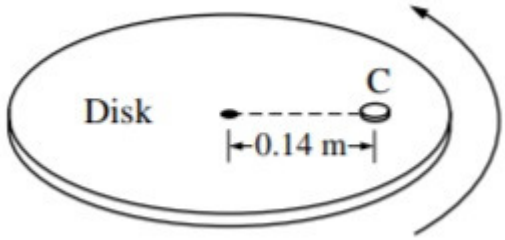
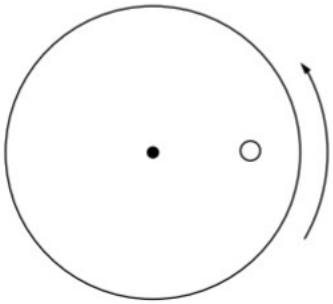
## chapter3

**1.**




A coin C of mass 0.0050 kg is placed on a horizontal disk at a distance of 0.14 m from the center, as shown above. The disk rotates at a constant rate in a counterclockwise direction as seen from above. The coin does not slip, and the time it takes for the coin to make a complete revolution is 1.5 s.

a. The figure below shows the disk and coin as viewed from above. Draw and label vectors on the figure below to show the instantaneous acceleration and linear velocity vectors for the coin when it is at the position shown.



b. Determine the linear speed of the coin.

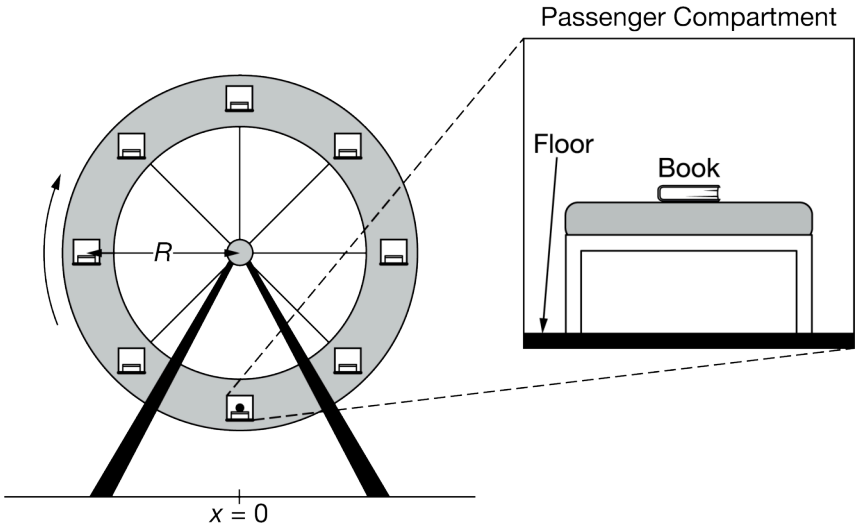
c. The rate of rotation of the disk is gradually increased. The coefficient of static friction between the coin and the disk is 0.50. Determine the linear speed of the coin when it just begins to slip.

d. If the experiment in part (c) were repeated with a second, identical coin glued to the top of the first coin, how would this affect the answer to part (c) ? Explain your reasoning.

**chapter3**

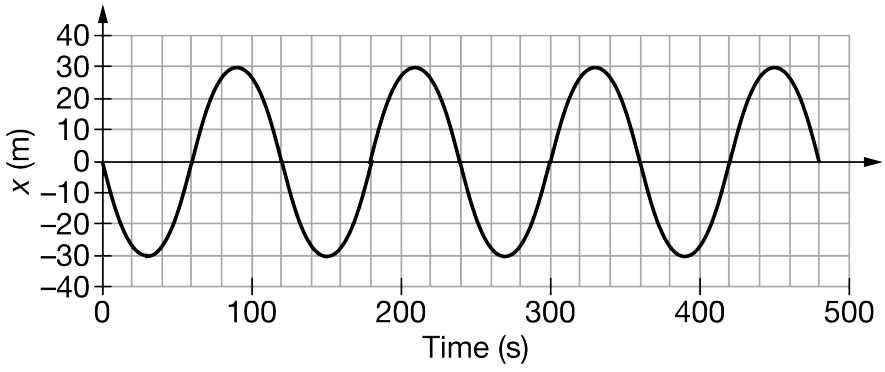
**2.** This question is a long free-response question. Show your work for each part of the question.

(12 points, suggested time 25 minutes)




A passenger compartment of a rotating amusement park ride contains a bench on which a book of mass $m_b$ is placed, as indicated by the dot in the left figure above. The compartment moves with a constant angular speed about the center of the ride along a circular path of radius $R$. The bench remains horizontal throughout the compartment's motion. The right figure above shows a magnified view of the compartment.

The graph below shows the horizontal $(x)$ component of the book's position as a function of time, where the $+x$ -direction is to the right.




(a)

i. Determine the period of revolution of the book.

ii. Calculate the tangential speed $v_b$ (not the angular speed) of the book.

**chapter3**

(b)

i. On the dot below, which represents the book, draw and label the forces (not components) that act on the book at the lowest point of its circular path. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



ii. At the lowest point of the circular path, the book is moving only in the horizontal direction. In what direction, if any, is the net vertical force on the book?

_____ Up _____ Down _____ No direction, since the net vertical force is equal to zero

Without deriving any equations, briefly explain your reasoning in terms of the book's motion.

(c) Derive an algebraic equation for the vertical force that the bench exerts on the book at the lowest point of the circular path in terms of the book's mass $m_b$, tangential speed $v_b$, radius $R$ of the path, and physical constants, as appropriate. Do not substitute any numerical values for variables or physical constants.

(d) At the lowest point of the circular path, is the force that the bench exerts on the book greater than, less than, or equal to the weight of the book?

_____ Greater Than _____ Less Than _____ Equal to

Briefly explain how your answers in (b)ii and (c) support your selection.

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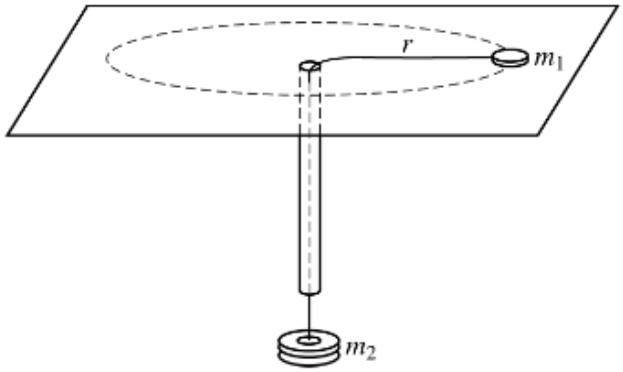
**3.**




An experiment is performed using the apparatus above. A small disk of mass $m_1$ on a frictionless table is attached to one end of a string. The string passes through a hole in the table and an attached narrow, vertical plastic tube. An object of mass $m_2$ is hung at the other end of the string. A student holding the tube makes the disk rotate in a circle of constant radius r, while another student measures the period P.

(a) Derive the equation $P = 2\pi\sqrt{\frac{m_1 r}{m_2 g}}$ that relates $P$ and $m_2$.

The procedure is repeated, and the period P is determined for four different values of $m_2$, where $m_1 = 0.012$ kg and $r = 0.80$ m. The data, which are presented below, can be used to compute an experimental value for $g$.

| | | | | |
|---|---|---|---|---|
| $m_2(kg)$ | 0.020 | 0.040 | 0.060 | 0.080 |
| $P$ (s) | 1.40 | 1.05 | 0.80 | 0.75 |
| | | | | |

(b) What quantities should be graphed to yield a straight line with a slope that could be used to determine $g$ ?

(c) On the grid below, plot the quantities determined in part (b), label the axes, and draw the best-fit line to the data. You may use the blank rows above to record any values you may need to calculate.

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(d) Use your graph to calculate the experimental value of $g$.

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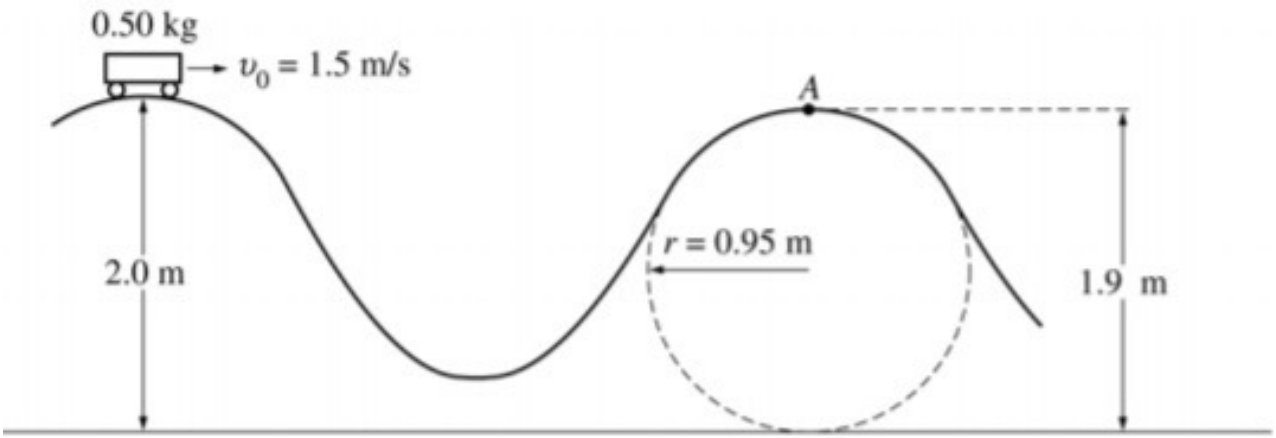
**4.**




A designer is working on a new roller coaster, and she begins by making a scale model. On this model, a car of total mass 0.50 kg moves with negligible friction along the track shown in the figure above. The car is given an initial speed $v_0 = 1.5$ m/s at the top of the first hill of height 2.0 m. Point *A* is located at a height of 1.9 m at the top of the second hill, the upper part of which is a circular arc of radius 0.95 m.

a. Calculate the speed of the car at point *A*.

b. On the figure of the car below, draw and label vectors to represent the forces on the car at point *A*.



c. Calculate the magnitude of the force of the track on the car at point *A*.

d. In order to stop the car at point A, some friction must be introduced. Calculate the work that must be done by the friction force in order to stop the car at point *A*.

e. Explain how to modify the track design to cause the car to lose contact with the track at point *A* before descending down the track. Justify your answer.

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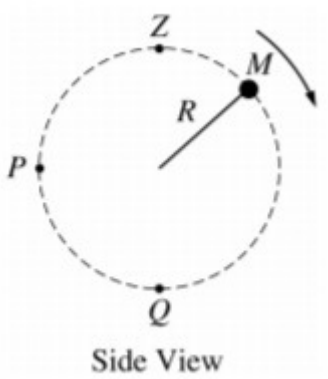
**5.**




A ball of mass $M$ is attached to a string of length $R$ and negligible mass. The ball moves clockwise in a vertical circle, as shown above. When the ball is at point $P$, the string is horizontal. Point $Q$ is at the bottom of the circle and point $Z$ is at the top of the circle. Air resistance is negligible. Express all algebraic answers in terms of the given quantities and fundamental constants.

a. On the figures below, draw and label all the forces exerted on the ball when it is at points $P$ and $Q$, respectively.



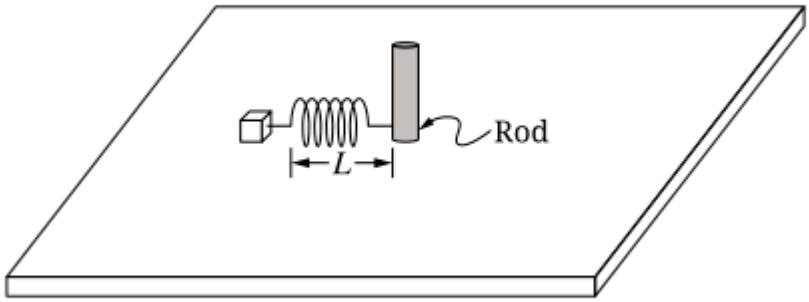

b. Derive an expression for $v_{min}$ , the minimum speed the ball can have at point $Z$ without leaving the circular path.

c. The maximum tension the string can have without breaking is $T_{max}$ . Derive an expression for $v_{max}$, the maximum speed the ball can have at point $Q$ without breaking the string.

d. Suppose that the string breaks at the instant the ball is at point $P$. Describe the motion of the ball immediately after the string breaks.

**chapter3**

**6.** This question is a long free-response question. Show your work for each part of the question.




(12 points, suggested time 25 minutes)

A small block of mass $m_0$ is attached to the end of a spring of spring constant $k_0$ that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length $L$, as shown. All frictional forces are negligible.

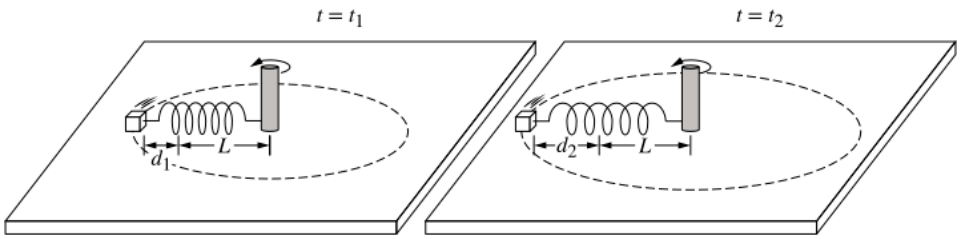



Figure 1 Figure 2

(a) At time $t = t_1$, the rod is spinning such that the block moves in a circular path with a constant tangential speed $v_1$ and the spring is stretched a distance $d_1$ from the spring's unstretched length, as shown in Figure 1. At time $t = t_2$, the rod is spinning such that the block moves in a circular path with a constant tangential speed $v_2$ and the spring is stretched a distance $d_2$ from the spring's unstretched length, where $d_2 > d_1$, as shown in Figure 2.

i. On the following dots, which represent the block at the locations shown in Figure 1 and Figure 2, draw the force that is exerted on the block by the spring at times $t = t_1$ and $t = t_2$. The spring force must be represented by a distinct arrow starting on, and pointing away from, the dot.
Note: Draw the relative lengths of the vectors to reflect the relative magnitudes of the forces exerted by the spring at both times.

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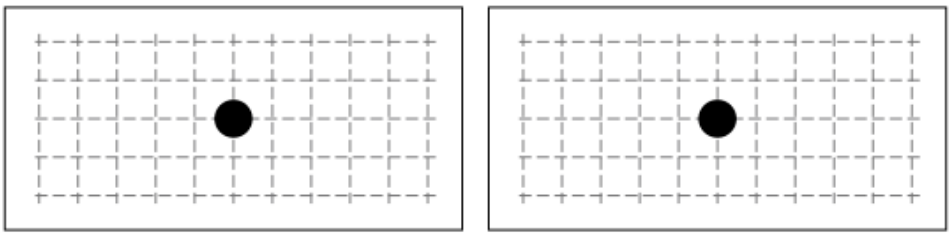


ii. Referencing $d_1$ and $d_2$, describe your reasoning for drawing the arrows the length that you did in part (a)(i).

iii. Is the tangential speed $v_1$ of the block at time $t = t_1$ greater than, less than, or equal to the tangential speed $v_2$ of the block at time $t = t_2$?

_____ $v_1 > v_2$ _____ $v_1 < v_2$ _____ $v_1 = v_2$

Justify your answer without using equations.

(b) Consider a scenario where the block travels in a circular path where the spring is stretched a distance $d$ from its unstretched length $L$.

i. Determine an expression for the magnitude of the net force $F_{\text{net}}$ exerted on the block. Express your answer in terms of $m_0$, $k_0$, $L$, $d$, and fundamental constants, as appropriate.

ii. Derive an equation for the tangential speed $v$ of the block. Express your answer in terms of $m_0$, $k_0$, $L$, $d$, and fundamental constants, as appropriate.

(c) Does your equation for the tangential speed $v$ of the block from part (b)(ii) agree with your reasoning from part (a) ?

____ Yes ____ No

Explain your reasoning.

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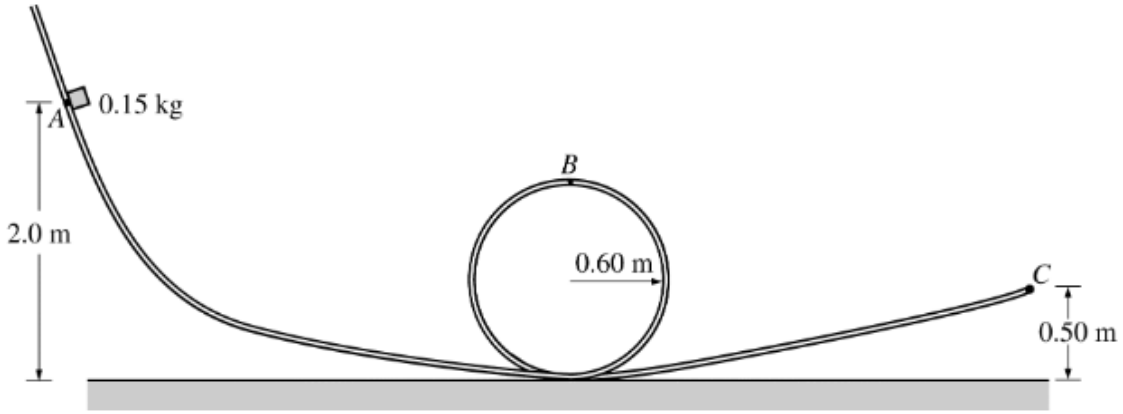
**7.**




A small block of mass 0.15 kg is placed at point *A* at a height 2.0 m above the bottom of a track, as shown in the figure above, and is released from rest. It slides with negligible friction down the track, around the inside of the loop of radius 0.60 m, and leaves the track at point *C* at a height 0.50 m above the bottom of the track.

(a) Calculate the speed of the block when it leaves the track at point *C*.

(b) On the figure below, draw and label the forces (not components) that act on the block when it is at the top of the loop at point *B*.

(c) Calculate the minimum speed the block can have at point *B* without losing contact with the track.

(d) Calculate the minimum height $h_{min}$ above the bottom of the track at which the block can be released and still go around the loop without losing contact with the track.

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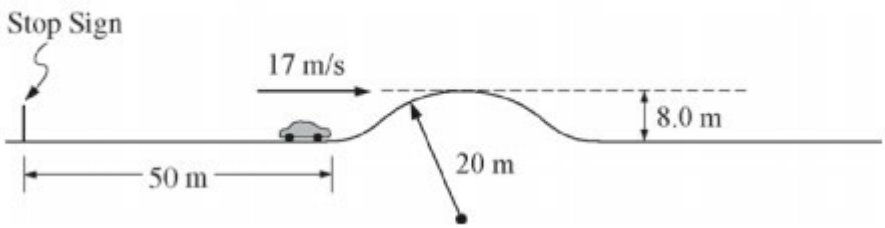
**8.**




You are assigned to do some calculations for a movie stunt that involves a car on a straight road. The road, pictured above, has a hill that rises 8.0 m above the flat region. The top of the hill is a circular arc of radius 20 m. You need to determine whether a car traveling under certain conditions will lose contact with the road at the top of the hill. There is a stop sign 50 m from the beginning of the hill. You are to assume that a car of mass 1600 kg accelerates uniformly from rest at the stop sign, has a speed of when it reaches the beginning of the hill, and then coasts with the engine off. Assume energy losses due to friction and air resistance are negligible.

a. Calculate the magnitude of the acceleration of the car during the first 50 m.

b. Calculate the time it takes the car to reach the beginning of the hill

c. Calculate the magnitude of the net force required to accelerate the car during the first 50 m.

d. On the dot below that represents the car, draw and label the forces (not components) that act on the car at the top of the hill if it travels over the hill without losing contact.



e. Calculate the minimum speed the car must have at the top of the hill to momentarily lose contact with the road. If you need to draw anything other than what you have shown in part (d) to assist in your solution, use the space below. Do NOT add anything to the figure in part (d).

f. Calculate the speed the car must have at the beginning of the hill in order to have the speed at the top of the hill you calculated in part (e).

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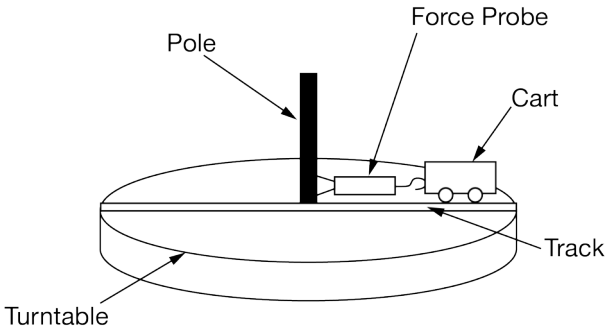
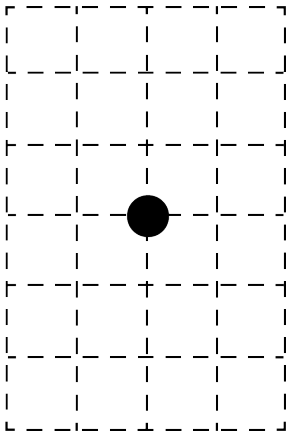
**9.** This question is a long free-response question. Show your work for each part of the question.




Students want to verify that the centripetal acceleration of an object undergoing uniform circular motion with tangential speed $v$ and radius $r$ can be described by the equation $a_c = \frac{v^2}{r}$ . In the experiment, the students use a variable speed turntable that can rotate about a pole at its center. A track is attached to the top of the turntable such that a cart of mass $m$ can freely move toward and away from the center pole of the turntable. One end of a force probe is attached to the pole, and the other end is attached to the cart, as shown above. All frictional forces are negligible.

(a) The dot below represents the cart while undergoing uniform circular motion and at the location shown in the figure above. On the dot, draw and label the forces (not components) exerted on the cart. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.
Note: Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces.



(b) Using the experimental apparatus shown above, design an experimental procedure the students could use to verify that the centripetal acceleration of an object undergoing uniform circular motion with tangential speed $v$ and radius $r$ can be described by the equation $a_c = \frac{v^2}{r}$ .

i. In the table below, list the quantities and associated symbols that would be measured in the experiment and the equipment used to measure them. Also list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

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| Quantity to Be Measured | Symbol for Quantity | Equipment for Measurement |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |

ii. Describe an experimental procedure to verify that the centripetal acceleration of an object undergoing uniform circular motion with tangential speed $v$ and radius $r$ can be described by the equation $a_c = \frac{v^2}{r}$ . Give enough detail so that another student could replicate the experiment. As needed, include a diagram of the experimental setup. Assume equipment usually found in a school physics laboratory is available.

iii. State how the measured quantities could be analyzed to verify that the centripetal acceleration of an object undergoing uniform circular motion with tangential speed $v$ and radius $r$ can be described by the equation $a_c = \frac{v^2}{r}$ .

(c) In a separate experiment, the students take force probe readings for different turntable speeds, and also measure the time it takes the turntable to make one revolution at each speed. The table shows the results of the students' measurements. The cart's center of mass moves in a circle of radius $R = 0.10\,\text{m}$.

| Period $(\text{s})$ | Force $(\text{N})$ | | |
|---|---|---|---|
| 0.75 | 3.5 | | |
| 1.25 | 1.3 | | |
| 2.0 | 0.49 | | |
| 2.5 | 0.32 | | |

Indicate two quantities that could be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass of the cart.

Vertical axis _____
Horizontal axis _____

Use the remaining columns in the table, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

On the grid, plot the data points you indicated. Clearly scale and label both axes, including units as appropriate.

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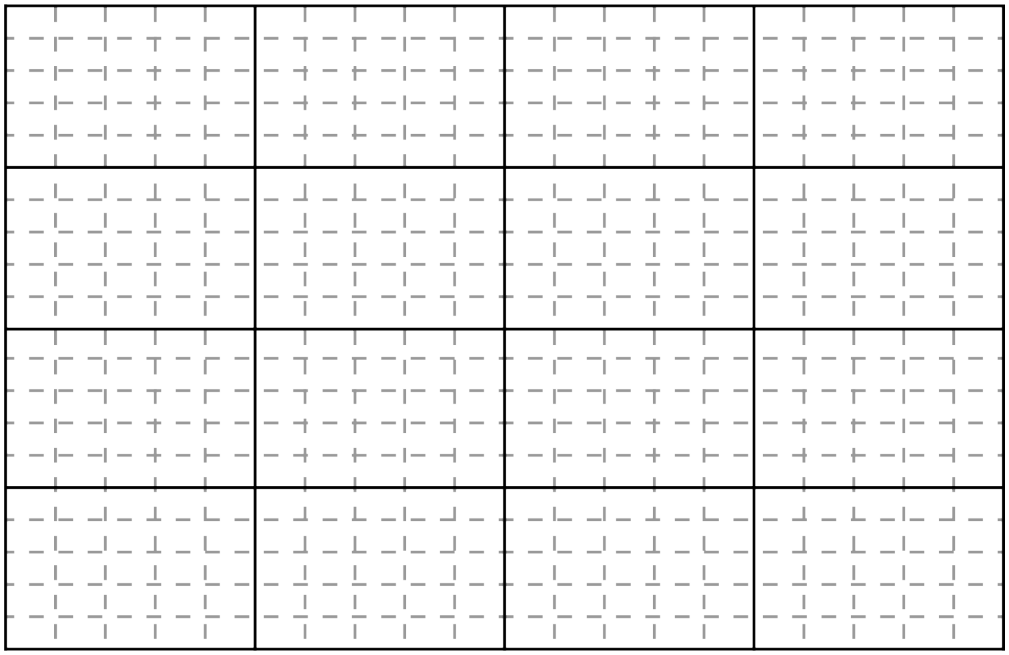
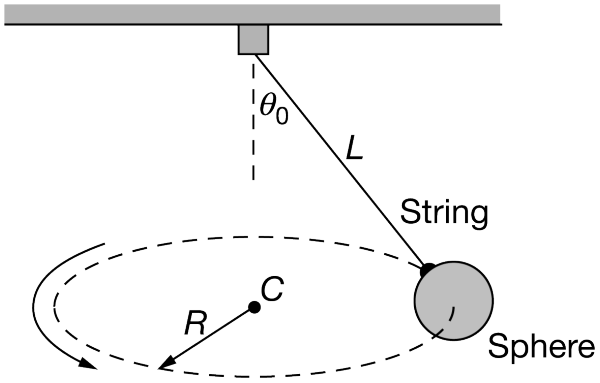


**chapter3**

**10.** This question is a long free-response question. Show your work for each part of the question.




(12 points, suggested time 25 minutes)

A small sphere of mass $M$ is suspended by a string of length $L$. The sphere is made to move in a horizontal circle of radius $R$ at a constant speed, as shown above. The center of the circle is labeled point $C$, and the string makes an $\theta_0$ with the vertical.

(a) Two students are discussing the motion of the sphere and make the following statements.

Student 1: None of the forces exerted on the sphere are in the direction of point C, the center of the circular path. Therefore, I don't see how there can be a centripetal force exerted on the sphere to make it move in a circle.

Student 2: I see another problem. The tension force exerted by the string is at an angle from the vertical. Therefore, its vertical component must be less than the weight $Mg$ of the sphere. That means the net force on the sphere has a downward vertical component, and the sphere should move downward as well as moving around in a circle.

i. What is one aspect of Student 1's reasoning that is incorrect?

ii. What is one aspect of Student 2's reasoning that is incorrect?

(b)

i. Derive an equation for the magnitude of the net force exerted on the sphere. Express your answer in terms of $M$, $\theta_0$, and physical constants, as appropriate.

ii. Describe one aspect or step in your derivation of part (b)(i) that can be correctly linked to your answer to either part (a)(i) or part (a)(ii).

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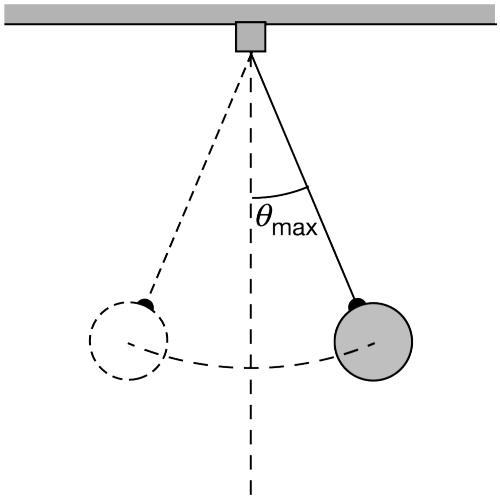
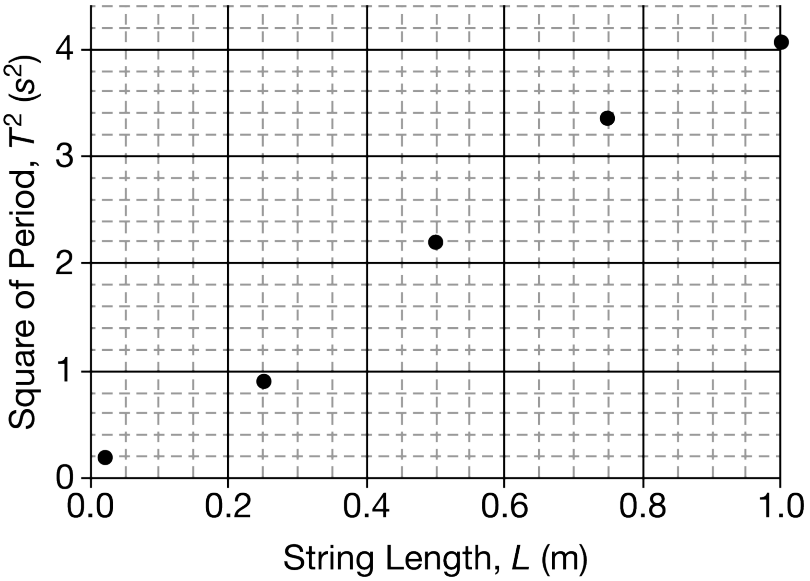



Instead of moving in a horizontal circle, the sphere now moves in a vertical plane so that it is a simple pendulum, as shown above. The maximum angle $\theta_{max}$ that the string makes from the vertical can be assumed to be small. The graph below shows data for the square of the pendulum period $T$ as a function of string length $L$.




(c) On the graph above, draw a best-fit line for the data. Then use the line to calculate a numerical value for the gravitational acceleration $g$.

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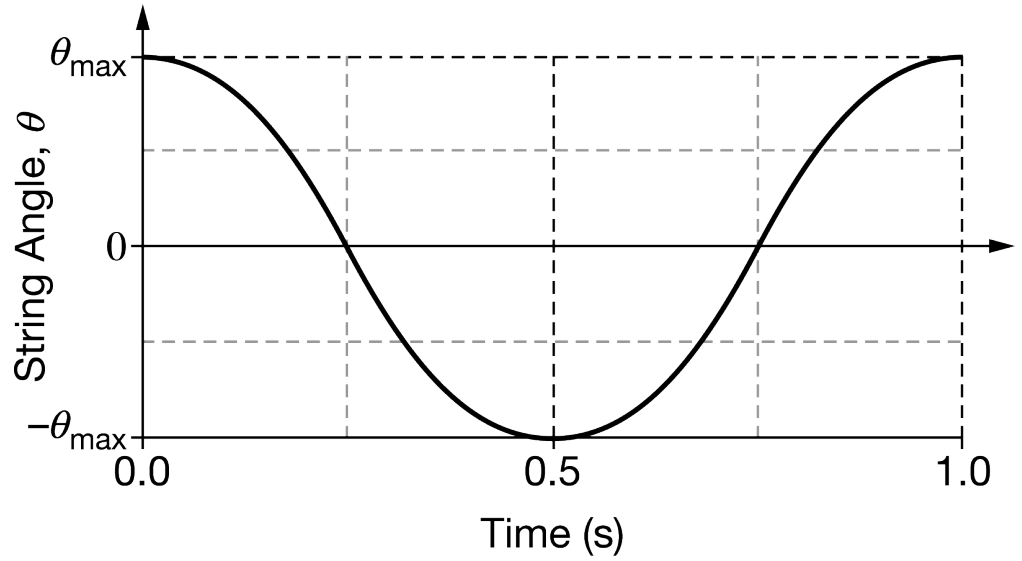
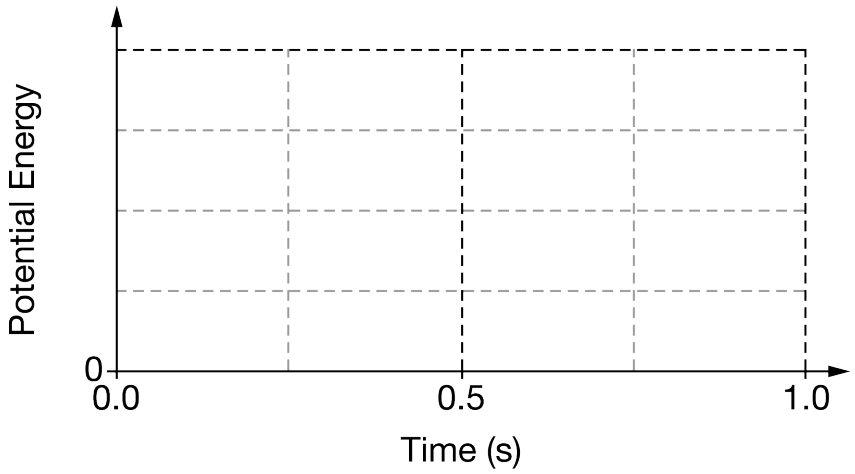
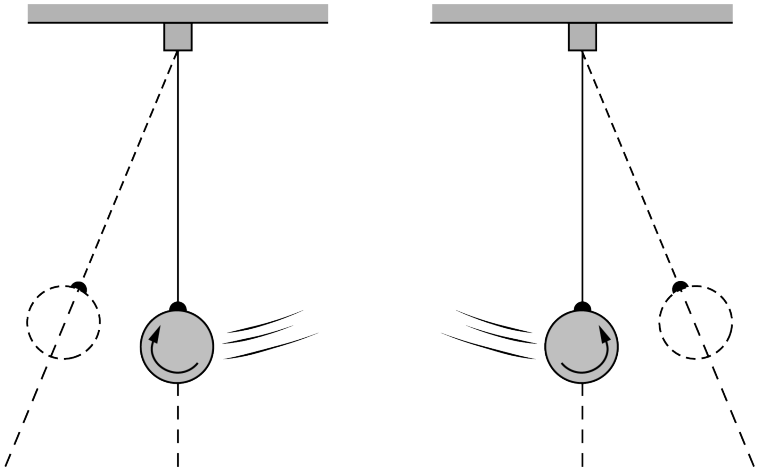



(d) The graph above shows the angle $\theta$ from the vertical as a function of time for the pendulum. On the axes below, sketch a graph of the gravitational potential energy of the sphere-Earth system for the same time interval. Take the zero of potential energy to be when the potential energy has its minimum value.




(e) As the sphere swings back and forth, it must also rotate a small amount during each swing. The figures below indicate the direction that the sphere rotates as it is swinging in each direction.

**chapter3**



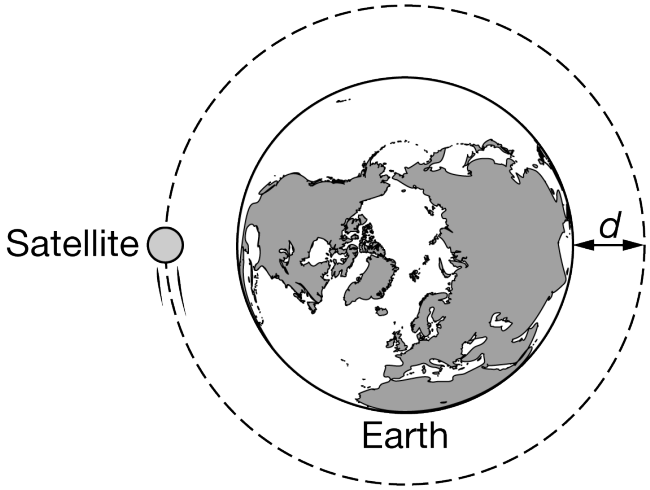
In order for the sphere's rotation to change direction, a torque must be exerted on the sphere. When the sphere is at its maximum rightward displacement, what is the direction of the torque exerted on the sphere with respect to the point of attachment between the sphere and string?

_____ Clockwise _____ Counterclockwise

Briefly state why the torque is in the direction you indicated.

**chapter3**

**11.** This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)




Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed $v$ and at a constant distance $d$ from Earth's surface, as shown above.

An identical satellite is set into orbit around a new planet. The new planet has the same radius as the radius of Earth but is significantly more massive than Earth. The satellite orbits the same distance $d$ from the surface of the new planet.

(a) Is the speed of the satellite orbiting the new planet greater than, less than, or the same as the speed of the satellite orbiting Earth? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

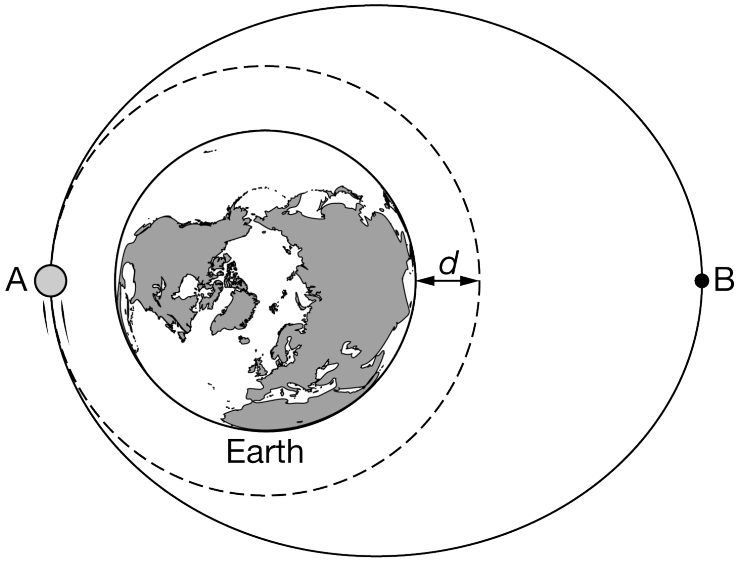
$$\frac{MT^2}{R^3} = C$$

In the equation above

$M$ is the mass of a planet;
$T$ is the orbital period of a satellite in a circular orbit;
$R$ is the distance from the planet's center to the satellite; and
$C$ is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

**chapter3**




Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point A to point B, and state the following:

**Student 1**: The satellite is moving faster at point A because the satellite's angular momentum is greater at point A than at point B.

**Student 2**: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is higher at point B than at point A.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

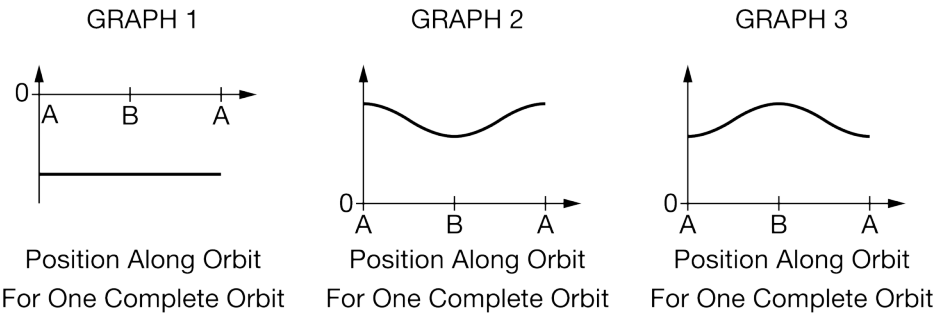
(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite as it completes one elliptical orbit, starting and ending at point A.

**chapter3**

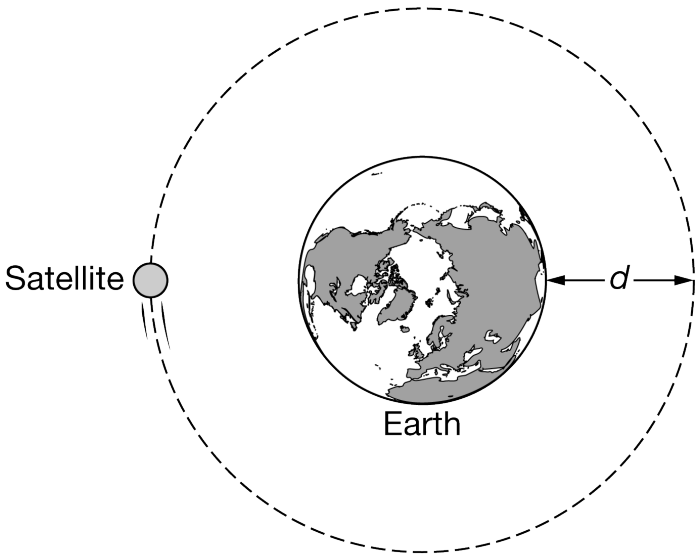



(h) Which of the above graphs best represents the magnitude of the gravitational field at the location of the satellite as it orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the total mechanical energy of the satellite-Earth system as the satellite orbits? Briefly explain your reasoning.

**chapter3**

**12.** This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)




Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed $v$ and at a constant distance $d$ from Earth's surface, as shown above.

An identical satellite is set into orbit around a new planet. The new planet's radius is the same as the radius of Earth but is significantly more massive than Earth. The satellite orbits the new planet with the same orbital period.

(a) Is the distance of the satellite from the surface of the new planet greater than, less than, or equal to $d$ ? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

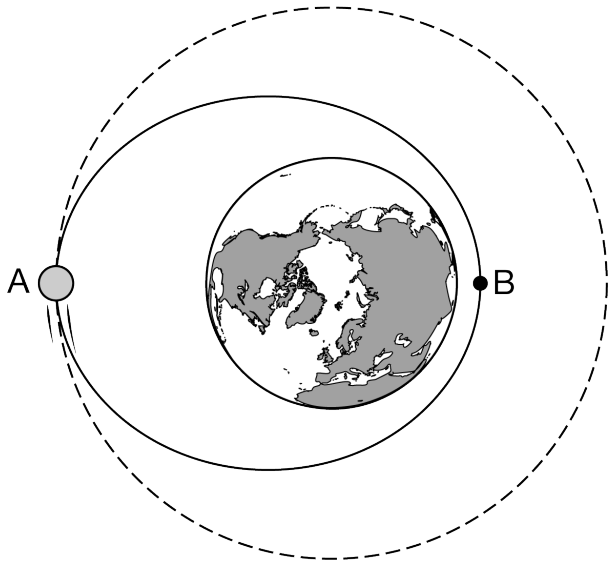
$\frac{Rv^2}{M} = C$

In the equation above

$R$ is the distance from a planet's center to a satellite in a circular orbit;
$v$ is the speed of the satellite;
$M$ is the mass of the planet; and
$C$ is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

**chapter3**




Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point A to point B, and state the following:

**Student 1**: The satellite is moving faster at point A because the total mechanical energy of the satellite-Earth system is constant.

**Student 2**: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is higher at point B than at point A.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

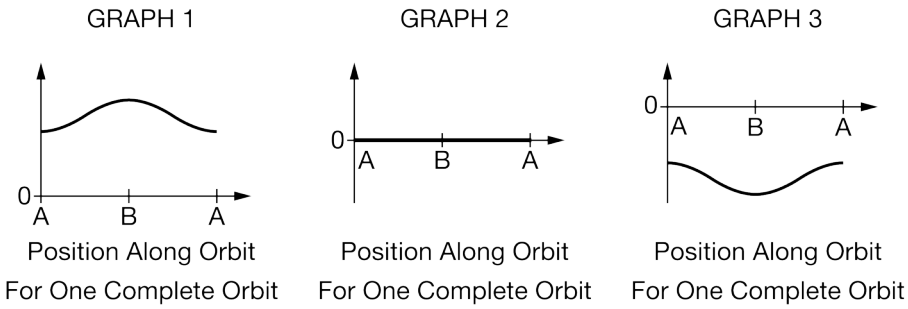
(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite

as it completes one elliptical orbit, starting and ending at point A.

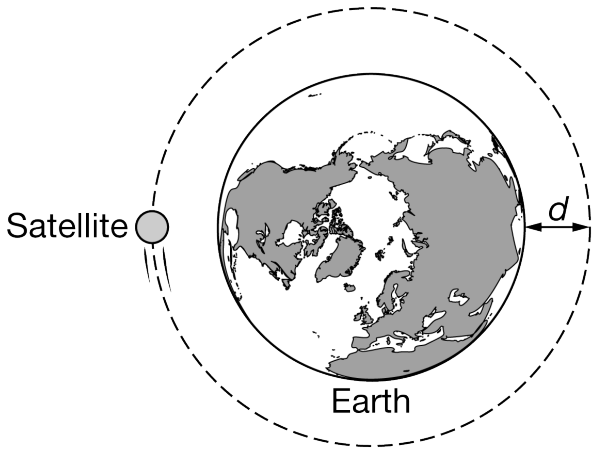



(h) Which of the above graphs best represents the potential energy of the satellite-Earth system as the satellite orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the torque about the center of Earth exerted on the satellite by Earth's gravitational force as the satellite orbits? Briefly explain your reasoning.

**chapter3**

**13.** This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)




Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed $v$ and at a constant distance $d$ from Earth's surface.

An identical satellite is set into orbit around a new planet. The new planet's radius is the same as the radius of Earth but the new planet is significantly more massive than Earth. The satellite orbits the new planet with the same speed $v$.

(a) Is the distance of the satellite from the surface of the new planet greater than, less than, or equal to $d$ ? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

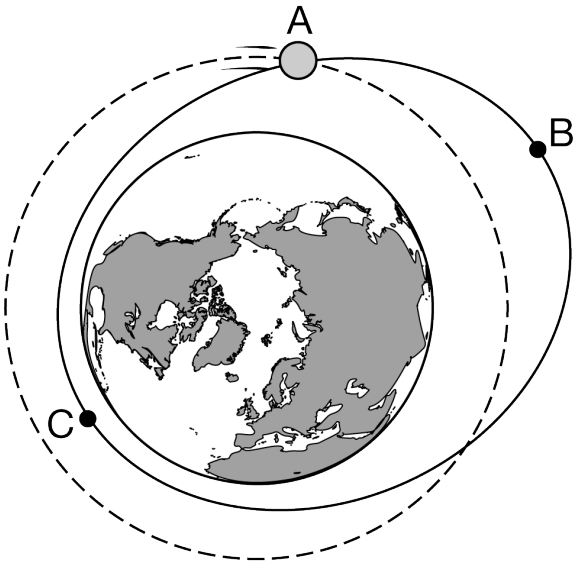
$$\frac{R^3}{MT^2} = C$$

In the equation above

$R$ is the distance from a planet's center to a satellite in a circular orbit;
$M$ is the mass of the planet;
$T$ is the orbital period of the satellite; and
$C$ is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

**chapter3**




Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit. In the modified orbit, the satellite's speed at point A, after the thrusters were fired, is the same as it was in the original orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point B to point C, and state the following:

**Student 1**: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is negative.

**Student 2**: The satellite is moving faster at point C because the total mechanical energy of the satellite-Earth system is greater at point C than at point B.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

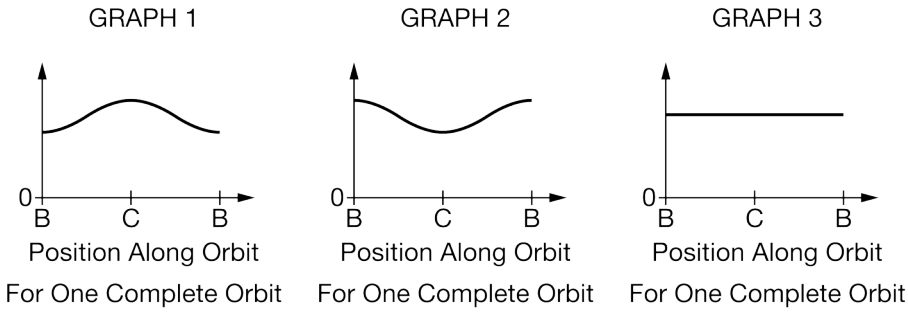
(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

**chapter3**

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite as it completes one elliptical orbit, starting and ending at point B.

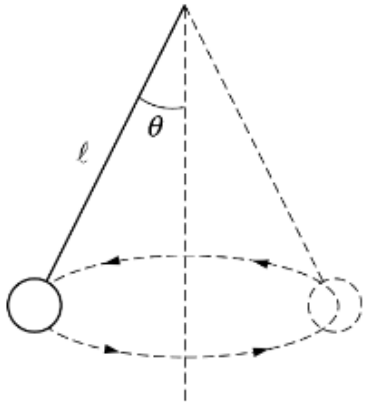



(h) Which of the above graphs best represents the magnitude of the acceleration of the satellite as it orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the magnitude of the angular momentum of the satellite with respect to the center of Earth as the satellite orbits? Briefly explain your reasoning.

## chapter3

**14.**




A ball attached to a string of length $l$ swings in a horizontal circle, as shown above, with a constant speed. The string makes an angle $\theta$ with the vertical, and $T$ is the magnitude of the tension in the string. Express your answers to the following in terms of the given quantities and fundamental constants.

a. On the figure below, draw and label vectors to represent all the forces acting on the ball when it is at the position shown in the diagram. The lengths of the vectors should be consistent with the relative magnitudes of the forces.



b. Determine the mass of the ball.

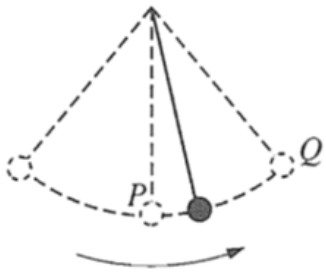
c. Determine the speed of the ball.

d. Determine the frequency of revolution of the ball.

e. Suppose that the string breaks as the ball swings in its circular path. Qualitatively describe the trajectory of the ball after the string breaks but before it hits the ground.

**chapter3**

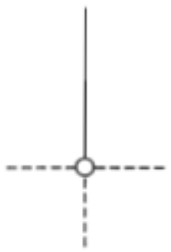
**15.**




A heavy ball swings at the end of a string as shown above, with negligible air resistance. Point *P* is the lowest point reached by the ball in its motion, and point *Q* is one of the two highest points.

(a) On the following diagrams draw and label vectors that could represent the velocity and acceleration of the ball at points *P* and *Q*. If a vector is zero, explicitly state this fact. The dashed lines indicate horizontal and vertical directions.

i. Point *P*



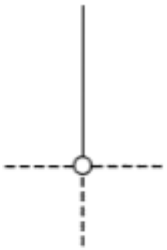
ii. Point *Q*



(b) After several swings, the string breaks. The mass of the string and air resistance are negligible. On the following diagrams, sketch the path of the ball if the break occurs when the ball is at point *P* or point *Q*. In each case, briefly describe the motion of the ball after the break.

## chapter3

i. Point $P$

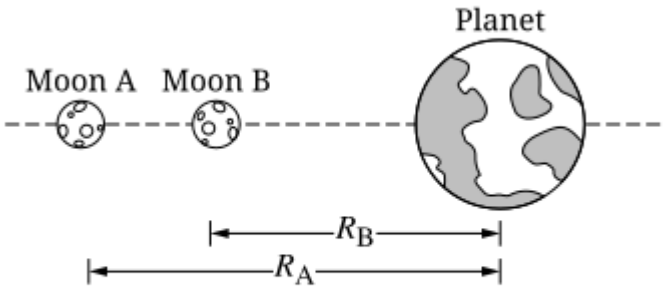


ii. Point $Q$



**chapter3**

**16.** This question is a long free-response question. Show your work for each part of the question.

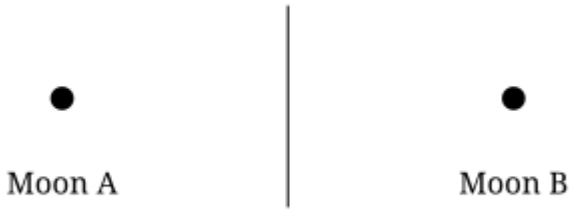



Note: Figure not drawn to scale.

(12 points, suggested time 25 minutes)

Two identical moons, Moon A and Moon B, orbit a planet. The mass $m_0$ of each moon is significant, but less than the mass $m_p$ of the planet. At some point in their orbits, the planet and the two moons are aligned as shown in the figure.

(a) The following dots represent the two moons when they are at the locations shown in the previous figure. On each dot, draw and label the forces (not components) exerted on Moon A and on Moon B. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.




(b) Consider the net gravitational force exerted on each moon due to the planet and the other moon.

i. Justify why the magnitude of the net force exerted on Moon A could be larger than the magnitude of the net force exerted on Moon B.

ii. Justify why the magnitude of the net force exerted on Moon B could be larger than the magnitude of the net force exerted on Moon A.

(c) Derive expressions for both of the following quantities. Express your answers in terms of $m_0$, $m_p$, $R_\mathrm{A}$, $R_\mathrm{B}$, and physical constants, as appropriate.

• The net force $F_\mathrm{A}$ exerted on Moon A

• The net force $F_\mathrm{B}$ exerted on Moon B

(d)

i. Could the expressions in part (c) support your reasoning in part (b)(i) ?

___ Yes ___ No

Explain your reasoning.

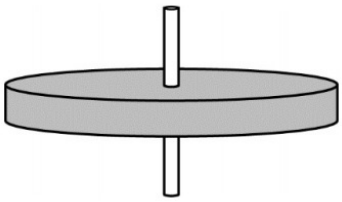
ii. Could the expressions in part (c) support your reasoning in part (b)(ii) ?

___ Yes ___ No

Explain your reasoning.

**chapter3**

**17.**



A disk-shaped platform has a known rotational inertia. The platform is mounted on a fixed axle and rotates in a horizontal plane, as shown above. A student wishes to determine the frictional torque exerted on the platform by the axle as the platform rotates. The student has access to equipment that would usually be found in a school physics laboratory.

a. Describe an experimental procedure the student could use to collect the data needed to find the frictional torque exerted on the platform while it rotates.

    i. What quantities would be measured?

    ii. What equipment would be used for the measurements, and how would that equipment be used?

    iii. Describe the overall procedure to be used, including any steps necessary to reduce experimental uncertainty. Give enough detail so that another student could replicate the experiment.

b. Describe how the data from the measurements could be analyzed to determine the frictional torque exerted on the rotating platform.

It is often assumed that the frictional force between two surfaces is independent of their relative speed. However, the details of the axle's construction are unknown, and it is possible that the frictional torque for the axle depends on the platform's angular speed.

c. Does the experiment described in parts (a) and (b) depend on the assumption that the frictional torque is independent of the platform's angular speed?

    ____ Yes ____ No

    Note: Either answer may be correct, depending on your experimental design.

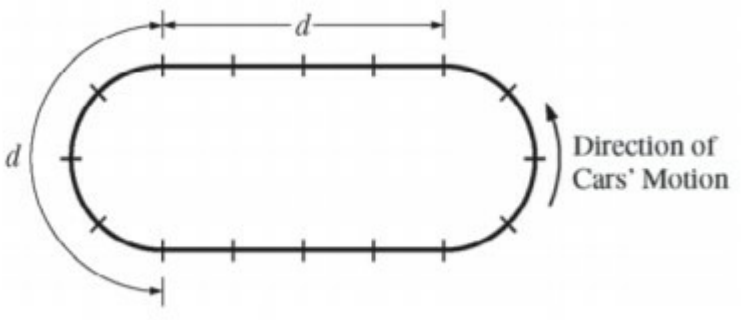
    If you answered yes, describe which part of your analysis depends on this assumption.

    If you answered no, explain how your analysis does not depend on this assumption.

d. Briefly describe how the experimental procedure described in parts (a) and (b) could be modified to determine whether the frictional torque stays constant as the angular speed changes. If no modification is necessary, state that explicitly. In either case, describe how the data could be analyzed to determine whether the frictional torque stays constant as the angular speed changes.

**chapter3**

**18.**




The figure above represents a racetrack with semicircular sections connected by straight sections. Each section has length $d$, and markers along the track are spaced $d/4$ apart. Two people drive cars counterclockwise around the track, as shown. Car $X$ goes around the curves at constant speed $v_c$, increases speed at constant acceleration for half of each straight section to reach a maximum speed of $2v_c$ , then brakes at constant acceleration for the other half of each straight section to return to speed $v_c$. Car $Y$ also goes around the curves at constant speed $v_c$, increases speed at constant acceleration for one-fourth of each straight section to reach the same maximum speed $2v_c$, stays at that speed for half of each straight section, then brakes at constant acceleration for the remaining fourth of each straight section to return to speed $v_c$.

(a) On the figures below, draw an arrow showing the direction of the net force on each of the cars at the positions noted by the dots. If the net force is zero at any position, label the dot with 0.




(b)

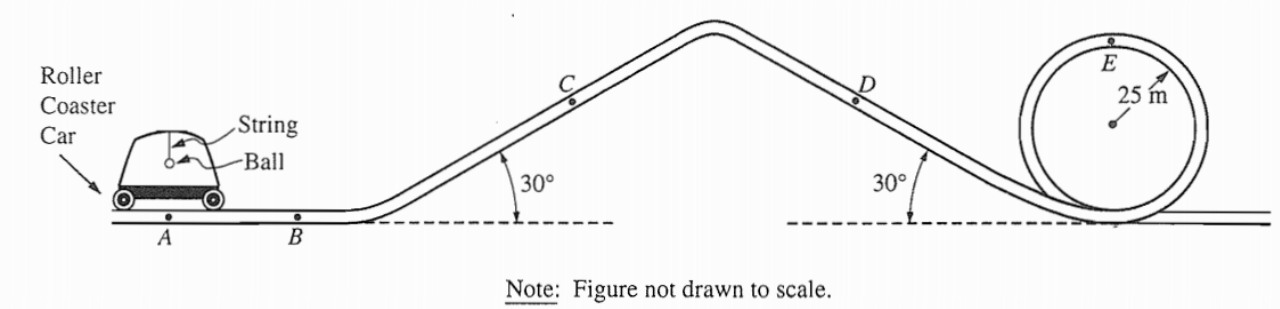
i. Indicate which car, if either, completes one trip around the track in less time, and justify your answer quantitatively without using equations.

ii. Justify your answer about which car, if either, completes one trip around the track in less time quantitatively with appropriate equations.

(c) Explain how your equations in part (b) ii reexpress your reasoning in part (b) i. Do not simply refer to any final results of your calculations, but instead indicate how terms in your equations correspond to concepts in your qualitative explanation.

## chapter3

**19.**




Part of the track of an amusement park roller coaster is shaped as shown above. A safety bar is oriented lengthwise along the top of each car. In one roller coaster car, a small 0.10-kilogram ball is suspended from this bar by a short length of light, inextensible string.

(a) Initially, the car is at rest a point A.

i. On the diagram to the right, draw and label all the forces acting on the 0.10-kilogram ball. 

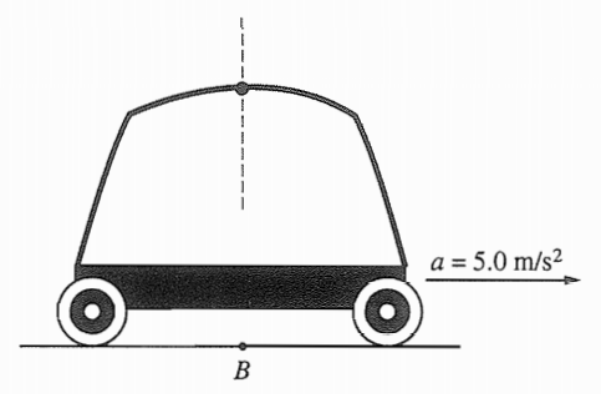
ii. Calculate the tension in the string.

The car is then accelerated horizontally, goes up a $30^o$ incline, goes down a $30^o$ incline, and then goes around a vertical circular loop of radius 25 meters. For each of the four situations described in parts (b) to (e), do all three of the following. In each situation, assume that the ball has stopped swinging back and forth.

• Determine the horizontal component $T_h$ of the tension in the string in newtons and record your answer in the space provided.

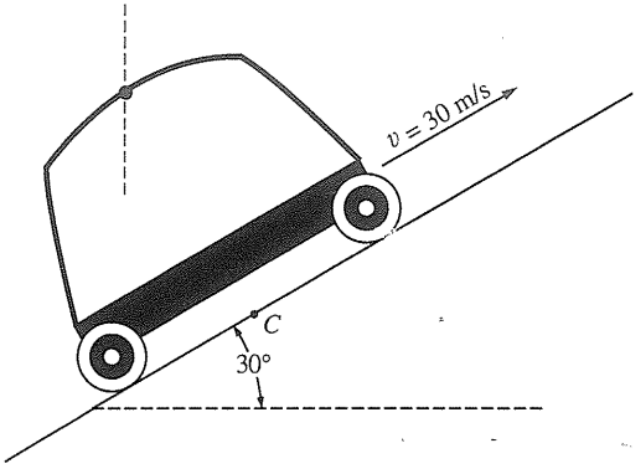
• Determine the vertical component $T_v$ of the tension in the string in newtons and record your answer in the space provided.

• Show on the adjacent diagram the approximate direction of the string with respect to the vertical. The dashed line shows the vertical in each situation.



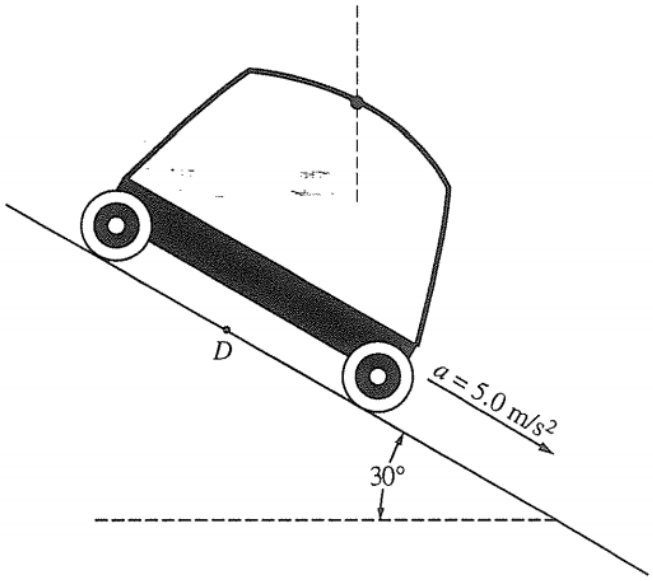
(b) The car is at point B moving horizontally to the right with an acceleration of 5.0 m/s$^2$.

$T_h$ =_____N $T_v$ =_____N




(c) The car is at point C and is being pulled up the 30$^o$ incline with a constant speed of 30 m/s

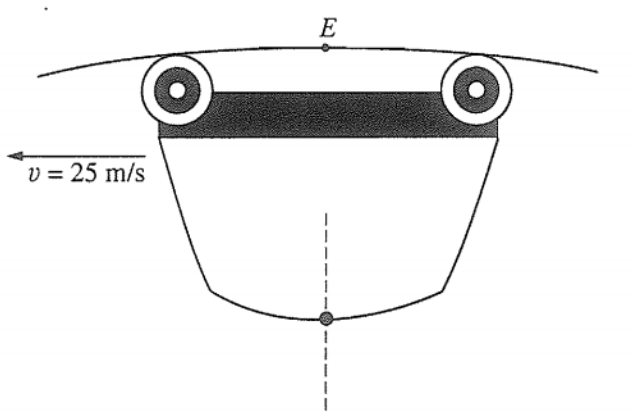
$T_h$ =_____N $T_v$ =_____N




(d) The car is at point D moving down the 30$^o$ incline with an acceleration of 5.0 m/s$^2$.

$T_h$ =_____N $T_v$ =_____N

**chapter3**

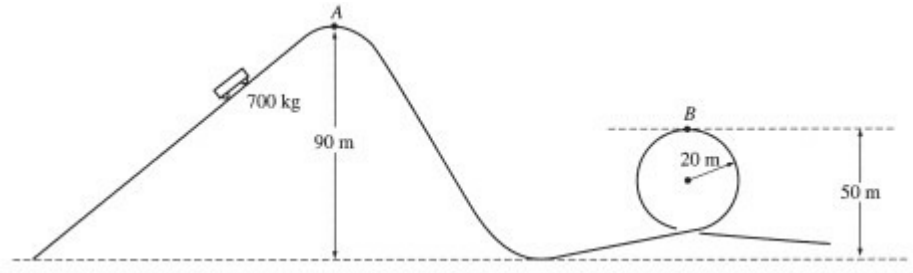



(e) The car is at point E moving upside down with an instantaneous speed of 25 m/s and no tangential acceleration at the top of the vertical loop of radius 25 meters.

$T_h$ =_____N $T_v$ =_____N

## chapter3

**20.**




A roller coaster ride at an amusement park lifts a car of mass 700 kg to point *A* at a height of 90 m above the lowest point on the track, as shown above. The car starts from rest at point *A*, rolls with negligible friction down the incline and follows the track around a loop of radius 20 m. Point *B*, the highest point on the loop, is at a height of 50 m above the lowest point on the track.

(a)

i. Indicate on the figure the point *P* at which the maximum speed of the car is attained.

ii. Calculate the value $v_{max}$ of this maximum speed.

(b) Calculate the speed $v_B$ of the car at point *B*.

(c)

i. On the figure of the car below, draw and label vectors to represent the forces acting on the car when it is upside down at point *B*.



ii. Calculate the magnitude of all the forces identified in (c)i.

(d) Now suppose that friction is not negligible. How could the loop be modified to maintain the same speed at the top of the loop as found in (b)? Justify your answer.

## chapter3

**21.**



The Sojourner rover vehicle shown in the sketch above was used to explore the surface of Mars as part of the Pathfinder mission in 1997. Use the data in the tables below to answer the questions that follow.

| Mars Data | Sojourner Data |
|---|---|
| Radius: 0.53 x Earth's radius<br>Mass: 0.11 x Earth's mass | Mass of Sojourner vehicle: 11.5 kg<br>Wheel diameter: 0.13 m<br>Stored energy available: 5.4 x $10^5$J<br>Power required for driving under average conditions: 10W<br>Land speed: 6.7 x $10^{-3}$ m/s |

1. Determine the acceleration due to gravity at the surface of Mars in terms of *g*, the acceleration due to gravity at the surface of Earth.

2. Calculate Sojournerís weight on the surface of Mars.

3. Assume that when leaving the Pathfinder spacecraft Sojourner rolls down a ramp inclined at 20° to the horizontal. The ramp must be lightweight but strong enough to support Sojourner. Calculate the minimum normal force that must be supplied by the ramp.

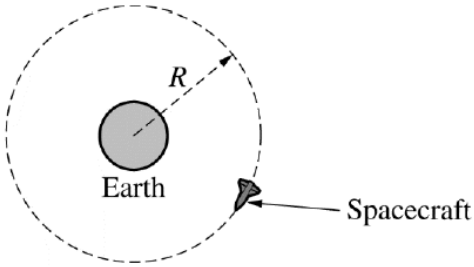
4. What is the net force on Sojourner as it travels across the Martian surface at constant velocity? Justify your answer.

5. Determine the maximum distance that Sojourner can travel on a horizontal Martian surface using its stored energy.

6. Suppose that 0.010% of the power for driving is expended against atmospheric drag as Sojourner travels on the Martian surface. Calculate the magnitude of the drag force.

**chapter3**

**22.**

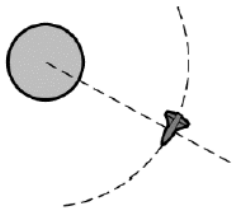



Note: Figure not drawn to scale.

A spacecraft of mass *m* is in a clockwise circular orbit of radius *R* around Earth, as shown in the figure above.

The mass of Earth is $M_E$.

a. In the figure below, draw and label the forces (not components) that act on the spacecraft. Each force must be represented by a distinct arrow starting on, and pointing away from, the spacecraft.



Note: Figure not drawn to scale.

b. i. Derive an equation for the orbital period *T* of the spacecraft in terms of *m, $M_E$, R*, and physical constants, as appropriate. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

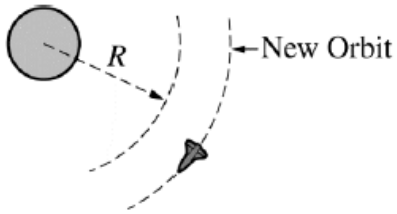
ii. A second spacecraft of mass *2m* is placed in a circular orbit with the same radius R. Is the orbital period of the second spacecraft greater than, less than, or equal to the orbital period of the first spacecraft?

____Greater than ____Less than ____Equal to

Briefly explain your reasoning.

c. The first spacecraft is moved into a new circular orbit that has a radius greater than *R*, as shown in the figure below.

**chapter3**




Note: Figure not drawn to scale.

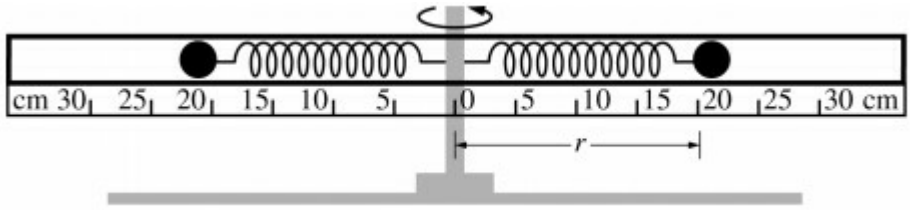
Is the speed of the spacecraft in the new orbit greater than, less than, or equal to the original speed?

____Greater than ____Less than ____Equal to

Briefly explain your reasoning.

## chapter3

**23.**




The apparatus shown above consists of two identical springs of negligible mass, each with spring constant $k$ = 20.4 N/m and each attached at one end to a vertical rotating pole. Identical small spheres of mass $m$ are attached to the other ends of the springs. The spheres are constrained to horizontal motion by horizontal guides of negligible friction, each of which has a ruler below it for measuring the radial position $r$ of the sphere. The system can be manually rotated about the pole's axis. In a lab experiment, a student adjusts the rotational speed so that the spheres move to a desired radius $r$. For each such value of $r$, the student measures the rotational period $T$. The student's partially completed data table is shown below. The length of each unstretched spring is $L$ = 0.15 m.

| Trial | Radial Position $r$ (m) | Period $T$ (s) | Acceleration $a_c$ $\left(\text{m/s}^2\right)$ | Spring Force $F_{\text{spring}}$ (N) |
|---|---|---|---|---|
| 1 | 0.300 | 0.440 | 61.2 | 3.06 |
| 2 | 0.270 | 0.475 | 47.2 | 2.45 |
| 3 | 0.240 | 0.530 | | |
| 4 | 0.210 | 0.570 | 25.5 | 1.22 |
| 5 | 0.180 | 0.795 | 11.2 | 0.61 |

(a) Calculate the missing values from the table above. Show your work in the space below

(b) On the axes below, plot the data points for the spring force $F_{\text{spring}}$ as a function of the acceleration $a_c$. Label the axes, including the scale. Draw a straight line that best represents the data

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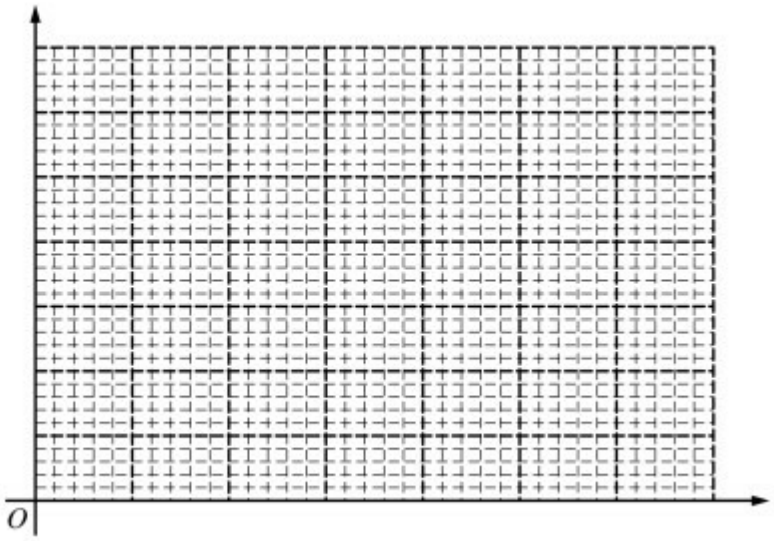
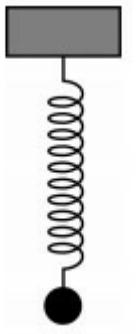



(c)

i. Calculate the slope of your line.

ii. Indicate what the slope calculated in part (c)-i represents.

(d) One sphere and one spring are removed from the rotation apparatus. They are hung vertically so that the sphere is now suspended from the spring, as shown below.



i. Describe a procedure you could use, and the measurements you would make, to verify the value obtained in part (c) using the setup shown above.

ii. Show how you would use the measurements described in part (d)-i to verify the value obtained in part (c).

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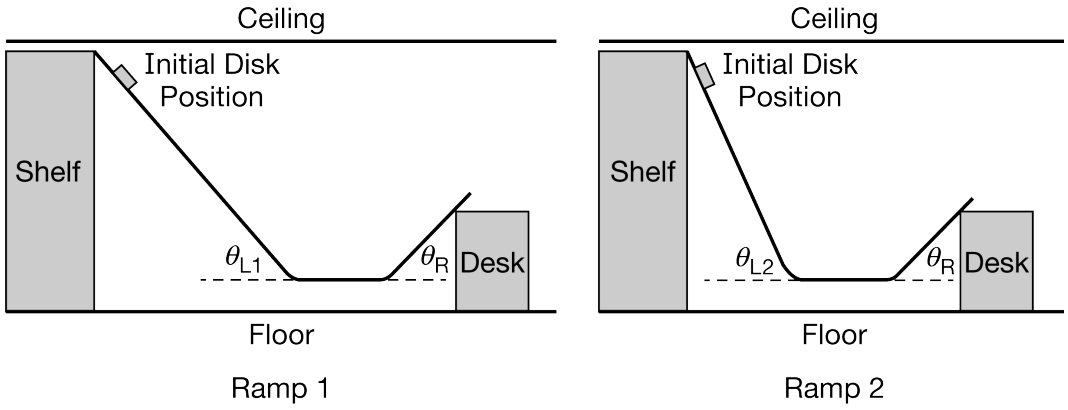
**24.** This question is a short free-response question. Show your work for each part of the question. (10 points, suggested time 15 minutes)

A class of physics students is investigating projectile motion using a variety of equipment. Assume air resistance can be ignored in all cases.

One group constructs a long ramp as shown in Ramp 1 below. In their experiment, they release a disk from the left side of the ramp and allow the disk to launch off the right side. The ramp is smooth enough that friction can be ignored. The left side of the ramp is supported by the top of a shelf, and the other side is supported by the top of a desk. The left and right sides of the ramp are at angles $\theta_{\mathrm{L1}}$ and $\theta_{\mathrm{R}}$ with the horizontal respectively.




Note: Figures not drawn to scale.

The students release a disk from Ramp 1 and measure the maximum height it reaches after leaving the right side of the ramp. The students then increase the angle of the left ramp so that $\theta_{\mathrm{L1}} < \theta_{\mathrm{L2}}$, and they keep the top of the ramp against the shelf as shown in the Ramp 2 configuration above so that the disk is released at the same height.

(a) Will the disk launched from Ramp 2 reach a maximum height that is greater than, less than, or equal to the maximum height reached by the disk launched from Ramp 1? Justify your answer.

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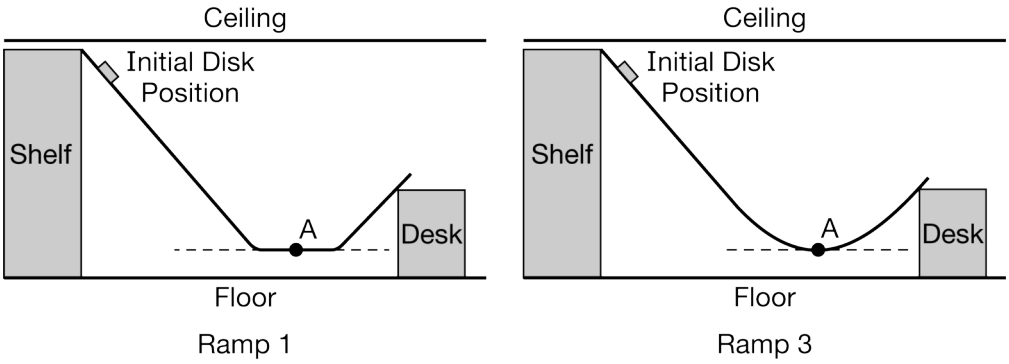
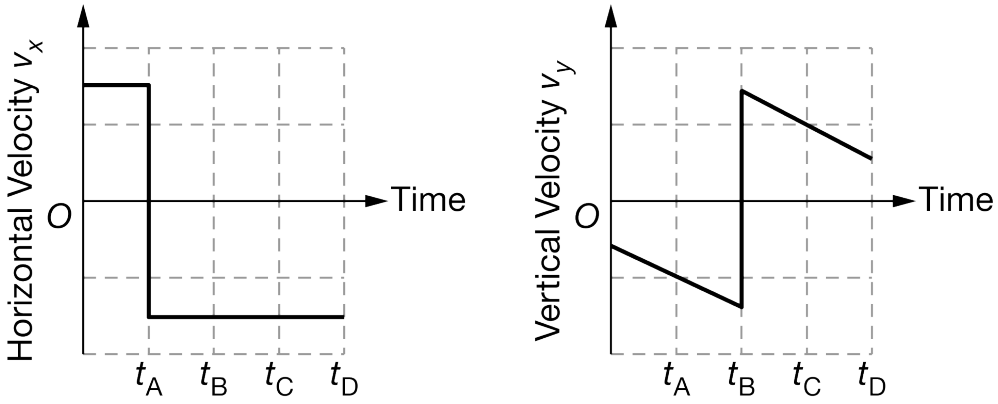



Note: Figures not drawn to scale.

Next, the students adjust the ramp into the configuration labeled Ramp 3 in which the lowest part of the ramp has been shaped into a circular curve. The lowest point of Ramp 3 is at the same height as the lowest section of Ramp 1. Both ramps are shown above.

(b) On which ramp, Ramp 1 or Ramp 3, is the force exerted by the ramp on the disk at point A greater? Explain your answer.




Another group uses a device that can launch a steel ball. The group launches the ball and it bounces off the ceiling, floor, and wall several times though not necessarily in that order. Each collision can be modeled as elastic, and friction between the ball and the surfaces is negligible. The graphs above show the horizontal and vertical components of the velocity of the ball during some time interval in the ball's motion. The positive direction is taken to be upward.

(c) During the time interval shown in the graphs, the ball has two collisions. In a clear, coherent, paragraph-length response, identify the times when each of these collisions occur using the times indicated on the graph, and explain how you know whether each collision is with the wall, the ceiling, or the floor.

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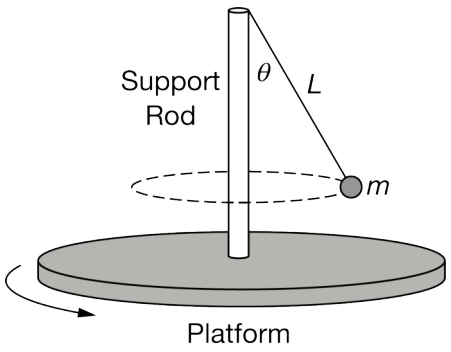
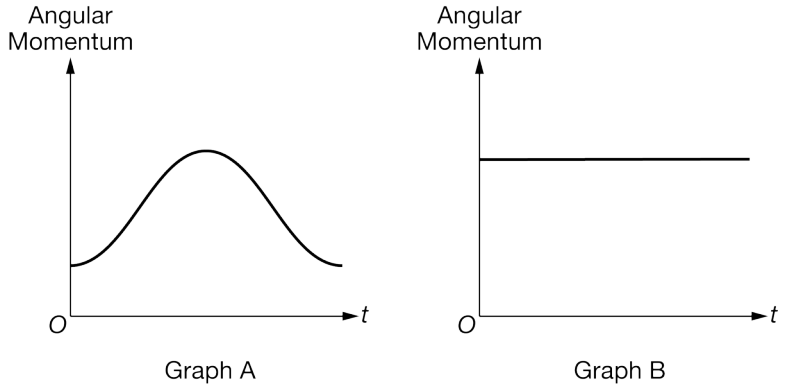
**25.** This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)




A vertical support rod is fixed at the center of a platform. A light string of length $L$ is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass $m$, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed $\omega$ (omega), the string makes an angle $\theta$ (theta) with the rod.




(a) The platform rotates with a constant angular speed. Which of the above graphs correctly shows the magnitude of the angular momentum of the platform as a function of time $t$ about a vertical axis at the center of the support rod? Briefly justify your answer.

(b) The platform rotates with a constant angular speed. Is the net torque exerted on the platform about its center clockwise, counterclockwise, or zero? Explain your reasoning.

(c) As the period of rotation becomes very small, what value does $\theta$ approach? Without writing any equations or formulas, justify your answer.

(d) A student derives an equation for the square of the period of revolution $T$ of the sphere as a function of $\theta$: $T^2 = \left(4\pi^2\right)\frac{L\cos\theta}{g}$. Indicate whether the equation is consistent with your answer in part (c) and explain why.

(e) In a series of experimental trials, a group of students changes $L$ and adjusts $T$ to keep $\theta$ constant. They record $L$ and $T$ for each trial. If the students use the equation given in part (d), what quantities could they graph to yield a straight line whose slope could be used to find the acceleration due to gravity, $g$ ?

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(f) As the angular speed of the platform increases, does the period of revolution of the sphere increase, decrease, or stay the same? Briefly explain your reasoning.

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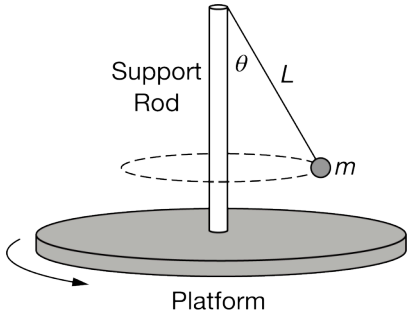
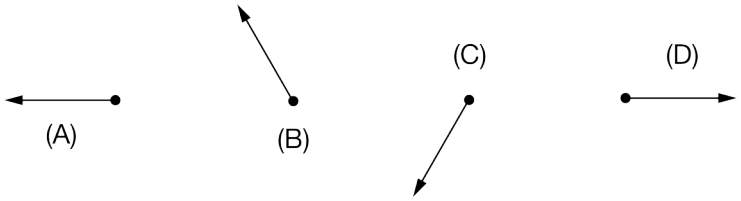
**26. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)**




A vertical support rod is fixed at the center of a platform. A light string of length $L$ is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass $m$, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed $\omega$ (omega), the string makes an angle $\theta$ (theta) with the rod.




(a) Identify which of the above vectors represents the net force exerted on the sphere at the moment shown above. Briefly justify your answer.

(b) Is the magnitude of the tension in the string greater than, less than, or equal to the weight of the sphere? Explain your reasoning.

(c) The string can never be completely horizontal no matter how fast the platform is rotating. Without writing any equations or formulas, explain why.

One of the following two equations for the square of the tangential speed $v$ of the sphere as a function of $\theta$ is consistent with the statement in part (c) that the string can never be horizontal as the platform is rotating.

**Equation 1:** $v^2 = \frac{gL(\sin^2\theta)}{(\cos\theta)}$ **Equation 2:** $v^2 = \frac{gL(\cos^2\theta)}{(\sin\theta)}$

(d) Indicate which equation is consistent with the statement in part (c), and explain why. Also explain why the other equation is not consistent with the statement in part (c).

(e) In a series of experimental trials, a group of students changes $L$ and adjusts $v$ to keep $\theta$ constant. They record $L$ and $v$ for each trial. Based on the equation in part (d) that you indicated as consistent with the statement in part (c), what quantities could the students graph to yield a straight line whose slope could be used to find the acceleration due to gravity, $g$ ?

(f) Let $I$ be the rotational inertia of the sphere-rod-platform system about a vertical axis through the center of the

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support rod. Does $I$ increase, decrease, or stay the same when $L$ increases (but $\theta$ stays constant)? Briefly explain your reasoning.

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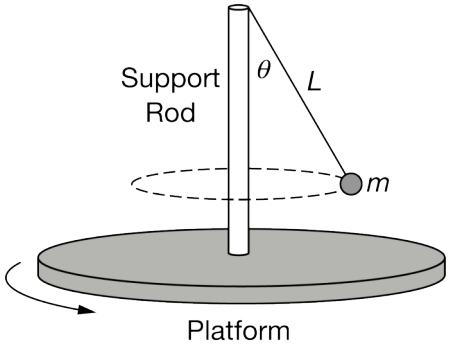
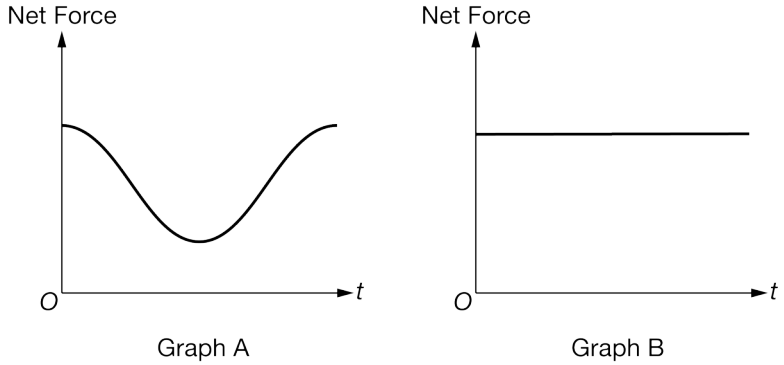
**27.** This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)




A vertical support rod is fixed at the center of a platform. A light string of length $L$ is attached to the top of the support rod, and the other end of the string is attached to a sphere of mass $m$, as shown above. The platform rotates with an angular speed that can be varied. When the platform rotates with a constant angular speed $\omega$ (omega), the string makes an angle $\theta$ (theta) with the rod.




(a) The platform rotates with a constant angular speed. Which of the above graphs correctly shows the magnitude of the net force exerted on the sphere as a function of time $t$ ? Briefly justify your answer.

(b) Is the magnitude of the tension in the string greater than, less than, or equal to the centripetal force exerted on the sphere? Explain your reasoning.

(c) The string is nearly vertical when the sphere's tangential speed is very small. Without writing any equations or formulas, explain why.

(d) A student derives an equation for the square of the tangential speed $v$ of the sphere as a function of $\theta$ : $v^2 = gL(\tan\theta)(\sin\theta)$. Indicate whether the equation is consistent with the statement in part (c) and explain why.

(e) In a series of experimental trials, a group of students adjusts $v$ and measures $\theta$ while keeping all other variables constant. They record $v$ and $\theta$ for each trial. If the students use the equation given in part (d), what quantities could they graph to yield a straight line whose slope could be used to find the acceleration due to gravity, $g$ ?

(f) Let $I$ be the rotational inertia of the sphere-rod-platform system about a vertical axis through the center of the

support rod. Does $I$ increase, decrease, or stay the same when $\theta$ decreases? Briefly explain your reasoning.

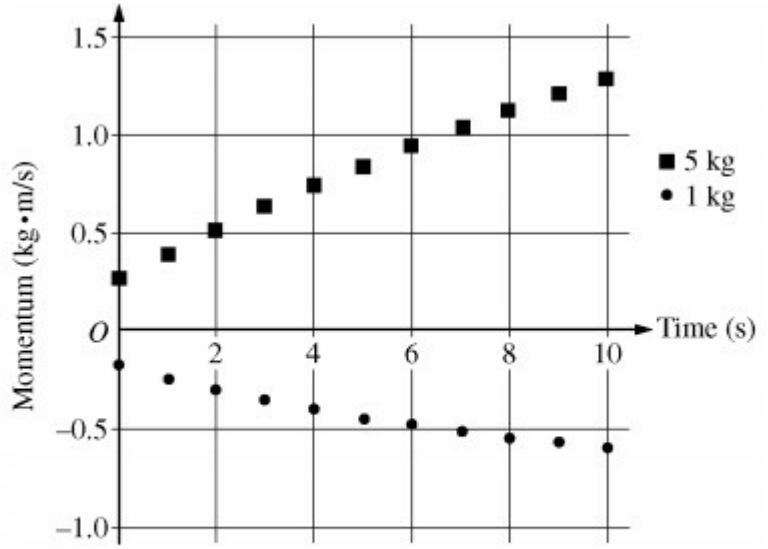
**28.**




Top View

The figure above represents two carts, with magnets attached, that make up a system. The mass of one cart and magnet is 1 kg and the mass of the other is 5 kg. The carts are initially at rest on a frictionless track. They are released from rest and exert a repulsive force on each other. The track is not quite horizontal, with the right side slightly lower than the left side.

The speeds of the carts are measured over a 10 s interval. The graph below shows the momentum of the two carts as a function of time for this interval as they move along the x-axis.




a. Based on the graph, were the measurements started at the instant the carts were released?

   Justify your answer.

b. Calculate the magnitude of the external force exerted on the system.

c. Suppose the experiment is repeated with different carts, so that the masses of cart plus magnet are 2 kg and 4 kg. Would your answer to part (b) be different with the new masses?

   Justify your answer.

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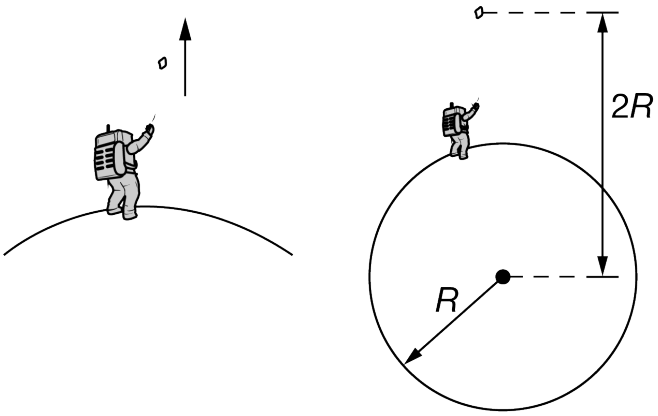
**29.** This question is a short free-response question. Show your work for each part of the question.

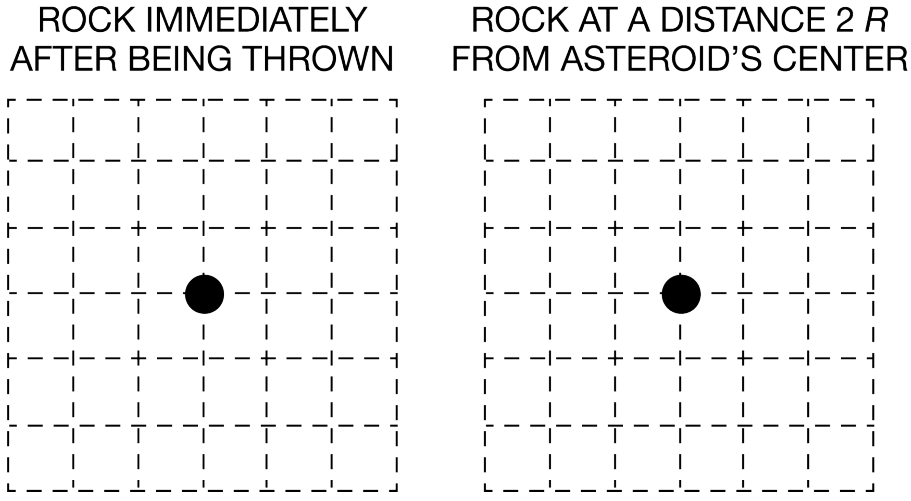



Note: Figures not drawn to scale.

An astronaut stands on the surface of a spherical asteroid that has a weak gravitational field but no atmosphere. The astronaut throws a small rock straight upward, as shown at left above. The rock travels upward until it reaches a distance of $2R$ from the center of the asteroid, where $R$ is the asteroid's radius, and then falls back down to the asteroid's surface.

(a) The dots below represent the rock immediately after being thrown and when it is a distance $2R$ from the asteroid's center. On each dot, draw and label the forces (not components) exerted on the rock at the indicated time. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Note: Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces at both times.




(b) The graph at left below shows the rock's velocity as a function of time after being thrown upward with an initial velocity $v_{01}$. The dashed line shows what the velocity would be if the rock experienced a constant acceleration, and the solid curve shows the rock's actual velocity.

After the rock lands back on the surface, the astronaut then throws the rock straight upward a second time but with

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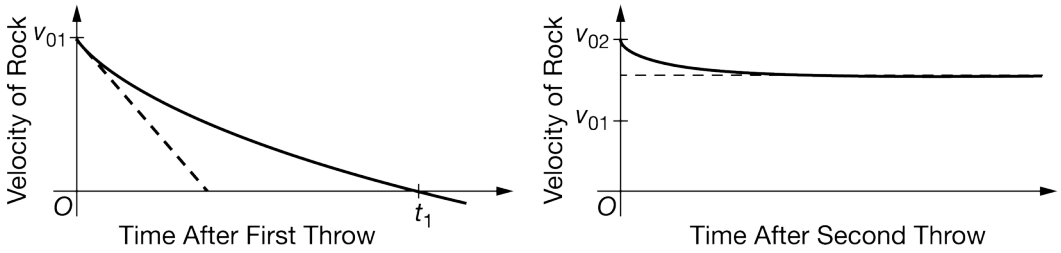
a greater force so that the rock's initial speed $v_{02}$ is greater than initial speed $v_{01}$ for the first throw. The graph at right below shows the rock's velocity as a function of time after being thrown upward the second time.




In a clear, coherent, paragraph-length response that may also contain equations and/or drawings, make claims for the following aspects of the rock's motion.

For the first throw, state a claim for what is happening at time $t_1$.
State a claim about the strength of the gravitational force exerted on the rock as the rock gets farther from the asteroid.
For the first throw, state a claim about the relationship about the strength of the gravitational force exerted on the rock as the rock gets farther from the asteroid and the behavior indicated between time 0 and $t_1$ in the velocity-time graph.
For the second throw, state a claim about why there is a horizontal asymptote for the velocity.