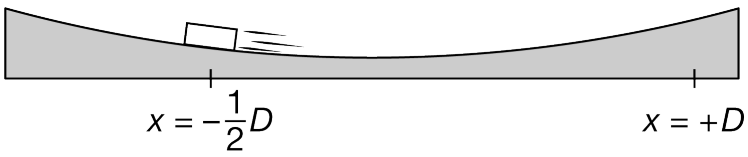
chapter4



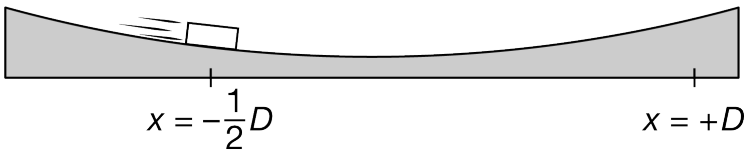
(c) Graph 3 above shows the total horizontal distance traveled by the block as a function of time as it completes two oscillations. Using the graph, determine the distance D between the midpoint of the ramp and the maximum position of the block on the ramp. Explain how you arrived at your answer.

(d) Suppose the entire setup was moved to the Moon, where the gravitational field strength is less than that on Earth. The center of mass of the block again oscillates between $x = -D$ and $x = +D$. Would the period of the block's oscillation on the Moon be greater than, less than, or equal to the period of oscillation on Earth? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{D^2}{2Hg}}$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?



Block on Track 2 at Time $t = T_1$



Block on Track 2 at Time $t = T_2$

Note: Figures not drawn to scale.

(f) Track 2 is identical to Track 1 except there is nonnegligible friction between the block and Track 2. The block

chapter4

is released from rest on Track 2 at position $x = +D$. At time $t = T_1$, the block moves to the left, passing through position $x = -\frac{1}{2}D$, as shown above. After the block reverses direction near $x = -D$, the block moves to the right, passing through position $x = -\frac{1}{2}D$ at time $t = T_2$. At which time, T_1 or T_2, if either, is the magnitude of the tangential acceleration of the block greater? Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that only Graph 1 represents acceleration or that only Graph 2 represents momentum.
- ☐ 1 point is earned for a valid justification correctly relating force to acceleration and including a reference to a specific feature of the graph.
- ☐ 1 point is earned for a valid justification relating velocity to momentum and including a reference to a specific feature of the graph.

Example Response:

Graph 1 represents the magnitude of the cart's horizontal acceleration, and Graph 2 represents the block's momentum. The horizontal component of the acceleration will look like the net force graph, which should be a restoring force, and so should be proportional to D. The momentum of the cart graph will look like the velocity graph which has a maximum at $x = 0$ *and is zero at D and* $-D$.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for any statement that between $x = 0$ and $x = -D$, the kinetic or mechanical energy decreases.
- ☐ 1 point is earned for a clear indication that the mechanical energy of the system consists of only the kinetic energy of the block.

Example Response:

chapter4

The mechanical energy of the block is only kinetic energy, so between $x = 0$ and $x = -D$, the mechanical energy (kinetic energy), decreases.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an attempt at relating the distance traveled to the amplitude.
- ☐ 1 point is earned for a correct answer: $D = 1.5\ \text{m}$.

Example Response:

D is equal to 1.5 meters. In two complete oscillations the block travels $8D$, in 8 seconds. From the graph we can see that the total distance traveled in 8 seconds is 12 meters, so the distance D is $(12\ \text{m})/8 = 1.5\ \text{m}$.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that the period of oscillation is longer on the moon.
- ☐ 1 point is earned for correct physical reasoning why the average speed or average acceleration is less on the moon.
- ☐ 1 point is earned for connecting smaller average speed or smaller average acceleration over the same distance to a longer time.

Example Response:

The period of oscillation on the moon would be greater. The average speed of the box on the moon would be smaller because the acceleration due to gravity is smaller, so for a smaller average speed, over the same distance, the time to oscillate would be larger.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence of *T* on *g*, with an attempt at justification.
- ☐ 1 point is earned for correctly indicating whether the dependence of *T* on *g* is consistent with the answer to part (d).
  Note: *T* should increase when *g* decreases. If part (d) was answered correctly, the equation is consistent.

Example Response:

Yes, this makes sense with my reasoning. Since the acceleration due to gravity, g, is in the denominator, when g is smaller, (like it is on the Moon), the period T is larger.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

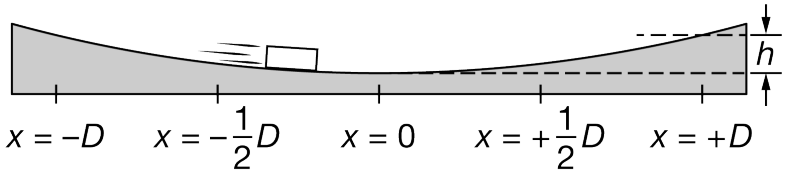
- ☐ 1 point is earned for a claim that the acceleration is greater at T_1.
- ☐ 1 point is earned for stating that both forces (component of gravity and friction) act on the block.
- ☐ 1 point is earned for correctly relating the sum of the forces on the block to the acceleration of the block at both T_1 OR T_2.

Example Response:

The tangential acceleration is greater at time T_1. At time T_1 both the component of gravity along the track and the friction force are in the same direction, while at T_2, the force of friction is in the opposite direction, so the force along the track is less at T_2, or greater at T_1. Since the force along the track is greater at T_1, the tangential acceleration of the block is greater at T_1.

chapter4

22. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

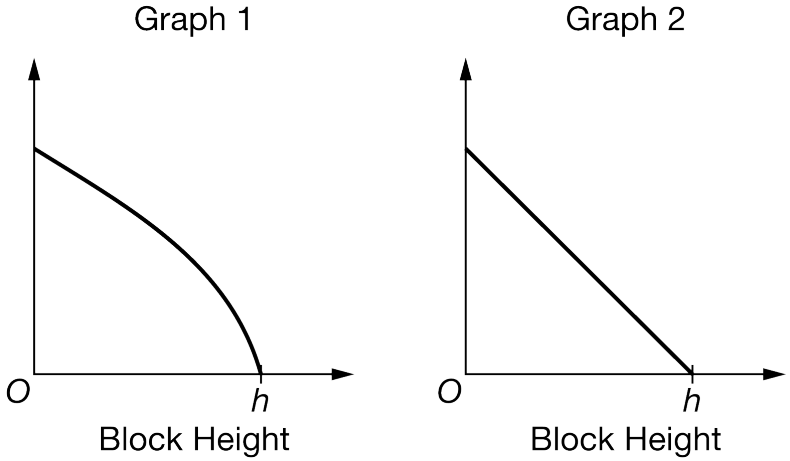


Track 1

Note: Figure not drawn to scale.

A block slides back and forth on a track with a slight circular curvature, where friction is negligible, as shown above. The maximum height h of the track at each end is much smaller than D.

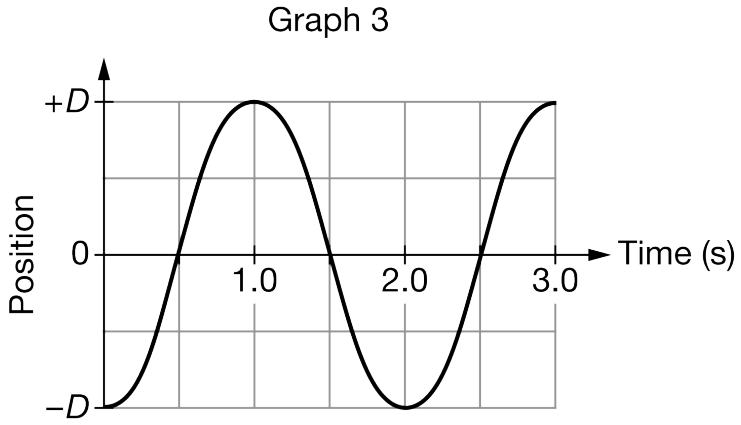
The center of mass of the block oscillates between $x = -D$ and $x = +D$, reaching a maximum height h every time the block is at $x = +D$ or $x = -D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. At time $t = 0$, the speed of the block is zero. The next time that the block's speed is zero is at time $t = 2.0$ s.



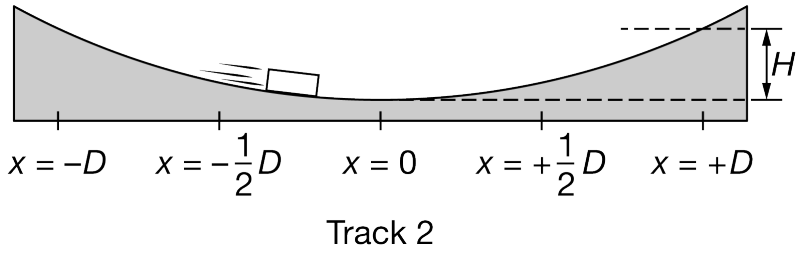
(a) Which of the graphs above, Graph 1 or Graph 2, could represent the kinetic energy of the block as a function of its height on the track, and which graph could represent the speed as a function of the block's height on the track? Justify both of your choices with physics principles.

(b) For the system consisting of the block and Earth, describe how the mechanical energy of the system changes, if at all, as the block moves from $x = +D$ to $x = 0$. Justify your answer.

chapter4



(c) Graph 3 above incorrectly represents the horizontal position of the block as a function of time. Identify a feature of the graph that does not correctly represent the motion of the block. Justify your answer.



Note: Figure not drawn to scale.

(d) Track 2 is similar to Track 1, but the maximum height H on Track 2 is greater than h and is still much smaller than D, as shown above. The center of mass of the block oscillates between $x = -D$ and $x = +D$, the same as on Track 1. How does the period of the block's oscillation on Track 2 compare to the period of the block's oscillation on Track 1? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{H^2}{2Dg}}$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?

(f) When the block reaches $x = 0$ at the bottom of the circular track, is the force exerted by the track on the block greater than, less than, or equal to the weight of the block? Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

chapter4

The response accurately includes **all** of the following criteria.

- 1 point is earned for a claim that only Graph 1 represents the speed as a function of height OR that only Graph 2 represents the kinetic energy as a function of height.
- 1 point is earned for a valid justification correctly relating speed to height and including a reference to a specific feature of the graph.
  Note: It acceptable to explain that speed is proportional to the square root of the kinetic energy.
- 1 point is earned for a valid justification that kinetic energy decreases linearly with increasing height and including a reference to a specific feature of the graph.

Example Response:

Graph 1 is the speed as a function of height and Graph 2 is the kinetic energy of the block as a function of height. Speed is equal to zero at h, but does not decrease linearly since the acceleration changes as the slope of the track changes. The kinetic energy however, changes linearly with height, since it is related to gravitational potential energy which changes linearly with height.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: The mechanical energy stays the same.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- 1 point is earned for a statement that the mechanical energy of the system consists of gravitational potential energy and kinetic energy of the block.
- 1 point is earned for a statement that no work is done on the system, so the mechanical energy is constant.

Example Response:

For this system, the total mechanical energy is both the kinetic energy of the block and the gravitational potential energy of the block-Earth system. Because there are no net external forces that do work on the system, the total mechanical energy of the system remains constant.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

chapter4

The response accurately includes **both** of the following criteria.

- 1 point is earned for identifying that a maxima or minima on the graph indicates zero velocity,
  OR
  for identifying that the period on the graph is incorrect (2.0 seconds),
  OR
  for identifying that the graph should not have a zero slope at 1.0 seconds.
- 1 point is earned for a claim that the period should be longer (4.0 seconds) with no incorrect statements,
  OR
  for a claim that the next maxima should occur at 2.0 seconds with no incorrect statements,
  OR
  for a claim that the velocity should be zero (or flat slope) at only 0 and 2.0 seconds with no incorrect statements,
  OR
  for a claim that the graph should be "stretched" to twice its width along the *x*-axis (or similar) with no incorrect statements.

Example Response:

The period in this graph is incorrect – the period should be 4 seconds, since it takes 2 seconds just to go from $-D$ *to D, which is only half a period. Also, the velocity (the slope of the graph) should be zero at zero seconds and 2 seconds, not at 1 second.*

The problem states that the speed of the block is zero at $t = 0$ s *and* $t = 2$ s*, so the graph should have a maximum at* $t = 2$ s *and not* $t = 1$ s.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- 1 point is earned for a claim that the period of oscillation is shorter on Track 2.
- 1 point is earned for correct reasoning why the average speed or average acceleration is greater on Track 2 (connecting increased total mechanical energy to a greater average speed or connecting greater average acceleration to a steeper track).
- 1 point is earned for connecting greater average speed or greater average acceleration over the same distance to a shorter time.

Example Response:

The period is shorter on track 2 because the average speed is greater on track 2 because the total mechanical energy is larger since H is larger. Because the average speed is larger, over the same distance the block takes less time to travel from D to $-D$ *and back.*

Part E

chapter4

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence of *T* on *H,* with an attempt at a justification.
- ☐ 1 point is earned for correctly indicating whether the dependence of *T* on *H* is consistent with the answer to part (d).

Example Response:

No this does not match my claim above. Because H is in the numerator, an increase in H should lead to an increase in T according to the equation. However, in part (d) I found that an increase in H leads to a shorter period (T).

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

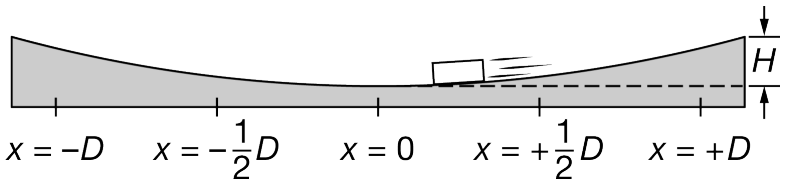
- ☐ 1 point is earned for a claim that the force exerted on the block by the track is greater than the weight.
- ☐ 1 point is earned for describing the net force in terms of the force of track on the block (or normal force) and weight only.
- ☐ 1 point is earned for describing that the net force is directed upward because the block is in circular motion.

Example Response:

The force exerted on the box by the track is greater than the weight at the lowest point, since the force from the track minus the weight of the block has to be equal to the centripetal force up.

chapter4

23. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

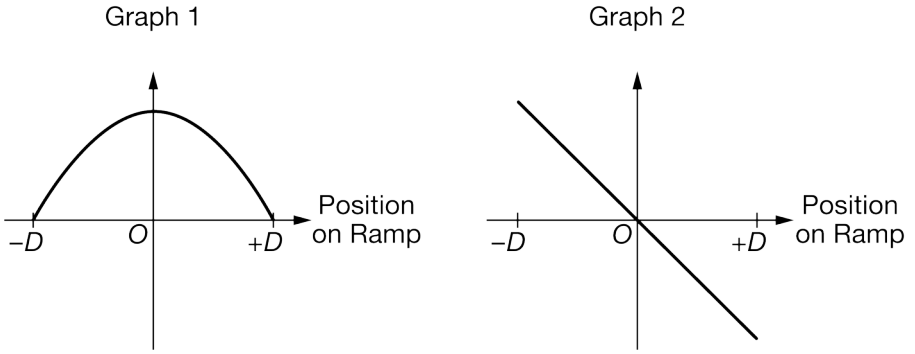


Track 1

Note: Figure not drawn to scale.

A block slides back and forth on a track with a slight circular curvature, where friction is negligible, as shown above. The maximum height H of the track at each end is much smaller than D.

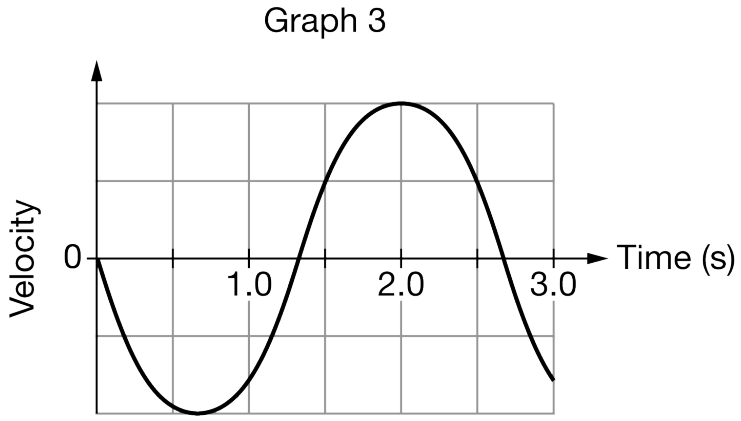
The center of mass of the block oscillates between $x = -D$ and $x = +D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. At time $t = 0$ the block is released from rest at position $x = +D$. The period of the block's oscillation is 2.0 seconds.



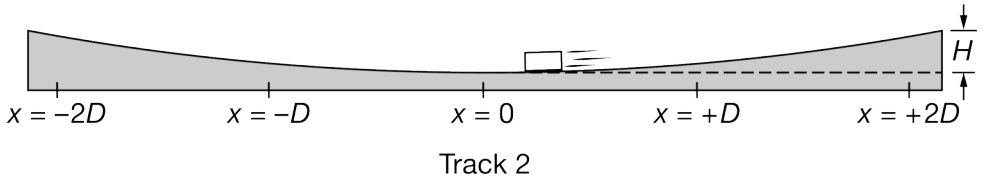
(a) Which of the graphs above, Graph 1 or Graph 2, could represent the block's kinetic energy as a function of its position on the circular track, and which graph could represent the horizontal component of the net force acting on the block as a function of its position on the track? Justify both of your choices with physics principles.

(b) Choose one of the graphs in part (a) above and explain how the graph shows evidence that a restoring force is acting on the block.

chapter4



(c) Graph 3 above incorrectly shows the horizontal velocity of the block as a function of time. Identify a feature of the graph that does not correctly represent the motion of the block. Justify your answer.

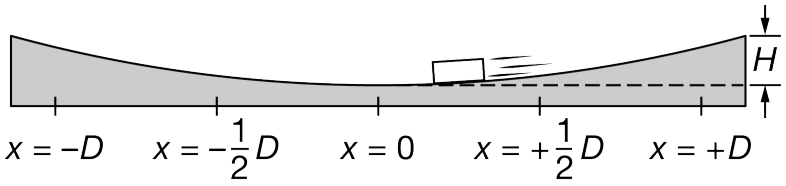


Note: Figure not drawn to scale.

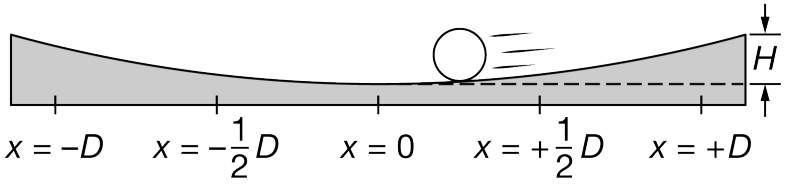
(d) Track 2, shown above, has a horizontal length that is greater than that of Track 1 and the same maximum height H. The block is released from rest at position $x = +2D$ Is the period of the block's oscillation on Track 2 greater than, less than, or equal to the period of oscillation on Track 1? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{h^2}{2dg}}$, where d is the maximum displacement of the block from equilibrium and h is the height of the track at $x = d$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?

chapter4



Track 1



Track 3

Note: Figures not drawn to scale.

(f) On Track 1, shown above again for reference, friction is negligible and the center of mass of the block oscillates between a horizontal position of $+D$ and $-D$ with a period of 2.0 seconds. If a sphere with a mass equal to that of the block were to roll without slipping between $+D$ and $-D$ along Track 3 shown above, which is identical to Track 1 but has nonnegligible friction, would the period of the motion of the sphere be greater than, less than, or equal to 2.0 seconds? Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that only Graph 1 represents kinetic energy or that only Graph 2 represents the horizontal component of the net force.
- ☐ 1 point is earned for a valid justification relating the slope of the track to the horizontal force or describing the positive/negative values of the force on each side of the graph.
- ☐ 1 point is earned for a valid justification referring to the speed of the block and its kinetic energy and including a reference to a specific feature of the graph.

Example Response:

Graph 1 represents the kinetic energy of the block and Graph 2 represents the horizontal component of the net force. The kinetic energy is a maximum at $x = 0$ and a minimum at $x = -D$ and $x = +D$. The horizontal component of the net force is zero at $x = 0$ and the magnitude is a maximum at $x = -D$ and $x = +D$. The horizontal component of the net

chapter4

force for small oscillations is linear and is proportional to displacement.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for identifying an appropriate feature of one of the graphs consistent with the answer in part (a).
- ☐ 1 point is earned for correctly explaining how a feature of the graph shows evidence of a restoring force.

Example Response:

Graph 2 shows evidence of a restoring force – a restoring force has a magnitude that is proportional to the displacement and opposite in direction to the displacement which is what graph 2 is showing.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for identifying that the period is incorrect,
  OR
  for identifying that the curve should intersect the horizontal axis at 1.0 seconds.
- ☐ 1 point is earned for a claim that the period is too long or that the graph should be compressed horizontally.

Example Response:

The period of the graph shown is incorrect. The problem states that the period is 2 seconds, but the graph shows a period that is longer than 2 seconds.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a claim that the period of oscillation is longer on Track 2.
- ☐ 1 point is earned for correct reasoning why the average acceleration or average speed is less on Track 2.
- ☐ 1 point is earned for connecting the smaller average acceleration or average speed over the longer distance to a larger time.

Example Response:

The period on track 2 is longer. The block will have the same maximum speed but spread out over a much longer track means that the time to travel the length of the track and back is larger.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence of T on d, with an attempt at a justification.
- ☐ 1 point is earned for correctly indicating whether the dependence of T on d is consistent with the answer to part (d).
  Note: T should increase with d. If part (d) was answered correctly, the equation is inconsistent.

Example Response:

No this does not match my reasoning. In the equation when d is in the denominator, as d increases, T decreases, which means that if the ramp gets longer, the period should decrease, which is not what will happen.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

chapter4

The response accurately includes **all** of the following criteria.

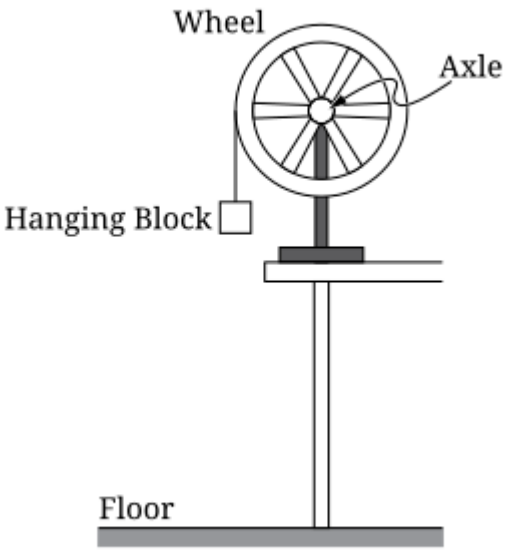
- 1 point is earned for a claim that the period of the sphere would be greater than the period of the block.
- 1 point is earned for a claim that maximum total kinetic energy of the sphere is equal to the maximum kinetic energy of the block,
  OR
  for a claim that the net force on the sphere will be less than that on the block because a frictional force is directed opposite the sphere's motion.
- 1 point is earned for recognizing that some portion of the sphere's kinetic energy is rotational,
  OR
  for recognizing that a smaller net force would result in a smaller average acceleration.

Example Response:

The period of the sphere would be greater. For both the block and the sphere the gravitational potential at the top are equal and so the kinetic energy as each passes through $x = 0$ *are equal. But for the sphere, some of that KE is turned into rotational kinetic energy, therefore it will be traveling, on average, slower than the block and so will take longer to travel the same distance.*

chapter4

24. This question is a long free-response question. Show your work for each part of the question.



(12 points, suggested time 25 minutes)

A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

(a) Design an experimental procedure that the students could use to compare the increase in the block's translational kinetic energy with the decrease in the gravitational potential energy of the block-Earth system as the block falls.

In the table, list the quantities that would be measured in your experiment. Define a symbol to represent each quantity and list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

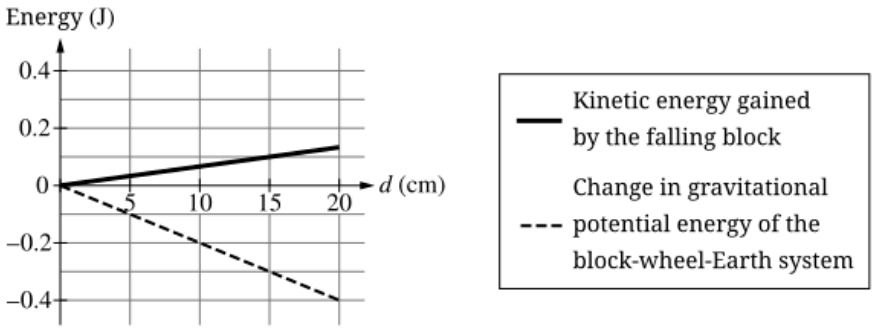
In the space to the right of the table, describe the overall procedure. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table.

If needed, you may include a simple diagram of the setup with your procedure.

chapter4

Quantity to Be Measured	Symbol for Quantity	Equipment for Measurement	Procedure (and diagram, if needed)

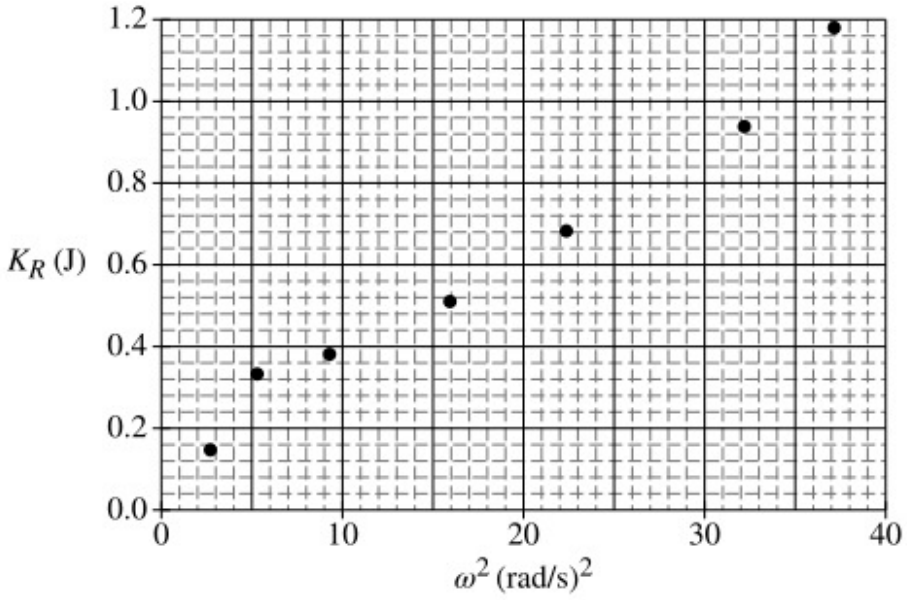
(b) Explain how the students could determine the kinetic energy of the block immediately before it reaches the floor using the quantities you indicated in the table in part (a).



(c) The graph above represents both the change in the gravitational potential energy of the block-wheel-Earth system and the translational kinetic energy gained by the block as functions of the block's falling distance d. On the graph, draw a line or curve to represent the rotational kinetic energy of the wheel as a function of the block's falling distance d.

(d) The students also measure the angular velocity ω of the wheel as the block falls and determine the rotational kinetic energy K_R of the wheel. The students then make a graph of K_R as a function of ω^2, as shown.

chapter4



i. On the above graph, draw a straight line that best represents the data.

ii. Using the line you drew for part (d)(i), calculate an experimental value for the rotational inertia of the wheel.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5 ✓

The response accurately includes **all** the following criteria.

- ☐ 1 point is earned for listing relevant equipment that matches the measured quantities.
- ☐ 1 point is earned for listing measurements of quantities sufficient to determine the kinetic energy of the block.
- ☐ 1 point is earned for listing measurements of quantities sufficient to determine the gravitational potential energy of the block-Earth system.
- ☐ 1 point is earned for a plausible procedure (i.e., can be done in a typical school physics lab).
- ☐ 1 point is earned for attempting to reduce uncertainty.

Example Response:

chapter4

Quantity to Be Measured	Symbol for Quantity	Equipment for Measurement
Mass of block	m_B	Mass balance
Distance that block falls (initial height above floor)	d	Meterstick
Time for block to fall	t_B	Stopwatch

1. Measure the mass of the block with the mass balance.

2. Hold the block in place with the string taut and measure the distance d with the meterstick.

3. Release the block and start the stopwatch.

4. Stop the stopwatch when the block hits the floor.

5. Record d and t_B.

6. Repeat steps $3-5$ *to get* 3 *separate trials at the same starting distance d.*

7. Repeat steps $2-6$ *for several different starting distances d.*

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that mass and velocity are needed.
  Scoring Note: This need not be the final velocity.
- ☐ 1 point is earned a valid explanation of how the final kinetic energy could be determined.
  Scoring Note: This needs to be the FINAL kinetic energy.

Example Response:

You can calculate the block's average speed by dividing $\frac{d}{t_B}$. *The block's final speed* v_{F} *is twice the average,* $\frac{2d}{t_B}$. *The kinetic energy K can then be calculated from* $K = \frac{1}{2}m_B(v_{\mathrm{F}})^2$.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

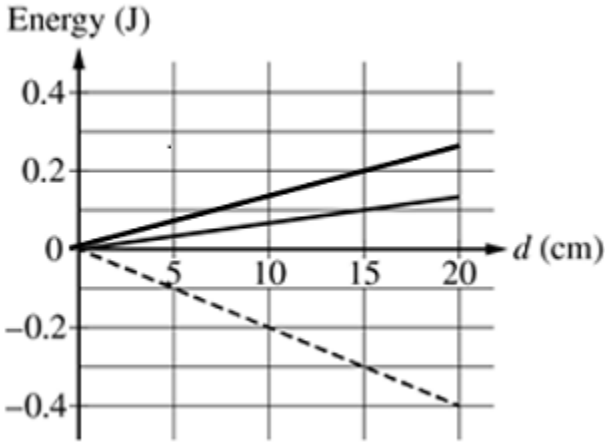
0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for drawing a straight line that passes through the origin and has a positive slope.
- ☐ 1 point is earned for drawing a line that yields a total energy sum of zero at all values of d.

Scoring Note: The correct line passes through the origin and $(15\ \text{cm}, 0.2\ \text{J})$.

Example Response:



Part D

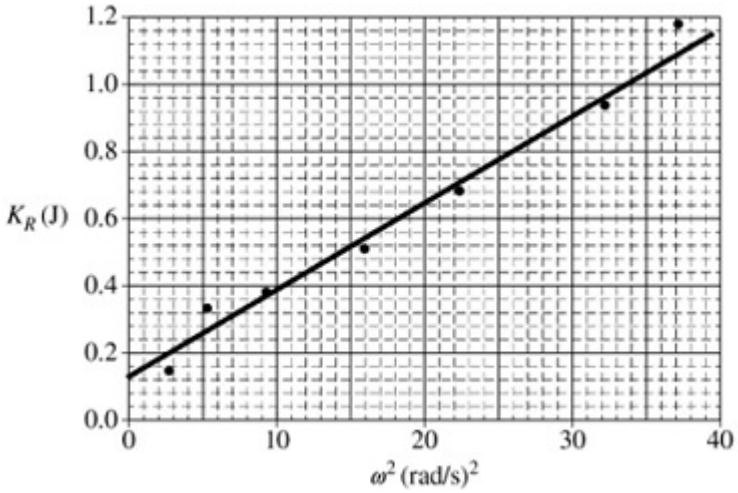
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** the following criteria.

- ☐ 1 point is earned for drawing a reasonable best-fit line.
  Example Response:

chapter4



- 1 point is earned for correctly calculating a value for the slope using points on the line drawn, or a statement that a calculator was used to do a linear regression.
- 1 point is earned for **both** of the following:
  · correctly relating the slope of the graph to the rotational inertia.
  · a value of the rotational inertia I consistent with the calculated slope with correct units.

Example Response:

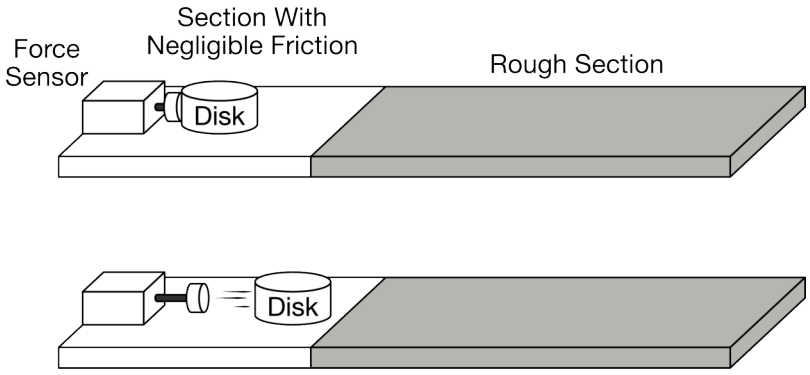
$\text{slope} = \frac{1.04-0.20}{35-2.5} = 0.0258\ \text{kg} \cdot \text{m}^2$

From $K = \frac{1}{2} I\omega^2$*, we have* $\text{slope} = \frac{1}{2} I$

The rotational inertia is $I = 2 \times \text{slope} = 0.0517\ \text{kg} \cdot \text{m}^2$

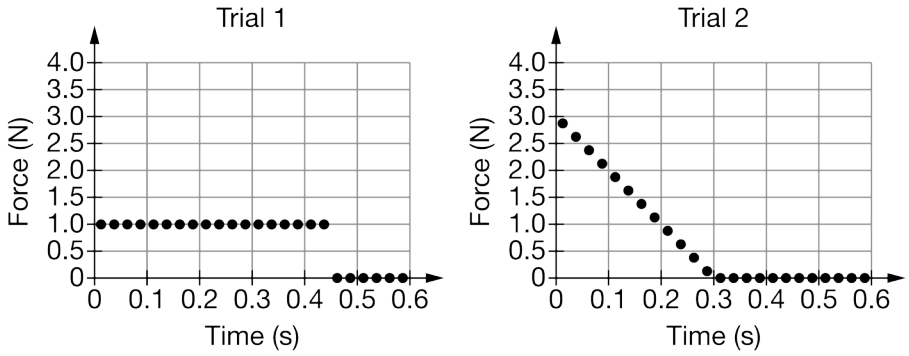
chapter4

25. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Students use a force sensor to push a disk of mass m across a horizontal table. The force sensor pushes the disk over a short time interval. The magnitude of the force and the duration of the time interval can be set by the students. After being pushed by the sensor, the disk first slides along a surface with negligible friction and then enters a rough section where friction is not negligible, as shown above.

The students perform two trials in which the magnitude of the force exerted by the force sensor and the time interval are different. They create the graphs of force as a function of time shown below. The disk enters the rough section in each trial after $t = 0.5$ s.



(a) In which of the two trials, if either, is the change in momentum of the disk between 0 seconds and 0.5 seconds greater? Explain your reasoning.

(b) In both trials, the disk comes to rest in the rough section before reaching the edge of the table. In which trial, if either, does the disk travel a greater distance in the rough section? Explain your reasoning.

(c) The students repeat Trial 1 with a disk that is made from the same material as the original disk and has the same mass as the original disk but that has a larger radius (and therefore more surface area is in contact with the surface). The same force is exerted on the new disk over the same time interval as in Trial 1. Would the distance traveled in the rough section by the new disk be greater than, less than, or the same as the distance traveled by the original disk in Trial 1? Explain your reasoning.

Students derive the following equation, which may or may not be correct, to describe the distance D traveled by

the original disk on the rough section: $D = \frac{(F_{\text{avg}}{}^2)t^2}{2\mu mg}$, where F_{avg} is the average force exerted by the sensor, t is the time that the disk is in contact with the sensor, μ (mu) is the coefficient of kinetic friction, and m is the mass of the disk.

(d) Without algebraic manipulation of equations or attempting to derive this equation, indicate whether the above equation is consistent with your answer in part (b). Explain your reasoning.

A student measures the distance x_0 that the disk travels while in contact with the force sensor in Trial 1. The disk then travels a distance D along the rough section. The student claims that if a force with twice the magnitude of the original force was exerted on the disk for a distance x_0, the disk would travel a distance $2D$ in the rough section.

(e) Is the above claim correct or incorrect? Justify your answer without citing or manipulating equations.

The student finds the following equation on the Internet describing the relationship between the magnitude F of the force exerted on the disk, the distance x the disk is pushed, and the distance D it slides in the rough section: $D = C\frac{Fx}{\mu mg}$, where C is a constant with appropriate units.

(f) Indicate whether or not the equation above is consistent with your statement in part (e) relating D and x. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the impulse or $\mathrm{F(t)}$ or area under the curve is the impulse or change in momentum
  OR
  For recognizing that the acceleration can be determined from the force.
- ☐ 1 point is earned for correctly indicating the area is equal to the $\mathrm{F_{ave}(t)}$ or $\mathrm{F_t/2}$ or $0.45\ \mathrm{Ns}$
  OR
  For addressing that the mass value is the same or need not be known.
- ☐ 1 point is earned for indicating that both disks have the same change in momentum
  OR
  For correct application of kinematics including the use of the time.

Example Response:

The area under each curve is the change in momentum, and since both graphs have the same area under the curve, $0.45\ Ns$*, both have the same change in momentum.*

chapter4

OR

The change in momentum is the same for both pucks because the same average force was applied to the same mass for the same time frame causing the same launch velocity.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks.
- ☐ 1 point is earned for a correct relationship between the speed OR momentum OR kinetic energy and the distance traveled in the rough section consistent with their answer to part (a).

Example Response:

Both disks experience the same frictional force in both trials, so the acceleration will be the same for both disks, and since both disks start with the same initial speed, they will slide the same distance.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating both disks would have the same launch speed with the same force over the same time interval.
  OR
  For indicating that both disks will have the same kinetic energy OR momentum when a force is applied over the same time.
- ☐ 1 point is earned for indicating that the surface area of the disk does not affect the frictional force.
- ☐ 1 point is earned for indicating that the new block will travel the same distance before stopping.

Example Response:

Both disks will have the same launch speed because they experience the same force and have the same mass. The surface area of the disks does not affect the frictional force, so the new block will travel the same distance before stopping.

chapter4

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an answer consistent with part (b), with an attempt to use the functional dependence of D on F_{avg} and/or t.
- ☐ 1 point is earned for correct reasoning (i.e., that $F_{\text{avg}}t$ is the same in both trials, so D is the same).

Example Response:

The equation is consistent with my reasoning in part (b). In the equation D is proportional to the square of the product of F_{avg} times t, which is the same for both trials, so D is the same.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: “Correct”.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a disk that is pushed with twice the force will have twice the original energy.
- ☐ 1 point is earned for a correct application of conservation of energy or work.
- ☐ 1 point is earned for indicating that since the work to stop the disk is also proportional to the distance to stop it, the disk will travel twice the distance before stopping.

Example Response:

A disk that is pushed from rest with twice the force for the same distance will have twice the work done one it, and so will have twice the original energy. The work done to stop the disk is equal to the energy that the disk has after being pushed. Work is also proportional to the distance to stop it; the new disk will travel twice the distance before stopping.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1	2 ✓

The response accurately includes **both** of the following criteria.

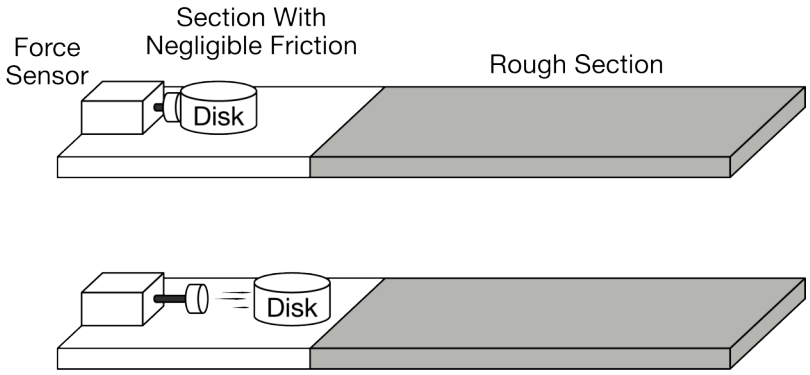
- ☐ 1 point is earned for indicating the equation is consistent with their statement in part (e) AND an attempt at a justification using functional dependence.
- ☐ 1 point is earned for correctly indicating D is proportional to F.

Example Response:

The given equation is consistent with my reasoning in part (e). Because D in the equation is directly proportional to F, pushing with twice the force will result in a distance D that is twice as large.

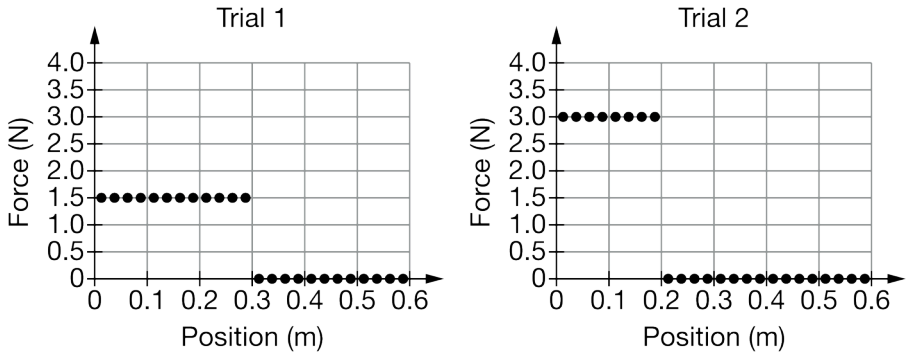
chapter4

26. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Students use a force sensor to push a disk of mass m across a horizontal table. The force sensor pushes the disk over a short distance. The magnitude of the force and the distance the disk is pushed can be set by the students. After being pushed by the sensor, the disk first slides along a surface with negligible friction and then enters a rough section where friction is not negligible, as shown above.

The students perform two trials in which the magnitude of the force exerted by the force sensor and the distance the disk is pushed are different. They create the graphs of force as a function of position shown below. The disk enters the rough surface in each trial after moving 0.6 meters.



(a) In which of the two trials, if either, is the change in energy of the disk the greatest during the interval shown in the graphs? Explain your reasoning.

(b) In both trials, the disk comes to rest in the rough section before reaching the edge of the table. In which trial, if either, does the disk travel a shorter distance in the rough section? Explain your reasoning.

(c) The students repeat Trial 2 with a disk that is made from the same material as the original disk but that is less massive. The same constant force is exerted on the less massive disk over the same distance as in Trial 2. Would the distance traveled in the rough section by the less massive disk be greater than, less than, or the same as the distance traveled in the rough section by the original disk in Trial 2? Explain your reasoning.

Students derive the following equation, which may or may not be correct, to describe the distance D traveled by

the original disk in Trial 2 on the rough section: $D = \frac{Fx}{\mu mg}$, where F is the force exerted by the sensor, x is the distance over which the sensor exerts a force on the disk, μ (mu) is the coefficient of kinetic friction, and m is the mass of the disk.

(d) Without algebraic manipulation of equations or attempting to derive this equation, indicate whether the equation is consistent with your answer in part (b). Explain your reasoning.

A student measures the time t that the disk is in contact with the force sensor in Trial 2. The disk then travels a distance D along the rough section. The student claims that if the same force is exerted on the disk for a time $2t$, the disk would travel a distance $2D$ in the rough section.

(e) Is the above claim correct or incorrect? Justify your answer without citing or manipulating equations.

The student finds the following equation on the Internet describing the relationship between the time t the disk is pushed and the distance D it slides in the rough section: $D = C\frac{F^2t^2}{2\mu m^2 g}$, where C is a constant with appropriate units.

(f) Indicate whether or not the above equation is consistent with your statement in part (e) relating D and t. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that trial 2 has the graph with the greatest area under the curve
  OR
  For recognizing that the acceleration can be determined from the force.
- ☐ 1 point is earned for indicating the area represents the work done
  OR
  For addressing that the mass value is the same OR need not be known.
- ☐ 1 point is earned for relating the greatest work done to the greatest change in energy
  OR
  For correct application of kinematics including the use of the distance traveled.

Example Response:

Trial 2. The work done on the disk is represented by the area under the force vs position graph, so the graph with the largest area under the curve has the greatest work done on it. Graph 2 has the greatest area, and so the disk in trial 2 had the greatest amount of work done on it.

Part B

chapter4

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks.
- ☐ 1 point is earned for indicating that the disk with the smaller launch speed will travel a shorter distance in the rough section.

Example Response:

Trial 1. The disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks. This means that the disk with the smaller speed after the push will travel a smaller distance.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a less massive disk would have a greater launch speed with the same force over the same distance.
- ☐ 1 point is earned for indicating that the distance traveled in the rough section, for a given speed, does not depend on the mass of the disk.
- ☐ 1 point is earned for indicating that the less massive disk will travel farther before stopping.

Example Response:

Farther than the original disk in trial 2. When pushed by the same force over the same distance, a less massive disk would have a larger launch speed. Since the mass of the disk doesn't affect the distance traveled through the rough section, the less massive disk will travel farther before stopping.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an answer consistent with part (b), with an attempt to use the functional dependence of D on F and/or x.
- ☐ 1 point is earned for correct reasoning (i.e., that Fx is greater in Trial 2, so D is greater).

Example Response:

This equation matches my reasoning in part (b). Since D is inversely proportional mass, if the mass increases, the distance D decreases, since m is in the numerator.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: “Incorrect”.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a disk that is pushed for twice the time will have twice the speed.
- ☐ 1 point is earned for a correct application of conservation of energy or work for the disk in the rough section.
- ☐ 1 point is earned for indicating that the stopping distance is proportional to kinetic energy, not speed.

Example Response:

A disk that is pushed from rest with the same force for twice the time will have twice the momentum. Since the mass of the disk doesn’t change, the second disk will have twice the speed. However, the stopping distance is proportional to the kinetic energy which is proportional to the velocity squared.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1	2 ✓

The response accurately includes **both** of the following criteria.

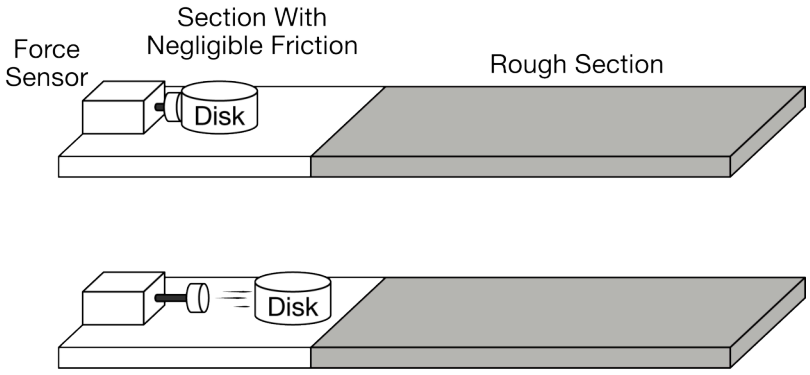
- ☐ 1 point is earned for indicating the equation is consistent (or inconsistent) with the statement in part (e) AND an attempt at a justification using functional dependence.
- ☐ 1 point is earned for indicating that the distance is proportional to *t* squared.

Example Response:

It is consistent. If t is doubled, the distance traveled should quadruple, because D is proportional to t squared.

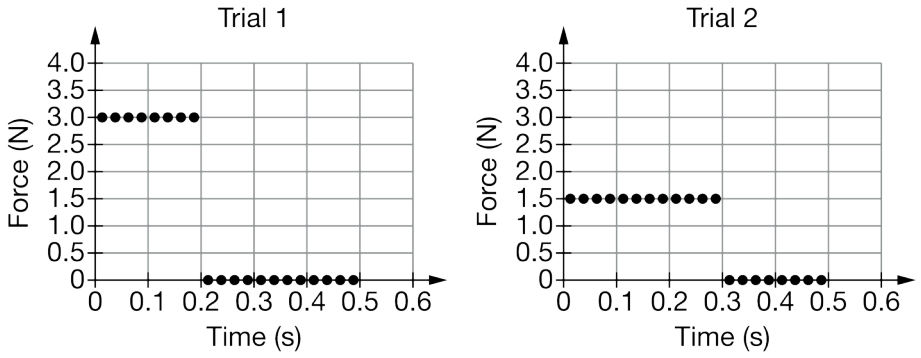
chapter4

27. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Students use a force sensor to push a disk of mass m across a horizontal table. The force sensor pushes the disk with a force of constant magnitude over a short time interval. The magnitude of the force and the duration of the time interval can be set by the students. After being pushed by the sensor, the disk first slides along a surface with negligible friction and then enters a rough section where friction is not negligible, as shown above.

The students perform two trials in which the magnitude of the force exerted by the force sensor and the time interval are different. They create the graphs of force as a function of time shown below. The disk enters the rough surface in each trial after $t = 0.5$ s.



(a) In which of the two trials, if either, is the speed of the disk the greatest at time 0.5 seconds? Explain your reasoning.

(b) In both trials, the disk comes to rest in the rough section before reaching the edge of the table. In which trial, if either, does the disk travel a greater distance in the rough section? Explain your reasoning.

(c) The students repeat Trial 1 with a disk that is made from the same material as the original disk but that is more massive. The same force is exerted on the more massive disk over the same time interval as in Trial 1. Would the distance traveled in the rough section by the more massive disk be greater than, less than, or the same as the distance traveled by the original disk in Trial 1? Explain your reasoning.

Students derive the following equation, which may or may not be correct, to describe the distance D traveled by

the original disk on the rough section: $D = \frac{F^2t^2}{2\mu mg}$, where F is the force exerted by the sensor, t is the time that the disk is in contact with the sensor, μ (mu) is the coefficient of kinetic friction, and m is the mass of the disk.

(d) Without algebraic manipulation of equations or attempting to derive this equation, indicate whether the above equation is consistent with your answer in part (b). Explain your reasoning.

A student measures the distance x_0 that the disk travels while in contact with the force sensor in Trial 1. The disk then travels a distance D along the rough section. The student claims that if the same force is exerted on the disk for a distance $2x_0$, the disk would travel a distance $2D$ in the rough section.

(e) Is the above claim correct or incorrect? Justify your answer without citing or manipulating equations.

The student finds the following equation on the Internet describing the relationship between the distance x the disk is pushed and the distance D it slides in the rough section: $D = C\frac{Fx^2}{\mu mg}$, where C is a constant with appropriate units.

(f) Indicate whether or not the equation above is consistent with your statement in part (e) relating D and x. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that trial 1 has the graph with the greatest area under the curve OR impulse OR $F(t)$ OR addressing the numerical values of 0.45 Ns vs 0.6 Ns
  OR
  For recognizing that the acceleration can be determined from the force.
- ☐ 1 point is earned for indicating the area represents the change in momentum
  OR
  For addressing that the mass value is the same OR need not be known
- ☐ 1 point is earned for relating the greatest change in momentum to the greatest change in speed
  OR
  For correct application of kinematics including the use of the time.

Example Response:

Trial 1 has the biggest area under the curve, and since the area under a force vs time graph is impulse or change in momentum, the trial with the biggest area will have the biggest change in momentum. Both disks start at rest so the disk with the biggest change in momentum has the biggest change in speed.

OR

chapter4

Because the mass of the puck in both trials is the same, the trial with the largest force has the largest acceleration. Since both pucks start at rest, the puck with the largest acceleration will have the largest speed when pushed for the same amount of time.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks.
- ☐ 1 point is earned for a correct relationship between the speed OR momentum OR kinetic energy and the distance traveled in the rough section consistent with their answer to part (a).

Example Response:

In both trials the disk experiences the same frictional force, so the acceleration is the same in both trials. The puck with the greatest launch speed travels the furthest before coming to rest because it will take a longer time to stop with the same acceleration.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a more massive disk would have a smaller launch speed OR momentum with the same force over the same time interval
  OR
  For indicating that a more massive disk will have less kinetic energy when a force is applied over the same time.
- ☐ 1 point is earned for indicating that the mass of the disk does not affect the distance traveled in the rough section.
- ☐ 1 point is earned for indicating that the more massive block will travel less far before stopping.

Example Response:

The more massive disk will have a smaller launch speed after being pushed with the same force for the same time. The pucks will have the same acceleration in the rough section of the track, regardless of the mass, so the more massive block

will not travel as far before stopping.

OR

The disk would have less kinetic energy over the same time frame, because the force would be applied over a smaller distance. Since the masses are equal, the F_f on the disks would be the same in the rough patch, for a disc with less kinetic energy entering the rough region, and experiencing the same F_f, the disk would not go as far.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an answer consistent with part (b), with an attempt to use the functional dependence of *D* on *F* and/or *t*.
- ☐ 1 point is earned for correct reasoning (i.e., that F_t is greater in trial 1, so *D* is greater).

Example Response:

The equation is consistent with my answer to part (b). D increases with the product F_t, so if this product increases, the distance D will also increase.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a disk that is pushed for twice the original distance will have twice the original energy.
- ☐ 1 point is earned for a correct application of conservation of energy or work.
- ☐ 1 point is earned for indicating that since the work to stop the disk is also proportional to the distance to stop it, the disk will travel twice the distance before stopping.

Example Response:

A disk that is pushed from rest for twice the original distance will have twice the work done on it and so will have twice the energy. The work done to stop the disk is equal to the energy that the disk has after being pushed. Work is also proportional to the distance to stop it; the disk will travel twice the distance before stopping.

chapter4

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

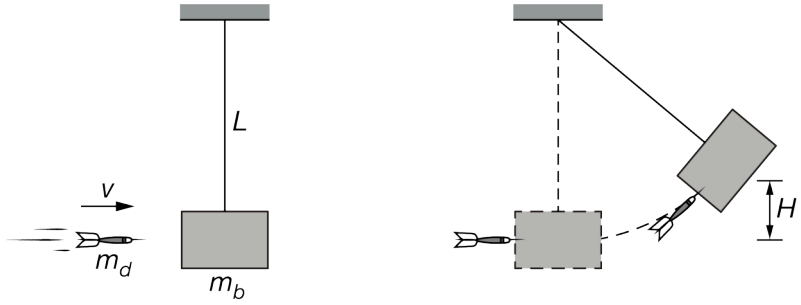
- ☐ 1 point is earned for indicating that the equation is not consistent (or consistent) with their statement in part (e) AND an attempt at justification using functional dependence.
- ☐ 1 point is earned for correctly indicating that D is proportional to x.

Example Response:

The given equation is not consistent with my statement in (e). In the equation D is directly proportional to x squared. But in part (e) I stated that if the distance x doubles, D will also double.

chapter4

28. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown at left above. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown at right above. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, if either, experiences a net horizontal force of larger magnitude during the collision. If the net horizontal force is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the experiment was repeated with a dart with the same horizontal speed but with mass smaller than m_d, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the motion of the dart-block system at the instant it first reaches a height of $H/2$, and they make the following statements:

Student 1: When the dart-block system is at a height of $H/2$, the kinetic energy of the system is half of its maximum value, and therefore the speed of the system is half of its maximum speed.

Student 2: When the dart-block system reaches a height of $H/2$, the centripetal force is equal to the tension in the string. Therefore, the acceleration of the system must point toward the fixed end of the string.

(c) Which underlined phrase or phrases, if either, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if either, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if either, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

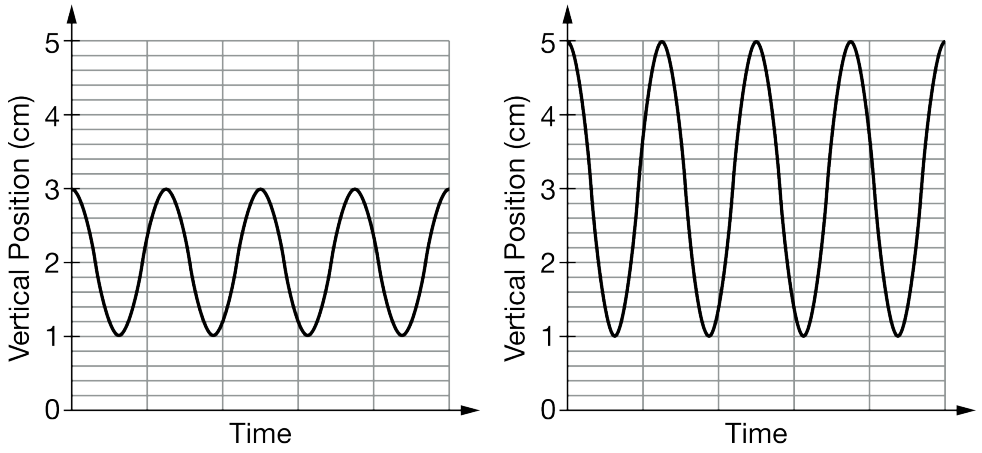
(f) Which underlined phrase or phrases, if either, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

chapter4

(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{v}{1+\frac{m_b}{m_d}}\right)^2$. Explain how this equation supports your claim in part (b).

(h) Could the masses of the dart and block both be changed so that the maximum height *H* reached by the dart-block system would remain the same as in the original setup? Justify your answer by referring to the equation in part (g).

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart mass or the string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a statement that both objects experience the same net force because of Newton's third law.

chapter4

Example Response:

Both the dart and the block experience the same force because they exert equal and opposite forces on each other because of Newton's third law.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the final height is smaller.
- ☐ 1 point is earned for a statement of conservation of momentum to indicate that the final speed of the dart-block system is smaller **AND** an indication of conservation of energy leading to a smaller height.

Example Response:

Because the momentum of the dart before the collision must be equal to the momentum of the dart-block system after the collision, a smaller dart mass (a smaller dart momentum) will lead to a smaller dart-block speed after the collision (a smaller dart-block momentum). Because the dart-block is traveling slower after the collision, it has less kinetic energy which means that the dart-block will not swing as high.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Responses:

Student 1 is correct in saying that the kinetic energy of the system is half of its maximum at $H/2$.

OR

The first statement

Part D

chapter4

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for an incorrect statement and no correct statements.

Example Responses:

Student 1 is incorrect in saying that the speed of the system is half of its maximum speed

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for indicating student 2 has no correct statements.

Example Response:

None

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for indicating both of Student 2's statements are incorrect.

chapter4

Example Responses:

Student 2 is incorrect in saying that centripetal force is equal to the tension in the string, AND incorrect in saying that the acceleration of the system must point toward the center of the circle.

OR

Both statements

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement including functional dependence of the variables.
- ☐ 1 point is earned for correctly asserting that as the mass of the dart decreases in the equation *H* also decreases.

Example Response:

This equation matches my reasoning in part (b)– if the mass of the dart decreases, the height of the swing decreases because a decrease in the mass of the dart means an increase in the ratio $(block\ mass)/(dart\ mass)$*, which is in the denominator of the expression.*

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for “yes”, with an attempt at explanation.
- ☐ 1 point is earned for a statement that both either increase or decrease.
- ☐ 1 point is earned for a statement indicating the factor in increase/decrease (or ratio of masses) is the same.

Example Response:

Yes, if the masses were both doubled, the maximum height would still be H – it works as long as the masses both increase by the same factor .

chapter4

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for recognizing that the amplitude of the graph for group 2 is larger.
- ☐ 1 point is earned for a statement discussing the dependence of amplitude on dart mass.
- ☐ 1 point is earned for indicating that the mass of the dart could be larger.

Example Response:

If group 2 used a dart with a larger mass, the dart would have had more momentum, and so the dart-block system would have gone higher (making the amplitude larger), but the string length is the same and so the period would still be the same.

chapter4

29. This question is a long free-response question. Show your work for each part of the question.

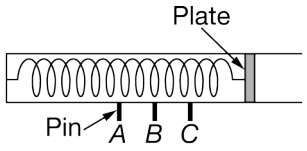


Figure 1. Uncompressed spring

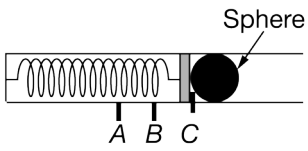


Figure 2. Compressed spring

(12 points, suggested time 25 minutes)

A projectile launcher consists of a spring with an attached plate, as shown in Figure 1. When the spring is compressed, the plate can be held in place by a pin at any of three positions A, B, or C. For example, Figure 2 shows a steel sphere placed against the plate, which is held in place by a pin at position C. The sphere is launched upon release of the pin.

A student hypothesizes that the spring constant of the spring inside the launcher has the same value for different compression distances.

(a) The student plans to test the hypothesis by launching the sphere using the launcher.

i. State a basic physics principle or law the student could use in designing an experiment to test the hypothesis.

ii. Using the principle or law stated in part (a)(i), determine an expression for the spring constant in terms of quantities that can be obtained from measurements made with equipment usually found in a school physics laboratory.

(b) Design an experimental procedure to test the hypothesis in which the student uses the launcher to launch the sphere. Assume equipment usually found in a school physics laboratory is available.

In the table below, list the quantities and associated symbols that would be measured in your experiment. Also list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

Quantity to be Measured	Symbol for Quantity	Equipment for Measurement

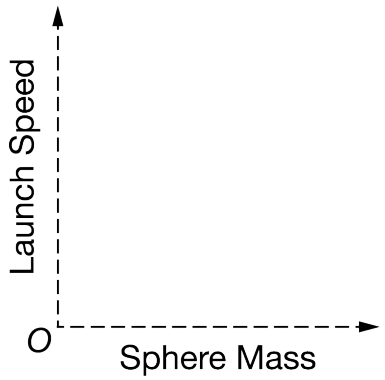
Describe the overall procedure to be used to test the hypothesis that the spring constant of the spring inside the launcher has the same value for different compression distances, referring to the table. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental

chapter4

uncertainty. As needed, use the symbols defined in the table and/or include a simple diagram of the setup.

(c) Describe how the experimental data could be analyzed to confirm or disconfirm the hypothesis that the spring constant of the spring inside the launcher has the same value for different compression distances.

(d) Another student uses the launcher to consecutively launch several spheres that have the same diameter but different masses, one after another. Each sphere is launched from position A. Consider each sphere's launch speed, which is the speed of the sphere at the instant it loses contact with the plate. On the axes below, sketch a graph of launch speed as a function of sphere mass.



Part A(i) & Part A(ii)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **three** of the following criteria.

- ☐ An equation that is consistent with a relevant principle or law as written in (a)(i).
- ☐ A valid equation that contains measurable quantities and includes spring constant.
- ☐ A correct and valid algebraic expression for spring constant. The expression must be solved for k .

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: The first, second, and fifth points of part (b) can be earned if the sphere is not launched.

0	1	2	3	4	5 ✓

chapter4

The response accurately includes **five** of the following criteria.

- [ ] Listing relevant/appropriate equipment that matches all measured quantities in the experimental procedure.
- [ ] Describing measurements of quantities sufficient to determine the spring constant.
- [ ] A plausible procedure (i.e., can be done in a typical school physics lab) that involves launching the sphere to determine the spring constant.
- [ ] Launching the sphere from at least 2 different initial positions.
- [ ] Attempting to reduce uncertainty (e.g., multiple trials at a pin setting).

Example Response:

The mass of the sphere is measured with a triple beam balance. The launcher is aimed horizontally on a level surface toward a motion sensor. The spring is compressed to pin position A and the spring compression distance is measured. The mass is launched. The motion sensor measures launch speed. The process is repeated three times at position A. The procedure is repeated with the spring compressed to pin positions B and C.

The launcher is aimed horizontally at a height above the ground so that the sphere will follow a projectile path and land on the floor. The spring is compressed to pin position A and the sphere is launched. Measure the mass of the sphere, the initial spring compression, and the vertical and horizontal displacements of the sphere from release to landing position. Repeat three times at pin position A. The procedure is repeated with the spring compressed to pin positions B and C.

Aim the launcher vertically. Compress the spring to pin position A. Launch the sphere vertically. Measure the mass of the sphere, the initial spring compression, and vertical displacement of the sphere above the release position. Repeat three times at pin position A. Repeat the procedure with the spring compressed to pin positions B and C.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: The second point of part (c) is not earned for saying “if the numbers are the same” or similar phrasing that does not address experimental uncertainty.

0	1	2 ✓

The response accurately includes **both** of the following criteria:

- [ ] Comparing the measurements of the spring constant (or a suitable proxy) at all three possible compression distances (A, B, C).
- [ ] Considering uncertainties in confirming the hypothesis (e.g., “If numbers match within experimental uncertainty,” or “If the numbers are about the same”).

Example Response:

For each pin position, take the average $v_{\text{L-avg}}$ of the launch speeds measured at that position. Calculate the spring constant k using the energy conservation relation $\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m_S {v_{\text{L-avg}}}^2$ or $k = \frac{m_S {v_{\text{L-avg}}}^2}{(\Delta x)^2}$. Then compare the k values for

chapter4

each spring position. If the values agree within experimental uncertainty, then the hypothesis is confirmed.

For each pin position, take the averages Δx_{avg} and Δy_{avg} of the horizontal and vertical sphere displacements. Calculate the time interval Δt using the kinematics equation $\Delta y_{\text{avg}} = \frac{1}{2} g (\Delta t)^2$, and then calculate the launch speed $v_L = \frac{\Delta x_{\text{avg}}}{\Delta t}$. Calculate the spring constant using the relation $k = \frac{m_S {v_L}^2}{d^2}$. Compare the k values for each spring position. If the values agree within experimental uncertainty, then the hypothesis is confirmed.

For each pin position, take the average Δy_{avg} of the maximum vertical sphere displacement. Use conservation of energy to calculate a value for the spring constant k from the equation

· $\frac{1}{2} k d^2 = mg \Delta y_{\text{avg}}$ (if measuring height from the release (pin) position)

· $\frac{1}{2} k d^2 = mg(\Delta y_{\text{avg}} + d)$ (if measuring height from the spring's uncompressed position)

Compare the k values for each spring position. If the values agree within experimental uncertainty, then the hypothesis is confirmed.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

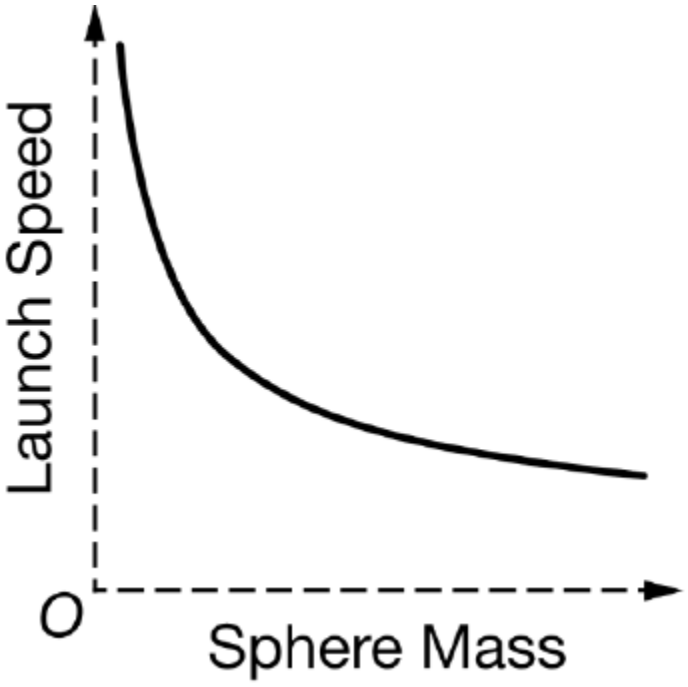
0	1	2 ✓

The response accurately includes **both** of the following criteria:

- ☐ A curve where launch speed always decreases with increasing sphere mass.
- ☐ A curve that is entirely concave up AND has the launch speed always decreasing with increasing sphere mass.

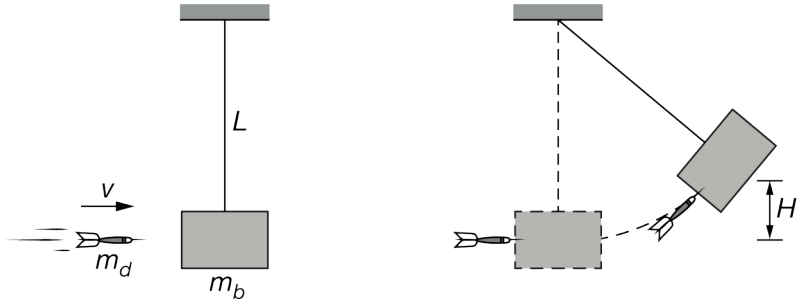
Example Response:

chapter4



chapter4

30. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown above at left. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown above at right. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, if either, experiences an acceleration of larger magnitude during the collision. If the acceleration is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the experiment were repeated with a block with mass greater than m_b, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the tension in the string at the moment immediately after the collision and state the following:

Student 1: The tension in the string immediately after the collision is greater than the weight of the dart-block system because the acceleration is upward at that instant.

Student 2: The tension in the string is equal to the weight of the block-dart system because there is no vertical acceleration immediately after the collision.

(c) Which underlined phrase or phrases, if any, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if any, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if any, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

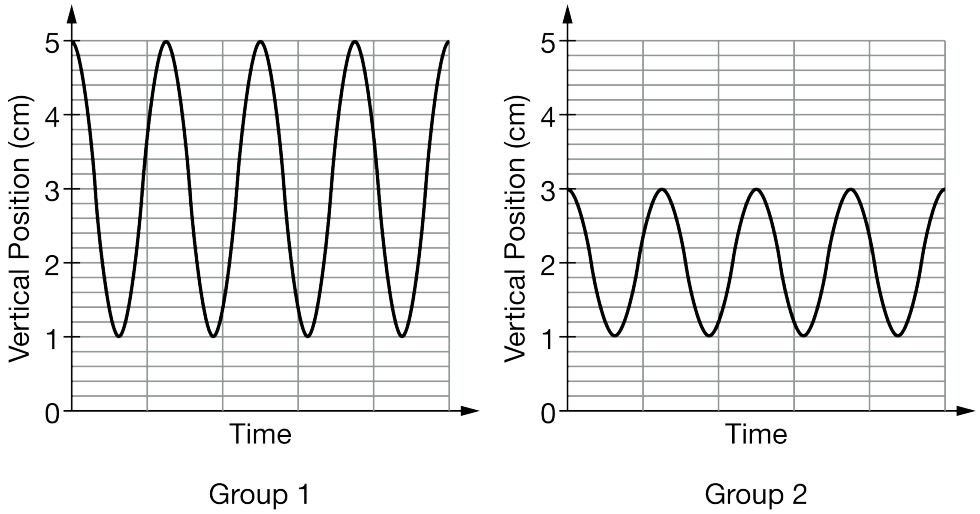
(f) Which underlined phrase or phrases, if any, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{m_d v}{m_d + m_b}\right)^2$. Explain how this equation supports your claim in part (b).

chapter4

(h) Consider the case in which the collision between the dart and the block is perfectly elastic (the dart bounces off the block to the left). In this case, is the speed of the block immediately after the collision greater than, less than, or the same as the speed of the block in the original case? Justify your answer.

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart speed or the string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a statement that the dart experiences the larger acceleration because it has the smaller mass with equal force exerted on it because of Newton's third law.

Example Response:

Both the dart and the block experience the same force because they exert equal and opposite forces on each other because of Newton's third law, however the mass of the dart is smaller and so experiences a larger acceleration.

chapter4

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the final height is smaller.
- ☐ 1 point is earned for a statement of conservation of momentum to indicate that the final speed of the dart-block system is smaller **AND** an indication of conservation of energy leading to a smaller height.

Example Response:

Because the momentum of the dart before the collision must be equal to the momentum of the dart-block system after the collision, a larger block mass will lead to a smaller dart-block speed after the collision because of conservation of momentum. Because the dart-block system is traveling slower after the collision, it has less kinetic energy which means that the dart-block system will not swing as high.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating both of Student 1's statements are correct.

Example Responses:

Student 1's entire statement is correct

OR

Both statements

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating student 1 has no incorrect statements.

Example Response:

None

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating student 2 has no correct statements.

Example Response:

None

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating both of Student 2's statements are incorrect.

Example Responses:

Student 2's entire statement is incorrect.

OR

chapter4

Both statements

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement including functional dependence of the variables.
- ☐ 1 point is earned for correctly asserting that as the mass of the block increases in the equation H decreases.

Example Response:

This equation matches my reasoning in part b – if the mass of the block increases, the height of the swing decreases because the mass of the block is in the denominator.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for "greater", with an attempt at an explanation.
- ☐ 1 point is earned for a statement indicating a larger change in momentum of the dart.
- ☐ 1 point is earned for a statement using conservation of momentum to show that the block momentum/ speed is greater.

Example Response:

When the dart bounces off the block, it has a larger change in momentum (going from v to $-v$ instead of v to a smaller v), which means that the block also has a larger change in momentum (since both objects receive the same impulse and experience the same change in momentum). Because the block has a greater change in momentum, it has a larger speed after the collision than in the original case.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

chapter4

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

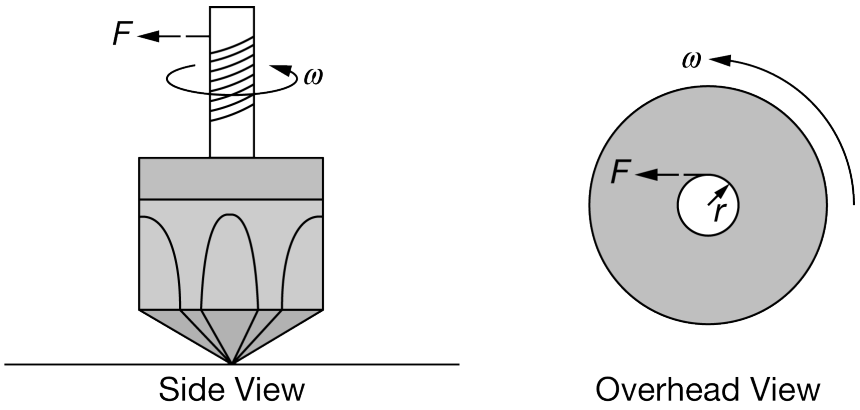
- ☐ 1 point is earned for recognizing that the amplitude of the graph for group 2 is smaller.
- ☐ 1 point is earned for a statement discussing the dependence of amplitude on dart momentum.
- ☐ 1 point is earned for indicating that the speed of the dart could be smaller.

Example Response:

If group 2 used a dart with a smaller initial speed, the dart would have had less momentum, and so the dart-block system would not have gone as high (making the amplitude smaller), but the string length is the same and so the period would still be the same.

chapter4

31. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A light string of total length L is wrapped completely around the axle of a spinning toy top. The radius of the axle is r and the rotational inertia of the top is I. The top is initially at rest and the axle is held vertically with respect to a horizontal table. At time $t = 0$ a force with constant magnitude F pulls the string outwards from the axle of the top, which causes the top to rotate. After a time T the string loses contact with the axle of the top and the top spins on the table with an angular speed ω (omega). Assume the top always remains upright and there is negligible friction between the top and the table.

(a) The original string is replaced with a new string of length $2L$. The string is again pulled with a constant force of magnitude F. When the string loses contact with the top, the angular speed of the top is ω_A. Is ω_A greater than, less than, or equal to ω? Justify your answer without citing or manipulating equations.

(b) The original string of length L is used to spin a second top that also has an axle with radius r, but a larger rotational inertia $I_B = 2I$. The string is pulled with the same constant force of magnitude F. When the string loses contact with the top, the angular speed of the second top is ω_B. Is ω_B greater than, less than, or equal to ω? Justify your answer without citing or manipulating equations.

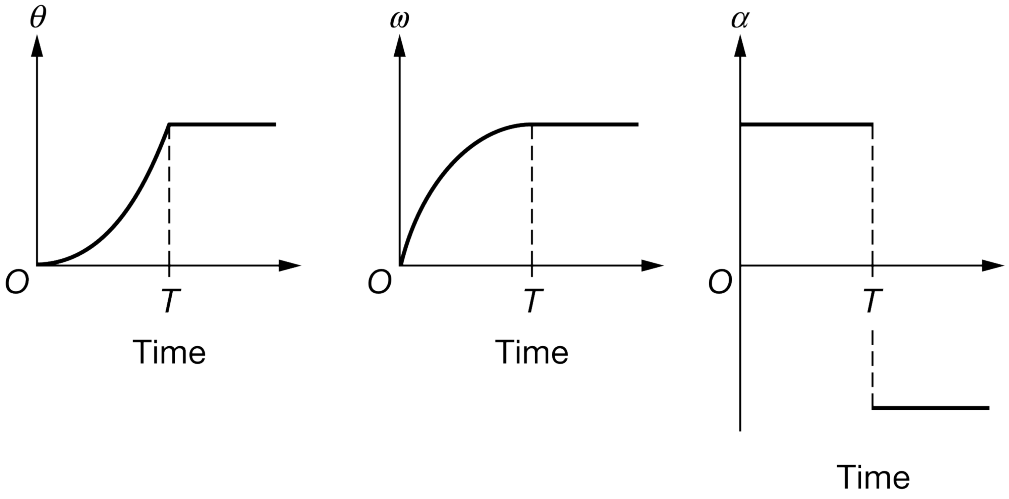
(c) A student derives the following equation to predict the final angular speed of the top: $\omega = \sqrt{\frac{2FL}{I}}$. Whether or not this equation is correct, does it match your reasoning in parts (a) and (b)?

Once the string has been pulled for a certain amount of time T, the string comes off and the top spins on the table.

(d) Describe any changes in the top's angular momentum about a vertical axis through the center of the top after the string loses contact with the top. Explain your answer.

A student makes sketches of the angular displacement θ, angular velocity ω, and angular acceleration α of the top as functions of time. The graphs below represent the time interval from when the string is first pulled until after the string loses contact with the top at time T. Consider the parts of the graph for the time before T and the time after T. Each graph is consistent with the motion of the top for one part and inconsistent for the other part.

chapter4



(e) For which part of the graph, either the time before T or the time after T, is the graph of the angular displacement θ inconsistent with the motion of the top as described directly above the graphs? Explain how to correct the part of the graph that is inconsistent.

(f) For which part of the graph, either the time before T or the time after T, is the graph of the angular velocity ω inconsistent with the motion of the top as described directly above the graphs? Explain how to correct the part of the graph that is inconsistent.

(g) For which part of the graph, either the time before T or the time after T, is the graph of the angular acceleration α inconsistent with the motion of the top as described directly above the graphs? Explain how to correct the part of the graph that is inconsistent.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that ω_A is greater than ω
- ☐ 1 point is earned for a correct justification of why ω_A is greater than ω

Example Response:

The angular speed will be larger because a longer string will result in the pulling force acting over a larger distance, which will do more work. The larger work will change the rotational kinetic energy more, which means a larger angular speed.

Part B

chapter4

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that ω_B is less than ω
- ☐ 1 point is earned for a correct justification of why ω_B is less than ω

Example Responses:

The same force and string length will result in the same work done to the top, and therefore the same change in rotational kinetic energy. But with a larger rotational inertia, the angular speed will be less.

OR

While the string is being pulled, both tops will experience the same torque and rotate through the same angular displacement. According to the rotational analogue of Newton's second law, the top with greater rotational inertia will experience less angular acceleration. Lower angular acceleration through the same displacement means a lower angular speed.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for an answer that includes any reference to functional relationships between ω and L **OR** ω and I **AND** correctly evaluating whether this matches the reasoning in part (a) or (b)
- ☐ 1 point is earned for correctly identifying that the relationship for length is correct – that ω is proportional to the square root of L, so a larger L results in a larger ω
- ☐ 1 point is earned for correctly identifying that the relationship for I is correct – that ω is proportional to $1/\sqrt{I}$, so if I increases, omega decreases

Example Response:

Yes. In part (a) I said the longer the string the larger the angular speed. Length is in the numerator so a larger L results in a larger angular speed. In part (b) I said the larger the rotational inertia the smaller the angular speed. Rotational inertia is in the denominator so a larger rotational inertia will result in a smaller angular speed.

Part D

chapter4

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that includes that the angular momentum will stay the same
- ☐ 1 point is earned for a correct explanation of why the angular momentum remains the same

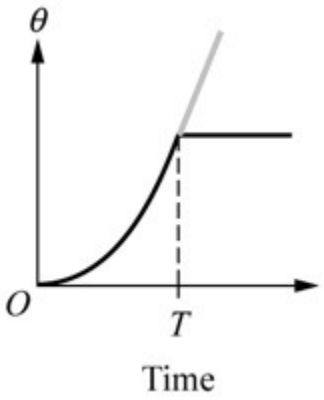
Example Response:

If there are no external forces, like friction, then there are no torques applied to the top so its angular momentum will not change.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Corrected graph, with correction shown in gray:



0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the time after *T* is inconsistent
- ☐ 1 point is earned for a statement that the slope should be positive, not zero, because the top is still spinning

Example Response:

The graph in the segment after the time T is inconsistent with the motion of the top. The top maintains a constant speed

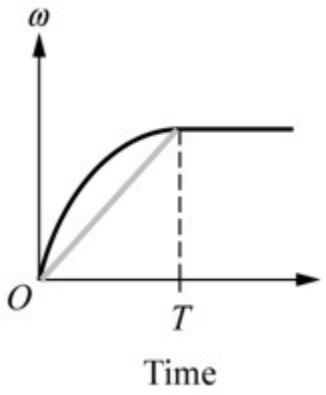
chapter4

after the time T, and the horizontal line indicates that it would be at rest. The line should have a positive slope.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Corrected graph, with correction shown in gray:



0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the time before *T* is inconsistent
- ☐ 1 point is earned for a statement that the slope should be constant because the angular acceleration, force, or torque is constant

Example Response:

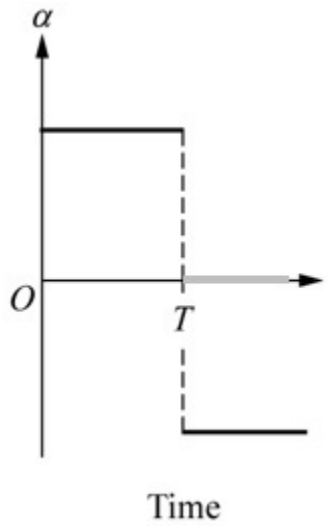
The graph in the segment before the time T is inconsistent with the motion of the top because the rate at which the speed increases is constant since the force is constant. The graph should show a straight line with a positive slope.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Corrected graph, with correction shown in gray:

chapter4



0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the time after *T* is inconsistent
- ☐ 1 point is earned for a statement that the angular acceleration should be zero because there is no net torque on the system

Example Response:

The graph in the segment after the time T is inconsistent with the motion of the top because the velocity is constant as there is no torque exerted on the top and therefore the acceleration should be zero. The graph should show a horizontal line directly on the time axis.