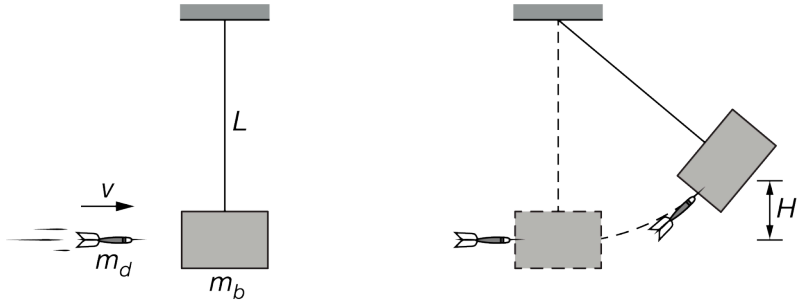
chapter4

1. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown at left above. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown at right above. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, the dart or the block, if either, experiences an impulse of larger magnitude during the collision. If the impulse is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the speed of the dart as it embeds itself into the block is greater than *v*, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the motion of the dart-block system at the instant it reaches height *H*. The students make the following statements:

Student 1: The speed of the system at height *H* is zero. Since it is moving in circular motion and centripetal acceleration is given by $a_c = \frac{v^2}{r}$, the acceleration must also be zero.

Student 2: The acceleration of the system at height *H* can't be zero, or the system would not swing back down. The acceleration must be equal to *g* at height *H*.

(c) Which underlined phrase or phrases, if either, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if either, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if either, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases, if either, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{m_d v}{m_d + m_b}\right)^2$. Explain how this equation supports your claim in part (b).

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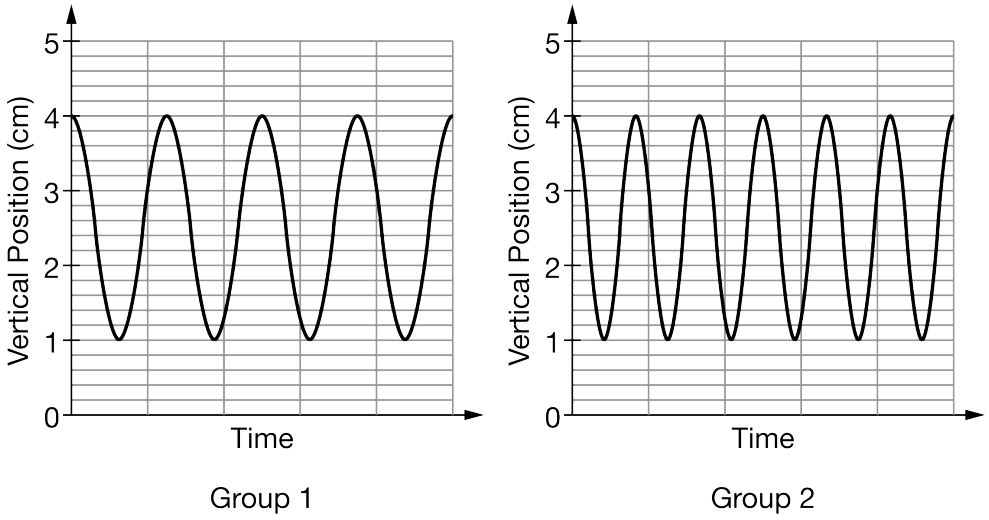
The students perform several experimental trials. In each trial, a different speed v is used, and the resulting maximum height reached by the center of mass of the dart-block system is measured.

(h) Which quantities could the students graph to produce a straight line whose slope can be used to find the acceleration due to gravity, g ?

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart mass, initial dart speed, block mass, or string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

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2. A new kind of toy ball is advertised to "bounce perfectly elastically" off hard surfaces. A student suspects, however, that no collision can be perfectly elastic. The student hypothesizes that the collisions are very close to being perfectly elastic for low-speed collisions but that they deviate more and more from being perfectly elastic as the collision speed increases.

a. Design an experiment to test the student's hypothesis about collisions of the ball with a hard surface. The student has equipment that would usually be found in a school physics laboratory.

   i. What quantities would be measured?

   ii. What equipment would be used for the measurements, and how would that equipment be used?

   iii. Describe the procedure to be used to test the student's hypothesis. Give enough detail so that another student could replicate the experiment.

b. Describe how you would represent the data in a graph or table. Explain how that representation would be used to determine whether the data are consistent with the student's hypothesis.

c. A student carries out the experiment and analysis described in parts (a) and (b). The student immediately concludes that something went wrong in the experiment because the graph or table shows behavior that is elastic for low-speed collisions but appears to violate a basic physics principle for high-speed collisions.

   i. Give an example of a graph or table that indicates nearly elastic behavior for low-speed collisions but appears to violate a basic physics principle for high-speed collisions.

   ii. State one physics principle that appears to be violated in the graph or table given in part (c)i. Several physics principles might appear to be violated, but you only need to identify one.

   Briefly explain what aspect of the graph or table indicates that the physics principle is violated, and why.

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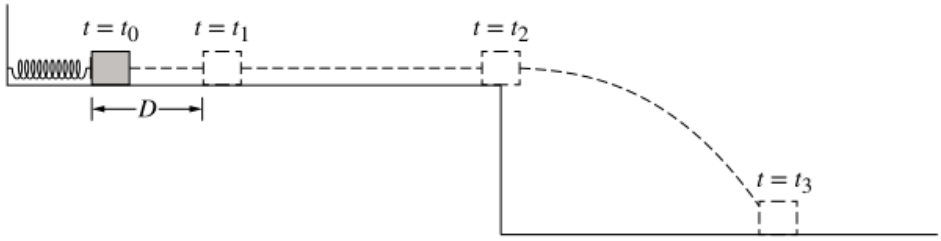
3. This question is a long free-response question. Show your work for each part of the question.



(12 points, suggested time 25 minutes)

A block is launched from a table by a compressed spring that is attached to a wall. The spring is initially compressed a distance D before the block is launched, as shown above. There is negligible friction between the block and the table.

At time $t = t_0$ the block is released from rest.
At $t = t_1$ the block loses contact with the spring.
At $t = t_2$ the block leaves the edge of the table.
At $t = t_3$ the block lands on the floor.

The graph below shows the total mechanical energy of the system containing the block and Earth from $t = t_0$ to immediately before $t = t_3$. The gravitational potential energy is defined to be zero when the block is on the floor.

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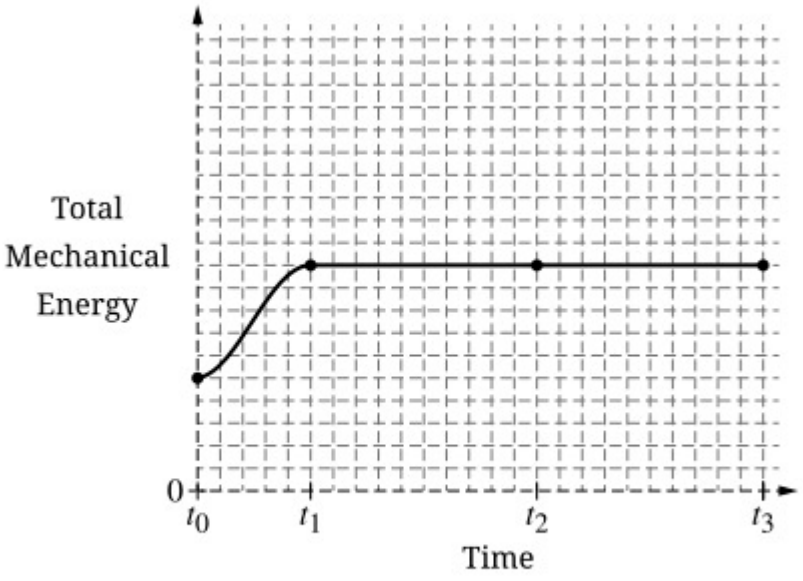

Note: Time axis not drawn to scale.

(a) On the same axes, sketch a graph of the total mechanical energy of just the block from time t_0 to time t_3.

The procedure is repeated with a second block, Block 2, which is more massive than the original block, Block 1. Block 2 is placed on the table, and the spring is compressed the same distance D as before.

(b) For Block 2, is the horizontal distance traveled from the edge of the table to where it hits the ground greater than, less than, or equal to the corresponding distance for Block 1?

___ Greater than ___ Less than ___ Equal to

Justify your answer.

(c) Derive a general expression for X, the horizontal distance traveled by a block of mass m_B, from the edge of the table to where it hits the floor. Let k_0 represent the spring constant and h_0 represent the height of the tabletop above the floor. Express your answer in terms of D, m_B, k_0, h_0, and physical constants, as appropriate.

(d) Does your expression derived in part (c) support your reasoning in part (b)?

___ Yes ___ No

Justify your answer.

(e) Is the kinetic energy of Block 2 immediately after it leaves the spring the same as or different than the kinetic energy of Block 1 immediately after it leaves the spring?

___ Same ___ Different

Briefly justify your answer.

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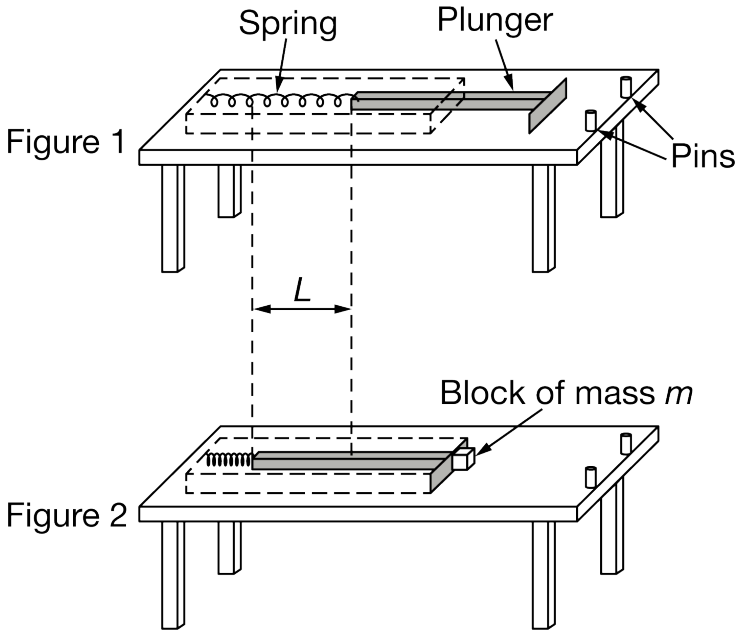
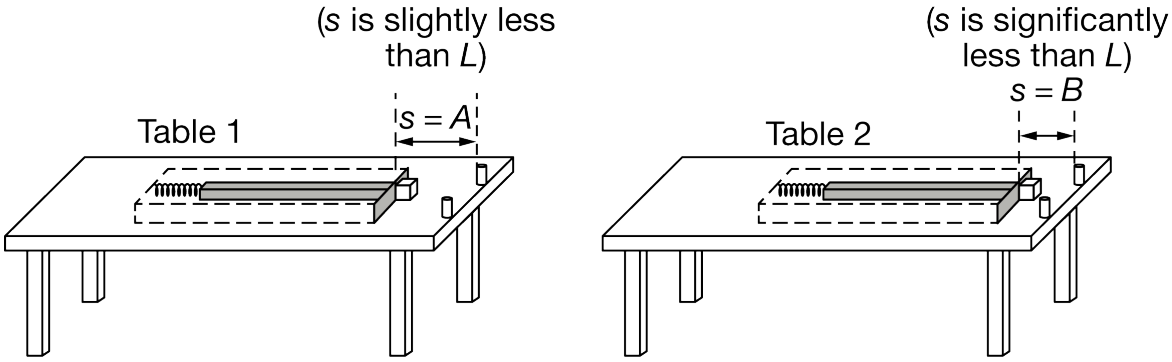
4. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A spring-loaded launcher can be securely attached to a tabletop. When the spring is at its uncompressed length, the plunger is in the position shown above in Figure 1. A small wood block can be pressed against the end of the plunger, compressing the spring a distance L, as shown in Figure 2. When the block is released, the plunger pushes the block along the tabletop. Two pins are attached to the edge of the table to prevent the plunger from extending beyond the table. When the plunger hits the pins, the block then leaves the table. There is nonnegligible friction between the block and the tabletop.



The launcher can be moved closer to or farther from the edge of the tabletop. Students set up tables 1 and 2, as shown above, with tabletops that are made of the same material. On table 1, the launcher is positioned so that the distance $s = A$ between the launcher and the edge of the tabletop is slightly less than L. On table 2, the launcher is positioned so that the distance $s = B$ between the launcher and the edge of the tabletop is significantly less than L. On both tables, the block is released from rest and loses contact with the plunger at the moment the plunger reaches the pins. Tables 1 and 2 are the same height.

k is the spring constant of the spring.
m is the mass of the block.
L is the distance the spring is compressed.
μ (mu) is the coefficient of kinetic friction between the block and the tabletop.
s is the distance between the launcher and the edge of the tabletop.

Assume the spring is ideal, the plunger has negligible mass, the block loses contact with the plunger immediately after it reaches the end of the table, and there is non-negligible friction between the tabletops and the blocks.

(a) Without manipulating equations, explain why the block launched from table 1 could land farther from the table than the block launched from table 2 does.

(b) Does the block launched from table 1 spend more, less, or the same amount of time in the air than the block launched from table 2 does? Explain your reasoning.

(c) Consider the Earth-block system. Determine the change, if any, in the total mechanical energy of the system from the instant the block leaves the table to the instant immediately before it reaches the ground.

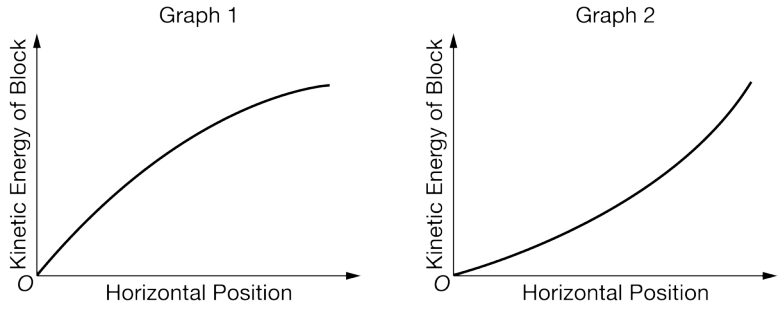
The students correctly derive an equation for the horizontal distance D from the edge of the tabletop that the block lands, in terms of s (the distance between the launcher and the edge of the tabletop):

$$D = t\sqrt{\frac{k}{m}(2Ls - s^2) - 2\mu g s},$$

where t is the time interval between the instant the block leaves the table and the instant immediately before it reaches the ground.

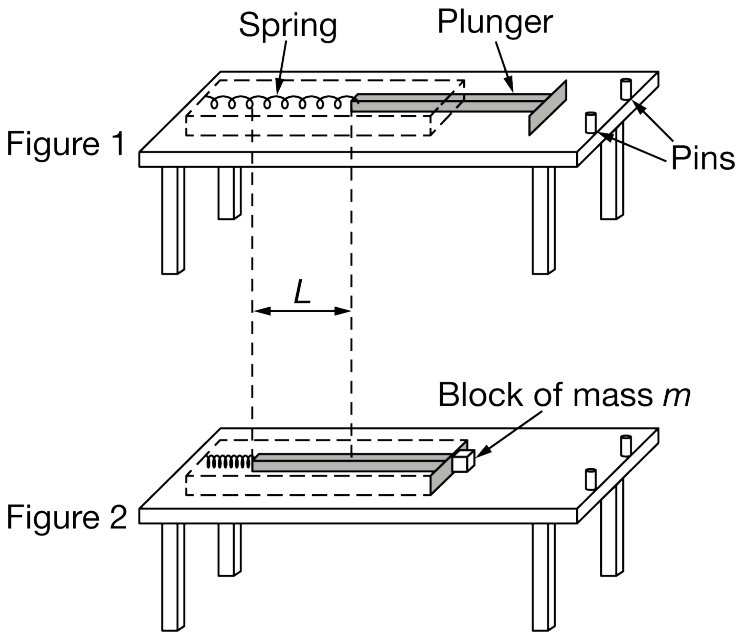
(d) Does this equation for D support your argument from part (a) that the block launched from table 1 could land farther from the table than the block launched from table 2? Briefly explain why or why not.

(e) The students now use the same setups to launch blocks that are identical to each other but more massive than the original blocks. The blocks are made of the same material as the original blocks. Does this change in the mass of the blocks make it more likely or less likely that the block launched from table 1 goes farther than the block launched from table 2 does? Briefly explain your reasoning.

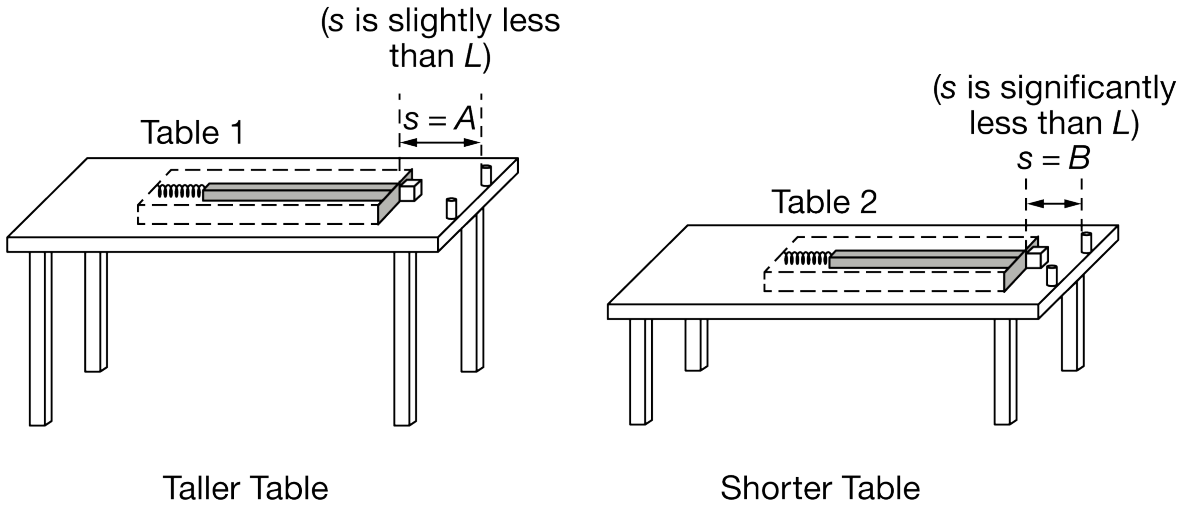


(f) Two students each try to sketch a graph of the kinetic energy of the block as a function of its position from launch to the time the block reaches the edge of the table. Their graphs are shown above. Which graph best represents the kinetic energy of the block? Justify your selection.

5. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A spring-loaded launcher can be securely attached to a tabletop. When the spring is at its uncompressed length, the plunger is in the position shown above in Figure 1. A small wood block can be pressed against the end of the plunger, compressing the spring a distance L as shown in Figure 2. When the block is released, the plunger pushes the block along the tabletop. Two pins are attached to the edge of the table to prevent the plunger from extending beyond the table. When the plunger hits the pins, the block then leaves the table. There is nonnegligible friction between the block and the tabletop.



The launcher can be moved closer to or farther from the edge of the tabletop. Students set up tables 1 and 2, as shown above, with tabletops that are made of the same material. On table 1, which is taller than table 2, the

launcher is positioned so that the distance $s = A$ between the launcher and the edge of the tabletop is slightly less than L. On table 2, the launcher is positioned so that the distance $s = B$ between the launcher and the edge of the tabletop is significantly less than L. On both tables, the block is released from rest and loses contact with the plunger at the moment the plunger reaches the pins.

k is the spring constant of the spring.
m is the mass of the block.
L is the distance the spring is compressed.
μ (mu) is the coefficient of kinetic friction between the block and the tabletop.
s is the distance between the launcher and the edge of the tabletop.

Assume the spring is ideal, the plunger has negligible mass, the block loses contact with the plunger immediately after it reaches the end of the table, and there is non-negligible friction between the tabletops and the blocks.

(a) Without manipulating equations, explain why the block launched from table 2 could have a greater speed when it leaves the table than the block launched from table 1 does.

(b) Does the block launched from table 2 spend more, less, or the same amount of time in the air than the block launched from table 1 does? Explain your reasoning.

(c) Consider the system consisting of only the block. Determine the change, if any, in the total mechanical energy of the system from the instant the block leaves the table to the instant immediately before it reaches the ground.

The students correctly derive an equation for the speed v of the block when it is at the edge of the tabletop, in terms of s (the distance between the launcher and the edge of the tabletop):

$v = \sqrt{\frac{k}{m}(2Ls - s^2) - 2\mu g s}$.

(d) Does this equation for v support your argument from part (a) that the block launched from table 2 could have a larger speed when it leaves the table than the block on table 1 does? Briefly explain why or why not.

(e) The students now repeat the scenario with identical setups except that the coefficient of kinetic friction between the blocks and the table is larger. Does this change in the coefficient of friction make it more likely or less likely that the block launched from table 1 has a greater speed when it leaves the table than the block on table 2 does? Briefly explain your reasoning.

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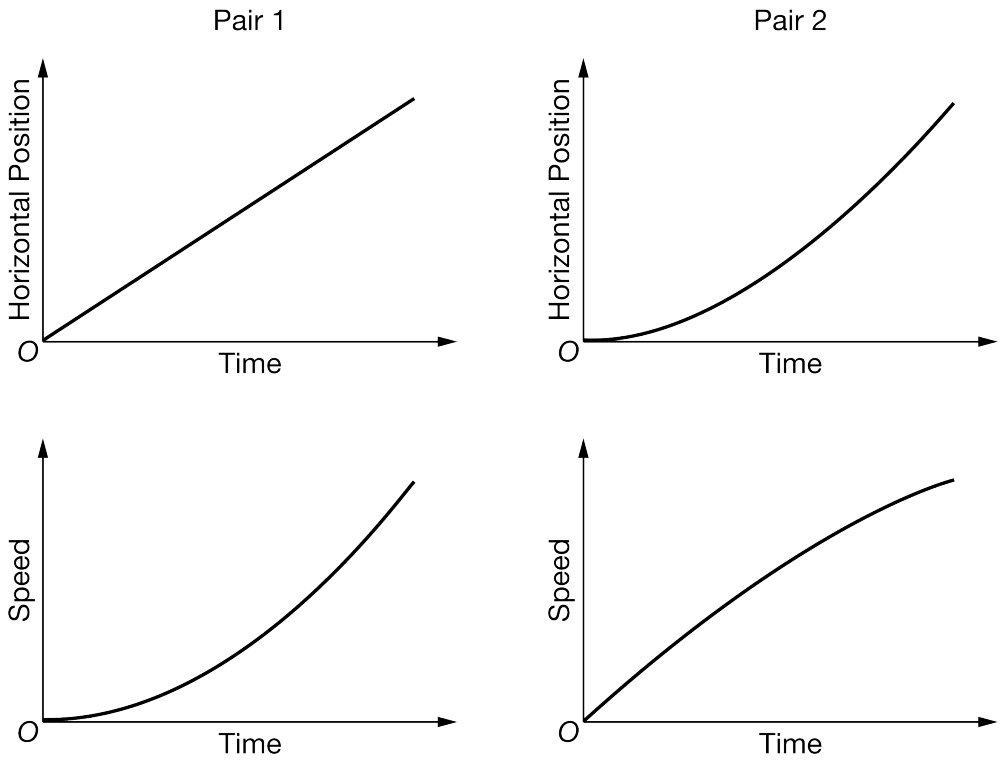

(f) Two students each try to sketch a graph of the horizontal position and speed of the block as a function of time while the block is being pushed by the plunger. Their graphs are shown above. In which pair of graphs are the two graphs consistent with each other? Justify your selection.

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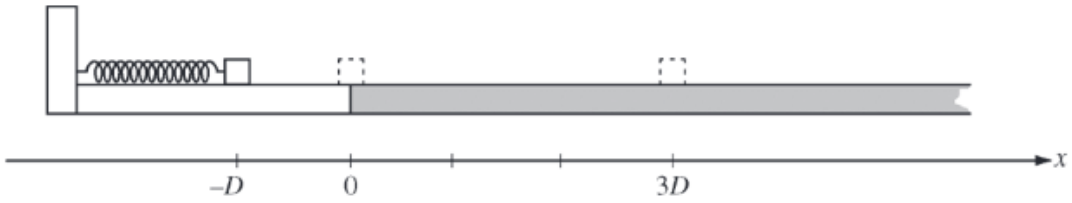
6.

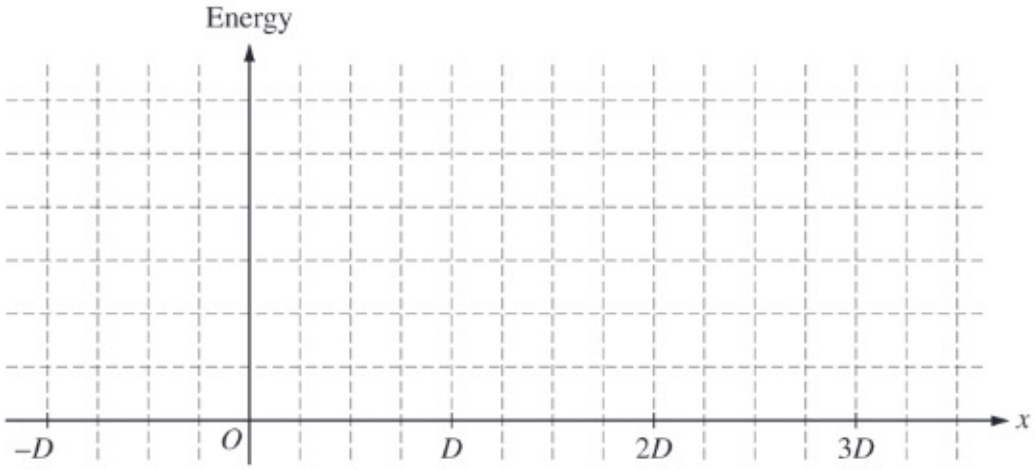


A block is initially at position x = 0 and in contact with an uncompressed spring of negligible mass. The block is pushed back along a frictionless surface from position *x = 0 to x = -D*, as shown above, compressing the spring by an amount $\Delta x = D$. The block is then released. At *x = 0* the block enters a rough part of the track and eventually comes to rest at position *x = D*. The coefficient of kinetic friction between the block and the rough track is μ.

(a) On the axes below, sketch and label graphs of the following two quantities as a function of the position of the block between x = -D and x = 3D. You do not need to calculate values for the vertical axis, but the same vertical scale should be used for both quantities.

i. The kinetic energy *K* of the block

ii. The potential energy *U* of the block-spring system



The spring is now compressed twice as much, to $\Delta x = 2D$. A student is asked to predict whether the final position of the block will be twice as far at *x = 6D*. The student reasons that since the spring will be compressed twice as much as before, the block will have more energy when it leaves the spring, so it will slide farther along the track before stopping at position *x = 6D*.

b)

i. Which aspects of the student's reasoning, if any, are correct? Explain how you arrived at your answer.

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ii. Which aspects of the student's reasoning, if any, are incorrect? Explain how you arrived at your answer.

(c) Use quantitative reasoning, including equations as needed, to develop an expression for the new final position of the block. Express your answer in terms of D.

(d) Explain how any correct aspects of the student's reasoning identified in part (b) are expressed by your mathematical relationships in part (c). Explain how your relationships in part (c) correct any incorrect aspects of the student's reasoning identified in part (b). Refer to the relationships you wrote in part (c), not just the final answer you obtained by manipulating those relationships.

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7. This question is a long free-response question. Show your work for each part of the question.

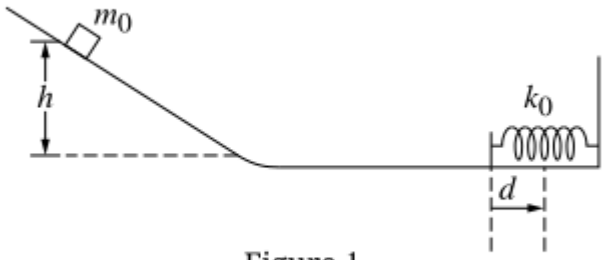

Figure 1

(12 points, suggested time 25 minutes)

In an experiment, a block is released from rest on a ramp, as shown in Figure 1. The block slides down an incline and across a horizontal surface, where it compresses a spring and then momentarily comes to rest. Frictional forces between the block and surfaces are negligible.

(a) The experiment is repeated, but the block is released from a greater height on the ramp. Will the new compression distance of the spring be greater than, less than, or equal to the original compression distance?

_____ Greater than _____ Less than _____ Equal to

Briefly justify your selection without deriving or using equations.

(b) Derive an expression for the distance d that the spring compresses. Express your answer in terms of the mass of the block m_0, the height h from which the block is released, the spring constant k_0, and physical constants, as appropriate.

(c) Does the equation you derived in part (b) agree with your reasoning in part (a)? Justify your answer.

(d) After the block compresses the spring, the block remains attached to the spring and oscillates. A student finds an incorrect equation for the period of oscillation T for the block-spring system on the Internet, $T = C\sqrt{\frac{h}{m_0 g}}$, where C is a constant with appropriate units. Explain why this equation does not make physical sense.

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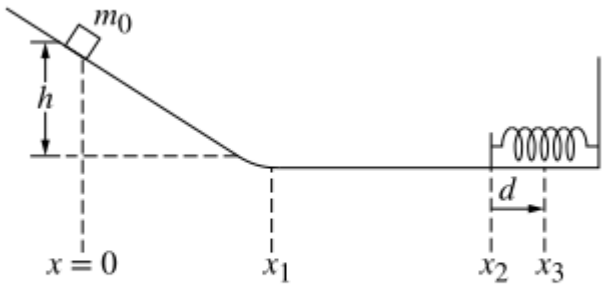


Figure 2

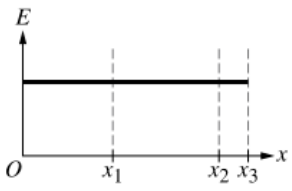
Figure 2 shows the same experiment with four horizontal positions indicated.

(e) Consider the following systems:

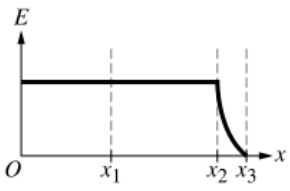
System A	Block only
System B	Block and Earth only
System C	Block, Earth, and spring

Each of the following graphs represents the mechanical energy E as a function of the horizontal position x of the block for one of the systems.

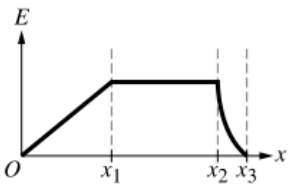
i. Under each graph, write the letter "A," "B," or "C" to indicate the system that is represented by the graph.



System_____



System_____



System_____

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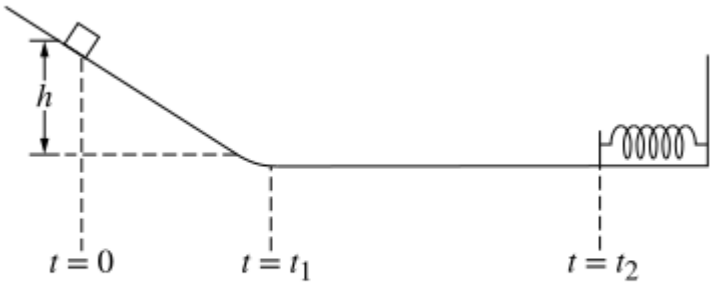
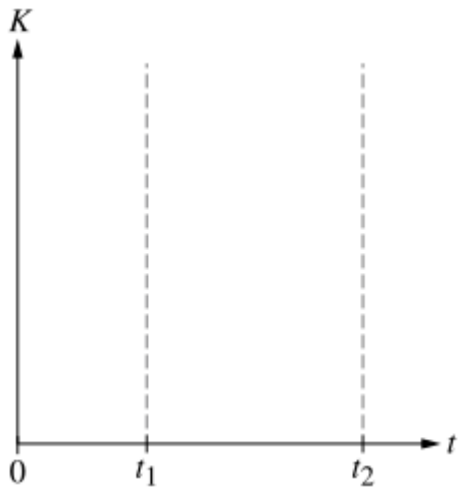


Figure 3

Figure 3 indicates the times the block is at three specific positions during the same experiment. The block is released from rest at $t = 0$. At time t_1, the block reaches the horizontal surface. Time t_2 indicates the time immediately before the block makes contact with the spring.

ii. On the following axes, sketch a graph of the kinetic energy K of the block as a function of time t from time $t = 0$ to time $t = t_2$.



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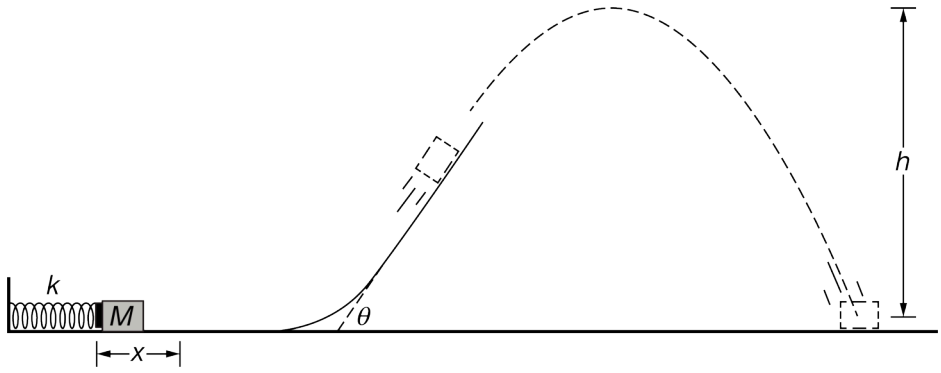
8. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A box is pressed against a horizontal spring, compressing the spring from its relaxed length. The box is then released, and the spring launches the box horizontally along a track that ends in a ramp, as shown above. The box has enough speed to leave the ramp, and the box reaches a maximum vertical height above the floor. Assume there is negligible friction between the box and the track and air resistance is negligible.

k is the spring constant of the spring.
x is the distance the spring is compressed.
M is the mass of the box.
θ (theta) is the angle of the ramp from the horizontal.
h is the maximum height reached by the box.

The scenario is repeated using a stiffer spring with a spring constant that has a greater value for k than that of the original spring. The stiffer spring is compressed the same distance x, and the same box is used.

(a) Indicate how using a stiffer spring would affect the maximum height reached by the box, and explain why without mathematically deriving a relation for h.

(b) Students derive an equation for the maximum height, $h = \frac{(\sin^2\theta)kx^2}{2Mg}$, which may or may not be correct. Is this equation consistent with your claim in part (a)? Explain why or why not.

Two students are discussing the motion of the box when it is at its maximum height and make the following statements:

Student 1: The speed of the block at its maximum height is zero because the kinetic energy is a minimum at the top of the trajectory.

Student 2: Only the vertical component of the velocity is zero at the maximum height, because the vertical component of the acceleration is zero at the top of the trajectory.

(c) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

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(d) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

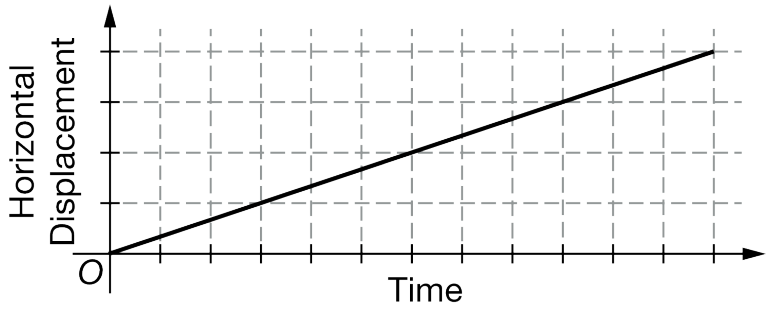
(e) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(g) Consider the system consisting of the box and the spring, but not Earth. How does the energy of the system when the spring is fully compressed compare to the energy of the system at the moment immediately before the box hits the ground? Justify your answer.

(h) If the friction between the track and the box were not negligible, state one way in which the setup could be changed such that the maximum height of the box was the same as in the original setup.

The graph below shows the horizontal component of the displacement of the box as a function of time. The graph represents the entire time the box is in the air after the box is launched from the ramp at an angle θ to the horizontal.



(i) How would the slope of the graph be different, if at all, if the ramp angle were larger than θ, but less than 90 degrees, and the height of the end of the ramp were the same? Justify your response.

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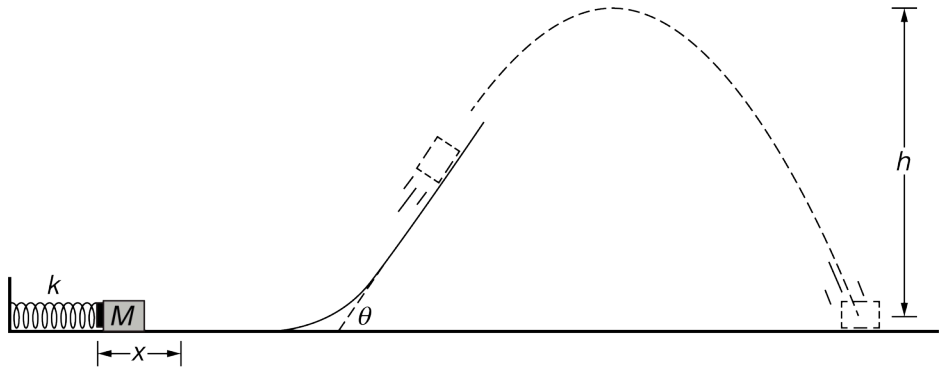
9. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A box is pressed against a horizontal spring, compressing the spring from its relaxed length. The box is then released and the spring launches the box horizontally along a track that ends in a ramp, as shown above. The box has enough speed to leave the ramp, and the box reaches a maximum vertical height above the floor. Assume there is negligible friction between the box and the track and air resistance is negligible.

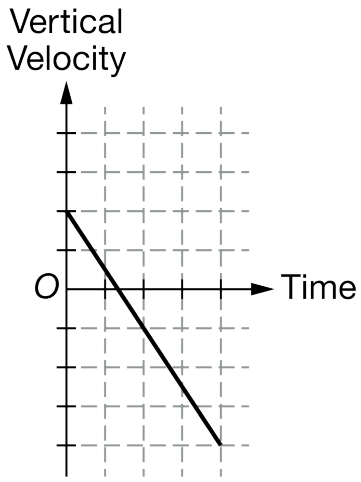
k is the spring constant of the spring.
x is the distance the spring is compressed.
M is the mass of the box.
θ (theta) is the angle of the ramp from the horizontal.
h is the maximum height reached by the box.

The scenario is repeated with the box compressing the spring a distance that is larger than the original distance x.

(a) Indicate how h in this second scenario compares to h in the original scenario, and explain why without mathematically deriving a relation for h.

(b) Students derive an equation for the maximum height h in the original scenario, $h = \frac{(\sin^2\theta)kx^2}{2Mg}$, which may or may not be correct. Is this equation consistent with your claim in part (a)? Explain why or why not.

chapter4



Two students, student 1 and student 2, are discussing the graph shown above, which shows the vertical component of the velocity of the box from when it leaves the ramp to immediately before it hits the floor. The students make the following statements.

Student 1: This graph shows that the vertical acceleration of the box is constant, and that the box's speed decreases for the entire time shown.

Student 2: The area between the graph and the time axis is the box's vertical displacement, but the graph is incorrect because the positive displacement should be equal to the negative displacement.

(c) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(g) Consider the system consisting of only the box, but not the spring or Earth. How does the energy of the system when the spring is fully compressed compare to the energy of the system at the moment immediately before the box hits the ground? Justify your answer.

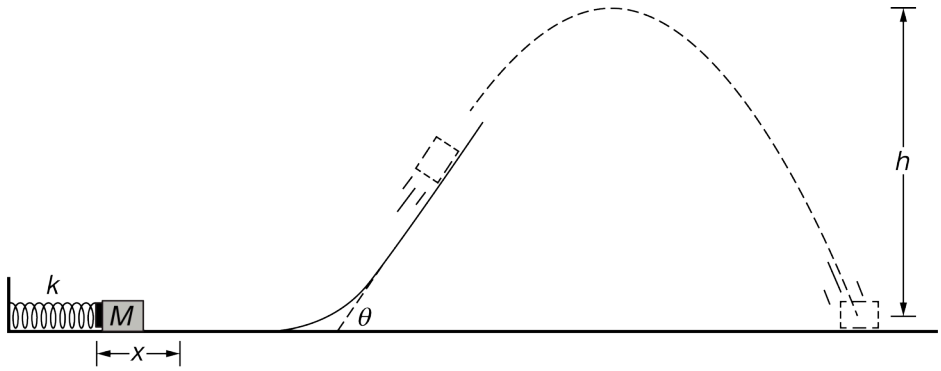
The angle of the ramp is changed such that it is larger than θ but less than 90 degrees, while the height of the end of the ramp remains the same.

(h) How would the maximum height of the box compare to the original scenario in which the ramp angle was equal to θ ? Justify your answer.

(i) Would the slope of the graph of vertical velocity as a function of time shown above part (c) be different for this scenario in which the box is launched from the larger angle? Justify your response.

chapter4

10. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A box is pressed against a horizontal spring, compressing the spring from its relaxed length. The box is then released and the spring launches the box horizontally along a track that ends in a ramp, as shown above. The box has enough speed to leave the ramp, and the box reaches a maximum vertical height above the floor. Assume there is negligible friction between the box and the track and air resistance is negligible.

k is the spring constant of the spring.
x is the distance the spring is compressed.
M is the mass of the box.
θ (theta) is the angle of the ramp from the horizontal.
h is the maximum height reached by the box.

The scenario is repeated using a different box with a greater mass. The spring is compressed the same distance x.

(a) Indicate how h in this second scenario compares to h in the original scenario, and explain why without mathematically deriving a relation for h.

(b) Students derive an equation for h in the original scenario, $h = \frac{(\sin^2\theta)kx^2}{2Mg}$, which may or may not be correct. Is this equation consistent with your claim in part (a)? Explain why or why not.

Two students are discussing the work done on the system consisting of only the box between the instant just before the spring begins accelerating the box and the instant the box reaches the end of the ramp. The students make the following statements.

Student 1: At first, the mechanical energy of the system increases because the spring does positive work on the system. Then, the mechanical energy of the system increases again as the box travels up the ramp because the box's potential energy increases.

Student 2: The mechanical energy of the system remains constant while the spring pushes the box because the work done on the box by the spring cancels the work done by the spring on the box. Then, when the block is on the ramp, the mechanical energy of the system decreases because the box slows down.

(c) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***,

write “none.”

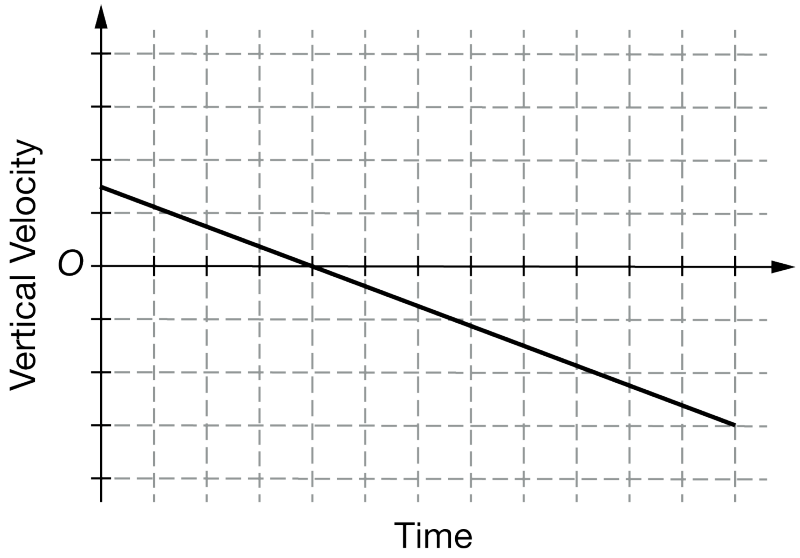
(d) Which underlined phrase or phrases in student 1’s statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write “none.”

(e) Which underlined phrase or phrases in student 2’s statement are ***correct***, if either? If neither phrase is ***correct***, write “none.”

(f) Which underlined phrase or phrases in student 2’s statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write “none.”

(g) Does the mechanical energy of the box-spring-Earth system increase, decrease, or remain the same from the moment the spring is released until just before the box hits the ground? Justify your answer.

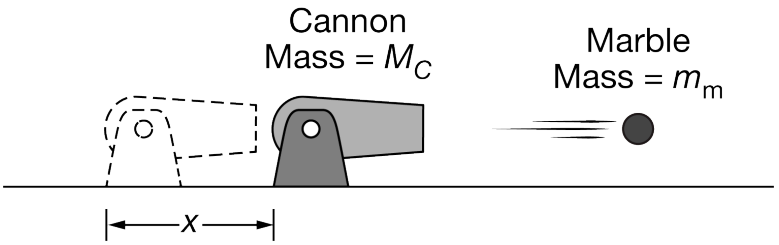
(h) If air resistance were not negligible, how would that affect your answer to part (g), if at all? Justify your answer.



(i) The graph above shows the vertical component of the velocity of the box as a function of time. The graph represents the entire time the box is in the air when the box is launched from the ramp at an angle θ to the horizontal. How would the total area under the curve be different, if at all, if the ramp angle θ were increased but still less than 90 degrees, without changing the height of the end of the ramp?

chapter4

11. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch marbles of mass m_{m} horizontally from a toy cannon of mass M_C, where M_C is always greater than m_{m}, as shown above. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students use a new marble with a different mass launched at the same speed v relative to the ground. The students record the distance x the cannon slides backward for each launch.

(a) Without manipulating equations, briefly describe what happens to the distance x that the cannon slides backward as students increase the mass of the marble. Explain your reasoning.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\mathrm{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) Without manipulating or stating equations, briefly describe whether it would take more work to stop the marble or to stop the cannon after the marble has been launched. Justify your answer.

(d) Students correctly derive the following equation relating the work W done on an object initially at rest, the momentum p of the object, and the mass m of the object: $W = \frac{p^2}{2m}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble-cannon system during the short time interval in which the marble is accelerated inside the cannon and make the following claims:

Student 1: The cannon is pushing the marble forward, and the marble is pushing back with equal and opposite force. Since the work done by the cannon cancels out the work done by the marble, the mechanical energy of the system is constant.

Student 2: The mechanical energy of the system increases while the marble is being accelerated. The marble gains kinetic energy to the right, and the cannon gains an equal amount of kinetic energy to the left.

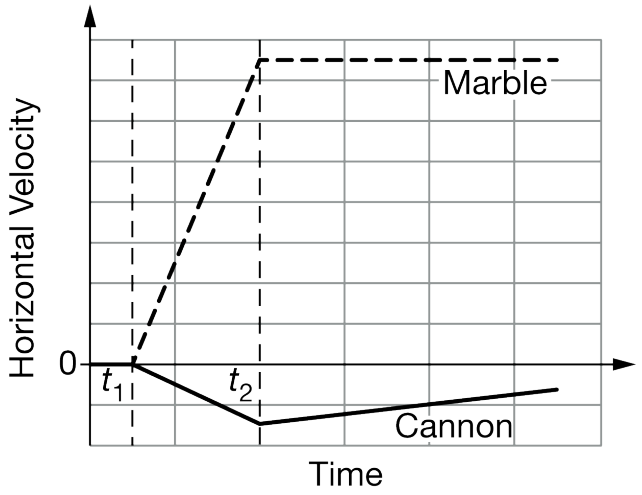
(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is

incorrect, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

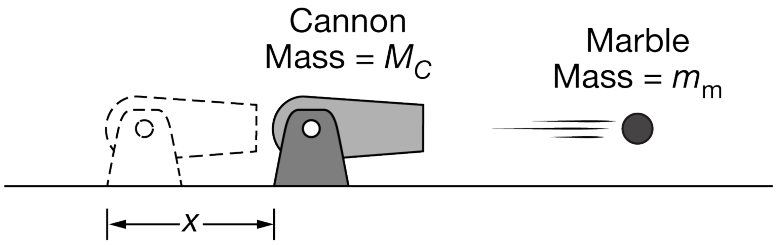


The graph above shows the horizontal component of the velocity of the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Provide three pieces of evidence that together explain why the magnitude of the rate of change of the cannon's velocity is less than the magnitude of the rate of change of the marble's velocity during the time interval from t_1 to t_2.

chapter4

12. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch identical marbles of mass m_{m} horizontally from a toy cannon of varying mass M_C, where M_C is always greater than m_{m}, as shown above. The students can attach weights to the cannon to change M_C. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students vary the mass of the cannon and launch a marble with the same speed v relative to the ground. The students record the distance x that the cannon slides backward.

(a) Without manipulating equations, explain how the distance x that the cannon slides backward changes, if at all, when marbles are launched from more massive cannons.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\text{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) If constant horizontal forces were exerted over the same distance on the cannon and on the marble after the launch, which object would require a force with a greater magnitude to completely stop it? Without manipulating or referencing equations, briefly explain your reasoning.

(d) Students correctly derive the following equation relating the force F exerted on an object initially at rest over a distance d, the mass m of the object, and the final momentum p of the object: $F = \frac{p^2}{2md}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble after it has been launched by the cannon but before it reaches the ground. Both students agree that air resistance is negligible and make the following claims.

Student 1: The mechanical energy of the marble is constant. There are no external forces exerted on the marble, so its energy does not change.

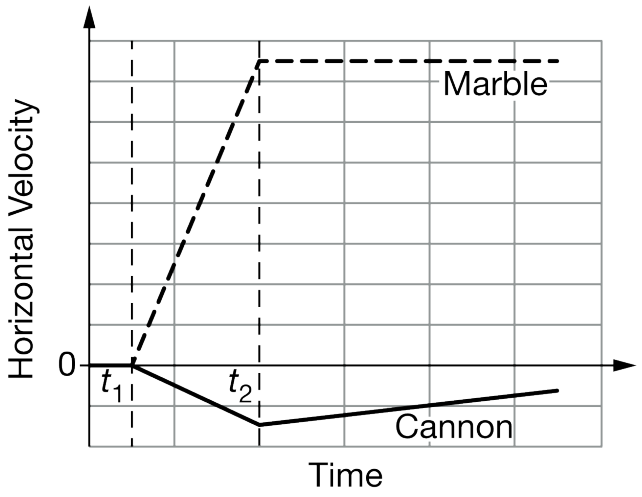
Student 2: The mechanical energy decreases as the marble falls. This happens because the gravitational force is doing work on the marble.

(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

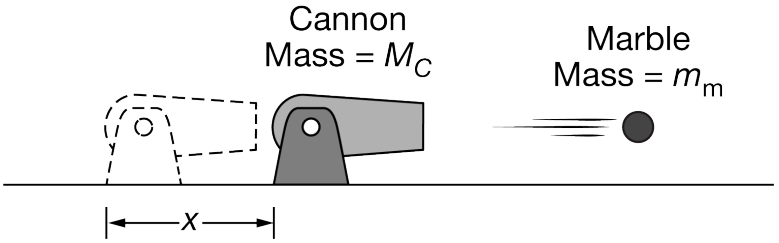


The graph above shows the horizontal component of the velocity of the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Describe the motion of the center of mass of the marble-cannon system during the time interval from t_1 to t_2. Explain your reasoning.

chapter4

13. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

Students launch identical marbles of mass m_{m} horizontally from a toy cannon of mass M_C, where $M_C > m_{\mathrm{m}}$, as shown above. The cannon can be adjusted to change the launch speed v of the marble relative to the ground. Each time a marble is launched, the cannon slides backward before coming to rest. The coefficient of kinetic friction between the cannon and the ground is μ (mu). For each launch, the students vary the launch speed v and record the distance x the cannon slides backward for each launch speed.

(a) Without manipulating equations, briefly describe what happens to the distance x that the cannon slides backward as students increase the launch speed. Explain your reasoning.

(b) Students find the following equation on the Internet to describe the distance x that the cannon slides backward: $x = \frac{m_{\mathrm{m}}^2 v^2}{2g\mu M_C^2}$. Whether or not the equation is correct, is it consistent with your prediction in part (a)? Briefly justify your answer.

(c) Without manipulating or stating equations, briefly describe whether the cannon or the marble has more kinetic energy immediately after the marble leaves the cannon. Justify your answer.

(d) Students correctly derive an equation relating the kinetic energy K, the momentum p, and the mass m of an object: $K = \frac{p^2}{2m}$. Is this equation consistent with your description in part (c)? Justify your answer.

Students then discuss the mechanical energy of the marble after it has been launched by the cannon but before it reaches the ground. Both students agree that air resistance is negligible and make the following claims.

Student 1: There are two external forces exerted on the marble while it is in the air, the gravitational force and the force exerted by the cannon. Therefore, the mechanical energy of the marble changes while it is in the air.

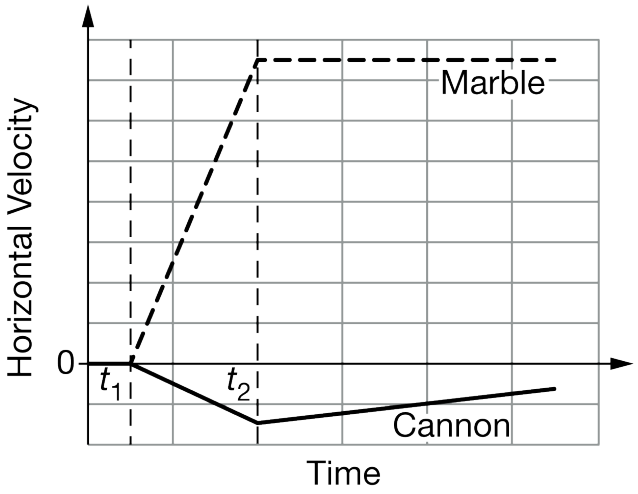
Student 2: The cannon does not exert a force on the marble after the launch. Therefore, the mechanical energy of the marble is constant.

(e) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(h) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."



The graph above shows the horizontal component of the velocity for the marble (dashed line) and the cannon (solid line) relative to the ground. Before time t_1 the marble-cannon system is at rest. At time t_1 the marble begins accelerating within the cannon, and at time t_2 the marble leaves the cannon.

(i) Describe how to find the change in the distance between the center of mass of the cannon and the center of mass of the marble from time t_1 to time t_2. Assume that before time t_1 the center of mass of the marble is at the same position as the center of mass of the cannon.

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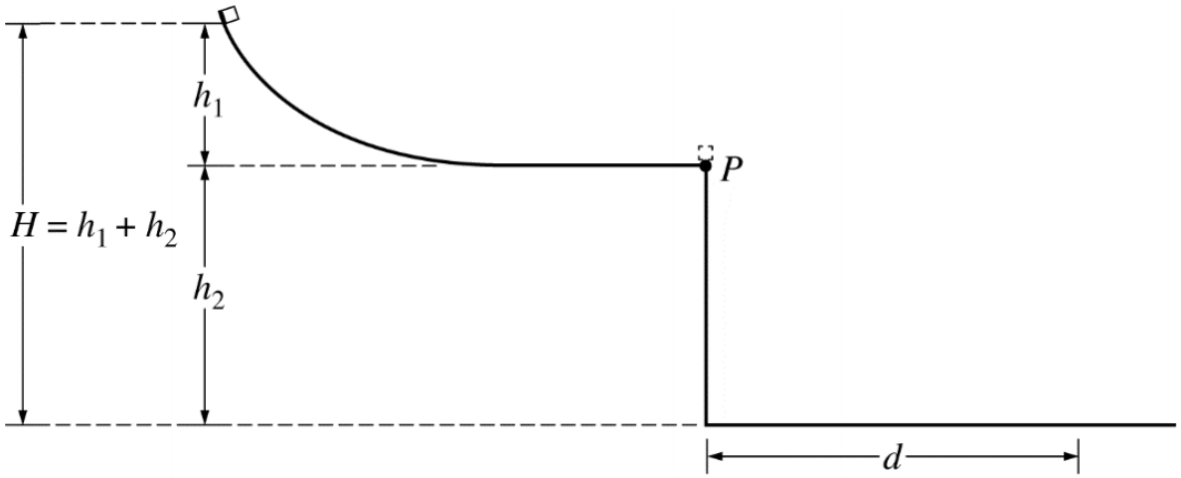
14.



A student releases a block of mass m from rest at the top of a slide of height h_1. The block moves down the slide and off the end of a table of height h_2 , landing on the floor a horizontal distance d from the edge of the table. Friction and air resistance are negligible. The overall height H of the setup is determined by the height of the room. Therefore, if h_1 is increased, h_2 must decrease by the same amount so that the sum $h_1 + h_2$ remains equal to H. The student wants to adjust h_1 and h_2 to make d as large as possible.

a. i. Without using equations, explain why making h_1 very small would cause d to be small, even though h_2 would be large.

ii. Without using equations, explain why making h_2 very small would cause d to be small, even though h_1 would be large

b. Derive an equation for d in terms of h_1, h_2, m, and physical constants, as appropriate.

c. i. Write the equation or step in your derivation in part (b) (not your final answer) that supports your reasoning in part (a)i.

Briefly explain your choice.

ii. Write the equation or step in your derivation in part (b) (not your final answer) that supports your reasoning in part (a)ii.

Briefly explain your choice.

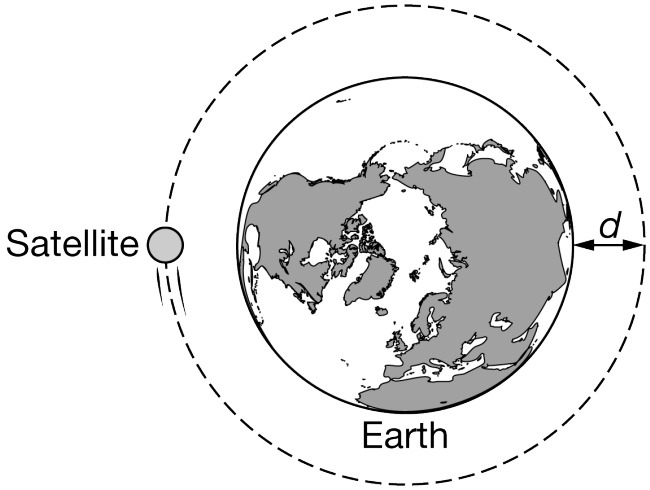
d. If the experiment is repeated on the Moon without changing h_1 or h_2 , will the new landing distance d be greater than, less than, or the same as the landing distance when the experiment is performed on Earth?

_____Greater than _____Less than _____The same as

Briefly explain your choice.

chapter4

15. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface, as shown above.

An identical satellite is set into orbit around a new planet. The new planet has the same radius as the radius of Earth but is significantly more massive than Earth. The satellite orbits the same distance d from the surface of the new planet.

(a) Is the speed of the satellite orbiting the new planet greater than, less than, or the same as the speed of the satellite orbiting Earth? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

$$\frac{MT^2}{R^3} = C$$

In the equation above

M is the mass of a planet;
T is the orbital period of a satellite in a circular orbit;
R is the distance from the planet's center to the satellite; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

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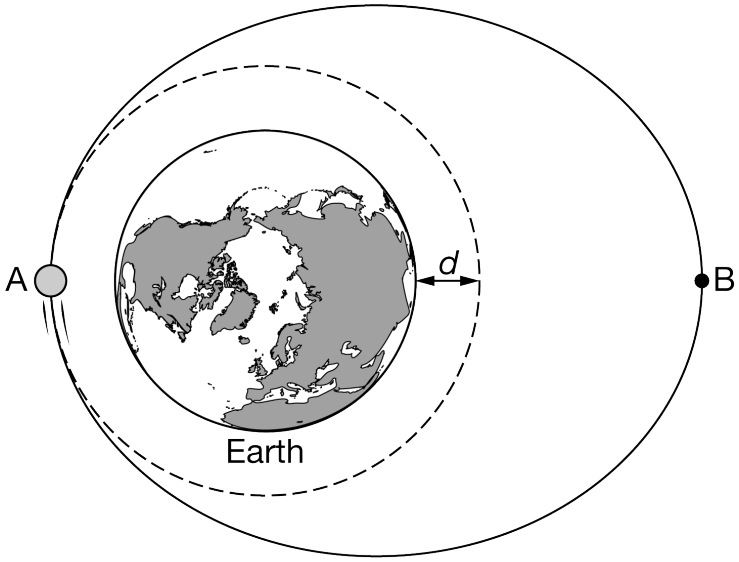


Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point A to point B, and state the following:

Student 1: The satellite is moving faster at point A because the satellite's angular momentum is greater at point A than at point B.

Student 2: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is higher at point B than at point A.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

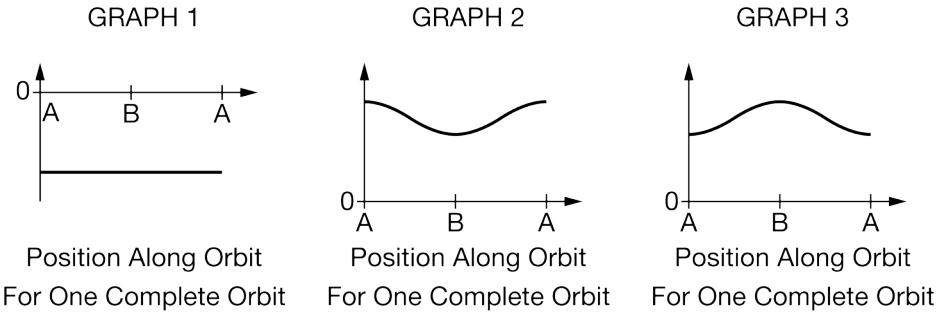
(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite as it completes one elliptical orbit, starting and ending at point A.

chapter4

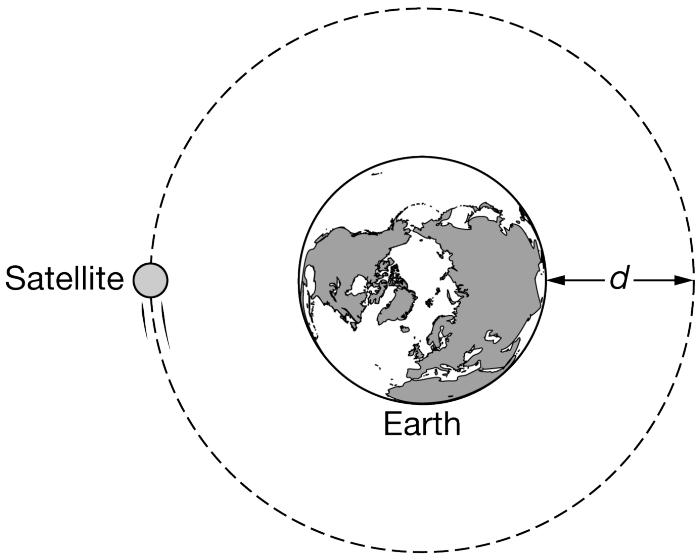


(h) Which of the above graphs best represents the magnitude of the gravitational field at the location of the satellite as it orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the total mechanical energy of the satellite-Earth system as the satellite orbits? Briefly explain your reasoning.

chapter4

16. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface, as shown above.

An identical satellite is set into orbit around a new planet. The new planet's radius is the same as the radius of Earth but is significantly more massive than Earth. The satellite orbits the new planet with the same orbital period.

(a) Is the distance of the satellite from the surface of the new planet greater than, less than, or equal to d ? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

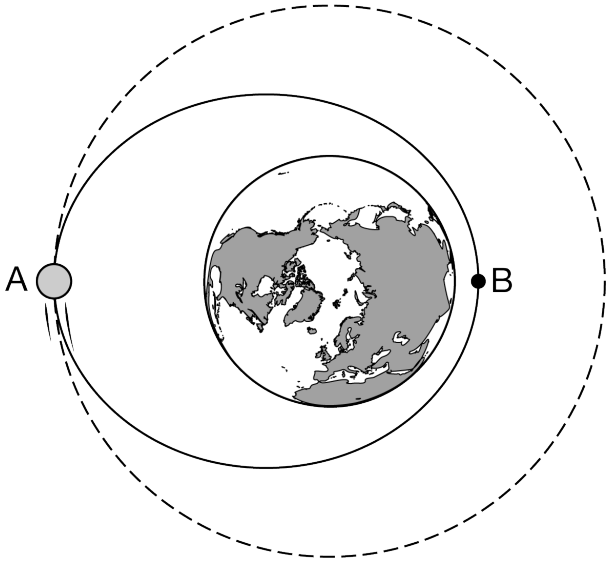
$\frac{Rv^2}{M} = C$

In the equation above

R is the distance from a planet's center to a satellite in a circular orbit;
v is the speed of the satellite;
M is the mass of the planet; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

chapter4



Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point A to point B, and state the following:

Student 1: The satellite is moving faster at point A because the total mechanical energy of the satellite-Earth system is constant.

Student 2: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is higher at point B than at point A.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

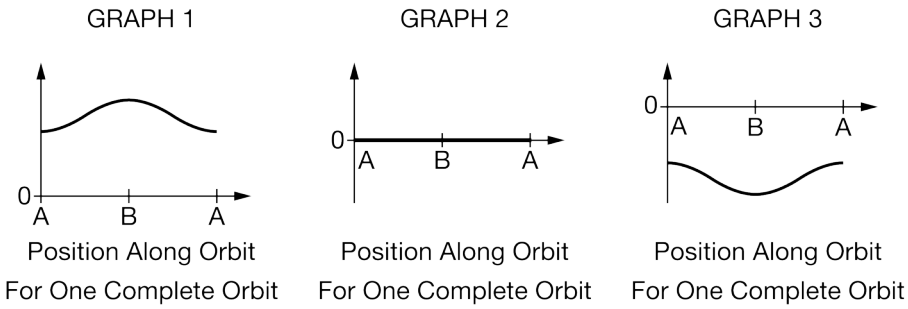
(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite

as it completes one elliptical orbit, starting and ending at point A.

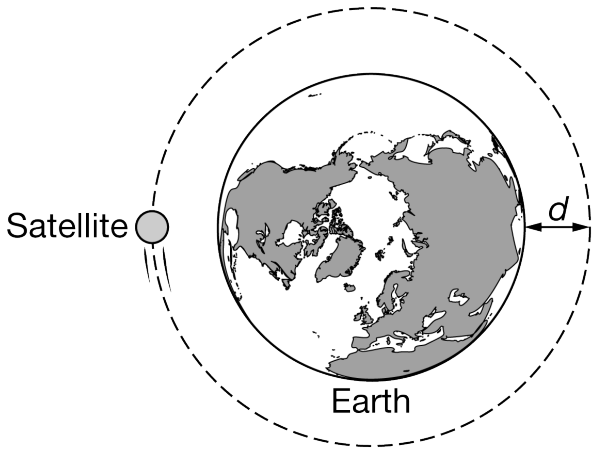


(h) Which of the above graphs best represents the potential energy of the satellite-Earth system as the satellite orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the torque about the center of Earth exerted on the satellite by Earth's gravitational force as the satellite orbits? Briefly explain your reasoning.

chapter4

17. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Note: Figure not drawn to scale.

A satellite is traveling around Earth in a circular orbit at a constant speed v and at a constant distance d from Earth's surface.

An identical satellite is set into orbit around a new planet. The new planet's radius is the same as the radius of Earth but the new planet is significantly more massive than Earth. The satellite orbits the new planet with the same speed v.

(a) Is the distance of the satellite from the surface of the new planet greater than, less than, or equal to d ? Explain your reasoning.

Students find the following equation on the Internet to describe circular orbits in general. The equation may or may not be correct.

$$\frac{R^3}{MT^2} = C$$

In the equation above

R is the distance from a planet's center to a satellite in a circular orbit;
M is the mass of the planet;
T is the orbital period of the satellite; and
C is a constant with appropriate units.

(b) Is this equation consistent with your reasoning in part (a)? Explain your reasoning.

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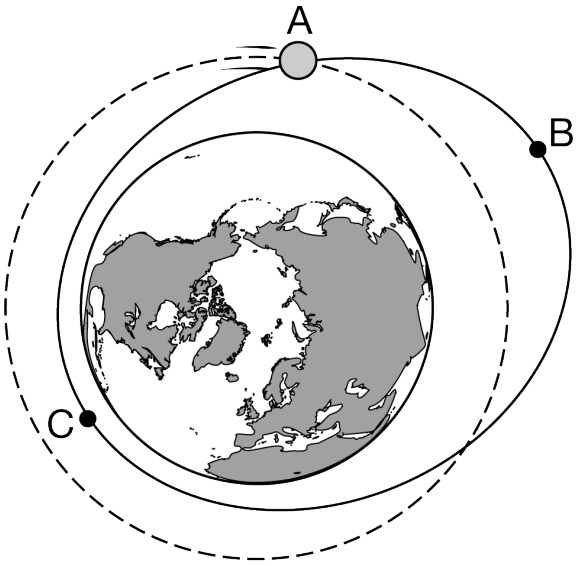

Note: Figure not drawn to scale.

The original satellite is orbiting clockwise around Earth. When the satellite is at point A in the above figure, it fires a thruster (which exerts a force on the satellite) for a short time, modifying the orbital path. The dashed line in the figure shows the original orbit of the satellite, and the solid line shows the modified orbit. In the modified orbit, the satellite's speed at point A, after the thrusters were fired, is the same as it was in the original orbit.

(c) Consider the force exerted by the thruster at point A that modifies the satellite's orbit from the dashed-line original orbit to the solid-line new orbit. Was the force exerted in the same direction as, in the opposite direction of, or perpendicular to the satellite's velocity at point A? Explain your answer.

The satellite has now been in its new orbit for a long time. Two students are discussing the satellite in its new clockwise, elliptical orbit as it travels from point B to point C, and state the following:

Student 1: The satellite is moving faster at point B because the gravitational potential energy of the satellite-Earth system is negative.

Student 2: The satellite is moving faster at point C because the total mechanical energy of the satellite-Earth system is greater at point C than at point B.

(d) Which underlined phrase or phrases in student 1's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(e) Which underlined phrase or phrases in student 1's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

(f) Which underlined phrase or phrases in student 2's statement are ***correct***, if either? If neither phrase is ***correct***, write "none."

(g) Which underlined phrase or phrases in student 2's statement are ***incorrect***, if either? If neither phrase is ***incorrect***, write "none."

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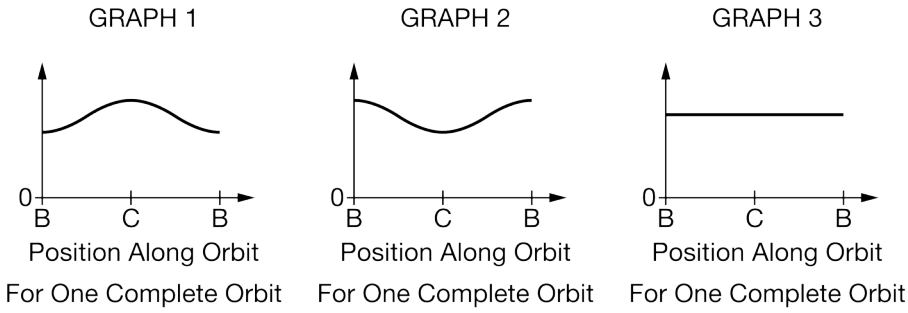
In each of the graphs shown below, the vertical axis represents a quantity that could be related to the satellite and/or Earth while the satellite is in the new elliptical orbit. The horizontal axis represents the position of the satellite as it completes one elliptical orbit, starting and ending at point B.

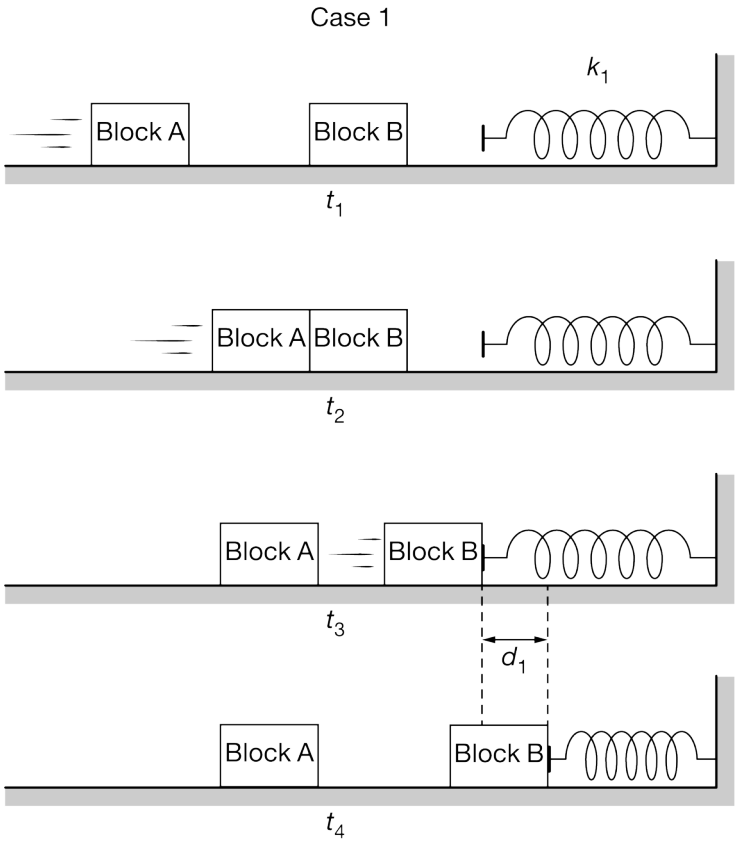


(h) Which of the above graphs best represents the magnitude of the acceleration of the satellite as it orbits? Briefly explain your reasoning.

(i) Which of the above graphs best represents the magnitude of the angular momentum of the satellite with respect to the center of Earth as the satellite orbits? Briefly explain your reasoning.

chapter4

18. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



The figure above shows Case 1 of an experiment, in which the following events occur.

- At time t_1, block A slides to the right along a horizontal surface toward identical block B, which is at rest.
- At time t_2, block A collides elastically with block B and block A comes to rest.
- At time t_3, the right side of block B reaches an ideal, horizontal spring with negligible mass and spring constant k_1. The spring is initially at its relaxed (uncompressed) length and fixed to a wall.
- At time t_4, the spring is compressed a maximum distance d_1 and block B is instantaneously at rest.

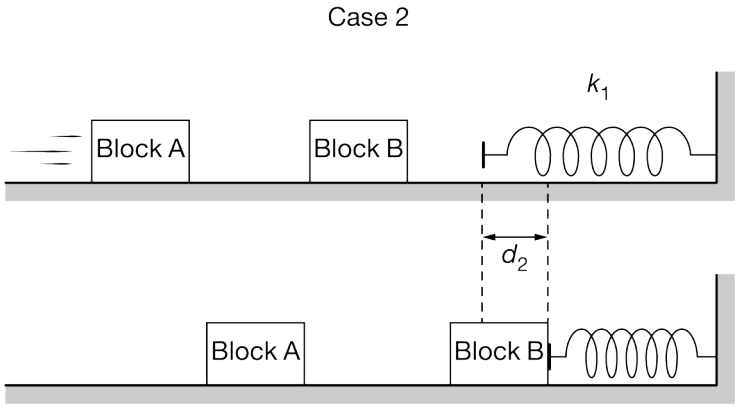
Assume there is negligible friction between each block and the surface. For parts (a)-(d), consider the system consisting of block A, block B, and the spring.

(a) Indicate whether the linear momentum of the two-block–spring system is either constant or not constant during the time interval from t_1 to immediately before t_3, and explain why.

(b) Indicate whether the mechanical energy of the two-block–spring system is either constant or not constant during the time interval from t_1 to immediately before t_3, and explain why.

(c) Indicate whether the linear momentum of the two-block–spring system is either constant or not constant during the time interval t_3 from to t_4, and explain why.

(d) Indicate whether the mechanical energy of the two-block–spring system is either constant or not constant during the time interval from t_3 to t_4, and explain why.



In Case 2 shown above, the setup is identical to Case 1 except that the speed of block A before colliding with block B is greater than in Case 1. Block A again comes to rest immediately after colliding with block B. The spring compresses a maximum distance d_2.

(e) How does the new maximum spring compression d_2 compare to d_1, the spring compression in Case 1? (Is d_2 greater than d_1, the same as d_1, or less than d_1 ?)

(f) Briefly explain your reasoning for part (e).

Students attempt to determine a general equation for the maximum compression distance d of a spring with spring constant k. Blocks A and B each have mass m and block A has initial speed v. The students come up with the following two equations.

Equation 1: $d = C_1\sqrt{\frac{m}{vk}}$ **Equation 2:** $d = C_2\sqrt{\frac{m}{k}}v$

In the two equations, C_1 and C_2 are positive constants with appropriate units.

(g) Without algebraic manipulation of equations or deriving the correct equation, explain which equation better supports your explanation for the compression distance in part (f).

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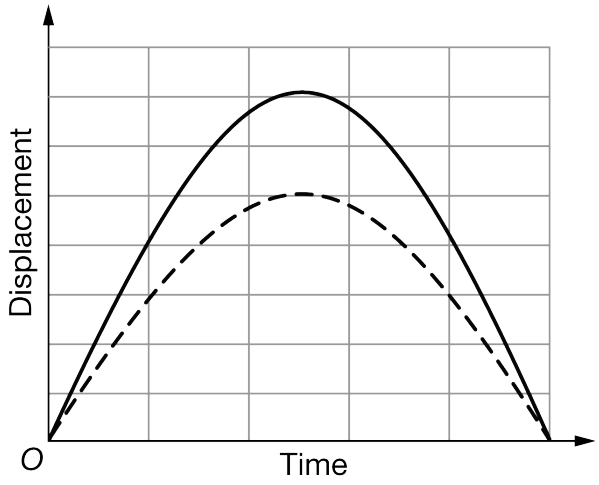

(h) The graph above shows the displacement of the blocks while block B is in contact with the spring. The solid line corresponds to either Case 1 or Case 2, while the dashed line corresponds to the other case. Does the dashed-line curve correspond to Case 1 (slower block A) or Case 2 (faster block A)? Explain your reasoning.

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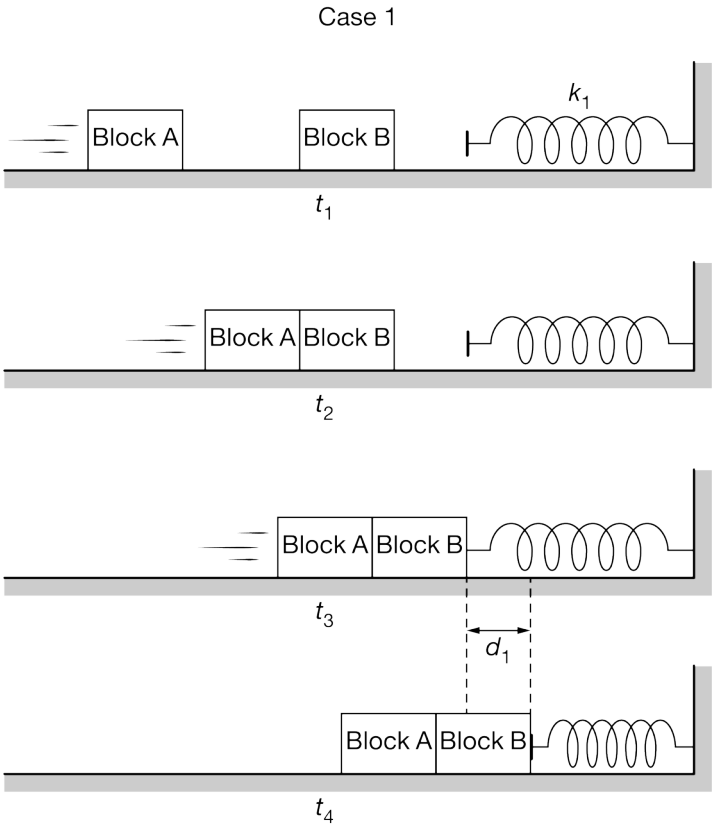
19. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



The figure above shows Case 1 of an experiment, in which the following events occur.

- At time t_1, block A slides to the right along a horizontal surface toward identical block B, which is at rest.
- At time t_2, block A collides with and sticks to block B.
- At time t_3, block A and block B are still stuck together and the right side of block B reaches an ideal, horizontal spring with negligible mass and spring constant k_1. The spring is initially at its relaxed (uncompressed) length and fixed to a wall.
- At time t_4, the spring is compressed a maximum distance d_1 and the blocks are instantaneously at rest.

Assume there is negligible friction between each block and the surface.

A student observes the experiment and makes the following claim.

"*Since some mechanical energy is dissipated when the blocks stick together, the momentum of the two-block system decreases.*"

(a) Indicate whether the claim is ***correct*** or ***incorrect*** and explain why.

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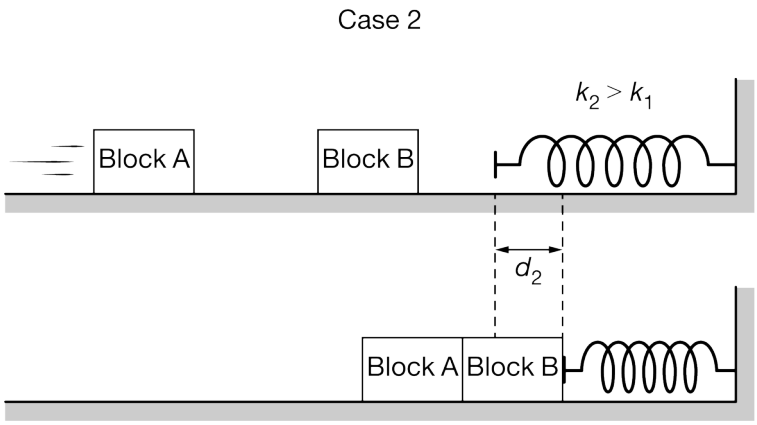
Another student makes the following claim about the collision between block A and block B.

"*When the blocks collide the total momentum of the two-block system doesn't change, so the kinetic energy of the system must also be constant.*"

(b) Indicate whether the claim is ***correct*** or ***incorrect*** and explain why.

(c) During the time interval from t_3 to t_4, is the magnitude of the acceleration of the two-block system increasing, decreasing, or constant? Briefly explain why.

(d) During the time interval from t_3 to t_4, does the spring do positive, negative, or zero work on the two-block system? Briefly explain your answer.



In Case 2 shown above, the setup is identical to Case 1 except that a stiffer spring is used. The spring in Case 2 has a spring constant k_2 that is greater than k_1. Block A slides with the same speed as in Case 1 and the spring compresses a maximum distance d_2.

(e) How does the new maximum spring compression distance d_2 in Case 2 compare to d_1 in Case 1? (Is d_2 greater than d_1, the same as d_1, or less than d_1 ?)

(f) Briefly explain your reasoning for part (e).

Students attempt to determine a general equation for the maximum compression distance d of a spring with spring constant k. Block A has mass m_A and initial speed v, and block B has mass m_B. The students come up with the following two equations.

Equation 1: $d = \frac{C_1 m_A v}{\sqrt{(k)(m_A+m_B)}}$ **Equation 2:** $d = C_2\left(k\sqrt{m_A + m_B}\right)v$

In the two equations, C_1 and C_2 are positive constants with appropriate units.

(g) Without algebraic manipulation of equations or deriving the correct equation, explain which equation better supports your explanation for the compression distance in part (f).

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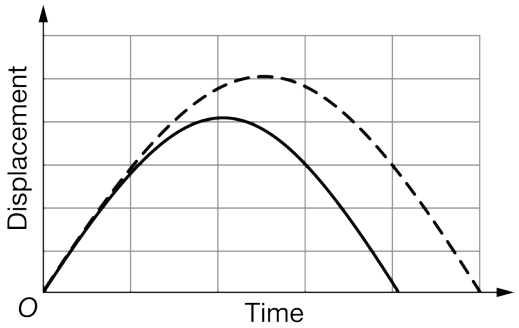

(h) The graph above shows the displacement of the blocks while block B is in contact with the spring. The solid line corresponds to either Case 1 or Case 2, while the dashed line corresponds to the other case. Does the solid-line curve correspond to Case 1 (with a smaller spring constant) or Case 2 (with a larger spring constant)? Explain your reasoning.

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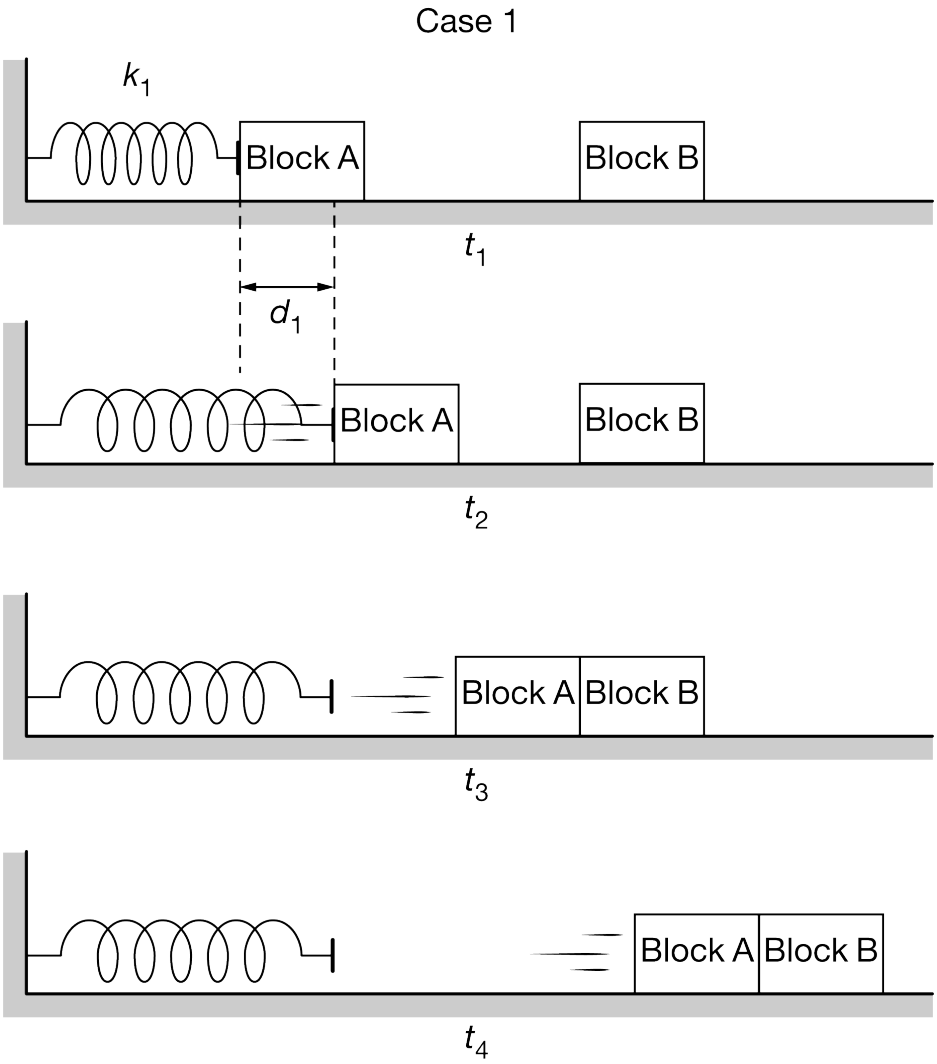
20. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



The figure above shows Case 1 of an experiment. Initially, block A is held in place while in contact with a horizontal spring with spring constant k_1 that is compressed a distance d_1 and fixed to a wall. The following events occur.

- At time t_1, block A is released from rest.
- At time t_2, the ideal spring reaches its uncompressed length, and block A leaves the spring.
- At time t_3, block A collides with and sticks to identical block B, which is initially at rest.
- At time t_4, block A and block B are sliding together along the surface.

Assume there is negligible friction between each block and the surface.

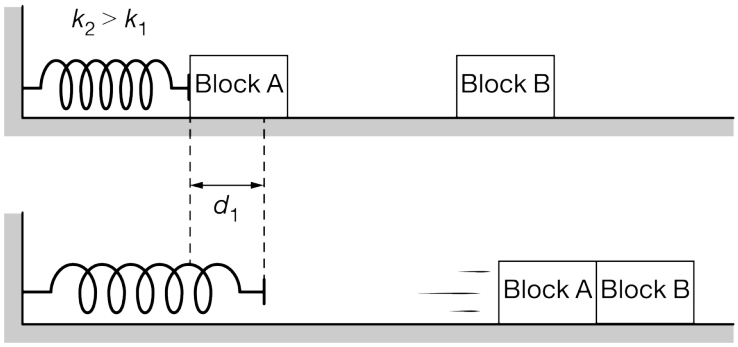
(a) Indicate whether the speed of the center of mass of the two-block system is either constant or not constant during the time interval from t_1 to t_2, and explain why.

(b) Indicate whether the mechanical energy of the two-block system is either constant or not constant during the time interval from t_1 to t_2, and explain why.

(c) Indicate whether the speed of the center of mass of the two-block system is either constant or not constant during the time interval from t_2 to t_4, and explain why.

(d) Indicate whether the mechanical energy of the two-block system is either constant or not constant during the time interval from t_2 to t_4, and explain why.

Case 2



In Case 2 shown above, the setup is identical to Case 1 except that a stiffer spring is used. The spring in Case 2 has a spring constant k_2 that is greater than k_1. The spring is compressed the same distance, d_1, before block A is released. As in Case 1, the blocks stick together when they collide.

(e) Is the speed of the blocks after the collision in Case 2 greater than, less than, or the same as the speed of the blocks after the collision in Case 1?

(f) Briefly explain your reasoning for part (e).

Students attempt to determine a general equation for the speed v of the blocks after the collision. A spring with spring constant k is initially compressed a distance d, block A has mass m_A, and block B has mass m_B. The students come up with the following two equations.

Equation 1: $v = C_1\left(\frac{m_B+m_A}{\sqrt{km_A}}\right)d$ **Equation 2:** $v = C_2\left(\frac{\sqrt{km_A}}{m_A+m_B}\right)d$

In the two equations, C_1 and C_2 are positive constants with appropriate units.

(g) Without algebraic manipulation of equations or deriving the correct equation, explain which equation better supports your explanation for the speed of the two blocks in part (f).

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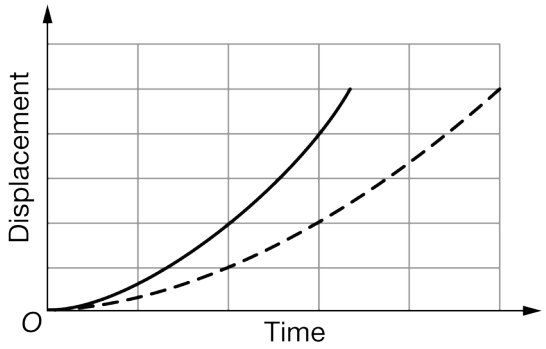

The graph above shows the displacement of block A while it is in contact with the spring for Case 1 and Case 2. The solid line corresponds to either Case 1 or Case 2, while the dashed line corresponds to the other case.

(h) Does the solid-line curve best correspond to Case 1 (with a smaller spring constant) or to Case 2 (with a larger spring constant)? Explain your reasoning.

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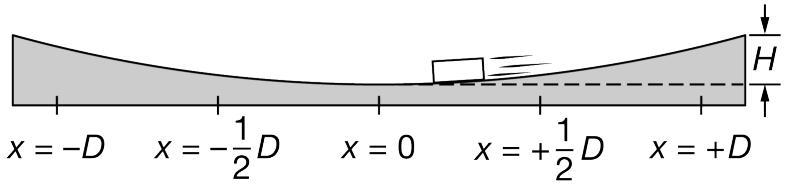
21. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

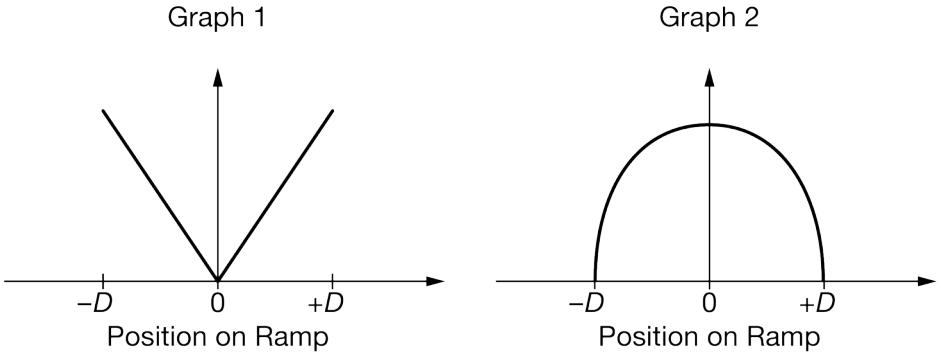


Track 1

Note: Figure not drawn to scale.

A block slides back and forth without any appreciable loss of energy on a track with a slight circular curvature, as shown above. The maximum height H of the track at each end is much smaller than D.

The center of mass of the block oscillates between $x = -D$ and $x = +D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. The block undergoes two full oscillations in 8.0 seconds.



(a) Which of the graphs above, Graph 1 or Graph 2, could represent the magnitude of the horizontal component of the block's acceleration as a function of its position on the ramp, and which graph could represent the magnitude of the block's momentum as a function of its position on the ramp? Justify both your choices with physics principles.

(b) For the system consisting of only the block, describe how the mechanical energy of the system changes, if at all, as the block moves from $x = 0$ to $x = -D$. Justify your answer.

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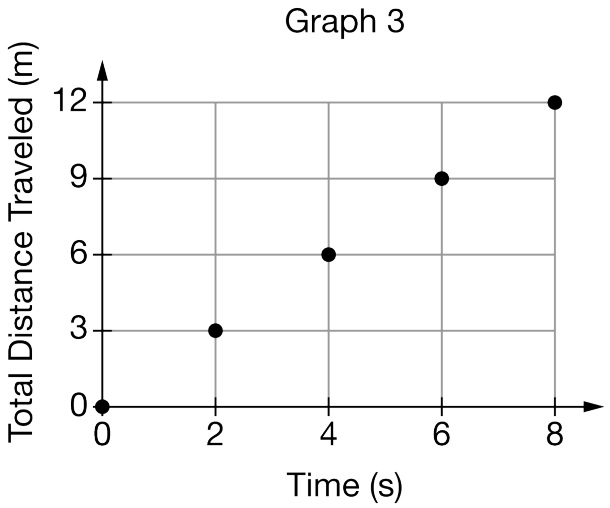
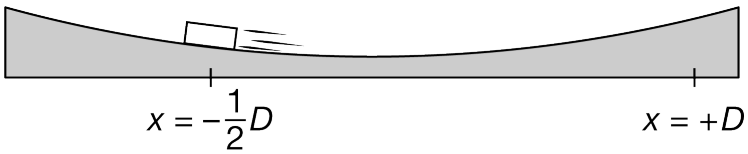


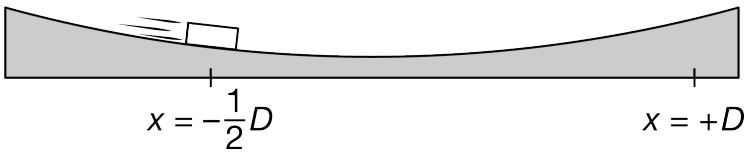
(c) Graph 3 above shows the total horizontal distance traveled by the block as a function of time as it completes two oscillations. Using the graph, determine the distance D between the midpoint of the ramp and the maximum position of the block on the ramp. Explain how you arrived at your answer.

(d) Suppose the entire setup was moved to the Moon, where the gravitational field strength is less than that on Earth. The center of mass of the block again oscillates between $x = -D$ and $x = +D$. Would the period of the block's oscillation on the Moon be greater than, less than, or equal to the period of oscillation on Earth? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{D^2}{2Hg}}$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?



Block on Track 2 at Time $t = T_1$



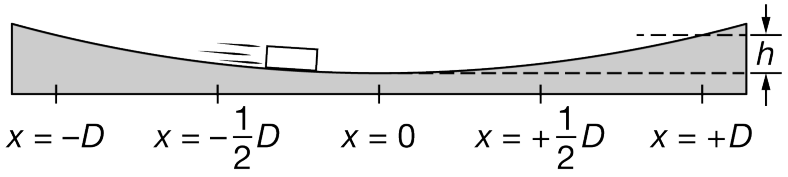
Block on Track 2 at Time $t = T_2$

Note: Figures not drawn to scale.

(f) Track 2 is identical to Track 1 except there is nonnegligible friction between the block and Track 2. The block

is released from rest on Track 2 at position $x = +D$. At time $t = T_1$, the block moves to the left, passing through position $x = -\frac{1}{2}D$, as shown above. After the block reverses direction near $x = -D$, the block moves to the right, passing through position $x = -\frac{1}{2}D$ at time $t = T_2$. At which time, T_1 or T_2, if either, is the magnitude of the tangential acceleration of the block greater? Justify your answer.

22. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)

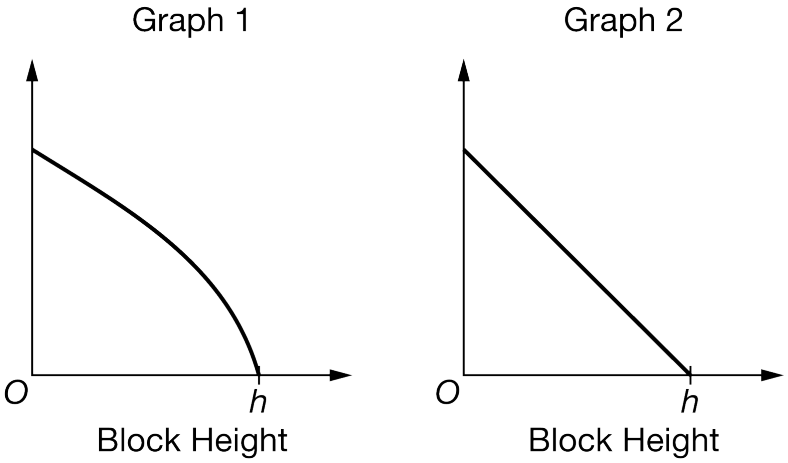


Track 1

Note: Figure not drawn to scale.

A block slides back and forth on a track with a slight circular curvature, where friction is negligible, as shown above. The maximum height h of the track at each end is much smaller than D.

The center of mass of the block oscillates between $x = -D$ and $x = +D$, reaching a maximum height h every time the block is at $x = +D$ or $x = -D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. At time $t = 0$, the speed of the block is zero. The next time that the block's speed is zero is at time $t = 2.0$ s.



(a) Which of the graphs above, Graph 1 or Graph 2, could represent the kinetic energy of the block as a function of its height on the track, and which graph could represent the speed as a function of the block's height on the track? Justify both of your choices with physics principles.

(b) For the system consisting of the block and Earth, describe how the mechanical energy of the system changes, if at all, as the block moves from $x = +D$ to $x = 0$. Justify your answer.

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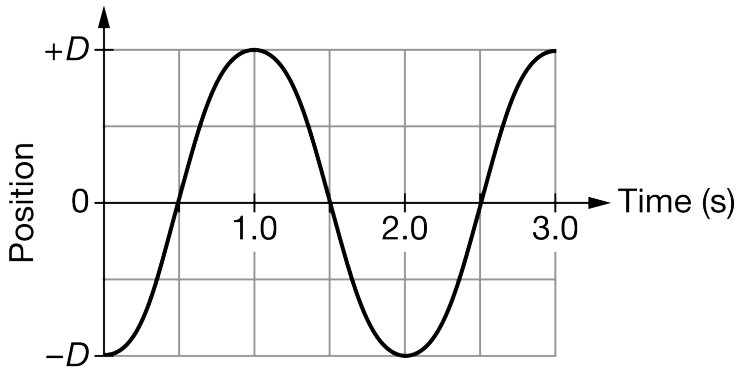
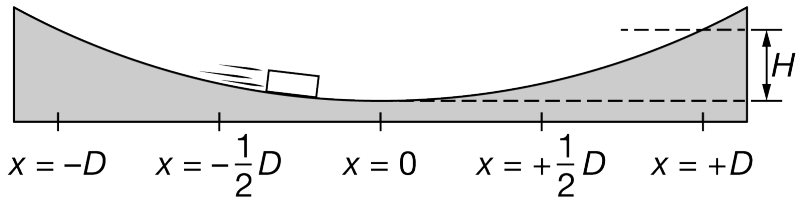
Graph 3



(c) Graph 3 above incorrectly represents the horizontal position of the block as a function of time. Identify a feature of the graph that does not correctly represent the motion of the block. Justify your answer.



Track 2

Note: Figure not drawn to scale.

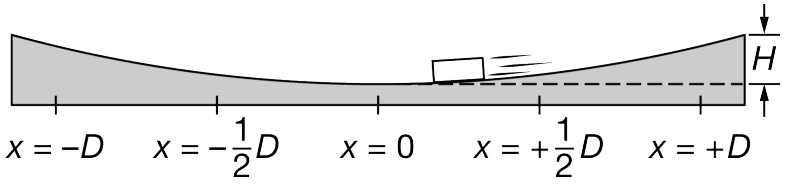
(d) Track 2 is similar to Track 1, but the maximum height H on Track 2 is greater than h and is still much smaller than D, as shown above. The center of mass of the block oscillates between $x = -D$ and $x = +D$, the same as on Track 1. How does the period of the block's oscillation on Track 2 compare to the period of the block's oscillation on Track 1? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{H^2}{2Dg}}$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?

(f) When the block reaches $x = 0$ at the bottom of the circular track, is the force exerted by the track on the block greater than, less than, or equal to the weight of the block? Justify your answer.

chapter4

23. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



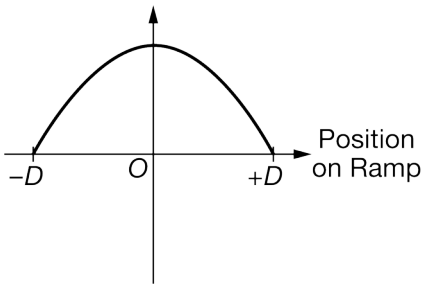
Track 1

Note: Figure not drawn to scale.

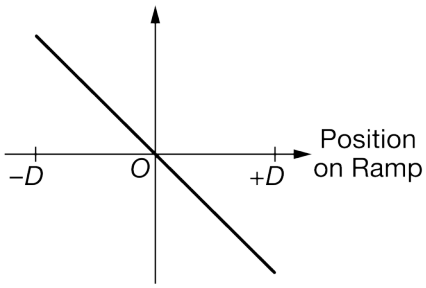
A block slides back and forth on a track with a slight circular curvature, where friction is negligible, as shown above. The maximum height H of the track at each end is much smaller than D.

The center of mass of the block oscillates between $x = -D$ and $x = +D$. The curve is so gradual that the total distance traveled along the curved portion of the track by the center of mass of the block can be considered to be $2D$. At time $t = 0$ the block is released from rest at position $x = +D$. The period of the block's oscillation is 2.0 seconds.

Graph 1



Graph 2



(a) Which of the graphs above, Graph 1 or Graph 2, could represent the block's kinetic energy as a function of its position on the circular track, and which graph could represent the horizontal component of the net force acting on the block as a function of its position on the track? Justify both of your choices with physics principles.

(b) Choose one of the graphs in part (a) above and explain how the graph shows evidence that a restoring force is acting on the block.

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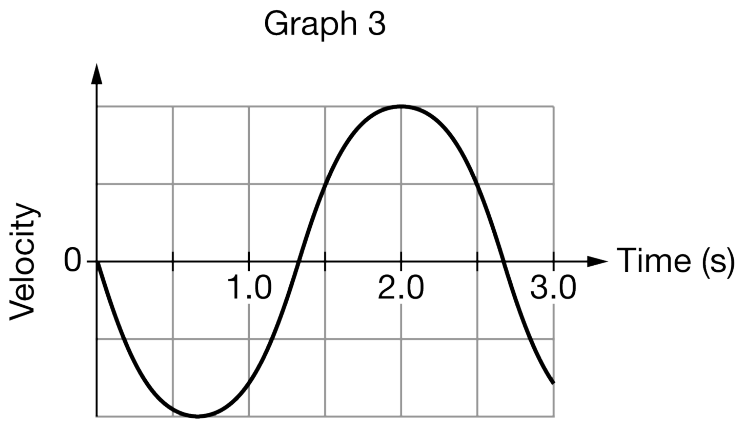
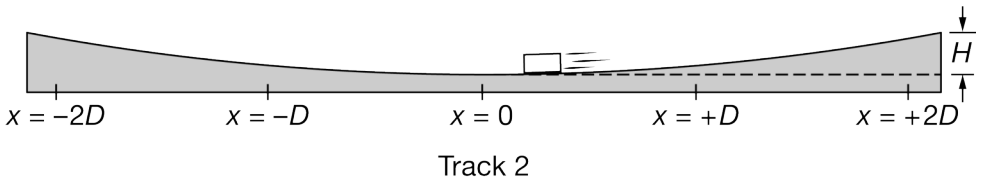


(c) Graph 3 above incorrectly shows the horizontal velocity of the block as a function of time. Identify a feature of the graph that does not correctly represent the motion of the block. Justify your answer.



Track 2

Note: Figure not drawn to scale.

(d) Track 2, shown above, has a horizontal length that is greater than that of Track 1 and the same maximum height H. The block is released from rest at position $x = +2D$ Is the period of the block's oscillation on Track 2 greater than, less than, or equal to the period of oscillation on Track 1? Justify your answer.

(e) A student derives the following equation for the period T of the block's oscillation: $T = 2\pi\sqrt{\frac{h^2}{2dg}}$, where d is the maximum displacement of the block from equilibrium and h is the height of the track at $x = d$. Whether or not this equation is correct, is the equation consistent with your answer to part (d)?

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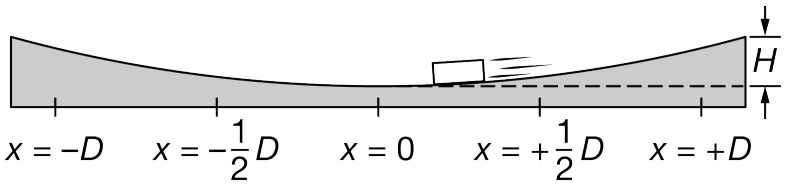
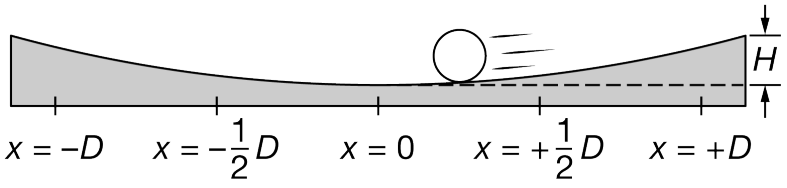


Track 1



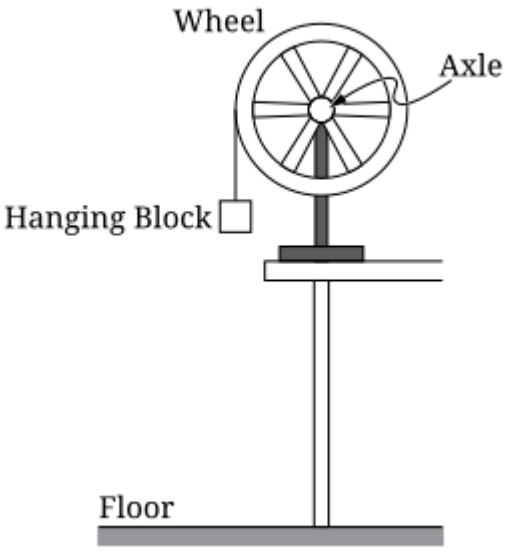
Track 3

Note: Figures not drawn to scale.

(f) On Track 1, shown above again for reference, friction is negligible and the center of mass of the block oscillates between a horizontal position of $+D$ and $-D$ with a period of 2.0 seconds. If a sphere with a mass equal to that of the block were to roll without slipping between $+D$ and $-D$ along Track 3 shown above, which is identical to Track 1 but has nonnegligible friction, would the period of the motion of the sphere be greater than, less than, or equal to 2.0 seconds? Justify your answer.

chapter4

24. This question is a long free-response question. Show your work for each part of the question.



(12 points, suggested time 25 minutes)

A wheel is mounted on a horizontal axle. A light string is attached to the wheel's rim and wrapped around it several times, and a small block is attached to the free end of the string, as shown in the figure. When the block is released from rest and begins to fall, the wheel begins to rotate with negligible friction.

Two students are discussing how different forms of energy change as the block falls. One student says that the kinetic energy of the block increases as it falls. The second student says that this is because gravitational potential energy is converted to kinetic energy. The students decide to test whether the decrease in gravitational potential energy is equal to the increase in the block's kinetic energy from when the block starts moving to immediately before it reaches the floor.

(a) Design an experimental procedure that the students could use to compare the increase in the block's translational kinetic energy with the decrease in the gravitational potential energy of the block-Earth system as the block falls.

In the table, list the quantities that would be measured in your experiment. Define a symbol to represent each quantity and list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

In the space to the right of the table, describe the overall procedure. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table.

If needed, you may include a simple diagram of the setup with your procedure.

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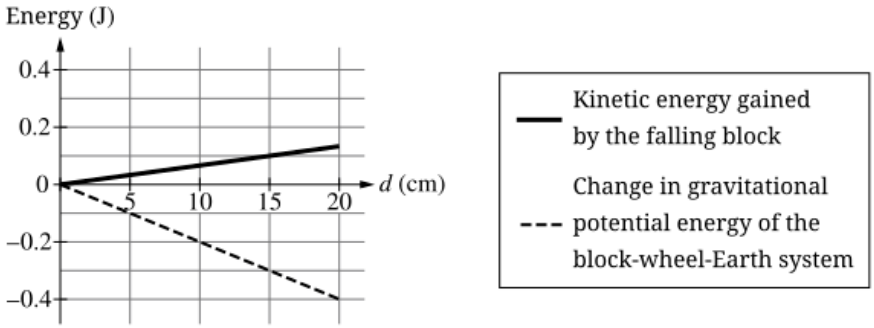
Quantity to Be Measured	Symbol for Quantity	Equipment for Measurement	Procedure (and diagram, if needed)

(b) Explain how the students could determine the kinetic energy of the block immediately before it reaches the floor using the quantities you indicated in the table in part (a).



(c) The graph above represents both the change in the gravitational potential energy of the block-wheel-Earth system and the translational kinetic energy gained by the block as functions of the block's falling distance d. On the graph, draw a line or curve to represent the rotational kinetic energy of the wheel as a function of the block's falling distance d.

(d) The students also measure the angular velocity ω of the wheel as the block falls and determine the rotational kinetic energy K_R of the wheel. The students then make a graph of K_R as a function of ω^2, as shown.

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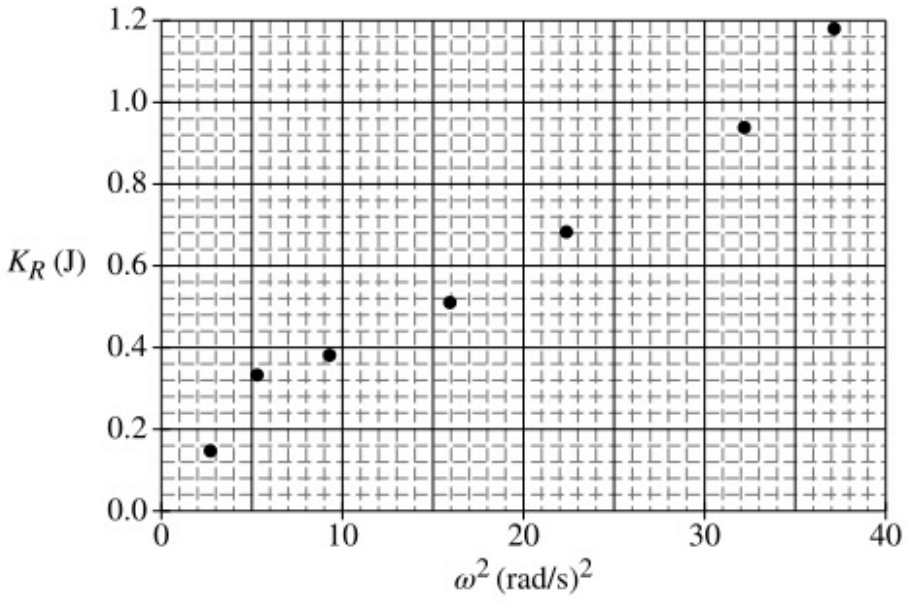
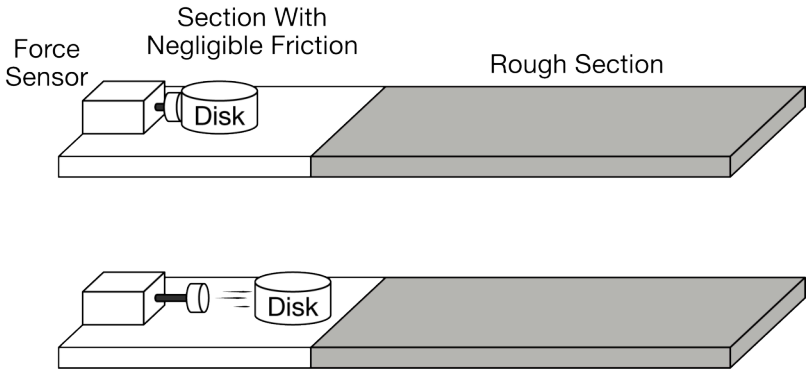


i. On the above graph, draw a straight line that best represents the data.

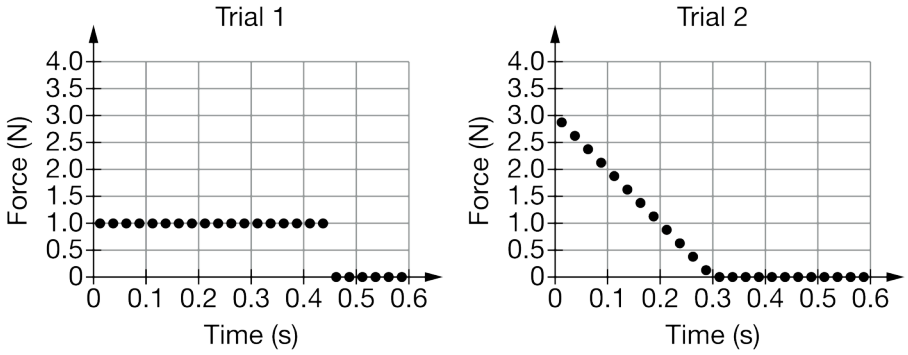
ii. Using the line you drew for part (d)(i), calculate an experimental value for the rotational inertia of the wheel.

25. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Students use a force sensor to push a disk of mass m across a horizontal table. The force sensor pushes the disk over a short time interval. The magnitude of the force and the duration of the time interval can be set by the students. After being pushed by the sensor, the disk first slides along a surface with negligible friction and then enters a rough section where friction is not negligible, as shown above.

The students perform two trials in which the magnitude of the force exerted by the force sensor and the time interval are different. They create the graphs of force as a function of time shown below. The disk enters the rough section in each trial after $t = 0.5$ s.



(a) In which of the two trials, if either, is the change in momentum of the disk between 0 seconds and 0.5 seconds greater? Explain your reasoning.

(b) In both trials, the disk comes to rest in the rough section before reaching the edge of the table. In which trial, if either, does the disk travel a greater distance in the rough section? Explain your reasoning.

(c) The students repeat Trial 1 with a disk that is made from the same material as the original disk and has the same mass as the original disk but that has a larger radius (and therefore more surface area is in contact with the surface). The same force is exerted on the new disk over the same time interval as in Trial 1. Would the distance traveled in the rough section by the new disk be greater than, less than, or the same as the distance traveled by the original disk in Trial 1? Explain your reasoning.

Students derive the following equation, which may or may not be correct, to describe the distance D traveled by

the original disk on the rough section: $D = \frac{(F_{\text{avg}}{}^2)t^2}{2\mu mg}$, where F_{avg} is the average force exerted by the sensor, t is the time that the disk is in contact with the sensor, μ (mu) is the coefficient of kinetic friction, and m is the mass of the disk.

(d) Without algebraic manipulation of equations or attempting to derive this equation, indicate whether the above equation is consistent with your answer in part (b). Explain your reasoning.

A student measures the distance x_0 that the disk travels while in contact with the force sensor in Trial 1. The disk then travels a distance D along the rough section. The student claims that if a force with twice the magnitude of the original force was exerted on the disk for a distance x_0, the disk would travel a distance $2D$ in the rough section.

(e) Is the above claim correct or incorrect? Justify your answer without citing or manipulating equations.

The student finds the following equation on the Internet describing the relationship between the magnitude F of the force exerted on the disk, the distance x the disk is pushed, and the distance D it slides in the rough section: $D = C\frac{Fx}{\mu mg}$, where C is a constant with appropriate units.

(f) Indicate whether or not the equation above is consistent with your statement in part (e) relating D and x. Explain your reasoning.

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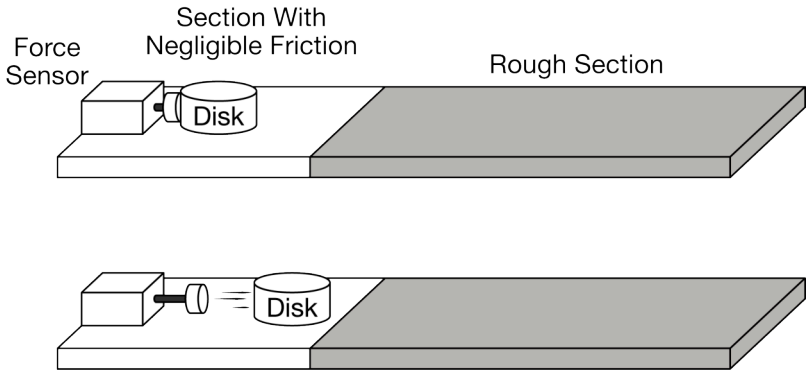
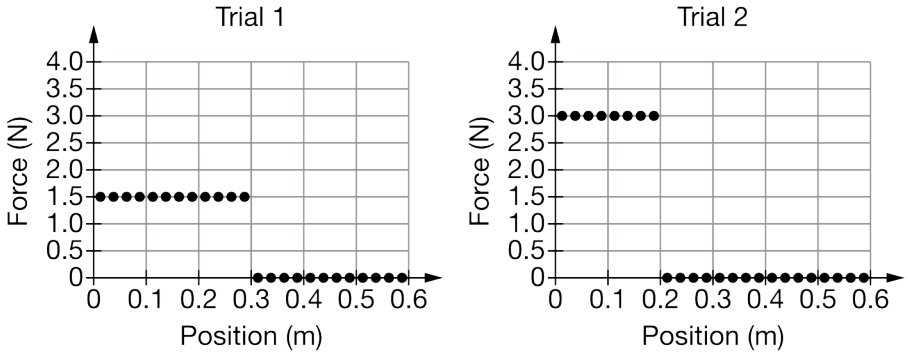
26. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Students use a force sensor to push a disk of mass m across a horizontal table. The force sensor pushes the disk over a short distance. The magnitude of the force and the distance the disk is pushed can be set by the students. After being pushed by the sensor, the disk first slides along a surface with negligible friction and then enters a rough section where friction is not negligible, as shown above.

The students perform two trials in which the magnitude of the force exerted by the force sensor and the distance the disk is pushed are different. They create the graphs of force as a function of position shown below. The disk enters the rough surface in each trial after moving 0.6 meters.



(a) In which of the two trials, if either, is the change in energy of the disk the greatest during the interval shown in the graphs? Explain your reasoning.

(b) In both trials, the disk comes to rest in the rough section before reaching the edge of the table. In which trial, if either, does the disk travel a shorter distance in the rough section? Explain your reasoning.

(c) The students repeat Trial 2 with a disk that is made from the same material as the original disk but that is less massive. The same constant force is exerted on the less massive disk over the same distance as in Trial 2. Would the distance traveled in the rough section by the less massive disk be greater than, less than, or the same as the distance traveled in the rough section by the original disk in Trial 2? Explain your reasoning.

Students derive the following equation, which may or may not be correct, to describe the distance D traveled by the original disk in Trial 2 on the rough section: $D = \frac{Fx}{\mu mg}$, where F is the force exerted by the sensor, x is the

distance over which the sensor exerts a force on the disk, μ (mu) is the coefficient of kinetic friction, and m is the mass of the disk.

(d) Without algebraic manipulation of equations or attempting to derive this equation, indicate whether the equation is consistent with your answer in part (b). Explain your reasoning.

A student measures the time t that the disk is in contact with the force sensor in Trial 2. The disk then travels a distance D along the rough section. The student claims that if the same force is exerted on the disk for a time $2t$, the disk would travel a distance $2D$ in the rough section.

(e) Is the above claim correct or incorrect? Justify your answer without citing or manipulating equations.

The student finds the following equation on the Internet describing the relationship between the time t the disk is pushed and the distance D it slides in the rough section: $D = C\frac{F^2t^2}{2\mu m^2 g}$, where C is a constant with appropriate units.

(f) Indicate whether or not the above equation is consistent with your statement in part (e) relating D and t. Explain your reasoning.

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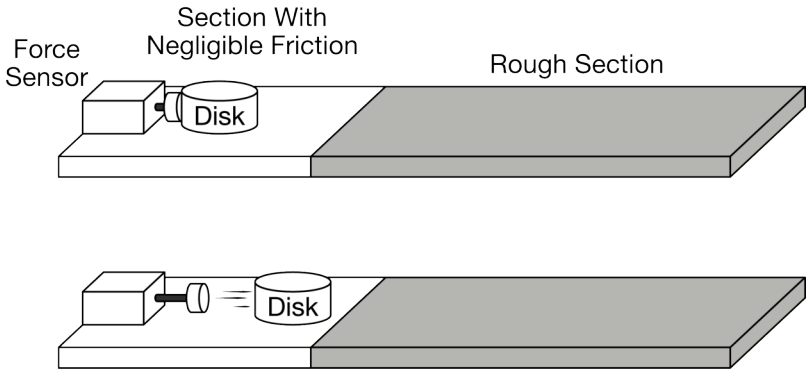
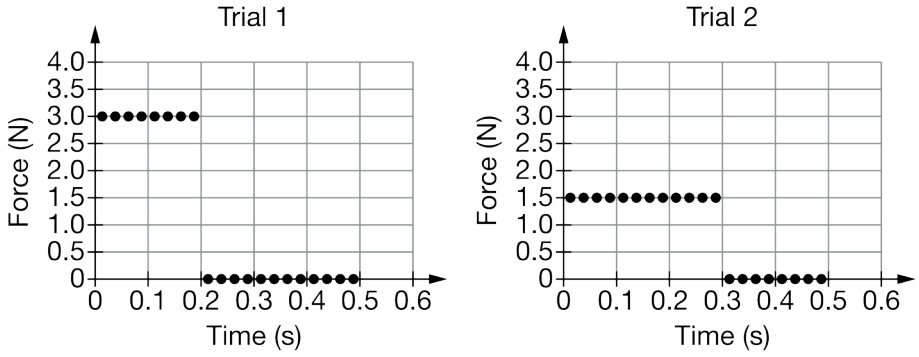
27. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



Students use a force sensor to push a disk of mass m across a horizontal table. The force sensor pushes the disk with a force of constant magnitude over a short time interval. The magnitude of the force and the duration of the time interval can be set by the students. After being pushed by the sensor, the disk first slides along a surface with negligible friction and then enters a rough section where friction is not negligible, as shown above.

The students perform two trials in which the magnitude of the force exerted by the force sensor and the time interval are different. They create the graphs of force as a function of time shown below. The disk enters the rough surface in each trial after $t = 0.5$ s.



(a) In which of the two trials, if either, is the speed of the disk the greatest at time 0.5 seconds? Explain your reasoning.

(b) In both trials, the disk comes to rest in the rough section before reaching the edge of the table. In which trial, if either, does the disk travel a greater distance in the rough section? Explain your reasoning.

(c) The students repeat Trial 1 with a disk that is made from the same material as the original disk but that is more massive. The same force is exerted on the more massive disk over the same time interval as in Trial 1. Would the distance traveled in the rough section by the more massive disk be greater than, less than, or the same as the distance traveled by the original disk in Trial 1? Explain your reasoning.

Students derive the following equation, which may or may not be correct, to describe the distance D traveled by the original disk on the rough section: $D = \frac{F^2 t^2}{2\mu m g}$, where F is the force exerted by the sensor, t is the time that

the disk is in contact with the sensor, μ (mu) is the coefficient of kinetic friction, and m is the mass of the disk.

(d) Without algebraic manipulation of equations or attempting to derive this equation, indicate whether the above equation is consistent with your answer in part (b). Explain your reasoning.

A student measures the distance x_0 that the disk travels while in contact with the force sensor in Trial 1. The disk then travels a distance D along the rough section. The student claims that if the same force is exerted on the disk for a distance $2x_0$, the disk would travel a distance $2D$ in the rough section.

(e) Is the above claim correct or incorrect? Justify your answer without citing or manipulating equations.

The student finds the following equation on the Internet describing the relationship between the distance x the disk is pushed and the distance D it slides in the rough section: $D = C\frac{Fx^2}{\mu mg}$, where C is a constant with appropriate units.

(f) Indicate whether or not the equation above is consistent with your statement in part (e) relating D and x. Explain your reasoning.

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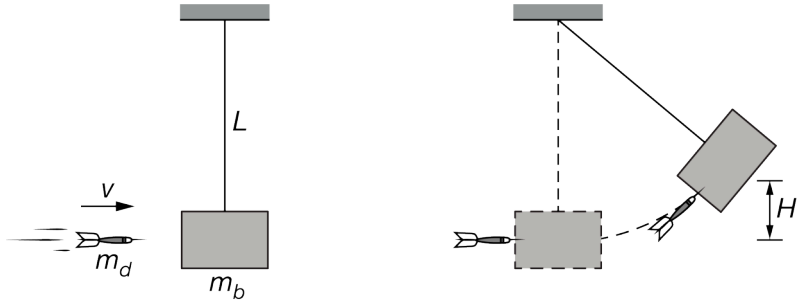
28. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown at left above. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown at right above. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, if either, experiences a net horizontal force of larger magnitude during the collision. If the net horizontal force is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the experiment was repeated with a dart with the same horizontal speed but with mass smaller than m_d, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the motion of the dart-block system at the instant it first reaches a height of $H/2$, and they make the following statements:

Student 1: When the dart-block system is at a height of $H/2$, the kinetic energy of the system is half of its maximum value, and therefore the speed of the system is half of its maximum speed.

Student 2: When the dart-block system reaches a height of $H/2$, the centripetal force is equal to the tension in the string. Therefore, the acceleration of the system must point toward the fixed end of the string.

(c) Which underlined phrase or phrases, if either, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if either, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if either, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

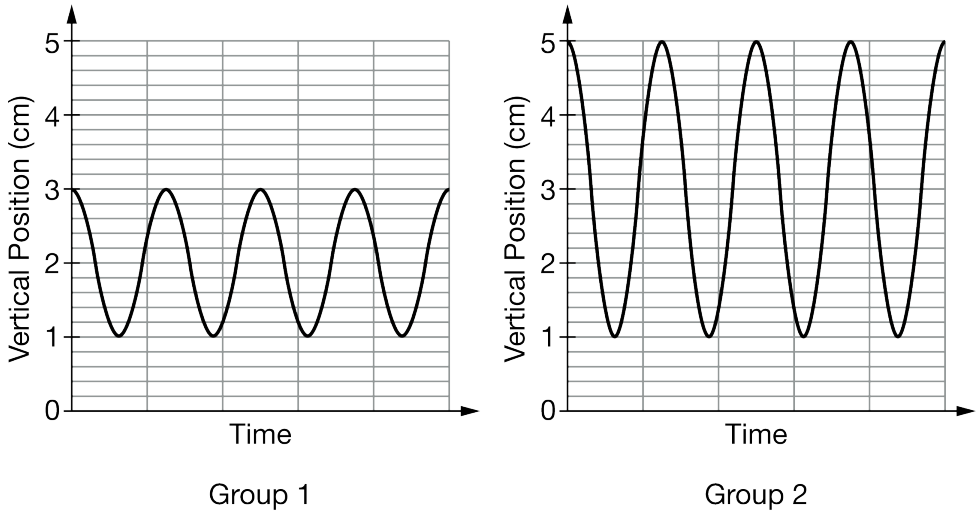
(f) Which underlined phrase or phrases, if either, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{v}{1+\frac{m_b}{m_d}}\right)^2$. Explain how this

equation supports your claim in part (b).

(h) Could the masses of the dart and block both be changed so that the maximum height *H* reached by the dart-block system would remain the same as in the original setup? Justify your answer by referring to the equation in part (g).

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart mass or the string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

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29. This question is a long free-response question. Show your work for each part of the question.

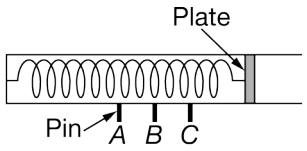


Figure 1. Uncompressed spring

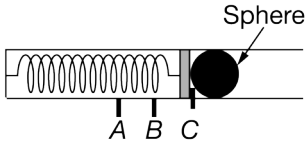


Figure 2. Compressed spring

(12 points, suggested time 25 minutes)

A projectile launcher consists of a spring with an attached plate, as shown in Figure 1. When the spring is compressed, the plate can be held in place by a pin at any of three positions A, B, or C. For example, Figure 2 shows a steel sphere placed against the plate, which is held in place by a pin at position C. The sphere is launched upon release of the pin.

A student hypothesizes that the spring constant of the spring inside the launcher has the same value for different compression distances.

(a) The student plans to test the hypothesis by launching the sphere using the launcher.

i. State a basic physics principle or law the student could use in designing an experiment to test the hypothesis.

ii. Using the principle or law stated in part (a)(i), determine an expression for the spring constant in terms of quantities that can be obtained from measurements made with equipment usually found in a school physics laboratory.

(b) Design an experimental procedure to test the hypothesis in which the student uses the launcher to launch the sphere. Assume equipment usually found in a school physics laboratory is available.

In the table below, list the quantities and associated symbols that would be measured in your experiment. Also list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

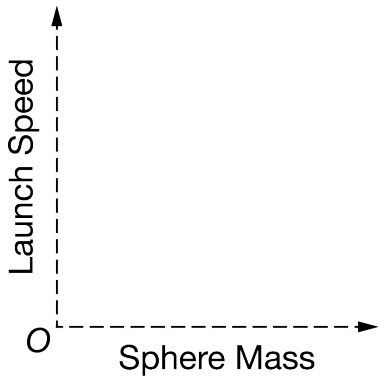
Quantity to be Measured	Symbol for Quantity	Equipment for Measurement

Describe the overall procedure to be used to test the hypothesis that the spring constant of the spring inside the launcher has the same value for different compression distances, referring to the table. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental

uncertainty. As needed, use the symbols defined in the table and/or include a simple diagram of the setup.

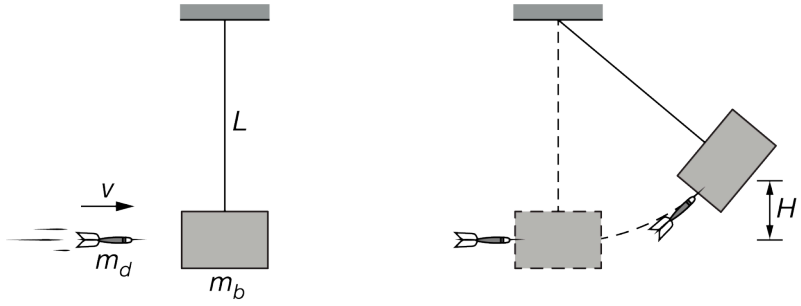
(c) Describe how the experimental data could be analyzed to confirm or disconfirm the hypothesis that the spring constant of the spring inside the launcher has the same value for different compression distances.

(d) Another student uses the launcher to consecutively launch several spheres that have the same diameter but different masses, one after another. Each sphere is launched from position A. Consider each sphere's launch speed, which is the speed of the sphere at the instant it loses contact with the plate. On the axes below, sketch a graph of launch speed as a function of sphere mass.



chapter4

30. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A dart with mass m_d is launched toward a block of mass m_b that is suspended from a string of length *L*, as shown above at left. The dart is moving horizontally with speed *v* immediately before it strikes the block and remains embedded. The dart-block system then swings up to a point at which its center of mass reaches a maximum height *H* above its starting position, as shown above at right. The block's mass m_b is greater than the dart's mass m_d.

(a) Indicate which object, if either, experiences an acceleration of larger magnitude during the collision. If the acceleration is the same magnitude for both objects, state this explicitly. Briefly explain your reasoning.

(b) If the experiment were repeated with a block with mass greater than m_b, how would the maximum height reached by the center of mass of the dart-block system compare to *H* ? Explain your response without deriving or manipulating equations.

Two students are discussing the tension in the string at the moment immediately after the collision and state the following:

Student 1: The tension in the string immediately after the collision is greater than the weight of the dart-block system because the acceleration is upward at that instant.

Student 2: The tension in the string is equal to the weight of the block-dart system because there is no vertical acceleration immediately after the collision.

(c) Which underlined phrase or phrases, if any, are ***correct*** about student 1's statement? If neither phrase is ***correct***, write "none."

(d) Which underlined phrase or phrases, if any, are ***incorrect*** about student 1's statement? If neither phrase is ***incorrect***, write "none."

(e) Which underlined phrase or phrases, if any, are ***correct*** about student 2's statement? If neither phrase is ***correct***, write "none."

(f) Which underlined phrase or phrases, if any, are ***incorrect*** about student 2's statement? If neither phrase is ***incorrect***, write "none."

(g) The students correctly derive an equation for the maximum height: $H = \frac{1}{2g}\left(\frac{m_d v}{m_d + m_b}\right)^2$. Explain how this equation supports your claim in part (b).

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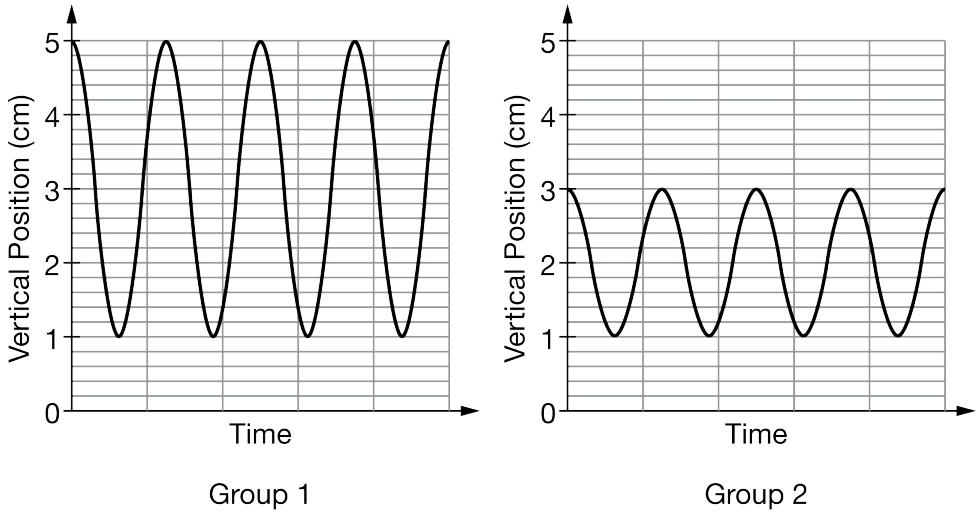
(h) Consider the case in which the collision between the dart and the block is perfectly elastic (the dart bounces off the block to the left). In this case, is the speed of the block immediately after the collision greater than, less than, or the same as the speed of the block in the original case? Justify your answer.

After the collision, the dart-block system is then allowed to swing freely, and the students produce the graph shown below at left, labeled Group 1, of the vertical position of the center of mass of the dart-block system as a function of time. A second group of students makes a single change to the setup and then repeats the experiment. Their graph, labeled Group 2, of the vertical position of the center of mass of the dart-block system as a function of time is shown below at right.



Note: The time scale is the same for both graphs.

(i) Indicate the single difference in either the dart speed or the string length between group 1's setup and group 2's setup that could lead to the difference between the two graphs. Justify your answer.

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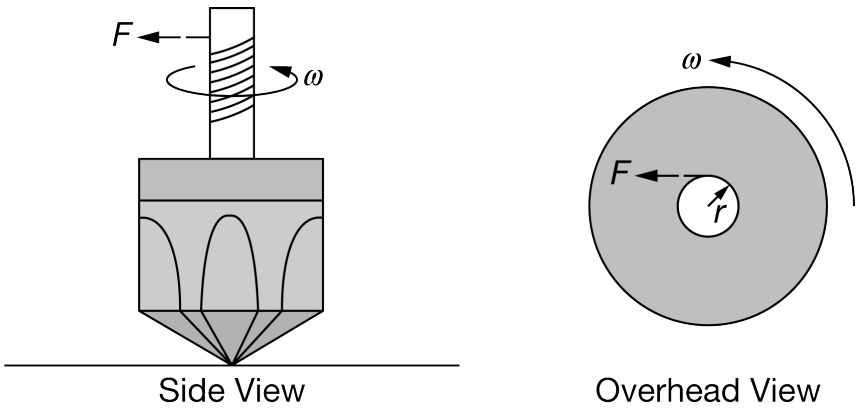
31. This question is a long free-response question. Show your work for each part of the question. (15 points, suggested time 25 minutes)



A light string of total length L is wrapped completely around the axle of a spinning toy top. The radius of the axle is r and the rotational inertia of the top is I. The top is initially at rest and the axle is held vertically with respect to a horizontal table. At time $t = 0$ a force with constant magnitude F pulls the string outwards from the axle of the top, which causes the top to rotate. After a time T the string loses contact with the axle of the top and the top spins on the table with an angular speed ω (omega). Assume the top always remains upright and there is negligible friction between the top and the table.

(a) The original string is replaced with a new string of length $2L$. The string is again pulled with a constant force of magnitude F. When the string loses contact with the top, the angular speed of the top is ω_A. Is ω_A greater than, less than, or equal to ω? Justify your answer without citing or manipulating equations.

(b) The original string of length L is used to spin a second top that also has an axle with radius r, but a larger rotational inertia $I_B = 2I$. The string is pulled with the same constant force of magnitude F. When the string loses contact with the top, the angular speed of the second top is ω_B. Is ω_B greater than, less than, or equal to ω? Justify your answer without citing or manipulating equations.

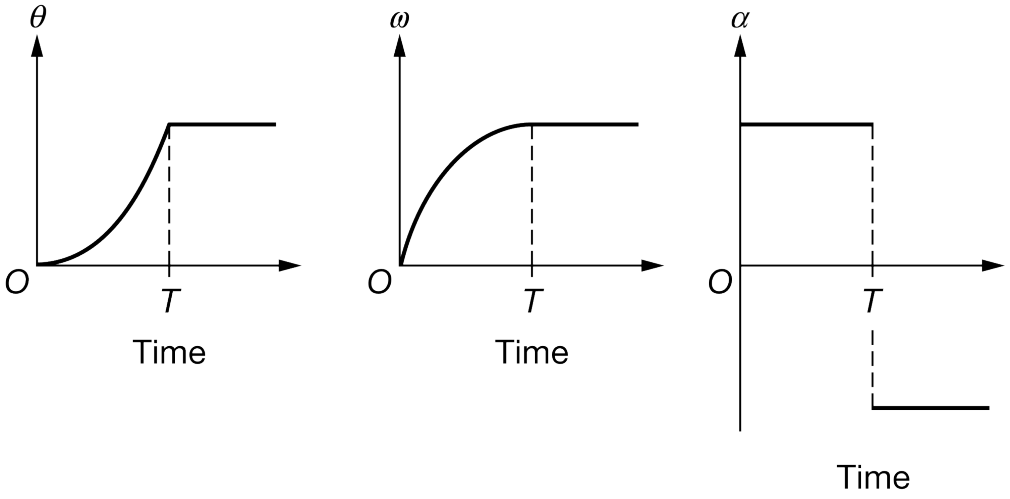
(c) A student derives the following equation to predict the final angular speed of the top: $\omega = \sqrt{\frac{2FL}{I}}$. Whether or not this equation is correct, does it match your reasoning in parts (a) and (b)?

Once the string has been pulled for a certain amount of time T, the string comes off and the top spins on the table.

(d) Describe any changes in the top's angular momentum about a vertical axis through the center of the top after the string loses contact with the top. Explain your answer.

A student makes sketches of the angular displacement θ, angular velocity ω, and angular acceleration α of the top as functions of time. The graphs below represent the time interval from when the string is first pulled until after the string loses contact with the top at time T. Consider the parts of the graph for the time before T and the time after T. Each graph is consistent with the motion of the top for one part and inconsistent for the other part.

chapter4



(e) For which part of the graph, either the time before T or the time after T, is the graph of the angular displacement θ inconsistent with the motion of the top as described directly above the graphs? Explain how to correct the part of the graph that is inconsistent.

(f) For which part of the graph, either the time before T or the time after T, is the graph of the angular velocity ω inconsistent with the motion of the top as described directly above the graphs? Explain how to correct the part of the graph that is inconsistent.

(g) For which part of the graph, either the time before T or the time after T, is the graph of the angular acceleration α inconsistent with the motion of the top as described directly above the graphs? Explain how to correct the part of the graph that is inconsistent.