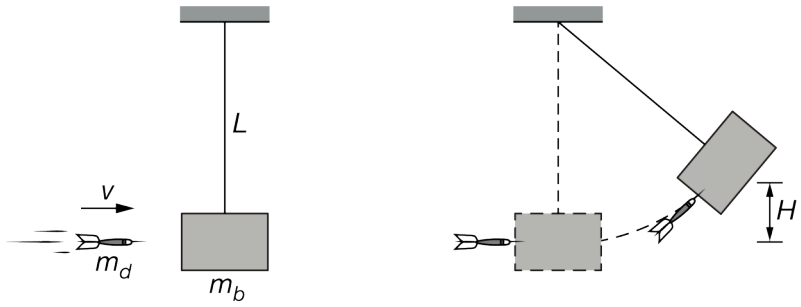
chapter5

1. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown at left above. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown at right above. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, the dart or the block, if either, experiences an impulse of larger magnitude during the collision. If the impulse is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the speed of the dart as it embeds itself into the block is greater than *v*, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the motion of the dart-block system at the instant it reaches height *H*. The students make the following statements:

Student 1: The speed of the system at height *H* is zero. Since it is moving in circular motion and centripetal acceleration is given by $a_c = \frac{v^2}{r}$, the acceleration must also be zero.

Student 2: The acceleration of the system at height *H* can't be zero, or the system would not swing back down. The acceleration must be equal to *g* at height *H*.

(c) Which underlined phrase or phrases, if either, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if either, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if either, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases, if either, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

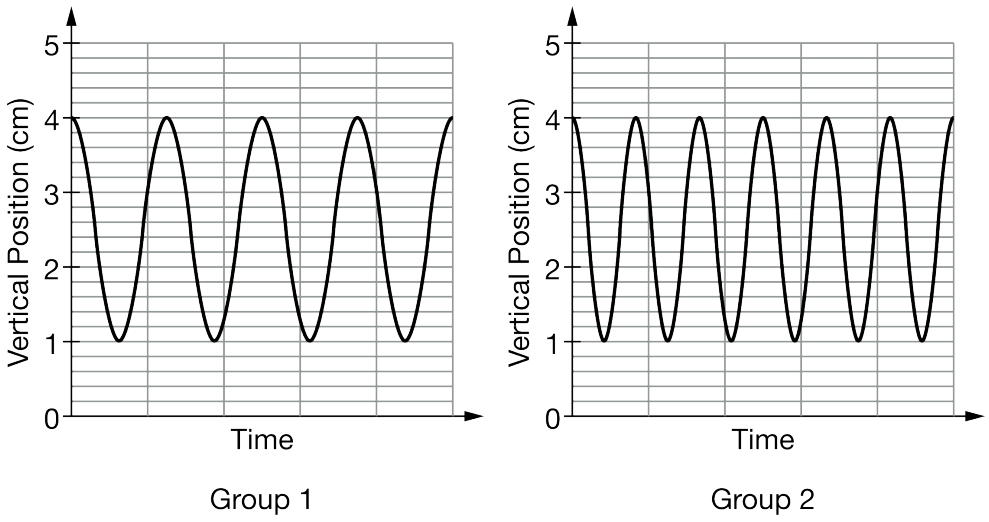
(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{m_d v}{m_d + m_b}\right)^2$. Explain how this

equation supports your claim in part (b).

The students perform several experimental trials. In each trial, a different speed v is used, and the resulting maximum height reached by the center of mass of the dart-block system is measured.

(h) Which quantities could the students graph to produce a straight line whose slope can be used to find the acceleration due to gravity, g ?

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart mass, initial dart speed, block mass, or string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct explanation, in terms of force & time (using Newton's third law)
  OR
  a correct explanation in terms of change in momentum (using conservation of momentum).

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Example Response:

Both the dart and the block experience the same impulse because they exert equal and opposite forces on each other because of Newton's third law, and the forces are exerted for equal times.

OR

By conservation of momentum, the decrease in the momentum of the dart must be equal and opposite to the increase in the momentum of the block, and therefore the impulse is the same in magnitude for both objects.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the final height is greater.
- ☐ 1 point is earned for a statement of conservation of momentum to indicate that the final speed of the dart-block system is greater **AND** an indication of conservation of energy leading to a greater height.

Example Response:

Because the momentum of the dart before the collision must be equal to the momentum of the dart-block system after the collision, a higher dart speed (a higher dart momentum) will lead to a higher dart-block speed after the collision (a higher dart-block momentum). Because the dart-block is traveling faster after the collision, it has more kinetic energy which means that the dart-block will swing higher.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Responses:

Student 1 is correct in saying that the speed at height H is zero

OR

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The first statement.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Responses:

Student 1 is incorrect in saying that the acceleration is zero at the height H

OR

The second statement.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Responses:

Student 2 is correct in saying the acceleration of the system at height H can't be zero or the system would not swing back down

OR

The first statement.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Responses:

Student 2 is incorrect in saying that the acceleration is g at height H

OR

The second statement.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement including functional dependence of the variables.
- ☐ 1 point is earned for correctly asserting that as the speed increases in the equation *H* also increases .

Example Response:

This equation matches my reasoning in part (b) – if the speed of the dart increases, the height of the swing increases because v is in the numerator.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for plotting a variant of *v* on one axis.

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- [ ] 1 point is earned for plotting a variant of H on the other axis.
- [ ] 1 point is earned for a correct power relationship between the two variables, i.e., H vs. v squared.

Example Response:

Students could graph H as a function of v squared.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- [ ] 1 point is earned for recognizing that the period in the second graph is shorter.
- [ ] 1 point is earned for a statement about the dependence of the period on the length of the pendulum.
- [ ] 1 point is earned for indicating that the length of the pendulum must be shorter for a shorter period.

Example Response:

Since the period is shorter for group 2, and the period is related to the length, if the period became shorter with no other changes, the length of the string must have been shortened.

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2. A new kind of toy ball is advertised to "bounce perfectly elastically" off hard surfaces. A student suspects, however, that no collision can be perfectly elastic. The student hypothesizes that the collisions are very close to being perfectly elastic for low-speed collisions but that they deviate more and more from being perfectly elastic as the collision speed increases.

a. Design an experiment to test the student's hypothesis about collisions of the ball with a hard surface. The student has equipment that would usually be found in a school physics laboratory.

i. What quantities would be measured?

ii. What equipment would be used for the measurements, and how would that equipment be used?

iii. Describe the procedure to be used to test the student's hypothesis. Give enough detail so that another student could replicate the experiment.

b. Describe how you would represent the data in a graph or table. Explain how that representation would be used to determine whether the data are consistent with the student's hypothesis.

c. A student carries out the experiment and analysis described in parts (a) and (b). The student immediately concludes that something went wrong in the experiment because the graph or table shows behavior that is elastic for low-speed collisions but appears to violate a basic physics principle for high-speed collisions.

i. Give an example of a graph or table that indicates nearly elastic behavior for low-speed collisions but appears to violate a basic physics principle for high-speed collisions.

ii. State one physics principle that appears to be violated in the graph or table given in part (c)i. Several physics principles might appear to be violated, but you only need to identify one.

Briefly explain what aspect of the graph or table indicates that the physics principle is violated, and why.

Part A

4 points total

Parts i, ii, and iii are scored as a unit

1 point is earned: For an overall plan in which quantities are measured that could be used to compare mechanical energy before and after a collision with a hard surface

1 point is earned: For a conceptually plausible plan to measure pre- and post-collision positions and/or speeds that could be used to compare pre- and post-collision mechanical energies, without extraneous equipment and/or measurements

1 point is earned: For having lab equipment and measurement procedures well specified

1 point is earned: For a procedure that includes trials of different pre-collision speeds, ranging from low speed to high speed (as is needed to test the student's hypothesis)

Example 1:

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i. The drop height of the ball and the bounce height.

ii. A meterstick to measure the heights and a video camera to record the ball's motion.

iii. Place the meterstick upright against the wall. Drop the ball from 10 different drop heights, using the video camera to record the bounce heights.

Example 2:

i. The speed of the ball immediately before and immediately after it bounces.

ii. A photogate near the floor, at a height just above the diameter of the ball, to measure the ball's speed.

iii. Drop the ball through the photogate. Record the speeds measured by the photogate before and after the bounce. Change the drop height of the ball at least five times, covering a range of heights from "low" to "high."

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Parts i, ii, and iii are scored as a unit

1 point is earned: For an overall plan in which quantities are measured that could be used to compare mechanical energy before and after a collision with a hard surface

1 point is earned: For a conceptually plausible plan to measure pre- and post-collision positions and/or speeds that could be used to compare pre- and post-collision mechanical energies, without extraneous equipment and/or measurements

1 point is earned: For having lab equipment and measurement procedures well specified

1 point is earned: For a procedure that includes trials of different pre-collision speeds, ranging from low speed to high speed (as is needed to test the student's hypothesis)

Example 1:

1. The drop height of the ball and the bounce height.

2. A meterstick to measure the heights and a video camera to record the ball's motion.

3. Place the meterstick upright against the wall. Drop the ball from 10 different drop heights, using the video camera to record the bounce heights.

Example 2:

1. The speed of the ball immediately before and immediately after it bounces.

2. A photogate near the floor, at a height just above the diameter of the ball, to measure the ball's speed.

3. Drop the ball through the photogate. Record the speeds measured by the photogate before and after the bounce. Change the drop height of the ball at least five times, covering a range of heights from "low" to "high."

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Part B

4 points total

1 point is earned: For describing how to plot or otherwise represent the data in a way that could be used to test the hypothesis

1 point is earned: For describing how to compare the post-collision to pre-collision mechanical energy (or a plausible alternative) to quantify the elasticity of the collision

1 point is earned: For comparing the low-speed versus high-speed results

1 point is earned: For addressing the hypothesis with an analysis such as a slope, ratio, or difference

Example 1:

> Make a graph of the bounce height h_f as a function of the drop height h_i. If the data are consistent with the hypothesis, then the data will (1) lie close to the line $h_f = h_i$ for low drop heights, and (2) lie below this line for high drop heights

Example 2:

> Make a graph of v_f^2-v_i^2 as a function of v_i, where v_f and v_i are the ball's speed just after and just before the bounce, respectively. If the data are consistent with the hypothesis, then v_f^2-v_i^2 will (1) be close to zero for low speeds, and (2) be negative for high speeds.

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

1 point is earned: For describing how to plot or otherwise represent the data in a way that could be used to test the hypothesis

1 point is earned: For describing how to compare the post-collision to pre-collision mechanical energy (or a plausible alternative) to quantify the elasticity of the collision

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Example 1:

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Example 2:

> Make a graph of v_f^2-v_i^2 as a function of v_i, where v_f and v_i are the ball's speed just after and just before the

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bounce, respectively. If the data are consistent with the hypothesis, then v_f^2-v_i^2 will (1) be close to zero for low speeds, and (2) be negative for high speeds.

Part C

4 points total

Part(i)

1 point is earned: For drawing a graph or table that shows that the low-speed collisions are nearly perfectly elastic

1 point is earned: For drawing a graph or table that shows a violation of a physics principle for higher-speed collisions

Example for energy conservation:

A graph of the ratio (Post-collision mechanical energy / Pre-collision mechanical energy) as a function of precollision speed, in which the graph stays near 1.0 for low initial speeds but becomes greater than 1.0 for high-speed collisions.

Part(ii)

1 point is earned: For a correct description of the aspect of the graph or table that shows a violation of the physical principle indicated

1 point is earned: For a correct explanation of why the representation shows a violation of the physical principle indicated

Example for energy conservation using the graph described above:

The value of the energy ratio shows a violation of conservation of energy when it becomes greater than 1.0 because the final energy cannot be greater than the initial energy.

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Part(i)

1 point is earned: For drawing a graph or table that shows that the low-speed collisions are nearly perfectly elastic

1 point is earned: For drawing a graph or table that shows a violation of a physics principle for higher-speed collisions

Example for energy conservation:

A graph of the ratio (Post-collision mechanical energy / Pre-collision mechanical energy) as a function of precollision speed, in which the graph stays near 1.0 for low initial speeds but becomes greater than 1.0 for high-speed collisions.

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Example for energy conservation using the graph described above:

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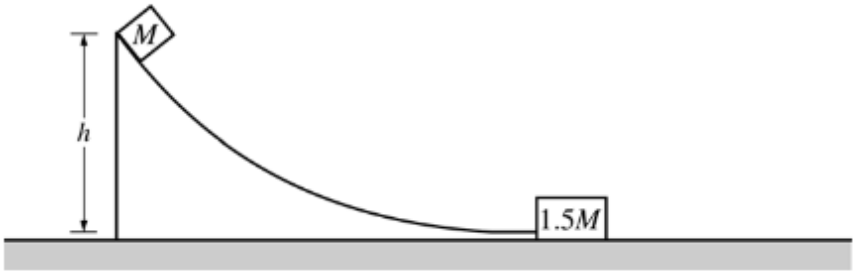
3.



A small block of mass *M* is released from rest at the top of the curved frictionless ramp shown above. The block slides down the ramp and is moving with a speed $3.5v_0$ when it collides with a larger block of mass 1.5M at rest at the bottom of the incline. The larger block moves to the right at a speed $2v_0$ immediately after the collision. Express your answers to the following questions in terms of the given quantities and fundamental constants.

(a) Determine the height *h* of the ramp from which the small block was released.

(b) Determine the speed of the small block after the collision.

(c) The larger block slides a distance *D* before coming to rest. Determine the value of the coefficient of kinetic friction μ between the larger block and the surface on which it slides.

(d) Indicate whether the collision between the two blocks is elastic or inelastic. Justify your answer.

Part A

1 point is earned: For any use of conservation of energy

For example, initially the small block has only potential energy, and it is all converted to kinetic energy when it reaches the bottom of the ramp.

1 point is earned: For a correct expression for the initial potential energy

1 point is earned: For a correct expression for the kinetic energy at the bottom of the ramp

$Mgh = \frac{1}{2}M(3.5v_0)^2$

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1 point is earned: For the correct answer

$h = \frac{3.5^2}{2}\frac{v_0^2}{g}$ or equivalent

0	1	2	3	4 ✓

The student response earns all of the following points:

1 point is earned: For any use of conservation of energy

For example, initially the small block has only potential energy, and it is all converted to kinetic energy when it reaches the bottom of the ramp.

1 point is earned: For a correct expression for the initial potential energy

1 point is earned: For a correct expression for the kinetic energy at the bottom of the ramp

$Mgh = \frac{1}{2}M(3.5v_0)^2$

1 point is earned: For the correct answer

$h = \frac{3.5^2}{2}\frac{v_0^2}{g}$ or equivalent

Part B

1 point is earned: For any use of conservation of momentum

1 point is earned: For a correct expression for the initial momentum of the blocks

1 point is earned: For a correct expression for the final momentum of the blocks

M(3.5*v0*) = *Mv* + (1.5*M*)(2*v0*)

v = 3.5*v0* - 3*v0*

1 point is earned: For the correct answer

v = 0.5*v0*

0	1	2	3	4 ✓

The student response earns all of the following points:

1 point is earned: For any use of conservation of momentum

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1 point is earned: For a correct expression for the initial momentum of the blocks

1 point is earned: For a correct expression for the final momentum of the blocks

$M(3.5v0) = Mv + (1.5M)(2v0)$

$v = 3.5v0 - 3v0$

1 point is earned: For the correct answer

$v = 0.5v0$

Part C

1 point is earned: For a correct relationship between friction and the acceleration of the block

$$\sum F = ma = f_{fric}$$

1 point is earned: For a correct kinematic equation relating acceleration and distance that does not contain time

$$v_f^2 = v_i^2 - 2aD$$

1 point is earned: For using the correct initial speed of the block

$$0 = 4v_0^2 - 2aD$$
$$a = 2v_0^2/D$$

Substituting expressions for *a* and *f* into the first equation above

$$(1.5M)2v_0^2/D = \mu(1.5M)g$$

1 point is earned: For the correct answer

$$\mu = \frac{2v_0^2}{Dg}$$

Alternate solution

1 point is earned: *For any indication that the work done on the block as it slides is equal to its initial kinetic energy*

$$f_{fric}d = \frac{1}{2}mv_i^2$$

1 point is earned: *For a correct expression for the work done on the block*

1 point is earned: *For a correct expression for the initial kinetic energy of the block*

$$\mu(1.5M)gD = \frac{1}{2}(1.5M)(2v_0)^2$$

1 point is earned: *For the correct answer*

$$\mu = \frac{2v_0^2}{Dg}$$

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0	1	2	3	4 ✓

The student response earns all of the following points:

1 point is earned: For a correct relationship between friction and the acceleration of the block

$$\sum F = ma = f_{fric}$$

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$$(1.5M)2v_0^2/D = \mu(1.5M)g$$

1 point is earned: For the correct answer

$$\mu = \frac{2v_0^2}{Dg}$$

Alternate solution

1 point is earned: *For any indication that the work done on the block as it slides is equal to its initial kinetic energy*

$$f_{fric}d = \frac{1}{2}mv_i^2$$

1 point is earned: *For a correct expression for the work done on the block*

1 point is earned: *For a correct expression for the initial kinetic energy of the block*

$$\mu(1.5M)gD = \frac{1}{2}(1.5M)(2v_0)^2$$

1 point is earned: *For the correct answer*

$$\mu = \frac{2v_0^2}{Dg}$$

Part D

1 point is earned: For indicating that the collision is inelastic

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1 point is earned: For indicating that the reason it is inelastic is because the change in kinetic energy is not zero, or because kinetic energy is lost in the collision

1 point is earned: For showing that the change in kinetic energy is not zero

$$\Delta K = K_f - K_i = \left[\tfrac{1}{2}M(0.5v_0)^2 + \tfrac{1}{2}(1.5M)(2v_0)^2\right] - \tfrac{1}{2}M(3.5v_0)^2$$
$$\Delta K = -3Mv_0^2$$

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For indicating that the collision is inelastic

1 point is earned: For indicating that the reason it is inelastic is because the change in kinetic energy is not zero, or because kinetic energy is lost in the collision

1 point is earned: For showing that the change in kinetic energy is not zero

$$\Delta K = K_f - K_i = \left[\tfrac{1}{2}M(0.5v_0)^2 + \tfrac{1}{2}(1.5M)(2v_0)^2\right] - \tfrac{1}{2}M(3.5v_0)^2$$
$$\Delta K = -3Mv_0^2$$

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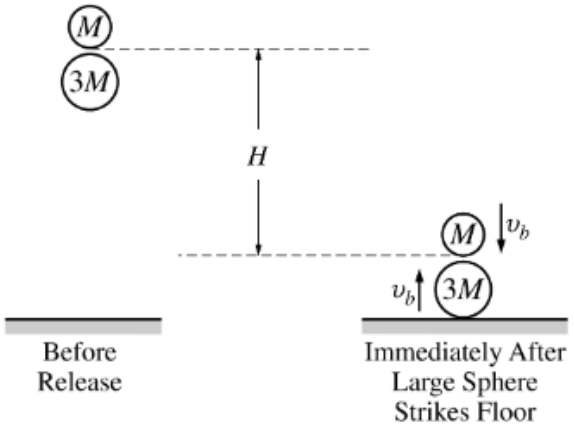
4.



A small and a large sphere, of mass *M* and *3M* respectively, are arranged as shown on the left side of the figure above. The spheres are then simultaneously dropped from rest. When the large sphere strikes the floor, the spheres have fallen a height *H*. Assume air resistance is negligible. Express all answers in terms of *M, H*, and fundamental constants, as appropriate.

(a) Derive an expression for the speed v_b with which the large sphere strikes the floor.

Immediately after striking the floor, the large sphere is moving upward with speed v_b and collides head-on with the small sphere, which is moving downward with identical speed v_b at that instant. Immediately after the collision, the small sphere moves upward with speed v_S and the large sphere has speed v_L.

(b) Derive an equation that relates v_b, v_S, and v_L.

In this particular situation $v_L = 0$.

(c) Use your relationship from part (b) to determine the speed of the small sphere in terms of v_b.

(d) Indicate whether the collision is elastic. Justify your answer using your results from parts (b) and (c).

(e) Determine the height *h* that the small sphere rises above its lowest position, in terms of the original height *H*.

Part A

1 point is earned: For a statement that shows the conservation of energy for the large sphere

$\Delta U_{3M} = \Delta K_{3M}$
$3MgH = \frac{1}{2}(3M)v_b^2$

1 point is earned: For a correct answer (or equivalent expression for vb)

$v_b = \sqrt{2gH}$

Alternate solution

1 point is earned: *For using a proper kinematic approach*

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$v_f^2 = v_0^2 + 2a\Delta y$
$v_b^2 = 2gH$

1 point is earned: *For a correct answer*

$v_b = \sqrt{2gH}$

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For a statement that shows the conservation of energy for the large sphere

$\Delta U_{3M} = \Delta K_{3M}$
$3MgH = \frac{1}{2}(3M)v_b^2$

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Alternate solution

1 point is earned: *For using a proper kinematic approach*

$v_f^2 = v_0^2 + 2a\Delta y$
$v_b^2 = 2gH$

1 point is earned: *For a correct answer*

$v_b = \sqrt{2gH}$

Part B

1 point is earned: For stating or showing the conservation of momentum applied to the collision

m1v1i + m2v2i = m1v1f + m2v2f

1 point is earned: For stating or showing that the spheres are initially traveling in opposite directions

3*Mvb* + *M*(-vb) = 3*M*vL + *M*vS

2vb = 3vL + vS

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0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For stating or showing the conservation of momentum applied to the collision

m1v1i + m2v2i = m1v1f + m2v2f

1 point is earned: For stating or showing that the spheres are initially traveling in opposite directions

3*Mvb* + *M*(-vb) = 3*M*vL + *M*vS

2vb = 3vL + vS

Part C

Substituting the given zero value into the answer from part (b)

2vb = 3vL + vS

2vb = 0 + vS

1 point is earned: For a correct answer

vS = 2vb

0	1 ✓

The student response earns all of the following points:

Substituting the given zero value into the answer from part (b)

2vb = 3vL + vS

2vb = 0 + vS

1 point is earned: For a correct answer

vS = 2vb

Part D

1 point is earned: For any correct attempt to compare total kinetic energy before the collision to total kinetic energy after the collision

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Ki = K3Mi + KMi and Kf = K3Mf + KMf

1 point is earned: For correct substitutions of vb, the expression for vS from part (c), and the correct masses in the kinetic energy terms

$K_i = \frac{1}{2}(3M)(v_b)^2 + \frac{1}{2}(M)(-v_b)^2 = 2Mv_b^2$
$K_f = \frac{1}{2}(M)(2v_b)^2 = 2Mv_b^2$

1 point is earned: For correctly stating that the collision is elastic (or inelastic if consistent with the comparison of initial and final kinetic energies)

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For any correct attempt to compare total kinetic energy before the collision to total kinetic energy after the collision

Ki = K3Mi + KMi and Kf = K3Mf + KMf

1 point is earned: For correct substitutions of vb, the expression for vS from part (c), and the correct masses in the kinetic energy terms

$K_i = \frac{1}{2}(3M)(v_b)^2 + \frac{1}{2}(M)(-v_b)^2 = 2Mv_b^2$
$K_f = \frac{1}{2}(M)(2v_b)^2 = 2Mv_b^2$

1 point is earned: For correctly stating that the collision is elastic (or inelastic if consistent with the comparison of initial and final kinetic energies)

Part E

1 point is earned: For a statement of conservation of energy for ball *M* as it rises to the new height *h*

$U_{gf} = K_0$
$Mgh = \frac{1}{2}Mv_S^2$
$h = \frac{(2v_b)^2}{2g} = \frac{4v_b^2}{2g} = \frac{2\left(\sqrt{2gH}\right)^2}{g}$

1 point is earned: For a correct answer consistent with the expression for vb obtained in part (a)

h = 4H

Alternate solution

1 point is earned: *For using a correct kinematic approach to solve for maximum height of ball M*

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$v_f^2 = v_0^2 + 2a\Delta y$

$v_0 = v_S = 2v_b = 2\sqrt{2gH}$

$0 = \left(2\left(\sqrt{2gH}\right)\right)^2 - 2gH$

1 point is earned: *For a correct answer consistent with the expression for vb obtained in part (a)*

h = 4H

Note: Both points are awarded for any correctly determined value of h without any written justification.

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For a statement of conservation of energy for ball *M* as it rises to the new height *h*

$U_{gf} = K_0$

$Mgh = \frac{1}{2}Mv_S^2$

$h = \frac{(2v_b)^2}{2g} = \frac{4v_b^2}{2g} = \frac{2\left(\sqrt{2gH}\right)^2}{g}$

1 point is earned: For a correct answer consistent with the expression for vb obtained in part (a)

h = 4H

Alternate solution

1 point is earned: *For using a correct kinematic approach to solve for maximum height of ball M*

$v_f^2 = v_0^2 + 2a\Delta y$

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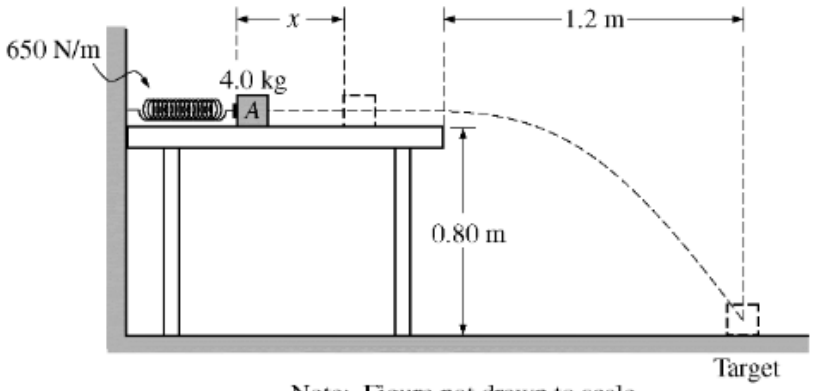
1 point is earned: *For a correct answer consistent with the expression for vb obtained in part (a)*

h = 4H

Note: Both points are awarded for any correctly determined value of h without any written justification.

chapter5

5.



Note: Figure not drawn to scale.

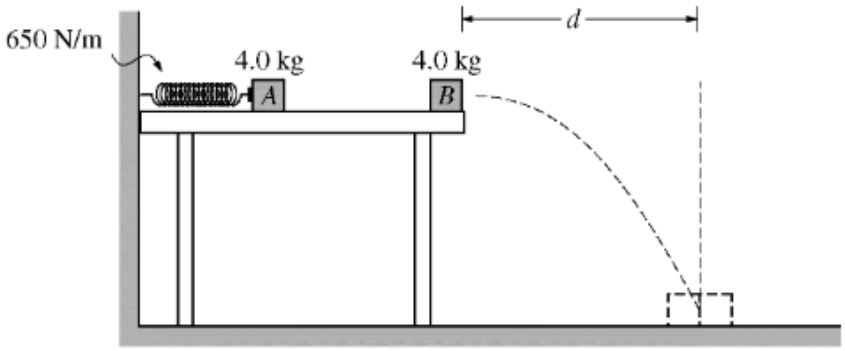
Block A of mass 4.0 kg is on a horizontal, frictionless tabletop and is placed against a spring of negligible mass and spring constant 650 N m. The other end of the spring is attached to a wall. The block is pushed toward the wall until the spring has been compressed a distance x, as shown above. The block is released and follows the trajectory shown, falling 0.80 m vertically and striking a target on the floor that is a horizontal distance of 1.2 m from the edge of the table. Air resistance is negligible.

(a) Calculate the time elapsed from the instant block A leaves the table to the instant it strikes the floor.

(b) Calculate the speed of the block as it leaves the table.

(c) Calculate the distance x the spring was compressed.

Block B, also of mass 4.0 kg, is now placed at the edge of the table. The spring is again compressed a distance x, and block A is released. As it nears the end of the table, it instantaneously collides with and sticks to block B. The blocks follow the trajectory shown in the figure below and strike the floor at a horizontal distance d from the edge of the table.



Note: Figure not drawn to scale.

(d) Calculate *d* if *x* is equal to the value determined in part (c).

(e) Consider the system consisting of the spring, the blocks, and the table. How does the total mechanical energy E_2 of the system just before the blocks leave the table compare to the total mechanical energy E_1 of the system just before block A is released?

____ $E_1 < E_2$ ____ $E_2 = E_1$ ____ $E_2 > E_1$

Justify your answer.

Part A

1 point is earned: For a correct relationship between the vertical distance and time

$h = \frac{1}{2}gt^2$

1 point is earned: For substitution of the vertical height and the acceleration due to gravity

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(0.80m)}{9.8m/s^2}}$

1 point is earned: For the correct answer

t = 0.40 s

Note: Credit was awarded for an alternate solution using

$$v_y^2 = 2gh$$

with appropriate substitutions to find the vertical velocity when the block lands, followed by substitution of this velocity into

$$v_y = gt$$

(or equivalent) to find the time

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For a correct relationship between the vertical distance and time

$h = \frac{1}{2}gt^2$

1 point is earned: For substitution of the vertical height and the acceleration due to gravity

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$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(0.80m)}{9.8m/s^2}}$

1 point is earned: For the correct answer

t = 0.40 s

Note: Credit was awarded for an alternate solution using

$$v_y^2 = 2gh$$

with appropriate substitutions to find the vertical velocity when the block lands, followed by substitution of this velocity into

$$v_y = gt$$

(or equivalent) to find the time

Part B

1 point is earned: For a correct relationship between the horizontal distance and time

x = *vt*

1 point is earned: For a consistent substitution of time from part (a) into the correct equation

$v = \frac{x}{t} = \frac{1.2m}{0.40s}$
$v = 3.0m/s$

0	1	2

The student response earns all of the following points:

1 point is earned: For a correct relationship between the horizontal distance and time

x = *vt*

1 point is earned: For a consistent substitution of time from part (a) into the correct equation

$v = \frac{x}{t} = \frac{1.2m}{0.40s}$
$v = 3.0m/s$

Part C

1 point is earned: For any statement of conservation of energy

1 point is earned: For correct use of appropriate energy equations

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

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1 point is earned: For a consistent substitution of velocity from part (b) into the correct equation

$x = \sqrt{\frac{m}{k}}v = \sqrt{\frac{4kg}{650N/m}}(3.0m/s)$

$x = 0.24m$

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For any statement of conservation of energy

1 point is earned: For correct use of appropriate energy equations

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

1 point is earned: For a consistent substitution of velocity from part (b) into the correct equation

$x = \sqrt{\frac{m}{k}}v = \sqrt{\frac{4kg}{650N/m}}(3.0m/s)$

$x = 0.24m$

Part D

1 point is earned: For any statement of conservation of momentum

mAvi = (mA + mB)vf

1 point is earned: For substitution of both masses into the equation

1 point is earned: For substitution of the velocity from part (b) into the equation

$v_f = \left(\frac{m_A}{m_A+m_B}\right)v_i = \left(\frac{4kg}{4kg+4kg}\right)3.0m/s = 1.5m/s$

1 point is earned: For substitution of time from part (a) into a correct relationship between the horizontal distance and time

d = *vft* = (1.5 m/s)(0.40 s)

d = 0.60 m

0	1	2	3	4 ✓

The student response earns all of the following points:

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1 point is earned: For any statement of conservation of momentum

mAvi = (mA + mB)vf

1 point is earned: For substitution of both masses into the equation

1 point is earned: For substitution of the velocity from part (b) into the equation

$$v_f = \left(\frac{m_A}{m_A+m_B}\right)v_i = \left(\frac{4kg}{4kg+4kg}\right)3.0m/s = 1.5m/s$$

1 point is earned: For substitution of time from part (a) into a correct relationship between the horizontal distance and time

d = vft = (1.5 m/s)(0.40 s)

d = 0.60 m

Part E

1 point is earned: For indicating that *E2 < E1*

1 point is earned: For a correct justification stating one of the following:

- the kinetic energy (or energy) is transformed into other forms of energy during the collision (e.g., by reference to heat, internal energy, sound)
- the kinetic energy is not conserved in an inelastic collision
- a numerical calculation of the relevant energies

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For indicating that *E2 < E1*

1 point is earned: For a correct justification stating one of the following:

- the kinetic energy (or energy) is transformed into other forms of energy during the collision (e.g., by reference to heat, internal energy, sound)
- the kinetic energy is not conserved in an inelastic collision
- a numerical calculation of the relevant energies

Units

1 point is earned: For correct units on all completed answers

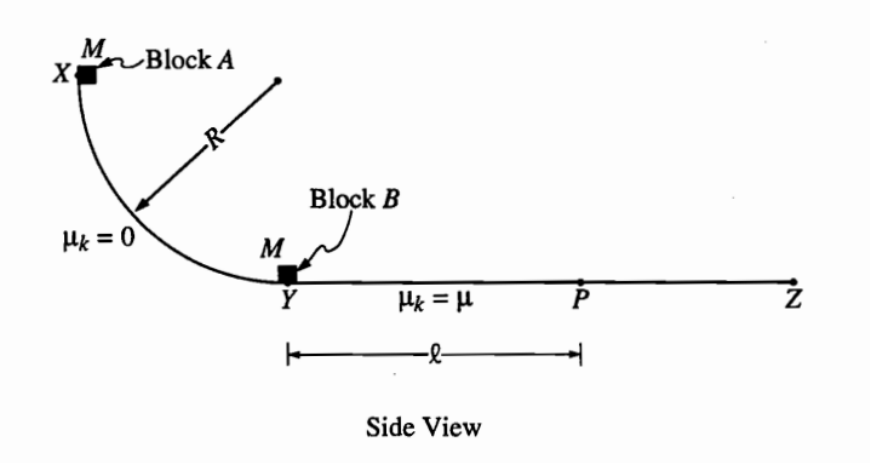
chapter5

0	1 ✓

The student response earns all of the following points:

1 point is earned: For correct units on all completed answers

6.



Side View

A track consists of a frictionless arc *XY*, which is a quarter-circle of radius *R*, and a rough horizontal section *YZ*. Block *A* of mass *M* is released from rest at point *X,* slides down the curved section of the track, and collides instantaneously and in elastically with identical block *B* at point *Y*. The two blocks move together to the right, sliding past point *P*, which is a distance *L* from point *Y*. The coefficient of kinetic friction between the blocks and the horizontal part of the track is μ. Express your answers in terms of *M*, *L*, μ, *R*, and *g*.

(a) Determine the speed of block *A* just before it hits block *B*.

(b) Determine the speed of the combined blocks immediately after the collision.

(c) Determine the amount of kinetic energy lost due to the collision.

(d) The specific heat of the material used to make the blocks is *c*. Determine the temperature rise that results from the collision in terms of *c* and the other given quantities. (Assume that no energy is transferred to the track or to the air surrounding the blocks.)

(e) Determine the additional thermal energy that is generated as the blocks move from *Y* to *P*.

Part A

1 point is earned : For any indication that energy is conserved

MgR = ½Mv 2

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1 point is earned : v 2 = 2gR

1 point is earned : $v = \sqrt{2gR}$

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned : For any indication that energy is conserved

MgR = ½Mv 2

1 point is earned : v 2 = 2gR

1 point is earned : $v = \sqrt{2gR}$

Part B

1 point is earned : Momentum is conserved during the collision

1 point is earned : Mv i = 2Mv f

$v_f = \frac{Mv_i}{2M} = \frac{v_i}{2}$

But v i is the speed calculated in part (a).

1 point is earned : $v_f = \frac{\sqrt{2gR}}{2}$

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned : Momentum is conserved during the collision

1 point is earned : Mv i = 2Mv f

$v_f = \frac{Mv_i}{2M} = \frac{v_i}{2}$

But v i is the speed calculated in part (a).

1 point is earned : $v_f = \frac{\sqrt{2gR}}{2}$

Part C

chapter5

1 point is earned : ΔK = K t - K f

1 point is earned : For the following two equations

$K_t = \frac{1}{2}Mv_i^2 = \frac{1}{2}M(2gR) = MgR$
$K_f = \frac{1}{2}(2M)v_f^2 - M\left(\frac{2gR}{4}\right) = \frac{MgR}{2}$

1 point is earned :

ΔK = MgR - MgR/2

ΔK = MgR/2

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned : ΔK = K t - K f

1 point is earned : For the following two equations

$K_t = \frac{1}{2}Mv_i^2 = \frac{1}{2}M(2gR) = MgR$
$K_f = \frac{1}{2}(2M)v_f^2 - M\left(\frac{2gR}{4}\right) = \frac{MgR}{2}$

1 point is earned :

ΔK = MgR - MgR/2

ΔK = MgR/2

Part D

The kinetic energy lost goes into heating the blocks.

1 point is earned: ΔK = Q

1 point is earned: Q = (2M)cΔT [This point also awarded for Q = Mc ΔT]

1 point is earned:

ΔT = ΔK/2Mc = MgR/2(2Mc)

ΔT = gR/4c

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0	1	2	3 ✓

The student response earns three of the following points:

The kinetic energy lost goes into heating the blocks.

1 point is earned: $\Delta K = Q$

1 point is earned: $Q = (2M)c\Delta T$ [This point also awarded for $Q = Mc\ \Delta T$]

1 point is earned:

$\Delta T = \Delta K/2Mc = MgR/2(2Mc)$

$\Delta T = gR/4c$

Part E

As the blocks move from Y to P, a frictional force $Ff = \mu N$ acts on them.

The work W done by F, equals the additional thermal energy generated, Qadd.

1 point is earned: Qadd = W

1 point is earned: W = FfL

Substituting the expression $Ff = \mu N = \mu(2Mg)$:

$Qadd = \mu(2Mg)L$

1 point is earned: $= 2\ \mu MgL$

Alternate solution

The equation for frictional force causes the block to decelerate.

The equation showing the termal energy generated

Solving for Qadd

0	1	2	3 ✓

The student response earns three of the following points:

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As the blocks move from Y to P, a frictional force Ff = μN acts on them.

The work W done by F, equals the additional thermal energy generated, Qadd.

1 point is earned: Qadd = W

1 point is earned: W = FfL

Substituting the expression Ff = μN = μ(2Mg):

Qadd = μ(2Mg)L

1 point is earned: = 2 μMgL

Alternate solution

The equation for frictional force causes the block to decelerate.

The equation showing the termal energy generated

Solving for Qadd

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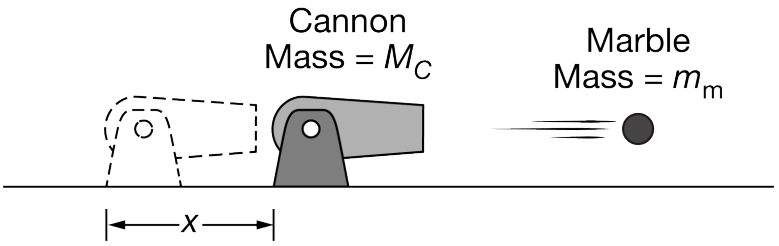
7. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch marbles of mass m_{m} horizontally from a toy cannon of mass M_C, where M_C is always greater than m_{m}, as shown above. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students use a new marble with a different mass launched at the same speed v relative to the ground. The students record the distance x the cannon slides backward for each launch.

(a) Without manipulating equations, briefly describe what happens to the distance x that the cannon slides backward as students increase the mass of the marble. Explain your reasoning.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\mathrm{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) Without manipulating or stating equations, briefly describe whether it would take more work to stop the marble or to stop the cannon after the marble has been launched. Justify your answer.

(d) Students correctly derive the following equation relating the work W done on an object initially at rest, the momentum p of the object, and the mass m of the object: $W = \frac{p^2}{2m}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble-cannon system during the short time interval in which the marble is accelerated inside the cannon and make the following claims:

Student 1: The cannon is pushing the marble forward, and the marble is pushing back with equal and opposite force. Since the work done by the cannon cancels out the work done by the marble, the mechanical energy of the system is constant.

Student 2: The mechanical energy of the system increases while the marble is being accelerated. The marble gains kinetic energy to the right, and the cannon gains an equal amount of kinetic energy to the left.

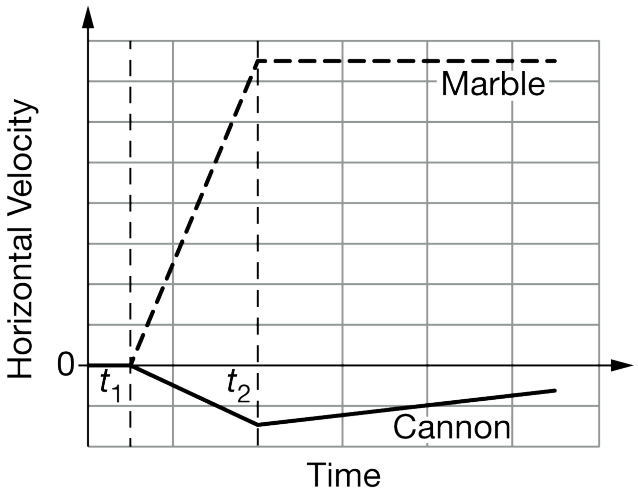
(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is

incorrect, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity of the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Provide three pieces of evidence that together explain why the magnitude of the rate of change of the cannon's velocity is less than the magnitude of the rate of change of the marble's velocity during the time interval from t_1 to t_2.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to indicate that a larger marble mass means a larger cannon speed
  OR
  For using a combination of Newton's 3rd law leading to Newton's 2nd law to indicate that a larger marble mass means a larger cannon speed.
- ☐ 1 point is earned for explaining why the larger cannon speed results in sliding a greater distance.

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Example Response:

The initial momentum of the marble-cannon system is zero, so it will be zero after the marble is launched as well. A greater marble mass then means that the backward cannon speed is greater. Since the acceleration due to friction is the same regardless of cannon speed, a greater cannon speed means that the cannon will slide a greater distance.

OR

While the marble is in the cannon, the force on the marble has the same magnitude as the force on the cannon (according to Newton's 3rd law). If the final speed of the marble is the same each time but the mass of the marble increases, the net force on the marble must increase (according to Newton's 2nd law) and, therefore, the net force on the cannon must increase as well (the force of friction remains constant) giving the cannon a larger final speed resulting in a greater distance.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence between the mass m_m and x.
- ☐ 1 point is earned for a claim that as m_m in the equation increases, x in the equation also increases.

Example Response:

Yes, the equation matches that as the mass of the marble increases the slide distance of the cannon also increases. The mass of the marble is in the numerator of the equation so if the numerator increases the slide distance will also increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim that the marble requires more work to stop, with attempt at a justification.
- ☐ 1 point is earned for a valid justification.

Example Response:

The marble requires more work to stop because the marble has a greater kinetic energy thus a greater change in kinetic

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energy in order to stop.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim consistent with the answer to (c).
- ☐ 1 point is earned for a claim that both the marble and cannon have equal momentum when the marble is launched and for recognizing that the marble's smaller mass in the denominator means it has more work done on it.

Example Response:

Yes. By conservation of momentum, the cannon and marble have the same magnitude of momentum immediately after the launch. Since p is the same, but m is smaller for the marble, the kinetic energy is greater for the marble since m is in the denominator, which means that the marble had more work done on it.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 1's statement that is correct is the marble is pushing back with equal and opposite force

OR

The first statement

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student 1's statement that is incorrect is work done by the cannon cancels out the work done by the marble and mechanical energy is constant.

OR

The second statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If a student considered the cannon to include a spring launching mechanism, then the first statement would be incorrect. A student who states "none" in part (g) and "both" in part (h) earns full credit for both parts.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

Student 2's statement that is correct is mechanical energy increases while the marble is being accelerated.

OR

The first statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

Example Response:

Student's 2 statement that is incorrect is the cannon gains an equal amount of kinetic energy to the left.

OR

The second statement

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Though the slope change in the graph occurs at t_2 (rather than "in the interval from t_1 to t_2 "), a student who indicates the effect of force of friction by discussing the slope change at t_2 is awarded this point.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for recognizing that the same force is exerted on both the marble and cannon.
- ☐ 1 point is earned for indicating that the larger mass of the cannon results in its acceleration being less.
- ☐ 1 point is earned for indicating the force of friction acting opposite the cannon's direction of motion will further reduce the acceleration of the cannon.

Example Response:

Even the though the force on the cannon and the marble is the same due to Newton's third law, the cannon's acceleration is smaller. So, its velocity increases less than the marble because the cannon's mass is larger. The cannon also has a frictional force opposing its motion, which decreases the net force and therefore the acceleration.

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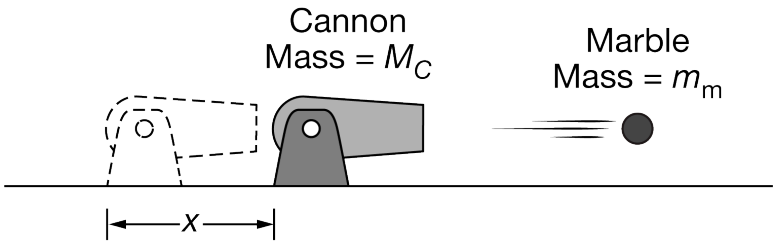
8. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch identical marbles of mass m_{m} horizontally from a toy cannon of varying mass M_C, where M_C is always greater than m_{m}, as shown above. The students can attach weights to the cannon to change M_C. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students vary the mass of the cannon and launch a marble with the same speed v relative to the ground. The students record the distance x that the cannon slides backward.

(a) Without manipulating equations, explain how the distance x that the cannon slides backward changes, if at all, when marbles are launched from more massive cannons.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\text{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) If constant horizontal forces were exerted over the same distance on the cannon and on the marble after the launch, which object would require a force with a greater magnitude to completely stop it? Without manipulating or referencing equations, briefly explain your reasoning.

(d) Students correctly derive the following equation relating the force F exerted on an object initially at rest over a distance d, the mass m of the object, and the final momentum p of the object: $F = \frac{p^2}{2md}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble after it has been launched by the cannon but before it reaches the ground. Both students agree that air resistance is negligible and make the following claims.

Student 1: The mechanical energy of the marble is constant. There are no external forces exerted on the marble, so its energy does not change.

Student 2: The mechanical energy decreases as the marble falls. This happens because the gravitational force is doing work on the marble.

(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

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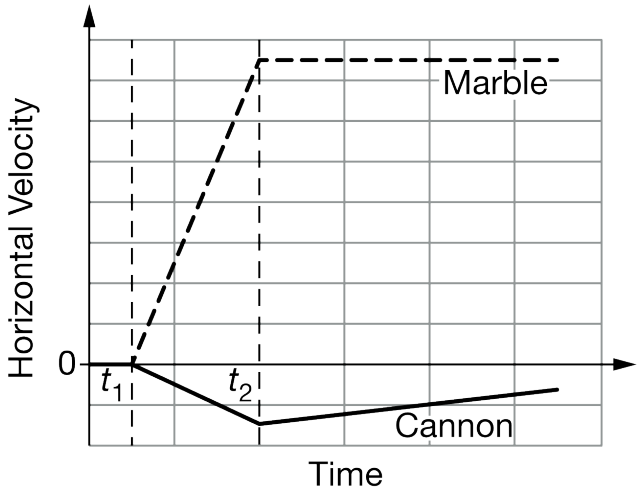
(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity of the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Describe the motion of the center of mass of the marble-cannon system during the time interval from t_1 to t_2. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: Students may earn the points for a correct discussion regarding the force of friction. A more massive cannon leads to a greater normal force increasing the force of friction. This would cause the net force on the cannon to the left to be smaller while the marble was in the cannon (giving it an initially smaller speed once the marble has left the cannon) resulting in a smaller distance traveled.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to indicate that a larger cannon mass means a smaller cannon speed

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OR

For using a combination of Newton's 3rd law leading to Newton's 2nd law to indicate that a larger cannon mass means a smaller cannon speed.

☐ 1 point is earned for explaining why the smaller cannon speed results in sliding a shorter distance.

Example Response:

The initial momentum of the marble-cannon system is zero, so it will be zero after the marble is launched as well. A more massive cannon then means that the backward cannon speed is less. A smaller cannon speed means that the cannon will slide a shorter distance.

OR

While the marble is in the cannon, the force on the marble has the same magnitude as the force on the cannon (according to Newton's 3rd law). If the mass of the marble stays constant but the mass of the cannon increases, the cannon will have smaller acceleration and smaller speed (according to Newton's 2nd law) resulting in a smaller distance.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

☐ 1 point is earned for using functional dependence between the mass and the distance x.

☐ 1 point is earned for a claim that as M_c in the equation increases, x in the equation decreases.

Example Response:

Yes, the equation matches the prediction that as the mass of the cannon increases the distance the cannon slides backwards decreases. From the equation, the mass of the cannon and the distance it slides backwards are inversely proportional. The mass of the cannon is in the denominator of the equations so as the mass increases the distance the cannon slides backwards will decreases.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

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- [ ] 1 point is earned for a claim that it would take more force to stop the marble, with an attempt at a justification.
- [ ] 1 point is earned for a valid justification using either work/energy or momentum.

Example Response:

It would take more force to stop the marble because the change in kinetic energy to stop the marble is greater so the work on the marble needs to be greater.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for a claim consistent with the answer to (c).
- [ ] 1 point is earned for a claim that both the marble and cannon have equal momentum when the marble is launched and for recognizing that the marble's smaller mass in the denominator means it requires more force to stop.

Example Response:

Yes. By conservation of momentum, the cannon and marble have the same magnitude of momentum immediately after the launch. Since p is the same, but m is smaller for the marble, the force is greater for the marble since m is in the denominator.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for a statement of "None".

Example Response:

None

Part F

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating both statements are incorrect.

Example Response:

Both parts of Student 1's statement are incorrect.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

The gravitational force is doing work on the marble

OR

The second statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for an incorrect statement and no correct statements.

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Example Response:

The mechanical energy decreases as the marble falls.

OR

The first statement

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating the center of mass moves to the right.
- ☐ 1 point is earned for indicating that there is an external force on the system due to friction.
- ☐ 1 point is earned for indicating that the net force on the system is needed to move the center on mass.

Example Response:

During the launch, the only horizontal external force exerted on the marble-cannon system is the frictional force exerted on the cannon. The friction force is directed to the right, so the net force on the system is to the right. This means the center of mass of the system is accelerating to the right.

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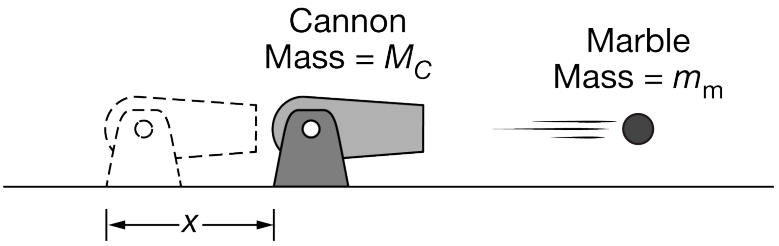
9. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch identical marbles of mass m_{m} horizontally from a toy cannon of mass M_C, where $M_C > m_{\text{m}}$, as shown above. The cannon can be adjusted to change the launch speed v of the marble relative to the ground. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students vary the launch speed v and record the distance x the cannon slides backward for each launch speed.

(a) Without manipulating equations, briefly describe what happens to the distance x that the cannon slides backward as students increase the launch speed. Explain your reasoning.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\text{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) Without manipulating or stating equations, briefly describe whether the cannon or the marble has more kinetic energy immediately after the marble leaves the cannon. Justify your answer.

(d) Students correctly derive an equation relating the kinetic energy K, the momentum p, and the mass m of an object: $K = \frac{p^2}{2m}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble after it has been launched by the cannon but before it reaches the ground. Both students agree that air resistance is negligible and make the following claims.

Student 1: There are two external forces exerted on the marble while it is in the air, the gravitational force and the force exerted by the cannon. Therefore, the mechanical energy of the marble changes while it is in the air.

Student 2: The cannon does not exert a force on the marble after the launch. Therefore, the mechanical energy of the marble is constant.

(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

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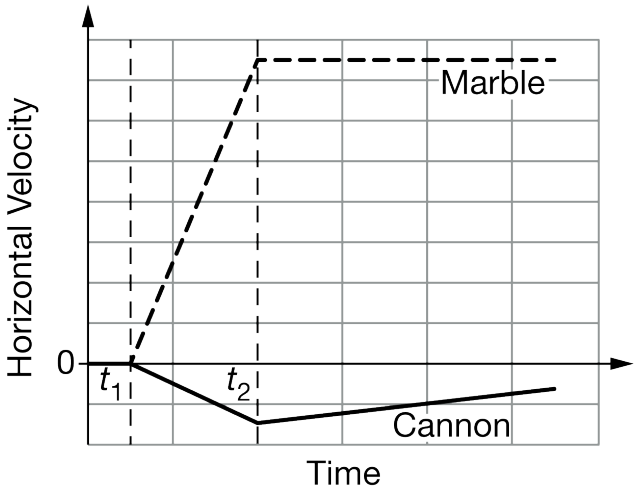
(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity for the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Describe how to find the change in the distance between the center of mass of the cannon and the center of mass of the marble from time t_1 to time t_2. Assume that before time t_1 the center of mass of the marble is at the same position as the center of mass of the cannon.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to indicate that a larger marble speed means a larger cannon speed
  OR
  For using a combination of Newton's 3rd law leading to Newton's 2nd law to indicate that a larger marble speed means a larger cannon speed.
- ☐ 1 point is earned for correctly explaining why the larger cannon speed results in sliding a greater distance.

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Example Response:

The initial momentum of the marble-cannon system is zero, so it will be zero after the marble is launched as well. A greater marble speed then means that the backward cannon speed is greater. Since the acceleration due to friction is the same regardless of cannon speed, a greater cannon speed means that the cannon will slide a greater distance.

OR

While the marble is in the cannon, the force on the marble has the same magnitude as the force on the cannon (according to Newton's 3rd law). If the mass of the marble stays constant but the final speed of the marble increases, the net force on the marble must increase (according to Newton's 2nd law) and, therefore, the net force on the cannon must increase as well (the force of friction remains constant) giving the cannon a larger final speed resulting in a greater distance.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using functional dependence.
- ☐ 1 point is earned for a claim that as v in the equation increases, x in the equation also increases.

Example Response:

Yes, the equation matches the prediction that as the velocity of the marble increases the distance the cannon moves backward also increases. In the equation as velocity increases the distance would increase. Since the velocity is in the numerator of the equation, as the numerator of a fraction increases the distance x would increase.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim that the marble has more kinetic energy, with an attempt at a justification.
- ☐ 1 point is earned for a valid justification.

Example Response:

By conservation of momentum, mass and velocity are inversely proportional. Since velocity is squared when determining

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kinetic energy, a greater speed increases the kinetic energy by a larger factor than a smaller mass decreases the energy.

OR

The cannon and marble exert forces of equal magnitude on each other. However, the marble will move a greater distance than the cannon while these forces are exerted. So, the marble has more work done on it, and therefore a greater kinetic energy.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a claim consistent with the answer to (c).
- ☐ 1 point is earned for a claim that both the marble and cannon have equal momentum when the marble is launched and for recognizing that the marble's smaller mass in the denominator means it has more kinetic energy.

Example Response:

Yes. By conservation of momentum, the cannon and marble have the same magnitude of momentum immediately after the launch. Since p is the same, but m is smaller for the marble, the kinetic energy is greater for the marble since m is in the denominator.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Response:

The correct statement from student 1 is that mechanical energy of the marble changes while it is in the air

OR

The second statement

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Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for an incorrect statement and no correct statements.

Example Response:

The incorrect statement for student 1 is that the cannon exerts a force on the marble while it is in the air

OR

The first statement

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for a correct statement and no incorrect statements.

Example Response:

The correct statement for student 2 is that the cannon no longer exerts a force on the marble

OR

The first statement

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1 ✓

The response accurately includes

- [ ] 1 point is earned for an incorrect statement and no correct statements.

Example Response:

The incorrect statement for student 2 is that the mechanical energy of the marble is constant

OR

The second statement

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- [ ] 1 point is earned for recognizing that displacement can be calculated by finding the area under a *v* vs. *t* graph.
- [ ] 1 point is earned for recognizing that the total displacement will be represented by combining the areas under each *v* vs. *t* graph.
- [ ] 1 point is earned for indicating that, for the cannon, either the area (which is negative) is to be subtracted
  OR
  the absolute value of the area is to be added, to the area under the marble graph.

Example Response:

The area under the velocity vs. time graph represents the displacement of an object. To find the change in the distance, subtract the areas of the two graphs between t_1 *and* t_2.

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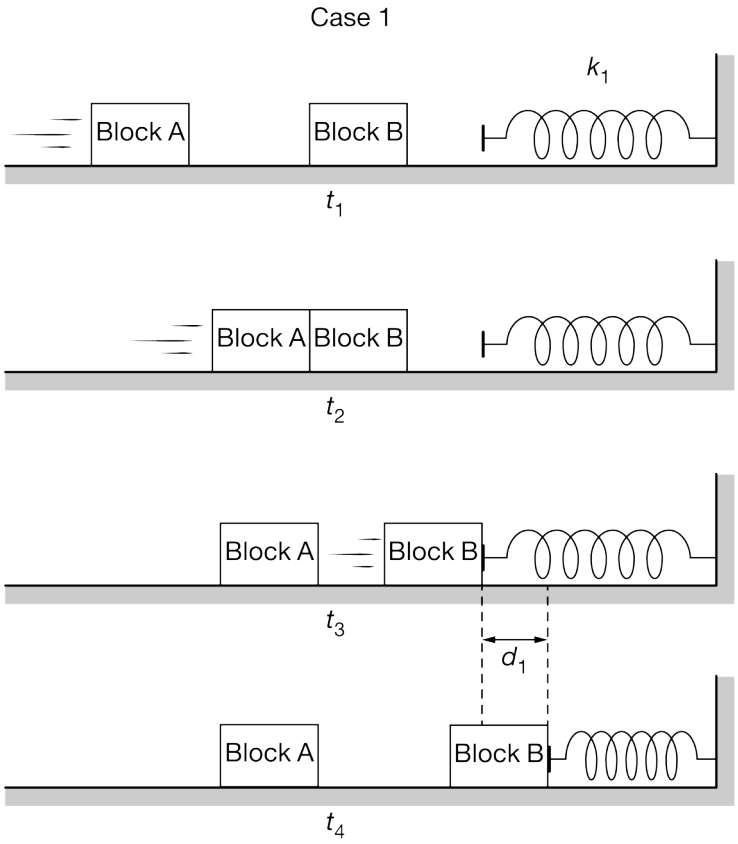
10. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



The figure above shows Case 1 of an experiment, in which the following events occur.

- At time t_1, block A slides to the right along a horizontal surface toward identical block B, which is at rest.
- At time t_2, block A collides elastically with block B and block A comes to rest.
- At time t_3, the right side of block B reaches an ideal, horizontal spring with negligible mass and spring constant k_1. The spring is initially at its relaxed (uncompressed) length and fixed to a wall.
- At time t_4, the spring is compressed a maximum distance d_1 and block B is instantaneously at rest.

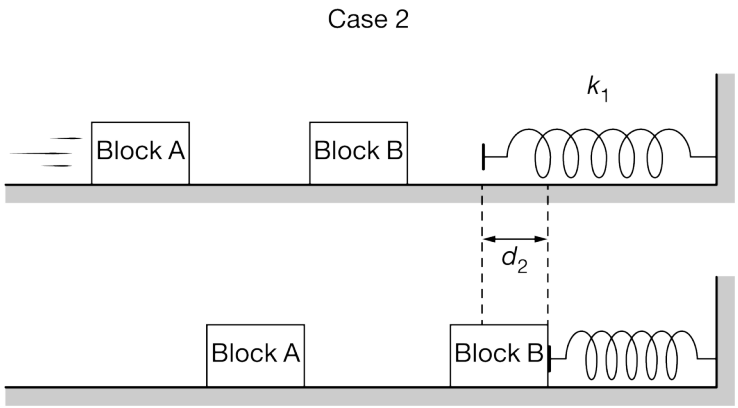
Assume there is negligible friction between each block and the surface. For parts (a)-(d), consider the system consisting of block A, block B, and the spring.

(a) Indicate whether the linear momentum of the two-block–spring system is either constant or not constant during the time interval from t_1 to immediately before t_3, and explain why.

(b) Indicate whether the mechanical energy of the two-block–spring system is either constant or not constant during the time interval from t_1 to immediately before t_3, and explain why.

(c) Indicate whether the linear momentum of the two-block–spring system is either constant or not constant during the time interval t_3 from to t_4, and explain why.

(d) Indicate whether the mechanical energy of the two-block–spring system is either constant or not constant during the time interval from t_3 to t_4, and explain why.



In Case 2 shown above, the setup is identical to Case 1 except that the speed of block A before colliding with block B is greater than in Case 1. Block A again comes to rest immediately after colliding with block B. The spring compresses a maximum distance d_2.

(e) How does the new maximum spring compression d_2 compare to d_1, the spring compression in Case 1? (Is d_2 greater than d_1, the same as d_1, or less than d_1 ?)

(f) Briefly explain your reasoning for part (e).

Students attempt to determine a general equation for the maximum compression distance d of a spring with spring constant k. Blocks A and B each have mass m and block A has initial speed v. The students come up with the following two equations.

Equation 1: $d = C_1\sqrt{\frac{m}{vk}}$ **Equation 2:** $d = C_2\sqrt{\frac{m}{k}}v$

In the two equations, C_1 and C_2 are positive constants with appropriate units.

(g) Without algebraic manipulation of equations or deriving the correct equation, explain which equation better supports your explanation for the compression distance in part (f).

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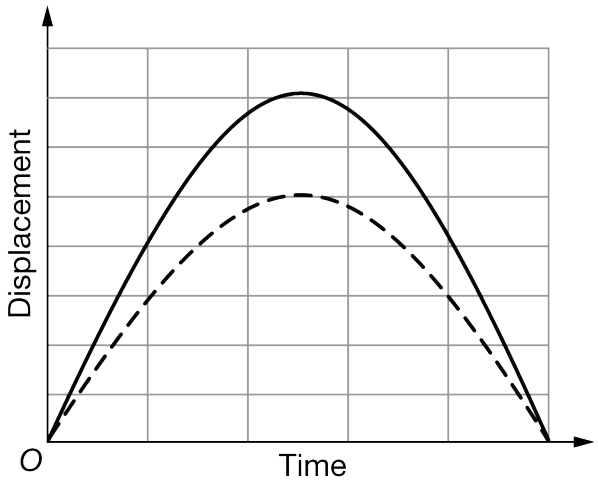

(h) The graph above shows the displacement of the blocks while block B is in contact with the spring. The solid line corresponds to either Case 1 or Case 2, while the dashed line corresponds to the other case. Does the dashed-line curve correspond to Case 1 (slower block A) or Case 2 (faster block A)? Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that linear momentum is constant, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the linear momentum is constant.

Example Response:

No net external forces are exerted on the two-block–spring system, so the momentum of the system is constant.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

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- [ ] 1 point is earned for indicating that mechanical energy is constant, with an attempt at an explanation.
- [ ] 1 point is earned for a correct explanation of why the mechanical energy is constant.

Example Response:

The mechanical energy of the system is constant because the collision is elastic.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for indicating that linear momentum is not constant, with an attempt at an explanation.
- [ ] 1 point is earned for a correct explanation of why the linear momentum is not constant.

Example Response:

The linear momentum is not constant, because a net external force is exerted on the two-block–spring system.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for indicating that mechanical energy is constant, with an attempt at an explanation.
- [ ] 1 point is earned for a correct explanation of why the mechanical energy is constant.

Example Response:

The mechanical energy is constant because no work is being done on the system by an external force.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1 ✓

The response accurately includes

- ☐ 1 point is earned for stating the compression distance is larger.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that block A transfers all of its kinetic energy to block B (i.e., block B has more kinetic energy when it reaches the spring than in Case 1).
- ☐ 1 point is earned for indicating that the greater kinetic energy means a larger compression because the elastic potential energy is equal to the initial kinetic energy.

Example Response:

Block A transfers all of its kinetic energy to block B in the elastic collision, so block B has more kinetic energy than in case 1. This means that there is more spring potential energy when the spring is compressed, so it is compressed a larger amount.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If part (g) is answered with a correct interpretation of Equation 1 consistent with an incorrect response in part (f), both points can be earned in part (g).

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that Equation 2 best models the compression with an attempt at an explanation that addresses functional dependence.
- ☐ 1 point is earned for a correct explanation that addresses the dependence of d on v.

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Example Response:

Since v is proportional to d in equation 2, as v increases d will increase, as predicted in part (f).

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the dashed line corresponds to Case 1, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation regarding the slope of the displacement vs. time graph
  OR
  For a correct explanation addressing the amplitude (energy).

Example Response:

The dashed-line has an initially smaller slope representing a smaller initial speed of block B. Both graphs have zero slope where the block B momentarily stops at the spring's maximum displacement. The maximum displacement is smaller for the initially slower Block B. Therefore, the dashed-line graph corresponds to Case 1.
OR
The smaller maximum displacement for the dashed line is consistent with less energy being stored in the spring. This would indicate that block B had less initial kinetic energy when it collided with the spring corresponding to Case 1.

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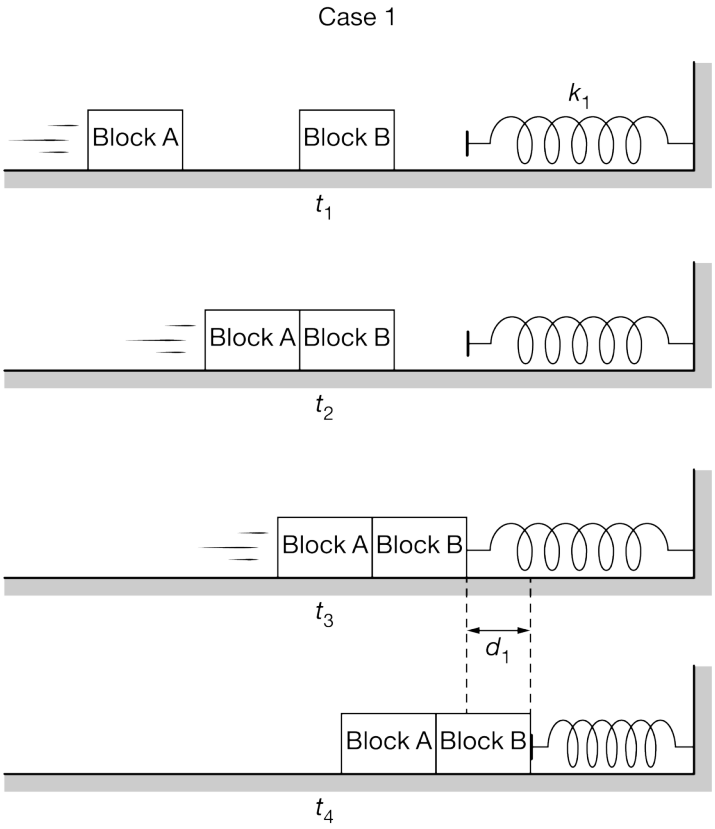
11. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



The figure above shows Case 1 of an experiment, in which the following events occur.

- At time t_1, block A slides to the right along a horizontal surface toward identical block B, which is at rest.
- At time t_2, block A collides with and sticks to block B.
- At time t_3, block A and block B are still stuck together and the right side of block B reaches an ideal, horizontal spring with negligible mass and spring constant k_1. The spring is initially at its relaxed (uncompressed) length and fixed to a wall.
- At time t_4, the spring is compressed a maximum distance d_1 and the blocks are instantaneously at rest.

Assume there is negligible friction between each block and the surface.

A student observes the experiment and makes the following claim.

"*Since some mechanical energy is dissipated when the blocks stick together, the momentum of the two-block system decreases.*"

(a) Indicate whether the claim is ***correct*** or ***incorrect*** and explain why.

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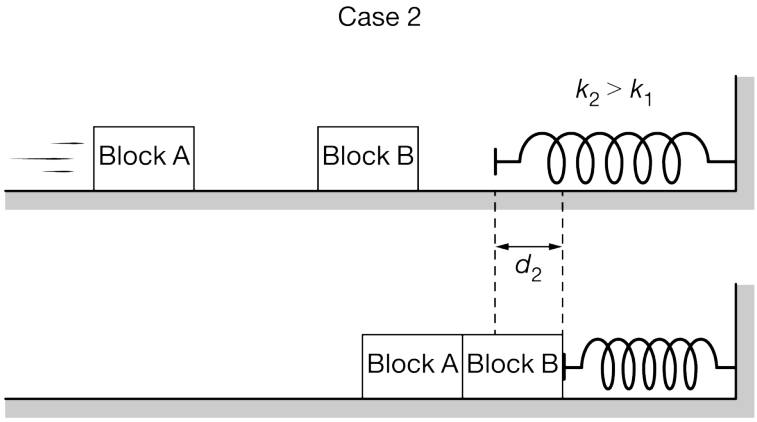
Another student makes the following claim about the collision between block A and block B.

"*When the blocks collide the total momentum of the two-block system doesn't change, so the kinetic energy of the system must also be constant.*"

(b) Indicate whether the claim is ***correct*** or ***incorrect*** and explain why.

(c) During the time interval from t_3 to t_4, is the magnitude of the acceleration of the two-block system increasing, decreasing, or constant? Briefly explain why.

(d) During the time interval from t_3 to t_4, does the spring do positive, negative, or zero work on the two-block system? Briefly explain your answer.



In Case 2 shown above, the setup is identical to Case 1 except that a stiffer spring is used. The spring in Case 2 has a spring constant k_2 that is greater than k_1. Block A slides with the same speed as in Case 1 and the spring compresses a maximum distance d_2.

(e) How does the new maximum spring compression distance d_2 in Case 2 compare to d_1 in Case 1? (Is d_2 greater than d_1, the same as d_1, or less than d_1 ?)

(f) Briefly explain your reasoning for part (e).

Students attempt to determine a general equation for the maximum compression distance d of a spring with spring constant k. Block A has mass m_A and initial speed v, and block B has mass m_B. The students come up with the following two equations.

Equation 1: $d = \frac{C_1 m_A v}{\sqrt{(k)(m_A+m_B)}}$ **Equation 2:** $d = C_2\left(k\sqrt{m_A + m_B}\right)v$

In the two equations, C_1 and C_2 are positive constants with appropriate units.

(g) Without algebraic manipulation of equations or deriving the correct equation, explain which equation better supports your explanation for the compression distance in part (f).

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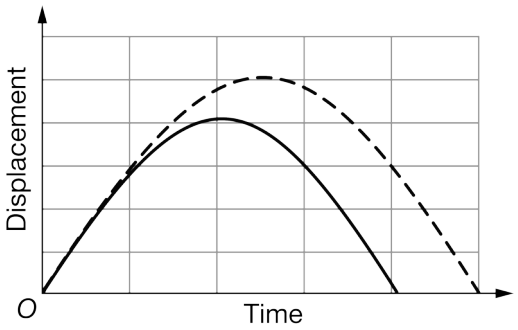

(h) The graph above shows the displacement of the blocks while block B is in contact with the spring. The solid line corresponds to either Case 1 or Case 2, while the dashed line corresponds to the other case. Does the solid-line curve correspond to Case 1 (with a smaller spring constant) or Case 2 (with a larger spring constant)? Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the claim is incorrect, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the momentum does not change.

Example Response:

The claim is incorrect. The momentum does not change because no net external force is exerted on the two-block system.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the claim is incorrect, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation.

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Example Response:

The claim is incorrect. Energy is dissipated during the collision and therefore the kinetic energy decreases.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the magnitude of the acceleration is increasing, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation.

Example Response:

The force exerted by the spring increases with increasing compression.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the spring does negative work on the two-block system, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why negative work is done.

Example Response:

The spring does negative work on the two-block system because the spring exerts a net external force opposite the direction of motion of the system.

OR

The spring force does negative work on the two-block system because the force exerted from the spring decreases the kinetic energy of the system.

Part E

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for stating the compression distance is smaller.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the two-block system has the same kinetic energy before reaching the spring as in Case 1 (or that the maximum elastic potential energy in the spring is the same as in Case 1).
- ☐ 1 point is earned for indicating that the elastic potential energy is directly related to both spring constant and compression distance, or that a greater spring constant means a smaller compression distance for the same potential energy.

Example Response:

The two-block system has the same kinetic energy when it reaches the spring. All the kinetic energy is converted to spring potential energy. Since the spring constant is now greater, the maximum compression distance must be less in order to have the same amount of potential energy at maximum compression.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If part (g) is answered with a correct interpretation of Equation 2 consistent with an incorrect response in part (f), both points can be earned in part (g).

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that Equation 1 best models the compression with an attempt at an

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explanation that addresses functional dependence.

- [ ] 1 point is earned for a correct explanation that addresses the dependence of d on k.

Example Response:

Since k is in the denominator of Equation 1, as k increases d will decrease, as predicted in part (f).

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

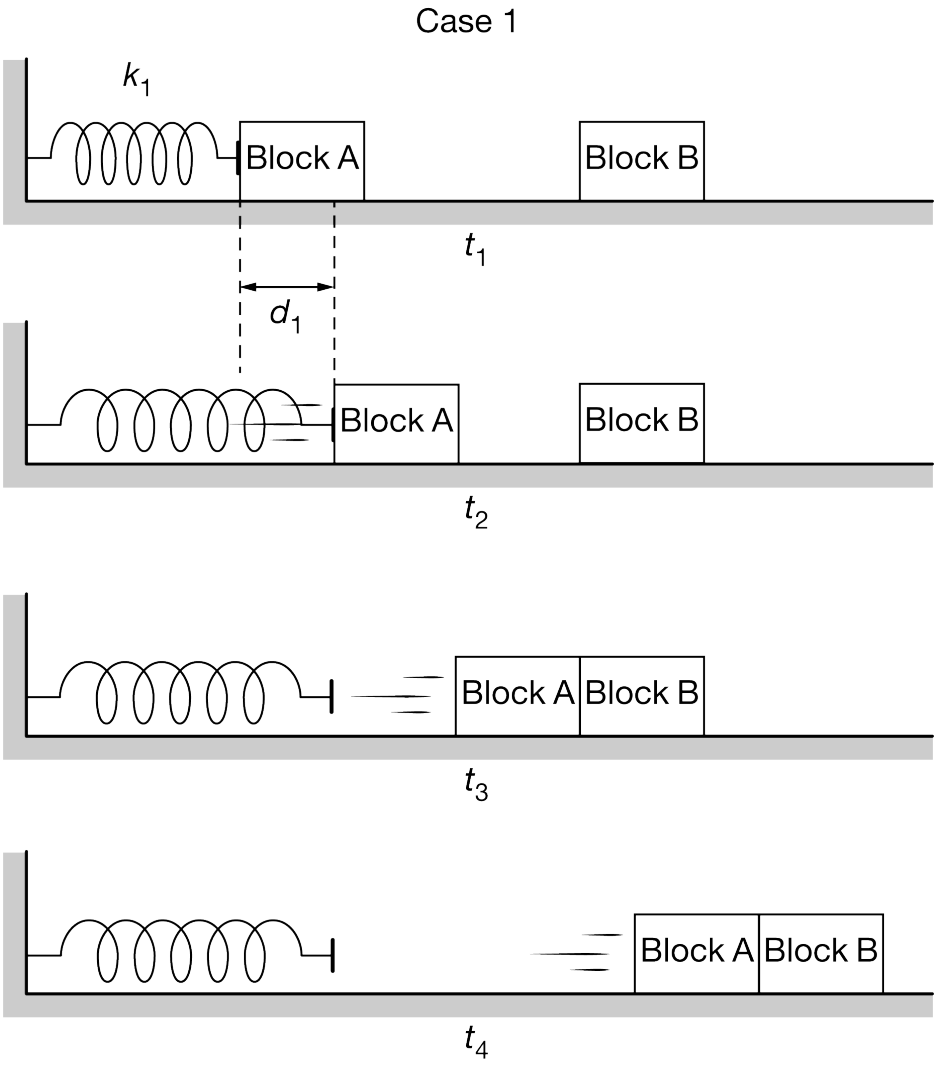
- [ ] 1 point is earned for indicating the solid line corresponds to Case 2, with an attempt at an explanation.
- [ ] 1 point is earned for a correct explanation addressing either the amplitude or period (or both).

Example Responses:

The smaller maximum displacement for the solid line is consistent with the larger spring constant in case 2.
OR
The period of an object oscillating on a spring is shorter when the spring constant is larger, so the solid line represents case 2.

chapter5

12. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



The figure above shows Case 1 of an experiment. Initially, block A is held in place while in contact with a horizontal spring with spring constant k_1 that is compressed a distance d_1 and fixed to a wall. The following events occur.

- At time t_1, block A is released from rest.
- At time t_2, the ideal spring reaches its uncompressed length, and block A leaves the spring.
- At time t_3, block A collides with and sticks to identical block B, which is initially at rest.
- At time t_4, block A and block B are sliding together along the surface.

Assume there is negligible friction between each block and the surface.

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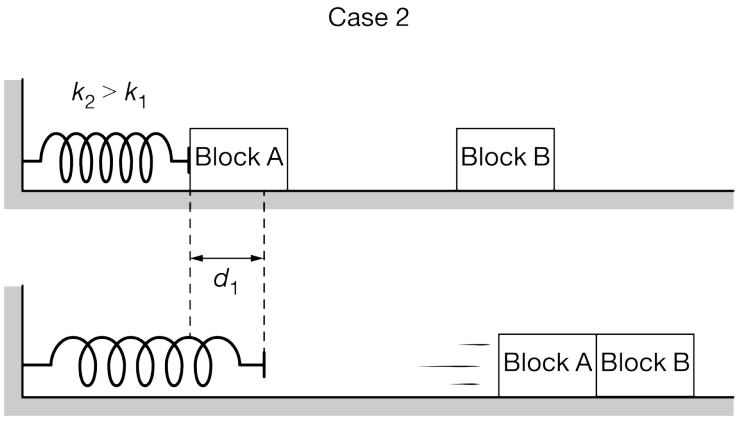
(a) Indicate whether the speed of the center of mass of the two-block system is either constant or not constant during the time interval from t_1 to t_2, and explain why.

(b) Indicate whether the mechanical energy of the two-block system is either constant or not constant during the time interval from t_1 to t_2, and explain why.

(c) Indicate whether the speed of the center of mass of the two-block system is either constant or not constant during the time interval from t_2 to t_4, and explain why.

(d) Indicate whether the mechanical energy of the two-block system is either constant or not constant during the time interval from t_2 to t_4, and explain why.



In Case 2 shown above, the setup is identical to Case 1 except that a stiffer spring is used. The spring in Case 2 has a spring constant k_2 that is greater than k_1. The spring is compressed the same distance, d_1, before block A is released. As in Case 1, the blocks stick together when they collide.

(e) Is the speed of the blocks after the collision in Case 2 greater than, less than, or the same as the speed of the blocks after the collision in Case 1?

(f) Briefly explain your reasoning for part (e).

Students attempt to determine a general equation for the speed v of the blocks after the collision. A spring with spring constant k is initially compressed a distance d, block A has mass m_A, and block B has mass m_B. The students come up with the following two equations.

Equation 1: $v = C_1\left(\frac{m_B+m_A}{\sqrt{km_A}}\right)d$ **Equation 2:** $v = C_2\left(\frac{\sqrt{km_A}}{m_A+m_B}\right)d$

In the two equations, C_1 and C_2 are positive constants with appropriate units.

(g) Without algebraic manipulation of equations or deriving the correct equation, explain which equation better supports your explanation for the speed of the two blocks in part (f).

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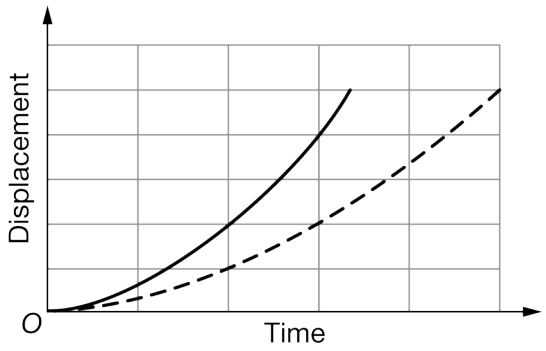

The graph above shows the displacement of block A while it is in contact with the spring for Case 1 and Case 2. The solid line corresponds to either Case 1 or Case 2, while the dashed line corresponds to the other case.

(h) Does the solid-line curve best correspond to Case 1 (with a smaller spring constant) or to Case 2 (with a larger spring constant)? Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the speed of the center of mass is not constant, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the center of mass speed is not constant.

Example Response:

The speed of the center of mass is not constant, because a net external force is exerted on the two-block system.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

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- ☐ 1 point is earned for indicating that mechanical energy is not constant, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the mechanical energy is not constant.

Example Response:

The mechanical energy is not constant, because the spring is doing work on the two-block system

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the speed of the center of mass of the system is constant, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the speed of the center of mass of the system is constant.

Example Response:

The speed of the center of mass of the system is constant, because no net external force is exerted on the two-block system.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that mechanical energy is not constant, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation of why the mechanical energy is not constant.

Example Response:

The mechanical energy is not constant, because the collision is inelastic and therefore the kinetic energy decreases.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1 ✓

The response accurately includes

- ☐ 1 point is earned for stating the speed in Case 2 is greater than that in Case 1.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that more energy is transferred to block A by the spring.
- ☐ 1 point is earned for indicating that a larger kinetic energy means larger initial speed for block A before the collision with block B and, therefore, a larger final speed for the two blocks after the collision.

Example Response:

Because the spring constant is greater in case 2, more energy is transferred to block A by the spring and block A has more kinetic energy than in case 1. The larger kinetic energy means a larger initial speed for block A before the collision with block B, so there is more momentum and a greater final speed.

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If (g) is answered with a correct interpretation of Equation 1 consistent with an incorrect response in part (f), both points can be earned in part (g)

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that Equation 2 best models the compression with an attempt at an explanation that addresses functional dependence.
- ☐ 1 point is earned for a correct explanation that addresses the dependence of v on k.

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Example Response:

Since v and k are directly related (k is in the numerator of the expression), a larger k means that v will be larger, as predicted in part (f).

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the solid line corresponds to Case 2, with an attempt at an explanation.
- ☐ 1 point is earned for a correct explanation that the block will spend less time in contact with the spring with a stiffer spring constant (or that the speed/slope will be larger).

Example Response:

The solid line has a greater slope at the end, which means a greater speed, consistent with the larger spring constant in case 2.
OR
The solid line corresponds to a shorter period of a block attached to a spring, consistent with the larger spring constant in case 2.

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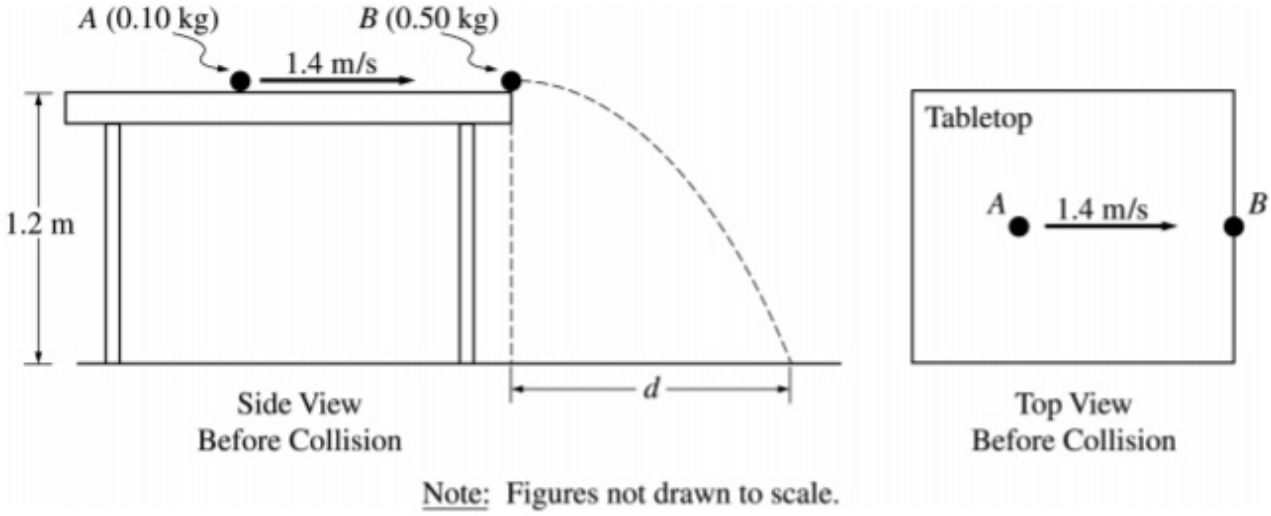
13.

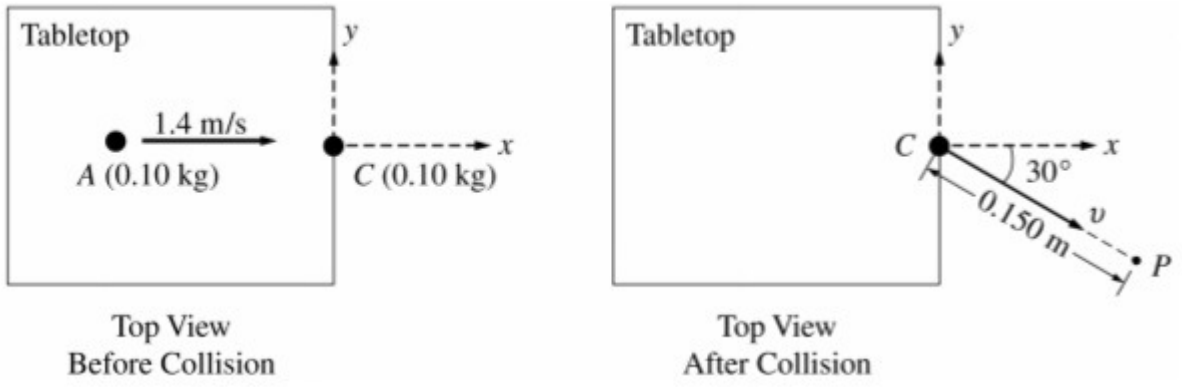


An incident ball *A* of mass 0.10 kg is sliding at 1.4 m/s on the horizontal tabletop of negligible friction shown above. It makes a head-on collision with a target ball *B* of mass 0.50 kg at rest at the edge of the table. As a result of the collision, the incident ball rebounds, sliding backwards at 0.70 m/s immediately after the collision.

a. Calculate the speed of the 0.50 kg target ball immediately after the collision.

The tabletop is 1.20 m above a level, horizontal floor. The target ball is projected horizontally and initially strikes the floor at a horizontal displacement *d* from the point of collision.

b. Calculate the horizontal displacement *d*.



In another experiment on the same table, the target ball *B* is replaced by target ball *C* of mass 0.10 kg. The incident ball *A* again slides at 1.4 m/s, as shown above left, but this time makes a glancing collision with the target ball *C* that is at rest at the edge of the table. The target ball *C* strikes the floor at point *P*, which is at a horizontal displacement of 0.15 m from the point of the collision, and at a horizontal angle of 30° from the +*x*-axis, as shown above right.

c. Calculate the speed *v* of the target ball *C* immediately after the collision.

d. Calculate the *y*-component of incident ball *A*'s momentum immediately after the collision.

Part A

1 point is earned: For any indication that conservation of momentum is applicable

pbefore = *pafter*

mAvAi = *mBvBf* + *mAvAf*

Solving for *vBf*

$v_{Bf} = \frac{m_A(v_{Ai} - v_{Af})}{m_B}$

1 point is earned: For correct substitution of values before the collision

1 point is earned: For correct substitution of values after the collision, including *vAf* = –0.70 m/s

$v_{Bf} = \frac{(0.10\ \text{kg})(1.4\ \text{m/s} + 0.70\ \text{m/s})}{0.50\ \text{kg}}$

1 point is earned: For the correct answer

vBf = 0.42 m/s

✓

0	1	2	3	4

The student response earns all of the following points:

1 point is earned: For any indication that conservation of momentum is applicable

pbefore = *pafter*

mAvAi = *mBvBf* + *mAvAf*

Solving for *vBf*

$v_{Bf} = \frac{m_A(v_{Ai} - v_{Af})}{m_B}$

1 point is earned: For correct substitution of values before the collision

1 point is earned: For correct substitution of values after the collision, including *vAf* = –0.70 m/s

$v_{Bf} = \frac{(0.10\ \text{kg})(1.4\ \text{m/s} + 0.70\ \text{m/s})}{0.50\ \text{kg}}$

1 point is earned: For the correct answer

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vBf = 0.42 m/s

Part B

1 point is earned: For any statement of an equation that can be used to find the time *t* for the ball to fall

For example:

$h = h_0 + v_0 t + \frac{1}{2} a t^2$

1 point is earned: For correct substitution

$0 = 1.2\ \text{m} + 0(t) + \frac{1}{2}(-9.8\ \text{m/s}^2)t^2$

1 point is earned: For the correct value for *t*

t = 0.49 s (same value obtained using *g* = 10 m/s2)

1 point is earned: For substituting the value of *v* from part (a) into a correct equation for the horizontal distance (i.e., having initial position and acceleration both zero)

d = *vt* = (0.42 m/s)(0.49 s)

1 point is earned: For the correct answer

d = 0.21 m



0	1	2	3	4	5

The student response earns all of the following points:

1 point is earned: For any statement of an equation that can be used to find the time *t* for the ball to fall

For example:

$h = h_0 + v_0 t + \frac{1}{2} a t^2$

1 point is earned: For correct substitution

$0 = 1.2\ \text{m} + 0(t) + \frac{1}{2}(-9.8\ \text{m/s}^2)t^2$

1 point is earned: For the correct value for *t*

t = 0.49 s (same value obtained using *g* = 10 m/s2)

1 point is earned: For substituting the value of *v* from part (a) into a correct equation for the horizontal distance (i.e., having initial position and acceleration both zero)

d = *vt* = (0.42 m/s)(0.49 s)

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1 point is earned: For the correct answer

$d = 0.21$ m

Part C

1 point is earned: For any statement of the equation for speed

$vC = d/t$

1 point is earned: For substituting the time from part (b) or correctly recalculating it

$vC = 0.15$ m/0.49 s

1 point is earned: For the correct answer

$vC = 0.31$ m/s



0	1	2	3

The student response earns all of the following points:

1 point is earned: For any statement of the equation for speed

$vC = d/t$

1 point is earned: For substituting the time from part (b) or correctly recalculating it

$vC = 0.15$ m/0.49 s

1 point is earned: For the correct answer

$vC = 0.31$ m/s

Part D

1 point is earned: For an indication that the momentum in the *y*-direction is conserved

$\mathbf{p}Ay = -\mathbf{p}Cy$

1 point is earned: For an indication that the *y*-component of ball *C*'s momentum was calculated

$pAy = mCvC \sin 30°$

$pAy = (0.10 \text{ kg})(0.31 \text{ m/s})\sin 30°$

1 point is earned: For the correct answer

$pAy = 0.015 \text{ kg} \cdot \text{m/s}$

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0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For an indication that the momentum in the *y*-direction is conserved

$\mathbf{p}Ay = -\mathbf{p}Cy$

1 point is earned: For an indication that the *y*-component of ball *C*'s momentum was calculated

$pAy = mCvC \sin 30°$

$pAy = (0.10 \text{ kg})(0.31 \text{ m/s})\sin 30°$

1 point is earned: For the correct answer

$pAy = 0.015 \text{ kg} \cdot \text{m/s}$

chapter5

14. This question is a long free-response question. Show your work for each part of the question.

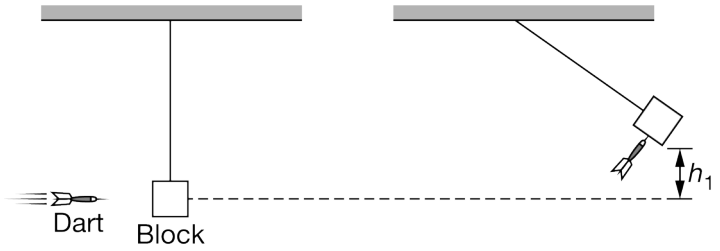


Figure I
Before Collision

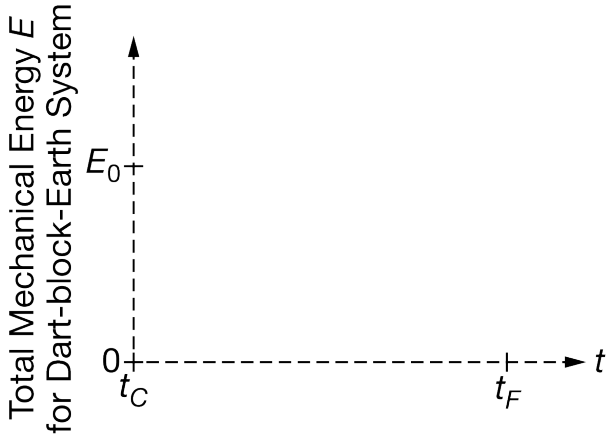
Figure II
Center of Mass at Height h_1

(12 points, suggested time 25 minutes)

A block with mass M_1 hangs at rest from a string that is attached to a ceiling. A dart with mass m_D is moving horizontally with speed v_0 immediately before colliding with the block, as shown in Figure I. The dart remains embedded in the block. The center of mass of the dart-block system swings up to a maximum height h_1 above its initial position before stopping momentarily, as shown in Figure II.

(a) The dart collides with the block at time t_C, and the center of mass of the dart-block system reaches its maximum height at time t_F.

i. Consider the dart-block-Earth system. E is the total mechanical energy of the dart-block-Earth system, and immediately before time t_C it is equal to E_0. On the axes below, sketch E as a function of time t for $t = t_C$ to $t = t_F$.



ii. Now consider the system containing just the block and the dart. The magnitude of the momentum of the dart-block system is p, and immediately before time t_C it is equal to p_0. On the axes below, sketch p as a function of time t for $t = t_C$ to $t = t_F$.

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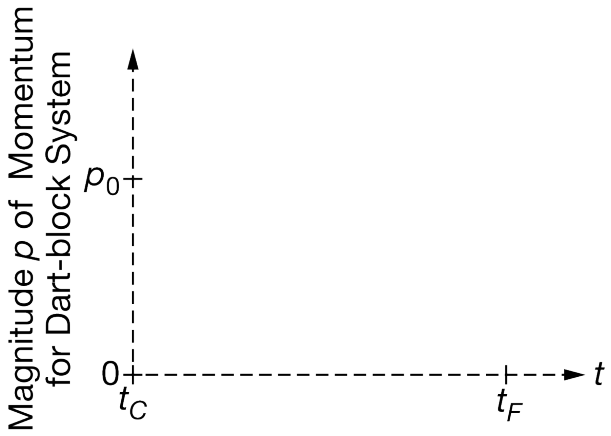

(b) The block is replaced with a different block of mass M_2, where M_2 is greater than M_1. The experiment is repeated with the same dart moving horizontally at the same speed v_0, and the center of mass of this dart-block system reaches a maximum height h_2 above its initial position. How does h_2 compare to h_1?

____ $h_2 < h_1$ ____ $h_2 = h_1$ ____ $h_2 > h_1$

Without including equations in your response, explain your reasoning.

(c) Consider the general case of a block of mass M_B hanging at rest and a dart of mass m_D moving horizontally with speed v_0. The dart hits and embeds in the block, and the center of mass of the dart-block system swings up to a maximum height h above its initial position. Derive an equation for h in terms of m_D, M_B, v_0, and physical constants, as appropriate.

(d) Consider the scenario in part (b) above, where the block has mass $M_2 > M_1$. Does your general equation for the maximum height from part (c) agree with your reasoning in part (b)?

____ Yes ____ No

Explain how your general equation from part (c) either supports or does not support your reasoning in part (b).

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a mechanical energy less than E_0 immediately after $t = t_C$
- ☐ 1 point is earned for a horizontal line showing constant mechanical energy from $t = t_C$ to $t = t_F$

Example Response for 2nd point:

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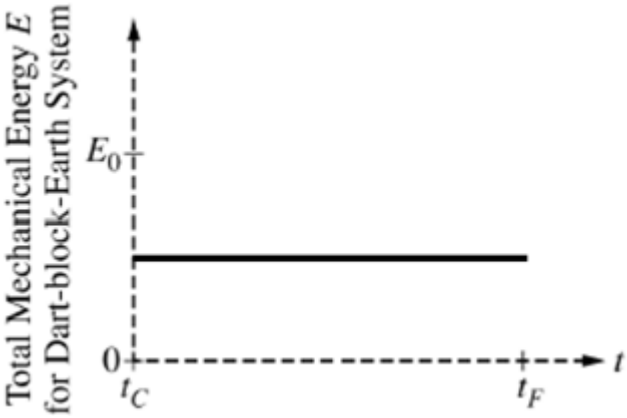
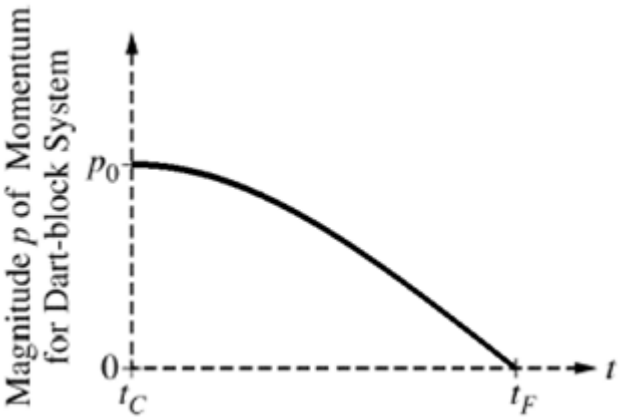


- [ ] 1 point is earned for a momentum equal to p_0 immediately after $t = t_C$
- [ ] 1 point is earned for a momentum that continually decreases and is zero at $t = t_F$

Example Response for 4th point:



Scoring Note: Graph does not have to be curved to earn this point.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

Scoring Note: Correct answer is $h_2 < h_1$; response is read even if incorrect selection is made.

- [ ] 1 point is earned for indicating that the post-collision speed of the system will be smaller than the original post-collision speed because the mass of the system is larger.
- [ ] 1 point is earned for indicating that the final height depends on the post-collision speed.

  Scoring Note: The 2nd point is earned for just indicating how the speed changes, along with a consistent selection for the height.

Part C

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for using conservation of momentum to determine the velocity of the block-arrow system after the collision.
- ☐ 1 point is earned for determining the velocity of the block-arrow system: $m_D v_0 = (m_D + M_B) v_f$ $v_f = \frac{m_D v_0}{(m_D + M_B)}$
- ☐ 1 point is earned for using a statement or expression for conservation of energy to determine the maximum height.
- ☐ 1 point is earned for substituting the velocity into a correct expression of energy conservation: $\frac{1}{2}(m_D + M_B)v_f^2 = (m_D + M_B)gh$

  $h = \frac{v_f^2}{2g} = \frac{1}{2g}\left(\frac{m_D v_0}{m_D + M_B}\right)^2$

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

Scoring Note: Correct answer is “Yes” (if parts (b) and (c) were correct); response is read even if “No” is selected.

- ☐ 1 point is earned for addressing the functional dependence between maximum height and block mass.
  Scoring Note: it is not necessary to use the functional dependence correctly to earn this point.
- ☐ 1 point is earned for correctly relating the functional dependence to the qualitative explanation in part (b).

chapter5

15. This question is a short free-response question. Show your work for each part of the question.

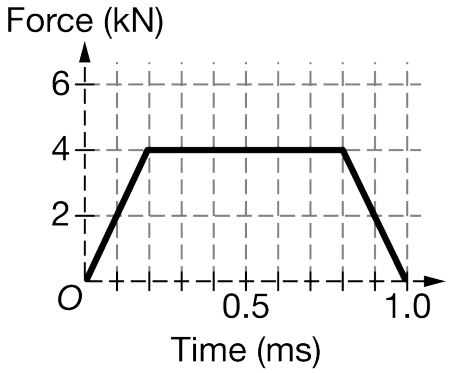
(7 points, suggested time 13 minutes)

A student in a physics lab drops a ball of mass $0.50\ \text{kg}$ from a height of $1.5\ \text{m}$ onto a force plate. The ball hits the force plate and rebounds upward. The student records the maximum height of the ball after it rebounds to be $1.1\ \text{m}$.

(a) The student uses their observations to correctly claim that the collision between the ball and the force plate is inelastic. Use the student's observations to justify their claim.

(b) Calculate the impulse exerted on the ball by the force plate.

The force plate records the following data for the magnitude of the force as a function of time for the $1\ \text{ms}$ the ball was in contact with the force sensor.



(c) Does the force plate data provide a reliable measurement of the impulse exerted on the ball by the force sensor? Justify your answer

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes the following criteria:

- ☐ For correctly relating the difference in maximum height of the ball to the kinetic energy of the ball before and after the collision.

Example Response:

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If the collision between the ball and the floor were elastic, the speed just before the ball hits would be equal to the speed of the ball just after it leaves the floor. Because of conservation of energy, since air resistance is negligible, if the speed before and after the collision were the same the ball should return to its starting height. Because it returns to a lower height, we must conclude that there was energy lost in the collision, making the collision inelastic.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Scoring note: Responses do not have to correctly calculate the change in momentum. A response must show that the momentum (or speeds) have different directions, and that the difference in these two are the algebraic sum of the two magnitudes.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

- ☐ For using conservation of energy or work energy principle to attempt to find the speed of the ball just before the ball hits the force sensor.
- ☐ For using conservation of energy or work energy principle to attempt to find the speed of the ball just after the ball hits the force sensor.
- ☐ For correctly addressing the vector nature of momentum (or velocity)

Example Response:

$KE = U_g$

$\frac{1}{2}mv_i^2 = mgh$

$v_i = \sqrt{2gh} = \sqrt{2\left(10\ \frac{\text{m}}{\text{s}^2}\right)(1.5\ \text{m})} = 5.5\ \frac{\text{m}}{\text{s}}$

$U_g = KE$

$mgh = \frac{1}{2}mv_f^2$

$v_f = \sqrt{2gh} = \sqrt{2\left(10\ \frac{\text{m}}{\text{s}^2}\right)(1.1\ \text{m})} = 4.7\ \frac{\text{m}}{\text{s}}$

$\Delta p = mv_f - mv_i$

$\Delta p = (0.50\ \text{kg})(4.7\ \frac{\text{m}}{\text{s}}) - (0.50\ \text{kg})(-5.5\ \frac{\text{m}}{\text{s}})$

$\Delta p = 5.1\ \text{Ns}$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: Both bars have the same angular momentum.

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0	1	2	3 ✓

The response accurately includes **all** of the following criteria:

- ☐ For attempting to calculate the area under the curve
- ☐ For comparing the area under the curve to the value of the change in momentum for part (b).
- ☐ For a correct justification that the impulse shown from the force sensor is not large enough to cause the ball to return to a height of 1.1 meters.

Example Response:

The area under the force as a function of time graph is equal to the impulse or change in momentum of the ball.

$\Delta p_{\text{observed}} = 5.1 \text{ Ns}$
$\Delta p_{\text{recorded}} = \frac{1}{2}(0.2 \text{ ms})(4 \text{ kN}) + (4 \text{ kN})(0.6 \text{ ms}) + \frac{1}{2}(0.2 \text{ ms})(4 \text{ kN})$
$\Delta p_{\text{recorded}} = 3.2 \text{ Ns}$

The change in momentum recorded by the force sensor is significantly smaller than the change in momentum determined from experimental data collected from the initial and final heights of the ball. It is likely that the force sensor reading cannot be trusted.

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16. This question is a long free-response question. Show your work for each part of the question.

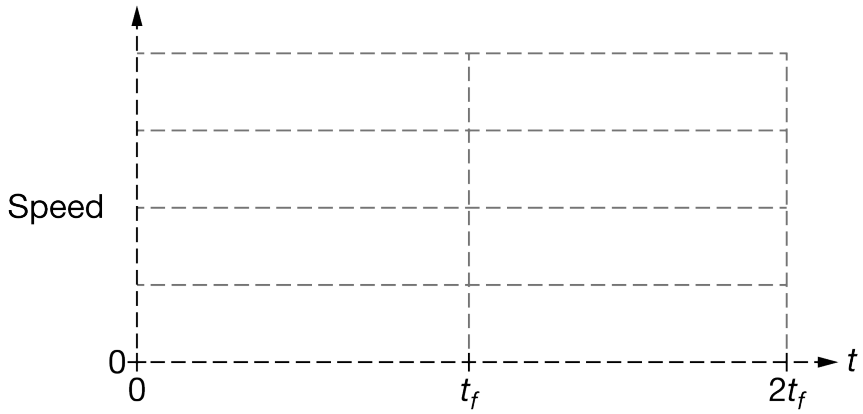
(12 points, suggested time 25 minutes)

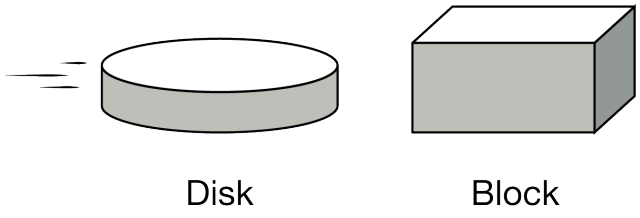
(a) A student of mass M_S, standing on a smooth surface, uses a stick to push a disk of mass M_D. The student exerts a constant horizontal force of magnitude F_H over the time interval from time $t = 0$ to $t = t_f$ while pushing the disk. Assume there is negligible friction between the disk and the surface.

i. Assuming the disk begins at rest, determine an expression for the final speed v_D of the disk relative to the surface. Express your answer in terms of F_H, t_f, M_S, M_D, and physical constants, as appropriate.

ii. Assume there is negligible friction between the student's shoes and the surface. After time t_f, the student slides with speed v_S. Derive an equation for the ratio v_D/v_S. Express your answer in terms of M_S, M_D, and physical constants, as appropriate.

(b) Assume that the student's mass is greater than that of the disk $(M_S > M_D)$. On the grid below, sketch graphs of the speeds of both the student and the disk as functions of time t between $t = 0$ and $t = 2t_f$. Assume that neither the disk nor the student collides with anything after $t = t_f$. On the vertical axis, label v_D and v_S. Label the graphs "S" and "D" for the student and the disk, respectively.





(c) The disk is now moving at a constant speed v_1 on the surface toward a block of mass M_B, which is at rest on the surface, as shown above. The disk and block collide head-on and stick together, and the center of mass of the disk-block system moves with speed v_{cm}.

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i. Suppose the mass of the disk is much greater than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

ii. Suppose the mass of the disk is much less than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

iii. Now suppose that neither object's mass is much greater than the other but that they are not necessarily equal. Derive an equation for v_{cm}. Express your answer in terms of v_1, M_D, M_B, and physical constants, as appropriate.

iv. Consider the scenario from part (c)(i), where the mass of the disk was much greater than the mass of the block. Does your equation for v_{cm} from part (c)(iii) agree with your reasoning from part (c)(i) ?

____ Yes ____ No

Explain your reasoning by addressing why, according to your equation, v_{cm} becomes (or approaches) a certain value when M_D is much greater than M_B.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for a correct answer $v_D = \frac{F_H t_f}{M_D}$ in part (a)(i).
- ☐ 1 point is earned for indicating the total momentum of the system is the same before and after the collision.
  Scoring Note: If the response only includes a correct final answer of $\frac{M_s}{M_D}$, the response earns this point but not the next point.
- ☐ 1 point is earned for correctly substituting the appropriate variables into a conservation of momentum equation
  AND

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an answer in the form $\frac{v_D}{v_s} = \ldots$.

Scoring Notes:
This point can be earned only if the first point is earned.
The answer need not be correct to earn this point.

Example Response for part (a)(ii):

$p_i = p_f$
$0 = M_s v_s - M_D v_D$
$\frac{v_D}{v_s} = \frac{M_s}{M_D}$

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for two functions that are straight segments for $t < t_f$,
  AND
  begin at the origin,
  AND
  have two different positive slopes.
- ☐ 1 point is earned for two functions that are horizontal functions for $t > t_f$
  AND
  are continuous over the entire time range $0 < t < 2t_f$.
- ☐ 1 point is earned for labeling values on the vertical axis with $v_D > v_S$.
  OR
  The curve labeled D is greater than the curve labeled S for all $t > 0$.
  Scoring note: This point can still be earned if the labels are not on the vertical axis but clearly indicate that $v_D > v_S$.

Example Response:

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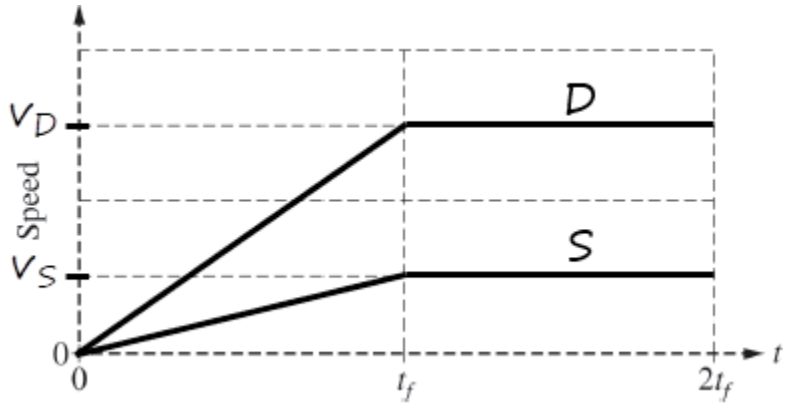

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for stating or mathematically representing that if the disk is much more massive, then the block will have little effect on the motion of disk 1
  OR
  For stating or mathematically representing that when $M_D >> M_B$, $v_{cm} = v_1$.
- ☐ 1 point is earned for correct reasoning in part (c)(ii).
  Correct answer: When $M_D << M_B$, $v_{cm} = 0$
- ☐ 1 point is earned for using conservation of momentum in part (c)(iii).
- ☐ 1 point is earned for a correct answer in part (c)(iii), $v_{cm} = \frac{m_D v_1}{m_D + m_B}$.
- ☐ 1 point is earned for an attempt to use limiting-case reasoning or functional dependence with the equation in part (c)(iii).
- ☐ 1 point is earned for recognizing the equation from (c)(iii) reduces to a simpler form and the simplified form is correctly compared to their answer in (c)(i)

Example response for part (c)(ii)

If the block is much more massive, then it will barely move when the disk collides and sticks to it.

Example response for part (c)(iv)

Yes. If M_B is very small, then the denominator of the equation simplifies to M_D, which then can cancel out of the equation leaving $v_{cm} = v_1$.

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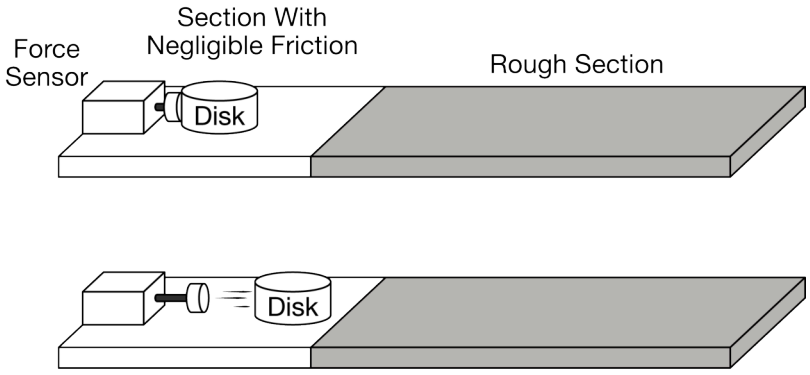
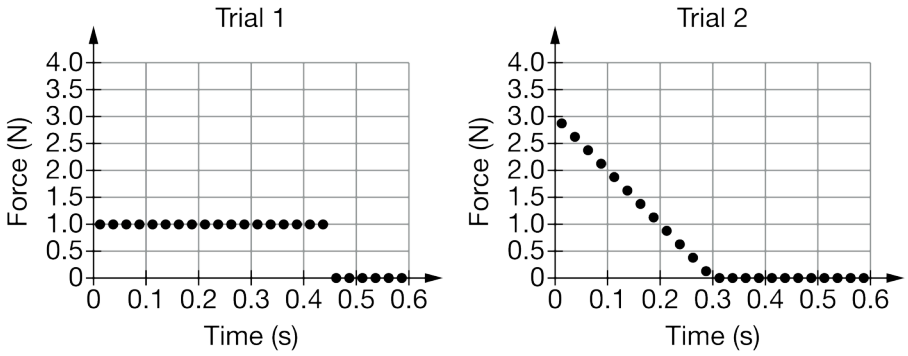
17. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Students use a force sensor to push a disk of mass m across a horizontal table. The force sensor pushes the disk over a short time interval. The magnitude of the force and the duration of the time interval can be set by the students. After being pushed by the sensor, the disk first slides along a surface with negligible friction and then enters a rough section where friction is not negligible, as shown above.

The students perform two trials in which the magnitude of the force exerted by the force sensor and the time interval are different. They create the graphs of force as a function of time shown below. The disk enters the rough section in each trial after $t = 0.5$ s.



(a) In which of the two trials, if either, is the change in momentum of the disk between 0 seconds and 0.5 seconds greater? Explain your reasoning.

(b) In both trials, the disk comes to rest in the rough section before reaching the edge of the table. In which trial, if either, does the disk travel a greater distance in the rough section? Explain your reasoning.

(c) The students repeat Trial 1 with a disk that is made from the same material as the original disk and has the same mass as the original disk but that has a larger radius (and therefore more surface area is in contact with the surface). The same force is exerted on the new disk over the same time interval as in Trial 1. Would the distance traveled in the rough section by the new disk be greater than, less than, or the same as the distance traveled by the original disk in Trial 1? Explain your reasoning.

Students derive the following equation, which may or may not be correct, to describe the distance D traveled by

the original disk on the rough section: $D = \frac{(F_{\text{avg}}{}^2)t^2}{2\mu mg}$, where F_{avg} is the average force exerted by the sensor, t is the time that the disk is in contact with the sensor, μ (mu) is the coefficient of kinetic friction, and m is the mass of the disk.

(d) Without algebraic manipulation of equations or attempting to derive this equation, indicate whether the above equation is consistent with your answer in part (b). Explain your reasoning.

A student measures the distance x_0 that the disk travels while in contact with the force sensor in Trial 1. The disk then travels a distance D along the rough section. The student claims that if a force with twice the magnitude of the original force was exerted on the disk for a distance x_0, the disk would travel a distance $2D$ in the rough section.

(e) Is the above claim correct or incorrect? Justify your answer without citing or manipulating equations.

The student finds the following equation on the Internet describing the relationship between the magnitude F of the force exerted on the disk, the distance x the disk is pushed, and the distance D it slides in the rough section: $D = C\frac{Fx}{\mu mg}$, where C is a constant with appropriate units.

(f) Indicate whether or not the equation above is consistent with your statement in part (e) relating D and x. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the impulse or $\mathrm{F(t)}$ or area under the curve is the impulse or change in momentum
  OR
  For recognizing that the acceleration can be determined from the force.
- ☐ 1 point is earned for correctly indicating the area is equal to the $\mathrm{F_{ave}(t)}$ or $\mathrm{F_t/2}$ or $0.45\ \mathrm{Ns}$
  OR
  For addressing that the mass value is the same or need not be known.
- ☐ 1 point is earned for indicating that both disks have the same change in momentum
  OR
  For correct application of kinematics including the use of the time.

Example Response:

The area under each curve is the change in momentum, and since both graphs have the same area under the curve, $0.45\ Ns$*, both have the same change in momentum.*

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OR

The change in momentum is the same for both pucks because the same average force was applied to the same mass for the same time frame causing the same launch velocity.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks.
- ☐ 1 point is earned for a correct relationship between the speed OR momentum OR kinetic energy and the distance traveled in the rough section consistent with their answer to part (a).

Example Response:

Both disks experience the same frictional force in both trials, so the acceleration will be the same for both disks, and since both disks start with the same initial speed, they will slide the same distance.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating both disks would have the same launch speed with the same force over the same time interval.
  OR
  For indicating that both disks will have the same kinetic energy OR momentum when a force is applied over the same time.
- ☐ 1 point is earned for indicating that the surface area of the disk does not affect the frictional force.
- ☐ 1 point is earned for indicating that the new block will travel the same distance before stopping.

Example Response:

Both disks will have the same launch speed because they experience the same force and have the same mass. The surface area of the disks does not affect the frictional force, so the new block will travel the same distance before stopping.

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Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an answer consistent with part (b), with an attempt to use the functional dependence of D on F_{avg} and/or t.
- ☐ 1 point is earned for correct reasoning (i.e., that $F_{\text{avg}}t$ is the same in both trials, so D is the same).

Example Response:

The equation is consistent with my reasoning in part (b). In the equation D is proportional to the square of the product of F_{avg} times t, which is the same for both trials, so D is the same.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: “Correct”.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a disk that is pushed with twice the force will have twice the original energy.
- ☐ 1 point is earned for a correct application of conservation of energy or work.
- ☐ 1 point is earned for indicating that since the work to stop the disk is also proportional to the distance to stop it, the disk will travel twice the distance before stopping.

Example Response:

A disk that is pushed from rest with twice the force for the same distance will have twice the work done one it, and so will have twice the original energy. The work done to stop the disk is equal to the energy that the disk has after being pushed. Work is also proportional to the distance to stop it; the new disk will travel twice the distance before stopping.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the equation is consistent with their statement in part (e) AND an attempt at a justification using functional dependence.
- ☐ 1 point is earned for correctly indicating D is proportional to F.

Example Response:

The given equation is consistent with my reasoning in part (e). Because D in the equation is directly proportional to F, pushing with twice the force will result in a distance D that is twice as large.

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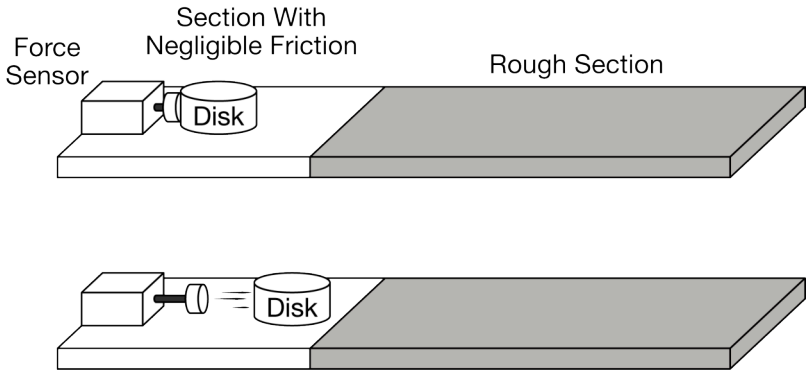
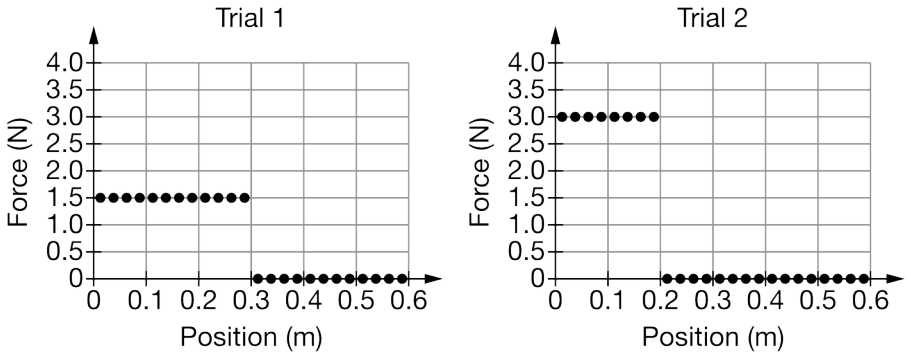
18. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Students use a force sensor to push a disk of mass m across a horizontal table. The force sensor pushes the disk over a short distance. The magnitude of the force and the distance the disk is pushed can be set by the students. After being pushed by the sensor, the disk first slides along a surface with negligible friction and then enters a rough section where friction is not negligible, as shown above.

The students perform two trials in which the magnitude of the force exerted by the force sensor and the distance the disk is pushed are different. They create the graphs of force as a function of position shown below. The disk enters the rough surface in each trial after moving 0.6 meters.



(a) In which of the two trials, if either, is the change in energy of the disk the greatest during the interval shown in the graphs? Explain your reasoning.

(b) In both trials, the disk comes to rest in the rough section before reaching the edge of the table. In which trial, if either, does the disk travel a shorter distance in the rough section? Explain your reasoning.

(c) The students repeat Trial 2 with a disk that is made from the same material as the original disk but that is less massive. The same constant force is exerted on the less massive disk over the same distance as in Trial 2. Would the distance traveled in the rough section by the less massive disk be greater than, less than, or the same as the distance traveled in the rough section by the original disk in Trial 2? Explain your reasoning.

Students derive the following equation, which may or may not be correct, to describe the distance D traveled by

the original disk in Trial 2 on the rough section: $D = \frac{Fx}{\mu mg}$, where F is the force exerted by the sensor, x is the distance over which the sensor exerts a force on the disk, μ (mu) is the coefficient of kinetic friction, and m is the mass of the disk.

(d) Without algebraic manipulation of equations or attempting to derive this equation, indicate whether the equation is consistent with your answer in part (b). Explain your reasoning.

A student measures the time t that the disk is in contact with the force sensor in Trial 2. The disk then travels a distance D along the rough section. The student claims that if the same force is exerted on the disk for a time $2t$, the disk would travel a distance $2D$ in the rough section.

(e) Is the above claim correct or incorrect? Justify your answer without citing or manipulating equations.

The student finds the following equation on the Internet describing the relationship between the time t the disk is pushed and the distance D it slides in the rough section: $D = C\frac{F^2t^2}{2\mu m^2 g}$, where C is a constant with appropriate units.

(f) Indicate whether or not the above equation is consistent with your statement in part (e) relating D and t. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that trial 2 has the graph with the greatest area under the curve
  OR
  For recognizing that the acceleration can be determined from the force.
- ☐ 1 point is earned for indicating the area represents the work done
  OR
  For addressing that the mass value is the same OR need not be known.
- ☐ 1 point is earned for relating the greatest work done to the greatest change in energy
  OR
  For correct application of kinematics including the use of the distance traveled.

Example Response:

Trial 2. The work done on the disk is represented by the area under the force vs position graph, so the graph with the largest area under the curve has the greatest work done on it. Graph 2 has the greatest area, and so the disk in trial 2 had the greatest amount of work done on it.

Part B

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks.
- ☐ 1 point is earned for indicating that the disk with the smaller launch speed will travel a shorter distance in the rough section.

Example Response:

Trial 1. The disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks. This means that the disk with the smaller speed after the push will travel a smaller distance.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a less massive disk would have a greater launch speed with the same force over the same distance.
- ☐ 1 point is earned for indicating that the distance traveled in the rough section, for a given speed, does not depend on the mass of the disk.
- ☐ 1 point is earned for indicating that the less massive disk will travel farther before stopping.

Example Response:

Farther than the original disk in trial 2. When pushed by the same force over the same distance, a less massive disk would have a larger launch speed. Since the mass of the disk doesn't affect the distance traveled through the rough section, the less massive disk will travel farther before stopping.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an answer consistent with part (b), with an attempt to use the functional dependence of D on F and/or x.
- ☐ 1 point is earned for correct reasoning (i.e., that Fx is greater in Trial 2, so D is greater).

Example Response:

This equation matches my reasoning in part (b). Since D is inversely proportional mass, if the mass increases, the distance D decreases, since m is in the numerator.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct answer: “Incorrect”.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a disk that is pushed for twice the time will have twice the speed.
- ☐ 1 point is earned for a correct application of conservation of energy or work for the disk in the rough section.
- ☐ 1 point is earned for indicating that the stopping distance is proportional to kinetic energy, not speed.

Example Response:

A disk that is pushed from rest with the same force for twice the time will have twice the momentum. Since the mass of the disk doesn’t change, the second disk will have twice the speed. However, the stopping distance is proportional to the kinetic energy which is proportional to the velocity squared.

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the equation is consistent (or inconsistent) with the statement in part (e) AND an attempt at a justification using functional dependence.
- ☐ 1 point is earned for indicating that the distance is proportional to *t* squared.

Example Response:

It is consistent. If t is doubled, the distance traveled should quadruple, because D is proportional to t squared.

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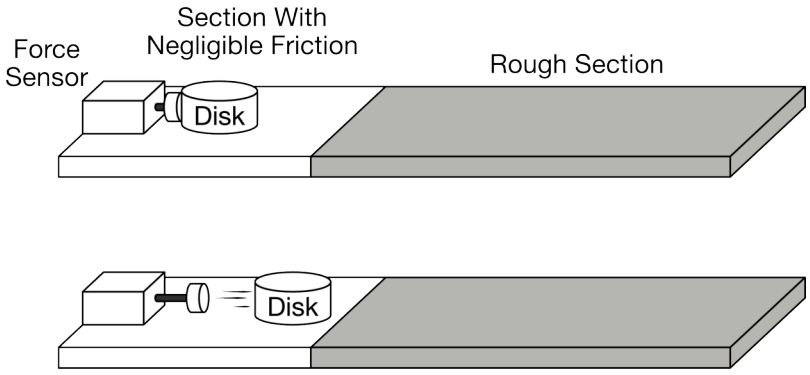
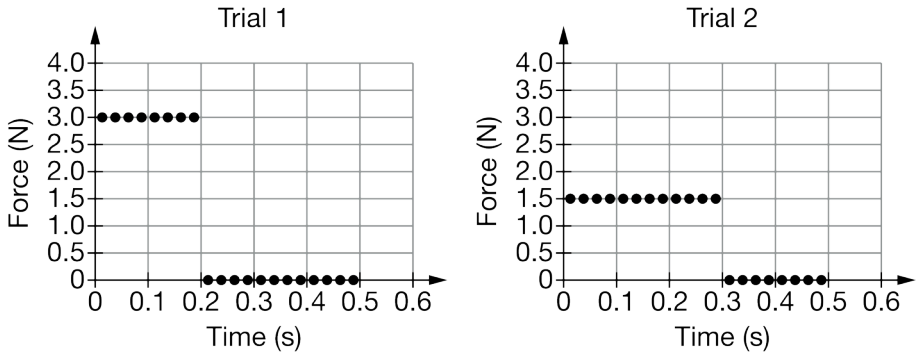
19. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Students use a force sensor to push a disk of mass m across a horizontal table. The force sensor pushes the disk with a force of constant magnitude over a short time interval. The magnitude of the force and the duration of the time interval can be set by the students. After being pushed by the sensor, the disk first slides along a surface with negligible friction and then enters a rough section where friction is not negligible, as shown above.

The students perform two trials in which the magnitude of the force exerted by the force sensor and the time interval are different. They create the graphs of force as a function of time shown below. The disk enters the rough surface in each trial after $t = 0.5$ s.



(a) In which of the two trials, if either, is the speed of the disk the greatest at time 0.5 seconds? Explain your reasoning.

(b) In both trials, the disk comes to rest in the rough section before reaching the edge of the table. In which trial, if either, does the disk travel a greater distance in the rough section? Explain your reasoning.

(c) The students repeat Trial 1 with a disk that is made from the same material as the original disk but that is more massive. The same force is exerted on the more massive disk over the same time interval as in Trial 1. Would the distance traveled in the rough section by the more massive disk be greater than, less than, or the same as the distance traveled by the original disk in Trial 1? Explain your reasoning.

Students derive the following equation, which may or may not be correct, to describe the distance D traveled by

the original disk on the rough section: $D = \frac{F^2t^2}{2\mu mg}$, where F is the force exerted by the sensor, t is the time that the disk is in contact with the sensor, μ (mu) is the coefficient of kinetic friction, and m is the mass of the disk.

(d) Without algebraic manipulation of equations or attempting to derive this equation, indicate whether the above equation is consistent with your answer in part (b). Explain your reasoning.

A student measures the distance x_0 that the disk travels while in contact with the force sensor in Trial 1. The disk then travels a distance D along the rough section. The student claims that if the same force is exerted on the disk for a distance $2x_0$, the disk would travel a distance $2D$ in the rough section.

(e) Is the above claim correct or incorrect? Justify your answer without citing or manipulating equations.

The student finds the following equation on the Internet describing the relationship between the distance x the disk is pushed and the distance D it slides in the rough section: $D = C\frac{Fx^2}{\mu mg}$, where C is a constant with appropriate units.

(f) Indicate whether or not the equation above is consistent with your statement in part (e) relating D and x. Explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that trial 1 has the graph with the greatest area under the curve OR impulse OR $F(t)$ OR addressing the numerical values of 0.45 Ns vs 0.6 Ns
  OR
  For recognizing that the acceleration can be determined from the force.
- ☐ 1 point is earned for indicating the area represents the change in momentum
  OR
  For addressing that the mass value is the same OR need not be known
- ☐ 1 point is earned for relating the greatest change in momentum to the greatest change in speed
  OR
  For correct application of kinematics including the use of the time.

Example Response:

Trial 1 has the biggest area under the curve, and since the area under a force vs time graph is impulse or change in momentum, the trial with the biggest area will have the biggest change in momentum. Both disks start at rest so the disk with the biggest change in momentum has the biggest change in speed.

OR

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Because the mass of the puck in both trials is the same, the trial with the largest force has the largest acceleration. Since both pucks start at rest, the puck with the largest acceleration will have the largest speed when pushed for the same amount of time.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the disk experiences the same frictional force in both trials, so the acceleration will be the same for both disks.
- ☐ 1 point is earned for a correct relationship between the speed OR momentum OR kinetic energy and the distance traveled in the rough section consistent with their answer to part (a).

Example Response:

In both trials the disk experiences the same frictional force, so the acceleration is the same in both trials. The puck with the greatest launch speed travels the furthest before coming to rest because it will take a longer time to stop with the same acceleration.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a more massive disk would have a smaller launch speed OR momentum with the same force over the same time interval
  OR
  For indicating that a more massive disk will have less kinetic energy when a force is applied over the same time.
- ☐ 1 point is earned for indicating that the mass of the disk does not affect the distance traveled in the rough section.
- ☐ 1 point is earned for indicating that the more massive block will travel less far before stopping.

Example Response:

The more massive disk will have a smaller launch speed after being pushed with the same force for the same time. The pucks will have the same acceleration in the rough section of the track, regardless of the mass, so the more massive block

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will not travel as far before stopping.

OR

The disk would have less kinetic energy over the same time frame, because the force would be applied over a smaller distance. Since the masses are equal, the F_f on the disks would be the same in the rough patch, for a disc with less kinetic energy entering the rough region, and experiencing the same F_f, the disk would not go as far.

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for an answer consistent with part (b), with an attempt to use the functional dependence of *D* on *F* and/or *t*.
- ☐ 1 point is earned for correct reasoning (i.e., that F_t is greater in trial 1, so *D* is greater).

Example Response:

The equation is consistent with my answer to part (b). D increases with the product F_t, so if this product increases, the distance D will also increase.

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating a disk that is pushed for twice the original distance will have twice the original energy.
- ☐ 1 point is earned for a correct application of conservation of energy or work.
- ☐ 1 point is earned for indicating that since the work to stop the disk is also proportional to the distance to stop it, the disk will travel twice the distance before stopping.

Example Response:

A disk that is pushed from rest for twice the original distance will have twice the work done on it and so will have twice the energy. The work done to stop the disk is equal to the energy that the disk has after being pushed. Work is also proportional to the distance to stop it; the disk will travel twice the distance before stopping.

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Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the equation is not consistent (or consistent) with their statement in part (e) AND an attempt at justification using functional dependence.
- ☐ 1 point is earned for correctly indicating that D is proportional to x.

Example Response:

The given equation is not consistent with my statement in (e). In the equation D is directly proportional to x squared. But in part (e) I stated that if the distance x doubles, D will also double.

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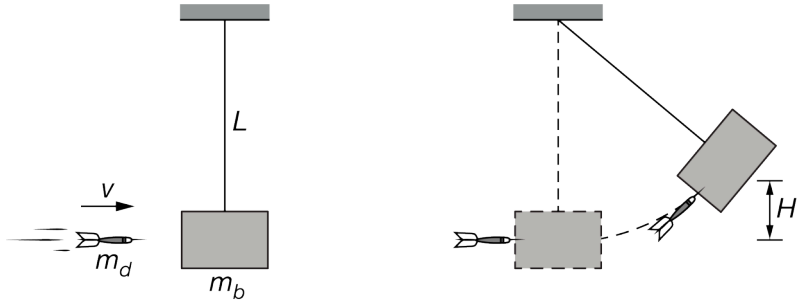
20. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown at left above. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown at right above. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, if either, experiences a net horizontal force of larger magnitude during the collision. If the net horizontal force is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the experiment was repeated with a dart with the same horizontal speed but with mass smaller than m_d, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the motion of the dart-block system at the instant it first reaches a height of $H/2$, and they make the following statements:

Student 1: When the dart-block system is at a height of $H/2$, the kinetic energy of the system is half of its maximum value, and therefore the speed of the system is half of its maximum speed.

Student 2: When the dart-block system reaches a height of $H/2$, the centripetal force is equal to the tension in the string. Therefore, the acceleration of the system must point toward the fixed end of the string.

(c) Which underlined phrase or phrases, if either, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if either, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if either, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases, if either, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

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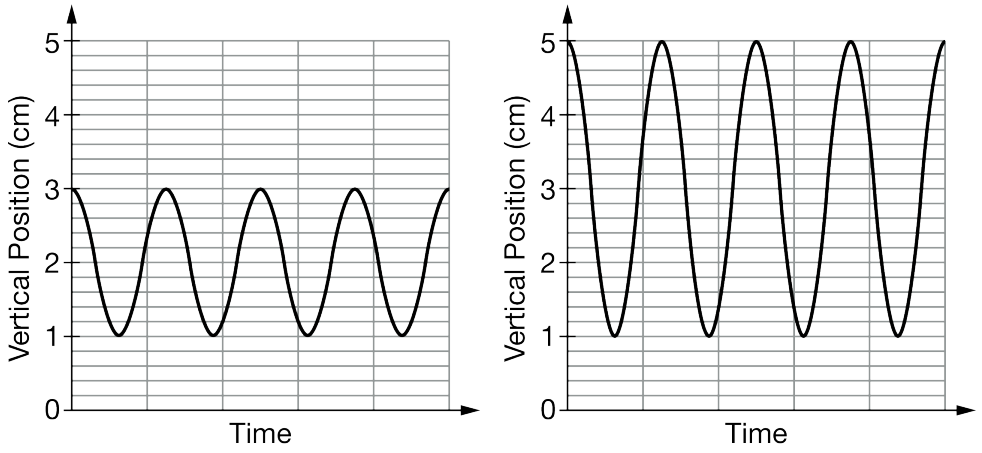
(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{v}{1+\frac{m_b}{m_d}}\right)^2$. Explain how this equation supports your claim in part (b).

(h) Could the masses of the dart and block both be changed so that the maximum height *H* reached by the dart-block system would remain the same as in the original setup? Justify your answer by referring to the equation in part (g).

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart mass or the string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a statement that both objects experience the same net force because of Newton's third law.

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Example Response:

Both the dart and the block experience the same force because they exert equal and opposite forces on each other because of Newton's third law.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the final height is smaller.
- ☐ 1 point is earned for a statement of conservation of momentum to indicate that the final speed of the dart-block system is smaller **AND** an indication of conservation of energy leading to a smaller height.

Example Response:

Because the momentum of the dart before the collision must be equal to the momentum of the dart-block system after the collision, a smaller dart mass (a smaller dart momentum) will lead to a smaller dart-block speed after the collision (a smaller dart-block momentum). Because the dart-block is traveling slower after the collision, it has less kinetic energy which means that the dart-block will not swing as high.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a correct statement and no incorrect statements.

Example Responses:

Student 1 is correct in saying that the kinetic energy of the system is half of its maximum at $H/2$.

OR

The first statement

Part D

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for an incorrect statement and no correct statements.

Example Responses:

Student 1 is incorrect in saying that the speed of the system is half of its maximum speed

OR

The second statement

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for indicating student 2 has no correct statements.

Example Response:

None

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- [ ] 1 point is earned for indicating both of Student 2's statements are incorrect.

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Example Responses:

Student 2 is incorrect in saying that centripetal force is equal to the tension in the string, AND incorrect in saying that the acceleration of the system must point toward the center of the circle.

OR

Both statements

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement including functional dependence of the variables.
- ☐ 1 point is earned for correctly asserting that as the mass of the dart decreases in the equation *H* also decreases.

Example Response:

This equation matches my reasoning in part (b)– if the mass of the dart decreases, the height of the swing decreases because a decrease in the mass of the dart means an increase in the ratio $(block\ mass)/(dart\ mass)$*, which is in the denominator of the expression.*

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for “yes”, with an attempt at explanation.
- ☐ 1 point is earned for a statement that both either increase or decrease.
- ☐ 1 point is earned for a statement indicating the factor in increase/decrease (or ratio of masses) is the same.

Example Response:

Yes, if the masses were both doubled, the maximum height would still be H – it works as long as the masses both increase by the same factor .

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Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for recognizing that the amplitude of the graph for group 2 is larger.
- ☐ 1 point is earned for a statement discussing the dependence of amplitude on dart mass.
- ☐ 1 point is earned for indicating that the mass of the dart could be larger.

Example Response:

If group 2 used a dart with a larger mass, the dart would have had more momentum, and so the dart-block system would have gone higher (making the amplitude larger), but the string length is the same and so the period would still be the same.

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21. This question is a long free-response question. Show your work for each part of the question.

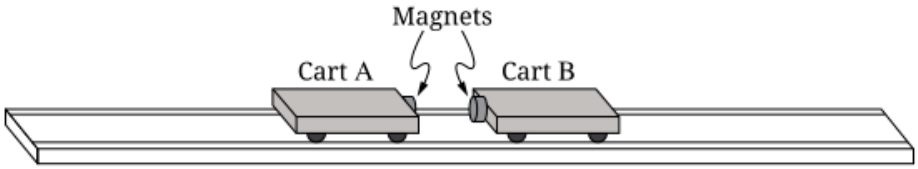

Figure 1

(12 points, suggested time 25 minutes)

Students are investigating collisions between Cart A and Cart B on a horizontal track, where frictional forces are negligible. A magnet is attached to each cart, as shown in Figure 1. The students give each cart a quick push so that the carts roll toward each other. As the carts approach each other, the magnetic interaction causes a slow collision in which the carts repel each other, momentarily come to a stop without touching, then reverse direction.

(a) The students wonder whether the collisions between the carts are elastic.

i. Design an experimental procedure that the students could use to determine whether the collisions are elastic.

In the first column of the table, list the quantities that would be measured in the experiment. In the next two columns of the table, define a symbol to represent each quantity and list the equipment that would be used to measure each quantity. Use as many rows as you need. Add rows to the space just below the table, if necessary.

In the space below the table, describe the overall procedure. Provide enough detail so that the students could replicate the experiment, including any steps necessary to reduce experimental uncertainty and increase confidence in the results. As needed, use the symbols defined in the table. You may include a simple diagram of the setup with the procedure.

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Quantity to Be Measured	Symbol for Quantity	Equipment for Measurement
Procedure (and diagram, if needed)		

ii. Describe the process of data analysis the students could use to decide whether the collisions between Cart A and Cart B are elastic.

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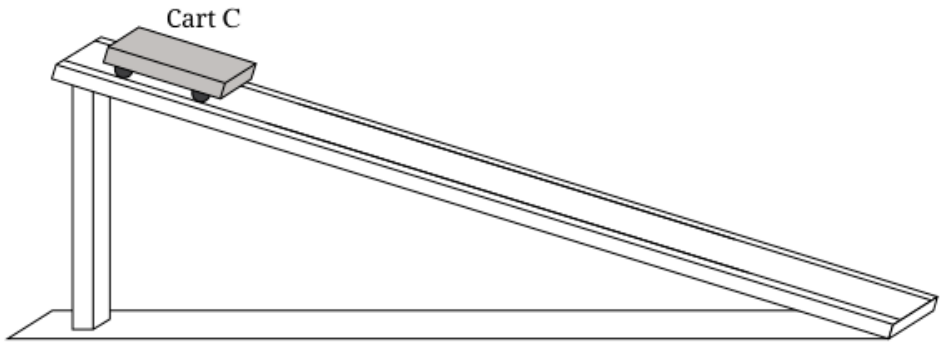

Figure 2

Another group of students perform an investigation where they roll a different cart, Cart C, from rest down a track that is angled above the horizontal, as shown in Figure 2. The students use their measurements to calculate the momentum of the cart at different times.

(b) The following table shows a sample of calculated momentum values of Cart C as it rolls down the track.

Time (s)	Momentum ($\text{kg} \cdot \text{m/s}$)
0.1	0.12
0.2	0.16
0.3	0.28
0.4	0.33
0.5	0.39

i. On the following grid, plot the points from the data table to create a graph that can be used to determine the net force exerted on Cart C as it rolls down the ramp. Clearly scale and label all axes (including units), as appropriate. Draw a straight line that best represents the data.

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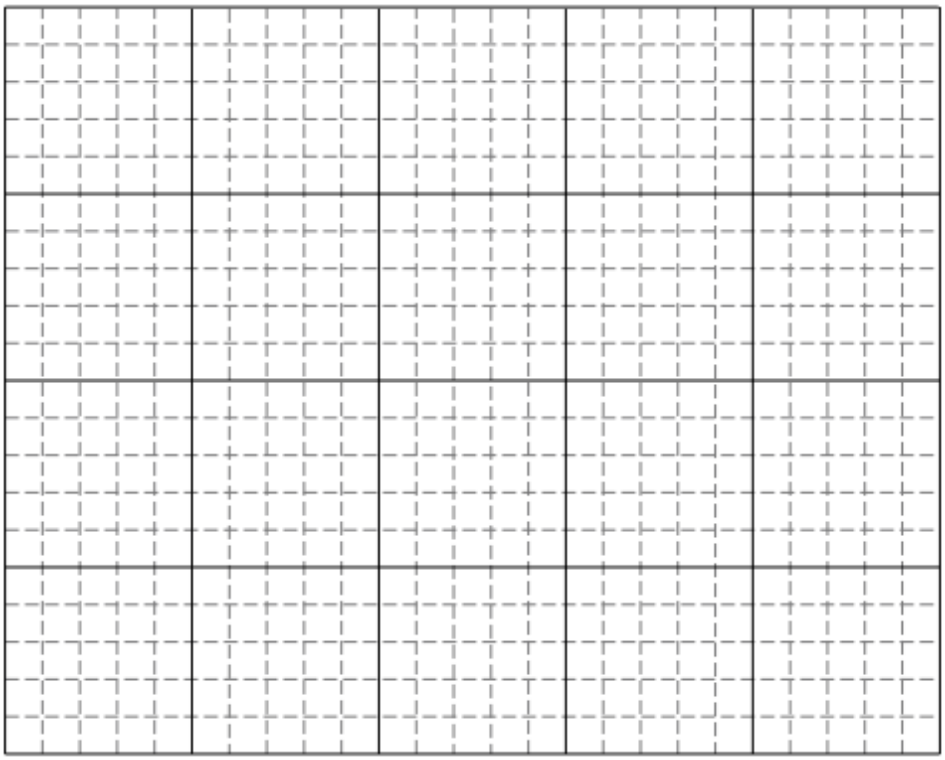

ii. What is the best way to use the graph to estimate the net external force exerted on Cart C as it rolls down the track?

iii. The students measure the mass of Cart C to be $m = 0.25\ \text{kg}$. Assuming resistive forces are negligible, use the graph to determine the angle of the ramp.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

Scoring Note: Part (a)(i) and (a)(ii) may be scored together.

- ☐ 1 point is earned for indicating that the mass of both carts should be measured using appropriate equipment such as a mass balance or scale.
- ☐ 1 point is earned for indicating measurements that could be used to determine the velocity of each cart and the tool that is used for that measurement.
  Accept **one** of the following:

- Speed of both carts before/after the collision with a motion sensor, motion detector, smart carts, or video analysis
- Time carts take to travel a known distance before/after collision with a stopwatch

- [ ] 1 point is earned for any indication that multiple collisions are analyzed.
- [ ] 1 point is earned for explaining how the measurements made can be used to determine the initial and final velocity of each cart (this could be described in (a)(i)).
- [ ] 1 point is earned for indicating that the total kinetic energy of the system would need to be calculated.
- [ ] 1 point is earned for indicating that if the total kinetic energy after the collision is meaningfully different than before, then the collision was not elastic.

Example Response

Quantities to Be Measured: mass of Cart A*, mass of Cart* B*, speed of Cart* A*, speed of Cart* B

Symbols for Quantities: m_{A}*,* m_{B}*,* v_{A}*,* v_{B}

Equipment for Measurements: 2 digital scales, 2 motion detectors

Procedure: Place each cart separately on a digital scale and record the mass of the individual carts. Attach one motion detector on either side of the track so that each motion detector can measure the velocities of one of the carts. Turn the motion detectors on so that they start collecting data and give each cart a quick push so that they roll toward each other. After the carts have approached each other and then reversed direction, turn the data collection off for the motion detectors. Record the initial speeds of the carts from when they move toward each other and the final speed from once they have changed direction. Repeat these measurements for several different pushes.

Find the initial kinetic energy of the two-cart system by calculating: $K_i = \frac{1}{2}m_{\mathrm{A}}{v_{\mathrm{A}i}}^2 + \frac{1}{2}m_{\mathrm{B}}{v_{\mathrm{B}i}}^2$ *where* $v_{\mathrm{A}i}$ *and* $v_{\mathrm{B}i}$ *are the speeds of carts A and B, respectively, recorded by the motion detectors at the same moment in time when the carts rolled toward each other.*

Find the final kinetic energy of the two-cart system by calculating: $K_f = \frac{1}{2}m_{\mathrm{A}}{v_{\mathrm{A}f}}^2 + \frac{1}{2}m_{\mathrm{B}}{v_{\mathrm{B}f}}^2$ *where* $v_{\mathrm{A}f}$ *and* $v_{\mathrm{B}f}$ *are the speeds of carts A and B, respectively, recorded by the motion detectors at the same moment in time when the carts rolled away from each other.*

Compare the initial and final kinetic energies. If they are nearly the same, then the collision can be classified as elastic. If the final kinetic energy of the two-cart system is less than the initial, then the collision was inelastic.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** the following criteria.

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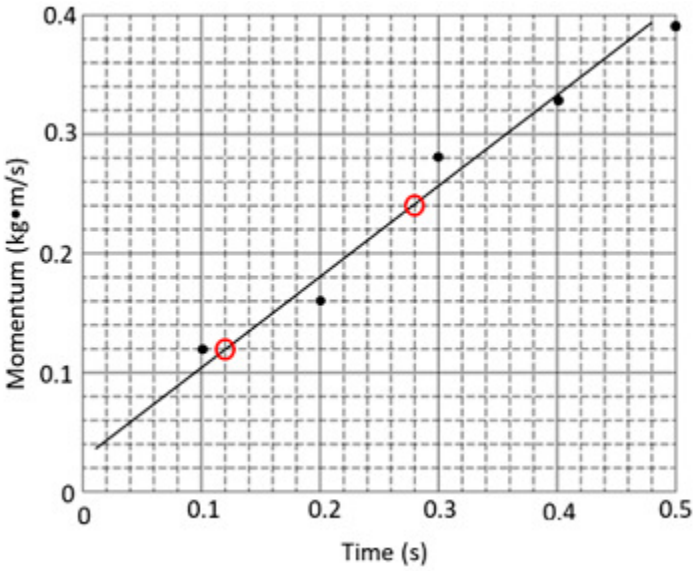
- 1 point is earned for scaling and identifying the axes (labels **AND** units) so that when graphed correctly, the data will span more than half of the horizontal and vertical axes.
- 1 point is earned for plotting at least four of the points correctly.
- 1 point is earned for a line that approximates the trend of the data.
- 1 point is earned for indicating **one** of the following:
  - Slope = net force (if momentum vs time is plotted)
  - Inverse of the slope = net force (if time vs momentum is plotted)
- 1 point is earned for calculating the slope of the best-fit line.
  Scoring Note: An indication that a calculator was used to determine the value of the slope may earn this point.
- 1 point is earned for a correct calculated answer within the range of $15^\circ - 21^\circ$.
  Scoring Note: A correct angle without supporting calculations does not earn this point.

Example Response



The external force exerted on Cart C *while it rolls down the track can be found by calculating the slope of the best-fit line.*

$\text{slope} = \text{net external force} = \Sigma F$

$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{0.24\,\text{kg·m/s} - 0.12\,\text{kg·m/s}}{0.28\,\text{s} - 0.12\,\text{s}} = 0.75\ \frac{\text{kg·m/s}}{\text{s}} = 0.75\ \text{N}$

$0.75\ \text{N} = \Sigma F = mg\sin\theta$

$\sin\theta = \frac{0.75\ \text{N}}{mg} = \frac{0.75\ \text{N}}{(0.25\,\text{kg})(9.8\,\text{m/s}^2)} = 0.31$

$\theta = \sin^{-1}(0.31)$

$\theta = 18^\circ$

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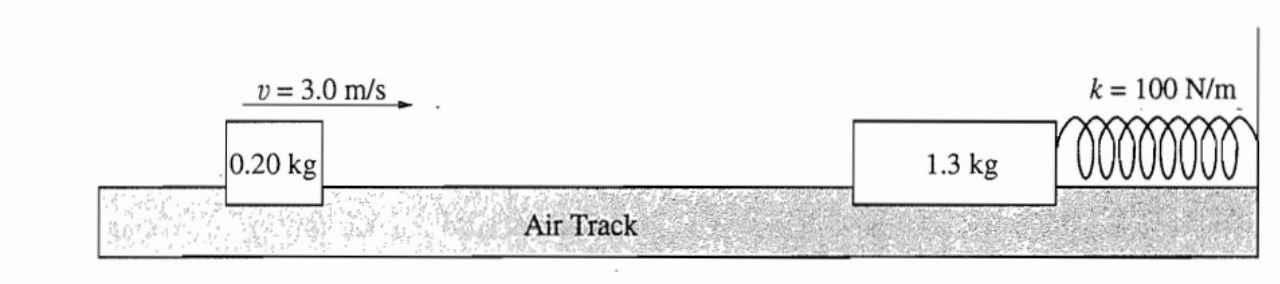
22.



As shown above, a 0.20-kilogram mass is sliding on a horizontal friction less air track with a speed of 3.0 meters per second when it instantaneously hits and sticks to a 1.3-kilogram mass initially at rest on the track. The 1.3-kilogram mass is connected to one end of a mass less spring, which has a spring constant of 100 newtons per meter. The other end of the spring is fixed.

(a) Determine the following for the 0.20-kilogram mass immediately before the impact.

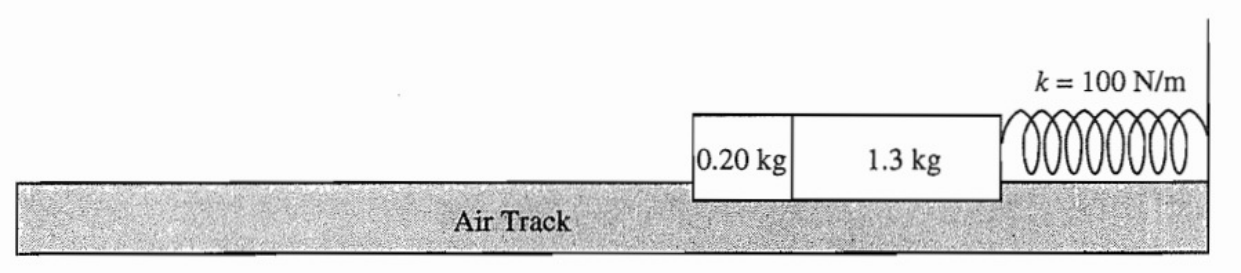
i. Its linear momentum

ii. Its kinetic energy

(b) Determine the following for the combined masses immediately after the impact

i. The linear momentum

ii. The kinetic energy



After the collision, the two masses undergo simple harmonic motion about their position at impact.

(c) Determine the amplitude of the harmonic motion.

(d) Determine the period of the harmonic motion.

Part A

(i)

1 point is earned: For a correct expression for momentum

p = mv

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pi=(0.2kg)(3.0 m/s)

1 point is earned: For the correct answer

pi = 0.6kg * m/s

(ii)

1 point is earned: For a correct expression for kinetic energy

K = ½ mv2

Ki = ½(0.2 kg)(3.0 m/s)2

1 point is earned: For the correct answer

Ki = 0.9J

0	1	2	3	4 ✓

The student response earns four of the following points:

(i)

1 point is earned: For a correct expression for momentum

p = mv

pi=(0.2kg)(3.0 m/s)

1 point is earned: For the correct answer

pi = 0.6kg * m/s

(ii)

1 point is earned: For a correct expression for kinetic energy

K = ½ mv2

Ki = ½(0.2 kg)(3.0 m/s)2

1 point is earned: For the correct answer

Ki = 0.9J

Part B

(i)

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1 point is earned : For any statement of conservation of momentum

p f = p i

1 point is earned : For the correct answer

p f = 0.6kg * m/s

(ii)

Knowing the final momentum, the final velocity can be calculated

p f = m f v f

v f = p f / m f

$= \frac{0.6kg \cdot m/s}{(1.3+0.2)kg}$

1 point is earned : For the correct value of the final velocity

v f = 0.4 m/s

Using the expression for kinetic energy

K f = ½m f v f 2

1 point is earned : For using the sum of the two masses

K f = ½(1.5kg)(0.4 m/s) 2

1 point is earned : For the correct answer

K f = 0.12J

0	1	2	3	4	5 ✓

The student response earns five of the following points:

(i)

1 point is earned: For any statement of conservation of momentum

pf = pi

1 point is earned: For the correct answer

pf = 0.6kg * m/s

(ii)

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Knowing the final momentum, the final velocity can be calculated

pf = mfvf

vf = pf / mf

$= \frac{0.6kg{\cdot}m/s}{(1.3+0.2)kg}$

1 point is earned: For the correct value of the final velocity

vf = 0.4 m/s

Using the expression for kinetic energy

Kf = ½mfvf2

1 point is earned: For using the sum of the two masses

Kf = ½(1.5kg)(0.4 m/s)2

1 point is earned: For the correct answer

Kf = 0.12J

Part C

1 point is earned : For any statement of conservation of energy

The kinetic energy after the impact is equal to the maximum energy stored in the spring.

K f = ½ kA 2

Solving for the amplitude of the motion:

$A = \sqrt{2K_f/k}$

1 point is earned : For substituting the answer to (b)ii for the kinetic energy

$A = \sqrt{2(0.12J)/(100N/m)}$

1 point is earned : For the correct answer

A = 0.05 m

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned : For any statement of conservation of energy

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The kinetic energy after the impact is equal to the maximum energy stored in the spring.

K f = ½ kA 2

Solving for the amplitude of the motion:

$A = \sqrt{2K_f/k}$

1 point is earned : For substituting the answer to (b)ii for the kinetic energy

$A = \sqrt{2(0.12J)/(100N/m)}$

1 point is earned : For the correct answer

A = 0.05 m

Part D

1 point is earned : For the correct expression for the period

$T = 2\pi\sqrt{m/k}$
$T = 2\pi\sqrt{(1.5kg)(100N/m)}$

1 point is earned : For the correct answer

T = 0.77 s

0	1	2 ✓

The student response earns two of the following points:

1 point is earned : For the correct expression for the period

$T = 2\pi\sqrt{m/k}$
$T = 2\pi\sqrt{(1.5kg)(100N/m)}$

1 point is earned : For the correct answer

T = 0.77 s

Units points

1 point is earned: For having both one correct energy unit and one correct momentum unit on answers in parts (a) and (b), and no incorrect units.

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0	1 ✓

The student response earns one of the following points:

1 point is earned: For having both one correct energy unit and one correct momentum unit on answers in parts (a) and (b), and no incorrect units.

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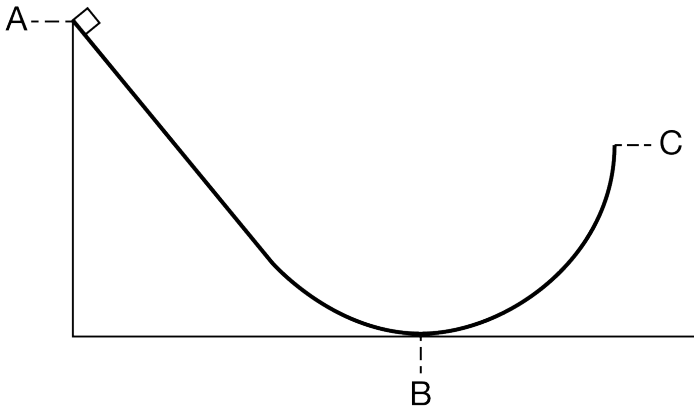
23. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)



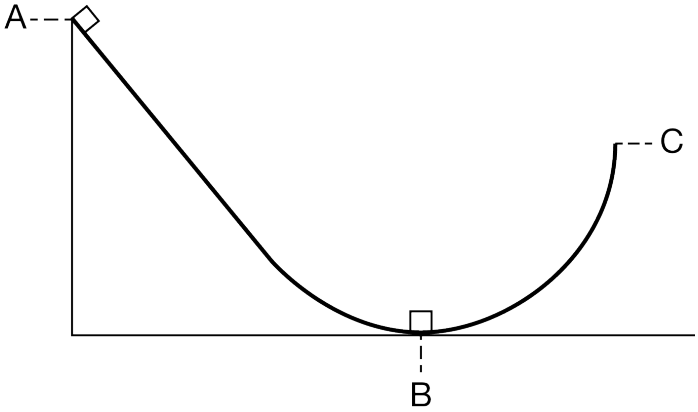
The figure above shows a track consisting of a straight section that connects to a circular section. The bottommost point of the track is at location B, and the track ends at location C, where the track is vertical. A block is released from rest at location A. There is negligible friction between the track and the box.

(a) Is the magnitude of the normal force exerted on the block by the track when the block is at location B greater than, less than, or equal to the normal force immediately before the block reaches location C? Justify your answer.

(b) In a coherent, paragraph-length response, describe the relative angle between the direction of the normal force exerted on the block by the track and the direction of the block's acceleration at the following times.

Immediately after the block is released at location A
When the block is at location B
Immediately before the block reaches location C

Specifically, indicate whether the normal force is parallel to the acceleration, perpendicular to the acceleration, or neither, at each location. Use physical principles to justify your responses.



(c) The scenario is repeated, but an identical block is at rest at location B, as shown above. When the block at

location A is released from rest, it slides down the track and collides with and sticks to the identical block. The two blocks then continue sliding up the ramp. Is the maximum height reached by the two blocks greater than, less than, or equal to the maximum height reached by the block in the original scenario? Briefly explain your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the normal force at B is greater than it is immediately before C.
- ☐ 1 point is earned for a correct explanation that correctly uses either or both of the following effects:
  · At B, the speed *v* is greater so the net force $F_{\text{net}} = mv^2/r$ is greater
  · At B, the normal force must "overcome" the downward weight force, so the normal force $F_n = F_{\text{net}} + mg$ is greater.

Example Response:

At B, the speed v is greater, so the net force mv^2/r is greater, resulting in a greater normal force than at just before location C.

OR

At B, the normal force must "overcome" the downward weight force, so the normal force $(= F_{\text{net}} + mg)$ is greater than at just before location C.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the normal force is always perpendicular to the surface of the track.
- ☐ 1 point is earned for relating the acceleration to the net force.
- ☐ 1 point is earned for indicating the acceleration at location A is along the ramp and thus perpendicular to the normal force.
- ☐ 1 point is earned for indicating that at location B the acceleration is parallel to the normal force.

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- [ ] 1 point is earned for indicating that at position just before location C the acceleration is neither perpendicular nor parallel to the normal force.
- [ ] 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response.

Example Response:

The normal force is always perpendicular to the track surface. At location A, the block is accelerating from rest in a straight line down the ramp, so the acceleration is along the ramp and thus perpendicular to the normal force.

At B, the acceleration is upward, so it is in the same direction as the normal force.

Just before C, the forces of gravity (downward) and the normal force (perpendicular to the track) by vector summation result in the net force, which is in the direction of the acceleration. –The acceleration therefore is at an angle to the normal force, that is neither parallel nor perpendicular.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- [ ] 1 point is earned for indicating the height of the two blocks is less than that of the original block.
- [ ] 1 point is earned for a correct explanation of why the height is less than the original block.

Example Response:

In an inelastic collision linear momentum is constant and kinetic energy is not constant. The two-block system has less energy than the first block had at the same position. All of the kinetic energy at that point can now be converted to gravitational potential energy. Which means that the two-block system will not go as high as the first block.

OR

By conservation of momentum, the speed of the two blocks after the collision is half the speed of the original block. Since the blocks are going slower, they will not go as high as the original block did.

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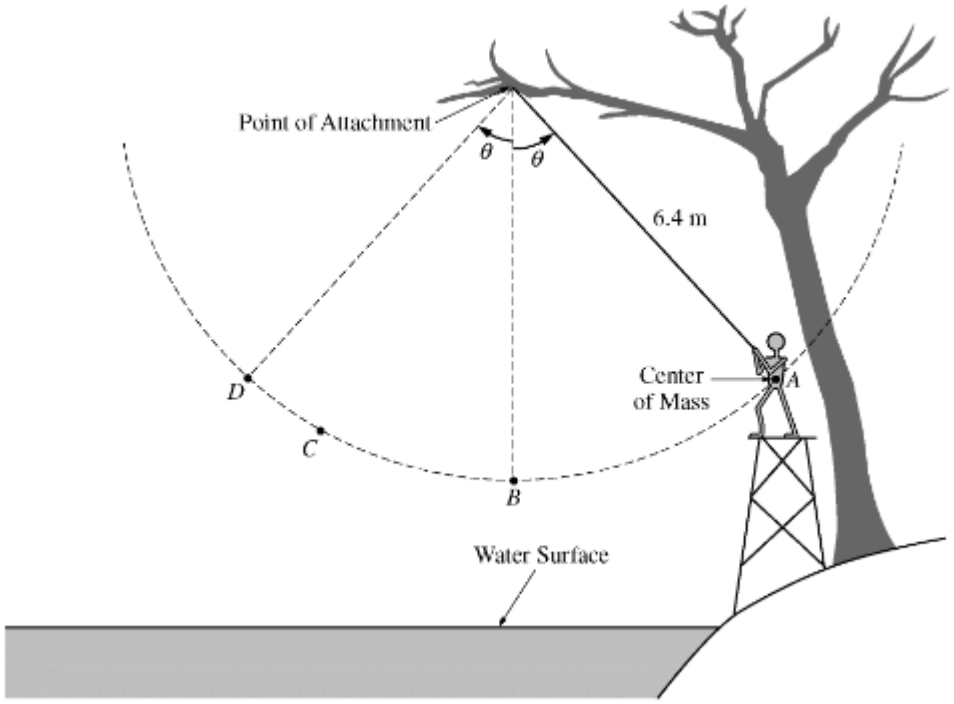
24.



Starting from rest at point A, a 50 kg person swings along a circular arc from a rope attached to a tree branch over a lake, as shown in the figure above. Point D is at the same height as point A. The distance from the point of attachment to the center of mass of the person is 6.4 m. Ignore air resistance and the mass and elasticity of the rope.

(a) The person swings two times, each time letting go of the rope at a different point.

i. On the first swing, the person lets go of the rope when first arriving at point C. Draw a solid line to represent the trajectory of the center of mass after the person releases the rope.

ii. A second time, the person lets go of the rope at point D. Draw a dashed line to represent the trajectory of the center of mass after the person releases the rope.

(b) The center of mass of the person standing on the platform is at point A, 4.1 m above the surface of the water. Calculate the gravitational potential energy when the person is at point A relative to when the person is at the surface of the water.

(c) The center of mass of the person at point B, the lowest point along the arc, is 2.4 m above the surface of the water. Calculate the person's speed at point B.

(d) Suppose that the person swings from the rope a third time, letting go of the rope at point B. Calculate R, the horizontal distance moved from where the person releases the rope at point B to where the person hits the water.

(e) If the person does not let go of the rope, how does the magnitude of the person's momentum p_C at point C compare with the magnitude of the person's momentum p_B at point B ?

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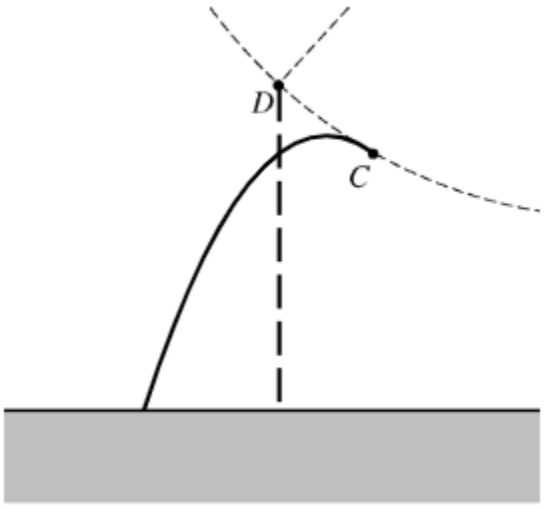
____ $p_C > p_B$ ____ $p_C < p_B$ ____ $p_C = p_B$

Provide a physical explanation to justify your answer.

Part A



1 point is earned: For showing a parabolic path from point *C* that is tangent to the arc at *C*

1 point is earned: For a maximum height of the parabolic path that is lower than point *D*

1 point is earned: For showing the person traveling straight downward from point *D*

Note: the two paths may, but need not, cross each other.

0	1	2	3 ✓

The student response earns all of the following points:

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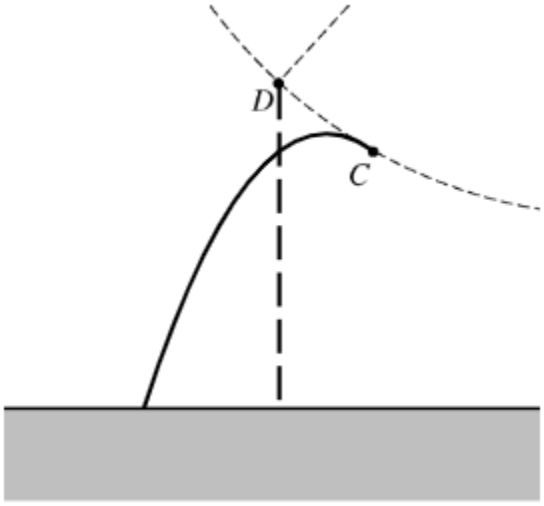

1 point is earned: For showing a parabolic path from point *C* that is tangent to the arc at *C*

1 point is earned: For a maximum height of the parabolic path that is lower than point *D*

1 point is earned: For showing the person traveling straight downward from point *D*

Note: the two paths may, but need not, cross each other

Part B

1 point is earned: For using the gravitational potential energy equation

$\Delta U_g = mgh = (50kg)\left(9.8m/s^2\right)(4.1m)$

1 point is earned: For a correct answer

$\Delta U_g = 2010J\left(or, 2050J, u \sin g, g = 10m/s^2\right)$

✓

0	1	2

The student response earns all of the following points:

1 point is earned: For using the gravitational potential energy equation

$\Delta U_g = mgh = (50kg)\left(9.8m/s^2\right)(4.1m)$

1 point is earned: For a correct answer

$\Delta U_g = 2010J\left(or, 2050J, u \sin g, g = 10m/s^2\right)$

Part C

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1 point is earned: For using an expression of conservation of energy

$U_{g1} = U_{g2} + K_2$
$mgh_1 = mgh_2 + \frac{1}{2}mv^2$

1 point is earned: For correctly substituting values

$gh_1 = gh_2 + \frac{1}{2}v^2$
$v = \sqrt{2g(h_1 - h_2)}$
$v = \sqrt{2(9.8m/s^2)(4.1m - 2.4m)}$

1 point is earned: For a correct answer

v = 5.8 m/s

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For using an expression of conservation of energy

$U_{g1} = U_{g2} + K_2$
$mgh_1 = mgh_2 + \frac{1}{2}mv^2$

1 point is earned: For correctly substituting values

$gh_1 = gh_2 + \frac{1}{2}v^2$
$v = \sqrt{2g(h_1 - h_2)}$
$v = \sqrt{2(9.8m/s^2)(4.1m - 2.4m)}$

1 point is earned: For a correct answer

v = 5.8 m/s

Part D

1 point is earned: For using a correct expression to solve for the time of fall

$\Delta y = v_0 t + \frac{1}{2}at^2$

1 point is earned: For correctly solving for the time of fall

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$\Delta y = \frac{1}{2}at^2$

$t = \sqrt{\frac{2\Delta y}{a}}$

$t = \sqrt{\frac{2(2.4m)}{(9.8m/s^2)}}$

$t = 0.70s\left(or, 0.69s, u\sin g, g = 10m/s^2\right)$

In the horizontal direction, the acceleration is zero.

R = *vxt*

1 point is earned: For substituting the speed from part (c) into the constant-velocity equation for horizontal motion

R = (5.8 m/s)(0.70 s)

R = 4.1 m (or 4.0 m using either unrounded speed of 5.77 m/s or g = 10 m/s2)

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For using a correct expression to solve for the time of fall

$\Delta y = v_0 t + \frac{1}{2}at^2$

1 point is earned: For correctly solving for the time of fall

$\Delta y = \frac{1}{2}at^2$

$t = \sqrt{\frac{2\Delta y}{a}}$

$t = \sqrt{\frac{2(2.4m)}{(9.8m/s^2)}}$

$t = 0.70s\left(or, 0.69s, u\sin g, g = 10m/s^2\right)$

In the horizontal direction, the acceleration is zero.

R = *vxt*

1 point is earned: For substituting the speed from part (c) into the constant-velocity equation for horizontal motion

R = (5.8 m/s)(0.70 s)

R = 4.1 m (or 4.0 m using either unrounded speed of 5.77 m/s or g = 10 m/s2

Units

1 point is earned: For correct units in the calculated answers to parts (b), (c), and (d)

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0	1 ✓

The student response earns all of the following points:

1 point is earned: For correct units in the calculated answers to parts (b), (c), and (d)

Part E

1 point is earned: For selecting " $pC < pB$ "

1 point is earned: For any indication that the speed at C is less than the speed at B

1 point is earned: For an explanation of why the speed decreases from B to C, which includes the following or equivalent statements:

by conservation of energy: U increases as the person moves from B to C, therefore K decreases, resulting in a decrease in speed

OR gravitational force opposes motion as the person moves from B to C, which results in a negative acceleration and a decrease in speed

OR by the impulse-momentum theorem: momentum decreases because the force of gravity provides impulse opposing the motion of the person

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For selecting " $pC < pB$ "

1 point is earned: For any indication that the speed at C is less than the speed at B

1 point is earned: For an explanation of why the speed decreases from B to C, which includes the following or equivalent statements:

by conservation of energy: U increases as the person moves from B to C, therefore K decreases, resulting in a decrease in speed

OR gravitational force opposes motion as the person moves from B to C, which results in a negative acceleration and a decrease in speed

OR by the impulse-momentum theorem: momentum decreases because the force of gravity provides impulse opposing the motion of the person

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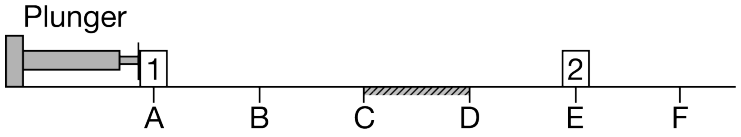
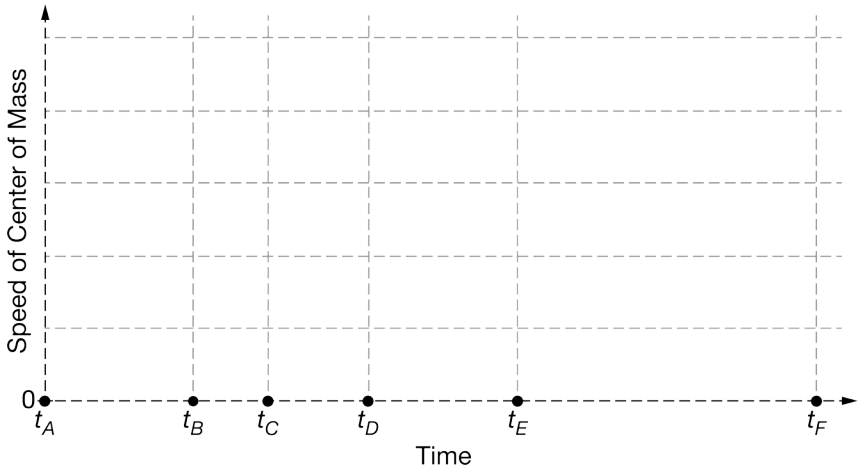
25. This question is a short free-response question. Show your work for each part of the question.

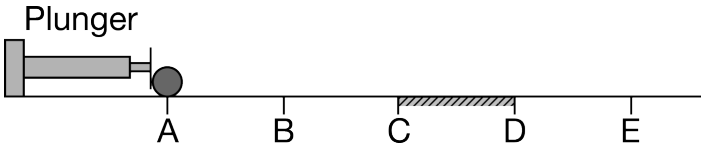


(7 points, suggested time 13 minutes)

Identical blocks 1 and 2 are placed on a horizontal surface at points A and E, respectively, as shown. The surface is frictionless except for the region between points C and D, where the surface is rough. Beginning at time t_A, block 1 is pushed with a constant horizontal force from point A to point B by a mechanical plunger. Upon reaching point B, block 1 loses contact with the plunger and continues moving to the right along the horizontal surface toward block 2. Block 1 collides with and sticks to block 2 at point E, after which the two-block system continues moving across the surface, eventually passing point F.

(a) On the axes below, sketch the speed of the center of mass of the two-block system as a function of time, from time t_A until the blocks pass point F at time t_F. The times at which block 1 reaches points A through F are indicated on the time axis.





(b) The plunger is returned to its original position, and both blocks are removed. A uniform solid sphere is placed at point A, as shown. The sphere is pushed by the plunger from point A to point B with a constant horizontal force that is directed toward the sphere's center of mass. The sphere loses contact with the plunger at point B and continues moving across the horizontal surface toward point E. In which interval(s), if any, does the sphere's angular momentum about its center of mass change? Check all that apply.

_____ A to B _____ B to C _____ C to D _____ D to E

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_____ None

Briefly explain your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Note: If the speed changes at t_E , the fourth point is not earned while the fifth point of part (a) may still be earned.

Note: No credit is earned for a horizontal line along the t axis.

0	1	2	3	4	5 ✓

The response accurately includes **five** of the following criteria.

- ☐ For a straight line that begins at zero at t_A and increases between t_A and t_B .
- ☐ For a segment that is horizontal and nonzero between t_B and t_C .
- ☐ For a segment that decreases linearly between t_C and t_D .
- ☐ For a segment that is horizontal, nonzero, and constant (but different value than segment from t_B to t_C) from t_D through t_F (with no change at t_E).
- ☐ For a curve that is continuous from t_A through t_F , with the possible exception of t_E .

Example Response:

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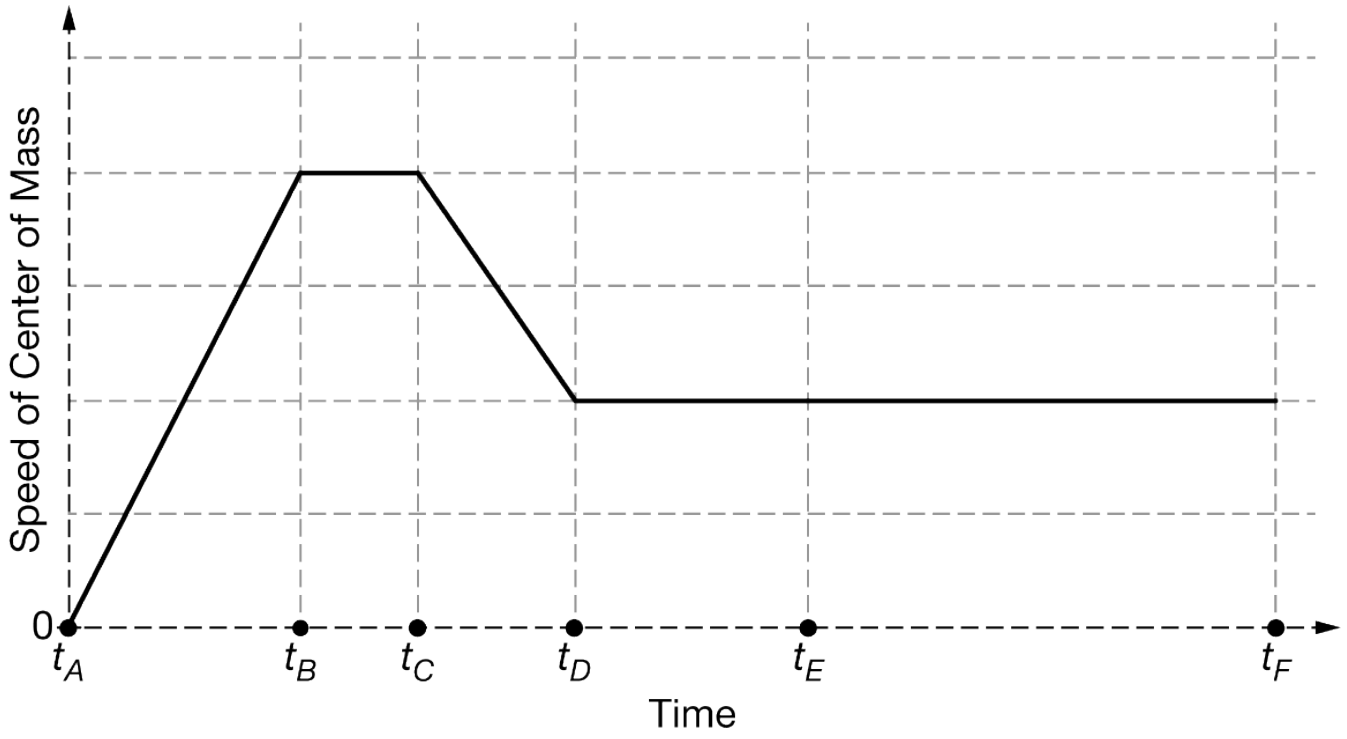

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Correct Answer: “C to D ”

Note: The second point of part (b) is not earned if a statement is made that the angular momentum or angular speed decreases between C and D or that the sphere stops rotating at point D .

0	1	2 ✓

The response accurately includes **both** of the following criteria:

- ☐ For reasoning that a change in angular momentum is caused by a net external torque.
- ☐ For correctly indicating that friction from C to D is the only force producing an external torque over the entire interval from A to E .

Example Response:

Claim: The sphere’s angular momentum about its center of mass changes in the interval C to D .

Evidence: There is friction between points C to D .

Reasoning Statements: Friction applies a torque in region C to D about the central axis of the cylinder to increase/ change its angular momentum.

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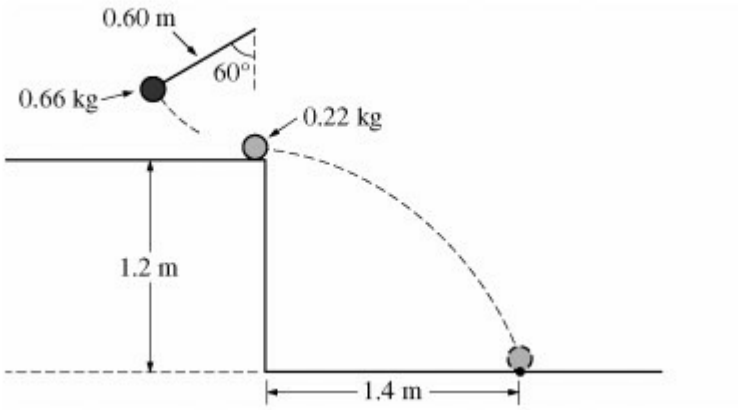
26.



The experimental apparatus shown in the figure above contains a pendulum consisting of a 0.66 kg ball attached to a string of length 0.60 m. The pendulum is released from rest at an angle of 60 degrees and collides with a ball of mass 0.22 kg initially at rest at the edge of a table. The 0.22 kg ball hits the floor a distance of 1.4 m from the edge of the table.

a. Calculate the speed of the 0.66 kg ball just before the collision.

b. Calculate the speed of the 0.22 kg ball immediately after the collision.

c. Calculate the speed of the 0.66 kg ball immediately after the collision.

d. Indicate the direction of motion of the 0.66 kg ball immediately after the collision.

____ To the left ____ To the right

e. Calculate the height to which the 0.66 kg ball rises after the collision.

f. Based on your data, is the collision elastic?

____ Yes ____ No

Justify your answer.

Part A

One point is earned for work indicating use of conservation of energy

$U_{g1} = K_2$

$mgh_1 = \frac{1}{2}mv_2^2$

One point is earned for a correct expression for determining the vertical drop-distance

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$$h = L - L\cos 60^\circ$$
$$h = 0.60m - (0.60m)\cos 60^\circ$$
$$h = 0.30m$$

One point is earned for substitution of a drop distance (other than 0.60 m) into a correct expression for the speed of the 0.66 kg ball just before the collision

$v = \sqrt{2gh} = \sqrt{2(9.8m/s^2)(0.30)}$
$v = 2.4m/s$

0	1	2	3 ✓

The student earns all of the following points:

One point is earned for work indicating use of conservation of energy

$U_{g1} = K_2$
$mgh_1 = \frac{1}{2}mv_2^2$

One point is earned for a correct expression for determining the vertical drop-distance

$$h = L - L\cos 60^\circ$$
$$h = 0.60m - (0.60m)\cos 60^\circ$$
$$h = 0.30m$$

One point is earned for substitution of a drop distance (other than 0.60 m) into a correct expression for the speed of the 0.66 kg ball just before the collision

$v = \sqrt{2gh} = \sqrt{2(9.8m/s^2)(0.30)}$
$v = 2.4m/s$

Part B

The speed immediately after the collision equals the horizontal speed of the 0.22 kg ball as it leaves the table, which is the horizontal distance it travels divided by the time it takes to fall.

One point is earned for a correct expression for the time it takes the 0.22 kg ball to fall

$$\triangle y = \frac{1}{2}gt^2$$
$$t = \sqrt{2 \triangle y/g}$$
$$t = \sqrt{2(1.2m)/(9.8m/s^2)}$$
$$t = 0.49s (or\ 0.50s \text{ using } g = 10m/s^2)$$

One point is earned for consistent substitution of the time into a correct expression for the speed of the 0.22 kg ball after

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the collision

$v_{0.22} = x/t = (1.4m)/(0.49s)$

One point is earned for the correct numerical value of the speed

$$v_{0.22} = 2.8m/s \ (or \ 2.9m/s \text{ using } g = 10m/s^2)$$

0	1	2	3 ✓

The student earns all of the following points:

The speed immediately after the collision equals the horizontal speed of the 0.22 kg ball as it leaves the table, which is the horizontal distance it travels divided by the time it takes to fall.

One point is earned for a correct expression for the time it takes the 0.22 kg ball to fall

$$\triangle y = \tfrac{1}{2} g t^2$$
$$t = \sqrt{2 \triangle y / g}$$
$$t = \sqrt{2(1.2m)/(9.8m/s^2)}$$
$$t = 0.49s (or \ 0.50s \text{ using } g = 10m/s^2)$$

One point is earned for consistent substitution of the time into a correct expression for the speed of the 0.22 kg ball after the collision

$v_{0.22} = x/t = (1.4m)/(0.49s)$

One point is earned for the correct numerical value of the speed

$$v_{0.22} = 2.8m/s \ (or \ 2.9m/s \text{ using } g = 10m/s^2)$$

Part C

One point is earned for work indicating use of conservation of momentum

$m_{0.66} v_{0.66i} = m_{0.66} v_{0.66f} + m_{0.22} v_{0.22f}$

One point is earned for consistent substitution of the answer from part (a) as *v0.66i* in a correct expression for *v0.66f*

$v_{0.66f} = \frac{m_{0.66} v_{0.66i} - m_{0.22} v_{0.22f}}{m_{0.66}} = \frac{(0.66kg)(2.4m/s) - (0.22kg)(2.8m/s)}{(0.66kg)}$

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One point is earned for the correct numerical value of the speed

$$v_{0.66f} = 1.5m/s \text{ } (or \text{ } 1.4m/s \text{ using } g = 10m/s^2)$$

0	1	2	3 ✓

The student earns all of the following points:

One point is earned for work indicating use of conservation of momentum

$m_{0.66}v_{0.66i} = m_{0.66}v_{0.66f} + m_{0.22}v_{0.22f}$

One point is earned for consistent substitution of the answer from part (a) as *v0.66i* in a correct expression for *v0.66f*

$v_{0.66f} = \frac{m_{0.66}v_{0.66i} - m_{0.22}v_{0.22f}}{m_{0.66}} = \frac{(0.66kg)(2.4m/s) - (0.22kg)(2.8m/s)}{(0.66kg)}$

One point is earned for the correct numerical value of the speed

$$v_{0.66f} = 1.5m/s \text{ } (or \text{ } 1.4m/s \text{ using } g = 10m/s^2)$$

Part D

One point is earned for indicating “To the right” as the direction of motion of the 0.66 kg ball after the collision

0	1 ✓

The student earns all of the following points:

One point is earned for indicating “To the right” as the direction of motion of the 0.66 kg ball after the collision

Part E

One point is earned for an appropriate conservation of energy statement

$K_1 = U_{g2}$
$\frac{1}{2}mv_{0.66f}^2 = mgh'$

One point is earned for consistent substitution of *v0.66f* from part (c) into a correct expression for h'

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$h' = \frac{v_{0.66f}^2}{2g} = \frac{(1.5m/s)^2}{(2)(9.8m/s^2)}$
$h' = 0.11m$

0	1	2 ✓

The student earns all of the following points:

One point is earned for an appropriate conservation of energy statement

$K_1 = U_{g2}$
$\frac{1}{2}mv_{0.66f}^2 = mgh'$

One point is earned for consistent substitution of *v0.66f* from part (c) into a correct expression for *h′*

$h' = \frac{v_{0.66f}^2}{2g} = \frac{(1.5m/s)^2}{(2)(9.8m/s^2)}$
$h' = 0.11m$

Part F

One point is earned for any indication of comparing kinetic energies of the system before and after the collision

One point is earned for calculating the two values of kinetic energy

$K_i = \frac{1}{2}(0.66kg)\text{msup} = 1.9J$
$K_f = \frac{1}{2}(0.66kg)\text{msup} + \frac{1}{2}(0.22kg)(2.8m/s^2) = 1.6J$

One point is earned for indication that the difference in the kinetic energies means an inelastic collision

0	1	2	3 ✓

The student earns all of the following points:

One point is earned for any indication of comparing kinetic energies of the system before and after the collision

One point is earned for calculating the two values of kinetic energy

$K_i = \frac{1}{2}(0.66kg)\text{msup} = 1.9J$
$K_f = \frac{1}{2}(0.66kg)\text{msup} + \frac{1}{2}(0.22kg)(2.8m/s^2) = 1.6J$

One point is earned for indication that the difference in the kinetic energies means an inelastic collision

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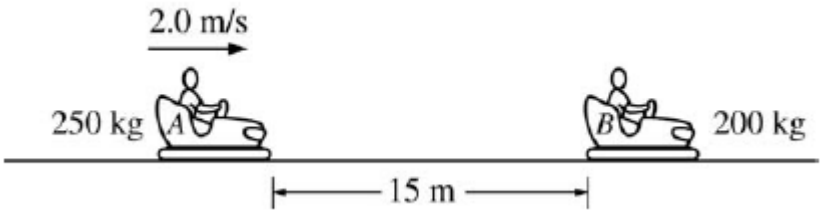
27.



Several students are riding in bumper cars at an amusement park. The combined mass of car A and its occupants is 250 kg. The combined mass of car B and its occupants is 200 kg. Car A is 15 m away from car B and moving to the right at 2.0 m/s, as shown, when the driver decides to bump into car B, which is at rest.

(a) Car A accelerates at 1.5 m/s^2 to a speed of 5.0 m/s and then continues at constant velocity until it strikes car B. Calculate the total time for car A to travel the 15 m.

(b) After the collision, car B moves to the right at a speed of 4.8 m/s .

i. Calculate the speed of car A after the collision.

ii. Indicate the direction of motion of car A after the collision.

____ To the left ____ To the right ____ None; car A is at rest.

(c) Is this an elastic collision?

____ Yes ____ No

Justify your answer.

Part A

1 point is earned: For the correct value of , the time car A is accelerating

vf = vi + at1

t1 = (vf-vi)/a = (5.0 m/s - 2.0 m/s)/(1.5 m/s)

t1 = 2.0 s

1 point is earned:For a correct value of x1 , the distance car A travels while accelerating

x1 = vit1 + ½ at12

x1 = (2.0 m/s)(2.0 s)+ ½(1.5 m/s2)(2.0 s)2

x1 = 7.0 m

Note: The equation vf2 = vi2 + 2ax1 could also be used.

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1 point is earned:For a correct value of (x-x1), the distance car A travels at constant velocity

(x-x1) = (15.0 m - 7.0 m) = 8.0 m

1 point is earned:For correctly calculating t2, the time car A travels at constant velocity

x = x1 + vft2

t2 = (x-x1)/vf = (15.0 m - 7.0 m)/5.0 m/s

t2 = 1.6 s

ttot = t1 + t2 = 2.0 s + 1.6 s

ttot = 3.6 s



0	1	2	3	4

The student response earns four of the following points:

1 point is earned: For the correct value of , the time car A is accelerating

vf = vi + at1

t1 = (vf-vi)/a = (5.0 m/s - 2.0 m/s)/(1.5 m/s)

t1 = 2.0 s

1 point is earned:For a correct value of x1 , the distance car A travels while accelerating

x1 = vit1 + ½ at12

x1 = (2.0 m/s)(2.0 s)+ ½(1.5 m/s2)(2.0 s)2

x1 = 7.0 m

Note: The equation vf2 = vi2 + 2ax1 could also be used.

1 point is earned:For a correct value of (x-x1), the distance car A travels at constant velocity

(x-x1) = (15.0 m - 7.0 m) = 8.0 m

1 point is earned:For correctly calculating t2, the time car A travels at constant velocity

x = x1 + vft2

t2 = (x-x1)/vf = (15.0 m - 7.0 m)/5.0 m/s

t2 = 1.6 s

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ttot = t1 + t2 = 2.0 s + 1.6 s

ttot = 3.6 s

Part B(i,ii)

(i)

1 point is earned : For any clear statement that momentum is conserved

m A v Ai = m A v Af + m B v B

$v_{Af} = \frac{m_A v_{Ai} - m_B v_B}{m_A} = \frac{(250\text{kg})(5.0\text{m/s}) - (200\text{kg})(4.8\text{m/s})}{250\text{kg}}$

1 point is earned : For a correct answer

v Af = 1.2 m/s

(ii)

1 point is earned : For indicating a direction of car A after the collision that is consistent with the calculation of v Af

Note: A correct calculation yields a direction to the right.

0	1	2	3 ✓

The student response earns three of the following points:

(i)

1 point is earned : For any clear statement that momentum is conserved

m A v Ai = m A v Af + m B v B

$v_{Af} = \frac{m_A v_{Ai} - m_B v_B}{m_A} = \frac{(250\text{kg})(5.0\text{m/s}) - (200\text{kg})(4.8\text{m/s})}{250\text{kg}}$

1 point is earned : For a correct answer

v Af = 1.2 m/s

(ii)

1 point is earned : For indicating a direction of car A after the collision that is consistent with the calculation of v Af

Note: A correct calculation yields a direction to the right.

Part C

1 point is earned : For correctly indicating that the collision is not elastic

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1 point is earned : For a statement that kinetic energy is not conserved

1 point is earned : For clearly showing that K f <K i , implying the collision is non-elastic

$K_i = \frac{1}{2} m_A v_{Ai}^2 = \frac{1}{2}(250\text{kg})(5.0\text{m/s})^2$

K i = 3125 J

$K_f = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_B = \frac{1}{2}(250\text{kg})(1.2\text{m/s})^2 + \frac{1}{2}(200\text{kg})(4.8\text{m/s})^2$

K f = 2484 J

Note: Two points were awarded for checking “yes” with a clear, correct explanation that it is a partially elastic collision

0	1	2	3 ✓

The student response earns three of the following points:

1 point is earned : For correctly indicating that the collision is not elastic

1 point is earned : For a statement that kinetic energy is not conserved

1 point is earned : For clearly showing that K f <K i , implying the collision is non-elastic

$K_i = \frac{1}{2} m_A v_{Ai}^2 = \frac{1}{2}(250\text{kg})(5.0\text{m/s})^2$

K i = 3125 J

$K_f = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_B = \frac{1}{2}(250\text{kg})(1.2\text{m/s})^2 + \frac{1}{2}(200\text{kg})(4.8\text{m/s})^2$

K f = 2484 J

Note: Two points were awarded for checking “yes” with a clear, correct explanation that it is a partially elastic collision

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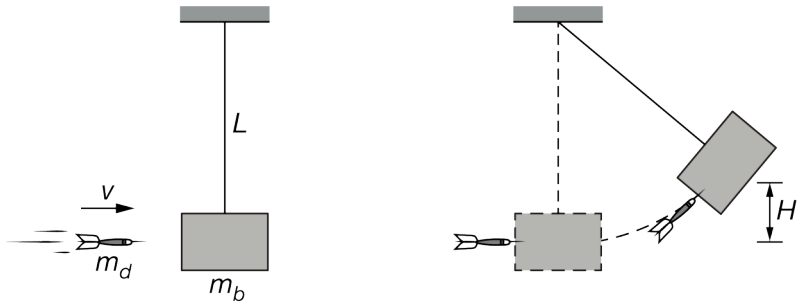
28. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown above at left. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown above at right. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, if either, experiences an acceleration of larger magnitude during the collision. If the acceleration is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the experiment were repeated with a block with mass greater than m_b, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the tension in the string at the moment immediately after the collision and state the following:

Student 1: The tension in the string immediately after the collision is greater than the weight of the dart-block system because the acceleration is upward at that instant.

Student 2: The tension in the string is equal to the weight of the block-dart system because there is no vertical acceleration immediately after the collision.

(c) Which underlined phrase or phrases, if any, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if any, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if any, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases, if any, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{m_d v}{m_d + m_b}\right)^2$. Explain how this equation supports your claim in part (b).

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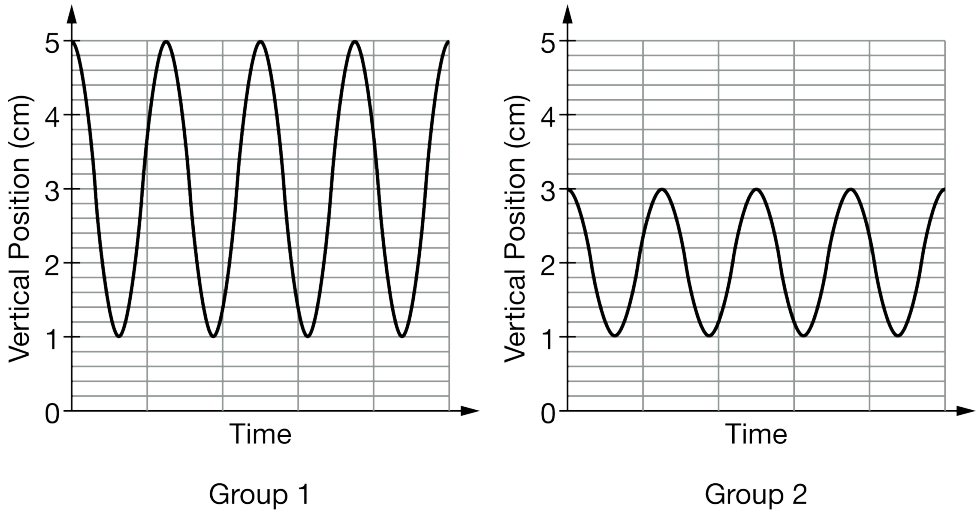
(h) Consider the case in which the collision between the dart and the block is perfectly elastic (the dart bounces off the block to the left). In this case, is the speed of the block immediately after the collision greater than, less than, or the same as the speed of the block in the original case? Justify your answer.

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart speed or the string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for a statement that the dart experiences the larger acceleration because it has the smaller mass with equal force exerted on it because of Newton's third law.

Example Response:

Both the dart and the block experience the same force because they exert equal and opposite forces on each other because of Newton's third law, however the mass of the dart is smaller and so experiences a larger acceleration.

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Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement that the final height is smaller.
- ☐ 1 point is earned for a statement of conservation of momentum to indicate that the final speed of the dart-block system is smaller **AND** an indication of conservation of energy leading to a smaller height.

Example Response:

Because the momentum of the dart before the collision must be equal to the momentum of the dart-block system after the collision, a larger block mass will lead to a smaller dart-block speed after the collision because of conservation of momentum. Because the dart-block system is traveling slower after the collision, it has less kinetic energy which means that the dart-block system will not swing as high.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating both of Student 1's statements are correct.

Example Responses:

Student 1's entire statement is correct

OR

Both statements

Part D

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating student 1 has no incorrect statements.

Example Response:

None

Part E

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating student 2 has no correct statements.

Example Response:

None

Part F

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

The response accurately includes

- ☐ 1 point is earned for indicating both of Student 2's statements are incorrect.

Example Responses:

Student 2's entire statement is incorrect.

OR

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Both statements

Part G

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for a statement including functional dependence of the variables.
- ☐ 1 point is earned for correctly asserting that as the mass of the block increases in the equation H decreases.

Example Response:

This equation matches my reasoning in part b – if the mass of the block increases, the height of the swing decreases because the mass of the block is in the denominator.

Part H

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for "greater", with an attempt at an explanation.
- ☐ 1 point is earned for a statement indicating a larger change in momentum of the dart.
- ☐ 1 point is earned for a statement using conservation of momentum to show that the block momentum/ speed is greater.

Example Response:

When the dart bounces off the block, it has a larger change in momentum (going from v to $-v$ instead of v to a smaller v), which means that the block also has a larger change in momentum (since both objects receive the same impulse and experience the same change in momentum). Because the block has a greater change in momentum, it has a larger speed after the collision than in the original case.

Part I

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for recognizing that the amplitude of the graph for group 2 is smaller.
- ☐ 1 point is earned for a statement discussing the dependence of amplitude on dart momentum.
- ☐ 1 point is earned for indicating that the speed of the dart could be smaller.

Example Response:

If group 2 used a dart with a smaller initial speed, the dart would have had less momentum, and so the dart-block system would not have gone as high (making the amplitude smaller), but the string length is the same and so the period would still be the same.

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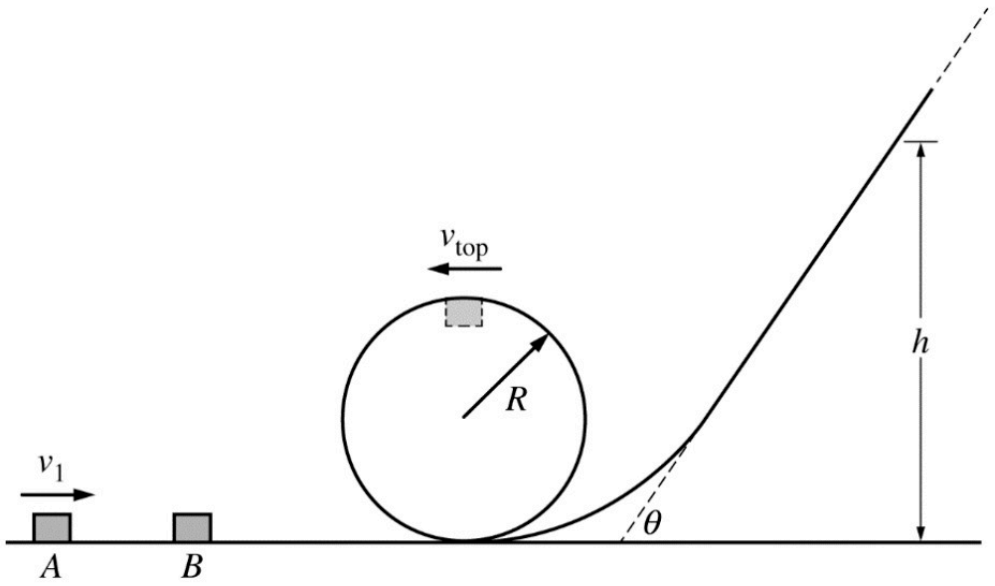
29.



Note: Figure not drawn to scale.

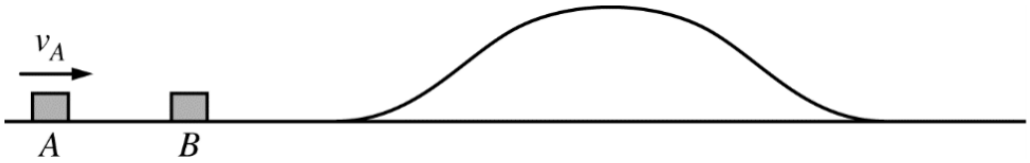
Two blocks A and B of equal mass m are on a frictionless track, as shown in the figure above. Block A, initially moving with speed v_1, has a perfectly elastic collision with block B. Block B has a speed v_1 immediately after the collision, and then it travels around a circular loop of radius R, where R is much larger than the size of the blocks. The speed of block B at the top of the loop is v_{top}. Block B then slides up a ramp until it comes momentarily to rest at a height h above the floor.

a. Derive an equation for the height *h* in terms of *m*, *R, v_{top}.* and *g,* as appropriate.

b. Suppose that the speed v_1 of block A is doubled. Will the final height of block *B* be greater than, less than, or equal to *2h*?

_____Greater than *2h* _____Less than *2h* _____ Equal to *2h*

Briefly explain how you arrived at your answer.



c. Blocks A and B (both of mass m) are moved to a different frictionless track, as shown above. Let v_A be the minimum initial speed for block A that allows block B to make it over the hump in the track. Block A is then replaced with a larger block, block C, which has mass 2m, as shown below.

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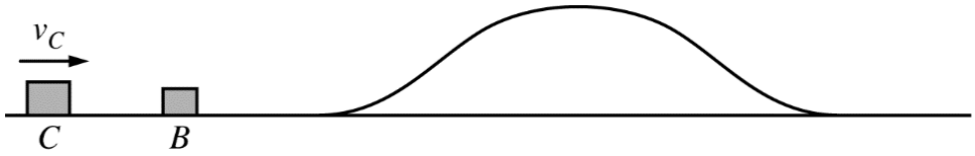

Block C makes a completely inelastic collision with block B so that both blocks stick together and travel along the track. What is the minimum initial speed v_C that block C must have so that the two-block system makes it over the hump? Express your answer in terms of v_A.

Part A

1 point is earned: For using an equation expressing the conservation of energy

$mgh = 2mgR + 1/2mv_{top}^2$ or $mgh = (1/2)mv_1^2$ or $mgh = (1/2)mv^2$

1 point is earned: For using two forms of energy when block B is at the top of the loop or using the initial kinetic energy

1 point is earned: For expressing the correct answer in terms of the listed quantities

$$h = \frac{2mgR + \frac{1}{2}mv_{top}^2}{mg}$$

$$h = 2R + \frac{v_{top}^2}{2g}$$

✓

0	1	2	3

Student response earns 3 of the following 3 point(s)

1 point is earned: For using an equation expressing the conservation of energy

$mgh = 2mgR + 1/2mv_{top}^2$ or $mgh = (1/2)mv_1^2$ or $mgh = (1/2)mv^2$

1 point is earned: For using two forms of energy when block B is at the top of the loop or using the initial kinetic energy

1 point is earned: For expressing the correct answer in terms of the listed quantities

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$$h = \frac{2mgR + \frac{1}{2}mv_{top}^2}{mg}$$

$$h = 2R + \frac{v_{top}^2}{2g}$$

Part B

1 point is earned: For a correct answer “Greater than 2h ” with an explanation

No point is earned for a correct selection without an explanation. If an incorrect answer is selected, the following point for using conservation of energy can still be earned.

1 point is earned: For using an equation or semi-quantitative reasoning expressing the conservation of energy Note: “semi-quantitative reasoning” here includes reasoning in terms of proportionality, squared relationships, etc.

mgh = 1/2 mv_1^2

h = v_1^2/2g

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

1 point is earned: For a correct answer “Greater than 2h ” with an explanation

No point is earned for a correct selection without an explanation. If an incorrect answer is selected, the following point for using conservation of energy can still be earned.

1 point is earned: For using an equation or semi-quantitative reasoning expressing the conservation of energy Note: “semi-quantitative reasoning” here includes reasoning in terms of proportionality, squared relationships, etc.

mgh = 1/2 mv_1^2

h = v_1^2/2g

Part C

Correct answer: $V_C = 3/2V_A$

Note: there is no “answer point”, so no credit is awarded for an answer with no work shown

1 point is earned: For using momentum conservation to relate the speeds before and after the completely inelastic collision between blocks B and C

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$2mv_C = (2m + m)V_{BC} = 3\ mv_{BC}$, where v_{BC} is the speed of blocks B and C after the collision

$2v_C = 2v_{BC}$

1 point is earned: For setting the speed of blocks B and C after the collision equal to the speed of block B after its elastic collision with block A, in order for blocks B and C to make it over the hump.

$2v_C = 3v_A$

$v_C = 3/2v_A$

0	1	2 ✓

Student response earns 2 of the following 2 point(s)

Correct answer: $V_C = 3/2V_A$

Note: there is no “answer point”, so no credit is awarded for an answer with no work shown

1 point is earned: For using momentum conservation to relate the speeds before and after the completely inelastic collision between blocks B and C

$2mv_C = (2m + m)V_{BC} = 3\ mv_{BC}$, where v_{BC} is the speed of blocks B and C after the collision

$2v_C = 2v_{BC}$

1 point is earned: For setting the speed of blocks B and C after the collision equal to the speed of block B after its elastic collision with block A, in order for blocks B and C to make it over the hump.

$2v_C = 3v_A$

$v_C = 3/2v_A$

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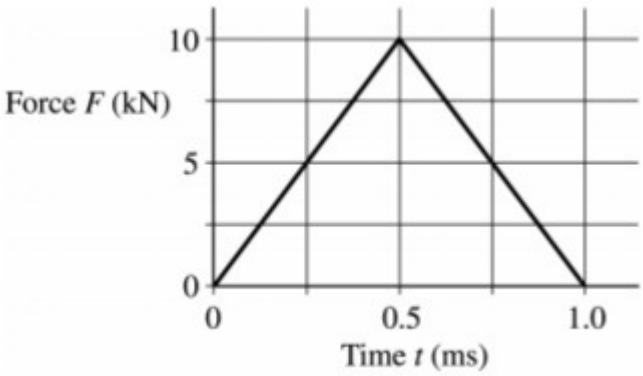
30.



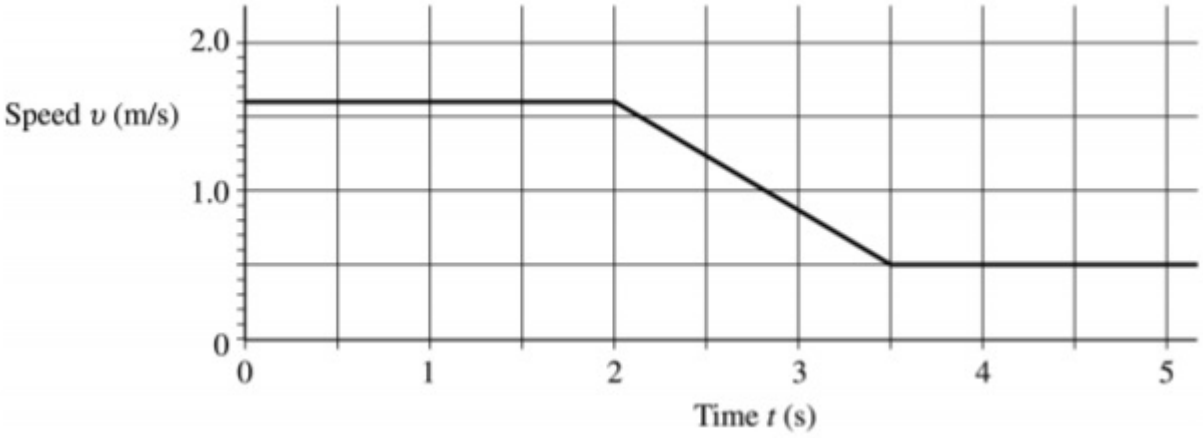
A 2.0 kg frictionless cart is moving at a constant speed of 3.0 m/s to the right on a horizontal surface, as shown above, when it collides with a second cart of undetermined mass m that is initially at rest. The force F of the collision as a function of time t is shown in the graph below, where $t = 0$ is the instant of initial contact. As a result of the collision, the second cart acquires a speed of 1.6 m/s to the right. Assume that friction is negligible before, during, and after the collision.



a. Calculate the magnitude and direction of the velocity of the 2.0 kg cart after the collision.

b. Calculate the mass m of the second cart.

After the collision, the second cart eventually experiences a ramp, which it traverses with no frictional losses. The graph below shows the speed v of the second cart as a function of time t for the next 5.0 s, where $t = 0$ is now the instant at which the carts separate.



c. Calculate the acceleration of the cart at $t = 3.0$ s.

d. Calculate the distance traveled by the second cart during the 5.0 s interval after the collision (0 s < t < 5.0 s).

e. State whether the ramp goes up or down and calculate the maximum elevation (above or below the initial height) reached by the second cart on the ramp during the 5.0 s interval after the collision (0 s < t < 5.0 s).

Part A

1 point is earned: For any statement indicating that impulse equals change in momentum

1 point is earned: For any statement indicating that impulse can be determined from the area under the graph

$\Delta p = F\Delta t$ = area under the curve

$$\Delta p = 2\left(\tfrac{1}{2}\right)(0.5 \times 10^{-3}\text{s})(10 \times 10^{3}\text{ N}) = 5\text{ N} \cdot \text{s}$$

1 point is earned: For recognizing that the impulse on the 2 kg cart is negative, and subtracting it from the initial momentum of the cart

p2 kg after = (2.0 kg)(3.0 m/s) – 5 N · s = 1 N · s

p2 kg after = (2 kg)v2 kg after = 1 N · s

1 point is earned: For the correct answer

p2 kg after = 0.5 m/s to the right

0	1	2	3	4 ✓

The student response earns all of the following points:

1 point is earned: For any statement indicating that impulse equals change in momentum

1 point is earned: For any statement indicating that impulse can be determined from the area under the graph

$\Delta p = F\Delta t$ = area under the curve

$$\Delta p = 2\left(\tfrac{1}{2}\right)(0.5 \times 10^{-3}\text{s})(10 \times 10^{3}\text{ N}) = 5\text{ N} \cdot \text{s}$$

1 point is earned: For recognizing that the impulse on the 2 kg cart is negative, and subtracting it from the initial momentum of the cart

p2 kg after = (2.0 kg)(3.0 m/s) – 5 N · s = 1 N · s

p2 kg after = (2 kg)v2 kg after = 1 N · s

1 point is earned: For the correct answer

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*v*2 kg after = 0.5 m/s to the right

Part B

1 point is earned: For any statement of conservation of momentum

*p*2 kg before = *p*2 kg after + *pm* after

1 point is earned: For correct substitutions

6 N · s = 1 N · s + *m*(1.6 m/s)

1 point is earned: For the correct answer

m = 3.1 kg

Alternate solution

The average acceleration is the average force divided by the mass

$\bar{a} = \overline{F}/m$

1 point is earned: For calculating the average acceleration

$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{1.6\ \text{m/s}}{1\times10^{-3}\ \text{s}} = 1600\ \text{m/s}^2$

1 point is earned: For calculating the average force

$\overline{F} = \frac{1}{2}(0 + 10,000)\ \text{N}{=}5000\ \text{N}$

m = (5000 N)/(1600 m/s2)

1 point is earned: For the correct answer

m = 3.1 kg

Note: An alternate solution is to do part (b) first using impulse, in which case the first two points noted above for part (a) could be earned. Then conservation of momentum can be used to solve part (a), so the first point noted above for part (b) could be earned.

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For any statement of conservation of momentum

*p*2 kg before = *p*2 kg after + *pm* after

1 point is earned: For correct substitutions

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6 N · s = 1 N · s + *m*(1.6 m/s)

1 point is earned: For the correct answer

m = 3.1 kg

Alternate solution

The average acceleration is the average force divided by the mass

$\bar{a} = \overline{F}/m$

1 point is earned: For calculating the average acceleration

$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{1.6\ \text{m/s}}{1\times10^{-3}\ \text{s}} = 1600\ \text{m/s}^2$

1 point is earned: For calculating the average force

$\overline{F} = \frac{1}{2}(0 + 10,000)\ \text{N} = 5000\ \text{N}$

m = (5000 N)/(1600 m/s2)

1 point is earned: For the correct answer

m = 3.1 kg

Note: An alternate solution is to do part (b) first using impulse, in which case the first two points noted above for part (a) could be earned. Then conservation of momentum can be used to solve part (a), so the first point noted above for part (b) could be earned.

Part C

1 point is earned: For using the slope of the graph to determine the acceleration

$a = \text{ slope} = \frac{(0.5-1.6)\ \text{m/s}}{(3.5-2)\ \text{s}}$

1 point is earned: For the correct answer

a = –0.73 m/s2

0	1	2 ✓

The student response earns all of the following points:

1 point is earned: For using the slope of the graph to determine the acceleration

$a = \text{ slope} = \frac{(0.5-1.6)\ \text{m/s}}{(3.5-2)\ \text{s}}$

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1 point is earned: For the correct answer

a = –0.73 m/s2

Part D

1 point is earned: For using the area under the curve or one of the equations $d = vt$ or $d = \bar{v}t$

1 point is earned: For calculating a distance for each segment of the graph

$d1 = (1.6 \text{ m/s})(2 \text{ s}) = 3.2 \text{ m}$

$d_2 = \frac{(1.6+0.5) \text{ m/s}}{2}(1.5 \text{ s})$ or $(1.6 \text{ m/s})(1.5 \text{ s}) + \frac{1}{2}(-0.73 \text{ m/s}^2)(1.5 \text{ s})^2$

$d2 = 1.6 \text{ m}$

$d3 = (0.5 \text{ m/s})(1.5 \text{ s}) = 0.8 \text{ m}$

$dtot = d1 + d2 + ds$

1 point is earned: For the correct answer

$dtot = 5.5 \text{ m}$

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned: For using the area under the curve or one of the equations $d = vt$ or $d = \bar{v}t$

1 point is earned: For calculating a distance for each segment of the graph

$d1 = (1.6 \text{ m/s})(2 \text{ s}) = 3.2 \text{ m}$

$d_2 = \frac{(1.6+0.5) \text{ m/s}}{2}(1.5 \text{ s})$ or $(1.6 \text{ m/s})(1.5 \text{ s}) + \frac{1}{2}(-0.73 \text{ m/s}^2)(1.5 \text{ s})^2$

$d2 = 1.6 \text{ m}$

$d3 = (0.5 \text{ m/s})(1.5 \text{ s}) = 0.8 \text{ m}$

$dtot = d1 + d2 + ds$

1 point is earned: For the correct answer

$dtot = 5.5 \text{ m}$

Part E

The acceleration is negative, so the cart must be moving opposite to the force of gravity, which is the only force acting on

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it.

1 point is earned: For indicating that the ramp goes up

1 point is earned: For using conservation of energy

$mgh = \frac{1}{2}m(v_i^2 - v_f^2)$

$h = \frac{1}{2(10\ \mathrm{m/s^2})}\left((1.6\ \mathrm{m/s})^2 - (0.5\ \mathrm{m/s})^2\right)$

1 point is earned: For the correct answer

h = 0.12 m

0	1	2	3 ✓

The student response earns all of the following points:

The acceleration is negative, so the cart must be moving opposite to the force of gravity, which is the only force acting on it.

1 point is earned: For indicating that the ramp goes up

1 point is earned: For using conservation of energy

$mgh = \frac{1}{2}m(v_i^2 - v_f^2)$

$h = \frac{1}{2(10\ \mathrm{m/s^2})}\left((1.6\ \mathrm{m/s})^2 - (0.5\ \mathrm{m/s})^2\right)$

1 point is earned: For the correct answer

h = 0.12 m

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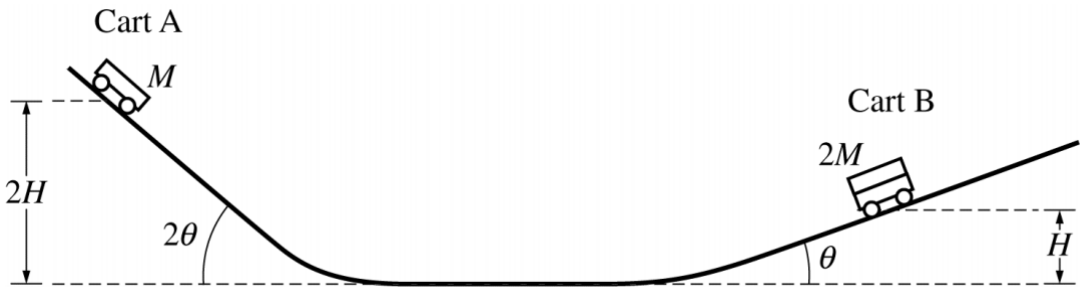
31.



Cart *A* has mass *M* and is released from rest at a height *2H* on a ramp making an angle 2θ with the horizontal, as shown above. Cart *B* has mass 2*M* and is released from rest at a height H on a ramp making an angle θ with the horizontal. The carts roll toward each other, have a head-on collision on the horizontal portion of the ramp, and stick together. The masses of the carts' wheels are negligible, as are any frictional or drag forces.

a. Indicate whether the carts remain at rest, move to the left, or move to the right after the collision.

____ Remain at rest ____ Move to the left ____ Move to the right

Explain how you arrived at your answer.

b. Consider the time interval from when the two carts are released until just after they collide.

i. For the system consisting of the two carts and Earth, indicate whether the total mechanical energy increases, decreases, or remains the same.

____ Increases ____ Decreases ____ Remains the same

Justify your answer.

ii. For the system consisting of only the two carts, indicate whether the total mechanical energy increases, decreases, or remains the same.

____ Increases ____ Decreases ____ Remains the same

Justify your answer.

Part A

1 point is earned: For any application of conservation of energy

1 point is earned: For correctly applying conservation of energy for each cart

1 point is earned: For any application of conservation of momentum

Example 1: The contributions of each cart to the potential energy of the carts-Earth system when the carts are at their points of release are the same, because one has twice the height and one has twice the mass of the other. All this potential energy is converted into kinetic energy when the carts reach the bottom of the ramp, so the

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carts have the same kinetic energy there. This means that the speed of cart A is 1.4 times the speed of Cart B. Before the collision, cart A has a momentum of about 1.4Mv to the right; cart B has a momentum of 2Mv to the left. The net momentum is to the left, so after the collision the carts move left

Example 2:

$$Mg(2H) = Mv_A^2/2; \quad v_A^2 = 4gH; \quad v_A = 2\sqrt{gH}$$
$$2Mg(H) = (2M)v_B^2/2; \quad v_B^2 = 2gH; \quad v_B = \sqrt{2gH}$$
$$M(2\sqrt{gH}) - 2M(\sqrt{2gH}) = (M+2M)v_F = 3Mv_F$$
$$v_F = \left(\tfrac{2-2\sqrt{2}}{3}\right)\sqrt{gH} = -\tfrac{0.83}{3}\sqrt{gH} = -0.28\sqrt{gH}$$

One earned point is deducted for any incorrect use of ramp angles.

0	1	2	3 ✓

Student response earns 3 of the following 3 point(s)

1 point is earned: For any application of conservation of energy

1 point is earned: For correctly applying conservation of energy for each cart

1 point is earned: For any application of conservation of momentum

Example 1: The contributions of each cart to the potential energy of the carts-Earth system when the carts are at their points of release are the same, because one has twice the height and one has twice the mass of the other. All this potential energy is converted into kinetic energy when the carts reach the bottom of the ramp, so the carts have the same kinetic energy there. This means that the speed of cart A is 1.4 times the speed of Cart B. Before the collision, cart A has a momentum of about 1.4Mv to the right; cart B has a momentum of 2Mv to the left. The net momentum is to the left, so after the collision the carts move left

Example 2:

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$$v_F = \left(\tfrac{2-2\sqrt{2}}{3}\right)\sqrt{gH} = -\tfrac{0.83}{3}\sqrt{gH} = -0.28\sqrt{gH}$$

One earned point is deducted for any incorrect use of ramp angles.

Part B

Part(i)

1 point is earned: For indicating that no energy is gained as the carts move down the ramps, or using the result of the

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reasoning in part (a) regarding the energy for this part of the motion

1 point is earned: For indicating that energy is dissipated in the collision and a comparison of initial and final energies consistent with the students' assumptions about energy before the collision

Part(ii)

1 point is earned: For indicating that the initial mechanical energy of the carts is zero

1 point is earned: For a comparison with the energy after the collision consistent with previous responses

Example: When the carts are released, they have no mechanical energy, since they are not moving. After the collision, the carts have kinetic energy. The total mechanical energy of the cart-cart system increases.

Alternate solution

1 point is earned: For identifying gravitational force as the only external force doing work

1 point is earned: For correct use of the work-kinetic energy theorem

> Example: For the system of just the two carts, there is no initial mechanical energy, since the carts are at rest. The gravitational force exerted on the carts by Earth is the only external force doing work, and the work on the system is positive. So the kinetic energy of the system increases, which increases the total mechanical energy

0	1	2	3	4 ✓

Student response earns 4 of the following 4 point(s)

Part(i)

1 point is earned: For indicating that no energy is gained as the carts move down the ramps, or using the result of the reasoning in part (a) regarding the energy for this part of the motion

1 point is earned: For indicating that energy is dissipated in the collision and a comparison of initial and final energies consistent with the students' assumptions about energy before the collision

Part(ii)

1 point is earned: For indicating that the initial mechanical energy of the carts is zero

1 point is earned: For a comparison with the energy after the collision consistent with previous responses

Example: When the carts are released, they have no mechanical energy, since they are not moving. After the collision, the carts have kinetic energy. The total mechanical energy of the cart-cart system increases.

Alternate solution

1 point is earned: For identifying gravitational force as the only external force doing work

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Example: For the system of just the two carts, there is no initial mechanical energy, since the carts are at rest. The gravitational force exerted on the carts by Earth is the only external force doing work, and the work on the system is positive. So the kinetic energy of the system increases, which increases the total mechanical energy

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32. This question is a short free-response question. Show your work for each part of the question.



Note: Figure not drawn to scale.

(7 points, suggested time 13 minutes)

Two carts are moving to the right on a straight, level track with negligible friction. At time $t = 0$, the carts are at the positions shown above. Cart 2 collides with and bounces off a wall at the right end of the track, after which the two carts collide.

Cart 1 has a mass of $0.20\ \text{kg}$ and an initial velocity of $3.0\ \text{m/s}$ to the right. Cart 2 has a mass of $0.5\ \text{kg}$ and an initial velocity of $0.50\ \text{m/s}$ to the right.

The graphs below show the velocity of each cart from $t = 0$ to sometime after the carts collide.

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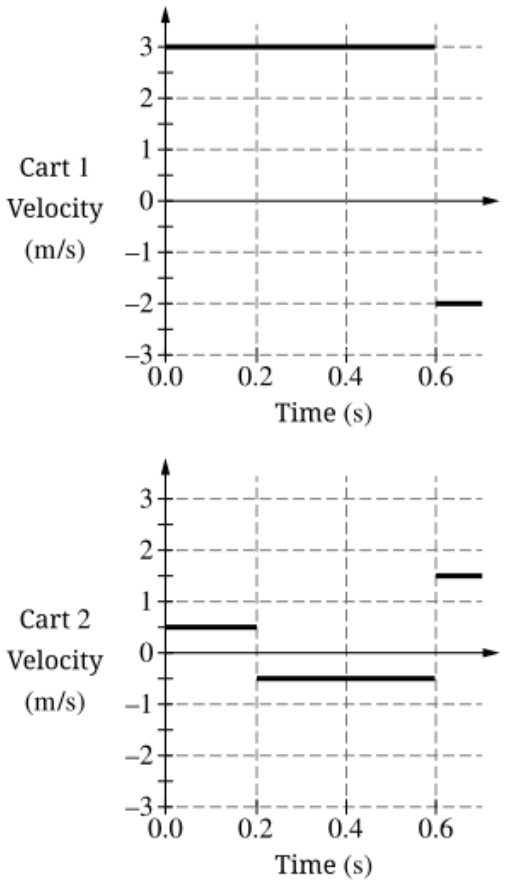

(a) Consider the system consisting of just Cart 2 when it bounces off the wall.

i. Is the momentum of Cart 2 the same before and after bouncing off the wall?

___ Yes ___ No

Briefly justify your answer.

ii. Is the mechanical energy of Cart 2 the same before and after bouncing off the wall?

___ Yes ___ No

Briefly justify your answer.

(b) Is the momentum of the two-cart system the same before and after the collision that occurs at $t = 0.6$ s?

___ Yes ___ No

Using calculations based on the information shown in the graphs, justify your answer.

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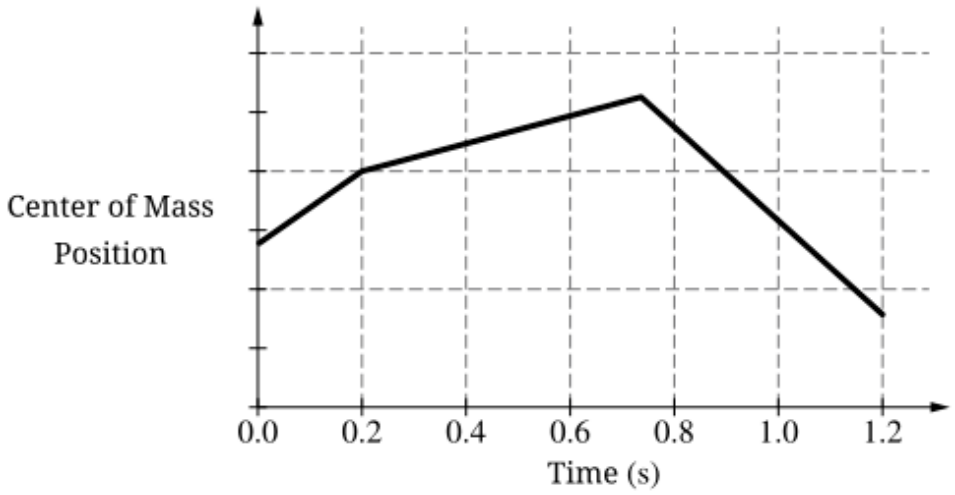

(c) The graph above shows the position of the center of mass of the two-cart system for a longer time interval $0 < t < 1.2$ s. The slope of the graph changes at time $t = 0.2$ s and also at time $t = 0.73$ s, which are the two times at which Cart 2 bounces off the wall. However, the slope does not change at time $t = 0.6$ s, when the two carts collide.

i. Use physics principles to explain why the slope of the graph changes at $t = 0.2$ s and $t = 0.73$ s.

ii. Use physics principles to explain why the slope of the graph does not change at $t = 0.6$ s.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for using selecting "no" and a justification that indicates **one** of the following:
  · the presence of a net force on the Cart 2 during the collision
  · the direction of either the momentum or the velocity changes for Cart 2
  Example Response:
  Momentum is a vector, and since the direction of the velocity changes from before the collision to after the collision, the momentum also changes.
- ☐ 1 point is earned for selecting "yes" and a justification that indicates **both** of the following:
  · the presence of kinetic energy only (this can be implicit, by not mentioning other forms of energy)
  · that there is no change in the kinetic energy of the cart.

Example Response:

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Mechanical energy of the cart is made up of only the kinetic energy of the cart which is a scalar. This has the same value before and after the collision because it doesn't depend on the direction of the speed.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for using the correct 4 velocities from the graph (Cart 1 & 2, before & after collision) in a correct momentum calculation
- ☐ 1 point is earned for marking a response of "yes" or "no" consistent with calculations **and** calculating **one** of the following:
  - the change in momentum of each object and examining whether they are equal and opposite
  - the initial and final momenta of the two-cart system

Example Response:

$$\Delta p_1 = p_{1f} - p_{1i} = (0.2\text{ kg})(-2\text{ m/s}) - (0.2\text{ kg})(3\text{ m/s}) = -1\text{ kg m/s}$$
$$\Delta p_2 = p_{2f} - p_{2i} = (0.5\text{ kg})(1.5\text{ m/s}) - (0.5\text{ kg})(-0.5\text{ m/s}) = 1\text{ kg m/s}$$

OR

$$\Delta p_{\text{2-cartsystem}} = (p_{2f} + p_{1f}) - (p_{2i} + p_{1i})$$
$$\Delta p_{\text{2-cartsystem}} = ((0.5\text{ kg})(1.5\text{ m/s})) + ((0.2\text{ kg})(-2\text{ m/s})) - ((0.5\text{ kg})(-0.5\text{ m/s}) + (0.2\text{ kg})(3\text{ m/s}))$$
$$\Delta p_{\text{2-cartsystem}} = 0$$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating **one** of the following:
  - there is an external force that changes the momentum OR velocity of the system (center of mass)
  - there is an external force that changes the momentum OR velocity of Cart 2

  Example Response:
  There is a net force from the wall on cart 2 *when cart* 2 *hits the wall. This external force changes the*

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speed of the center of mass. The speed of the center of mass of the system is equal to the slope of the center of mass position as a function of time graph.

☐ 1 point is earned for indicating that there is no net external force on the two-cart system

Example Response:

During the two cart collision, the center of mass speed remains the same (the slope of the center of mass position as a function of time) because there is no external force exerted on the system.

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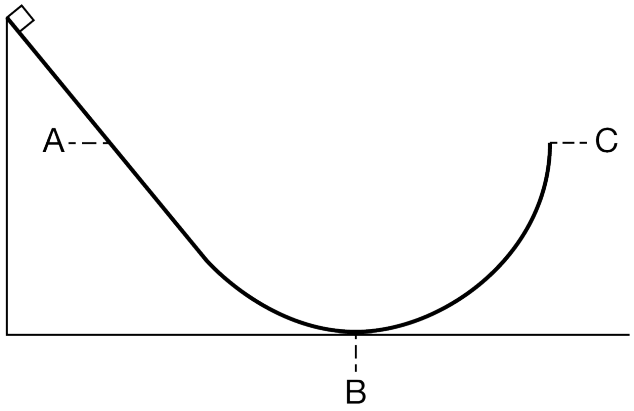
33. This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)



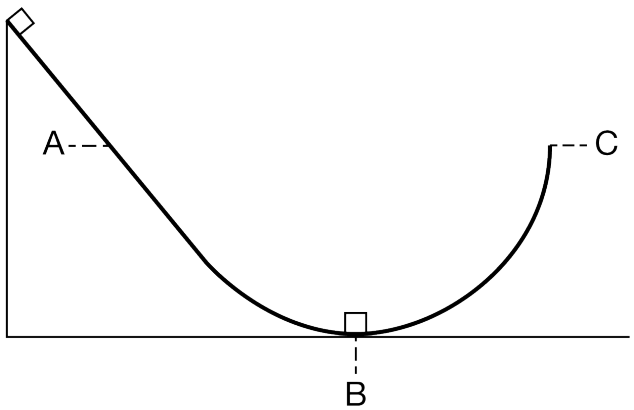
The figure above shows a track consisting of a straight section that connects to a circular section. Three locations are indicated: location A on the straight section of the track, location B at the bottommost point of the track, and location C at the point where the track is vertical. Locations A and C are at the same height. A block is released from rest at the top of the straight section of the track. There is negligible friction between the track and the block.

(a) Is the magnitude of the normal force exerted on the block by the track at location A greater than, less than, or equal to the normal force at location B? Justify your answer.

(b) In a coherent, paragraph-length response, describe the relative angle between the direction of the block's velocity and the direction of the block's acceleration at the following times.

When the block is at location A
When the block is at location B
Immediately before the block reaches location C

Specifically, indicate whether the velocity is parallel to the acceleration, perpendicular to the acceleration, or neither, at each location. Use physical principles to justify your responses.



(c) The scenario is repeated, but an identical block is at rest at location B, as shown above. When the block at the top of the straight section of the track is released from rest, it slides down the track and collides elastically with the identical block and comes to rest. The identical block then begins sliding up the ramp. Is the maximum height reached by the identical block greater than, less than, or equal to the maximum height reached by the block in the original scenario? Briefly explain your answer.

Part A

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating that the normal force at A is less than at B.
- ☐ 1 point is earned for a correct explanation including both locations A and B (e.g., at A the normal force is less than the weight, but at B it is greater than the weight).

Example Response:

At A, the block's acceleration is along the straight track, so the normal force must cancel the component of the gravitational force normal to the track and is therefore less in magnitude than the block's weight. At B, the block's acceleration and net force are upward, so the upward normal force must be greater than the block's weight.

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4	5	6 ✓

The response accurately includes **all** of the following criteria.

- ☐ 1 point is earned for indicating that the velocity is always parallel to the track.
- ☐ 1 point is earned for relating the acceleration to the net force.
- ☐ 1 point is earned for indicating the acceleration at location A is along the track and thus parallel to velocity.
- ☐ 1 point is earned for indicating that at location B the velocity is perpendicular to the acceleration.
- ☐ 1 point is earned for indicating that just before location C the acceleration is neither perpendicular nor parallel to the normal force.
- ☐ 1 point is earned for a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the

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paragraph-length response.

Example Response:

The velocity of the box is along the track.

At location A, the box is accelerating down the ramp because there is a net force exerted on the box down the ramp. The net force and acceleration always have the same direction. If the net force is down the track then the acceleration is down the track. So that makes the velocity and acceleration in the same direction at location A, therefore parallel.

At location B, the bottommost point, the box is moving in a circle. Here the velocity is still along the track. Since location B is the bottommost point of a circle, the net force and acceleration would be directed toward the center of that circle. So that makes the velocity and acceleration perpendicular at location B.

Just before location C, the box is both moving in a circle and slowing down as it rises above the ground. The velocity is in the direction of motion, which is upward. Since the box is slowing down, the acceleration must have a downward component. The acceleration would also have a component toward the center of the block's circular path. So, the acceleration is neither parallel nor perpendicular to the upward velocity at just before location C.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

The response accurately includes **both** of the following criteria.

- ☐ 1 point is earned for indicating the maximum height reached by the second block is the same as that of the original block.
- ☐ 1 point is earned for a correct explanation of why the second block reaches the same height.

Example Response:

Since the collision is elastic and the first block comes to rest, the second block has the same kinetic energy that the first block had just before the collision, which is the same as the first block's kinetic energy at the bottom of the ramp in the original scenario.