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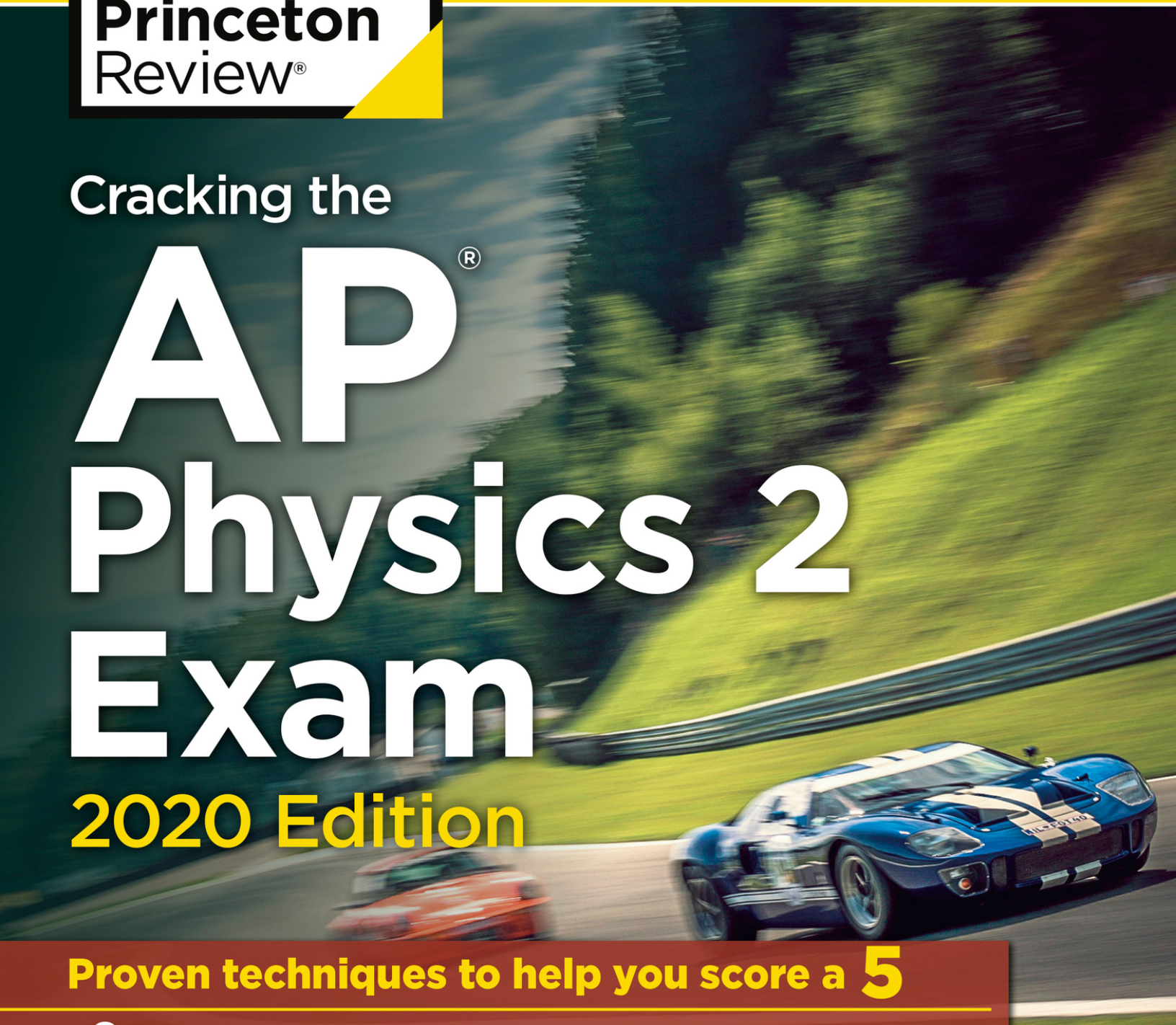

Cracking the AP® Physics 2 Exam

2020 Edition

Proven techniques to help you score a 5

- **2** full-length practice tests with answer explanations included
- Comprehensive review of key AP Physics 2 concepts
- Targeted strategies for every section of the exam
- Up-to-date info on the planned 2020 course changes via your online Student Tools

By the Staff of The Princeton Review

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The Princeton Review®

Cracking the AP® PHYSICS 2 EXAM

2020 Edition

By the Staff of The Princeton Review

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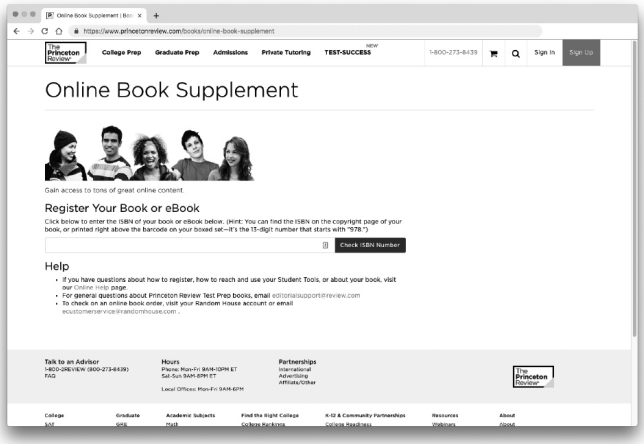

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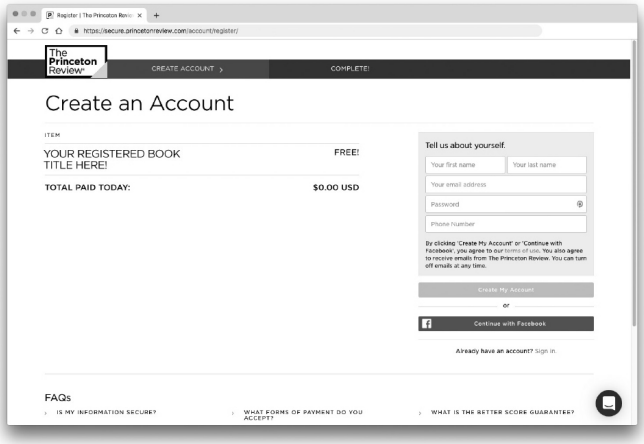

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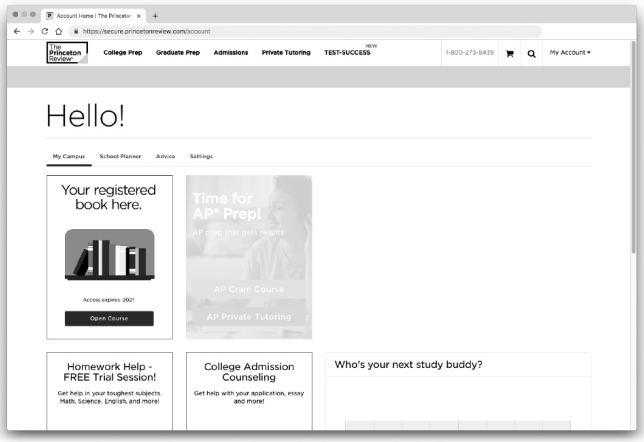

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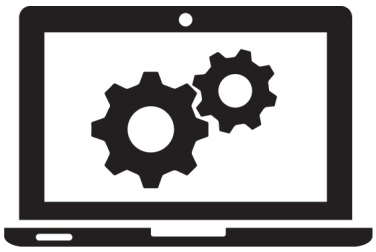

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- email address used to register the book
- full book title and ISBN
- Operating system (Mac/PC) and browser (Firefox, Safari, etc.)

Once you've registered, you can…

- Get our take on any recent or pending updates to the AP Physics 2 Exam
- Access comprehensive study guides and a variety of printable resources, including: bubble sheets for the practice tests in the book as well as important equations and formulas
- Take a full-length practice SAT and ACT
- Get valuable advice about the college application process, including tips for writing a great essay and where to apply for financial aid
- If you're still choosing between colleges, use our searchable rankings of *The Best 385 Colleges* to find out more information about your dream school.
- Check to see if there have been any corrections or updates to this edition

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ONLINE ARTICLES


PROVEN TECHNIQUES


MORE GREAT BOOKS


TIME-SAVING TIP

Part I

Using This Book to Improve Your AP Score

- Preview: Your Knowledge, Your Expectations
- Your Guide to Using This Book
- How to Begin

PREVIEW: YOUR KNOWLEDGE, YOUR EXPECTATIONS

Your route to a high score on the AP Physics 2 Exam will depend on how you plan to use this book. Please respond to the following questions.

1. Rate your level of confidence about your knowledge of the content tested by the AP Physics 2 Exam.

   A. Very confident—I know it all

   B. I'm pretty confident, but there are topics for which I could use help

   C. Not confident—I need quite a bit of support

   D. I'm not sure

2. If you have a goal score in mind, choose your goal score for the AP Physics 2 Exam.

   5 4 3 2 1 I'm not sure yet

3. What do you expect to learn from this book? Choose all that apply to you.

   A. A general overview of the test and what to expect

   B. Strategies for how to approach the test

   C. The content tested by this exam

   D. I'm not sure yet



Course Changes

For information on the College Board updates to the AP Physics 2 course, visit your Student Tools (see this page for details).

YOUR GUIDE TO USING THIS BOOK

This book is organized to provide as much—or as little—support as you need, so you can use this book in whatever way will be most helpful for improving your score on the AP Physics 2 Exam.

- The remainder of **Part I** will provide guidance on how to use this book and help you determine your strengths and weaknesses.

- **Part II** of this book contains your first practice test, answers and explanations, and a scoring guide. (Bubble sheets can be found in the very back of the book for easy tear-out.) This is where you should begin your test preparation in order to realistically determine:
    - your starting point right now
    - which question types you're ready for and which you might need to practice
    - which content topics you are familiar with and which you will want to carefully review

  Once you have nailed down your strengths and weaknesses with regard to this exam, you can focus your preparation and be efficient with your time.

- **Part III** of this book will:
    - provide information about the structure, scoring, and content of the AP Physics 2 Exam
    - help you to make a study plan
    - point you toward additional resources

- **Part IV** of this book will explore the following strategies:
    - how to attack multiple-choice questions
    - how to approach free-response questions

- how to manage your time to maximize the number of points available to you

- **Part V** of this book covers the content you need for the AP Physics 2 Exam.

- **Part VI** of this book contains Practice Test 2, its answers and explanations, and a scoring guide. (Bubble sheets can be found in the very back of the book for easy tear-out.) If you skipped Practice Test 1, we recommend that you do both (with at least a day or two between them) so that you can compare your progress between the two. Additionally, this will help to identify any external issues: if you get a certain type of question wrong both times, you probably need to review it. If you only got it wrong once, you may have run out of time or been distracted by something. In either case, this will allow you to focus on the factors that caused the discrepancy in scores and to be as prepared as possible on the day of the test.



Don't Forget!

When you register your book online following the instructions on this page, you'll gain access to a wealth of other helpful Student Tools, including study guides and printable bubble sheets.

You may choose to use some parts of this book over others, or you may work through the entire book. This will depend on your needs and how much time you have. Let's now look at how to make this determination.

Equation Sheet Sidebars

Throughout the content review chapters of this book you'll see the words "Equation Sheet" pop up in the sidebars. These words will be seen next to physics equations that are featured on the equation sheet that the College Board gives you during your AP Physics 2 Exam. So these are equations that are extremely important for you to be familiar with.



Access your Students Tools online for a free diagnostic answer key for Practice Test 1, which includes targeted study recommendations.

HOW TO BEGIN

1. **Take Practice Test 1**

   Before you can decide how to use this book, you need to take a practice test. Doing so will give you insight into your strengths and weaknesses, and the test will also help you make an effective study plan. If you're feeling test-phobic, remind yourself that a practice test is a tool for diagnosing yourself—it's not how well you do that matters but how you use information gleaned from your performance to guide your preparation.

   So, before you read further, take AP Physics 2 Practice Test 1 starting at this page of this book. Be sure to do so in one sitting, following the instructions that appear before the test.

2. **Check Your Answers**

   Using the answer key on this page, count how many multiple-choice questions you got right and how many you missed. Don't worry about the explanations for now, and don't worry about why you missed questions. We'll get to that soon.

3. **Reflect on the Test**

   After you take your first test, respond to the following questions:

   - How much time did you spend on the multiple-choice questions?
   - How much time did you spend on each free-response question?
   - How many multiple-choice questions did you miss?

- Do you feel you had the knowledge to address the subject matter of the free-response questions?

4. Read Part III of this Book and Complete the Self-Evaluation

Part III will provide information on how the test is structured and scored. It will also set out areas of content that are tested.

As you read Part III, re-evaluate your answers to the questions above. At the end of Part III, you will revisit the questions above and refine your answers to them. You will then be able to make a study plan, based on your needs and time available, that will allow you to use this book most effectively.

5. Engage with Parts IV and V as Needed

Notice the word *engage*. You'll get more out of this book if you use it intentionally than if you read it passively, hoping for an improved score through osmosis.

Strategy chapters will help you think about your approach to the question types on this exam. Part IV will open with a reminder to think about how you approach questions now and then close with a reflection section asking you to think about how/whether you will change your approach in the future.

Content chapters are designed to provide a review of the content tested on the AP Physics 2 Exam, including the level of detail you need to know and how the content is tested. You will have the opportunity to assess your mastery of the content of each chapter through test-appropriate questions and a reflection section.

6. Take Practice Test 2 and Assess Your Performance

Once you feel you have developed the strategies you need and gained the knowledge you lacked, you should take Practice Test 2. You should do so in one sitting, following the instructions at the beginning of the test.

When you are done, check your answers to the multiple-choice sections. See if a teacher will read your free-response answers and provide feedback.

Once you have taken the test, reflect on what areas you still need to work on, and revisit the chapters in this book that address those deficiencies. Through this type of reflection and engagement, you will continue to improve.

7. Keep Working

After you have revisited certain chapters in this book, continue the process of testing, reflecting, and engaging with the next practice test in this book. Consider what additional work you need to do and how you will change your strategic approach to different parts of the test. You can continue to explore areas that can stand to be improved and engage in those areas right up to the day of the test. As we will discuss in Part III, there are other resources available to you, including a wealth of information online at AP Students.

Part II
Practice Test 1

- Practice Test 1
- Practice Test 1: Answers and Explanations

Practice Test 1

Click here to download the PDF of Practice Test 1.

The Exam

AP® Physics 2 Exam

SECTION I: Multiple-Choice Questions

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
90 minutes
Number of Questions
50
Percent of Total Grade
50%
Writing Instrument
Pen required

Instructions

Section I of this examination contains 50 multiple-choice questions. Fill in only the ovals for numbers 1 through 50 on your answer sheet.

CALCULATORS MAY BE USED IN BOTH SECTIONS OF THE EXAMINATION.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding oval on the answer sheet. Give

only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question

Chicago is a
(A) state
(B) city
(C) country
(D) continent

Sample Answer



Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all the multiple-choice questions.

About Guessing

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. Multiple choice scores are based on the number of questions answered correctly. Points are not deducted for incorrect answers, and no points are awarded for unanswered questions. Because points are not deducted for incorrect answers, you are encouraged to answer all multiple-choice questions. On any questions you do not know the answer to, you should eliminate as many choices as you can, and then select the best answer among the remaining choices.

ADVANCED PLACEMENT PHYSICS 2 TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg	1 electron volt,	1 eV $= 1.60 \times 10^{-19}$ J
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg	Speed of light,	$c = 3.00 \times 10^{8}$ m/s
Avogadro's number,	$N_A = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant,	$G = 6.67 \times 10^{-11}$ m^3/kg·s^2
Universal gas constant,	$R = 8.31$ J/(mol·K)	Acceleration due to gravity at Earth's surface,	$g = 9.8$ m/s^2
Boltzmann's constant,	$k_B = 1.38 \times 10^{23}$ J/K		

1 unified atomic mass unit,	1 u $= 1.66 \times 10^{-27}$ kg $= 931$ MeV/c^2
Planck's constant,	$h = 6.63 \times 10^{-34}$ J·s $= 4.14 \times 10^{-15}$ eV·s
	$hc = 1.99 \times 10^{-25}$ J·m $= 1.24 \times 10^{3}$ eV·nm
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/N·m^2
Coulomb's law constant,	$k = 1/4\pi\varepsilon_0 = 9.0 \times 10^{9}$ N·m^2/C^2
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ (T·m)/A
Magnetic constant,	$k' = \mu_0/4\pi = 1 \times 10^{-7}$ (T·m)/A
1 atmosphere pressure,	1 atm $= 1.0 \times 10^{5}$ N/m^2 $= 1.0 \times 10^{5}$ Pa

UNIT SYMBOLS	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	1/2	3/5	$\sqrt{2}/2$	4/5	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	4/5	$\sqrt{2}/2$	3/5	1/2	0
$\tan\theta$	0	$\sqrt{3}/3$	3/4	1	4/3	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. The frame of reference of any problem is assumed to be inertial unless otherwise stated.
II. In all situations, positive work is defined as work done on a system.
III. The direction of current is conventional current: the direction in which positive charge would drift.
IV. Assume all batteries and meters are ideal unless otherwise stated.
V. Assume edge effects for the electric field of a parallel plate capacitor unless otherwise stated.
VI. For any isolated electrically charged object, the electric potential is defined as zero at infinite distance from the charged object.

ADVANCED PLACEMENT PHYSICS 2 EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\left|\vec{F}_f\right| \le \mu \left|\vec{F}_n\right|$

$a_c = \frac{v^2}{r}$

$\vec{p} = m\vec{v}$

$\Delta\vec{p} = \vec{F}\,\Delta t$

$K = \frac{1}{2} m v^2$

$\Delta E = W = F_{\|} d = F d \cos\theta$

$P = \frac{\Delta E}{\Delta t}$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega = \omega_0 + \alpha t$

$x = A\cos(\omega t) = A\cos(2\pi f t)$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$\tau = r_\perp F = rF\sin\theta$

$L = I\omega$

$\Delta L = \tau\,\Delta t$

$K = \frac{1}{2} I \omega^2$

$\left|\vec{F}_s\right| = k\left|\vec{x}\right|$

a = acceleration
A = amplitude
d = distance
E = energy
F = force
f = frequency
I = rotational inertia
K = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
r = radius or separation
T = period
t = time
U = potential energy
v = speed
W = work done on a system
x = position
y = height
α = angular acceleration
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed

$U_s = \frac{1}{2} k x^2$

$\Delta U_g = mg\,\Delta y$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$

$\vec{g} = \frac{\vec{F}_g}{m}$

$U_G = -\frac{G m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$\left|\vec{F}_E\right| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$

$\Delta U_E = q\Delta V$

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

$\left|\vec{E}\right| = \left|\frac{\Delta V}{\Delta r}\right|$

$\Delta V = \frac{Q}{C}$

$C = \kappa\varepsilon_0 \frac{A}{d}$

$E = \frac{Q}{\varepsilon_0 A}$

$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$

$I = \frac{\Delta Q}{\Delta t}$

$R = \frac{\rho\ell}{A}$

$P = I\,\Delta V$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$B = \frac{\mu_0}{2\pi}\frac{I}{r}$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
ℓ = length
P = power
Q = charge
q = point charge
R = resistance
r = separation
t = time
U = potential (stored) energy
V = electric potential
v = speed
κ = dielectric constant
ρ = resistivity
θ = angle
Φ = flux

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\left|\vec{F}_M\right| = |q\vec{v}|\left|\sin\theta\right|\left|\vec{B}\right|$

$\vec{F}_M = I\vec{\ell} \times \vec{B}$

$\left|\vec{F}_M\right| = \left|I\vec{\ell}\right|\left|\sin\theta\right|\left|\vec{B}\right|$

$\Phi_B = \vec{B} \cdot \vec{A}$

$\Phi_B = \left|\vec{B}\right|\cos\theta\left|\vec{A}\right|$

$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t}$

$\mathcal{E} = B\ell v$

ADVANCED PLACEMENT PHYSICS 2 EQUATIONS

FLUID MECHANICS AND THERMAL PHYSICS

$\rho = \frac{m}{V}$

$P = \frac{F}{A}$

$P = P_0 + \rho g h$

$F_b = \rho V g$

$A_1 v_1 = A_2 v_2$

$P_1 + \rho g y_1 + \frac{1}{2}\rho {v_1}^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho {v_2}^2$

$\frac{Q}{\Delta t} = \frac{kA\,\Delta T}{L}$

$PV = nRT = Nk_BT$

$K = \frac{3}{2}k_BT$

$W = -P\Delta V$

$\Delta U = Q + W$

A = area
F = force
h = depth
k = thermal conductivity
K = kinetic energy
L = thickness
m = mass
n = number of moles
N = number of molecules
P = pressure
Q = energy transferred to a system by heating
T = temperature
t = time
U = internal energy
V = volume
v = speed
W = work done on a system
y = height
ρ = density

MODERN PHYSICS

$E = hf$

$K_{\max} = hf - \phi$

$\lambda = \frac{h}{p}$

$E = mc^2$

E = energy
f = frequency
K = kinetic energy
m = mass
p = momentum
λ = wavelength
ϕ = work function

WAVES AND OPTICS

$\lambda = \frac{v}{f}$

$n = \frac{c}{v}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$

$|M| = \left|\frac{h_i}{h_o}\right| = \left|\frac{s_i}{s_o}\right|$

$\Delta L = m\lambda$

$d\sin\theta = m\lambda$

d = separation
f = frequency or focal length
h = height
L = distance
M = magnification
m = an integer
n = index of refraction
s = distance
v = speed
λ = wavelength
θ = angle

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = \frac{1}{2}bh$

Circle
$A = \pi r^2$
$C = 2\pi r$

Rectangular solid
$V = \ell wh$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r\ell + 2\pi r^2$

Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

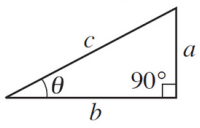
Right triangle

$c^2 = a^2 + b^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



Section I

PHYSICS 2
SECTION I
Time—90 minutes
50 Questions

Note: To simplify calculations, you may use $g = 10\ \text{m/s}^2$ in all problems.

Directions: Each of the questions or incomplete statements below is followed by four suggested answers or completions. Select the one that is best in each case and mark it on your sheet.

Questions 1 and 2 refer to the following situation.

A proton is travelling along a straight line at a constant speed through a uniform electric field near the surface of the Earth.

1. Which of the following choices correctly describes the direction of the electric field and the relative magnitudes of the electric and gravitational fields?

	Electric Field Direction	Field Strength
(A)	Down	$E_g = E_{el}$
(B)	Down	$E_g > E_{el}$
(C)	Up	$E_g = E_{el}$
(D)	Up	$E_g > E_{el}$

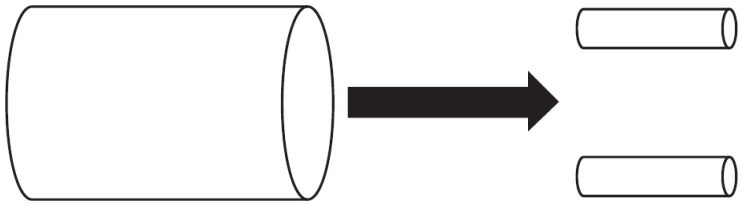
2. Which of the following describes the equipotential lines for the electric and gravitational fields that the proton experiences?

(A) Equipotential lines are straight, horizontal lines for both fields.

(B) Equipotential lines are straight, vertical lines for both fields.

(C) Equipotential lines are straight, horizontal lines for the electric field and curve upward for the gravitational field.

(D) Equipotential lines are straight, horizontal lines for the gravitational field and curve upward for the electric field.



3. A pipe with a diameter of D splits into two smaller, identical pipes with diameter d. If the speed of the water in the small pipes is v, what is the speed of the water in the large pipe?

(A) dv/D

(B) $2dv/D$

(C) d^2v/D^2

(D) $2d^2v/D^2$

4. A fluid container is shaped as a rectangular prism. The areas of the three faces of the prism are different. Which face of the container should be placed on a horizontal table so that the fluid pressure against the face touching the table is at the lowest value?

(A) The face with the smallest area

(B) The face with the median area

(C) The face with the largest area

(D) All three orientations will result in the same fluid pressure.

5. An ohmmeter is used to measure resistance. Measurements are made of the cross-sectional area, A, and length, L, of each

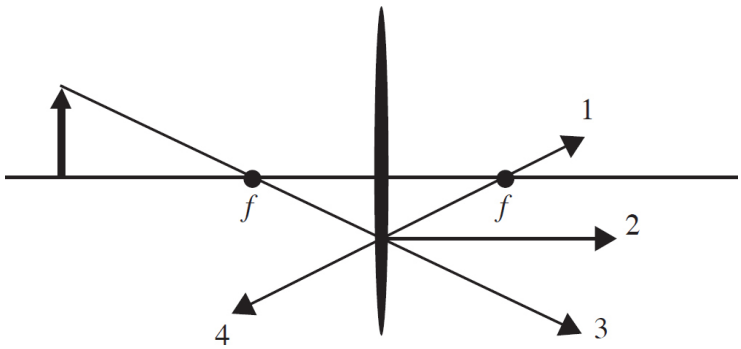
resistor. What should be plotted so that the slope of the plot will yield the resistivity, ρ, of the resistors?

(A) Resistance on the y-axis versus ratio of L to A on the x-axis
(B) Resistance on the y-axis versus ratio of A to L on the x-axis
(C) Ratio of L to A on the y-axis versus resistance on the x-axis
(D) Ratio of A to L on the y-axis versus resistance on the x-axis

6. An ideal gas is taken from an initial set of conditions with pressure P_i and volume V_i, to a final set of conditions, with pressure P_f and volume V_f, through several different processes. At the end of the process, the gas is at both a higher pressure and a larger volume than when it started. Which process requires the least amount of work on the system?

(A) First, expansion at constant pressure P_i from V_i to V_f followed by increasing pressure from P_i to P_f at constant volume V_f
(B) First, increasing pressure from P_i to P_f at constant V_i volume followed by expansion at constant pressure P_f from V_i to V_f
(C) A series of small increases in volume alternating with small increases in pressure, resulting in a nearly straight line on a PV graph from the beginning to the end of the process.
(D) Any set of steps will require the same amount of work because all gases have the same change in pressure and volume.

Questions 7 and 8 refer to the following diagram.



7. The image above shows a converging lens and an object represented as a bold vertical arrow. Which line correctly depicts the output path of the ray for the ray that is incident on the lens coming in through the focus?

(A) 1
(B) 2
(C) 3
(D) 4

8. The converging lens above has a focal length of 25 cm. The object is located at a distance of 65 cm from the lens. Where should a screen be placed so that the observer will see a focused image on the screen?

(A) 65 cm to the right side of the lens
(B) 40.6 cm to the right side of the lens
(C) At the focus of the lens
(D) The image in such an arrangement will be virtual and cannot be seen on a screen.

9. You are tasked with creating a real image using a concave mirror as your imaging system. Which of the following criteria is true about both the image and the object?

(A) A real image can be created only if the object is farther away from the lens than the radius of curvature of the lens.

(B) A real image can be created only with the object located between the center of the lens and the focal length of the lens.

(C) A real image can be created with the object located anywhere farther from the lens than the focal point.

(D) A real image cannot be created using only a concave lens.

10. An atom has its lowest four energy levels at −10 eV, −5 eV, −3.5 eV, −2 eV. Which of the following photons could not be absorbed by the atom?

(A) A 10 eV photon

(B) A 5 eV photon

(C) A 2.5 eV photon

(D) A 1.5 eV photon

11. A radioactive particle undergoes beta decay, emitting an electron from its nucleus. Which of the following explanations correctly explains why this process must also involve a neutron turning into a proton within the nucleus?

(A) The total charge of the system before and after is not the same if the proton is not created.

(B) The mass energy of the system is not conserved if the proton is not created.

(C) The momentum of the system could not be conserved without the generation of a proton.

(D) All nuclear decay processes involve the generation of two particles.

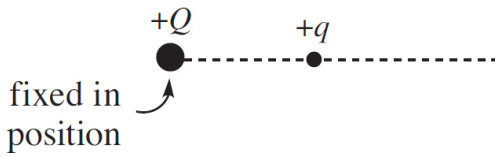
12. A thermodynamic process is conducted wherein an ideal gas is taken from State A to B to C to D to A. State A is at a pressure P and a volume 5V. State B is at pressure 4P and volume V. State C is at pressure 4P and volume 4V. State D is at pressure 2P and

volume 10V. Which step in the process requires the largest change in internal energy of the system?

(A) State A to State B
(B) State B to State C
(C) State C to State D
(D) State D to State A

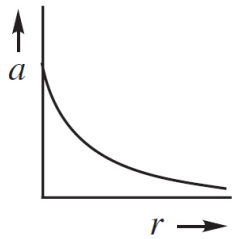
13. A circuit consists of a 50 V battery, a 100 Ω resistor and a 25 μF capacitor. Once the capacitor has become fully charged, how much energy is stored in the capacitor?

(A) 0.0013 J
(B) 0.031 J
(C) 0.062 J
(D) 0.125 J

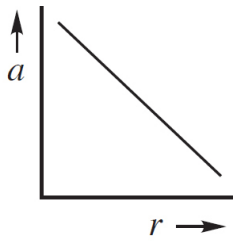


14. As shown above, the $+Q$ charge is fixed in position, and the $+q$ charge is brought close to $+Q$ and then released from rest. Which graph best shows the acceleration of the $+q$ charge as a function of its distance r from $+Q$?

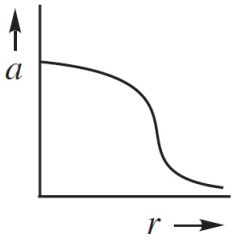
(A)



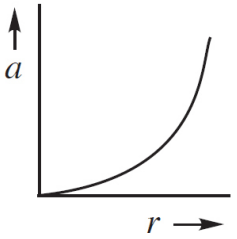
(C)



(B)



(D)



15. A charged particle with mass m is moving at a speed v at one particular instant in time. The particle is later found at a position with an electrical potential of ΔV higher than its initial position, and a gravitational potential equal to its original position. Which mathematical routine could be used to determine its speed in the final position?

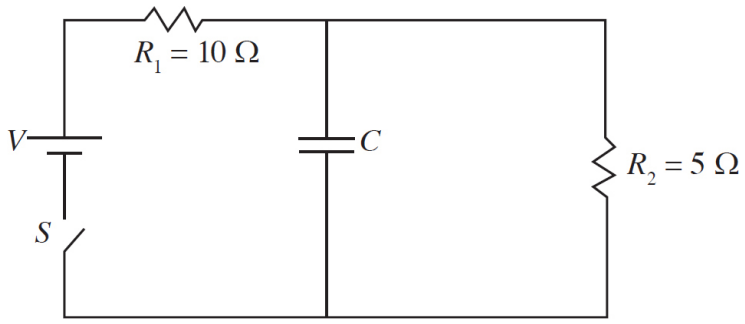
(A) Calculate the work from the electrical potential and use conservation of energy to find the final speed.

(B) Calculate the work from the gravitational potential and use conservation of energy to find the final speed.

(C) Calculate the impulse from the electric potential and use conservation of momentum to find the final speed.

(D) Calculate the impulse from the gravitational potential and use conservation of momentum to find the final speed.

Questions 16–18 refer to the following diagram.



16. Voltmeters are placed across the Resistor R_1, the capacitor C, and the resistor R_2. The switch S has been closed a long time. What is the rank of the value readings on the voltmeters?

(A) $V_{R_1} > (V_C = V_{R_2})$

(B) $(V_{R_1} = V_C = V_{R_2})$

(C) $(V_C = V_{R_2}) > V_{R_1}$

(D) $V_{R_1} > V_C < V_{R_2}$

17. The circuit is reset and the capacitor is discharged. Then, the switch is closed again. At what time will the current through the resistor R_2 be greatest?

(A) The current through the resistor will be constant.

(B) The current through the resistor will be greatest before closing the switch.

(C) The current through the resistor will be greatest immediate after closing the switch.

(D) The current through the resistor will be greatest a long time after closing the switch.

18. The capacitance of the capacitor is known initially. The capacitor is now altered to have a larger capacitance. Which of the following observations will occur with the new capacitor in the circuit a long time after the switch is closed as compared to

what was observed a long time after the switch was closed with the original capacitor?

(A) The current flowing in R_1 will be greater.

(B) The current flowing in R_2 will be greater.

(C) The current flowing into the capacitor will be greater.

(D) There will be no observed change in the circuit.

19. An ideal gas is confined within a cube shaped container. In addition to the length of the side of the container, which of the following sets of measurements will allow a student to determine the pressure of the gas in the container?

I The mass of gas in the container and the average speed of a gas molecule.

II The impulse delivered to the gas by a wall in a measured time period.

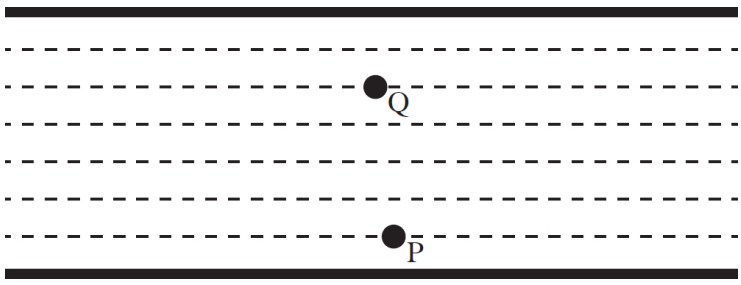
III The force of the gas against one of the walls.

(A) I only

(B) III only

(C) II or III

(D) I, II, or III



20. Points P and Q lie between the plates of a fully charged parallel plate capacitor as shown above. The lower plate is negatively charged and the upper plate is positively charged. How do the magnitudes of the electric fields at points P and Q compare?

(A) The field is 0 N/C at both points.

(B) The field is the same at both points, but not 0 N/C.

(C) $E_P > E_Q$

(D) $E_Q > E_P$

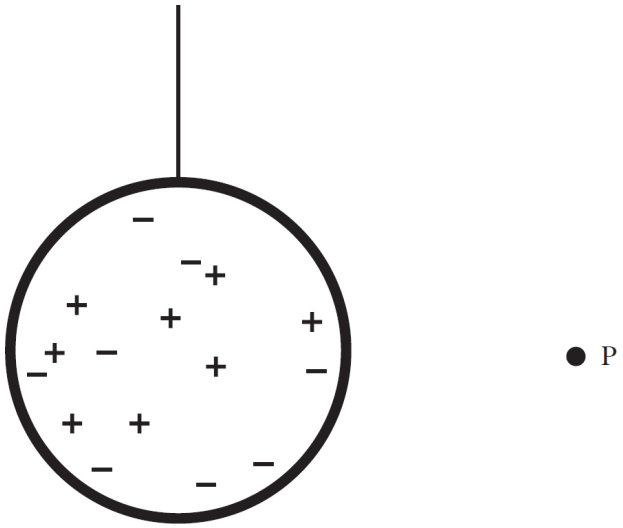
21. A wire carries a constant current to the right. A positively charged particle is a distance d above the wire, and it is moving in the same direction as the current. The particle will experience a magnetic force in which direction?

(A) To the right

(B) To the top of the page

(C) To the left

(D) To the bottom of the page



22. A neutral conducting sphere is hung from a thin insulating string. A positively charged object is brought to point P. The two objects are not allowed to touch. What is true about the string when the positively charged object is present at point P?

(A) The tension is the same as before the object was present at point P.

(B) The tension is greater than when the object was not at point P, and the string stretches to the left of its original orientation.

(C) The tension is greater than when the object was not at point P, and the string stretches to the right of its original orientation.

(D) The tension is less than when the object was not at point P, and the string stretches to the left of its original orientation.

23. An experiment is performed on a fixed volume of an ideal gas. The pressure, in pascals, of the gas is plotted on the vertical axis and the temperature of the gas, in degrees Kelvin, is plotted on the horizontal axis. During a second performance of the experiment at a greater volume, the pressure-temperature gas is expected to

(A) have a greater slope and the same intercept

(B) have a smaller slope and the same intercept

(C) have a greater slope and a greater intercept

(D) have a smaller slope and a greater intercept

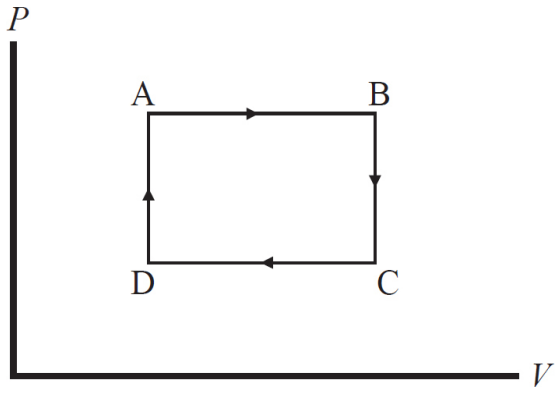
24. Two gas samples contain different gases. The first gas sample contains more massive molecules than the second. The molecules in both gas samples have the same average speed. When the samples are brought into contact, what is the flow of energy?

(A) No energy will flow because the average speed of the particles in each gas is the same.

(B) No energy will flow because the gases are made up of different molecules.

(C) Energy will flow away from the first sample because its molecules are more massive.

(D) Energy will flow away from the first sample because its molecules have more kinetic energy.



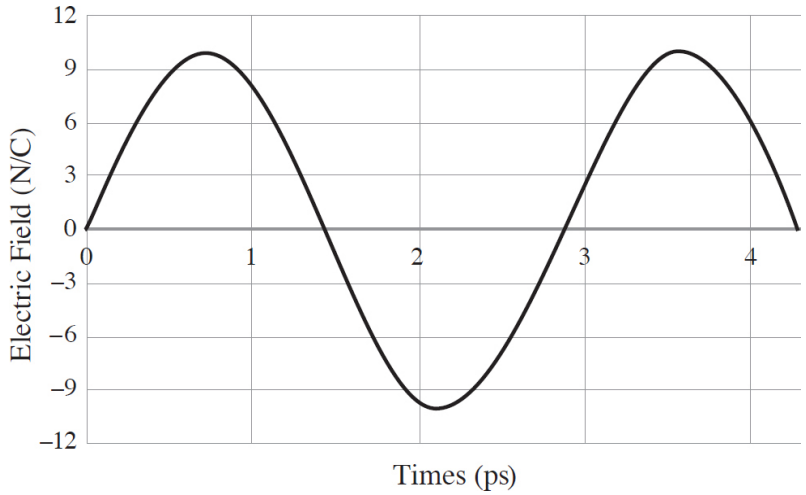
25. Which statement correctly characterizes the work done by the gas during the ABCDA cycle shown in the above *P*-*V* diagram?

(A) There is no work done by the gas because the system both starts and concludes in state A.

(B) There is no work done because the work done during the transition from A→B cancels out the work done in transition from C→D.

(C) The work done by the gas is positive because the work done during the transition from A→B is greater than the work done in transition from C→D.

(D) The work done by the gas is positive because the work done during the transition from B→C is greater than the work done in transition from D→A.



26. What is the equation for the electric field given by the above plot? The electric field crosses the axis at times 0 ps, 1.43 ps, 2.85 ps, and 4.28 ps.

(A) $E = \left(\frac{0\ \text{N}}{\text{C}}\right) \sin((2.85\ \text{THz})t)$

(B) $E = \left(\frac{10\ \text{N}}{\text{C}}\right) \sin((2.85\ \text{THz})t)$

(C) $E = \left(\frac{10\ \text{N}}{\text{C}}\right) \sin((2.20\ \text{THz})t)$

(D) $E = \left(\frac{10\ \text{N}}{\text{C}}\right) \cos((0.45\ \text{THz})t)$

27. A spherical balloon filled with helium is floating in air. If the balloon is inflated until its radius is doubled, how will the buoyant force on the balloon be affected?

(A) It will decrease by a factor of 4.
(B) It will increase by a factor of 4.
(C) It will increase by a factor of 8.
(D) It will not be affected.

28. Data is collected in an experiment performed on an ideal gas. In the experiment, temperature (in K) is the independent variable and volume (in m^3) is the dependent variable. If the data is graphed, which of the following is true about the slope and *y*-intercept of the graph?

(A) The slope will be directly proportional to the pressure of the gas and the intercept will be 0 m^3.
(B) The slope will be inversely proportional to the pressure of the gas and the intercept will be 0 m^3.
(C) The slope will be directly proportional to the pressure of the gas and the intercept will not be 0 m^3.
(D) The slope will be inversely proportional to the pressure of the gas and the intercept will not be 0 m^3.

29. Which of the following relationships, when plotted, will yield a curve which is inverse to the first power?

I The electric potential versus distance from a positive point particle.
II The volume versus pressure for an ideal gas.
III The magnetic field from a current carrying wire versus distance from the wire.

(A) I only
(B) I and III
(C) II only
(D) I, II, or III

30. A massive charged object is fixed in space with a separation of *d* meters. A quantity *R* is defined as the ratio of the gravitational potential energy to the electric potential energy. The separation is then increased slowly. What happens to *R*?

(A) *R* is constant for all separations.

(B) R increases as the separation increases.

(C) R decreases as the separation increases.

(D) R increases up to a distance D, then R decreases back to its original value.

31. An ideal gas is at a pressure P and a volume V. The gas is in a fixed volume, but is heated until the pressure doubles. What happens to the average speed of the molecules in the gas?

(A) The speed of the molecules on average remain unchanged.

(B) The speed of the molecules on average increases by a factor of $\sqrt{2}$.

(C) The speed of the molecules on average increases by a factor of 2.

(D) The speed of the molecules on average increases by a factor of 4.

32. Which of the following changes to a double-slit interference experiment would increase the widths of the fringes in the interference pattern that appears on the screen?

(A) Use light of a shorter wavelength.

(B) Move the screen closer to the slits.

(C) Move the slits closer together.

(D) Use light with a lower wave speed.

33. Tritium is an isotope of hydrogen consisting of one proton and two neutrons. The isotope has a mass of 5.008×10^{-27} kg. The mass of a proton is 1.673×10^{-27} kg and a neutron has a mass of 1.675×10^{-27} kg. What is the binding energy of tritium?

(A) 1.500×10^{-29} J

(B) 1.350×10^{-12} J

(C) 4.507×10^{-10} J

(D) 4.521×10^{-10} J

34. In an experiment designed to study the photoelectric effect, it is observed that low-intensity visible light of wavelength 550 nm produced no photoelectrons. Which of the following best describes what would occur if the intensity of this light were increased dramatically?

(A) Almost immediately, photoelectrons would be produced with a kinetic energy equal to the energy of the incident photons.
(B) Almost immediately, photoelectrons would be produced with a kinetic energy equal to the energy of the incident photons minus the work function of the metal.
(C) After several seconds, the electrons absorb sufficient energy from the incident light, photoelectrons would be produced with a kinetic energy equal to the energy of the incident photons minus the work function of the metal.
(D) Nothing would happen.

35. Radioactive carbon-14 undergoes beta-minus decay. The atomic number of carbon is 6. The number of nucleons in the products after the beta decay is

(A) 6
(B) 7
(C) 13
(D) 14

36. A pipe carries water with a density of $\rho = 1000$ kg/m^3. One end of the pipe has a diameter of 0.02 m. At that location, a pressure gauge says the pressure is $P = 1.25 \times 10^5$ Pa and the water is moving at a speed of 2.0 m/s. The other end of the pipe is 1.2 m higher than the end with the pressure gauge and a

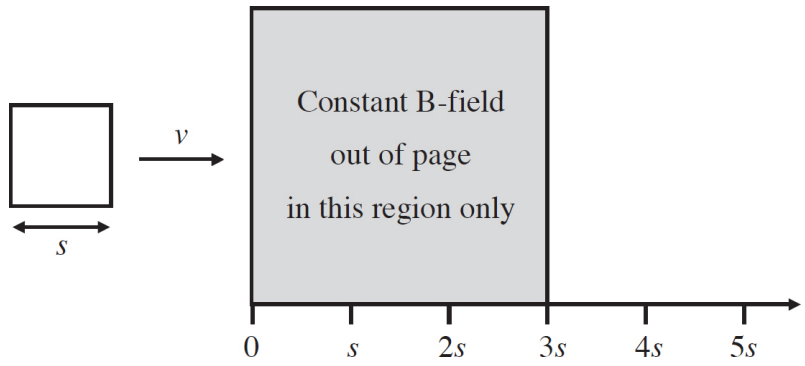
diameter of 0.01 m. What will a pressure gauge installed in the upper end of the pipe read?

(A) 7.90×10^4 Pa

(B) 8.30×10^4 Pa

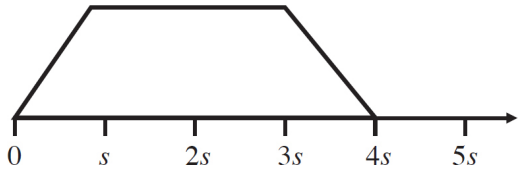
(C) 1.07×10^5 Pa

(D) 1.43×10^5 Pa

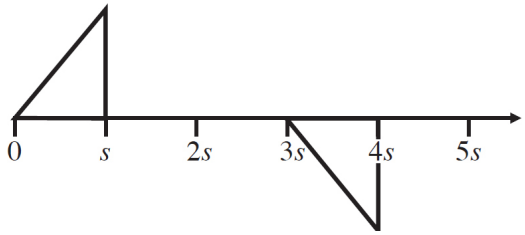


37. A square loop of conducting wire with side s is moved at a constant rate v to the right into a region where there is a constant magnetic field directed out of the page. Which of the following graphs shows the flux through the loop as a function of distance?

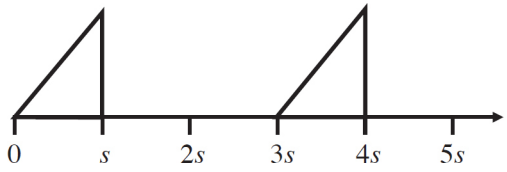
(A)

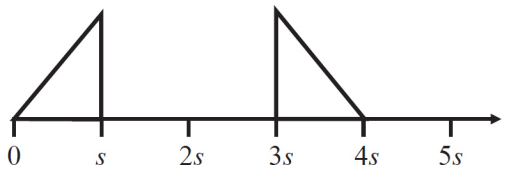


(B)



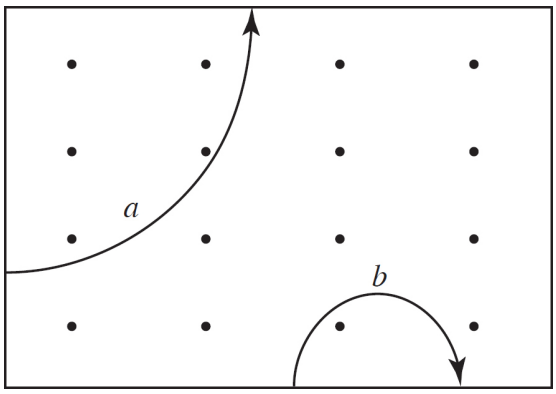
(C)



(D) 

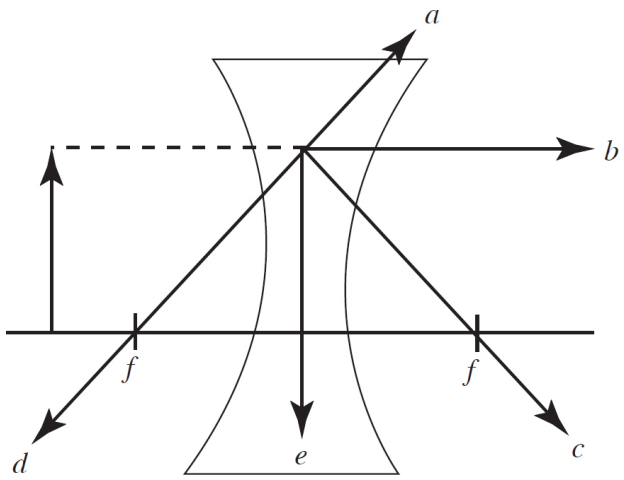
38. A hollow conducting sphere is placed around point P so that the center is at P. What happens to the electric field strength at point P?

(A) It doubles.
(B) It halves.
(C) It remains unchanged.
(D) It becomes zero.



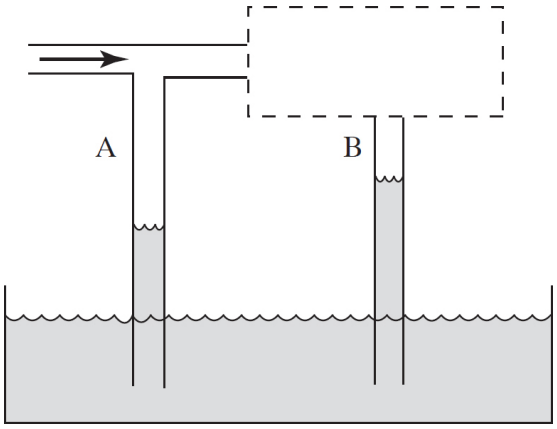
39. A machine shoots a proton, a neutron, or an electron into a magnetic field at various locations. The paths of two particles are shown above. Assume they are far enough apart so that they do not intersect and the magnetic field is going out of the page, as shown. What can you say about the paths that represent each particle?

(A) *a* is the proton and *b* is the electron.
(B) *b* is the proton and *a* is the electron.
(C) Either may be a neutron.
(D) You cannot make any conclusions without knowing the velocities.



40. In the figure above, a ray of light hits an object and travels parallel to the principal axis as shown by the dotted line. Which line shows the correct continuation of the ray after it hits the concave lens?

(A) *a*
(B) *b*
(C) *c*
(D) *d*



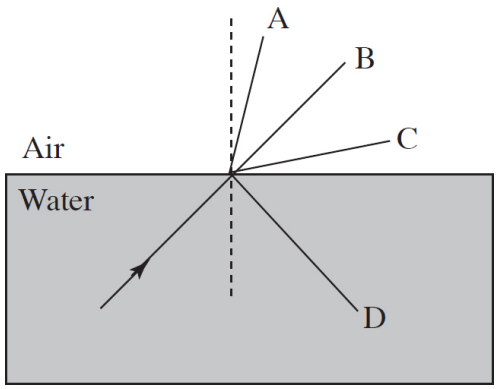
41. A tube with two T branches that has an open end is inserted in a liquid. However, the section of the tube above part B is hidden from view. The hidden section may be wider or narrower. Air is blown through the tube and the water levels rise as shown. You can conclude which of the following?

(A) The picture as drawn is impossible—A and B must be at equal heights.

(B) The tube is narrower and the air speed is greater above section B.

(C) The tube is narrower and the air speed is less above section B.

(D) The tube is wider and the air speed is greater above section B.



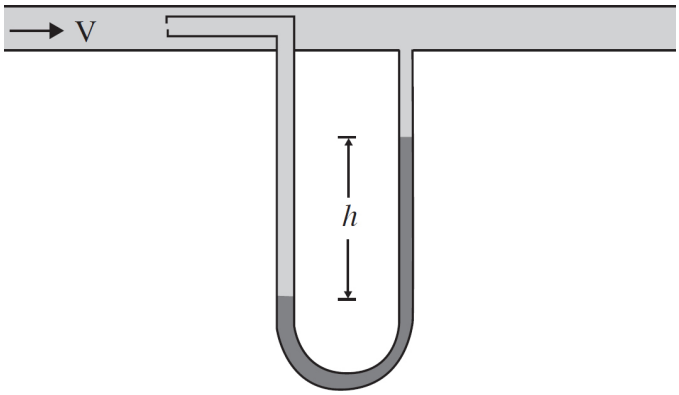
42. A beam of light goes from water to air. Depending on the actual angle that the light strikes the surface, which of the following rays are possible outcomes?

(A) A only

(B) B only

(C) A or D

(D) C or D



43. The instrument in an aircraft to measure airspeed is known as the pitot tube, shown in the figure above. The opening facing the incoming air (with the small aperture) is the part meant to capture the air at rest. The opening perpendicular to the flow of air (with the large aperture) is meant to capture air at speed. If h = 1 m and the fluid within the manometer is water, what is the airspeed? Take the density of air to be ρ_{air} = 1.2 kg/m^3.

(A) 27 m/s
(B) 68 m/s
(C) 95 m/s
(D) 128 m/s

44. Two charged, massive particles are isolated from all influence except those between the particles. They have charge and mass such that the net force between them is 0 N. Which of the following is NOT true?

(A) The particles must have the same sign of charge.
(B) If the distance between the particles changes, the force will no longer be 0 N.
(C) The particles must have the same mass.
(D) The particle cannot have the same magnitude of charge.

45. The following three step process refers to a simple RC circuit with a battery and an initially open switch.

Step 1: The switch is closed, allowing the capacitor to charge.
Step 2: After the capacitor has charged, a slab of dielectric material is inserted between the plates of the capacitor and time passes.
Step 3: The switch is opened, and the dielectric is removed.

Which of the following describes the change in potential energy stored in the capacitor during each step?

(A) $\Delta U_1 < 0$; $\Delta U_2 > 0$; $\Delta U_3 > 0$

(B) $\Delta U_1 > 0$; $\Delta U_2 > 0$; $\Delta U_3 > 0$

(C) $\Delta U_1 < 0$; $\Delta U_2 > 0$; $\Delta U_3 < 0$

(D) $\Delta U_1 > 0$; $\Delta U_2 < 0$; $\Delta U_3 < 0$

Directions: For questions 46–50 below, <u>two</u> of the suggested answers will be correct. Select the two answers that are best in each case, and then fill in both of the corresponding circles on the answer sheet.

46. N resistors ($N > 2$) are connected in parallel with a battery of voltage V_0. If one of the resistors is removed from the circuit, which of the following quantities will decrease? Select two answers.

(A) The voltage across any of the remaining resistors

(B) The current output by the battery

(C) The total power dissipated in the circuit

(D) The total resistance in the circuit

47. A metal conducting sphere of radius r in electrostatic equilibrium has a positive net charge of $+5e$. Which of the following is true? Select two answers.

(A) All of the charge will be located on the outside of the sphere.

(B) There will be a constant, non-zero electric field within the sphere.

(C) There will be a constant, non-zero electric potential within the sphere.

(D) When the sphere is connected to a ground, the $+5e$ of charge on the sphere will flow into the ground to neutralize the sphere.

48. A circuit is created with a battery of negligible internal resistance and three identical resistors with resistance of 10 Ω. The resistors are originally arranged so that one is in series with the battery, and the other two are in parallel with each other. Which of the following changes to the circuit will result in an increase in the amount of current drawn from the battery? Select two answers.

(A) Rearranging the resistors so that all three are in parallel
(B) Replacing every resistor with a resistor of half the resistance
(C) Removing one of the two parallel branches entirely from the circuit
(D) Replacing the battery with a battery with half the voltage

49. An ideal gas is in state 1, with P_1, V_1, and T_1. The final volume will be the same as V_1 for which of the following processes? Select two answers.

(A) Triple P_1 and decrease T_1 by $\frac{1}{3}$

(B) Triple T_1 and decrease P_1 by $\frac{1}{3}$

(C) Quadruple P_1 and T_1

(D) Decrease P_1 by $\frac{1}{3}$ and decrease T_1 by $\frac{1}{3}$

50. In a double slit experiment, students are attempting to increase the spacing of the fringes observed on the screen. Which modifications to the set up will result in increased fringe separation? Select two answers.

(A) Doubling the wavelength only
(B) Doubling the wavelength and doubling the slit separation
(C) Doubling the distance to the screen only

(D) Doubling the distance to the screen and doubling the slit separation

END OF SECTION I

Section II

PHYSICS 2
SECTION II
Time—90 minutes
4 Questions

Directions: Questions 1 and 2 are long free-response questions that require about 30 minutes to answer. Questions 3 and 4 are short answer questions that require about 15 minutes to answer. Show your work for each part in the space provided after that part.

1. An unplugged freezer is at room temperature. The door is closed and the freezer is plugged in.

   (a) Your friend observes, "When you open the freezer, the cold air comes out of the freezer. That didn't happen before the freezer was cooled down. It must therefore be the case that cold air is less dense than warm air, since the low density gas will expand more easily."

   i. Is there any part of your friend's statement you agree with? Why?

   ii. Is there any part of your friend's statement you disagree with? Why?

   (b) You set out to perform an experimental investigation of the relationship between gas density and temperature. The following equipment is available. Place an X beside each item you will need to use:

	Chamber with sliding piston lid with a known area
	Pressure monitor
	Meter stick
	Large tub of boiling water

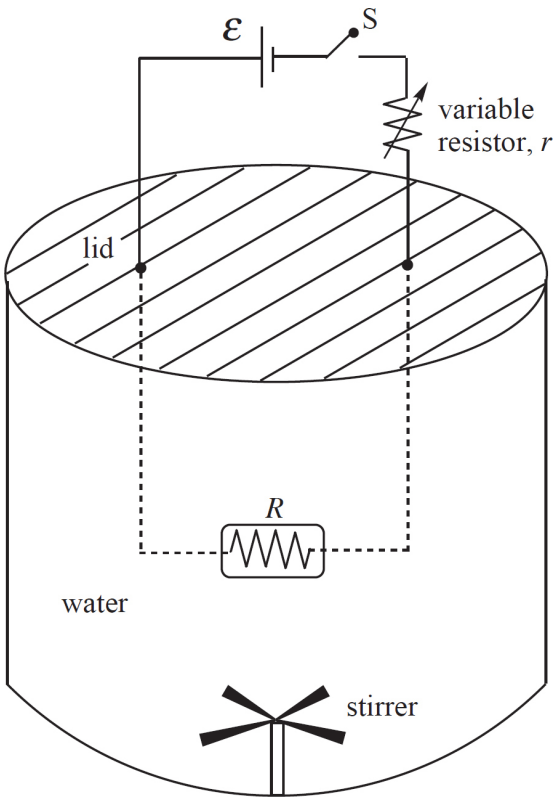
	Chamber with fixed lid with a known area
	Thermometer
	Stopwatch
	Large tub of ice water

(c) Write out a numbered procedure you will use to gather necessary data. Your description should be detailed enough that another student could reproduce your experiment.

(d) Your data analysis must include a graph. Explain what you would graph on the x-axis and the y-axis. Justify your decision and explain how your graph will help you understand the relationship between density of a gas and its temperature.

(e) What shape graph do you expect to see?

2. The figure below shows an electric circuit containing a source of emf, ε, a variable resistor (r) and a resistor of fixed resistance R. The resistor R is immersed in a sealed beaker containing a mass m of water, currently at temperature T_i. When the switch S is closed, current through the circuit causes the resistor in the water to dissipate heat, which is absorbed by the water. A stirrer at the bottom of the beaker simply ensures that the temperature is uniform throughout the water at any given moment. The apparatus is well-insulated (insulation not shown), and it may be assumed that no heat is lost to the walls or lid of the beaker or to the stirrer.



(a) Determine the current in the circuit once S is closed. Write your answer in terms of ε, r, and R.

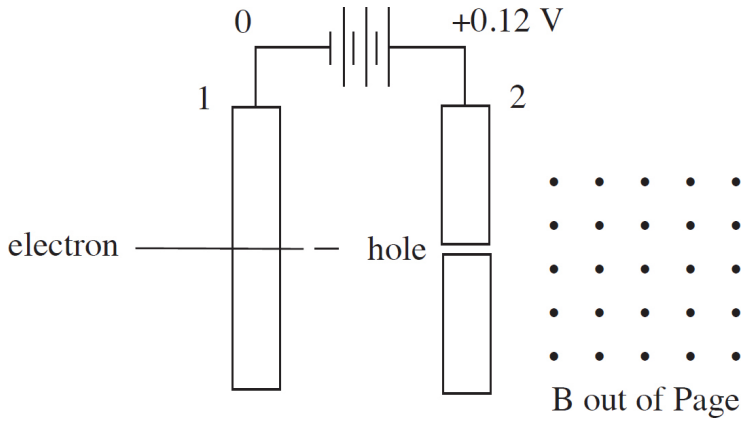
(b) Determine the power dissipated by the resistor R in terms of ε, r, and R.

(c) Explain at the microscopic level why the water heats up when the switch is closed and how the stirrer helps ensure a constant temperature throughout.

(d) Assume the stirrer has a knob, which changes its speed. How can the temperature of the water be increased more rapidly by adjusting the rotation rate of the stirrer?

(e) As the temperature of the water increases, whether from the resistor or from the stirrer rate, explain the microscopic interactions responsible for the changing pressure in the container.

3. In an experiment conducted, two tests are run. In both trials you may ignore the effect of gravity. The following is a diagram of the tests.



Test 1: There are two large parallel plates separated by a distance d = 0.5 m with a potential difference of 0.12 V across them. There is a uniform magnetic field **B** pointing perpendicularly out of the paper of strength 0.002 T starting to the right of plate 2. An electron is released from rest at plate 1 as shown above. It passes through a hole in plate 2 and enters the magnetic field and only experiences forces due to the magnetic field.

Test 2: The same set-up is run with the following two exceptions.

1. The battery is switched so that plate 1 becomes positive and plate 2 becomes negative.

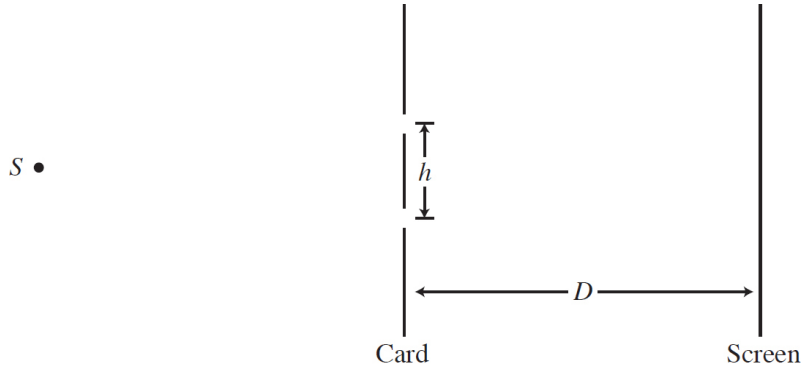
2. A proton is used instead of an electron.

(a) As the particles move from plate 1 toward plate 2 in each test, they experience unbalanced forces causing their speed to change. After plate 2, each particle still experiences unbalanced forces, but no longer change speed. Use the concept of work to explain how this occurs.

(b) Find the ratio of the speed of the proton as it emerges from the hole to the speed of the electron as it emerges from the hole.

(c) Make a sketch of each path each particle will follow after emerging from the hold in plate 2.

(d) A third test is conducted similar to test 2, except an alpha particle is used instead of a proton. Explain how the path of the alpha particle after emerging from the hole will differ from the proton path.



Note: Figure not drawn to scale.

4. In a double-slit interference experiment, a parallel beam of monochromatic light is needed to illuminate two narrow parallel slits of width w that are a distance h apart, where $h >> w$, in an opaque card as shown in the figure above. The interference pattern is formed on a screen a distance D from the slits, where $D >> h$.

(a) Draw the first three wavefronts that emerge from each slit after the card.

(b) In a clear, coherent, paragraph-length response, explain why a series of bright spots are seen on the screen after the light shines through the card.

(c) In the interference patterns on the screen, the distance from the central bright fringe to the third bright fringe on one side is measured to be y_3. In terms of D, H, y_3 and S, what is the wavelength of the light emitted from the source S?

(d) If the space between the slits and the screen was filled with a material having an index of refraction $n > 1$, would the distance between the bright fringes increase, decrease, or remain the same? Explain your reasoning.

STOP

END OF EXAM

Practice Test 1: Answers and Explanations

PRACTICE TEST 1 ANSWER KEY

1.	D	26.	C
2.	A	27.	C
3.	D	28.	B
4.	C	29.	D
5.	A	30.	A
6.	A	31.	B
7.	B	32.	C
8.	B	33.	B
9.	C	34.	D
10.	C	35.	D
11.	A	36.	B
12.	D	37.	A
13.	B	38.	D
14.	A	39.	A
15.	A	40.	A
16.	C	41.	B
17.	D	42.	D
18.	D	43.	D
19.	D	44.	A
20.	B	45.	B
21.	D	46.	B, C
22.	C	47.	A, C
23.	B	48.	A, B
24.	D	49.	C, D
25.	C	50.	A, C

Once you have checked your answers, remember to return to this page and respond to the Reflect questions.

Access your Student Tools for a free diagnostic answer key for Practice Test 1, which includes targeted study recommendations.

SECTION I

1. **D**

The forces must balance in order for the proton to travel along a straight line. If the forces balance, $m(E_g) = q(E_{el})$. Since m is less than q for a proton, the gravitation field is stronger. Gravity points down, so the electric force must be up, and for a positive particle, the field must also be up.

2. **A**

Equipotential lines will be perpendicular to the force lines. Gravitational force lines point vertically downward. The equipotential lines for the electric field must also be horizontal lines for the particle to move in a straight line at a constant speed.

3. **D**

The continuity equation, $A_1v_1 = A_2v_2$ applies here as $A_1v_1 = 2A_2v_2$ where position 1 is in the large pipe. Solving for $v_1 = 2\left(\pi\left(\frac{d}{2}\right)^2\right)v/\pi\left(\frac{D}{2}\right)^2$.

4. **C**

The fluid pressure is the force divided by the area. The force is the same regardless of the container orientation as the weight of the container is constant. To result in a minimum pressure, the largest face must be placed against the table.

5. **A**

$$R = \rho \frac{L}{A}$$

For any linear equation, $y = mx+b$, the slope is multiplied by the quantity which is graphed on the horizontal axis. In order to have the slope equal the resistivity, the horizontal axis must be the ratio of length to area.

6. **A**

All processes that end at higher volumes will require removing work to be done on the system. When pressure is constant, the work is found from $W = P\Delta V$, so to minimize the work required, you want the change in volume to occur at the lowest pressure possible.

7. **B**

A ray which travels into a converging lens along a line that goes through the focal point will refract to travel along a line parallel to the optical axis upon leaving the lens.

8. **B**

The screen must be placed at the image location in order for the image to appear in focus on the screen. Use the equation $\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$

$$\frac{1}{s_i} = \frac{1}{25\text{ cm}} - \frac{1}{65\text{ cm}} \rightarrow s_i = 40.6\text{ cm}$$

9. **C**

A concave mirror can only make a real image when the object is placed beyond the focus.

10. **C**

Any photon that has energy that matches an energy difference in that atom can be absorbed. The 10 eV photon can be absorbed if there is an electron in the −10 eV state by ionizing the atom. A 5 eV photon can excite a photon from the −10 eV state to the −5 eV state or ionize an electron from the −5 eV state. The 1.5 eV photon can cause a transition from the −3.5 eV state to the −2 eV state.

11. **A**

During nuclear decay processes, the total charge in the system before and after the decay must be the same.

12. **D**

The internal energy is proportional to the temperature of the system, and the Ideal Gas Law explains that the product of pressure and volume is also proportional to the temperature. State A is at 5PV, State B is at 4PV, State C is at 16PV, and State D is at 20PV. The change from State D to State A requires the largest change in energy.

13. **B**

The internal energy stored in a fully charged capacitor is $U_C = \frac{1}{2}CV^2$. The voltage of the battery will be the voltage across the capacitor when the capacitor is fully charged.

$$U_C = \frac{1}{2}(25 \times 10^{-6}\ \text{F})(50\ \text{V})^2 = 0.03125\ \text{J}$$

14. **A**

The acceleration of an object with charge $+q$ and mass m is given by $a = \frac{F_E}{m} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{mr^2}\right)$ Graph (A) shows an inverse square law.

15. **A**

There is a change in electrical potential and no change in gravitational potential. The work is done because of the change in the potential. Therefore, the Work–Energy Theorem, $\Delta W_{net} = \Delta KE$ can be used to find the change in kinetic energy. Finally, using $\Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ and the known initial speed and mass, the final speed can be determined.

16. **C**

A long time after the switch has been closed, the capacitor will be fully charged. There will be no current flowing across R_1 so it will have a voltage drop of 0 V. The capacitor and R_2 are both in parallel with the battery, and will have a voltage equal to the battery voltage.

17. **D**

You can eliminate (A) immediately. The current that flows into the capacitor changes over time, so the current in a parallel branch will also change over time. You can eliminate (B) as well, because before the switch is closed, no current flows in the circuit at all. Because the capacitor

is discharged, as soon as the switch is closed, the parallel branch with the capacitor will receive all of the current, eliminating (C). In both the cases of (B) and (C), the current through R_2 is zero. Once the capacitor is fully charged, current flows around the loop creating the outside of the circuit, and whatever value it has at that time is greater than 0A.

18. **D**

A fully charged capacitor acts like a wire regardless of the value of its capacitance, so a capacitance of a fully charged capacitor will have no effect on the circuit.

19. **D**

One way to find the pressure is with the Ideal Gas Law. The Ideal Gas Law states that $PV = nRT$, so in addition to the volume, found from the length, one needs n and T to find the pressure, so I is correct. The impulse and time lets us calculate the force applied against the gas in that time. Newton's Third Law states that this must be the same magnitude as the force applied to the wall. The force and area allow us to calculate the pressure. So II and III are also correct.

20. **B**

Between the plates of a parallel plate capacitor, the field will be constant and non-zero.

21. **D**

The magnetic field produced by the wire is found using the right hand rule. The current is to the right, so at a position above the wire, the magnetic field points outward. Then use the right hand rule again to find the force on the particle.

The particle moves to the right in a magnetic field pointed outward. The force is toward the bottom.

22. **C**

Negative charges will be drawn to the right side of the sphere, causing a force on the sphere to the right. The vertical tension will still have to balance with gravity, but there will also be a component of tension to the right.

23. **B**

The Ideal Gas Law is $PV = nRT$, or $P = (\frac{nR}{V})T$. Comparing this to $y = mx + b$, the intercept should be 0 and the slope should be related to $1/V$.

24. **D**

Energy will flow from the hotter gas to the cooler gas. The speeds are the same, but the masses are different, so the more massive gas has more kinetic energy.

25. **C**

Work is done when there is a change in volume. At constant pressure, the equation $W = -P\Delta V$ tells you that when P is higher (as it is at path A→B), a greater amount of negative work is done on the gas than at lower P. Thus, the work done by the gas during the entire cycle is positive, and the constant volume paths have no influence on the amount of work.

26. **C**

The field is a sine curve with an amplitude of 10 N/C. From the graph, the period, T, of the wave is 2.85 ps. The argument of sine needs to be $2\pi t/T = (2 \times 3.14/2.85 \text{ ps})t = (2.20T\text{Hz})t$.

27. **C**

The buoyant force is found from $F = \rho_{air} V_{sub} g$. Because the entire balloon is surrounded by air, the entire volume is V_{sub}. The volume of the spherical balloon is $V = \frac{4}{3}\pi r^3$. Doubling the radius increases the volume by a factor of 8.

28. **B**

The Ideal Gas Law equation is $PV = nRT$. Solving this for V (since volume is on the vertical axis of the graph described in the problem) yields $V = (\frac{nR}{P})T$. Comparing this to the equation for a line, $y = mx + b$, the slope of the line will be nR/P, which is inversely proportional to P, and $b = 0$ is the intercept.

29. **D**

The Ideal Gas Law is $P = (nRT)\left(\frac{1}{V}\right)$.

The potential from a point particle is $V = \frac{q_1}{4\pi\varepsilon_0}\left(\frac{1}{r}\right)$.

The magnetic field from a wire is $B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r}\right)$.

All of these are inverse to the first power.

30. **A**

The quantity R is defined as the ratio of the gravitational potential energy to the electric potential energy, so energy, so $R = \left(\frac{GMm}{r^2}\right) / \left(\frac{1}{4\pi\varepsilon_0}\left(\frac{Qq}{r^2}\right)\right)$

$$R = GMm / \left(\frac{1}{4\pi\varepsilon_0} Qq\right)$$

R is constant.

31. **B**

Doubling pressure at constant volume causes temperature to double. Doubling temperature doubles the average kinetic energy of the molecules. The speed squared is proportional to the kinetic energy, so the speed increases by a factor of $\sqrt{2}$.

32. **C**

Relative to the central maximum, the locations of the bright fringes on the screen are given by the expression

$m\left(\frac{\lambda L}{d}\right)$, where λ is the wavelength, L is the distance to the screen, d is the slit separation, and m is any integer. The width of a fringe is therefore $(m+1)\left(\frac{\lambda L}{d}\right) - m\left(\frac{\lambda L}{d}\right) = \left(\frac{\lambda L}{d}\right)$. The slit spacing will increase if there is a decrease in d.

33. **B**

The binding energy is the mass defect, the difference in the mass of the constituent components and the mass of the nucleus, multiplied by c^2. That mass defect is

$m_p + 2m_n - m_{Tr} = 1.673 \times 10^{-27} + 2 * 1.675 \times 10^{-27} - 5.008 \times 10^{-27} = 1.500 \times 10^{-29}$ kg

$E = \Delta mc^2 = 1.500 \times 10^{-29} * (3 \times 10^8)^2$

34. **D**

If the photons of the incident light have insufficient energy to liberate electrons from the metal's surface, then simply increasing the number of these weak photons (that is, increasing the intensity of the light) will do nothing. In order to produce photoelectrons, each photon of the incident light must have an energy at least as great as the work function of the metal.

35. **D**

Beta-minus decay occurs when a neutron turns into a proton and a beta particle. The number of nucleons does not change in this process.

36. **B**

Use the continuity equation to find the speeds.

$$A_1 v_1 = A_2 v_2$$

$$\pi \left(\frac{0.02 \text{ m}}{2} \right)^2 (2.0 \text{ m/s}) = \pi \left(\frac{0.01 \text{ m}}{2} \right)^2 v_2$$

$$v_2 = 8.0 \text{ m/s}$$

Then use Bernoulli's Equation

$$P_A + \frac{1}{2}\rho_{water} v_A^2 + \rho_{water} g y_A = P_B + \frac{1}{2}\rho_{water} v_B^2 + \rho_{water} g y_B$$

$$1.25 \times 10^5 + \frac{1}{2}1000(2.0)^2 + 1000(10)(0) = P_B + \frac{1}{2}1000(8.0)^2 + 1000(10)(1.2)$$

$$P_B = 83{,}000 \text{ Pa}$$

37. **A**

Flux is the product of the area enclosed by the loop of wire and the field strength. The flux will increase linearly up to a maximum value when the wire is completely within the field, and will begin to decrease once the wire begins to leaves the region within the field.

38. **D**

Within a conducting sphere, the electric field is always zero. Therefore, in the region of P, as well as everywhere else within the conducting sphere, there is zero electric field.

39. **A**

Path *a* follows the right-hand rule so must be positively charged. Path *b* is opposite the right-hand rule so must be

negatively charged. Neutrons would follow a straight line path, so it is impossible for either *a* or *b* to be a neutron.

40. **A**

The rules for ray-tracing diagrams state that a line parallel to the principle axis bends away from the focal point as shown.

41. **B**

Because the fluid is higher in column B, the air speed must be greater above column B. This must be due to a narrower tube.

42. **D**

Water has a higher index of refraction than air, so if light refracts through the water the light will bend away from the normal—so C is possible. Another possibility is D, total internal reflection when the angle in water is greater than the critical angle.

43. **D**

First, use Bernoulli's Equation. Call the point with the air at rest B and the point with the speeding air A.

$$P_A + \frac{1}{2}\rho_{air} v_A^2 + \rho_{air} g y_A = P_B + \frac{1}{2}\rho_{air} v_B^2 + \rho_{air} g y_B$$

You have $v_B = 0$ and $y_A = v_B$, so the equation reduces to

$$P_A + \frac{1}{2}\rho_{air} v_A^2 = P_B$$

The pressure difference, $P_B = P_A$ arises from the fluid depth $\rho_f gh$.

$$\rho_f gh = \frac{1}{2}\rho_{air} v_A^2$$

Solving for v_a and using the given values

$$v_a = \sqrt{\frac{2\rho_f gh}{\rho_{air}}}$$

$$v_a = \sqrt{\frac{2 * 1{,}000\ \text{kg/m}^3 * 9.8/\text{s}^2 * 1\ \text{m}}{1.2\ \text{kg/m}^3}}$$

$$v_a = 128\ \text{m/s}$$

44. **A**

It is possible for the gravitational force to balance the electrical force, making (D) false. In order for this to occur, since gravity is always an attractive force, the electrical force must be repulsed, and therefore the charges must have the same sign, making (A) true. Both the gravitational and electrical force are inverse square laws, so if the distance between the particles changes, both forces change by the same factor, making (B) false. And (C) is just false, both particles will experience the same gravitational force regardless of how the mass is distributed amongst them; the gravitational force depends on the product of the two masses.

45. **B**

Step 1: Potential energy rises because the magnitude of charge on each plate increases, $Q_C = CV_C$ and $U_C = \frac{1}{2}CV^2$. $\Delta U_1 > 0$, eliminate (A) and (C).

Step 2: Inserting a dielectric increases potential energy. Inserting a dielectric always increases capacitance. $Q_C = CV_C$ and $U_C = \frac{1}{2}CV^2$; therefore, potential energy increases. $\Delta U_2 > 0$, so eliminate (D).

Step 3: Capacitor is still charged so there is still potential energy.

46. **B, C**

The resistors are all in parallel, meaning their voltage is the same across each resistor and matches the voltage of the battery. Eliminate (A). The voltage provided by the battery does not increase or decrease, the battery still maintains the same voltage. Eliminate (D). The resistors are in parallel, so eliminating one of the resistors will increase the equivalent resistance (there is now one less path for current to go through so equivalent resistance goes up). Because $I = V/R$, this increase in resistance would decrease the current. Power's equation is $P = IV$. If the current decreases, the power decreases as well.

47. **A, C**

Conductors will always have all of their excess charge located at the surface. The entire volume of a conductor must be an equipotential in electrostatics because

otherwise the charges in the conductor would move as a result of the potential difference. Choice (B) is false because within a conductor in electrostatic equilibrium, there is an electric field of 0 N/C. Choice (D) is false because, while the sphere will neutralize when connected to a ground, it is not the + charges that flow out of the sphere, but – charges which will flow from the ground into the sphere.

48. **A, B**

From Ohm's Law, the current drawn out of the battery will increase when the total resistance of the circuit decreases, or when the total voltage supplied to the circuit increases, which makes (D) false. Resistors in parallel have a lower equivalent resistance than those in series, so (A) is true. To see that (B) decreases the overall resistance, think of a simpler version of the problem. Replacing a resistor with a wire will have the same effect of either increasing or decreasing the overall resistance, but with a larger effect. If any of the three resistors are replaced with a wire, the overall resistance decreases. Therefore, decreasing the resistance of any of the resistors will decrease the equivalent resistance, and replacing all of them most certainly will decrease the equivalent resistance.

49. **C, D**

The Ideal Gas Law states $PV = nRT$, so pressure is directly proportional to temperature. To keep volume constant, pressure and temperature must increase or decrease by the same amount.

50. **A, C**

For the double slit experiment, $m\lambda = dsin(\theta) \rightarrow m\lambda = dx\backslash L$.

The spacing of the fringes, x, increases for increasing λ and L and decreases by increasing d.

SECTION II

1. (a)

i) I agree with "When you open the freezer, the cold air comes out of the freezer. That didn't happen before the freezer was cooled down." The reason is that energy flows, according to the Second Law of Thermodynamics, due to a difference in temperature. When the freezer wasn't cooled down, there was no temperature difference.

ii) I disagree with "cold air is less dense than warm air, since the low-density gas will expand more easily." A low-density gas will have a lot of space between molecules, so a higher density gas will expand more easily into the low-density gas.

(b) Using the chamber with the sliding lid allows for a changing volume at a constant amount of air, so that density can be the dependent variable in the experiment. The area of the lid is constant, so you can determine volume as the piston length changes using the meter stick. Using the large tubs of boiling water and ice water to heat and cool the chamber allows the independent variable to be the temperature, measured with the thermometer.

X	Chamber with sliding piston lid with a known area
	Pressure monitor
X	Meter stick
X	Large tub of boiling water

	Chamber with fixed lid with a known area
X	Thermometer

	Stopwatch
X	Large tub of ice water

(c)

1. Place the thermometer in the chamber with the sliding piston.
2. Place the chamber in the ice water and let it cool.
3. Record the temperature with the thermometer and the piston length with the meter stick.
4. Heat the chamber by placing it in the hot water.
5. Record the temperature and the piston length.
6. Repeat steps 4 and 5 to get temperature and volume data.

(d) The density is n/V, so since you have data for Δx and you know area, plot $\frac{1}{\Delta x}$ versus temperature. The shape of the graph will tell you whether the increase in temperature causes the density to increase or decrease.

(e) Based on the Ideal Gas Law,

$$PV = nRT$$

$$P = (n/V)RT$$

$$(n/V)^{-1} = \left(\frac{R}{P}\right)T$$

you can expect to see an inverse relationship.

2. (a) The total resistance in the circuit is $r + R$. The current is then $I = \frac{\mathcal{E}}{r + R}$.

(b) Power is $P = IV = \left(\frac{\mathcal{E}}{r + R}\right)\mathcal{E} = \frac{\mathcal{E}^2}{r + R}$.

(c) Temperature increases when the average kinetic energy of a molecule increases. When the switch is closed, the electrons move through the circuit. As the electrons move within the metal of the circuit, they collide with the positive nuclei of the atoms. The nuclei are set into motion, so the resistor heats up. The Second Law of Thermodynamics states that energy will flow from high energy to low energy parts of a system. The molecules in the water are moving with a lower energy than the hot resistor, so as the water molecules collide with the resistor, energy transfers into the water molecules. The now heated water molecules transfer energy to one another through further collisions, with the colder, slower molecules heating up and speeding up. The stirrer helps to ensure that the average energy is equal throughout the water by dispersing the fast-moving water molecules so that they transfer energy throughout the container and not just locally near the resistor.

(d) Energy can be transferred to the water molecules by the motion of the stirrer as well as by the high temperature molecules in the resistor. If the stirrer spins at a rapid enough rate, an average water molecule will be sped up simply by its motion.

(e) The pressure in the container is a result of the collisions between the molecules within the container and its walls. As the molecules heat up and move more rapidly, they will collide more frequently with walls of the container and cause the pressure against the walls to increase.

3. (a) Between the plates, there is an electric field that points from one of the plates toward the other plate. As the particles change their position within a potential

difference, work is done on the particles, causing them to accelerate. After leaving plate 2, the particles are subjected to a magnetic field. Because the magnetic field is always perpendicular to the direction of motion, the magnetic field does no work and the particles do not change their speed.

(b) Using the Work–Energy Theorem for each particle

$$W = q\Delta V = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

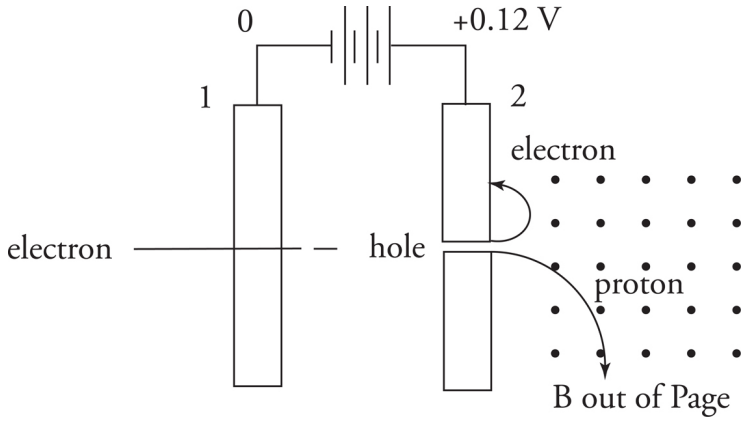
v_i is zero for each particle, and each particle has the same ΔV and q, so

$$v_f = \sqrt{\frac{2q\Delta V}{m}}$$

The ratio for the speed of the proton to the electron is the square root reciprocal ratio of the masses

$$\frac{v_{fp}}{v_{fe}} = \sqrt{\frac{m_e}{m_p}} = \sqrt{\frac{9.11 \times 10^{-31}\,\text{kg}}{1.67 \times 10^{-27}\,\text{kg}}} = 0.023$$

(c)



(d) The alpha particle will have twice the charge and four times the mass of the proton. Because

$$v_f = \sqrt{\frac{2q\Delta V}{m}}$$

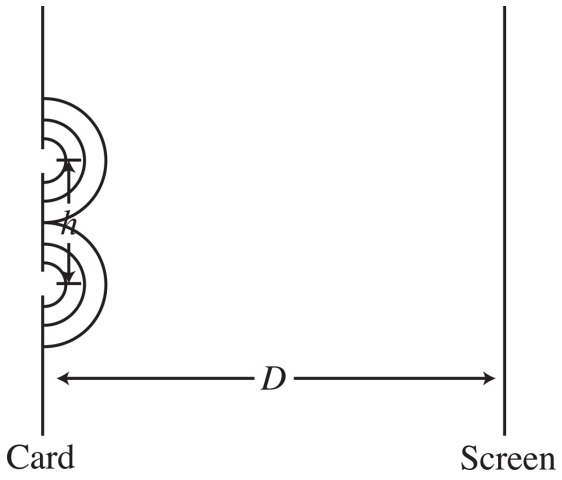
the final speed coming out of the hole will decrease by $\sqrt{2}$. The path curves because of the magnetic force being the centripetal force.

$$\frac{mv^2}{r} = qvB$$

$$r = \frac{mv}{qB} = \frac{4m_p\sqrt{2}v_p}{2q_pB} = 2\sqrt{2} * r_p$$

The radius of the circle will be $2\sqrt{2}$ times as large, but the curve will be in the same direction.

4. (a)



(b) The double slit experiment works by producing coherent spherical wavefronts that interfere with each other. A spherical wavefront is produced when a plane wave impinges upon a small slit, such as either of the slits in the

card, as shown in part (a). The two spherical wavefronts then propagate to the screen and because there are two waves, they interfere with one another. Bright spots occur whenever there is constructive interference and dark spots indicate that the interference is destructive. The interference will be constructive whenever the difference in the distance that the two waves travel is an integer multiple of the wavelength of the initial plane wave. There will be bright spots when the difference in travelled distance is 0 (in the middle), or ±1 wavelength, etc. There are a series of bright spots because there are many different possible locations on the card where constructive interference occurs.

(c) In the double slit experiment $m\lambda = dx/L$. Solving for λ,

$$\lambda = \frac{dx}{Lm}$$

d is the distance between the slits, x is the distance measured between fringes, L is the space from the slits to the screen and m is the "order" or the number of dark spots between fringes. Using the given variables,

$$\lambda = \frac{hy_3}{3D}$$

(d) When light enters a medium with a different index of refraction, its speed changes. The light wave will have a constant frequency, so this results from a change in its wavelength. When light goes from an index of $n = 1$ to an index of $n > 1$, the wave speed decreases, resulting in a decreased wavelength.

$$\lambda = \frac{dx}{Lm}$$

So x is directly proportional to λ, and the distance between fringes will decrease.

Part III
About the AP Physics 2 Exam

- The Structure of the AP Physics 2 Exam
- How the AP Exams Are Used
- Other Resources
- Designing Your Study Plan

THE STRUCTURE OF THE AP PHYSICS 2 EXAM

The AP Physics 2 Exam consists of two sections: a multiple-choice section and a free-response section. Questions in the multiple-choice section are each followed by four possible responses. In the single-select subsection, only one of these possible answers is correct. In the multi-select subsection, two of these possible answers are correct. It is your job to choose the right answer. Each correct answer is worth one point.

Section	Timing	Scoring	Question Type	Number of Questions
I: Multiple Choice	90 minutes	50% of exam score	Single-select (discrete questions and questions in sets with one correct answer)	45
			Multi-select (discrete questions with two correct answers)	5
				Total—50

Section	Timing	Scoring	Question Type	Number of Questions
II: Free Response	90 minutes	50% of exam score	Experimental Design	1
			Qualitative/ Quantitative/ Translation	1
			Short Answer	2
				Total—4



The free-response section requires you to write out your solutions, showing your work. The total amount of time for this section is 90 minutes, so you have an average of 22.5 minutes per question. Unlike the multiple-choice section, which is scored by computer, the free-response section is graded by high school and college teachers. They have guidelines for awarding partial credit, so you don't need to correctly answer every part to get points. You are allowed to use a calculator on the entire AP Physics 2 Exam (programmable or graphing calculators are okay, but ones with a typewriter-style keyboard are not) and a table of equations is

provided for your use. The two sections—multiple-choice and free-response—are weighted equally, so each is worth 50 percent of your grade.

Grades on the AP Physics 2 Exam are reported as a number: either 1, 2, 3, 4, or 5. The descriptions for each of these five numerical scores are as follows:

AP Exam Grade	Description
5	Extremely well qualified
4	Well qualified
3	Qualified
2	Possibly qualified
1	No recommendation

Colleges are generally looking for a 4 or 5, but some may grant credit for a 3. How well do you have to do to earn such a grade? Each test is curved, and specific cut-offs for each grade vary a little from year to year, but here's a rough idea of how many points you must earn—as a percentage of the maximum possible raw score—to achieve each of the grades 2 through 5:

AP Exam Grade	Percentage Needed
5	≥ 75%
4	≥ 60%
3	≥ 45%
2	≥ 25%

AP Physics 2: Algebra-Based Course Content

You may be using this book as a supplementary text as you take an AP Physics 2 course at your high school or you may be using it on your own. The College Board is very detailed in what they require your AP teacher to cover in his or her AP Physics 2 course. They

explain the course that you should know with the following summary.

Changes to the AP Physics 2 Course

The College Board recently announced that they are restructuring the AP Physics 2 course from the Big Ideas into seven content units. The exam itself is not changing, and the course content will likely remain the same—but the presentation of that content may be organized differently than it currently appears in this book. This new course structure will take effect for the 2019–2020 school year.

At the time of this book's printing, we are awaiting specifics on this; to stay up to date, register your book and check your online Student Tools, where we have listed the most recent information. See this page for details.

You can also visit the College Board's AP Central page for this course: https://apcentral.collegeboard.org/courses/ap-physics-1-and-2/course/updates-2019-20

Students explore principles of fluids, thermodynamics, electricity, magnetism, optics, and topics in modern physics. The course is based on seven Big Ideas, which encompass core scientific principles, theories, and processes that cut across traditional boundaries and provide a broad way of thinking about the physical world. The course also focuses on the seven Science Practices, which are the practices that allow for scientific reasoning.

The following are Big Ideas:

- Objects and systems have properties such as mass and charge. Systems may have internal structure.
- Fields existing in space can be used to explain interactions.
- The interactions of an object with other objects can be described by forces.
- Interactions between systems can result in changes in those systems.
- Changes that occur as a result of interactions are constrained by conservation laws.
- Waves can transfer energy and momentum from one location to another without the permanent transfer of mass and serve as a mathematical model for the description of other phenomena.
- The mathematics of probability can be used to describe the behavior of complex systems and to interpret the behavior of quantum mechanical systems.

The following are Science Practices:

- The student can use representations and models to communicate scientific phenomena and solve scientific problems.
- The student can use mathematics appropriately.
- The student can engage in scientific questioning to extend thinking or to guide investigations within the context of the AP course.
- The student can plan and implement data collection strategies in relation to a particular scientific question.
- The student can perform data analysis and evaluation of evidence.
- The student can work with scientific explanations and theories.
- The student is able to connect and relate knowledge across various scales, concepts, and representations in and across

domains.

Questions will address the content in a variety of ways. These will include mathematical calculations similar to those found in many textbooks, proportional reasoning, verbal descriptions, and analysis of graphs including interpretations of slopes and x- or y-intercepts. Naturally, it's important to be familiar with the topics—to understand the basics of the theory, to know the definitions of the fundamental quantities, and to recognize and be able to use the equations. Then, you must acquire practice at applying what you've learned to answering questions like you'll see on the exam. This book is designed to review all of the content areas covered on the exam, illustrated by hundreds of examples. Also, each content review chapter is followed by practice multiple-choice and free-response questions, and perhaps even more important, *answers and explanations are provided for every example and question in this book*. You'll learn as much—if not more—from actively reading the solutions as you will from reading the text and examples. Also, two full-length practice tests (with solutions) are provided in this book. The difficulty level of the examples and questions in this book is at or slightly above AP level, so if you have the time and motivation to attack these questions and learn from the solutions, you should feel confident that you can do your very best on the real thing.

The College Board also gives us a list of Physics 2 Principles that they describe as foundational physics principles. If you are taking an AP Physics 2 course, you will likely recognize these categories from your textbook sections. Be sure that you're familiar with the following principles as you prepare for your exam:

- Thermodynamics: Laws of Thermodynamics, Ideal Gases, Kinetic Theory
- Fluid Statics and Dynamics
- Electrostatics: Electric Force, Electric Field, and Electric Potential

- DC Circuits and RC Circuits (steady-state only)
- Magnetism and Electromagnetic Induction
- Geometric and Physical Optics
- Quantum Physics, Atomic, and Nuclear Physics

HOW AP EXAMS ARE USED

Different colleges use AP Exams in different ways, so it is important that you go to a particular college's website to determine how it uses AP Exams. The three items below represent the main ways in which AP Exam scores can be used:

- **College Credit.** Some colleges will give you college credit if you score well on an AP Exam. These credits count toward your graduation requirements, meaning that you can take fewer courses while in college. Given the cost of college, this could be quite a benefit, indeed.
- **Satisfy Requirements.** Some colleges will allow you to "place out" of certain requirements if you do well on an AP Exam, even if they do not give you actual college credits. For example, you might not need to take an introductory-level course, or perhaps you might not need to take a class in a certain discipline at all.
- **Admissions Plus.** Even if your AP Exam will not result in college credit or even allow you to place out of certain courses, most colleges will respect your decision to push yourself by taking an AP Course or even an AP Exam outside of a course. A high score on an AP Exam shows mastery of more difficult content than is taught in many high school courses, and colleges may take that into account during the admissions process.



Are You Preparing for College?

Check out all of the useful books from The Princeton Review, including *The Best 385 Colleges, Cracking the SAT, Cracking the ACT*, and more!

OTHER RESOURCES

There are many resources available to help you improve your score on the AP Physics 2 Exam, not the least of which are your **teachers**. If you are taking an AP class, you may be able to get extra attention from your teacher, such as obtaining feedback on your essays. If you are not in an AP course, reach out to a teacher who teaches AP Physics 2 and ask if the teacher will review your essays or otherwise help you with content.

Another wonderful resource is **AP Students**, the official site of the AP Exams. The scope of the information at this site is quite broad and includes:

- a Course Description, which includes details on what content is covered and sample questions
- sample questions from the AP Physics 2 Exam

The AP Students home page address is: **http://apstudent.collegeboard.org**.

Finally, The Princeton Review offers tutoring for the AP Physics 2 Exam. Our expert instructors can help you refine your strategic approach and add to your content knowledge. For more information, call 1-800-2REVIEW.



Break up your review into manageable portions. Download our helpful study guide for this book, once you register online (see this page for details).

DESIGNING YOUR STUDY PLAN

As part of the Introduction, you identified some areas of potential improvement. Let's now delve further into your performance on Test 1, with the goal of developing a study plan appropriate to your needs and time commitment.

Read the answers and explanations associated with the multiple-choice questions (starting at this page). After you have done so, respond to the following questions:

- Review the list of AP Physics 2 Principles on this page. Next to each topic, indicate your rank of the topic as follows: "1" means "I need a lot of work on this," "2" means "I need to beef up my knowledge," and "3" means "I know this topic well."
- How many days/weeks/months away is your exam?
- What time of day is your best, most focused study time?
- How much time per day/week/month will you devote to preparing for your exam?
- When will you do this preparation? (Be as specific as possible: Mondays and Wednesdays from 3:00 P.M. to 4:00 P.M., for example.)
- Based on the answers above, will you focus on strategy (Part Two) or content (Part Three) or both?
- What are your overall goals in using this book?

Part IV
Test-Taking Strategies for the AP Physics 2 Exam

- Preview

1 How to Approach Multiple-Choice Questions

2 How to Approach Free-Response Questions

PREVIEW

Review your Practice Test 1 results and then respond to the following questions:

- How many multiple-choice questions did you miss even though you knew the answer?
- On how many multiple-choice questions did you guess blindly?
- How many multiple-choice questions did you miss after eliminating some answers and guessing based on the remaining answers?
- Did you find any of the free-response questions easier or harder than the others—and, if so, why?

HOW TO USE THE CHAPTERS IN THIS PART

For the following Strategy chapters, think about what you are doing now before you read the chapters. As you read and engage in the directed practice, be sure to appreciate the ways you can change your approach.

Chapter 1
How to Approach Multiple-Choice Questions

CRACKING THE MULTIPLE-CHOICE SECTION

All the multiple-choice questions will have a similar format: each will be followed by four answer choices. At times, it may seem that there could be more than one possible correct answer. For the single choice problems, which represent 45 out of your 50 multiple-choice problems, there is only one correct answer! For the last 5 problems of the multiple-choice section, there will be two correct answers per question. Keep in mind that this section is identified before you start it. Answers resulting from common mistakes are often included in the four answer choices to trap you.

Use the Answer Sheet

For the multiple-choice section, you write the answers not in the test booklet but on a separate answer sheet (very similar to the ones we've supplied at the very end of this book). Four oval-shaped answer bubbles follow the question number, one for each possible answer. Don't forget to fill in all your answers on the answer sheet. Don't just mark them in the test booklet. Marks in the test booklet will not be graded. Also, make sure that your filled-in answers correspond to the correct question numbers! Check your answer sheet after every five answers to make sure you haven't skipped any bubbles by mistake.

The Two-Pass System

The AP Physics 2 Exam covers a broad range of topics. There's no way, even with our extensive review, that you will know everything about every topic in algebra-based physics. So, what should you do?

Adopt a two-pass system. The two-pass system entails going through the test and answering the easy questions first. Save the more time-consuming questions for later. (Don't worry—you'll have time to do them later!) First, read the question and decide if it is a "now" or "later" question. If you decide this is a "now" question, answer it in

the test booklet. If it is a “later” question, come back to it. Once you have finished all the “now” questions on a double page, transfer the answers to your bubble sheet. Flip the page and repeat the process.

Once you’ve finished all the “now” questions, move on to the “later” questions. Start with the easier questions first. These are the ones that require calculations or that require you to eliminate the answer choices (in essence, the correct answer does not jump out at you immediately). Transfer your answers to your bubble sheet as soon as you answer these “later” questions.



Use the two-pass system to save time!

Watch Out for Those Bubbles!

Because you’re skipping problems, you need to keep careful track of the bubbles on your answer sheet. One way to accomplish this is by answering all the questions on a page and then transferring your choices to the answer sheet. If you prefer to enter them one by one, make sure you double-check the number beside the ovals before filling them in. We’d hate to see you lose points because you forgot to skip a bubble!

Know How to Linearize Graphs to Equations

Graphs, data tables, and mathematical relations (equation of fit lines) are all separate ways to represent the outcome of experiments, and the AP Physics 2 Exam will expect you to know how to use all of these representations. For the multiple-choice portion of the AP

Physics 2 Exam (as well as the free-response portion), you are supplied with an equation sheet. You should know what graphs of these equations look like (a graphing calculator may be helpful for this), but most often you will be asked which variables need to be graphed to produce a direct proportion. For Coulomb's Law, $F_{el} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}$, there are several data that would produce a linear graph. If F_{el} and Q_1 are data, then a graph of F_{el} versus Q_1 will produce a linear plot with slope of $\frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r^2}$, but if F_{el} and r are data, then a graph of F_{el} versus $\frac{1}{r^2}$ produces a linear plot with slope of $\frac{1}{4\pi\varepsilon_0}Q_1 Q_2$. Producing a linear graph for data or from an equation is called linearization.

Process of Elimination (POE)

On most tests, you need to know your material backward and forward in order to get the right answer. In other words, if you don't know the answer beforehand, you probably won't answer the question correctly. This is particularly true of fill-in-the-blank and essay questions. We're taught to think that the only way to get a question right is by knowing the answer. However, that's not the case on Section I of the AP Physics 2 Exam. You can get a perfect score on this portion of the test without knowing a single right answer—provided you know all the wrong answers!



Proven Technique

Use POE to boost your chances of scoring well.

What are we talking about? This is perhaps the most important technique to use on the multiple-choice section of the exam. Let's take a look at an example.

41. A gas undergoes an expansion-compression cycle. If, plotted on a P–V diagram, the cycle is counterclockwise and the work is 300 J in magnitude, what was the heat transfer during this cycle?

(A) 300 J into the system
(B) 300 J out of the system
(C) 600 J into the system
(D) 600 J out of the system



Apply the Strategy

Which answer choices can you eliminate first?

Now, if this were a fill-in-the-blank-style question, you might be in a heap of trouble. But let's take a look at what we've got. If we can eliminate a few of the answer choices, we can get closer to making an educated guess.

For a cycle, $\Delta U = 0$, so we know the magnitude of the work and heat transfer have to be the same. This eliminates (C) and (D). Since the cycle is counterclockwise on a P–V diagram, the work has to be positive, so the heat transfer is negative. A negative heat transfer is heat leaving the system. Choice (B) is correct.

We think we've illustrated our point: Process of Elimination is the best way to approach the multiple-choice questions. Even when you don't know the answer right off the bat, you'll surely know that two or three of the answer choices are not correct. What then?

Aggressive Guessing

As mentioned earlier, you are scored only on the number of questions you get right, so we know guessing can't hurt you. But can it help you? It sure can. Let's say you guess on four questions; odds are you'll get one right. So you've already increased your score by one point. Now, let's add POE into the equation. If you can eliminate as many as two answer choices from each question, your chances of getting them right increase, and so does your overall score. Remember, don't leave any bubbles blank on test day!

General Advice

Answering 50 multiple-choice questions in 90 minutes can be challenging. Make sure to pace yourself accordingly and remember that you do not need to answer every question correctly to do well. Exploit the multiple-choice structure of this section. For 45 of your 50 multiple-choice questions, there are three wrong answers and only a single correct one (for the multi-correct section, the last 5 questions, there are two wrong and two correct), so even if you don't know exactly which one is the right answer, you can eliminate some that you know for sure are wrong. Then you can make an educated guess from among the answers that are left and greatly increase your odds of getting that question correct.

Problems with graphs and diagrams are usually the fastest to solve and problems with an explanation for each answer usually take the

longest to work through. Do not spend too much time on any one problem or you may not get to easier problems further into the test.

These practice exams are written to give you an idea of the format of the test, the difficulty of the questions, and to allow you to practice pacing yourself. Take them in the same circumstances as you will encounter during the real exam.

Reflect

Respond to the following questions:

- How long will you spend on multiple-choice questions?
- How will you change your approach to multiple-choice questions?
- What is your multiple-choice guessing strategy?
- Will you seek further help, outside of this book (such as a teacher, tutor, or AP Students), on how to approach the questions that you will see on the AP Physics 2 Exam?

Chapter 2
How to Approach Free-Response Questions

FREE-RESPONSE SECTION

On the free-response section, be sure to show the graders what you're thinking. Write clearly—that is *very* important—and show your steps. If you make a mistake in one part and carry an incorrect result to a later part of the question, you can still earn valuable points if your method is correct. But the graders cannot give you credit for work they can't follow or can't read. And, where appropriate, be sure to include units on your final answers.

The most important advice we can give you for the free-response section of the AP Physics 2 Exam is to read the questions carefully and answer according to exactly what the questions are asking you to do. Credit for the answers depends not only on the quality of the solutions but also on how they are explained. On the AP Physics 2 Exam, the words "justify," "explain," "calculate," "what is," "determine," and "derive" have specific meanings, and the graders are looking for very precise approaches in your explanations in order to get maximum credit.



Taking time to read the questions—making sure you understand them—will save you time in the long run.

Questions that ask you to "justify" are looking for you both to show an understanding in words of the principles underlying physical phenomena and to perform the mathematical operations needed to arrive at the correct answer. The word "justify" as well as the word "explain" requires that you support your answers with text, equations, calculations, diagrams, or graphs. In some cases, the text or equations must elucidate physics fundamentals or laws, while in

other cases they will serve to analyze the behavior of different values or different types of variables in the equation.

The word “calculate” requires you to show numerical or algebraic work to arrive at the final answer. In contrast, “what is” and “determine” questions signify that full credit may be given without showing mathematical work. Just remember, showing work that leads to the correct answer is always a good idea when possible, especially since showing work may still earn you partial credit even if the answer is not correct.

“Derive” questions are looking for a more specific approach, which entails beginning the solution with one or more fundamental equations and then arriving at the final answer through the proper use of mathematics, usually involving some algebra.

CRACKING THE FREE-RESPONSE SECTION

Section II is worth 50 percent of your grade on the AP Physics 2 Exam. This section is composed of four free-response questions. You're given a total of 90 minutes for this section. There are three types of problems on the exam. One question is an Experimental Design worth 12 points, one is a Quantitative/Qualitative Translation worth 12 points, and the other two are Short Answer worth 10 points each. One of the Short Answer problems will require a paragraph-length argument. At the top of each question, next to the question number, is the number of points for the problem as well as the recommended amount of time to spend on that problem. The suggested times are 25 minutes each for 12-point problems and 20 minutes each for 10-point problems. Pacing is a personal decision, though, so feel free to build your own timeline for the free-response section.

Clearly Explain and Justify Your Answers

Remember that your answers to the free-response questions are graded by readers and not by computers. Communication is a very important part of AP Physics 2. Compose your answers in precise sentences. Just getting the correct numerical answer is not enough. The majority of questions will require a written explanation or justification either in lieu of a numerical answer or in addition to a calculation. You will need to explain your reasoning behind the technique that you selected and communicate your answer in the context of the problem. Many problems are graded where points are given "for indicating" some physical reality of the situation, so either a written justification or a calculated solution are acceptable.

Do not expect the graders to read between the lines. Your grader will be an experienced AP Physics 2 teacher, so you will not need to explain every detail of each step in your thought process, but you need to present your solution in a systematic manner using solid logic and appropriate language. The first step in almost any written

response should cite a specific physical principle. And remember: although you won't earn points for neatness, the graders can't give you a grade if they can't read and understand your solution!

Use Only the Space You Need

Do not try to fill up the space provided for each question. The space given is usually more than enough. The people who design the tests realize that some students write in big letters and some students make mistakes and need extra space for corrections. So if you have a complete solution, don't worry about the extra space. Writing more will not earn you extra credit. In fact, many students tend to go overboard and shoot themselves in the foot by making a mistake after they've already written the right answer.

Use the Numbering to Guide Your Answer!

Questions will typically have several subparts. When the subparts are related directly to one another, they will be numbered with letters and lowercase Roman numerals (for example, *a.i* and *a.ii*). Subparts which are not directly related to one another, but which are still related to the main stem of the problem are numbered with different letters (for example, *a* and *b*). It is not uncommon for a question to have a mixture of both types of subparts. (For example, a problem could be numbered *a.i, a.ii, b, c.i, c.ii.* Subparts *a.i* and *a.ii* would be related to each other, while part *b* would be related back to the main stem of the problem but not to either of the *a* parts. Then both of the *c* parts would be related to each other, but not to the *a* or *b* parts.) Try to answer all of the subparts. If one part is giving you trouble, look at the next subpart and see if you can find a way to approach it. Since the answer to *b* does not usually depend on part *a*, you can take a fresh approach to answering when the numbering of the subparts changes.

Read and Respond to the Entire Question

Many times, a single subpart of a question will ask you to explain multiple things. In order to earn full points, you must address every

item the question asks. Such answers typically require no more than a single sentence per item to fully explain. For example, a subpart in a circuit question may ask how, if a particular resistor were removed, the total current and the total resistance and the total voltage of the circuit would change. You could not receive full credit unless you addressed all three of those quantities in your answer.

Use Common Sense

Always use your common sense in answering questions. If your solutions to part (*a.i*) are incorrect, but are substituted correctly into part (*a.ii*), you will likely not be penalized for getting an incorrect answer to part (*a.ii*). However, if your answer to (*a.ii*) is physically impossible, such as getting a speed of light in vacuum greater than 3.00×10^8 m/s or finding a number of electrons of 3.5 will not receive credit even if they accurately rely on the answer to a previous part. To receive the highest possible score, make your errors easily recognizable and make sure all of your answers are physically reasonable.

Think Like a Grader

When answering questions, try to think about what kind of answer the grader is expecting. Look at past free-response questions and grading rubrics on the College Board website. These examples will give you some idea of how the answers should be phrased. The graders are told to keep in mind that there are two aspects to the scoring of free-response answers: showing statistical knowledge and communicating that knowledge. Again, responses should be written as clearly as possible in complete sentences. You don't need to show all the steps of a calculation, but you must explain how you got your answer and why you chose the technique you used.

Think Before You Write

Abraham Lincoln once said that if he had eight hours to chop down a tree, he would spend six of them sharpening his axe. Be sure to spend some time thinking about what the question is, what answers

are being asked for, what answers might make sense, and what your intuition is before starting to write. These questions aren't meant to trick you, so all the information you need is given. If you think you don't have the right information, you may have misunderstood the question. In some calculations, it is easy to get confused, so think about whether your answers make sense in terms of what the question is asking. If you have some idea of what the answer should look like before starting to write, then you will avoid getting sidetracked and wasting time on dead-ends.

Types of Free Response Questions and How To Answer Them

The AP Physics 2 Exam will assess your ability to perform an **experimental analysis.** This will be one of the two 12-point questions. During the experimental design question, you will be expected to do the following things:

- Draw a clearly labeled diagram for the experimental procedure that you will follow.
- Distinguish between quantities that are directly measured and those that will need to be calculated.
- Describe an experimental procedure using an appropriate amount of detail. A typical appropriate level of detail will include between 4 and 6 steps, describe the arrangements of materials and the measurements which must be taken and which equipment is used to make each measurement described.
    - Steps such as "gather material," "record data," and "analyze data" should be omitted as that are implicit in performing any experimental analysis.
    - Descriptions that involve too much detail should also be avoided.
- Draw or describe a graph from data collected in the experiment of a data table supplied for a similar experiment on supplied

graph paper.

- In drawing or describing a graph, you need to label the vertical and horizontal axes, including labeling values and units.
- If you need to produce a graph, you must scale the axes so the data is shown on the graph provided.
  - Your data points must take up at least half of the space on the available graph in order to receive full credit.
- Drawing or describing a best-fit line for linear data. If drawing the fit line:
  - The best fit line should contain approximately an equal number of points above the line as below the line.
  - You should avoid drawing the best-fit line through the first and last data points.
  - You may create a best-fit line using a linear regression on a graphing calculator, but you must explicitly write that you have done this in your answer.

- Describe how to use the slope of the best-fit line and relate it to a physical quantity.
  - Using any method other than finding the slope of the fit line will result in lost points.
- You may be asked to discuss which measurement or variable in a procedure contributes most to overall uncertainty in the final result and on conclusions drawn from a given data set.
- You should be able to review and critique an experimental design or procedure and decide whether the conclusions can be justified based on the procedure and the evidence presented.

The AP Physics 2 Exam will have a question about **translating quantitative measurements into qualitative explanations** or vice versa. This will be the other 12-point problem. This problem will

require performing some calculations, so being familiar with the equations on the equation sheet will help you answer this question. You will be asked to relate your calculations to a non-calculated qualitative description on the physical phenomenon. In optics, this could be a question about how the magnification calculated is shown in a ray diagram. In electrostatics, it could be a description of how an electric potential map relates to the calculated electric field.

The AP Physics 2 Exam will have a **Short Answer** problem that includes the instruction to answer "in a coherent, paragraph-length response." The requirement for full credit for a paragraph-length response in AP Physics 2 includes that the argument be presented primarily in prose. If the argument relies too heavily on diagrams, graphs, equations, or calculations, you will not receive full credit. You need to clearly demonstrate your line of reasoning. According to the College Board, your answer "should be a coherent, organized, and sequential description of the analysis of a situation." To construct such an argument, you should begin by citing applicable physical principles (this might include using conservation of energy and charge for a circuit description or Bernoulli's Principle for a fluid dynamics question). You then need to sequentially describe the situation as physics dictates it will evolve. Your response should be at least 4 sentences long. Your sentences should not rely heavily on use of equations or on descriptions of diagrams.

The AP Physics 2 Exam will have a **Short Answer** problem that has you analyze an argument amongst a pair of fictitious students. You may be asked specifically in separate sub-parts what parts of each students' argument is correct (as parts i and ii) and what parts of his or her argument is incorrect (as parts iii and iv). The question may also be a single subpart asking you to address the claims of each student and explain why the student is correct or incorrect. Regardless of how the question is phrased, your response needs to include a sentence restating the part of the argument made by the student that is correct or incorrect and a sentence explaining why the argument is valid or invalid.

Reflect

Respond to the following questions:

- How much time will you spend on the short free-response questions? What about the long free-response questions?
- What will you do before you begin writing your free-response answers?
- Will you seek further help, outside of this book (such as a teacher, tutor, or AP Students), on how to approach the questions that you will see on the AP Physics 2 Exam?

Part V
Content Review for the AP Physics 2 Exam

3 Fluid Mechanics
4 Thermal Physics
5 Electric Forces and Fields
6 Electric Potential and Capacitance
7 Direct Current Circuits
8 Magnetic Forces and Fields
9 Electromagnetic Induction
10 Optics
11 Modern Physics
12 Solutions to Chapter Review Questions

Chapter 3
Fluid Mechanics

INTRODUCTION

In this chapter, we'll discuss some of the fundamental concepts dealing with substances that can flow, which are known as **fluids**. The term *fluid* refers to both liquids and gases. While there are distinctions between liquids and gases, this chapter focuses on the similarities of all fluids.

DENSITY

Although the concept of *mass* is central to your study of mechanics (because of the all-important equation $F_{\text{net}} = ma$), it is the substance's *density* that turns out to be more useful in fluid mechanics.

By definition, the density of a substance is its mass per unit volume, and it's typically denoted by the letter ρ (the Greek letter *rho*):

Equation Sheet

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{V}$$

Note that this equation immediately implies that m is equal to ρV when ρ is a constant.

For example, if 10^{-3} m^3 of oil has a mass of 0.8 kg, then the density of this oil is

$$\rho = \frac{m}{V} = \frac{0.8 \text{ kg}}{10^{-3} \text{ m}^3} = 800 \frac{\text{kg}}{\text{m}^3}$$

PRESSURE

If we place an object in a fluid, the fluid exerts a contact force on the object. How that force is distributed over any small area of the object's surface defines the **pressure**:

Equation Sheet

$$\text{pressure} = \frac{\text{force}_{\perp}}{\text{area}}$$

$$P = \frac{F_{\perp}}{A}$$

The subscript $\perp$ (which means *perpendicular*) is meant to emphasize that the pressure is defined to be the magnitude of the force that acts perpendicular to the surface, divided by the area. Because force is measured in newtons (N) and area is expressed in square meters (m^2), the SI unit for pressure is the newton per square meter. This unit is given its own name: the **pascal**, abbreviated Pa:

$$\text{SI unit of pressure: } 1 \text{ pascal} = 1 \text{ Pa} = 1 \frac{\text{N}}{\text{m}^2}$$

One pascal is a very tiny amount of pressure; for example, a nickel on a table exerts about 140 Pa of pressure, just due to its weight alone. For this reason, you'll often see pressures expressed in kPa (kilopascals, where 1 kPa = 10^3 Pa) or even in MPa (megapascals, where 1 MPa = 10^6 Pa). Another common unit for pressure is the atmosphere (atm). At sea level, atmospheric pressure, P_{atm}, is about 101,300 Pa; this is 1 **atmosphere**.

Example 1 A vertical column made of cement has a base area of 0.5 m^2. If its height is 2 m, and the density of cement is 3000 kg/m^3, how much pressure does this column exert on the ground?

Solution. The force the column exerts on the ground is equal to its weight, mg, so we'll find the pressure it exerts by dividing this by the base area, A. The mass of the column is equal to ρV, which we calculate as follows:

$$m = \rho V = \rho A h = (3\times10^3 \ \frac{\text{kg}}{\text{m}^3})(0.5 \ \text{m}^2)(2 \ \text{m}) = 3\times10^3 \ \text{kg}$$

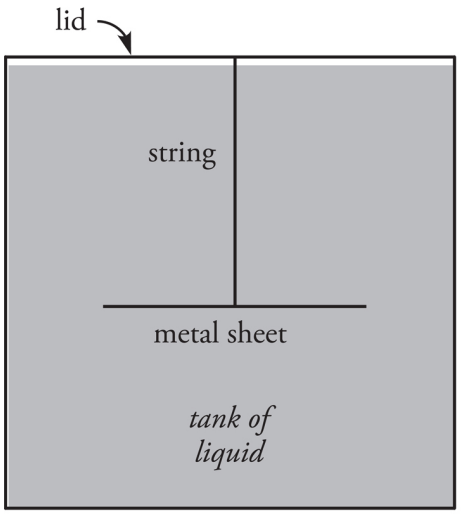
Therefore,

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{(3\times10^3 \ \text{kg})(10 \ \frac{\text{N}}{\text{kg}})}{0.5 \ \text{m}^2} = 6\times10^4 \ \text{Pa} = 60 \ \text{kPa}$$

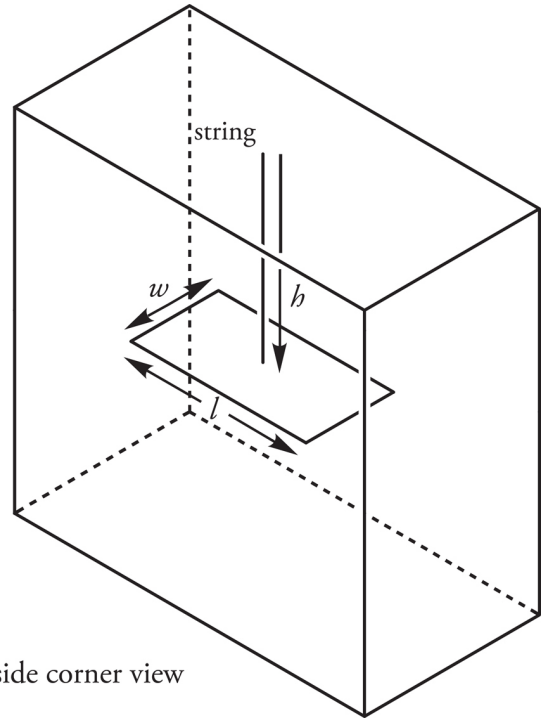
Now, you may be used to the unit m/s^2 for g. However, for this chapter, it is simpler to use the unit N/kg for g, as it is equivalent to m/s^2. So don't be confused when you see it later on.

HYDROSTATIC PRESSURE

Imagine that we have a tank with a lid on top, filled with some liquid. Suspended from this lid is a string, attached to a thin sheet of metal which hangs horizontally. The figures below show two views of this:



front view



side corner view

The weight of the liquid produces a force that pushes down on the metal sheet. If the sheet has length l and width w, and is at depth h below the surface of the liquid, then the weight of the liquid on top of the sheet is

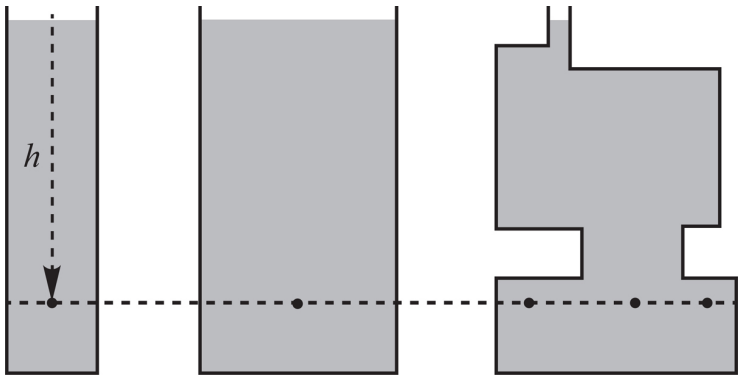
$$F_g = mg = \rho Vg = \rho(lwh)g$$

where ρ is the liquid's density. If we divide this weight by the area of the sheet ($A = lw$), we get the pressure due to the liquid:

$$P_{\text{liquid}} = \frac{\text{force}}{\text{area}} = \frac{F_{g\,\text{liquid}}}{A} = \frac{\rho(lwh)g}{lw} = \rho gh$$

Since the liquid is at rest, this is known as **hydrostatic pressure.**

Note that the hydrostatic pressure due to the liquid, $P_{\text{liquid}} = \rho gh$, depends only on the density of the liquid and the depth below the surface; in fact, it's proportional to both of these quantities. One important consequence of this is that the shape of the container doesn't matter. For example, if all the containers in the figure below are filled with the same liquid, then the pressure is the same at every point along the horizontal dashed line (and within a container), simply because every point on this line is at the same depth, h, below the surface of the liquid.



If the liquid in the tank were open to the atmosphere, then the total (or absolute) pressure at depth h would be equal to the pressure pushing down on the surface—the atmospheric pressure, P_{atm}—plus the pressure due to the liquid alone. In general, the pressure on the surface of the liquid is some pressure P_{o} and the total pressure is

total (absolute) pressure: $P_{\text{total}} = P_{\text{o}} + P_{\text{liquid}} = P_{\text{o}} + \rho gh$

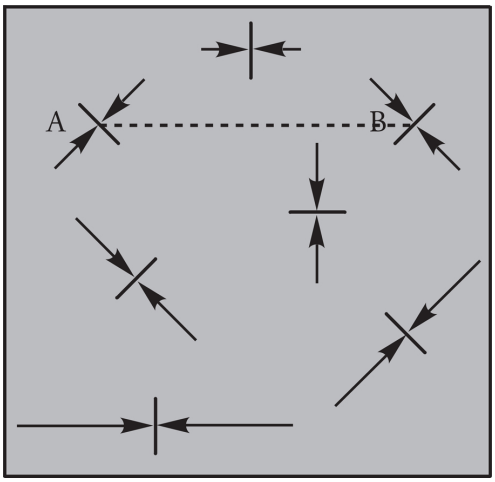
When the pressure at the top of the liquid is atmospheric pressure (as in the scenario above), then our equation for total pressure reduces to

$$P_{\text{total}} = P_{\text{atm}} + \rho gh$$

Because pressure is the *magnitude* of the force per area, pressure is a scalar. It has no direction. The direction of the force due to the pressure on any small surface is perpendicular to that surface. For example, in the figure below, the pressure at Point A is the same as the pressure at Point B, because they're at the same depth.

It's All About Depth

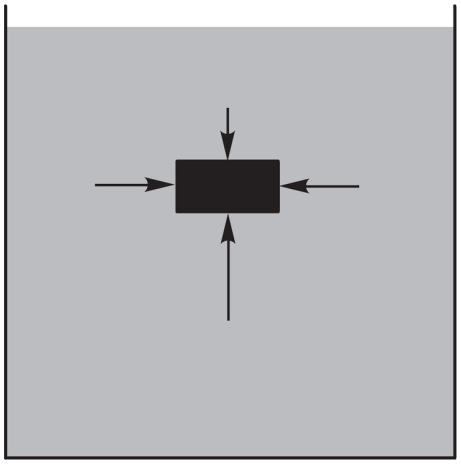
Pressure is proportional to depth of an object, not the mass of that object. An elephant and a feather at the same depth feel the same amount of pressure. How they react to this pressure is a different story.



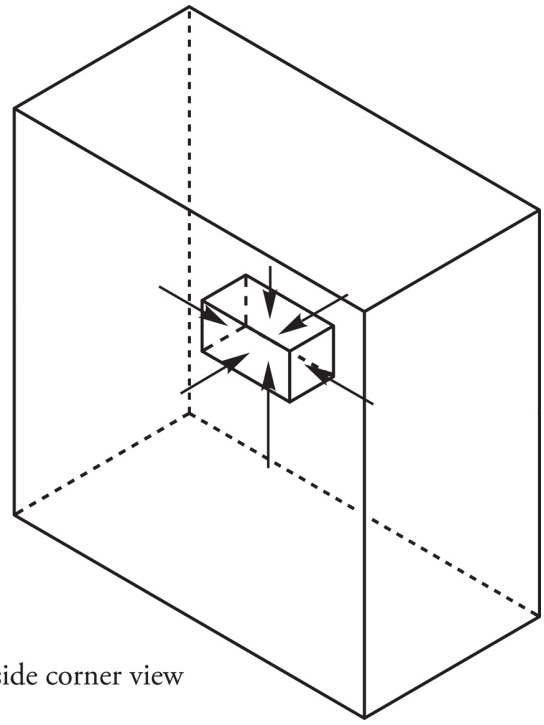
But, as you can see, the direction of the force due to the pressure varies depending on the orientation of the surface—and even which side of the surface—the force is pushing on.

BUOYANCY

Let's place a block in our tank of fluid. Because the pressure on each side of the block depends on its average depth, we see that there's more pressure on the bottom of the block than there is on the top. Because the block is rectangular and the top and bottom have the same area, there's a greater force pushing up on the block than there is pushing down on it. The forces due to the pressure on the other four sides cancel out (because they are at the same depth), so the net force on the block is upward.



front view



side corner view

This net upward force is called the **buoyant force** (or just **buoyancy** for short), denoted $\mathbf{F}_{\text{buoy}}$. We calculate the magnitude of the buoyant force using **Archimedes' principle**; in words, Archimedes' principle says

> The strength of the buoyant force on the object is equal to the weight of the fluid displaced by the object.

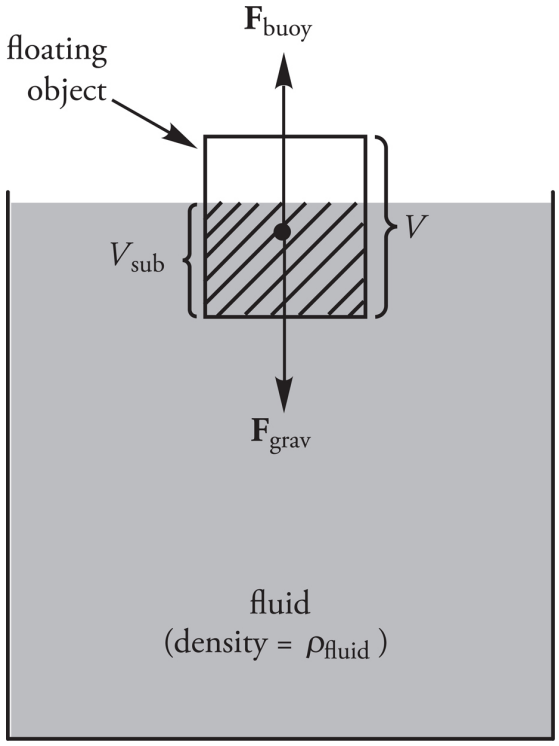
When an object is partially or completely submerged in a fluid, the volume of the object submerged, which we call V_{sub}, is the volume of the fluid displaced. By multiplying this volume by the density of the fluid, we get the mass of the fluid displaced; then, multiplying this mass by g gives us the weight of the fluid displaced. So, here's Archimedes' principle as a mathematical equation:

Buoyant force: $F_{\text{buoy}} = \rho_{\text{fluid}} V_{\text{sub}} g$

When an object floats, its submerged volume is just enough to make the buoyant force it feels balance its weight. So, if an object's density is ρ_{object} and its (total) volume is V, its weight will be $mg = \rho_{\text{object}} Vg$. The buoyant force it feels is $\rho_{\text{fluid}} V_{\text{sub}} g$. Setting these equal to each other, we find that

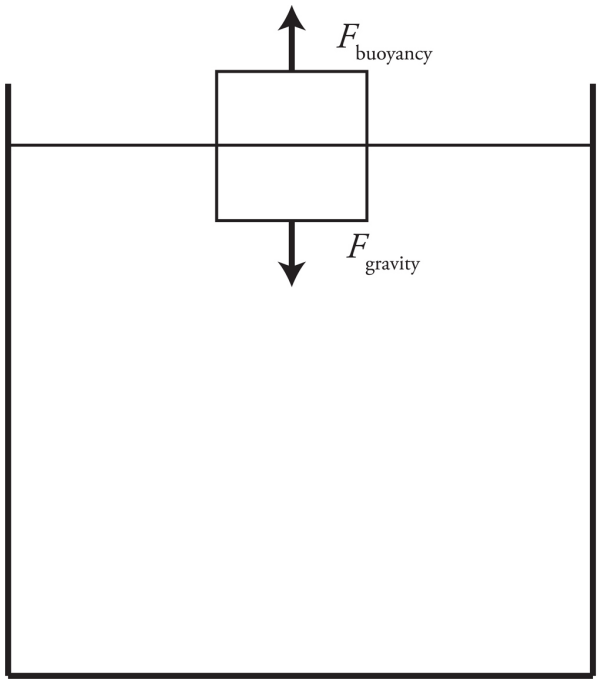
$$\frac{V_{\text{sub}}}{V} = \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}}$$

So, if $\rho_{\text{object}} < \rho_{\text{fluid}}$, then the object will float; and the fraction of its volume sub merged is the same as the ratio of its density to the fluid's density. For example, if the object's density is 2/3 the density of the fluid, then the object will float, and 2/3 of the object will be submerged.



If an object is denser than the fluid, it will sink. In this case, even if the entire object is submerged (in an attempt to maximize V_{sub} and the buoyant force), its weight is still greater than the buoyant force, and down it goes. And if an object just happens to have the same density as the fluid, it will be happy hovering (in static equilibrium) anywhere underneath the fluid.

Let's summarize, and go through the steps of an object floating and then an object as it sinks and hits the bottom of a tank. In the diagram on the next page, we have an object that is floating in a liquid.



$$\rho_{object} < \rho_{fluid}$$

$$F_{net} = 0$$

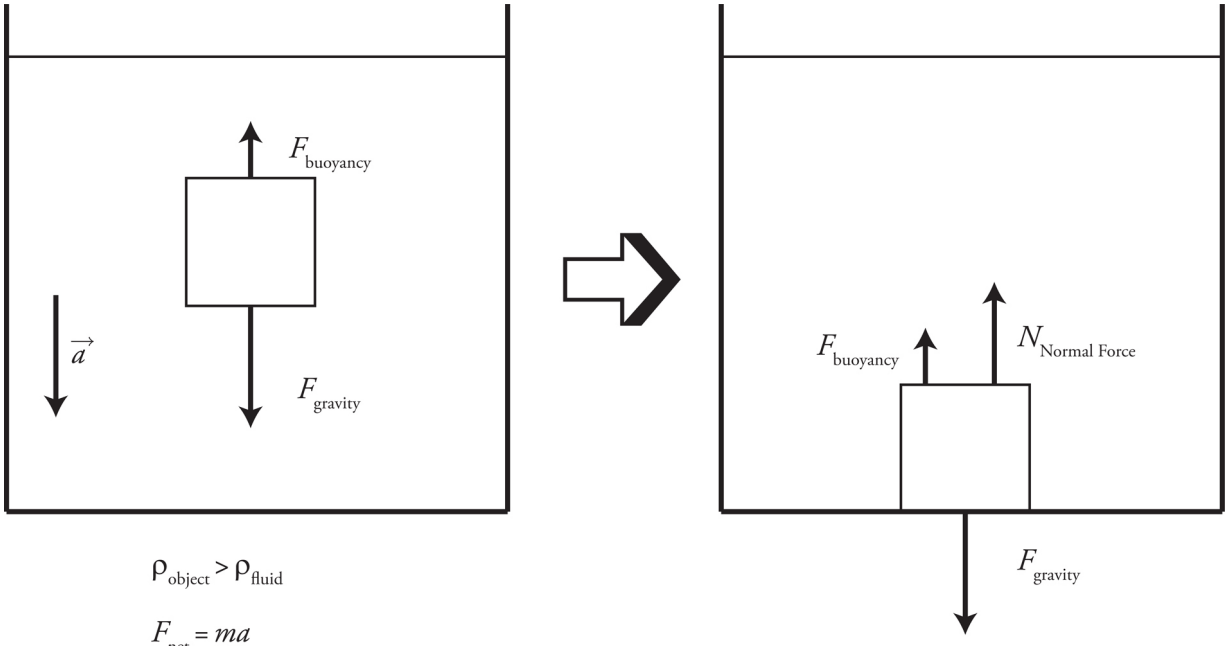
$$F_{buoyancy} = F_{gravity}$$

$$\rho_{fluid} V_{fluid} g = m_{object} g$$

$$\rho_{fluid} V_{sub} g = \rho_{object} V_{object} g$$

$$\frac{V_{sub}}{V_{obj.}} = \frac{\rho_{object}}{\rho_{fluid}}$$

Below is a diagram of an object with a density greater than the fluid density as it sinks to the bottom of the tank.



$\rho_{object} > \rho_{fluid}$

$F_{net} = ma$

$F_{net} = F_{gravity} - F_{buoyancy}$

$\rho_{object} > \rho_{fluid}$

$F_{net} = 0$

$F_{gravity} = F_{buoyancy} + N$

When we solve for N we get,

Apparent Weight

$N = F_{gravity} - F_{buoyancy}$

Because the buoyant force is always directed upward, and the force of gravity is defined to point downward, these forces are always directed in opposite directions. The result of the buoyant force opposing gravity is that any object in a fluid appears to weigh less than when it is not in the fluid, and as the fluid density increases, the apparent weight decreases. The apparent weight of an object is commonly referred to as the normal force of that object.

Example 2 An object with a mass of 150 kg and a volume of 0.75 m^3 is floating in ethyl alcohol, whose density is 800 kg/m^3. What fraction of the object's volume is above the surface of the fluid?

Solution. The density of the object is

$$\rho_{\text{object}} = \frac{m}{V} = \frac{150\ \text{kg}}{0.75\ \text{m}^3} = 200\ \frac{\text{kg}}{\text{m}^3}$$

The ratio of the object's density to the fluid's density is

$$\frac{\rho_{\text{object}}}{\rho_{\text{fluid}}} = \frac{200\ \text{kg/m}^3}{800\ \text{kg/m}^3} = \frac{1}{4}$$

This means that 1/4 of the object's volume is *below* the surface of the fluid; therefore, the fraction *above* the surface is 1 – (1/4) = 3/4. Make sure you know exactly what the question is asking. Typically you use this equation to find the volume submerged, but this question (Example 2) asks for the volume not submerged.

Example 3 A brick, of density 2000 kg/m^3 and volume 1.5×10^{-3} m^3, is dropped into a swimming pool full of water.

(a) Explain briefly why the brick will sink.

(b) When the brick is lying on the bottom of the pool, what is the apparent weight of the brick?

Solution.

(a) The brick has a greater density than the surrounding fluid (water), so it will sink.

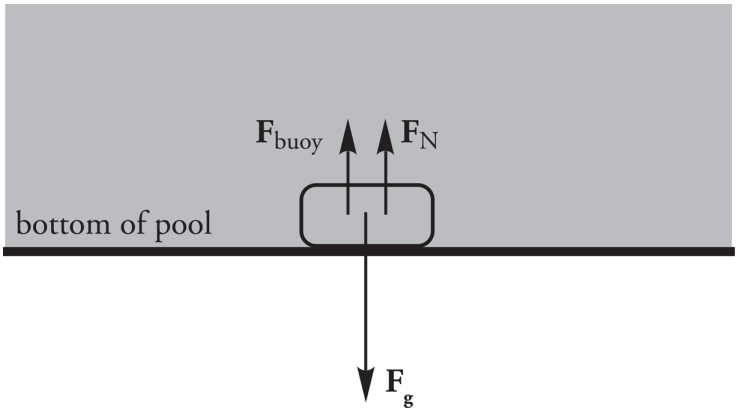
(b) When the brick is lying on the bottom surface of the pool, it is totally submerged, so $V_{\text{sub}} = V$; this means the buoyant force on the brick is

$$F_{\text{buoy}} = \rho_{\text{fluid}} V_{\text{sub}} g = \rho_{\text{water}} Vg$$
$$= (1000 \text{ kg/m}^3)(1.5 \times 10^{-3} \text{ m}^3)(10 \text{ N/kg})$$
$$= 15 \text{ N}$$

The weight of the brick is

$$F_g = mg = \rho_{\text{brick}} Vg$$
$$= (2000 \text{ kg/m}^3)(1.5 \times 10^{-3} \text{ m}^3)(10 \text{ N/kg})$$
$$= 30 \text{ N}$$

When the brick is lying on the bottom of the pool, the net force it feels is zero.



Therefore, we must have $F_{\text{buoy}} + F_N = F_g$, so $F_N = F_g - F_{\text{buoy}} = 30$ N − 15 N = 15 N. So the apparent weight of the brick, that is, the normal force it feels, is 15 N.

Example 4 A helium balloon has a volume of 0.03 m^3. What is the net force on the balloon if it's surrounded by air? (Note: The density of helium is 0.2 kg/m^3, and the density of air is 1.2 kg/m^3.)

Solution. The balloon will feel a buoyant force upward, and the force of gravity—the balloon's weight—downward. Because the

balloon is completely surrounded by air, $V_{sub} = V$, and the buoyant force is

$$\begin{aligned} F_{buoy} &= \rho_{fluid} V_{sub} g = \rho_{air} V g \\ &= (1.2 \text{ kg/m}^3)(0.03 \text{ m}^3)(10 \text{ N/kg}) \\ &= 0.36 \text{ N} \end{aligned}$$

The weight of the balloon is

Equation Sheet

$$\begin{aligned} F_g = mg = \rho_{helium} V g &= (0.2 \text{ kg/m}^3)(0.03 \text{ m}^3)(10 \text{ N/kg}) \\ &= 0.06 \text{ N} \end{aligned}$$

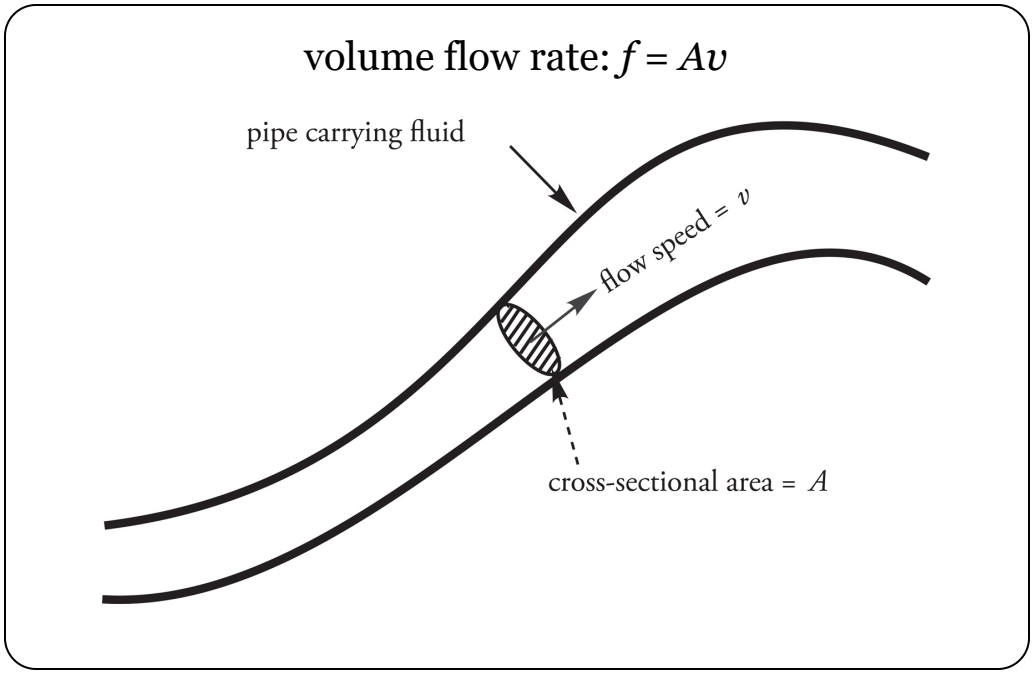
Because $F_{buoy} > w$, the net force on the balloon is upward and has magnitude

$$F_{net} = F_{buoy} - F_g = 0.36 \text{ N} - 0.06 \text{ N} = 0.3 \text{ N}$$

This is why a helium balloon floats away if you let go of its string.

FLOW RATE AND THE CONTINUITY EQUATION

Consider a pipe through which fluid is flowing. The **volume flow rate,** f, is the volume of fluid that passes a particular point per unit time. In SI units, flow rate is expressed in m^3/s. To find the volume flow rate, all we need to do is multiply the size of the pipe at a particular location by the average speed of the flow at that point. In this context, the size of the pipe means the area of a cross-section of the pipe that is perpendicular to the direction of the flow.



Be careful not to confuse *volume flow rate* with *flow speed*; volume flow rate tells us how *much* volume of fluid flows per unit time; flow speed tells us how *fast* it's moving. The volume flow rate is measured in cubic meters per second, while the flow speed is measured in meters per second.

If the pipe is carrying a fluid that is incompressible—that is, its density cannot be changed—then the volume flow rate must be the same everywhere along the pipe. A liquid is typically an incompressible fluid, whereas a gas is compressible. Rewriting this using $f = Av$, we get the **Continuity Equation:**

Equation Sheet

$$A_1 v_1 = A_2 v_2$$

Because the product Av is a constant, the flow speed will increase where the pipe narrows (decrease in area) and decrease where the pipe widens (increase in area). In fact, we can say that the flow speed is inversely proportional to the cross-sectional area—or to the square of the radius (in the case of a circular cross-section)—of the pipe.

Example 5 A circular pipe of non-uniform diameter carries water. At one point in the pipe, the radius is 2 cm and the flow speed is 6 m/s.

(a) What is the volume flow rate?

(b) What is the flow speed at a point where the pipe constricts to a radius of 1 cm?

Solution.

(a) At any point, the volume flow rate, f, is equal to the cross-sectional area of the pipe multiplied by the flow speed

$$f = Av = \pi r^2 v = \pi (2 \times 10^{-2} \text{ m})^2 (6 \text{ m/s}) \approx 75 \times 10^{-4} \text{ m}^3/\text{s} = 7.5 \times 10^{-3} \text{ m}^3/\text{s}$$

(b) By the Continuity Equation, we know that the volume flow rate must be the same at all points, so the volume flow rate must be the same as in part (a).

$$f = 7.510^{-3}\ \text{m}^3/\text{s} = Av = \pi r^2 = \pi\left(110^{-2}\,\text{m}\right)^2\left(v\right)$$

$$v = \frac{7.510^{-3}\,\text{m}^3/\text{s}}{\pi\left(110^{-2}\,\text{m}\right)^2} \approx 24\ \text{m/s}$$

A second approach to part (b) is to use proportional reasoning. From the Continuity Equation, we know that v, the flow speed, is inversely proportional to A, the cross-sectional area of the pipe. If the pipe's radius decreases by a factor of 2 (from 2 cm to 1 cm), then A decreases by a factor of 4, because A is proportional to r^2.

If A decreases by a factor of 4, then v will increase by a factor of 4. Therefore, the flow speed at a point where the pipe's radius is 1 cm will be 4 • (6 m/s) = 24 m/s.

BERNOULLI'S EQUATION

The most important equation in fluid mechanics is **Bernoulli's Equation**, which is the statement of conservation of energy for ideal fluid flow. First, let's describe the conditions that make fluid flow *ideal*.

- *The fluid is incompressible.*

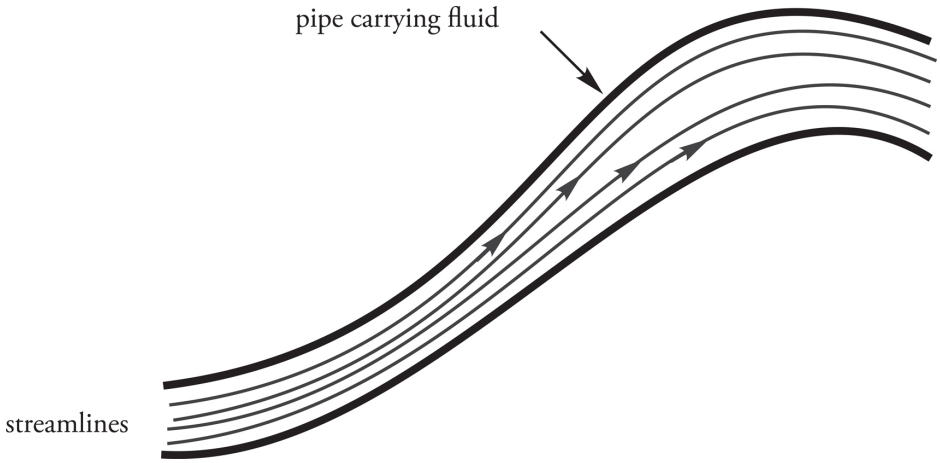
  This works very well for liquids and also applies to gases if the pressure changes are small.

- *The fluid's viscosity is negligible.*

  Viscosity is the force of cohesion between molecules in a fluid; think of viscosity as internal friction for fluids. For example, maple syrup is sticky and has a greater viscosity than water: there's more resistance to a flow of maple syrup than to a flow of water. While Bernoulli's Equation would give good results when applied to a flow of water, it would not give good results if it were applied to a flow of maple syrup.

- *The flow is streamline.*

  In a tube carrying a flowing fluid, a **streamline** is just what it sounds like: it's a *line* in the *stream*. If we were to inject a drop of dye into a clear glass pipe carrying, say, water, we'd see a streak of dye in the pipe, indicating a streamline.

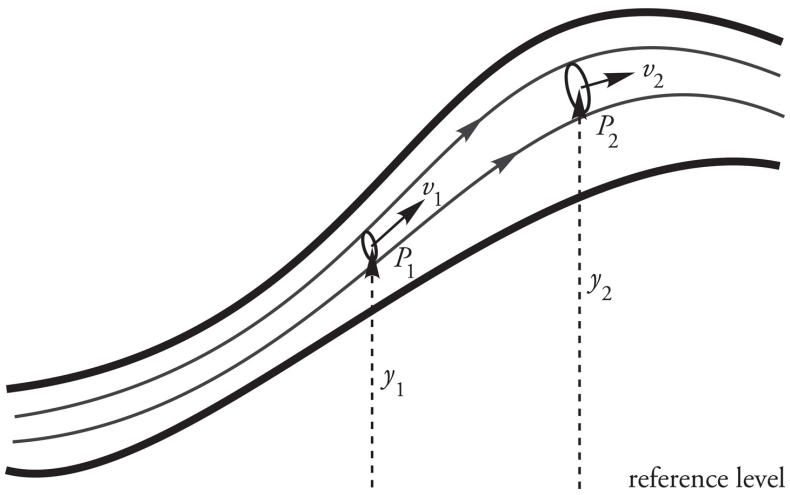


When the flow is streamline, the fluid moves smoothly through the tube. (The opposite of streamline flow is **turbulent** flow, which is characterized by rapidly swirling whirlpools; such chaotic flow is unpredictable.)

If the three conditions described above hold, and the flow rate, f, is steady (meaning it doesn't change with time), Bernoulli's Equation can be applied to any pair of points along a streamline within the flow. Let ρ be the density of the fluid that's flowing. Label the points we want to compare as Point 1 and Point 2. Choose a horizontal reference level, and let y_1 and y_2 be the heights of these points above this level. If the pressures at Points 1 and 2 are P_1 and P_2, and if the flow speeds at these points are v_1 and v_2, then Bernoulli's Equation says

Equation Sheet

$+ \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$



Conservation of Energy

This equations looks very similar to a previous equation dealing with Conservation of Energy with Total Mechanical Energy. In fact, Bernoulli's Equation deals with the Conservation of Energy for fluids.

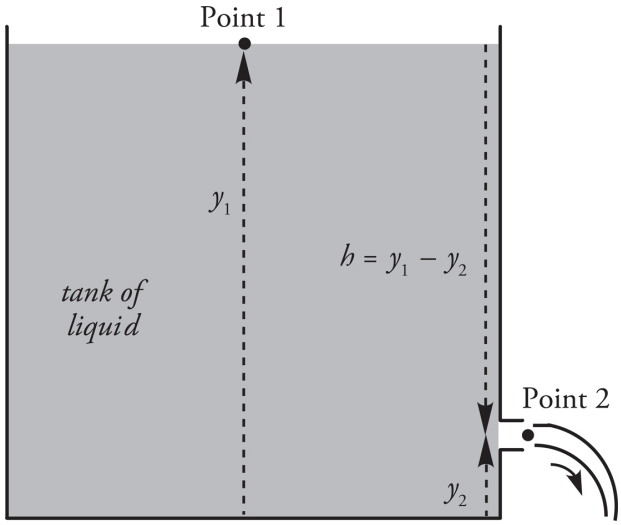
An alternative, but equivalent, way of stating Bernoulli's Equation is to say that the quantity

$$P + \rho g y + \tfrac{1}{2}\rho v^2$$

is constant along a streamline. We mentioned earlier that Bernoulli's Equation is a statement of conservation of energy. You should notice the similarity between $\rho g y$ and mgh (gravitational potential energy) as well as between $\frac{1}{2}\rho v^2$ and $\frac{1}{2}mv^2$ (kinetic energy).

Torricelli's Theorem

Imagine that we punch a small hole in the side of an open tank of liquid. We can use Bernoulli's Equation to figure out how fast the liquid will flow out of the hole.



Let the bottom of the tank be our horizontal reference level, and choose Point 1 to be at the surface of the liquid and Point 2 to be at the hole where the water shoots out. First, the pressure at Point 1 is atmospheric pressure since the tank is open; and the emerging stream at Point 2 is open to the air, so it's at atmospheric pressure too. Therefore, $P_1 = P_2$, and these terms cancel out of Bernoulli's Equation. Next, since the area at Point 1 is so much greater than at Point 2, we can assume that v_1, the speed at which the water level in the tank drops, is much lower than v_2, the speed at which the water shoots out of the hole. (Remember that by the Continuity Equation, $A_1v_1 = A_2v_2$; since $A_1 >> A_2$, we'll have $v_1 << v_2$.) Because $v_1 << v_2$, we can say that the water level at the top of the tank is approximately constant, and the speed of the water at the top is zero. So, Bernoulli's Equation becomes

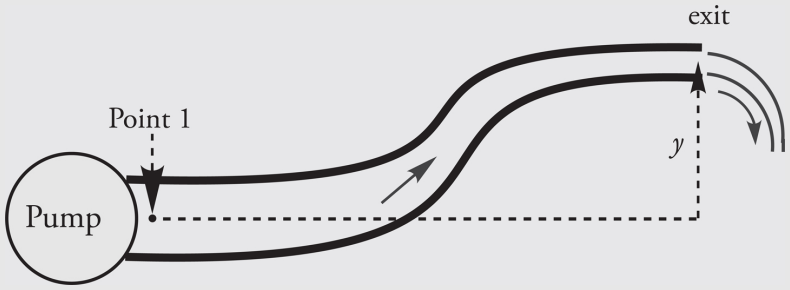
$$\rho g y_1 = \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

Solving for v_2, we get

$$v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2gh}$$

This is called Torricelli's Theorem.

Example 6 In the figure below, a pump forces water at a constant flow rate through a pipe whose cross-sectional area, A, gradually decreases: at the exit point, A has decreased to 1/3 its value at the beginning of the pipe. If y = 60 cm and the flow speed of the water just after it leaves the pump (Point 1 in the figure) is 1 m/s, what is the gauge pressure at Point 1?



Solution. We'll apply Bernoulli's Equation to Point 1 and the exit point, Point 2. We'll choose the level of Point 1 as the horizontal reference level; this makes $y_1 = 0$. Now, because the cross-sectional area of the pipe decreases by a factor of 3 between Points 1 and 2, the flow speed must increase by a factor of 3; that is, $v_2 = 3v_1$. Since the pressure at Point 2 is p_{atm}, Bernoulli's Equation becomes

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_{atm} + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

Now, P_1 is the total pressure at point 1. Recall that gauge pressure is $P_{tot} - P_{atm}$, so $P_{gauge} = P_1 - P_{atm}$.

Therefore,

$$\begin{aligned} P_1 - P_{\text{atm}} &= \rho g y_2 + \tfrac{1}{2}\rho v_2^2 - \tfrac{1}{2}\rho v_1^2 \\ &= \rho g y_2 + \tfrac{1}{2}\rho (3v_1)^2 - \tfrac{1}{2}\rho v_1^2 \\ &= \rho (g y_2 + 4 v_1^2) \\ &= (1000 \text{ kg/m}^3)[(10 \text{ m/s}^2)(0.6 \text{ m}) + 4(1 \text{ m/s})^2] \\ &= 10^4 \text{ Pa} \end{aligned}$$

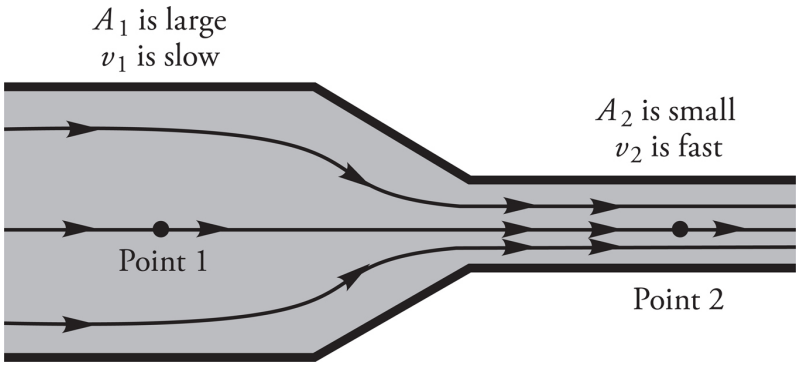
Example 7 The side of an aboveground pool is punctured, and water gushes out through the hole. If the total depth of the pool is 2.5 m, and the puncture is 1 m above ground level, what is the efflux speed of the water?

Solution. Torricelli's Theorem is $v = \sqrt{2gh}$, where h is the distance from the surface of the pool down to the hole. If the puncture is 1 m above ground level, then it's 2.5 − 1 = 1.5 m below the surface of the water (because the pool is 2.5 m deep). Therefore, the efflux speed will be

$$v = \sqrt{2gh} = \sqrt{2(10 \text{ m/s}^2)(1.5 \text{ m})} = \sqrt{30 \text{ m}^2/\text{s}^2} \approx 5.5 \text{ m/s}$$

The Bernoulli Effect

Consider the two points labeled in the pipe shown below:



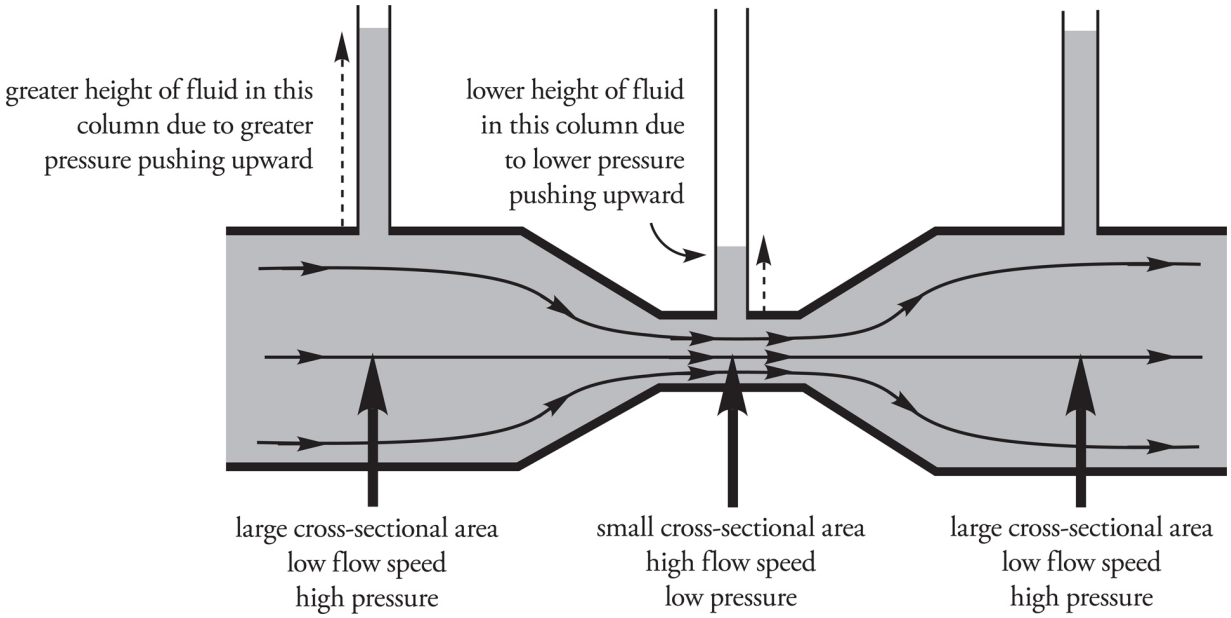
Since the heights y_1 and y_2 are equal in this case, the terms in Bernoulli's Equation that involve the heights will cancel, leaving us with

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

We already know from the Continuity Equation ($f = Av$) that the speed increases as the cross-sectional area of the pipe decreases; that is, since $A_2 < A_1$, we know that $v_2 > v_1$, so the equation above tells us that $P_2 < P_1$. This shows that

> At comparable heights, the pressure is lower where the flow speed is greater.

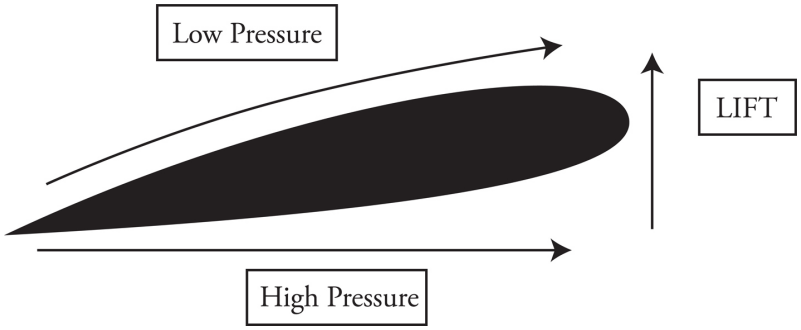
This is known as the **Bernoulli** (or **Venturi**) **Effect**, and is illustrated in the figure below.



The height of the liquid column above Point 2 is less than the height of the liquid column above Point 1, because the pressure at Point 2 is

lower than at Point 1, due to the fact that the flow speed at Point 2 is greater than at Point 1.

The Bernoulli Effect also accounts for many everyday phenomena. It's what allows airplanes to fly, curve balls to curve, and tennis balls hit with top spin to drop quickly. You may have seen sky divers or motorcycle riders wearing a jacket that seems to puff out as they move rapidly through the air. The essentially stagnant air trapped inside the jacket is at a much higher pressure than the air whizzing by outside, and as a result, the jacket expands outward. The drastic drop in air pressure that accompanies the high winds in a hurricane or tornado is yet another example. In fact, if high winds streak across the roof of a house whose windows are closed, the outside air pressure can be reduced so much that the air pressure inside the house (where the air speed is essentially zero) can be great enough to blow the roof off.



Air flow: The upper side of the wing is longer so air takes more time to reach the edge. If it takes the same time (but longer distance) [$\uparrow d = \uparrow v\ t$], then the top air is moving faster than bottom. The air on bottom has greater pressure and pushes up on the wing, giving airplanes lift force.

Chapter 3 Review Questions

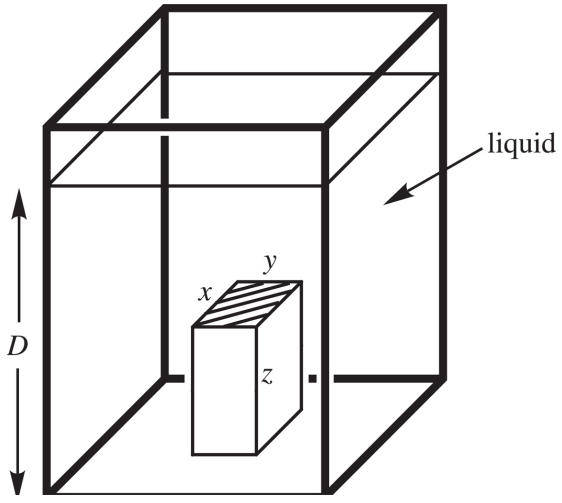
Click here to download the PDF of Chapter 3 Review Questions.

Solutions can be found in Chapter 12.

Section I: Multiple Choice

1. A large tank is filled with water to a depth of 6 m. If Point X is 1 m from the bottom and Point Y is 2 m from the bottom, how does p_X, the hydrostatic pressure due to the water at Point X, compare to p_Y, the hydrostatic pressure due to the water at Point Y?

   (A) $p_X = 2p_Y$
   (B) $2p_X = p_Y$
   (C) $5p_X = 4p_Y$
   (D) $4p_X = 5p_Y$



2. In the figure above, a box of dimensions x, y, and z rests on the bottom of a tank filled to depth D with a liquid of density ρ. If the tank is open to the atmosphere, what is the force on the (shaded) top of the box?

(A) $xy(p_{\text{atm}} + \rho g D)$

(B) $xyz[p_{\text{atm}} - \rho g(z - D)]$

(C) $xy[p_{\text{atm}} + \rho g(D - z)]$

(D) $xyz[p_{\text{atm}} + \rho g D]$

Questions 3 and 4 refer to the following material.

An experiment is performed where a cube is suspended from a spring scale. The cube is lowered into a beaker of water.

3. If the independent variable is the distance the cube is lowered and the dependent variable is the reading on the scale, what will a graph of the data show?

(A) The graph will show a positive correlation and be linear.
(B) The graph will show a negative correlation and be linear.
(C) The graph will show a positive correlation, but will be non-linear.
(D) The graph will show a negative correlation, but will be non-linear.

4. If the cube is replaced with a sphere and the same experiment is performed, which correctly describes the graph of the variables for the new experiment?

(A) The graph will be the same for both experiments because both experiments have the same independent and dependent variables.
(B) The graph will be different for both experiments because both experiments have the same independent and dependent variables.

(C) The graph will be the different for both experiments because they have different independent variables but the same dependent variable.

(D) The graph will be different for both experiments because they have different dependent variables but the same independent variable.

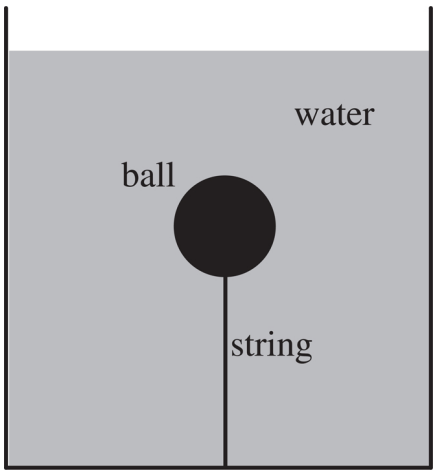
5. A block of Styrofoam, with a density of ρ_S and volume V, is pushed completely beneath the surface of a liquid whose density is ρ_L, and released from rest. Given that $\rho_L > \rho_S$, which of the following expressions gives the magnitude of the block's initial upward acceleration?

(A) $(\rho_L - \rho_S)g$

(B) $\left(\frac{\rho_L}{\rho_S} - 1\right)g$

(C) $\left(\frac{\rho_L}{\rho_S} + 1\right)g$

(D) $\left[\left(\frac{\rho_L}{\rho_S}\right)^2 - 1\right]g$



6. A ball is tied to a string and placed in a container of water as shown. The water is slowly drained from the container until the water level is below the position of the ball shown, but the draining stops while there is still water in the container. The tension in the string is measured while this occurs. Which describes why the tension measurements decrease?

(A) The tension decreases because there is less pressure on the ball when the water level drops because of the change in the water depth.

(B) The tension decreases because there is less pressure on the ball when the water level drops because the water is moving at a faster speed.

(C) The tension decreases because the force of gravity on the ball decreases.

(D) The tension decreases because the buoyant force on the ball decreases.

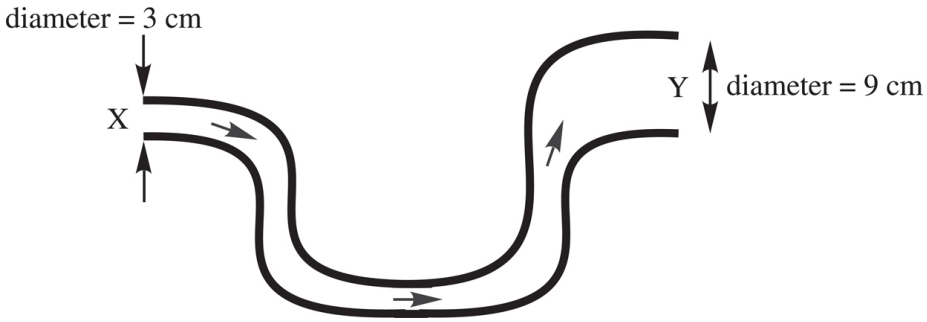
7. An object with a density of 2000 kg/m^3 weighs 100 N less when it's weighed while completely submerged in water than when it's weighed in air. What is the actual weight of this object?

(A) 200 N

(B) 300 N

(C) 400 N

(D) 600 N



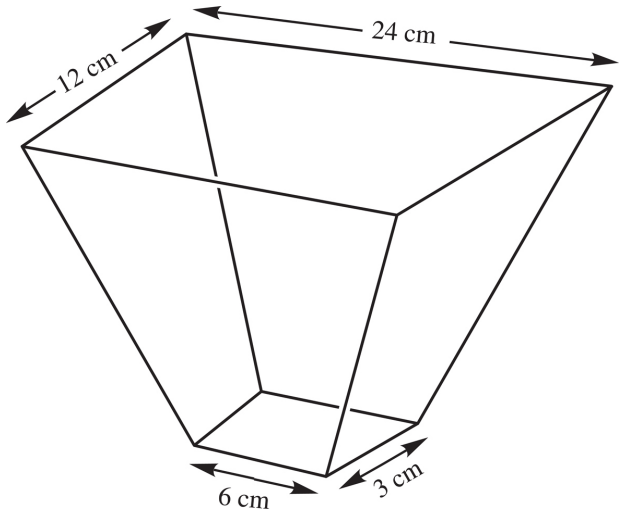
8. In the pipe shown above, which carries water, the flow speed at Point X is 6 m/s. What is the flow speed at Point Y?

(A) $\frac{2}{3}$ m/s

(B) 2 m/s

(C) 18 m/s

(D) 54 m/s



9. The figure above shows a portion of a conduit for water, one with rectangular cross sections. If the flow speed at the top is v, what is the flow speed at the bottom?

(A) $4v$
(B) $8v$
(C) $12v$
(D) $16v$

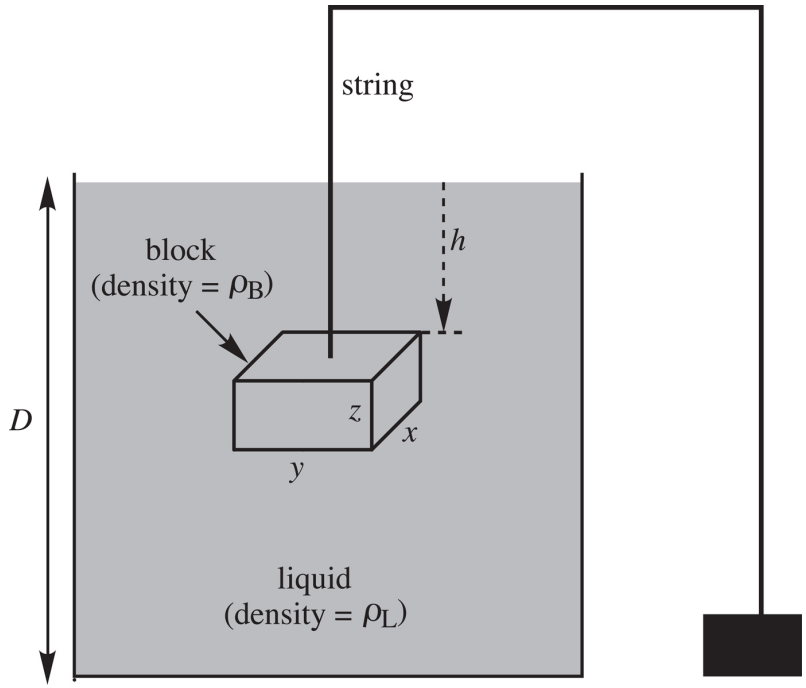
10. A pump is used to send water through a hose, the diameter of which is 10 times that of the nozzle through which the water exits. If the nozzle is 1 m higher than the pump, and the water

flows through the hose at 0.4 m/s, what is the difference in pressure between the pump and the atmosphere?

(A) 108 kPa
(B) 260 kPa
(C) 400 kPa
(D) 810 kPa

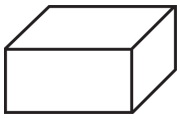
Section II: Free Response

1. The figure below shows a tank open to the atmosphere and filled to depth D with a liquid of density ρ_L. Suspended from a string is a block of density ρ_B (which is greater than ρ_L), whose dimensions are x, y, and z (meters). The top of the block is at depth h meters below the surface of the liquid.



In each of the following, write your answer in simplest form in terms of ρ_L, ρ_B, x, y, z, h, D, and g.

(a) Find the force due to the pressure on the top surface of the block and on the bottom surface. Sketch these forces in the diagram below:



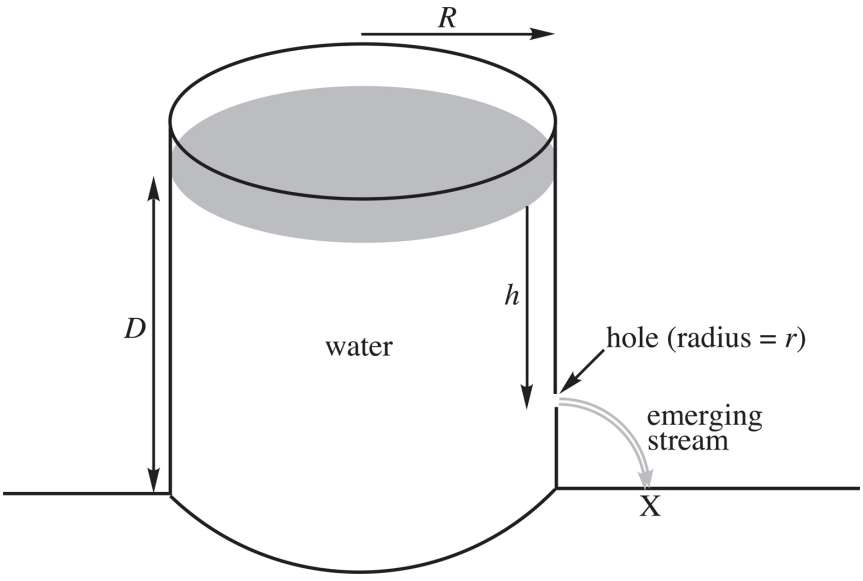
(b) What are the average forces due to the pressure on the other four sides of the block? Sketch these forces in the diagram above.

(c) What is the total force on the block due to the pressure?

(d) Find an expression for the buoyant force on the block. How does your answer here compare to your answer to part (c)?

(e) What is the tension in the string?

2. The figure below shows a large, cylindrical tank of water, open to the atmosphere, filled with water to depth D. The radius of the tank is R. At a depth h below the surface, a small circular hole of radius r is punctured in the side of the tank, and the point where the emerging stream strikes the level ground is labeled X.



In parts (a) through (c), assume that the speed with which the water level in the tank drops is negligible.

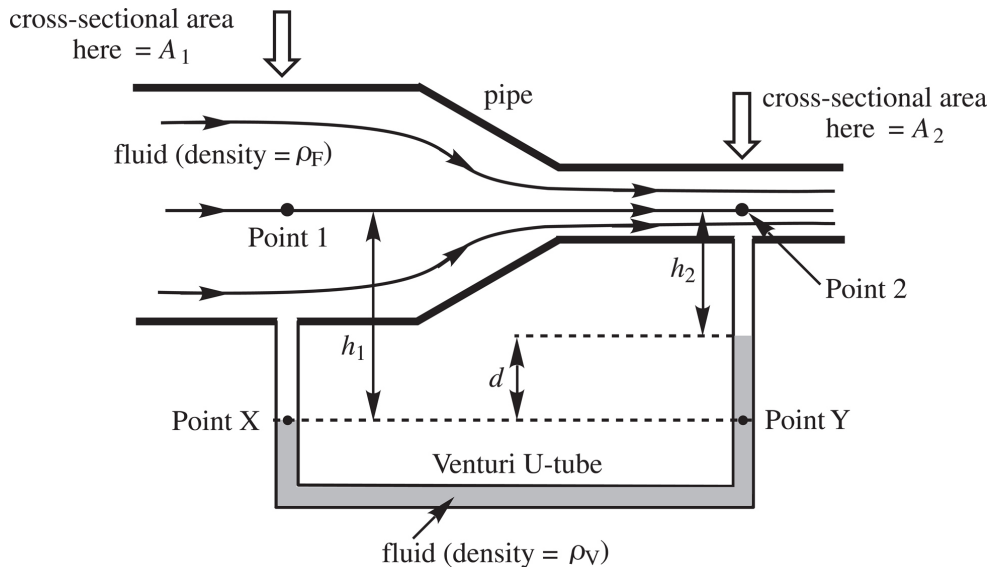
(a) At what speed does the water emerge from the hole?

(b) How far is point X from the edge of the tank?

(c) Assume that a second small hole is punctured in the side of the tank, a distance of $h/2$ directly above the hole shown in the figure. If the stream of water emerging from this second hole also lands at Point X, find h in terms of D.

(d) For this part, do *not* assume that the speed with which the water level in the tank drops is negligible, and derive an expression for the speed of efflux from the hole punctured at depth h below the surface of the water. Write your answer in terms of r, R, h, and g.

3. The figure below shows a pipe fitted with a Venturi U-tube. Fluid of density ρ_F flows at a constant flow rate and with negligible viscosity through the pipe, which constricts from a cross-sectional area A_1 at Point 1 to a smaller cross-sectional area A_2 at Point 2. The upper portion of both sides of the Venturi U-tube contain the same fluid that's flowing through the pipe, while the lower portion is filled with a fluid of density ρ_V (which is greater than ρ_F). At Point 1 in the pipe, the pressure is P_1 and the flow speed is v_1; at Point 2 in the pipe, the pressure is P_2 and the flow speed is v_2. All the fluid within the Venturi U-tube is stationary.



(a) What is P_X, the hydrostatic pressure at Point X? Write your answer in terms of P_1, ρ_F, h_1, and g.

(b) What is P_Y, the hydrostatic pressure at Point Y? Write your answer in terms of P_2, ρ_F, ρ_V, h_2, d, and g.

(c) Write down the result of Bernoulli's Equation applied to Points 1 and 2 in the pipe, and solve for $P_1 - P_2$.

(d) Since $P_X = P_Y$, set the expressions you derived in parts (a) and (b) equal to each other, and use this equation to find $P_1 - P_2$.

(e) Derive an expression for the flow speed, v_2, and the flow rate, f, in terms of A_1, A_2, d, ρ_F, ρ_V, and g. Show that v_2 and f are proportional to $\sqrt{d}$.

Summary

- Density is given by $\rho = \frac{m}{V}$. Pressure is given by $P = \frac{F}{A}$. Hydrostatic pressure can be found using $P = P_0 + \rho gh$, where ρgh is the pressure at a given depth below the surface of the fluid and P_0 is the pressure right above the surface of the fluid.

- The buoyant force is an upward force any object immersed in a fluid experiences due to the displaced fluid. The buoyant force is given by $F_{buoy} = \rho Vg$, where ρ is the density of the fluid and V is the volume of the fluid displaced.

- The Continuity Equation says that the flow rate through a pipe (area times velocity) is constant so that $A_1 v_1 = A_2 v_2$. This expresses the idea that a larger cross-sectional area of pipe will experience fluids traveling at a lower velocity.

- Bernoulli's Equation is a statement of conservation of energy. It states

  $$\mathbf{P} + \rho \mathbf{gy} + \frac{1}{2}\rho \mathbf{v}^2 = \mathbf{constant}$$

Chapter 4
Thermal Physics

INTRODUCTION

This chapter looks at heat and temperature, concepts that seem familiar from our everyday experience. Technically, **heat** is defined as thermal energy transmitted from one body to another. While an object can contain **thermal energy** (due to the random motion of its molecules), an object doesn't *contain* heat; heat is energy *in transit*. **Temperature**, on the other hand, is a measure of an object's internal thermal energy.

HEAT TRANSFER

There are three principal modes by which energy can be transferred.

Conduction

An iron skillet is sitting on a hot stove, and you accidentally touch the handle. You notice right away that there's been a transfer of thermal energy to your hand. The process by which this happens is known as conduction. The highly agitated atoms in the handle of the hot skillet bump into the atoms of your hand, making them vibrate more rapidly, thus heating up your hand.

Heat conducts from one point to another only if there is a temperature difference between the two objects. The rate at which heat is transferred is given by

Equation Sheet

$$\frac{Q}{\Delta t} = \frac{kA\Delta T}{L}$$

Convection

As the air around a candle flame warms, it expands, becomes less dense than the surrounding cooler air, and thus rises due to buoyancy. As a result, heat is transferred away from the flame by the large scale (from the atoms' point of view anyway) motion of a fluid (in the case, air). This is convection.

Radiation

Sunlight on your face warms your skin. Radiant energy from the Sun's fusion reactions is transferred across millions of kilometers of

essentially empty space via electromagnetic waves. Absorption of the energy carried by these light waves defines heat transfer by radiation.

THE KINETIC THEORY OF GASES

Unlike the condensed phases of matter—solid and liquid—the atoms or molecules that make up a gas do not move around relatively fixed positions. Rather, the molecules of a gas move freely and rapidly, in a chaotic swarm. A confined gas exerts a force on the walls of its container, because the molecules are zipping around inside the container, striking the walls and rebounding. The magnitude of the force per unit area is just the pressure, *P*, as defined in the previous chapter.

As we'll see, the faster the gas molecules are moving, the more pressure they exert.

THE IDEAL GAS LAW

Three physical properties—pressure (P), volume (V), and temperature (T)—describe a gas. At low densities, all gases approach *ideal* behavior; this means that these three variables are related by the equation

Equation Sheet

$$PV = nRT$$

where n is the number of moles of gas and R is a constant (8.31 J/mol·K) called the **universal gas constant**. This equation is known as the **Ideal Gas Law**. It can also be written as $PV = Nk_BT$, where k_B is Boltzmann's constant ($k_B = 1.38 \times 10^{-23}$ J/K). The Ideal Gas Law tells us how these variables are related to one another for a gas with no intermolecular forces. This is a bad assumption for a charged gas, where the individual charges would interact electrically (a strong interaction), but it is a good assumption for a neutral gas, where the atoms would only interact gravitationally (a very weak interaction).

An important consequence of this equation is that, for a fixed volume of gas, an increase in P gives a proportional increase in T. The pressure increases when the gas molecules strike the walls of their container with more force, which occurs if they move more rapidly. Using Newton's Second Law (*rate of change of momentum = force*), we can find that the pressure exerted by N molecules of gas in a

container of volume V is related to the average kinetic energy of the molecules by the equation $PV = \frac{2}{3}NK_{avg}$. Comparing this to the Ideal Gas Law, we see that $\frac{2}{3}NK_{avg} = nRT$. We can rewrite this equation in the form $\frac{2}{3}N_A K_{avg} = RT$, since, by definition, $N = nN_A$. The ratio R/N_A is a fundamental constant of nature called **Boltzmann's constant** ($k_B = 1.38 \times 10^{-23}$ J/K), so our equation becomes

Equation Sheet

$$K_{avg} = \frac{3}{2}k_B T$$

This tells us that the average translational kinetic energy of the gas molecules is directly proportional to the absolute temperature of the sample. Remember, this means you must use kelvins as your temperature unit.

Since the average kinetic energy of the gas molecules is $K_{avg} = \frac{1}{2}m(v^2)_{avg}$, the equation above becomes $\frac{1}{2}m(v^2)_{avg} = \frac{3}{2}k_B T$, so

$$\sqrt{(v^2)_{avg}} = \sqrt{\frac{3k_B T}{m}}$$

The quantity on the left-hand side of this equation, the square root of the average of the square of v, is called the **root-mean-square velocity**, v_{rms}, so

$$v_{\text{rms}} = \sqrt{\frac{3k_{\text{B}}T}{m}}$$

It's important to realize that the molecules in the container have a wide range of speeds; some are much slower and others are much faster than v_{rms}. The root-mean-square speed is important because it gives us a type of average speed that's easy to calculate from the temperature of the gas.

Example 1 In order for the rms velocity of the molecules in a given sample of gas to double, what must happen to the temperature?

Solution. Temperature is a measure of the average kinetic energy. The velocity is determined from the following equation:

$$v_{rms} = \sqrt{\frac{3k_b T}{m}}$$

Since v_{rms} is proportional to the square root of T, the temperature must quadruple, again, assuming the temperature is given in kelvins.

Example 2 A cylindrical container of radius 15 cm and height 30 cm contains 0.6 mole of gas at 433 K. How much force does the confined gas exert on the lid of the container?

Solution. The volume of the cylinder is $\pi r^2 h$, where r is the radius and h is the height. Since we know V and T, we can use the Ideal Gas

Law to find *P*. Because pressure is force per unit area, we can find the force on the lid by multiplying the gas pressure times the area of the lid.

$$P = \frac{nRT}{V} = \frac{(0.6\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})(433\ \text{K})}{\pi(0.15\ \text{m})^2(0.30\ \text{m})} = 1.018\times10^5\ \text{Pa}$$

So, since the area of the lid is πr^2, the force exerted by the confined gas on the lid is

$$F = PA = (1.018\times10^5\ \text{Pa})\cdot\pi(0.15\ \text{m})^2 = 7{,}200\ \text{N}$$

This is about 1,600 pounds of force, which seems like a lot. Why doesn't this pressure pop the lid off? Because, while the bottom of the lid is feeling a pressure (due to the confined gas) of 1.018×10^5 Pa that exerts a force upward, the top of the lid feels a pressure of 1.013×10^5 Pa (due to the atmosphere) that exerts a force downward. The net force on the lid is

$$F_{\text{net}} = (\Delta P)A = (0.005\times10^5\ \text{Pa})\cdot\pi(0.15\ \text{m})^2 = 35\ \text{N}$$

which is only about 8 pounds.

THE LAWS OF THERMODYNAMICS

We've learned about two ways in which energy may be transferred between a system and its environment. One is work, which takes place when a force acts over a distance. The other is heat, which takes place when energy is transferred due to a difference in temperature. The study of the energy transfers involving work and heat, and the resulting changes in internal energy, temperature, volume, and pressure is called **thermodynamics**.

The Zeroth Law of Thermodynamics

When two objects are brought into contact, heat will flow from the warmer object to the cooler one until they reach thermal equilibrium. This property of temperature is expressed by the Zeroth Law of Thermodynamics:

> If Objects 1 and 2 are each in thermal equilibrium with Object 3, then Objects 1 and 2 are in thermal equilibrium with each other.

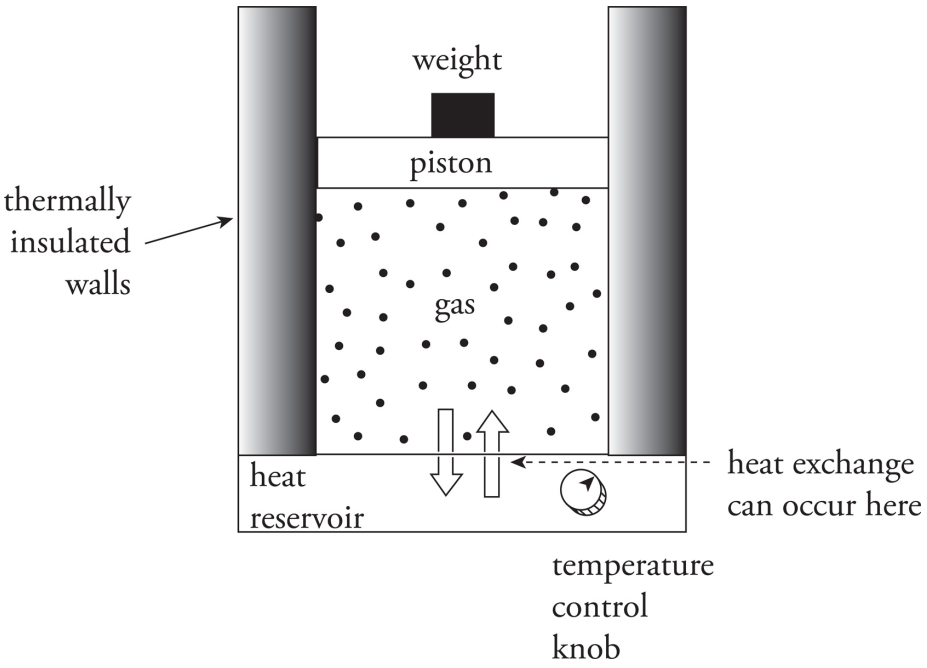
The First Law of Thermodynamics

Simply put, the First Law of Thermodynamics is a statement of the Conservation of Energy that includes heat.

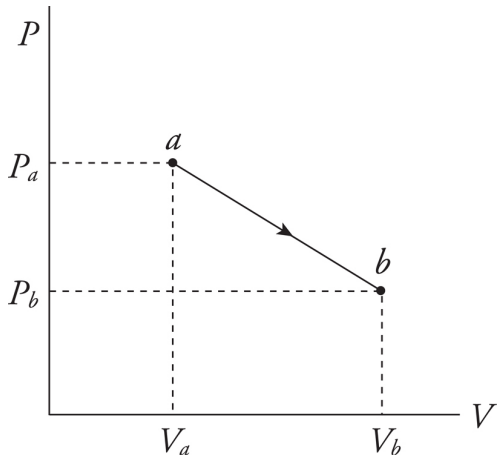
> Energy (in the form of heat) is neither created nor destroyed in any thermodynamic system.

Consider the following example, which is the prototype that's studied extensively in thermodynamics.

An insulated container filled with an ideal gas rests on a heat reservoir (that is, something that can act as a heat source or a heat sink). The container is fitted with a snug, but frictionless, weighted piston that can be raised or lowered. The confined gas is the *system*, and the piston and heat reservoir are the *surroundings*.



The **state** of the gas is given once its pressure, volume, and temperature are known, and the equation that connects these state variables is the Ideal Gas Law, $PV = nRT$. We'll imagine performing different experiments with the gas, such as heating it or allowing it to cool or increasing or decreasing the weight on the piston, and study the energy transfers (work and heat) and the changes in the state variables. If each process is carried out such that, at each moment, the system and its surroundings are in thermal equilibrium, we can plot the pressure (P) versus the volume (V) on a diagram. By following the path of this ***P–V* diagram**, we can study how the system is affected as it moves from one state to another.



Work is done on or by the system when the piston is moved and the volume of the gas changes. For example, imagine that the weight pushes the piston downward a distance d, causing a decrease in volume. Assume that the pressure stays constant at P. (Heat must be removed via the reservoir to accomplish this.) We can calculate the work done on the gas during this compression as $W = -Fd$, but since $F = PA$, we have $W = -PAd$, and because $Ad = \Delta V$, we have

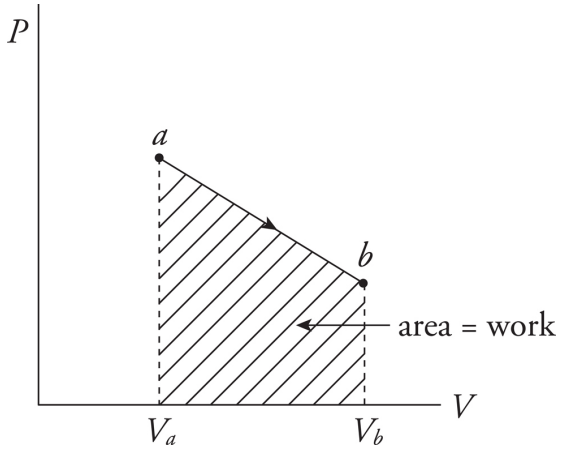
Equation Sheet

$$W = -P\Delta V$$

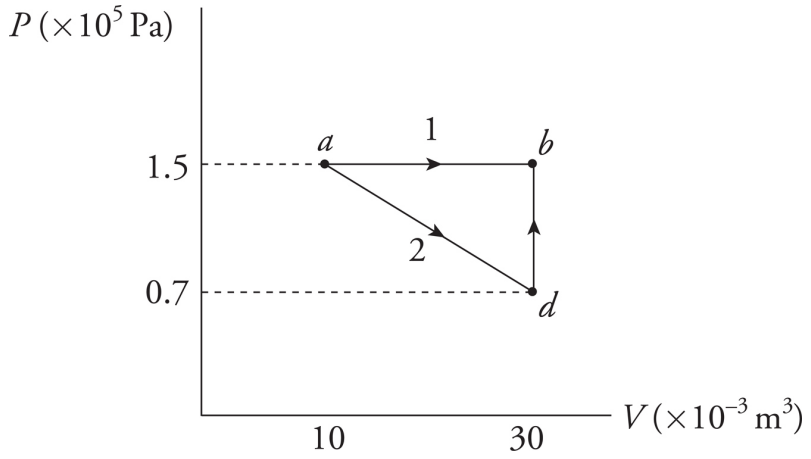
Textbooks differ about the circumstances under which work in thermodynamics is defined to be positive or negative. The negative signs we have included in the equations above are consistent with those used in the AP Physics 2 Exam (appearing in the equation sheet given for Section II). For the exam, work in thermodynamics is considered to be positive when the work is being done *on the system*. This means that the volume of the system is *decreasing*, ΔV is *negative*, and, in agreement with intuition, energy is being *added* to the system. In other words, when work is done in compressing a system, ΔV is *negative* and the work done, $W = -P\Delta V$, is *positive*. This also means, conversely, that when the system is doing work *on*

the surroundings (volume *increasing*, ΔV *positive*), the work is *negative*, in agreement with intuition that energy is leaving the system.

The equation $W = -P\Delta V$ assumes that the pressure P does not change during the process. If P *does* change, then the work is equal to the area under the curve in the P–V diagram; moving left to right as the volume increases gives a negative area (and negative work), while moving right to left as the volume decreases gives a positive area (and positive work). Because of this difference when moving left to right or right to left, processes in P–V diagrams are drawn with arrows to indicate which direction the process follows. In the diagram below, the process is taken from an initial state a, with a particular P and V value to a final state b, where P and V have changed.



Example 3 What's the value of W for the process ab following path 1 and for the same process following path 2 (from a to d to b), shown in the P–V diagram below?



Solution.

Path 1. Since, in path 1, P remains constant, the work done is just $-P\Delta V$:

$$W = -P\Delta V = -(1.5 \times 10^5 \text{ Pa})[(30 \times 10^{-3} \text{ m}^3) - (10 \times 10^{-3} \text{ m}^3)] = -3{,}000 \text{ J}$$

Path 2. If the gas is brought from state a to state b, along path 2, then work is done only along the part from a to d. From d to b, the volume of the gas does not change, so no work can be performed. The area under the graph from a to d is

$$\begin{aligned} W = -\frac{1}{2}h(b_1 + b_2) &= -\frac{1}{2}(\Delta V)(P_a + P_d) \\ &= -\frac{1}{2}(20 \times 10^{-3} \text{ m}^3)[(1.5 \times 10^5 \text{ Pa}) + (0.7 \times 10^5 \text{ Pa})] \\ &= -2{,}200 \text{ J} \end{aligned}$$

As this example shows, the amount of work done by a gas depends not only on the initial and final states of the system, but also on the path between the two. In general, different paths give different values for W.

Experiments have shown that the value of $Q + W$ is *not* path dependent; it depends only on the initial and the final state of the

system, so it describes a change in a fundamental property. This property is called the system's **internal energy**, denoted U, and the change in the system's internal energy, ΔU, is equal to $Q + W$. This is true regardless of the process that brought the system from its initial to final state. This statement is known as

Equation Sheet

The First Law of Thermodynamics

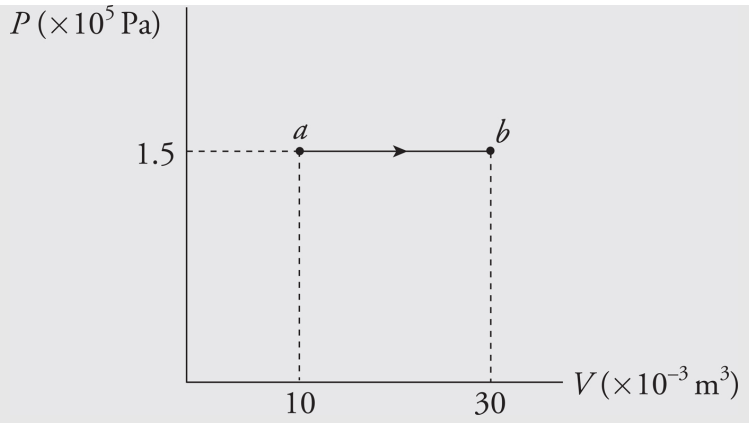
$$\Delta U = Q + W$$

ΔU depends only on temperature change

(If volume is constant, $W = 0$ and $U = Q$.)

This statement of the First Law is consistent with the interpretation of work ($W = -P\Delta V$) explained above. The First Law identifies W and Q as separate physical mechanisms for adding to or removing energy from the system, and the signs of both W and Q are defined consistently: both are positive when they are adding energy to the system and negative when they are removing energy from the system.

Example 4 A 0.5 mol sample of an ideal gas is brought from state *a* to state *b* when 7500 J of heat is added along the path shown in the following P–V diagram:



What are the values of each of the following?

(a) the temperature at *a*

(b) the temperature at *b*

(c) the work done by the gas during process *ab*

(d) the change in the internal energy of the gas

Solution.

(a, b) Both of these questions can be answered using the Ideal Gas Law, $T = PV/(nR)$:

$$T_a = \frac{P_a V_a}{nR} = \frac{(1.5\times10^5\ \text{Pa})(10\times10^{-3}\ \text{m}^3)}{(0.5\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})} = 360\ \text{K}$$

$$T_b = \frac{P_b V_b}{nR} = \frac{(1.5\times10^5\ \text{Pa})(30\times10^{-3}\ \text{m}^3)}{(0.5\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})} = 1{,}080\ \text{K}$$

(c) Since the pressure remains constant during the process, we can use the equation $W = -P\Delta V$. Because $\Delta V = (30 - 10) \times 10^{-3}\ \text{m}^3 = 20 \times 10^{-3}\ \text{m}^3$, we find that

$$W = -P\Delta V = (1.5 \times 10^5\ \text{Pa})(20 \times 10^{-3}\ \text{m}^3) = -3{,}000\ \text{J}$$

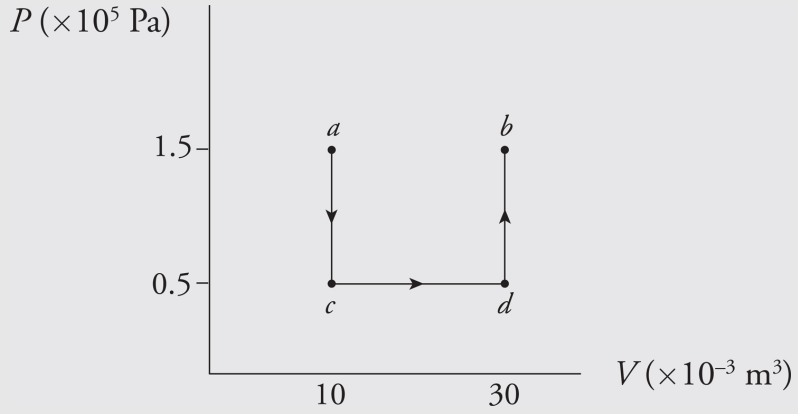
The expanding gas did negative work against its surroundings, pushing the piston upward. Important note: If

the pressure remains constant (which is designated by a horizontal line in the P–V diagram), the process is called **isobaric**.

(d) By the First Law of Thermodynamics,

$$\Delta U = Q + W = 7{,}500 - 3{,}000 \text{ J} = 4{,}500 \text{ J}$$

Example 5 A 0.5 mol sample of an ideal monatomic gas is brought from state a to state b along the path shown in the following P–V diagram:



What are the values of each of the following?

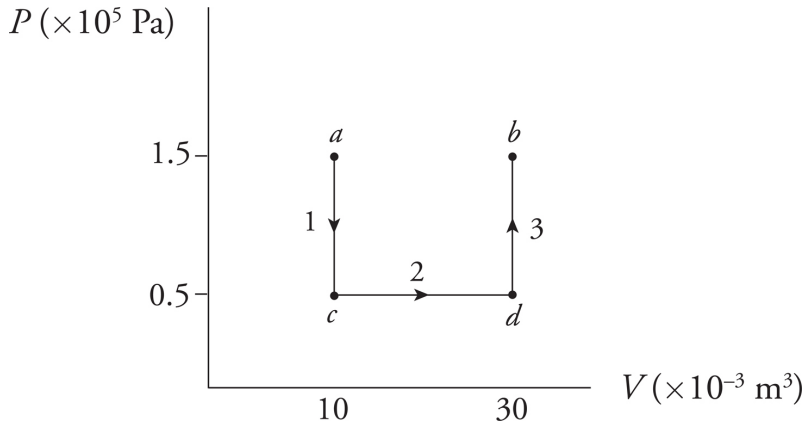
(a) the work done by the gas during process ab

(b) the change in the internal energy of the gas

(c) the heat added to the gas during process ab

Solution. Note that the initial and final states of the gas are the same as in the preceding example, but the path is different.

(a) Let's break the path into 3 pieces:



Over paths 1 and 3, the volume does not change, so no work is done. Work is done only over path 2:

$$W = -P\Delta V = -(0.5 \times 10^5 \text{ Pa})(20 \times 10^{-3} \text{ m}^3) = -1{,}000 \text{ J}$$

Once again, the expanding gas does negative work against its surroundings, pushing the piston upward.

(b) Because the initial and final states of the gas are the same here as they were in the preceding example, the change in internal energy, ΔU, *must* be the same. Therefore, $\Delta U = 4{,}500$ J.

(c) By the First Law of Thermodynamics, $\Delta U = Q + W$, so

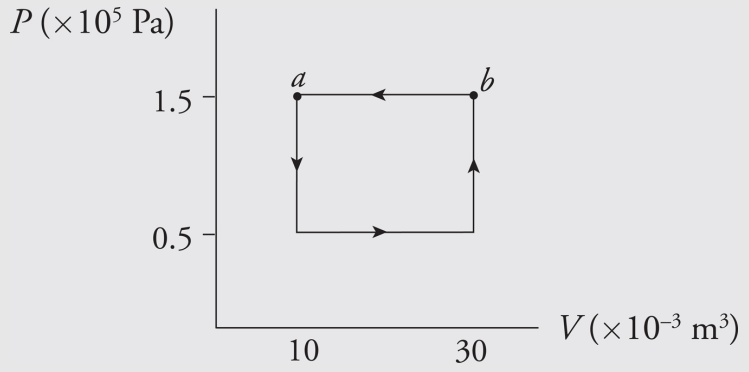
$$Q = \Delta U - W = 4{,}500 \text{ J} - (-1{,}000 \text{ J}) = 5{,}500 \text{ J}$$

Example 6 An **isochoric** process is one that takes place with no change in volume. What can you say about the change in the internal energy of a gas if it undergoes an isochoric change of state?

Solution. An isochoric process is illustrated by a vertical line in a P–V diagram and, since no change in volume occurs, $W = 0$. By the First Law of Thermodynamics, $\Delta U = Q + W = Q$. Therefore, the

change in internal energy is entirely due to (and equal to) the heat transferred. If heat is transferred into the system (positive Q), then ΔU is positive; if heat is transferred out of the system (negative Q), then ΔU is negative.

Example 7 A 0.5 mol sample of an ideal gas is brought from state a back to state a along the path shown in the following P–V diagram:



What are the values of each of the following?

(a) the change in the internal energy of the gas

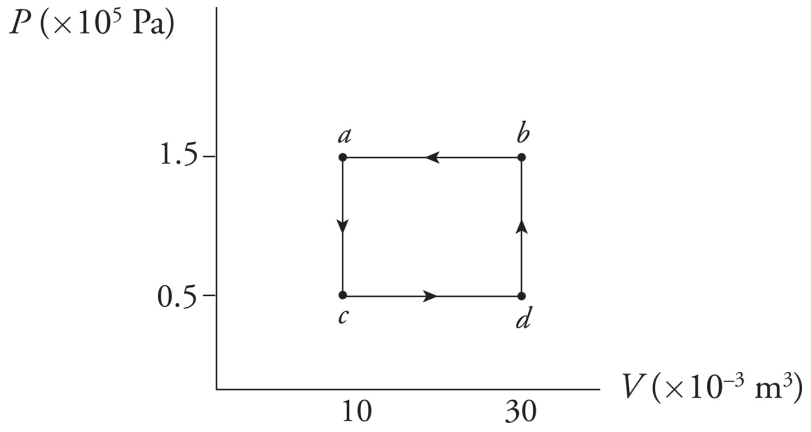
(b) the work done on the gas during the process

(c) the heat added to the gas during the process

Solution. A process such as this, which begins and ends at the same state, is said to be cyclical.

(a) Because the final state is the same as the initial state, the internal energy of the system cannot have changed, so $\Delta U = 0$.

(b) The total work involved in the process is equal to the work done from c to d plus the work done from b to a,



because only along these paths does the volume change. Along these portions, we find that

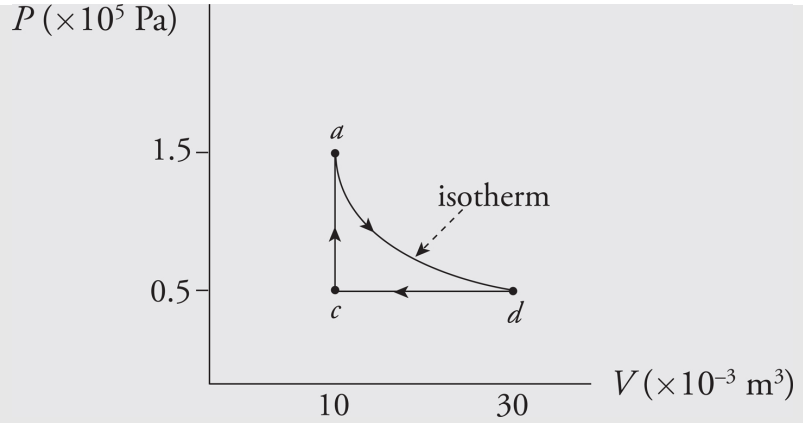
$$W_{cd} = -P\Delta V_{cd} = (0.5 \times 10^5 \text{ Pa})(+20 \times 10^{-3} \text{ m}^3) = -1{,}000 \text{ J}$$

$$W_{cd} = -P\Delta V_{ba} = (1.5 \times 10^5 \text{ Pa})(-20 \times 10^{-3} \text{ m}^3) = +3{,}000 \text{ J}$$

So the total work done is $W = +2{,}000$ J. The fact that W is positive means that, overall, work was done *on* the gas by the surroundings. Notice that for a cyclical process, the total work done is equal to the area enclosed by the loop, with clockwise travel taken as negative and counterclockwise travel taken as positive.

(c) The First Law of Thermodynamics states that $\Delta U = Q + W$. Since $\Delta U = 0$, it must be true that $Q = -W$ (which will always be the case for a cyclical process), so $Q = -2{,}000$ J.

Example 8 A 0.5 mol sample of an ideal gas is brought from state *a* to state *d* along an **isotherm**, then isobarically to state *c* and isochorically back to state *a*, as shown in the following *P*–*V* diagram:



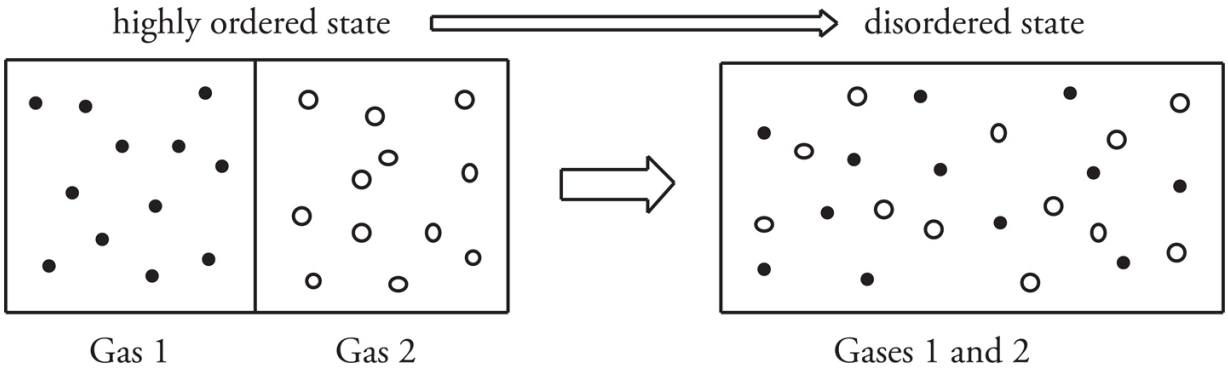
A process that takes place with no variation in temperature is said to be isothermal. Given that the work done during the **isothermal** part of the cycle is −1,650 J, how much heat is transferred during the isothermal process, from *a* to *d*?

Solution. Be careful that you don't confuse *isothermal* with *adiabatic*. A process is isothermal if the *temperature* remains constant; a process is **adiabatic** if $Q = 0$. You might ask, *How could a process be isothermal without also being adiabatic at the same time?* Remember that the temperature is determined by the internal energy of the gas, which is affected by changes in Q, W, or both. Therefore, it's possible for U to remain unchanged even if Q is not 0 (because there can be an equal but opposite W to cancel it out). In fact, this is the key to this problem. Since T doesn't change from *a* to *d*, neither can the internal energy, which depends entirely on T. Because $\Delta U_{ad} = 0$, it must be true that $Q_{ad} = -W_{ad}$. Since W_{ad} equals −1,650 J, Q_{ad} must be +1,650 J. The gas absorbs heat from the reservoir and uses all this energy to do negative work as it expands, pushing the piston upward.

The Second Law of Thermodynamics

Entropy

Consider a box containing two pure gases separated by a partition. What would happen if the partition were removed? The gases would mix, and the positions of the gas molecules would be random.



A closed system that shows a high degree of order tends to evolve in such a way that its degree of order decreases. In other words, disorder (or, as it's technically called, entropy) increases. The term "disorder" often makes the concept of entropy sound negative. Rather, entropy is better described as **increasing molecular freedom**. This is the reason why broken glass does not put itself back together and the reason why chemical reactions take place. If we started with the box on the right, containing the mixture of the gases, it would be virtually impossible that at any later time all the molecules of Gas 1 would happen to move to the left side of the box and, at the same time, the molecules of Gas 2 would spontaneously move to the right side of the box. If we were to watch a movie of this process, and saw the mixed-up molecules suddenly separate and move to opposite sides of the box, we'd assume that the film was running backward. In a way, the Second Law of Thermodynamics defines the direction of time. Time flows in such a way that ordered systems become disordered. Disordered states do not spontaneously become ordered without any other changes taking place. The following is the essence of one form of the Second Law of Thermodynamics:

The total amount of disorder—the total entropy—of a system plus its surroundings will never decrease.

It is possible for the entropy of a system to decrease, but it will always be at the expense of a greater increase in entropy in the surroundings. For example, when water freezes, its entropy decreases. The molecules making up an ice crystal have a more structured order than the random collection of water molecules in the liquid phase, so the entropy of the water decreases when it freezes. But when water freezes, it releases heat energy into its environment, which creates disorder in the surroundings. If we were to figure out the total change in entropy of the water plus its surroundings, we would find that although the entropy of the water itself decreased, it was more than compensated by a greater amount of entropy increase in the surroundings. So, the total entropy of the system and its surroundings increased, in agreement with the Second Law of Thermodynamics.

There are several equivalent statements of the second law. In addition to the entropy form, another form of the Second Law of Thermodynamics says that heat always flows from an object at higher temperature to an object at lower temperature, never the other way around. Heat always flows from hot to cold, never cold to hot. Another form, which will be considered more fully in a moment, says that a heat engine can never operate at 100% efficiency, or equivalently, that it is impossible to convert heat completely into work.

Entropy and Heat

An isolated system never decreases in entropy over time. A non-idealized (real world) isolated system always increases in entropy over time. This entropy is usually in the form of heat given off. When a system is not isolated, it is possible for the entropy of the system to decrease as long as the surroundings to the system increase in entropy by a greater amount than the decrease in the system. As a result, the entropy of the universe never decreases, and the entropy of the universe will increase over time as real world systems transfer heat and work.

Heat Engines

The form of the Second Law of Thermodynamics that we'll study for the AP Physics 2 Exam deals with heat engines.

Converting work to heat is easy—rubbing your hands together in order to warm them up shows that work can be completely converted to heat. What we'll look at is the reverse process: How efficiently can heat be converted into work? A device that uses heat to produce useful work is called a **heat engine**. The internal-combustion engine in a car is an example. In particular, we're only interested in engines that take their working substance (a mixture of air and fuel in this case) through a cyclic process, so that the cycle can be repeated. The basic components of any cyclic heat engine are simple: Energy in the form of heat comes into the engine from a high-temperature source, some of this energy is converted into useful work, the remainder is ejected as exhaust heat into a low-temperature sink, and the system returns to its original state to run through the cycle again.

Since we're looking at cyclic engines only, the system returns to its original state at the end of each cycle, so ΔU must be 0. Therefore, by the First Law of Thermodynamics, $Q_{\text{net}} = -W$. That is, the net heat absorbed by the system is equal to the work performed by the system. The heat absorbed from the high-temperature source is

denoted Q_H (H for *hot*), and the heat that is discharged into the low-temperature reservoir is denoted Q_C (C for *cold*). Because heat coming *in* is positive and heat going *out* is negative, Q_H is positive and Q_C is negative, and the net heat absorbed is $Q_H + Q_C$. Instead of writing Q_{net} in this way, it's customary to write it as $Q_H - |Q_C|$, to show explicitly that Q_{net} is less than Q_H.

All engines function to convert heat into work by exchanging energy from a reservoir at higher temperature to reservoirs of lower temperature. A good example of this is a refrigerator. In a fridge, a liquid moves through tubes to remove heat from the internal chamber (such as the freezer) to the outer air. Thus, cold is not actually a concept, but the sensation of low heat content. This is why the back of a refrigerator feels hot.

This is one of the forms of

The Second Law of Thermodynamics

For any cyclic heat engine, some exhaust heat is always produced. It's impossible to completely convert heat into useful work.

Example 9 A heat engine draws 800 J of heat from its high-temperature source and discards 450 J of exhaust heat into its cold-temperature reservoir during each cycle. How much work does this engine perform per cycle?

Solution. The absolute value of the work output per cycle is equal to the difference between the heat energy drawn in and the heat energy discarded:

$$|W| = Q_{\mathrm{H}} - |Q_{\mathrm{C}}| = 800\ \mathrm{J} - 450\ \mathrm{J} = 350\ \mathrm{J}$$

Chapter 4 Review Questions

Click here to download the PDF of Chapter 4 Review Questions.

Solutions can be found in Chapter 12.

Section I: Multiple Choice

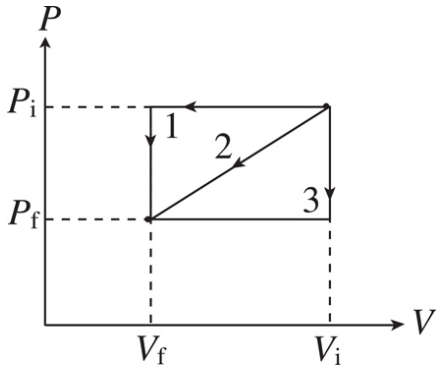
1. A container holds a mixture of two gases, CO_2 and H_2, in thermal equilibrium. Let K_C and K_H denote the average kinetic energy of a CO_2 molecule and an H_2 molecule, respectively. Given that a molecule of CO_2 has 22 times the mass of a molecule of H_2, the ratio K_C/ K_H is equal to

   (A) 1/22
   (B) 1
   (C) $\sqrt{22}$
   (D) 22

2. If the temperature and volume of a sample of an ideal gas are both doubled, then a PV diagram will show what sort of shape?

   (A) A horizontal line
   (B) A vertical line
   (C) A curve indicating a positive correlation
   (D) A curve indicating a negative correlation

3. In three separate experiments, a gas is transformed from state P_i, V_i to state P_f, V_f along the paths (1, 2, and 3) illustrated in the figure below:



The work done on the gas is

(A) greatest for path 1
(B) least for path 2
(C) greatest for path 2
(D) the same for all three paths

4. An ideal gas is compressed isothermally from 20 m^3 to 10 m^3. During this process, 5 J of work is done to compress the gas. What is the change of internal energy for this gas?

(A) –10 J
(B) –5 J
(C) 0 J
(D) 5 J

Questions 5 and 6 refer to the following material.

An ideal gas is confined in a container with a fixed volume. The amount of gas, *n*, is slowly increased in the container. This experiment is done in such a way that the temperature of the gas remains constant. Pressure data is collected.

5. Which describes a graph with pressure on the vertical axis and amount of gas on the horizontal axis?

(A) The graph will be linear and a fit line will go through the origin. (0, 0) will be a data point.

(B) The graph will be linear and a fit line will go through the origin. (0, 0) will not be a data point.

(C) The graph will be nonlinear. (0, 0) will be a data point.

(D) The graph will be nonlinear. (0, 0) will not be a data point.

6. How could the experiment be done so that as the amount of gas is increased, the temperature remains constant?

(A) Allow the pressure to change while n is changed so that T will remain constant.

(B) Allow work to be added or removed from the gas so that T will remain constant.

(C) Allow heat to be added or removed from the gas so that T will remain constant.

(D) Allow internal energy to be added or removed from the gas so that T will remain constant.

7. Through a series of thermodynamic processes, the internal energy of a sample of confined gas is increased by 560 J. If the net amount of work done on the sample by its surroundings is 320 J, how much heat was transferred between the gas and its environment?

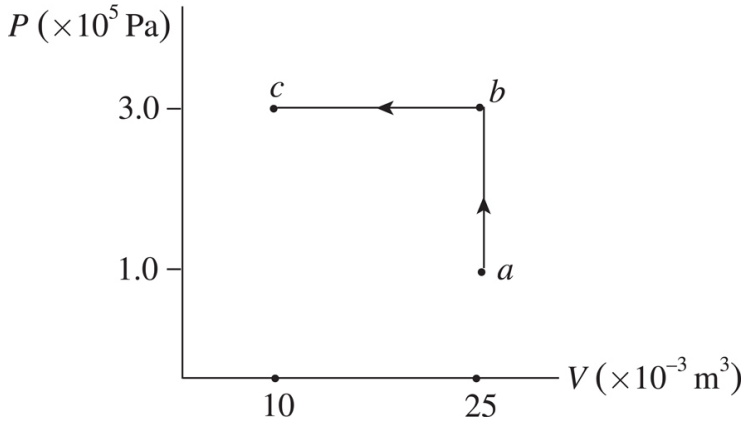
(A) 240 J absorbed

(B) 240 J dissipated

(C) 880 J absorbed

(D) 880 J dissipated

8. What is the total work performed on the gas as it is transformed from state a to state c, along the path indicated?



(A) 1,500 J
(B) 3,000 J
(C) 4,500 J
(D) 9,500 J

9. In one of the steps of the Carnot cycle, the gas undergoes an isothermal expansion. Which of the following statements is true concerning this step?

(A) No heat is exchanged between the gas and its surroundings, because the process is isothermal.
(B) The temperature decreases because the gas expands.
(C) The internal energy of the gas remains constant.
(D) The internal energy of the gas decreases due to the expansion.

10. A cup of hot coffee is sealed inside a perfectly thermally insulating container. A long time is allowed to pass. Which of the following correctly explains the final thermal configuration within the box?

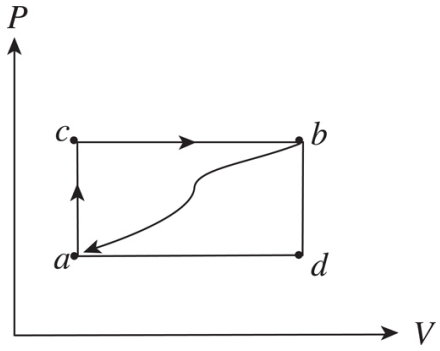
(A) The coffee has not changed temperature because the box is perfectly insulating.
(B) The coffee has gotten warmer and the air in the box has gotten cooler because of an exchange of thermal energy between the air and the coffee.

(C) The coffee has gotten cooler and the air in the box has gotten warmer because of an exchange of thermal energy between the air and the coffee.

(D) The coffee has gotten cooler but the air in the box has not changed its temperature. The energy from the coffee has caused an increase in entropy within the box.

Section II: Free Response

1. When a system is taken from state *a* to state *b* along the path *acb* shown in the figure below, 70 J of heat flows into the system, and the system does 30 J of work.



(a) When the system is returned from state *b* to state *a* along the curved path shown, 60 J of heat flows out of the system. Does the system perform work on its surroundings or do the surroundings perform work on the system? How much work is done?

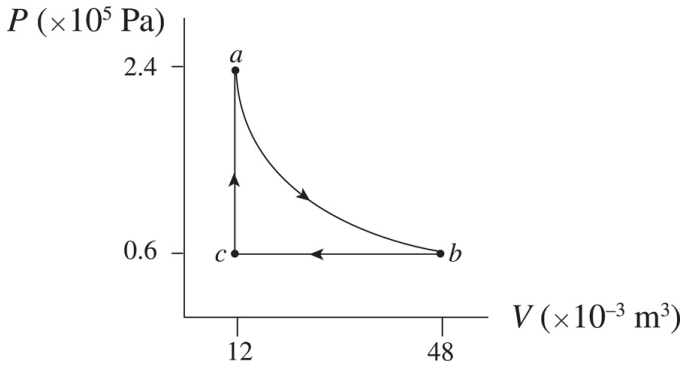
(b) If the system does 10 J of work in transforming from state *a* to state *b* along path *adb*, does the system absorb or does it emit heat? How much heat is transferred?

(c) If U_a = 0 J and U_d = 30 J, determine the heat absorbed in the processes *db* and *ad*.

(d) For the process *adbca*, identify each of the following quantities as positive, negative, or zero:

W = _____ Q = _____ U = _____

2. A 0.4 mol sample of an ideal diatomic gas undergoes slow changes from state *a* to state *b* to state *c* and back to *a* along the cycle shown in the *P*–*V* diagram below:



Path *ab* is an isotherm, and it can be shown that the work done by the gas as it changes isothermally from state *a* to state *b* is given by the equation

$$W_{ab} = -nRT \times \ln \frac{V_b}{V_a}$$

(a) What's the temperature of

i. state *a*?

ii. state *b*?

iii. state *c*?

(b) In order for step *ab* to be isothermal, the gas must be kept in thermal equilibrium with its surroundings at all times. Because heat flows spontaneously from hot to cold systems, does this imply that there is no heat flow during step *ab*? Explain.

(c) How much work, W_{ab}, is done by the gas during step *ab*?

(d) What is the total work done over cycle *abca*?

Summary

- In an isolated system, $Q_{\text{gained}} + Q_{\text{lost}} = 0$.
- The rate at which heat is transferred is given by $\Delta Q/\Delta t = \frac{kA\Delta T}{L}$, where k is the thermal conductivity (a property of the material), A is the cross-sectional area, ΔT is the temperature difference between the two sides, and L is the thickness or distance between the two ends of the material.
- For gases there are a few important ideas to understand:
  - Pressure is the result of the molecules colliding with one another and with the sides of the container. It is defined as the force per unit area ($P = F/A$) on the walls of the container.
  - The Ideal Gas Law is expressed as either $PV = nRT$ or $PV = Nk_{\text{B}}T$.
  - The average kinetic energy of the gas molecules is given by $K_{avg} = \frac{3}{2}k_bT$. This is related to the most likely speed a particle will be moving, but the particles in the gas will be moving with a large distribution of various speeds.
- The work done can be found by the area under a pressure-vs.-volume graph. If there is a volume change while the pressure remains constant, then the equation $W = -PV$ can be used to calculate the work.

- The First Law of Thermodynamics is $\Delta U = Q + W$ where ΔU depends only on the temperature change.

- W positive means energy is being added to the system, so work is done **on** the system **by** the surroundings. A negative W means that energy is being subtracted from the system, so work is done **by** the system **on** the surroundings.

- Q positive (or negative) means energy is being added to (or subtracted from) the system by means of a flow of heat from the higher temperature surroundings (or system) to the lower temperature system (or surroundings).

  Note that some textbooks define work in thermodynamics in a different way: Work is considered to be positive when work is done on the surroundings. This is consistent with the idea that the overall objective of a heat engine is to produce external (positive) work. Under this definition, the First Law must be written as $U = Q - W$ (or $U + W = Q$), and W must then be interpreted differently from Q. That is, while Q is still positive when heat is being added to the system, W is now positive when work is being done by the system on the surroundings (thus decreasing the internal energy of the system).

Chapter 5
Electric Forces and Fields

INTRODUCTION

The existence of mass in an object results in that object having the ability to experience the gravitational force, and thereby interact with other objects that also have mass. All objects in the universe that have been discovered thus far have mass (although the particle known as the neutrino has such a small mass that it is practically massless). Another fundamental property of objects is charge. A charged object will interact with another charged object through the electromagnetic force, similar to massive objects. The property of charge is as fundamental to physics as the property of mass.

ELECTRIC CHARGE

The basic components of atoms are protons, neutrons, and electrons. Protons and neutrons form the nucleus (and are referred to collectively as *nucleons*), while the electrons keep their distance, swarming around the nucleus. Most of an atom consists of empty space. In fact, if a nucleus were the size of the period at the end of this sentence, then the electrons would be 5 meters away. So what holds such an apparently tenuous structure together? One of the most powerful forces in nature: the *electromagnetic force*. Protons and electrons have a quality called **electric charge** that gives them an attractive force. Electric charge comes in two varieties: positive and negative. A positive particle always attracts a negative particle, and particles of the same charge always repel each other. Protons are positively charged, and electrons are negatively charged.

Protons and electrons are intrinsically charged, but bulk matter is not. This is because the amount of charge on a proton exactly balances the charge on an electron, which is quite remarkable in light of the fact that protons and electrons are very different particles. Since most atoms contain an equal number of protons and electrons, their overall electric charge is 0, because the negative charges cancel out the positive charges. Therefore, in order for matter to be **charged**, an imbalance between the numbers of protons and electrons must exist. This can be accomplished by either the removal or addition of electrons (that is, by the **ionization** of some of the object's atoms). If you remove electrons, then the object becomes positively charged, while if you add electrons, then it becomes negatively charged. Furthermore, charge is **conserved**. For example, if you rub a glass rod with a piece of silk, then the silk will acquire a negative charge and the glass will be left with an *equal* positive charge. *Net charge cannot be created or destroyed.* (*Charge* can be created or destroyed—it happens all the time—but *net* charge cannot.)

The magnitude of charge on an electron (and therefore on a proton) is denoted *e*. This stands for **elementary charge**, because it's the basic unit of electric charge. The charge of an ionized atom must be a whole number times *e*, because charge can be added or subtracted only in lumps of size *e*. For this reason we say that charge is **quantized**. To remind us of the quantized nature of electric charge, the charge of a particle (or object) is denoted by the letter *q*. In the SI system of units, charge is expressed in **coulombs** (abbreviated **C**). One coulomb is a tremendous amount of charge, about 10^{18} electrons. The value of *e* is about 1.6×10^{-19} C.

COULOMB'S LAW

The electric force between two charged particles obeys a law that is very similar to that describing the gravitational force between two masses: they are both inverse-square laws. The **electric force** between two particles with charges of q_1 and q_2, separated by a distance r, is given by the equation

Equation Sheet

$$F_E = k\frac{q_1 q_2}{r^2}$$

This is **Coulomb's Law**. We interpret a negative F_E as an attraction between the charges and a positive F_E as a repulsion. The value of the proportionality constant, k, depends on the material between the charged particles. In empty space (vacuum)—or air, for all practical purposes—it is called **Coulomb's constant** and has the approximate value $k_0 = 9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$. k_0 is sometimes written in terms of a fundamental constant known as the **permittivity of free space**, denoted ε_0:

$$k_0 = \frac{1}{4\pi\varepsilon_0}$$

Example 1 Consider the proton and electron in hydrogen. The proton has a mass of 1.6×10^{-27} kg and a charge of $+e$. The electron has a mass of 9.1×10^{-31} kg and a charge of $-e$. In hydrogen, they are separated by a Bohr radius, about 0.5×10^{-10} m. What is the

electric force between the proton and electron in hydrogen? What about the gravitational force between the proton and electron?

Solution. The electric force between the proton and the electron is given by Coulomb's Law:

$$F_{\mathrm{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = (9\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{(1.6\times10^{-19}\ \mathrm{C})(-1.6\times10^{-19}\ \mathrm{C})}{(0.5\times10^{-10}\ \mathrm{m})^2} = -9.24\times10^{-8}\ \mathrm{N}$$

The fact that F_{E} is negative means that the force is one of *attraction*, which we naturally expect, since one charge is positive and the other is negative. The force between the proton and electron is along the line that joins the charges, as we've illustrated below. The two forces shown form an action/reaction pair.



Now, the gravitational force between the two charges is given by Newton's Law of Gravitation:

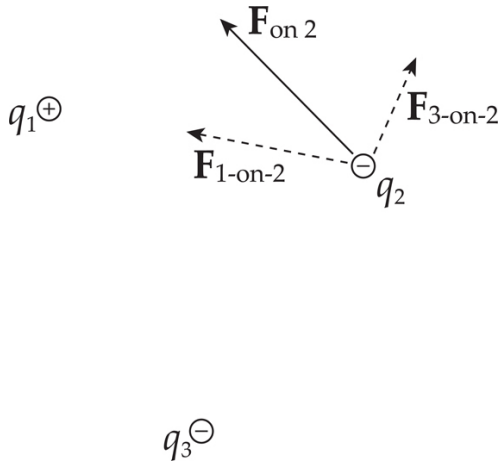
$$\mathbf{F}_G = G\times\frac{(m_1 m_2)}{r^2} = (6.67\times10^{-11}\frac{\mathrm{kg\cdot m^2}}{\mathrm{kg^2}})\times\frac{(1.67\times10^{-27}\,\mathrm{kg})(9.11\times10^{-31}\ \mathrm{kg})}{(0.5\times10^{-10}\,\mathrm{m})^2} = 4.06\times10^{-47}\,\mathrm{N}$$

Now, compare the orders of magnitude of the electric force to the gravitational force. The electric force has an order of magnitude of 10^{-8} N, but the gravitational force has an order of magnitude of 10^{-47} N! This means that the electric force is something like 10^{39} times larger! This is the reason that in problems where we calculate electrostatic forces, we often neglect gravitational forces.

Addition of Forces

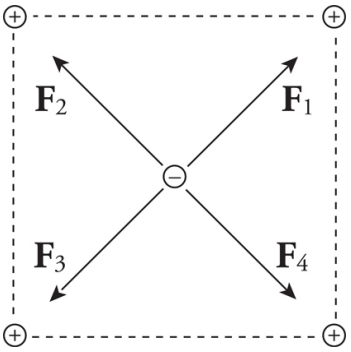
Consider three point charges: q_1, q_2, and q_3. The total electric force acting on, say, q_2 is simply the sum of $\mathbf{F}_{1\text{-on-}2}$, the electric force on q_2 due to q_1, and $\mathbf{F}_{3\text{-on-}2}$, the electric force on q_2 due to q_3:

$$\mathbf{F}_{\text{on }2} = \mathbf{F}_{1\text{-on-}2} + \mathbf{F}_{3\text{-on-}2}$$



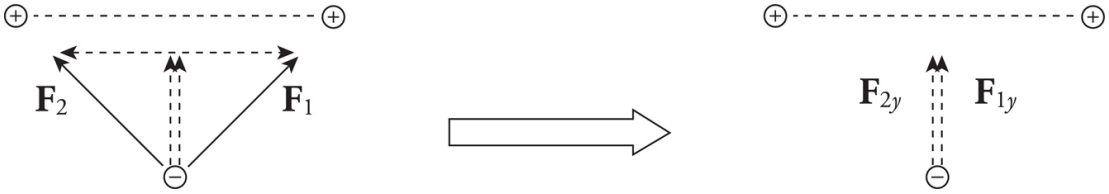
Example 2 Consider four equal, positive point charges that are situated at the vertices of a square. Find the net electric force on a negative point charge placed at the square's center.

Solution. Refer to the diagram below. The attractive forces due to the two charges on each diagonal cancel out: $F_1 + F_3 = 0$, and $F_2 + F_4 = 0$, because the distances between the negative charge and the positive charges are all the same and the positive charges are all equivalent. Therefore, by symmetry, the net force on the center charge is zero.



Example 3 If the two positive charges on the bottom side of the square in the previous example were removed, what would be the net electric force on the negative charge? Assume that each side of the square is 4.0 cm, each positive charge is 1.5 μC, and the negative charge is −6.2 nC. (μ is the symbol for "micro-," which equals 10^{-6}.)

Solution. If we break down $\mathbf{F}_1$ and $\mathbf{F}_2$ into horizontal and vertical components, then by symmetry the two horizontal components will cancel each other out, and the two vertical components will add:



Since the diagram on the left shows the components of $\mathbf{F}_1$ and $\mathbf{F}_2$ making right triangles with legs each of length 2 cm, it must be that $F_{1y} = F_1 \sin 45°$ and $F_{2y} = F_2 \sin 45°$. Also, the magnitude of $\mathbf{F}_1$ equals that of $\mathbf{F}_2$. So the net electric force on the negative charge is $F_{1y} + F_{2y}$

= $2F$ sin 45°, where F is the strength of the force between the negative charge and each of the positive charges.

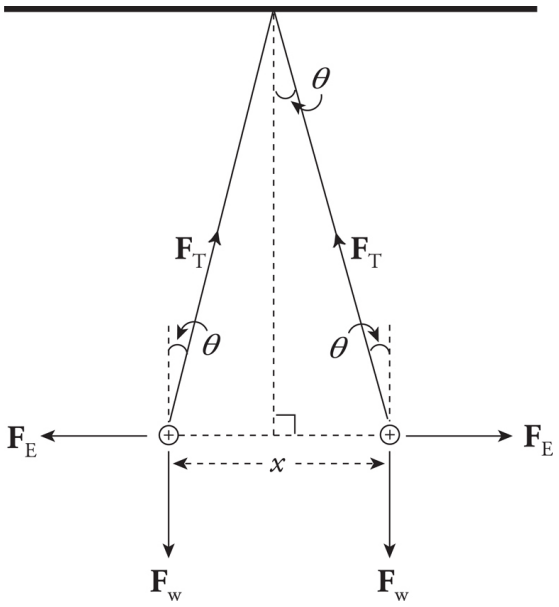
If s is the length of each side of the square, then the distance r between each positive charge and the negative charge is $r = \frac{1}{2}s\sqrt{2}$ and

$$\begin{aligned} F_{\mathrm{E}} = 2F\sin 45^\circ &= 2\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\sin 45^\circ \\ &= 2(9\times 10^9\ \mathrm{N\cdot m^2/C^2})\frac{(1.5\times 10^{-6}\ \mathrm{C})(6.2\times 10^{-9}\ \mathrm{C})}{(\frac{1}{2}\cdot 4.0\times 10^{-2}\cdot\sqrt{2}\ \mathrm{m})^2}\sin 45^\circ \\ &= 0.15\ \mathrm{N} \end{aligned}$$

The direction of the net force is straight upward, toward the center of the line that joins the two positive charges.

Example 4 Two pith balls of mass m are each given a charge of $+q$. They are hung side-by-side from two threads each of length L, and move apart as a result of their electrical repulsion. Find the equilibrium separation distance x in terms of m, q, and L. (Use the fact that if θ is small, then tan $\theta \approx$ sin θ.)

Solution. Three forces act on each ball: weight, tension, and electrical repulsion:



When the balls are in equilibrium, the net force each feels is zero. Therefore, the vertical component of $\mathbf{F}_T$ must cancel out $\mathbf{F}_w$ and the horizontal component of $\mathbf{F}_T$ must cancel out $\mathbf{F}_E$:

$$F_T \cos\theta = F_w \qquad \text{and} \qquad F_T \sin\theta = F_E$$

Dividing the second equation by the first, we get $\tan\theta = F_E/F_w$. Therefore,

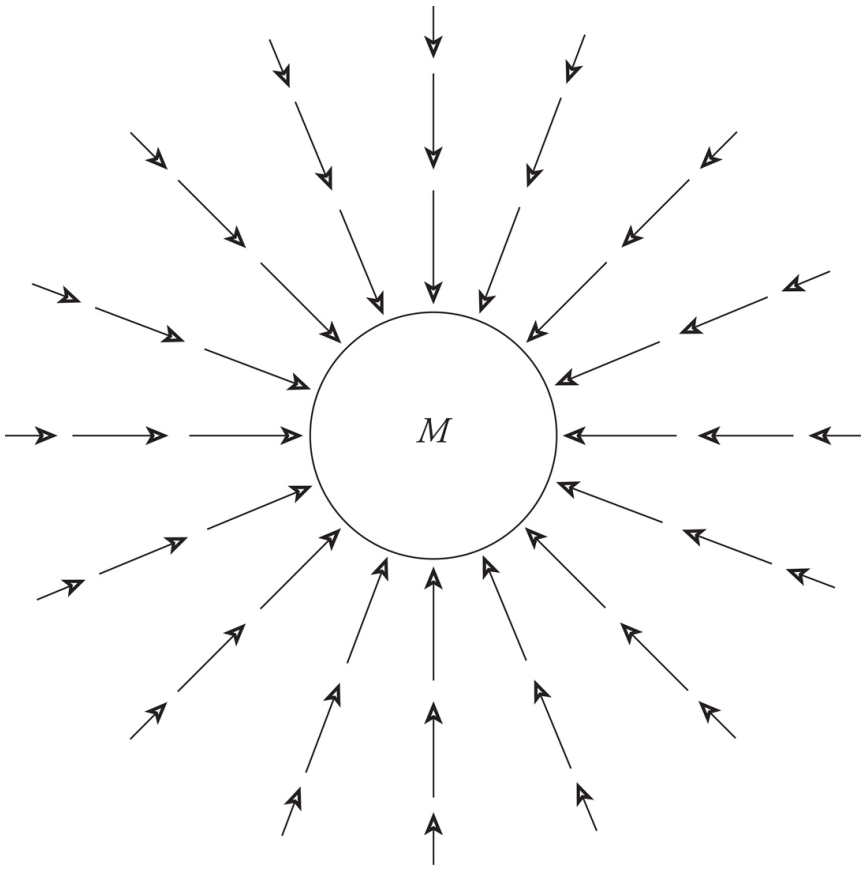
$$\tan\theta = \frac{k\dfrac{q^2}{x^2}}{mg} = \frac{kq^2}{mgx^2}$$

Now, to approximate: if θ is small, the $\tan\theta \approx \sin\theta$ and, from the diagram $\sin\theta \dfrac{1}{2} = x/L$. Therefore, the equation above becomes

$$\frac{\dfrac{1}{2}x}{L} = \frac{kq^2}{mgx^2} \quad \Rightarrow \quad \frac{1}{2}mgx^3 = kq^2L \quad \Rightarrow \quad x = \sqrt[3]{\frac{2kq^2L}{mg}}$$

THE ELECTRIC FIELD

The presence of a massive body such as the Earth causes objects to experience a gravitational force directed toward the Earth's center. For objects located outside the Earth, this force varies inversely with the square of the distance and directly with the mass of the gravitational source. A vector diagram of the gravitational field surrounding the Earth looks like this:



We can think of the space surrounding the Earth as permeated by a **gravitational field** created by the Earth. Any mass placed in this field then experiences a gravitational force due to an interaction with this field.

The same process is used to describe the electric force. Rather than having two charges reach out across empty space to each other to produce a force, we will instead interpret the interaction in the

following way: the presence of a charge creates an **electric field** in the space that surrounds it. Another charge placed in the field created by the first will experience a force due to the field.

Consider a point charge Q in a fixed position and assume that it's positive. Now imagine moving a tiny positive test charge q around to various locations near Q. At each location, measure the force that the test charge experiences, and call it $\mathbf{F}_{\text{on } q}$. Divide this force by the test charge q; the resulting vector is the **electric field vector, E,** at that location:

Equation Sheet

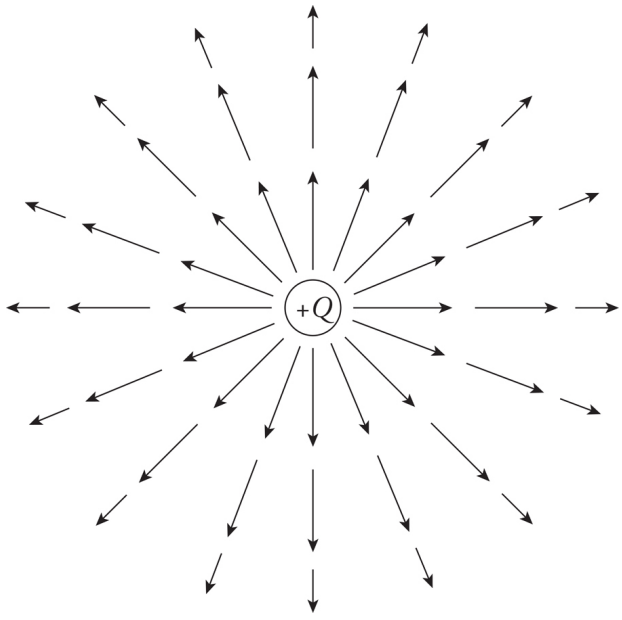
$$E = \frac{F_{\text{on } q}}{q}$$

The reason for dividing by the test charge is simple. If we were to use a different test charge with, say, twice the charge of the first one, then each of the forces **F** we'd measure would be twice as much as before. But when we divided this new, stronger force by the new, greater test charge, the factors of 2 would cancel, leaving the same ratio as before. So this ratio tells us the intrinsic strength of the field due to the source charge, independent of whatever test charge we may use to measure it.

There are three types of electric fields that you should expect to encounter on the AP Physics 2 Exam. The first type of field is a radial field. This occurs from a point charge (or from a charged sphere). The field points along lines that radiate outward (for positive charges) or point inward (for negative charges) toward the center of the circle. These fields are inversely proportionally to the distance from the point (or center of the sphere), just like the gravitational field. The second type of field is that generated by a collection of

point charges with specified locations and charges. This distribution of charge results in an electric field that is determined by superposition; the field at a particular observation location is determined from each charge separately, and then the vector sum of the individual fields is taken to determine the net field. This is identical to finding the vector sum of the forces of the charge distribution before dividing by the test charge. Lastly, an infinite sheet of charge will result in a field which is constant in both magnitude and direction. Because infinite sheets of charge are impossible to construct in the real world, the "infinite sheet of charge" is a useful approximation when the distance between the observation location and the sheet of charge is small compared to the distance between the observation location and the edges of the sheet.

First, let's examine the point charge. Since the test charge used to measure the field is positive, every electric field vector would point radially away from the source charge. *If the source charge is positive, the electric field vectors point away from it; if the source charge is negative, then the field vectors point toward it.* And, since the force decreases as we get farther away from the charge (as $1/r^2$), so does the electric field. This is why the electric field vectors farther from the source charge are shorter than those that are closer.

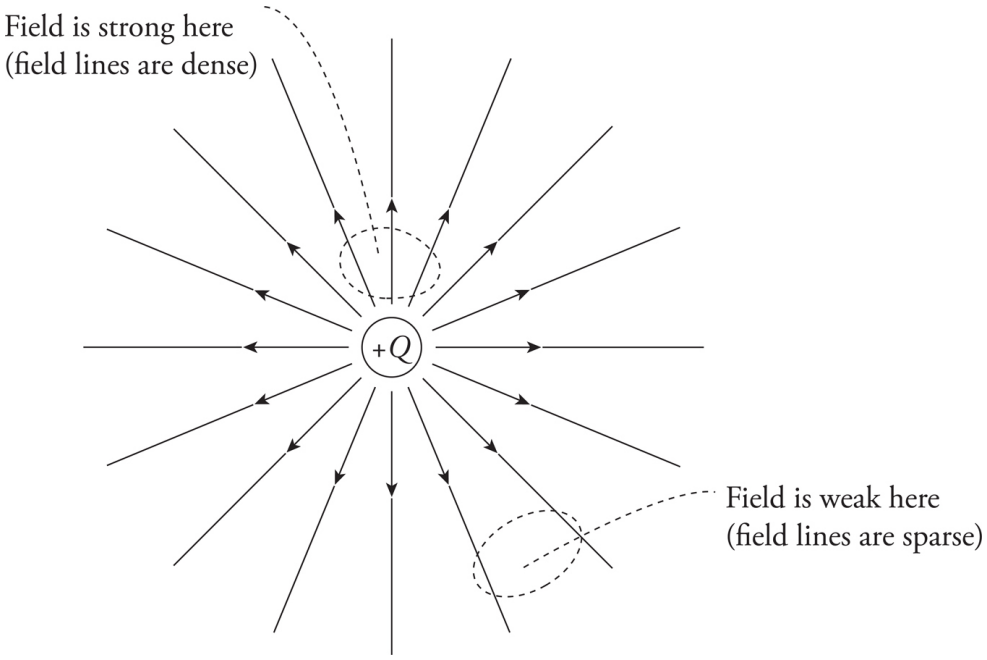


Since the force on any test charge q, due to some source Q, has a strength of $qQ/4\pi\varepsilon_0 r^2$, when we divide this by q, we get the expression for the strength of the electric field created by a point-charge source of magnitude Q:

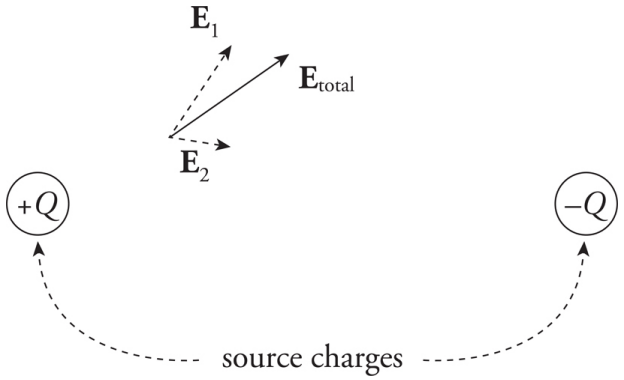
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

This is the electric field in the space surrounding for a **point charge.**

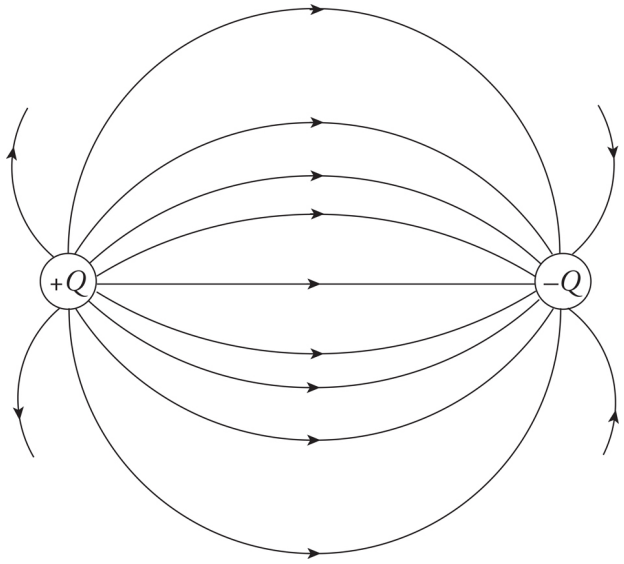
Sometimes electric fields are drawn as shown below, with arrows at various locations whose lengths indicate the field strength and which point in the field direction. However, sometimes they are sketched simply as continuous lines (not arrows) from the source such that the electric field vector is always tangent to the line everywhere it's drawn (a single arrowhead on the line indicates the direction to draw the field line along) and the strength is determined by how close the lines are together.



Electric fields obey the same addition properties as the electric force. If we had two source charges, their fields would overlap and effectively add; a third charge would feel the net influence of both charges. At each position in space (referred to as *observation locations*), add the electric field vector due to one of the charges to the electric field vector due to the other charge: $\mathbf{E}_{\text{total}} = \mathbf{E}_1 + \mathbf{E}_1$. This extends to any number of source charges. In the diagram below, $\mathbf{E}_1$ is the electric field vector at a particular location due to the charge $+Q$, and $\mathbf{E}_2$ is the electric field vector at that same location due to the other charge, $-Q$. Adding these vectors gives the overall field vector $\mathbf{E}_{\text{total}}$ at that location.

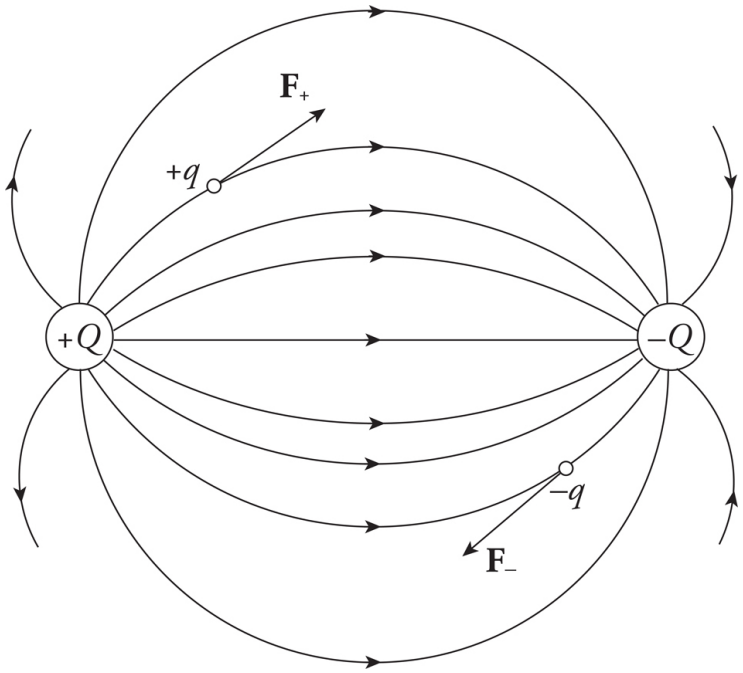


If this is done at enough locations, the electric field lines can be sketched.



Note that, like electric field vectors, electric field lines always point away from positive source charges and toward negative ones. Two equal but opposite charges, like the ones shown in the diagram above, form a pair called an **electric dipole.**

If a positive charge $+q$ were placed in the electric field above, it would experience a force that is tangent to, and in the same direction as, the field line passing through $+q$'s location. After all, electric field lines indicate the magnitude and direction of the force a positive test charge would experience. On the other hand, if a negative charge $-q$ were placed in the electric field, it would experience a force that is tangent to, but in the direction opposite from, the field line passing through $-q$'s location.



Finally, notice that electric field lines never cross.

Example 5 A charge q = +3.0 nC is placed at a location at which the electric field strength is 400 N/C. Find the force felt by the charge q.

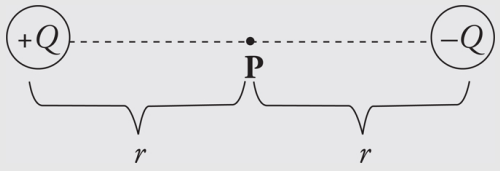
Solution. From the definition of the electric field, we have the following equation:

$$\mathbf{F}_{\text{on } q} = q\mathbf{E}$$

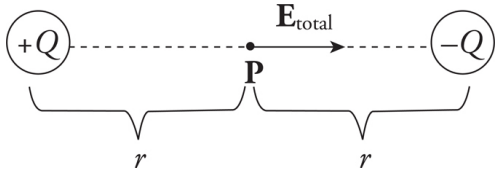
Therefore, in this case, $F_{\text{on } q} = qE = (3 \times 10^{-9}\ \text{C})(400\ \text{N/C}) = 1.2 \times 10^{-6}\ \text{N}$.

Example 6 A dipole is formed by two point charges, each of magnitude 4.0 nC, separated by a distance of

6.0 cm. What is the strength of the electric field at the point midway between them?



Solution. Let the two source charges be denoted $+Q$ and $-Q$. At Point P, the electric field vector due to $+Q$ would point directly away from $+Q$, and the electric field vector due to $-Q$ would point directly toward $-Q$. Therefore, these two vectors point in the same direction (from $+Q$ to $-Q$), so their magnitudes would add.



Using the equation for the electric field strength due to a single point charge, we find that

$$\mathbf{E}_{\text{total}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = 2\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$= 2(9\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4.0\times10^{-9}\ \text{C}}{\frac{1}{2}(6.0\times10^{-2}\ \text{m})^2}$$

$$= 8.0\times10^{4}\ \text{N/C}$$

Example 7 If a charge $q = -5.0$ pC were placed at the midway point described in the previous example, describe the force it would feel. ("p" is the abbreviation for "pico-," which means 10^{-12}.)

Solution. Since the field E at this location is known, the force felt by q is easy to calculate:

$$\mathbf{F}_{\text{on } q} = q\mathbf{E} = (-5.0 \times 10^{-12}\ \text{C})(8.0 \times 10^{4}\ \text{N/C to the right}) = 4.0 \times 10^{-7}\ \text{N to the } \textit{left}$$

Example 8 What can you say about the electric force that a charge would feel if it were placed at a location at which the electric field was zero?

Solution. Remember that $\mathbf{F}_{\text{on } q} = q\text{E}$. So if E = 0, then $\mathbf{F}_{\text{on } q} = 0$. (Zero field means zero force.)

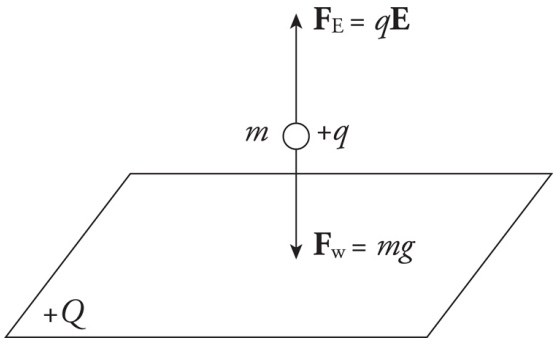
The Uniform Electric Field

An important subset of problems deal with uniform electric fields. One method of creating this uniform field is to have two large conducting sheets, each storing some charge Q, some distance d apart. Near the edges of each sheet, the field may not be uniform, but near the middle, for all practical purposes the field is uniform. Having a uniform field means having a constant force (and therefore a constant acceleration), so you can use kinematic equations just as if you had a uniform gravitational field (as you do near the surface of the Earth).

Example 9 Positive charge is distributed uniformly over a large, horizontal plate, which then acts as the source of a vertical electric field. An object of mass 5 g is placed at a distance of 2 cm above the plate. If the strength of the electric field at this location is 10^6 N/C, how much charge would the object need to have in

order for the electrical repulsion to balance the gravitational pull?

Solution. Clearly, since the plate is positively charged, the object would also have to carry a positive charge so that the electric force would be repulsive.



Let q be the charge on the object. Then, in order for F_E to balance mg, we must have

$$qE = mg \quad \Rightarrow \quad q = \frac{mg}{E} = \frac{(5\times10^{-3}\ \text{kg})(10\ \text{N/kg})}{10^{6}\ \text{N/C}} = 5\times10^{-8}\ \text{C} = 50\ \text{nC}$$

Example 10 A proton, neutron, and electron are in a uniform electric field of 20 N/C that is caused by two large charged plates that are 30 cm apart. The particles are far enough apart that they don't interact with each other. They are released from rest equidistant from each plate.

(a) What is the magnitude of the net force acting on each particle?

(b) What is the magnitude of the acceleration of each particle?

(c) How much work will be done on the particle as it moves to coincide with one of the charged

plates?

(d) What is the speed of each particle when it strikes the plate?

(e) How long does it take to reach the plate?

Solution.

(a) Since $E = \frac{F}{q}$, $F = qE$. Plugging in the values, we get

proton: $F = (1.6 \times 10^{-19}\ \text{C})(20\ \text{N/C}) = 3.2 \times 10^{-18}\ \text{N}$

electron: $F = (1.6 \times 10^{-19}\ \text{C})(20\ \text{N/C}) = 3.2 \times 10^{-18}\ \text{N}$

neutron: $F = (0\ \text{C})(20\ \text{N/C}) = 0\ \text{N}$

Note: Because the proton and electron have the same magnitude, they will experience the same force. If you're asked for the direction, the proton travels in the same direction as the electric field and the electron travels in the opposite direction as the electric field.

(b) Since $F = ma$, $a = \frac{F}{m}$. Plugging in the values, we get

proton: $a = \dfrac{3.2 \times 10^{-18}\ \text{N}}{1.67 \times 10^{-27}\ \text{kg}} = 1.9 \times 10^{9}\ \text{m/s}^2$

electron: $a = \dfrac{3.2 \times 10^{-18}\ \text{N}}{9.11 \times 10^{-31}\ \text{kg}} = 3.5 \times 10^{12}\ \text{m/s}^2$

neutron: $a = \dfrac{0\ \text{N}}{1.67 \times 10^{-27}\ \text{kg}} = 0\ \text{m/s}^2$

Notice that although the charges have the same magnitude of force, the electron experiences an acceleration almost 2000 times greater

due to its mass being almost 2000 times smaller than the proton's mass.

(c) Since $W = Fd$, we get $W = qEd$. Plugging in the values, recalling that the charges start midway between the plates, we get

proton: $W = (1.6 \times 10^{-19}\ \text{C})(20\ \text{N/C})\ (0.15\ \text{m}) = 4.8 \times 10^{-19}\ \text{J}$

electron: $W = (1.6 \times 10^{-19}\ \text{C})(20\ \text{N/C})\ (0.15\ \text{m}) = 4.8 \times 10^{-19}\ \text{J}$

neutron: $W = (0\ \text{C})\ (20\ \text{N/C})(0.15\ \text{m}) = 0\ \text{J}$

(d) From the Work–Energy Theorem,

$\Delta W = 1/2mv^2$, so $v = \sqrt{2\Delta W/m}$

Using the answer from part (c) for the work

proton: $v_f = \sqrt{(4.8 \times 10^{-19}\ \text{J})/(1.67 \times 10^{-27}\ \text{kg})} \rightarrow v_f = 24{,}000\ \text{m/s}$

electron: $v_f = \sqrt{(4.8 \times 10^{-19}\ \text{J})/(9.11 \times 10^{-31}\ \text{kg})} \rightarrow v_f = 1.0 \times 10^{6}\ \text{m/s}$

neutron: The neutron never hits the plate, so the question about what speed it hits the plate with is not well posed.

Notice that, even though the force is the same and the same work is done on both charges, there is a significant difference in final velocities due to the large mass difference. An alternative solution to this would be using kinematics. You would have obtained the same answers.

(e) Recall the definition of the final velocity of an accelerating object:

$$v_f = v_i + at \rightarrow t = \frac{v_f - v_i}{a} \rightarrow t = \frac{v_f}{a}$$

proton: $t = \dfrac{24{,}000\ \text{m/s}}{1.9 \times 10^9\ \text{m/s}^2} \rightarrow 1.3 \times 10^{-5}\ \text{s}$

electron: $t = \dfrac{1 \times 10^6\ \text{m/s}}{3.5 \times 10^{12}\ \text{m/s}^2} \rightarrow 2.9 \times 10^{-7}\ \text{s}$

neutron: The neutron never accelerates, so it will never hit the plate

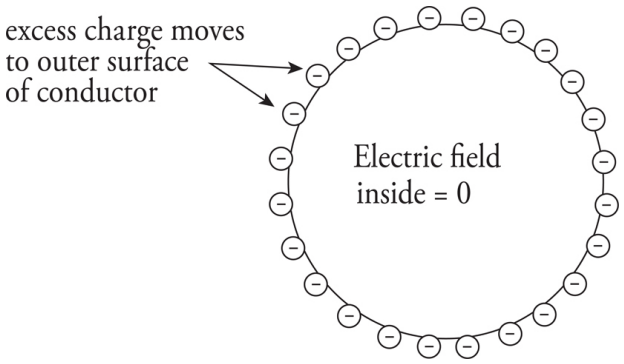
CONDUCTORS AND INSULATORS

Materials can be classified into broad categories based on their ability to permit the flow of charge. If electrons were placed on a metal sphere, they would quickly spread out and cover the outside of the sphere uniformly. These electrons would be free to flow through the metal and redistribute themselves, moving to get as far away from each other as they could. Materials that permit the flow of excess charge are called **conductors**; they conduct electricity. Metals are the best examples of conductors. Aqueous solutions that contain dissolved electrolytes (such as salt water) are also conductors. Metals conduct electricity because they bind all but their outermost electrons very tightly. That outermost electron is free to move about the metal. This creates a sort of sea of mobile (or conduction) electrons.

Insulators, on the other hand, closely guard their electrons—and even extra ones that might be added. Electrons are not free to roam throughout the atomic lattice. Examples of insulators are glass, wood, rubber, and plastic. If excess charge is placed on an insulator, it stays put.

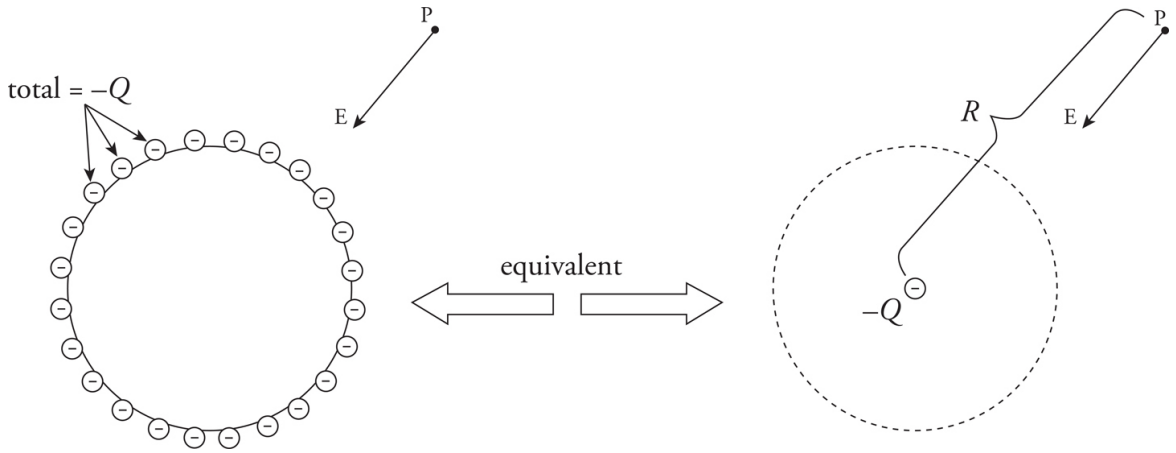
Example 11 A solid sphere of copper is given a negative charge. Discuss the electric field inside and outside the sphere.

Solution. The electric field inside a conductor is zero. Therefore, the excess electrons that are deposited on the sphere move quickly to the outer surface (copper is a great conductor). *Any excess charge on a conductor resides entirely on the outer surface.*

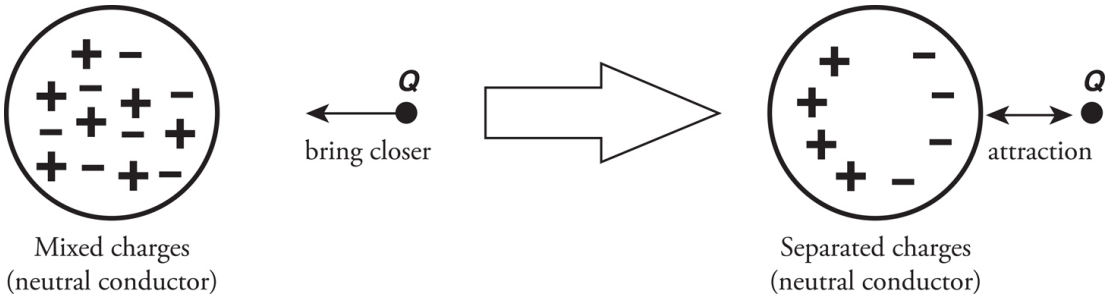


Once these excess electrons establish a uniform distribution on the outer surface of the sphere, there will be no net electric field within the sphere. Why not? Since there is no additional excess charge inside the conductor, there are no excess charges to serve as a source or sink of an electric field line cutting down into the sphere, because field lines begin or end on excess charges. *There can be no electrostatic field within the body of a conductor*.

In fact, you can shield yourself from electric fields simply by surrounding yourself with metal. Charges may move around on the outer surface of your cage, but within the cage, the electric field will be zero. For points outside the sphere, it can be shown that the sphere behaves as if all its excess charge were concentrated at its center. (Remember that this is just like the gravitational field due to a uniform spherical mass.) Also, *the electric field is always perpendicular to the surface, no matter what shape the surface may be*. See the diagram below.

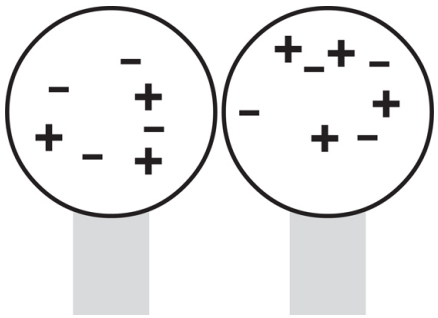


Now let's take our previous sphere and put it in a different situation. Start with a neutral metal sphere and bring a charge (a positive charge) *Q* nearby without touching the original metal sphere. What will happen? The positive charge will attract free electrons in the metal, leaving the far side of the sphere positively charged. Since the negative charge is closer to *Q* than the positive charge, there will be a net attraction between *Q* and the sphere. So, even though the sphere as a whole is electrically neutral, the separation of charge induced by the presence of *Q* will create a force of electrical attraction between them.



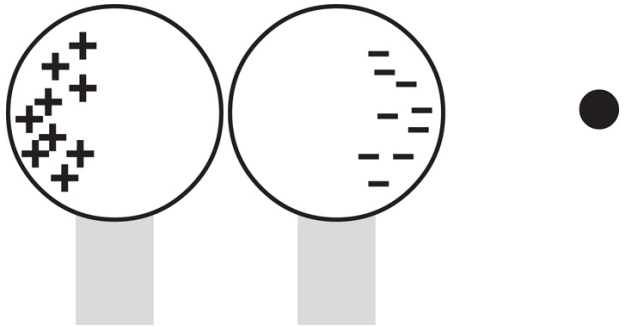
Charging by Induction

This process may be used to redistribute charges among a pair of neutrally charged spheres, so that in the end both spheres are charged. Imagine two neutrally charged spheres that are each set on an insulating stand. The spheres are arranged so that they are in contact with one another.

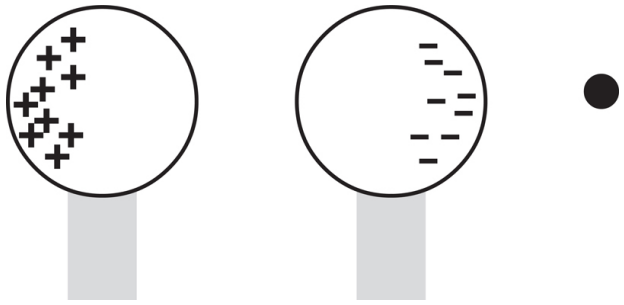


A charge is brought near the side of one of the spheres resulting in the same attraction as occurred with the single sphere. However, because we have two conducting spheres, the excess negative charges

are located on the right sphere and the positive excess charges are on the left sphere.



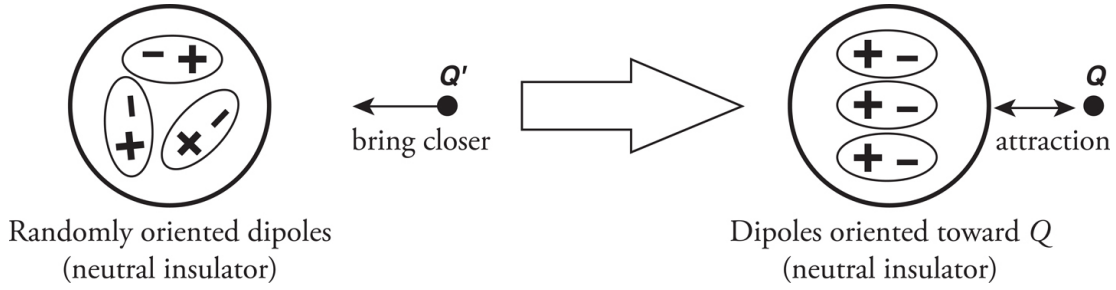
With the external charge still nearby, the two conducting spheres are separated. Because the conductors are no longer in contact, the negative charge has been trapped on the right sphere and the positive charge has been trapped on the left sphere.



Note that the net charge is unchanged between the two spheres—the distribution of the charges has simply been changed.

Now what if the sphere was made of glass (an insulator)? Although there aren't free electrons that can move to the near side of the sphere, the atoms that make up the sphere will become polarized. That is, their electrons will feel a tug toward *Q*, and so will spend more time on the side of the atom closer to *Q* than on the side opposite *Q*, causing the atoms to develop a distribution of charge that is more negative on the side nearby *Q* (and a partial positive charge on the other side from *Q*). The effect isn't as dramatic as the mass movement of free electrons in the case of a metal sphere, but the polarization is still enough to cause an electrical attraction between the sphere and *Q*. For example, if you comb your hair, the

comb will pick up extra electrons, making it negatively charged. If you place this electric field source near little bits of paper, the paper will become polarized and will then be attracted to the comb.



The same phenomenon, in which the presence of a charge tends to cause polarization in a nearby collection of charges, is responsible for a kind of intermolecular force. Dipole-induced forces are caused by a shifting of the electron cloud of a neutral molecule toward positively charged ions or away from a negatively charged ion; in either case, the resulting force between the ion and the atom is attractive.

Chapter 5 Review Questions

Click here to download the PDF of Chapter 5 Review Questions.

Solutions can be found in Chapter 12.

Section I: Multiple Choice

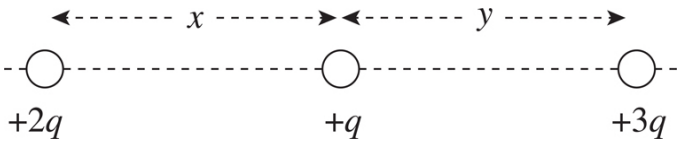
1. An experiment is conducted measuring the electrostatic force, F, on a test object at various distances, r. In order to create a plot with a straight line, what should be graphed?

   (A) F versus r^2
   (B) F versus r
   (C) F versus r^{-1}
   (D) F versus r^{-2}

2. Two 1 kg spheres each carry a charge of magnitude 1 C. How does F_E, the strength of the electric force between the spheres, compare to F_G, the strength of their gravitational attraction?

   (A) $F_E < F_G$
   (B) $F_E = F_G$
   (C) $F_E > F_G$
   (D) If the charges on the spheres are of the same sign, then $F_E > F_G$; but if the charges on the spheres are of the opposite sign, then $F_E < F_G$.

3. The figure below shows three point charges, all positive. If the net electric force on the center charge is zero, what is the value of y/x?

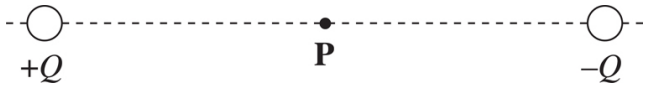


(A) $\frac{4}{9}$

(B) $\sqrt{\frac{2}{3}}$

(C) $\sqrt{\frac{3}{2}}$

(D) $\frac{3}{2}$



4. The figure above shows two point charges, $+Q$ and $-Q$. If the negative charge were absent, the electric field at Point P due to $+Q$ would have strength E. With $-Q$ in place, what is the strength of the total electric field at P, which lies at the midpoint of the line segment joining the charges?

(A) 0

(B) $\frac{E}{2}$

(C) E

(D) $2E$

5. A sphere of charge $+Q$ is fixed in position. A smaller sphere of charge $+q$ is placed near the larger sphere and released from

rest. The small sphere will move away from the large sphere with

(A) decreasing velocity and decreasing acceleration
(B) decreasing velocity and increasing acceleration
(C) increasing velocity and decreasing acceleration
(D) increasing velocity and increasing acceleration

6. An object of charge $+q$ feels an electric force $\mathbf{F}_E$ when placed at a particular location in an electric field, $\mathbf{E}$. Therefore, if an object of charge $-2q$ were placed at the same location where the first charge was, it would feel an electric force of

(A) $\frac{-\mathbf{F}_E}{2}$

(B) $-2\mathbf{F}_E$

(C) $-2q\mathbf{F}_E$

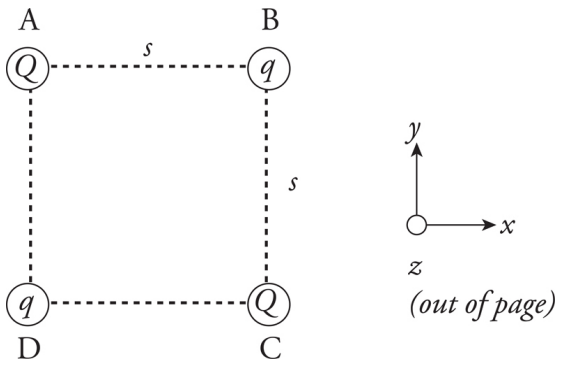
(D) $\frac{-2\mathbf{F}_E}{q}$

7. A charge of $-3Q$ is transferred to a solid metal sphere of radius r. How will this excess charge be distributed?

(A) $-Q$ at the center, and $-2Q$ on the outer surface
(B) $-3Q$ at the center
(C) $-3Q$ on the outer surface
(D) $-Q$ at the center, $-Q$ in a ring of radius $\frac{1}{2}r$, and $-Q$ on the outer surface

Section II: Free Response

1. In the figure shown, four charges are situated at the corners of a square of side length s. The charges on opposite corners are equal to one another and are labeled Q for corners A and C and labeled q for corners B and D. The charge on Q is positive for all experiments.



(a) In a first experiment, it is found that the force on the charge at position C is 0 N.

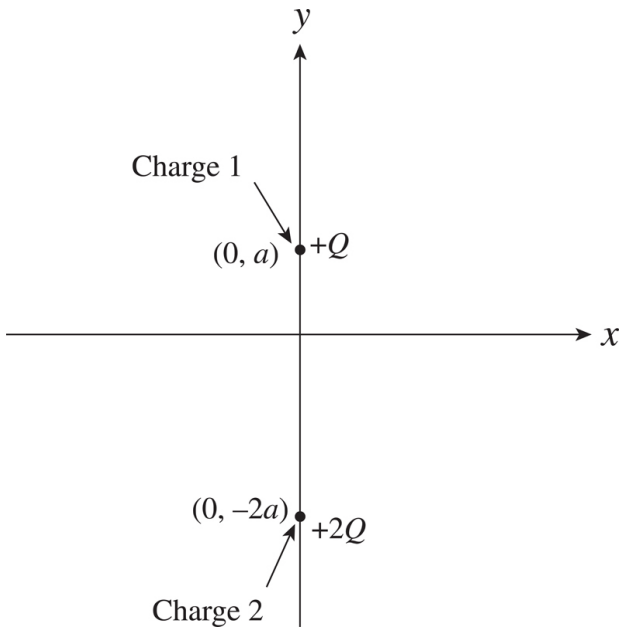
i. Justify the assertion that the charges q cannot have different magnitudes in this experiment.

ii. Derive an expression for the magnitude and sign of charge on q in this experiment.

(b) In a second experiment, the charge at point C is removed. If the charges q are positive, is there anywhere within the boundary square where the electric field could be 0 N/C?

(c) The charges are reassembled into their original positions. In a clear, coherent paragraph length response, explain why the electric field at the center of the square must be 0 N/C regardless of the magnitudes of the charges q or Q and regardless of their signs.

2. Two charges, $+Q$ and $+2Q$, are fixed in place along the y-axis of an xy-coordinate system as shown in the figure below. Charge 1 is at the point $(0, a)$, and Charge 2 is at the point $(0, -2a)$.



(a) Find the electric force (magnitude and direction) felt by Charge 1 due to Charge 2.

(b) Find the electric field (magnitude and direction) at the origin created by both Charges 1 and 2.

(c) Is there a point on the x-axis where the total electric field is zero? If so, where? If not, explain briefly.

(d) Is there a point on the y-axis where the total electric field is zero? If so, where? If not, explain briefly.

Summary

- Coulomb's Law describes the force acting on two point charges and is given by

  $$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

  where $\frac{1}{4\pi\varepsilon_0}$ is a constant equal to 9.0×10^9 N·m²/C².

- The electric field is given by $E = \frac{F}{q}$

- The electric field from a point charge is $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

- Both the electric force and field are vector quantities and therefore all the rules for vector addition apply.

Chapter 6
Electric Potential and Capacitance

INTRODUCTION

When an object moves in a gravitational field, it usually experiences a change in kinetic energy and in gravitational potential energy due to the work done on the object by gravity. Similarly, when a charge moves in an electric field, it generally experiences a change in kinetic energy and in electrical potential energy due to the work done on it by the electric field. By exploring the idea of electric potential, we can simplify our calculations of work and energy changes within electric fields.

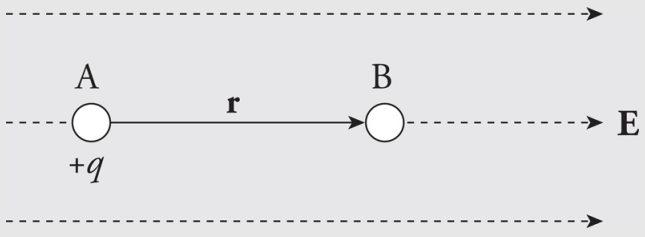
ELECTRICAL POTENTIAL ENERGY

When a charge moves in an electric field, then unless its displacement is always perpendicular to the field, the electric force does work on the charge. If W_E is the work done by the electric force, then the change in the charge's **electrical potential energy** is defined by

$$\Delta U_E = -W_E$$

Notice that this is the same equation that defined the change in the gravitational potential energy of an object of mass m undergoing a displacement in a gravitational field ($\Delta U_G = -W_G$).

Example 1 A positive charge $+q$ moves from position A to position B in a uniform electric field **E:**



What is its change in electrical potential energy?

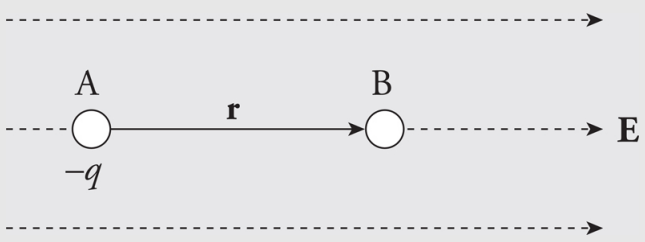
Solution. Since the field is uniform, the electric force that the charge feels, $\mathbf{F}_E = q\mathbf{E}$, is constant. Since q is positive, $\mathbf{F}_E$ points in the same direction as **E**, and, as the figure shows, they point in the same direction as the displacement, r. This makes the work done by the

electric field equal to $W_E = F_E r = qEr$, so the change in the electrical potential energy is

$$\Delta U_E = -qEr$$

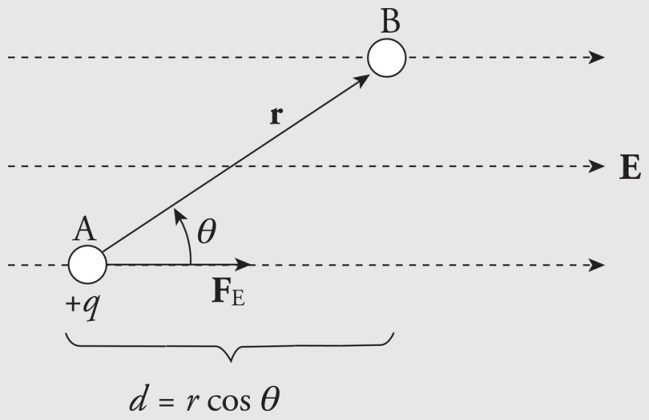
Note that the change in potential energy is negative, which means that potential energy has decreased; this always happens when the field does positive work. It's just like dropping a rock to the ground: Gravity does positive work, and the rock loses gravitational potential energy.

Example 2 Do the previous problem, but consider the case of a negative charge, $-q$.



Solution. In this case, an outside agent must be pushing the charge to make it move, because the electric force *naturally* pushes negative charges against field lines. Therefore, we expect that the work done by the electric field is negative. The electric force, $\mathbf{F}_E = (-q)\mathbf{E}$, points in the direction opposite to the displacement, so the work it does is $W_E = -F_E r = -qEr = -qEr$. Thus, the change in electrical potential energy is positive: $\Delta U_E = -W_E = -(-qEr) = qEr$. Because the change in potential energy is positive, the potential energy increases; this always happens when the field does negative work. It's like lifting a rock off the ground: Gravity does negative work, and the rock gains gravitational potential energy.

Example 3 A positive charge $+q$ moves from position A to position B in a uniform electric field **E:**



What is its change in electrical potential energy?

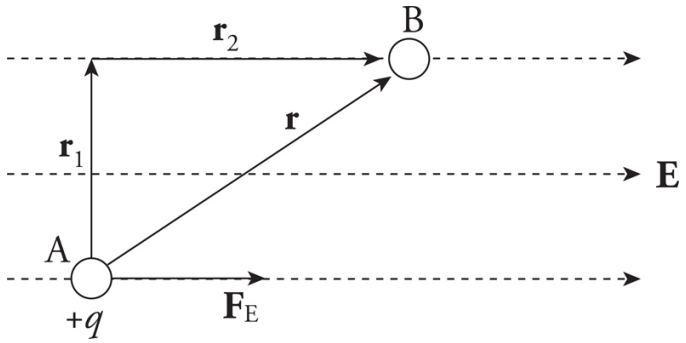
Solution. The electric force felt by the charge q is $\mathbf{F}_E = q\mathbf{E}$ and this force is parallel to **E**, because q is positive. In this case, because $\mathbf{F}_E$ is not parallel to r (as it was in Example 1), we will use the more general definition of work for a constant force:

$$W_E = \mathbf{F}_E \cdot \mathbf{r} = F_E\, r \cos\theta = qEr \cos\theta$$

But $r \cos\theta = d$, so

$$W_E = qEd \text{ and } \Delta U_E = -W_E = -qEd$$

Because the electric force is a conservative force, which means that the work done does not depend on the path that connects the positions A and B, the work calculated above could have been figured out by considering the path from A to B composed of the segments $\mathbf{r}_1$ and $\mathbf{r}_2$:



Along $\mathbf{r}_1$, the electric force does no work since this displacement is perpendicular to the force. Thus, the work done by the electric field as q moves from A to B is simply equal to the work it does along $\mathbf{r}_2$. And since the length of $\mathbf{r}_2$ is $d = r \cos \theta$, we have $W_E = F_E d = qEd$, just as before.

Electric Potential Energy from a Point Charge

Example 4 A positive charge $q_1 = +2 \times 10^{-6}$ C is held stationary, while a negative charge, $q_2 = -1 \times 10^{-8}$ C, is released from rest at a distance of 10 cm from q_1. Find the kinetic energy of charge q_2 when it's 1 cm from q_1.

Solution. The gain in kinetic energy is equal to the loss in potential energy; you know this from Conservation of Energy. Previously, we looked at constant electric fields and were able to use the equation for work from a constant force. However, when the field (and therefore the force) changes, we need another equation. Electrical potential energy required to move along the field lines surrounding a point charge is given by

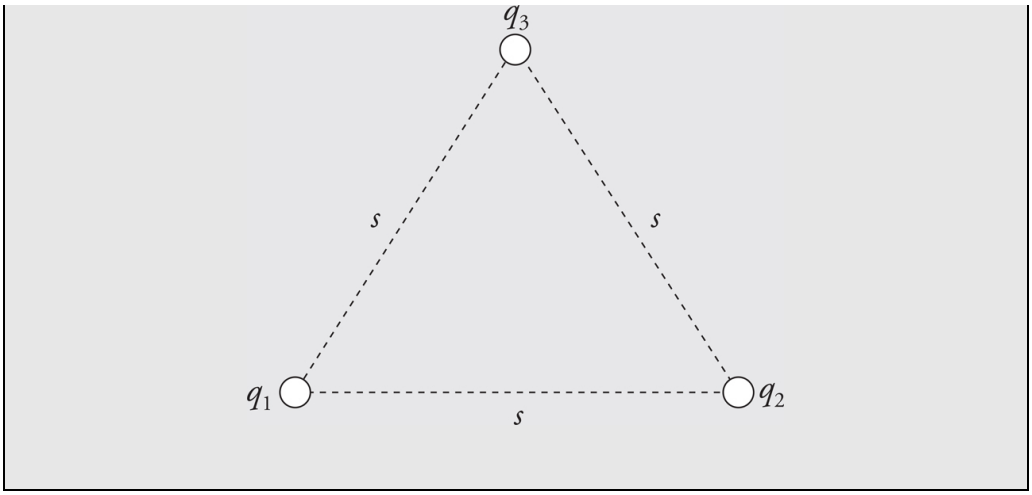
$$U_E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r}\right)$$

Therefore, if q_1 is fixed and q_2 moves from r_A to r_B, the change in potential energy is

$$\begin{aligned}
\Delta U_E &= U_B - U_A \\
&= \frac{q_1}{4\pi\varepsilon_0}\left(\frac{q_2}{r_B} - \frac{q_2}{r_A}\right) \\
&= \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \\
&= (9\times10^9\ \mathrm{N\cdot m^2/C^2})(+2\times10^{-6}\ \mathrm{C})(-1\times10^{-8}\ \mathrm{C})\left(\frac{1}{0.01\ \mathrm{m}} - \frac{1}{0.10\ \mathrm{m}}\right) \\
&= -0.016\ \mathrm{J}
\end{aligned}$$

Since q_2 lost 0.016 J of potential energy, the gain in kinetic energy is 0.016 J. Since q_2 started from rest (with no kinetic energy), this is the kinetic energy of q_2 when it's 1 cm from q_1.

Example 5 Two positive charges, q_1 and q_2, are held in the positions shown below. How much work would be required to bring (from infinity) a third positive charge, q_3, and place it so that the three charges form the corners of an equilateral triangle of side length s?



Solution. An external agent would need to do positive work, equal in magnitude to the negative work done by the electric force on q_3 as it is brought into place, so let's first compute this quantity. Let's first compute the work done *by the electric force* as q_3 is brought in. Since q_3 is fighting against both q_1's and q_2's electric field, the total work done on q_3 by the electric force, W_{E}, is equal to the work done on q_3 by q_1 ($W_{1\text{–}3}$) plus the work done on q_3 by q_2 ($W_{2\text{–}3}$). Using the equation $W_{\mathrm{E}} = -\Delta U_{\mathrm{E}}$ and the one we gave above for ΔU_{E}, we have

$$\begin{aligned} W_{1\text{-}3} + W_{2\text{-}3} &= -\Delta U_{1\text{-}3} + -\Delta U_{2\text{-}3} \\ &= -\frac{q_1 q_3}{4\pi\varepsilon_0}\left(\frac{1}{s} - 0\right) + -\frac{q_2 q_3}{4\pi\varepsilon_0}\left(\frac{1}{s} - 0\right) \\ &= -\left(\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{s} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{s}\right) \end{aligned}$$

Therefore, the work that an external agent must do to bring q_3 into position is

$$-W_{\mathrm{E}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{s} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{s}$$

NOT the Same

Electric Potential and Electric Potential Energy, although very closely related and having similar names, are not the same thing. Even the units are measured differently—electric potential is measured in joules per coulomb and electric potential energy in joules.

ELECTRIC POTENTIAL

Let W_E be the work done by the electric field on a charge q as it undergoes a displacement. If another charge, say $2q$, were to undergo the same displacement, the electric force would be twice as great on this second charge, and the work done by the electric field would be twice as much, $2W_E$. Since the work would be twice as much in the second case, the change in electrical potential energy would be twice as great as well, but the ratio of the change in potential energy to the charge would be the same: $W_E/q = (2W_E)/2q$. This ratio says something about the work done by the field and the *displacement* but not the charge that made the move. The change in **electric potential** ΔV, is defined as this ratio:

Equation Sheet

$$\Delta V = \frac{\Delta U_E}{q}$$

There is a similar concept with gravitational fields. The work done by gravity (near the Earth) on an object as it changes its position is $\Delta W = mgh$. If twice the mass were moved, the work would have to be $\Delta W = (2m)gh$. The ratio of the work to the mass moved is technically the "gravitational potential," but near the Earth, this quantity is simply the gravitational field strength, g, multiplied by the change in the height of the object. *Electric potential is the electrical equivalent of a change in height.*

Electric potential is electrical potential energy *per unit charge*; the units of electric potential are joules per coulomb. One joule per coulomb is called one **volt** (abbreviated V); so 1 J/C = 1 V.

Potential Difference

Finding the potential at a certain single point in space is meaningless. With potential, what is more important is an initial position and final position. Just like in gravity, finding the potential energy at a certain point is meaningless if there is no relation to another point. Once we establish this potential difference, when we move a charge across it we can generate energy. This will be key in the next chapter where potential difference is needed in order to have an electrical circuit.

Electric Potential from a Point Charge

Consider the electric field created by a point source charge Q. If a charge q moves from a distance r_A to a distance r_B from Q, then the change in the potential energy is

$$U_B - U_A = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

The difference in electric potential between positions A and B in the field created by Q is

$$V_B - V_A = \frac{U_B - U_A}{q} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

If we designate $V_A \to 0$ as $r_A \to \infty$ (an assumption that's stated on the AP Physics 2 Exam), then for a point charge Q, the electric potential at a distance r from Q is

Equation Sheet

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Note that the potential depends on the strength of the source charge making the field and the distance from the source charge.

Example 6 Let $Q = 2 \times 10^{-9}$ C. What is the potential at a Point P that is 2 cm from Q?

Solution. Relative to $V = 0$ at infinity, we have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \left(9\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\frac{2\times10^{-9}\ \mathrm{C}}{0.02\ \mathrm{m}} = 900\ \mathrm{V}$$

This means that the work done by the electric field on a charge of q coulombs brought to a point 2 cm from Q would be $-900q$ joules.

Note that, like potential energy, electric potential is a *scalar*. In the preceding example, we didn't have to specify the direction of the vector from the position of Q to the Point P, because it didn't matter. Imagine a sphere with a surface of 2 cm from Q; at any point on that sphere, the potential will be 900 V. These spheres around Q are called **equipotential surfaces,** and they're surfaces of constant potential. The equipotentials are always perpendicular to the electric field lines.

Remember

Equipotential surfaces are often imaginary. Whether we put a metal sphere in that location or whether we visualize a sphere that isn't really there doesn't change the electric potential in that region.

Example 7 How much work is done by the electric field as a charge moves along an equipotential surface?

Solution. If the charge always remains on a single equipotential, then, by definition, the potential, V, never changes. Therefore, $\Delta V = 0$, so $\Delta U_E = 0$. Since $W_E = -\Delta U_E$, the work done by the electric field is zero.

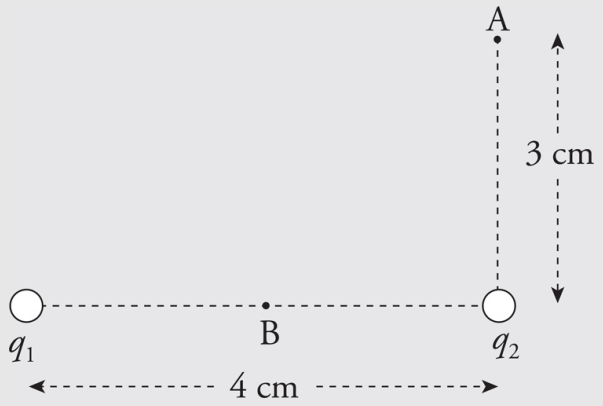
Addition of Electric Potential

The formula $V = kQ/r$ tells us how to find the potential due to a single point source charge, Q. Potential is scalar (we will not be concerned with orientation, just the sign of charge). When we add up individual potentials, we're simply adding numbers; we're not adding vectors.

Just like Gravitational Potential Energy

The electric force is conservative. All we care about is the change in position when calculating the change in potential energy.

Example 8 How much work would it take to move a charge $q = +1 \times 10^{-2}$ C from Point A to Point B (the point midway between q_1 and q_2)? Note: We are using the data from Example 8.



Solution. $\Delta U_{\mathrm{E}} = q\Delta V$, so if we calculate the potential difference between Points A and B and multiply by q, we will have found the change in the electrical potential energy: $\Delta U_{\mathrm{A}\to\mathrm{B}} = q\Delta V_{\mathrm{A}\to\mathrm{B}}$. Then, since the work by the electric field is $-\Delta U$, the work required by an external agent is ΔU.

First, we need the potential at point A, V_A. Since there are two charges q_1 and q_2 contributing to the potential at point A, we calculate the contribution of each using $V \frac{kQ}{r}$ and sum the results. Remember that the potential is a scalar quantity and so no vector addition is required here. It is very important to keep track of the positive and negative signs, however.

$$V_A = \Sigma k\left(\frac{q_i}{r_r}\right) = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) = \left(9 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)\left(\frac{4 \times 10^{-9}\,\mathrm{C}}{0.05\ \mathrm{m}} + \frac{-6 \times 10^{-9}\,\mathrm{C}}{0.03\ \mathrm{m}}\right)$$

$$V_A = -1080\ \mathrm{V}$$

A similar calculation can be carried out for point B, although the distance from the charges will be different for point B than they were for point A.

$$V_B = \Sigma k\left(\frac{q_i}{r_r}\right) = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) = \left(9 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)\left(\frac{4 \times 10^{-9}\mathrm{C}}{0.02\ \mathrm{m}} + \frac{-6 \times 10^{-9}\mathrm{C}}{0.02\ \mathrm{m}}\right)$$

$$V_B = -900\ \mathrm{V}$$

$\Delta V_{\mathrm{A}\,\rightarrow\,\mathrm{B}} = V_{\mathrm{B}} - V_{\mathrm{A}} = (-900\ \mathrm{V}) - (-1080\ \mathrm{V}) = +180\ \mathrm{V}$. This means that the change in electrical potential energy as q moves from A to B is

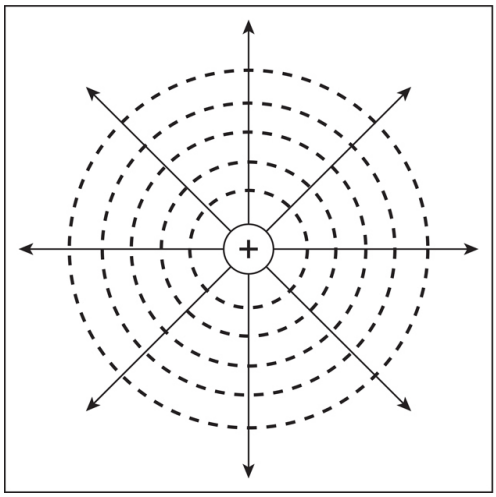
$$\Delta U_{\mathrm{A}\rightarrow\mathrm{B}} = q\Delta V_{\mathrm{A}\rightarrow\mathrm{B}} = (+1 \times 10^{-2}\ \mathrm{C})(+180\ \mathrm{V}) = 1.8\ \mathrm{J}$$

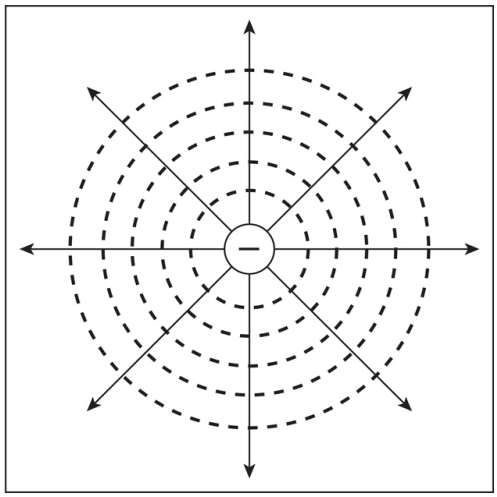
This is the work required by an external agent to move q from A to B.

EQUIPOTENTIAL CURVES AND EQUIPOTENIAL MAPS

Chapter 6 electric field diagrams are a simple way to represent a sometimes very complex result of the influence of a charge distribution created by several charges. A similar image may be formed using isolines of electric potential instead of electric field vectors. In a downward gravitational field, if an object were to move from one position on a line at a constant height to another position on the line, no work would be done because the displacement would be perpendicular to the gravitational field. Electric equipotential curves are similar. A charge may be moved from one position on an equipotential curve to another without any work being required. A drawing of several equipotential curves at various value of the potential for a charge distribution (which may or may not be specified) is called an **equipotential map.** These images may look somewhat familiar as they are very similar to topographical maps, which plot lines of constant height in a uniformly downward directed gravitational field.

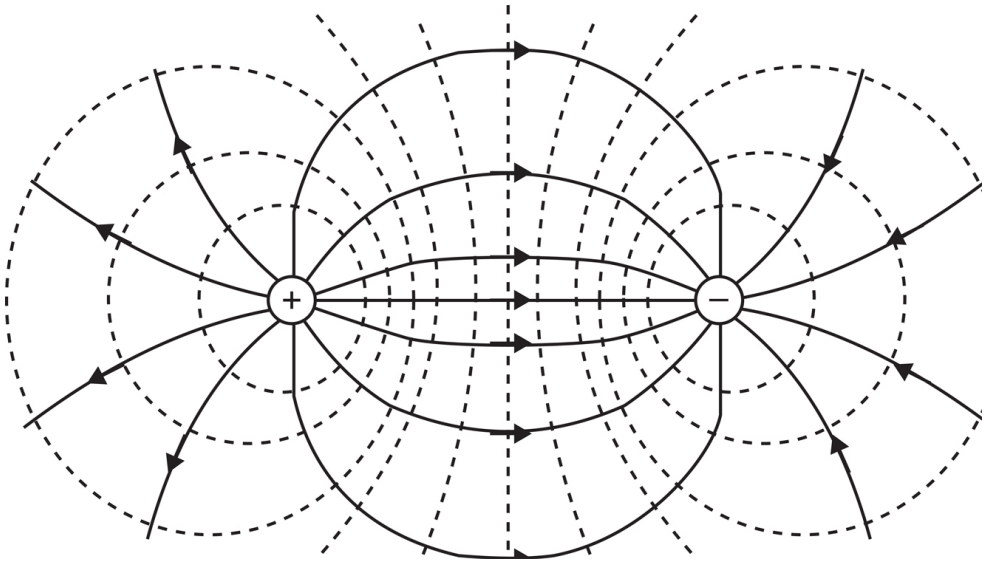
Here are the electric field (which should look very familiar from chapter 5) and some equipotential lines for isolated positive or negative point charges. The field lines are shown in solid and the equipotentials are shown as dashed lines.





The equipotential map for an isolated positive charge is exactly the same shape as for an isolated negative charge. Both equipotential maps are made out of concentric circles. Notice that everywhere the equipotential curves meet the field lines, the two types of curves meet at a right angle. This must be the case for an equipotential curve to require no work to move along. Imagine if a field component were directed opposite the motion along an equipotential curve. Then an external agent would have to push harder to maintain the same speed along the curve, and that would mean some outside work were being done. Despite the fields for positive and negative charges being identical, it is possible to determine whether the charge at the center is positive or negative whenever two or more of the equipotential lines were labeled with values. By definition, the potential will be higher (or more positive) close to a positive charge and lower (or more negative) close to a negative charge.

Below is the equipotential map and some electric field lines for an electric dipole.



Example 9 Describe the equipotential lines in areas where the electric field is relatively strong compared to the areas where the electric field is relatively weak.

Solution. The electric field is strong where the field lines, which are solid here, densely packed and weak where they are spread out. Therefore, using the dipole field shown above as a reference, the field is strong between the two charges and weak off to the sides. Comparing the equipotential lines at those two locations, we can conclude that where the equipotential lines are close together, the field is strong, and as the distance between adjacent equipotential lines increases, the field strength decreases. This is consistent with the relationship $|\vec{E}| = \left|\frac{\Delta V}{r}\right|$ as the equipotential map shows lines at a constant voltage difference ΔV, so the field strength, $|\vec{E}|$, is strong when the distance, r, between the lines is small.

THE ELECTRIC POTENTIAL OF A UNIFORM FIELD

Example 10 Consider a very large, flat plate that contains a uniform surface charge density σ. At points that are not too far from the plate, the electric field is uniform and given by the equation

$$E = \frac{\sigma}{2\varepsilon_0}$$

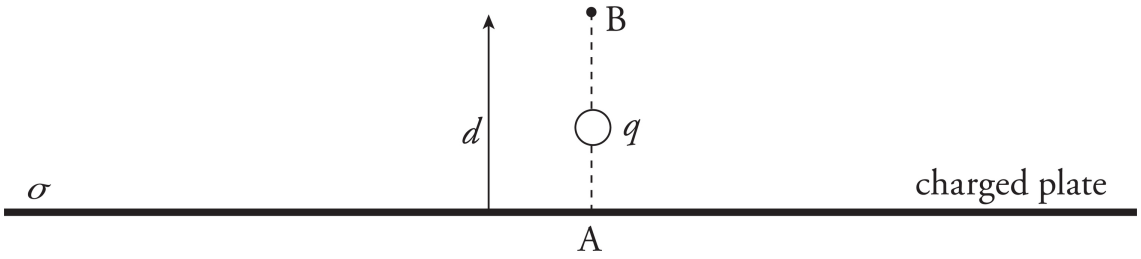
What is the potential at a point which is a distance d from the sheet (close to the plates), relative to the potential of the sheet itself?

Solution. Let A be a point on the plate and let B be a point a distance d from the sheet. Then

$$V_B - V_A = \frac{-W_{E,\,A\to B} \text{ on } q}{q}$$

Since the field is constant, the force that a charge q would feel is also constant, and is equal to

$$F_E = qE = q\frac{\sigma}{2\varepsilon_0}$$



Therefore,

$$W_{E,\,A\to B} = F_E d = \frac{q\sigma}{2\varepsilon_0} d$$

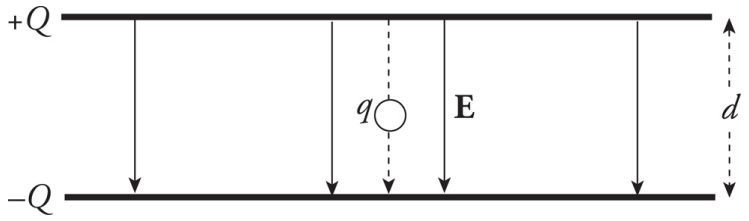
so applying the definition gives us

$$V_B - V_A = \frac{-W_{E,\,A\to B}}{q} = -\frac{\sigma}{2\varepsilon_0} d$$

This says that the potential decreases linearly as we move away from the plate.

Example 11 Two large flat plates—one carrying a charge of + Q, the other $-Q$—are separated by a distance d. The electric field between the plates, **E**, is uniform. Determine the potential difference between the plates.

Solution. Imagine a positive charge q moving from the positive plate to the negative plate:



Since the work done by the electric field is

$$W_{\mathrm{E},+\to-} = F_{\mathrm{E}}d = qEd$$

the potential difference between the plates is

$$V_- - V_+ = \frac{-W_{\mathrm{E},+\to-}}{q} = \frac{-qEd}{q} = -Ed$$

This tells us that the potential of the positive plate is greater than the potential of the negative plate, by the amount *Ed*. This equation can also be written as

$$E = -\frac{V_- - V_+}{d}$$

Therefore, if the potential difference and the distance between the plates are known, then the magnitude of the electric field can be determined quickly. The magnitude is simply

Equation Sheet

$$E = \left|\frac{\Delta V}{d}\right|$$

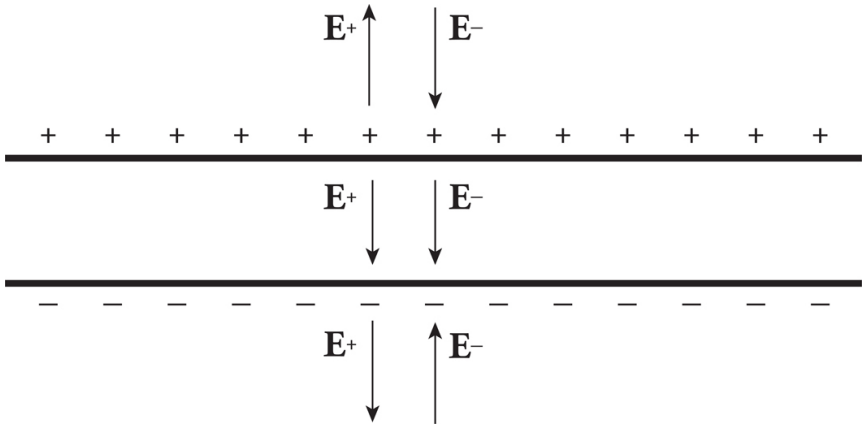
The River: Dam

In terms of a circuit (which we will cover in the next chapter), we can think of the capacitor as a dam in a river. Water builds up in one section of the balloon. When air pressure pushes on one side, air gets pushed into the balloon, but eventually the balloon gets filled up, unless you blow a lot harder, no more air will go into the capacitor.

CAPACITORS AND CAPACITANCE

Consider two conductors, separated by some distance, that carry equal but opposite charges, $+Q$ and $-Q$. Such a pair of conductors comprise a system called a **capacitor.** Work must be done to create this separation of charge, and, as a result, potential energy is stored. Capacitors are basically storage devices for electrical potential energy.

The conductors may have any shape, but the most common conductors are parallel metal plates or sheets. These types of capacitors are called **parallel-plate capacitors.** We'll assume that the distance d between the plates is small compared to the dimensions of the plates since, in this case, the electric field between the plates is uniform. The electric field due to *one* such plate, if its surface charge density is $\sigma = Q/A$, is given by the equation $E = \sigma/(2\varepsilon_0)$, with **E** pointing away from the sheet if σ is positive and toward the plate if σ is negative.



What Determines Capacitance?

Capacitance does NOT determine the charge on the capacitor, *Q*, or the potential difference, *V*, across it. It only shows a relationship between *Q* and *V*. The greater the voltage applied to a capacitor, the greater the amount of charge we can hold on the capacitor. Think of capacitance as a property of a physical object. Physically, capacitance is determined by two things: Area of the plates, *A*, and the separation between the plates, *d.*

Therefore, with two plates, one with surface charge density $+\sigma$ and the other $-\sigma$, the electric fields combine to give a field that's zero outside the plates and that has the magnitude

$$E_{\text{total}} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

in between.

Equation Sheet

$$E_{total} = \frac{Q}{\varepsilon_0 A}$$

In Example 11, we learned that the magnitude of the potential difference, ΔV, between the plates satisfies the relationship $\Delta V = Ed$, so combining this with the previous equation, we get

$$E = \frac{\sigma}{\varepsilon_0} \quad \Rightarrow \quad \frac{\Delta V}{d} = \frac{\sigma}{\varepsilon_0} \quad \Rightarrow \quad \frac{\Delta V}{d} = \frac{Q/A}{\varepsilon_0} \quad \Rightarrow \quad \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$

The ratio of *Q* to ΔV, for *any* capacitor, is defined as its **capacitance** (*C*),

Equation Sheet

$$C = Q/\Delta V$$

so for a parallel-plate capacitor, we get

Equation Sheet

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

The capacitance measures the capacity for holding charge. The greater the capacitance, the more charge can be stored on the plates at a given potential difference. The capacitance of any capacitor depends only on the size, shape, and separation of the conductors, and the "dielectric constant", κ, that depends on what material is between the plates of the capacitor. When there is nothing in the gap between the plates, then $\kappa = 1$. From the definition, $C = Q/\Delta V$, the units of C are coulombs per volt. One coulomb per volt is renamed one **farad** (abbreviated F): 1 C/V = 1 F.

Example 12 A 10-nanofarad parallel-plate capacitor holds a charge of magnitude 50 μC on each plate.

(a) What is the potential difference between the plates?

(b) If the plates are separated by a distance of 0.2 mm, what is the area of each plate?

Solution.

(a) From the definition, $C = Q/\Delta V$, we find that

$$\Delta V = \frac{Q}{C} = \frac{50 \times 10^{-6}\ \text{C}}{10 \times 10^{-9}\ \text{F}} = 5000\ \text{V}$$

(b) Because $\kappa = 1$, we have the equation $C = \varepsilon_0 A/d$, and can calculate the area, A, of each plate:

$$A = \frac{Cd}{\varepsilon_0} = \frac{(10 \times 10^{-9}\ \text{F})(0.2 \times 10^{-3}\ \text{m})}{8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2} = 0.23\ \text{m}^2$$

"Uniform electric field" is only approximately true.

The electric field is only constant everywhere in space for an infinitely large sheet of charge, and such a thing is impossible to construct. However, whenever the observation location is sufficiently close to the plates, or the edges of the plates are sufficiently far away, the constant electric field approximation holds true.

ELECTRIC FIELD AND CAPACITORS

When we studied electric fields in the preceding chapter, we noticed that the electric field created by one or more point source charges varied, depending on the location. For example, as we move farther away from the source charge, the electric field gets weaker. Even if we stay at the same distance from, say, a single source charge, the direction of the field changes as we move around. Therefore, we could never obtain an electric field that was constant in both magnitude and direction throughout some region of space from point-source charges. However, the electric field that is created between the plates of a charged parallel-plate capacitor is constant in both magnitude and direction throughout the region between the plates; in other words, a charged parallel-plate capacitor can create a uniform electric field. The electric field, **E**, always points from the positive plate toward the negative plate, and its magnitude remains the same at every point between the plates, whether we choose a point close to the positive plate, closer to the negative plate, or between them.

Because **E** is so straightforward (it's the same everywhere between the plates), the equation for calculating it is equally straightforward, $V = Ed$. The strength of **E** depends upon the voltage between the plates, ΔV.

The equation $F = qE$ showed us that the units of E are N/C. The formula $\Delta V = Ed$ tells us that the units of **E** are also V/m. You'll see both newtons-per-coulomb and volts-per-meter used as units for the electric field; it turns out these units are exactly the same.

Example 13 The charge on a parallel-plate capacitor is 4×10^{-6} C. If the distance between the plates is 2 mm and the capacitance is 1 μF, what's the strength of the electric field between the plates?

Solution. Since $C = Q/\Delta V$, we have $\Delta V = Q/C = (4 \times 10^{-6}\ \text{C})/(10^{-6}\ \text{F}) = 4\ \text{V}$. Now, using the equation $\Delta V = Ed$,

$$E = \Delta V/d = 4\Delta V/(2 \times 10^{-3}\ \text{m}) = 2000\ \text{V/m}$$

Example 14 The plates of a parallel-plate capacitor are separated by a distance of 2 mm. The device's capacitance is 1 μF. How much charge needs to be transferred from one plate to the other in order to create a uniform electric field whose strength is 10^4 V/m?

Solution. Because $Q = C\Delta V$ and $\Delta V = Ed$, we find that

$$Q = CEd = (1 \times 10^{-6}\ \text{F})(1 \times 10^{4}\ \text{V/m})(2 \times 10^{-3}\ \text{m}) = 2 \times 10^{-5}\ \text{C} = 20\ \mu\text{C}$$

THE ENERGY STORED IN A CAPACITOR

To figure out the electrical potential energy stored in a capacitor, imagine taking a small amount of negative charge off the positive plate and transferring it to the negative plate. This requires that positive work is done by an external agent, and this is the reason that the capacitor stores energy. If the final charge on the capacitor is Q, then we transferred an amount of charge equal to Q, fighting against the prevailing voltage at each stage. If the final voltage is ΔV, then the average voltage during the charging process is $\frac{1}{2}\Delta V$; so, because ΔU_{E} is equal to charge times voltage, we can write that for a capacitor $\Delta U_C = Q \cdot \frac{1}{2}\Delta V = \frac{1}{2}Q\Delta V$. At the beginning of the charging process, when there was no charge on the capacitor, we had $U_{\mathrm{i}} = 0$, so $\Delta U_C = U_{\mathrm{f}} - U_{\mathrm{i}} = U_{\mathrm{f}} - 0 = U_{\mathrm{f}}$; therefore, we have

Equation Sheet

$$U_C = \frac{1}{2} Q \Delta V$$

This is the electrical potential energy stored in a capacitor. Because of the definition $C = Q/\Delta V$, the equation for the stored potential energy can be written as

$$U_C = \frac{1}{2}(C\Delta V) \cdot \Delta V = \frac{1}{2} C (\Delta V)^2$$

or

$$U_C = \frac{1}{2} Q \cdot \frac{Q}{C} = \frac{Q^2}{2C}$$

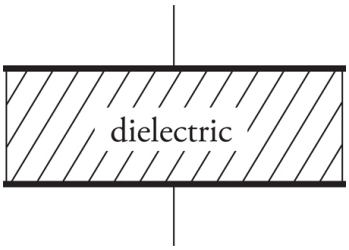
Interestingly, this work is typically done by a battery as the external agent. The battery supplies a total work of $U = QV$, but the capacitor only ends up storing half of this energy. The other 50% of the energy supplied by the battery is dissipated as heat during this charging process.

Insulator?

Why not a conductor? If we place a conductor, It defeats the purpose of a capacitor because now electrons are free to pass.

CAPACITORS AND DIELECTRICS

One method of keeping the plates of a capacitor apart, which is necessary to maintain charge separation and store potential energy, is to insert an insulator (called a dielectric) between the plates.



A dielectric always increases the capacitance of a capacitor.

Let's see why this is true. Imagine charging a capacitor to a potential difference of ΔV with charge $+Q$ on one plate and $-Q$ on the other. Now disconnect the capacitor from the charging source and insert a dielectric. What happens? Although the dielectric is not a conductor, the electric field that existed between the plates causes the molecules within the dielectric material to polarize; there is more electron density on the side of the molecule near the positive plate.

The effect of this is to form a layer of negative charge along the top surface of the dielectric and a layer of positive charge along the bottom surface; this separation of charge induces its own electric field ($\mathbf{E}_i$), within the dielectric, which opposes the original electric field, **E**, within the capacitor.

So the overall electric field has been reduced from its previous value: $\mathbf{E}_{total} = \mathbf{E} + \mathbf{E}_i$, and $\mathbf{E}_{total} = \mathbf{E} - \mathbf{E}_i$. Let's say that the electric field has been reduced by a factor of κ (the Greek letter *kappa*) from its original value as follows

$$\mathbf{E}_{\text{with dielectric}} = \mathbf{E}_{\text{without dielectric}} - \mathbf{E}_{\text{i}} = \frac{\mathbf{E}}{\kappa}$$

Since $\Delta V = Ed$ for a parallel-plate capacitor, we see that ΔV must have decreased by a factor of κ. But $C = C = \frac{Q}{\Delta V}$, so if ΔV decreases by a factor of κ, then C increases by a factor of κ:

$$C_{\text{with dielectric}} = \kappa C_{\text{without dielectric}}$$

The value of κ, called the **dielectric constant**, varies from material to material, but it's always greater than 1. In general, the capacitance of parallel-plate capacitors is

Equation Sheet

$$C = \kappa \varepsilon_0 A/d$$

Chapter 6 Review Questions

Click here to download the PDF of Chapter 6 Review Questions.

Solutions can be found in Chapter 12.

Section I: Multiple Choice

1. An experiment is conducted and data is gathered for the electric potential V at various positions r away from a uniformly charged sphere. All measurements are taken outside of the sphere. Which of the following graphs yields a straight line?

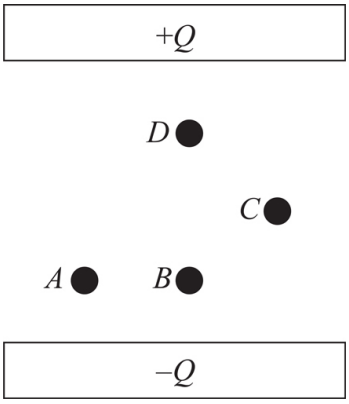
   (A) V as a function of $\frac{1}{r^2}$

   (B) V as a function of $\frac{1}{r}$

   (C) V as a function of r

   (D) V as a function of r^2

2. Below is shown a section near the center of a parallel plate capacitor. There are 4 labeled positions between the plates shown as A, B, C, and D. Relative to A, which point has the largest potential difference and why?



(A) Point A because the potential difference is infinite when the position between points is 0 m.

(B) Point B because it is the same distance from the $-Q$ plate as A.

(C) Point C because it is farther in distance from Point A.

(D) Point D because it is closest to the $+Q$ Plate.

3. Negative charges are accelerated by electric fields toward points

(A) at lower electric potential

(B) at higher electric potential

(C) where the electric field is weaker

(D) where the electric field is stronger

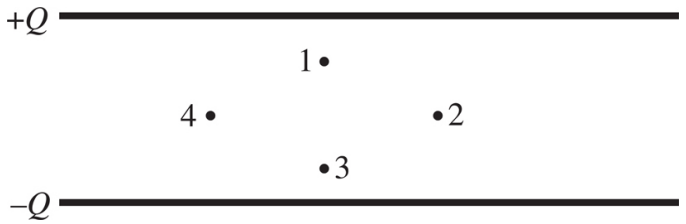
4. A charge q experiences a displacement within an electric field from Position A to Position B. The change in the electrical potential energy is ΔU_E, and the work done by the electric field during this displacement is W_E. Then

(A) $V_A - V_B = qW_E$

(B) $V_B - V_A = qW_E$

(C) $V_A - V_B = \Delta U_E/q$

(D) $V_B - V_B = \Delta U_E/q$



5. Which points in this uniform electric field (between the plates of the capacitor) shown above lie on the same equipotential?

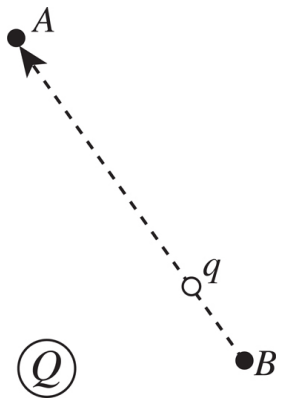
(A) 1 and 3 only

(B) 2 and 4 only

(C) None lie on the same equipotential.

(D) 1, 2, 3, and 4 all lie on the same equipotential since the electric field is uniform.

6. A charge Q creates an electric field through which a second charge q moves, as shown below. q is initially at point B and is moved to point A. The potential from Q at position A is V_A = 100 V and at B is V_B = 200 V. The charge on q is negative. What is the sign of Q and the sign of the work done by the electric field of Q as q is moved from B to A?



(A) Q is positive and the work done by the electric field is positive.

(B) Q is positive and the work done by the electric field is negative.

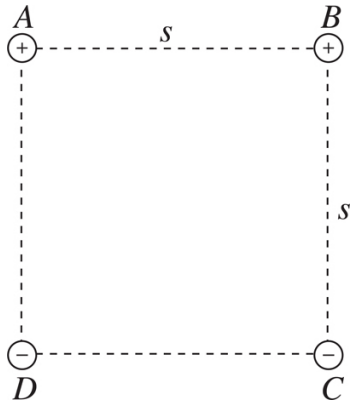
(C) Q is negative and the work done by the electric field is positive.

(D) Q is negative and the work done by the electric field is negative.

Section II: Free Response

1. In the figure shown below, four charges, each of magnitude Q, are situated at the corners of a square with side lengths s. The two charges on the top of the square are positively charged,

while the two on the bottom of the square are negatively charged.



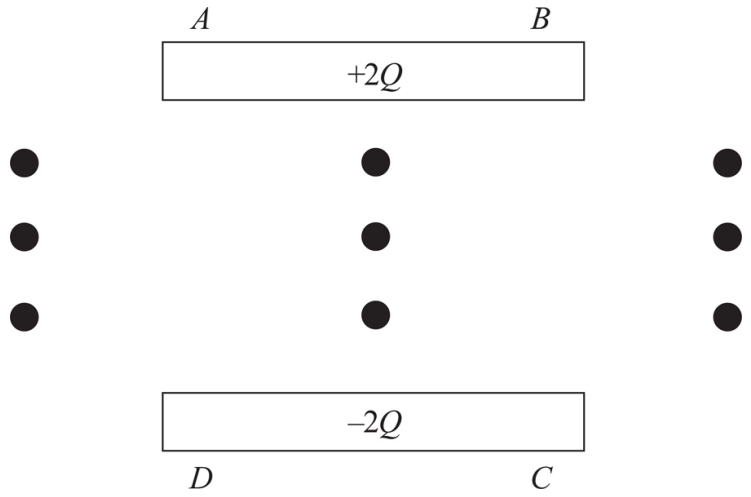
(a) These charges were assembled in order by first bringing in charge *A*, then bringing in charge *B*, then bringing in *C*, and finally bringing in charge *D*. Rank the amount of energy in the charge distribution in the present of charge only charge *A*, only charges *A* and *B*, only charges *A*, *B*, and *C*, and in the presence of all four charges. Negative numbers should be taken as smaller then positive numbers. Justify your answer.

Greatest (most positive) ____ ____ ____ ____ Least (most negative)

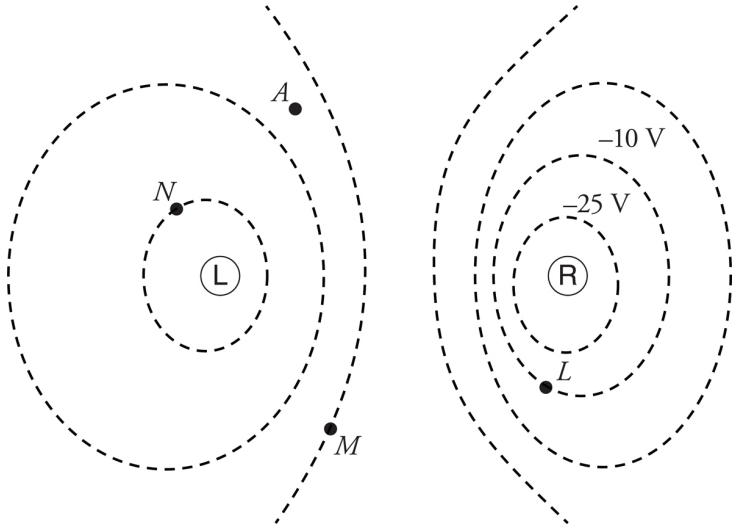
(b) Show the potential at the exact center of the square is 0 V by calculating the potential from each charge at that location.

(c) Sketch (on the diagram) the portion of the equipotential surface that lies in the plane of the figure and passes through the center of the square.

(d) As shown below, solid conducting bars are placed to connect points *A* to *B* and also to connect points *C* to *D*. The charge is allowed to distribute over these conductors. Sketch the electric field at each of the dots. Explain why the field is constant at the dots in the center but not at the dots on the edges.



2. The image below shows the isolines of electric potential surrounding two charges spheres, labeled L and R. The spheres carry opposite charges and the potential difference between adjacent pairs of lines is $\Delta V = 15$ V. The isolines with a voltages of −25 V and −10 V are indicated.



(a) Sketch in the line of potential 0 V on the drawing.

(b) Approximately what is the value of the potential at the points labeled *A*?

(c)

i. Which sphere, L or R, carries a negative charge? Explain your answer.

ii. Which sphere, L or R, carries a greater magnitude of charge? Explain how you know.

(d)

i. Draw arrows to indicate the electric fields at the points labeled *L, M*, and *N*.

ii. Rank the magnitude of the electric field strength at points *L, M*, and *N*. Explain your answer.

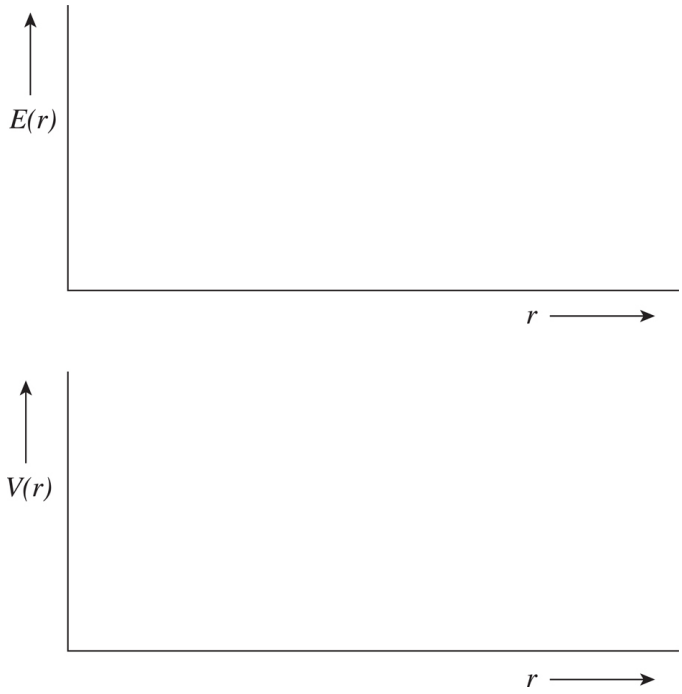
(e) If the sphere labeled L were replaced with another sphere to have the same magnitude of charge but of the opposite sign, would the value of the potential at the point labeled *N* be larger, smaller, or stay the same? Justify your answer.

3. A solid conducting sphere of radius a carries an excess charge of Q.

(a) Determine the electric field magnitude, $E(r)$, as a function of r, the distance from the sphere's center.

(b) Determine the potential, $V(r)$, as a function of r. Take the zero of potential at $r = \infty$.

(c) On the diagrams below, sketch $E(r)$ and $V(r)$. (Cover at least the range $0 < r < 2a$.)


E(r)
r
V(r)
r

Summary

- The electric potential (commonly referred to as the voltage) is defined as $V = \frac{\Delta U_E}{q}$.
- The electrical potential energy is defined by

  $\Delta U_E = -W_E$ or $U_E = qV$ or $U_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$.
- The work done moving a charge through an electric field is given by $W = qEd$.
- A capacitor is basically a storage device for electrical potential energy. Capacitance is given by $C = \frac{Q}{V}$. For parallel plate capacitors which are separated by vacuum, we have $C = \frac{\varepsilon_0 A}{d}$, where $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2, A is the area of the plate, and d is the distance between the plates.
- When a capacitor is filled with a dielectric, its capacitance increases from the capacitance it has when it had a vacuum between its plates.
- The electrical energy stored in a capacitor is given by either $U_C = \frac{1}{2}QV$ or $U_C = \frac{1}{2}CV^2$.

Chapter 7
Direct Current Circuits

INTRODUCTION

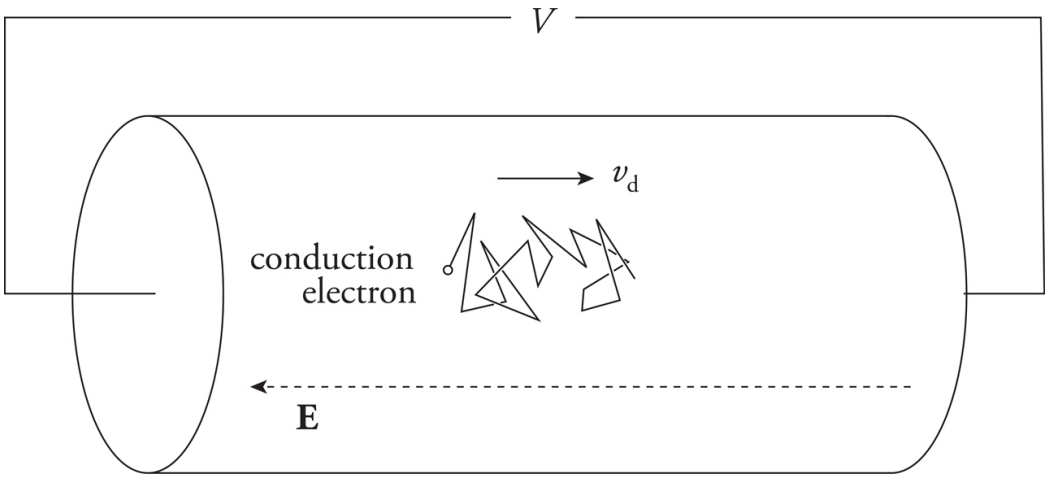
Today, electricity is well-known and appears all around us, from the lighting in our houses to the computing in our personal computers. All of these are powered by complex circuits. In this chapter, we will study the basics of simple direct current circuits.

A Note on Convention for this Chapter

In this chapter, we will use the standard convention of referring to a change in potential as a voltage, V. In the previous chapter, we referred to these changes in potential as ΔV Make note of this as you move forward so as to not be confused.

ELECTRIC CURRENT

Picture a piece of metal wire. Within the metal, electrons are zooming around at speeds of about a million m/s in random directions, colliding with other electrons and positive ions in the lattice. This constitutes charge in motion, but it doesn't constitute *net* movement of charge, because the electrons move randomly. If there's no net motion of charge, there's no current. However, if we were to create a potential difference between the ends of the wire, meaning if we set up an electric field, the electrons would experience an electric force, and they would start to drift through the wire. This is current. Although the electric field would travel through the wire at nearly the speed of light, the electrons themselves would still have to make their way through a crowd of atoms and other free electrons, so their **drift speed**, v_d, would be relatively slow, on the order of a millimeter per second.



Wind as an Analogy to Current

Think of current as the wind. Blowing wind is comprised of many air molecules moving randomly and colliding with one another and other things the air molecules encounter, but overall there is a flow of air in a particular direction. When talking about wind, the rate of motion of air molecules is being considered. When talking about current, the rate of motion of electric charge is under consideration.

To measure the current, we have to measure how much charge crosses a plane per unit time. If an amount of charge of magnitude ΔQ crosses an imaginary plane in a time interval Δt, then the **average current** is

Equation Sheet

$$I_{\text{avg}} = \frac{\Delta Q}{\Delta t}$$

Because current is charge per unit time, it's expressed in coulombs per second. One coulomb per second is an **ampere** (abbreviated **A**), or amp. So 1 C/s = 1 A.

Although the charge carriers that constitute the current within a metal are electrons, the direction of the current is taken to be the direction that *positive* charge carriers would move. (This is explicitly stated on the AP Physics 2 Exam.) So, if the conduction electrons drift to the right, we'd say the current points toward the left.

Benjamin Franklin

Conventional direction of current is “wrong” because Benjamin Franklin made an error when he first started investigating electrostatics. Current direction is in the flow of positive charges, but electrons flow opposite to this convention.

RESISTANCE AND RESISTIVITY

Let's say we had a copper wire and a glass fiber that had the same length and cross-sectional area, and that we hooked up the ends of the metal wire to a battery and measured the resulting current. If we were to do the same thing with the glass fiber, the current would probably be too small to measure, but why? Well, the glass provided more resistance to the flow of charge.

Resistance vs. Resistivity

Resistance and Resistivity are NOT the same. Resistance is the extent to which a resistor impedes the passage of electric current. Resistivity is a property of what molecules a material consists of and how the molecules that constitute a material are bound together. The units of resistivity are different as well.

The resistance of an object depends on two things: the material it's made of and its shape. For example, think again of the copper wire and glass fiber of the same length and area. They have the same shape, but their resistances are different because they're made of different materials. Glass has a much greater **resistivity** than copper does. Each material has its own characteristic resistivity, and resistance depends on this property as well as on how the material is shaped. For a wire of length L and cross-sectional area A made of a material with resistivity ρ, resistance is given by

Equation Sheet

$$R = \frac{\rho \ell}{A}$$

This equation only applies to shapes with constant cross-sectional area, such as a cylinder or a rectangular prism, and will not apply to those with a varying cross-sectional area, such as a sphere or a cone.

The resistivity of copper is around 10^{-8} Ω·m, while the resistivity of glass is *much* greater, around 10^{12} Ω·m.

Example 1 A wire of radius 1 mm and length 2 m is made of platinum (resistivity = 1×10^{-7} Ω·m).

Solution. First, the resistance of the wire is given by the equation

$$R = \frac{\rho \ell}{A} = \frac{\rho \ell}{\pi r^2} = \frac{(1 \times 10^{-7}\ \Omega \cdot \text{m})(2\ \text{m})}{\pi (0.001\ \text{m})^2} = 0.064\ \Omega$$

OHM'S LAW

The resistance of a wire depends on the resistivity, length, and cross-sectional area of the wire. Once these three quantities are known, the wire is commonly referred to as a "resistor." This is a misleading name because these are wires, and they conduct electricity.

There are two types of "resistors": those where the current is directly proportional to the voltage and those where the relationship between the current and the voltage is nonlinear. When the current is directly proportional to the voltage applied across the wire,

Equation Sheet

$$\frac{V}{I} = R \text{ or } V = IR$$

This is known as Ohm's Law, and devices which obey Ohm's Law are called Ohmic resistors. When performing calculation with resistors on the AP Physics 2 Exam, they will (almost) always will be Ohmic, but you may be asked to qualitatively describe non-Ohmic wires.

You can determine whether or not a resistive element in a circuit obeys Ohm's Law or not by making a plot of the voltage versus the current. A nonlinear plot indicates that the resistor in non-Ohmic, while a directly proportional graph will indicate that the wire is Ohmic. Moreover, that graph will have a slope equal to the constant resistance of the wire.

Because resistance is voltage divided by current, it is expressed in units of volts per amp. One volt per amp is one **ohm** (Ω, *omega*). So, 1 V/A = 1 Ω.

Example 1 revisited If a voltage of 9 V is applied between the ends of the wire from example 1, and the wire is Ohmic, what will be the resulting current?

Solution. From Ohm's Law, $I = V/R$, so we get

$$I = \frac{V}{R} = \frac{9\,\text{V}}{0.0064\,\Omega} = 140\ \text{A}$$

VOLTAGE

Now that we know how to measure current, the next question is, What causes it? Why does an electron drift through the circuit? One answer is to say that there's an electric field inside the wire, and since negative charges move in the direction opposite to the electric field lines, electrons would drift opposite the electric field.

Wind as an Analogy to Voltage

The reason that the wind blows is because of differences in air pressure. If air pressure throughout a location is the same, the wind will not blow. Voltage is the source of current flow in a way equivalent to the pressure difference being the source of wind blowing.

Another (equivalent) answer to the question is that there's a difference in electric potential—otherwise known as a potential difference or, most simply, a voltage—between the ends of the wire. Negative charges naturally move from regions of higher potential to lower potential.

Voltage across two points along a conductor creates a current.

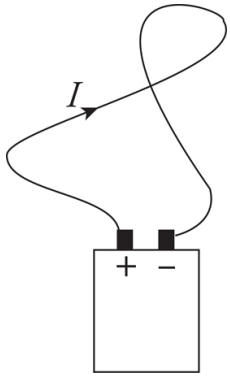
It is not uncommon to see the voltage that creates a current referred to as electromotive force (emf) ε, since it is the cause that sets the charges into motion in a preferred direction. The voltage is provided in a circuit by some source, which is typically a battery for direct current circuits.

Remember

emf is not a force. It is potential difference measured in volts which are J/C, not newtons.

ELECTRIC CIRCUITS

An electric current is maintained when the terminals of a voltage source (a battery, for example) are connected by a conducting pathway, in what's called a **circuit**. If the current always travels in the same direction through the pathway, it's called a **direct current.**



To try to imagine what's happening in a circuit in which a steady-state current is maintained, let's follow one of the charge carriers drifting through the pathway. (Remember we're pretending that the charge carriers are positive.) The charge is introduced by the positive terminal of the battery and enters the wire, where it's pushed by the electric field. It encounters resistance, bumping into the relatively stationary atoms that make up the metal's lattice and setting them into greater motion. So the electrical potential energy that the charge had when it left the battery is turning into thermal energy of the wire. By the time the charge reaches the negative terminal, all of its original electrical potential energy is lost. In order to keep the current going, the voltage source must do positive work on the charge, forcing it to move from the negative terminal toward the positive terminal. The charge is now ready to make another journey around the circuit.

Heat Dissipation

Power in a circuit is very closely associated with heat given off. If we were to touch a light bulb, we would notice it becomes hot. The greater the heat given off, the brighter the light bulb is.

Energy and Power

When a carrier of positive charge q drops by an amount V in potential, it loses potential energy in the amount qV. If this happens in time t, then the rate at which this energy is transformed is equal to $(qV)/t = (q/t)V$. But q/t is equal to the current, I, so the rate at which electrical energy is transferred is given by the equation

Equation Sheet

$$P = IV$$

This equation works for the power delivered by a battery to the circuit as well as for resistors. The power dissipated in a resistor, as electrical potential energy is turned into heat, is given by $P = IV$. When a resistor obeys Ohm's Law, then $V = IR$, we can express this in two other ways:

Power Dissipated by Ohmic Resistors

$$P = IV = I(IR) = I^2R$$

or

$$P = IV = \frac{V}{R} \cdot V = \frac{V^2}{R}$$

Whether or not a resistor obeys Ohm's Law, the power dissipated by the resistor is always $P = IV$. This power dissipation is the reason that resistors become hot when current passes through them.

CIRCUIT ANALYSIS

We will now develop a way of specifying the current, voltage, and power associated with each element in a circuit. Our circuits will contain three basic elements: batteries, resistors, and connecting wires. As we've seen, the resistance of an ordinary metal wire is negligible; resistance is provided by devices that control the current: **resistors.** Resistors are wires with specific length, area, and resistivity which result in a calculated value of R that is relatively high compared to the other wires in the circuit. You can model the wires as having $R = 0$ and then all the resistance of the system is concentrated in the resistors, which are symbolized in a circuit diagram by this symbol



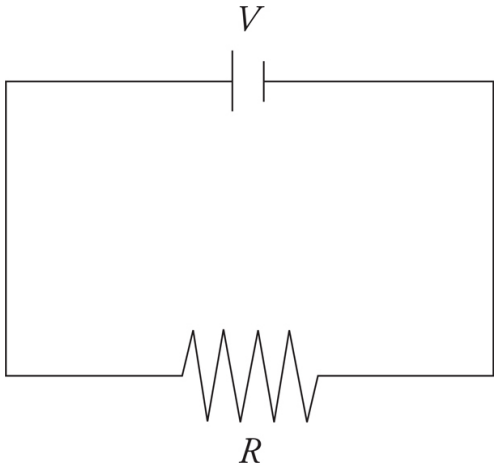
Batteries are denoted by the symbol



where the longer line represents the **positive** (higher potential) terminal, and the shorter line is the **negative** (lower potential) terminal. Sometimes a battery is denoted by more than one pair of such lines:



Here's a simple circuit diagram:

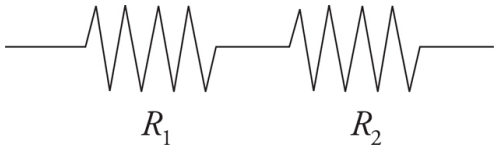


The electric potential (V) of the battery is indicated, as is the resistance (R) of the resistor. As stated before, before the electrons return to the battery, all their energy is lost in the circuit. Since the resistor is the only thing in this circuit that requires energy for charge to move through (when we treat wires as having a resistance of 0 Ω, the moving charges do not make any collisions within the wire and no energy is used to move from one side of the wire to the other), the change in energy of the charge from the battery must equal the change in energy from the resistor, and so the potential difference of the battery must be the same as the potential difference of the resistor as well. Determining the current in this case is straightforward. The equation $V = IR$ gives us

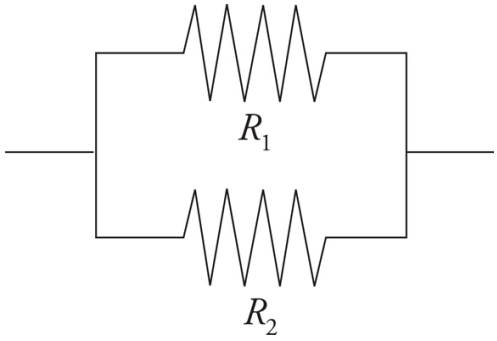
$$I = \frac{V}{R}$$

COMBINATIONS OF RESISTORS

Two common ways of combining resistors within a circuit is to place them either in **series** (one after the other),



or in **parallel** (that is, side-by-side):



EQUIVALENT RESISTANCE

The effect of any two resistors can be equaled by a single resistor with different properties. This single resistor is called an “equivalent resistance.” In order to simplify the circuit, your goal is to find the equivalent resistance of combinations. You can mathematically treat any two resistors as if they were a single equivalent resistor. The way you combine the resistors depends on how they are arranged. A pair of resistors can be arranged either in series or in parallel.

What’s the Same?

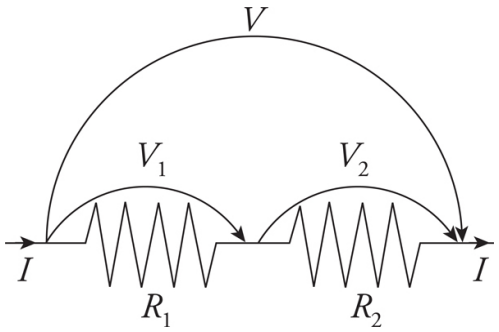
Remember, in a series branch the current is the same through each resistor—in parallel, the voltage drop is the same.

TWO RESISTORS IN SERIES

Resistors are in series if they are arranged one after the next. In such an arrangement, the current that flows into the first resistor must then flow into the second resistor. To construct an equivalent resistor of the same type of wire, the new wire would need to be longer to match this arrangement. From the relationship $R = \rho L / A$, when resistors are arranged in series the equivalent resistance is larger than the resistance of either original resistor. Resistors are said to be in series if they all share the same current and if the total voltage drop across them is equal to the sum of the individual voltage drops across each.

Which Would You Rather Travel?

It makes sense that in a set of parallel resistors a greater amount of current will want to travel through the path of least resistance—if two resistors are in parallel and the first resistor has the greater resistance, the second resistor will have more of the current running through it.



In this case, then, if V denotes the voltage drop across the combination, we have

$$V_{\text{eq}} = V_1 + V_2 = IR_1 + IR_2 = I\left(R_1 + R_2\right) = IR_{\text{eq}}$$

$$R_{\text{eq}} = R_1 + R_2$$

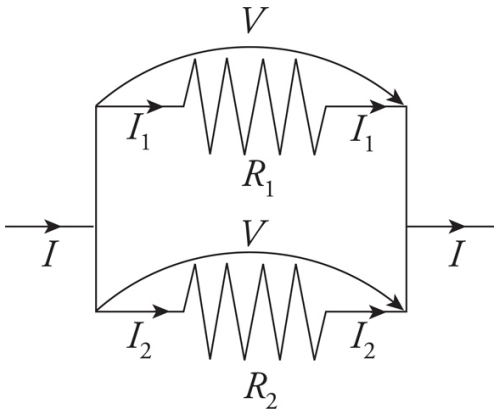
Pay Attention

A common mistake happens when calculating an equivalent resistor for a set of parallel resistors. Write out the equation. Students often forget to take the reciprocal as the final step in calculating the equivalent resistance.

TWO RESISTORS IN PARALLEL

Two resistors are said to be in parallel if the current that flows through one of the resistors cannot flow through the other because there is a junction before the resistors. To construct an equivalent resistor of the same type of wire to match this arrangement, the new wire would need to have a larger area since the charges are able to spread out along either of two paths. From the relationship $R = \rho L / A$, when resistors are arranged in parallel, the equivalent resistance is smaller than the resistance of either original resistor.

The total current entering the combination is split among the resistors. Imagine that a current I enters the combination. It splits; some of the current, I_1, would go through R_1, and the remainder, I_2, would go through R_2. Then the voltage is the same for each resistor and they are in parallel.



So if V is the voltage drop across the combination, we have

$$V = V_1 = V_2$$

$$\frac{V}{R_{eq}} = I = (I_1 + I_2) = \left(\frac{V}{R_1} + \frac{V}{R_2} \right)$$

$$\frac{1}{R_{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

COMBINING MORE THAN TWO RESISTORS

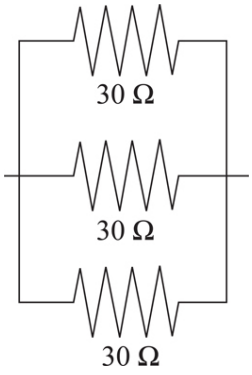
The idea of resistors in series can be applied to any number of resistors as long as they are all in series. which means that any charge that flows through one of the resistors must also flow through the rest. For example, the following three resistors have an equivalent resistance of 6 Ω.



In general, for any number of resistors in series, the equivalent resistance is the sum of the resistance of each resistor:

$$R_s = \Sigma R_i$$

Similarly, the ides of parallel resistors can be applied to any number of resistors that are all arranged in parallel. For example, the following three resistors have an equivalent resistance of 10 Ω.

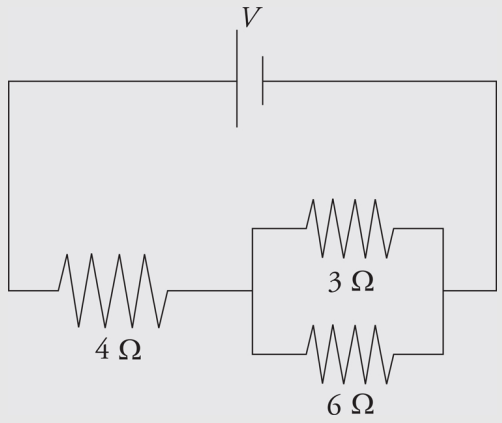


In general, for any number of resistors in parallel, the reciprocal of the equivalent resistance for resistors in parallel is equal to the sum of the reciprocals of the individual resistances:

$$\frac{1}{R_p} = \Sigma\left(\frac{1}{R_i}\right)$$

When a circuit has more than two resistors, it is possible that the combination of resistors are all in parallel or all in series, in which case the equations above give the equivalent resistance of the circuit. However, it is possible that the combination of three (or more) resistors to not be purely in parallel or purely in series, but some combination with parts in parallel and other parts in series. When this is the case, the equivalent resistance of the part of the circuit that is purely series (or parallel) must be found as a first step, and then that resistance is used for further calculations.

Example 2 Calculate the equivalent resistance for the following circuit:



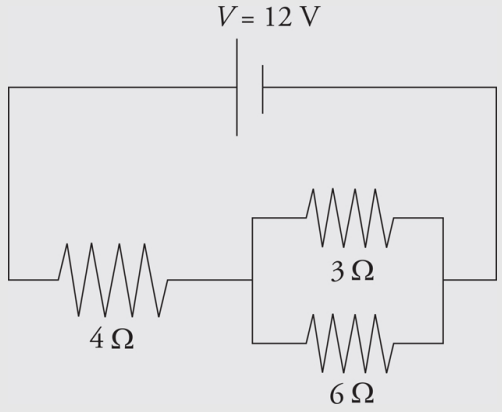
Solution. First, you see that the three resistors are not purely in parallel or purely in series. There must be a pair which is purely in parallel or series. The current that flows through the 4 Ω could flow through the 3 Ω or 6 Ω resistor, so none of the resistors are purely parallel. However, the no current that flows through the 3 Ω can flow through the 6 Ω because of the junction before that combination, so those two resistor are purely parallel. You find the equivalent resistance of the two parallel resistors:

$$\frac{1}{R_p} = \frac{1}{3\ \Omega} + \frac{1}{6\ \Omega} \rightarrow \frac{1}{R_p} = \frac{1}{2\ \Omega} \rightarrow R_p = 2\ \Omega$$

The current that flows into the 4 Ω must flow into this equivalent resistor, so this resistance is in series with the 4 Ω resistor. The total equivalent resistance in the circuit is R = 4 Ω + 2 Ω = 6 Ω.

The reason for considering these equivalent resistances is because the easiest problem to solve is the simple, one-resistor problem presented originally. You can represent many combinations of resistors in series or parallel as a single equivalent resistor, and then use Ohm's Law to solve for the current that flows out of the battery into the circuit.

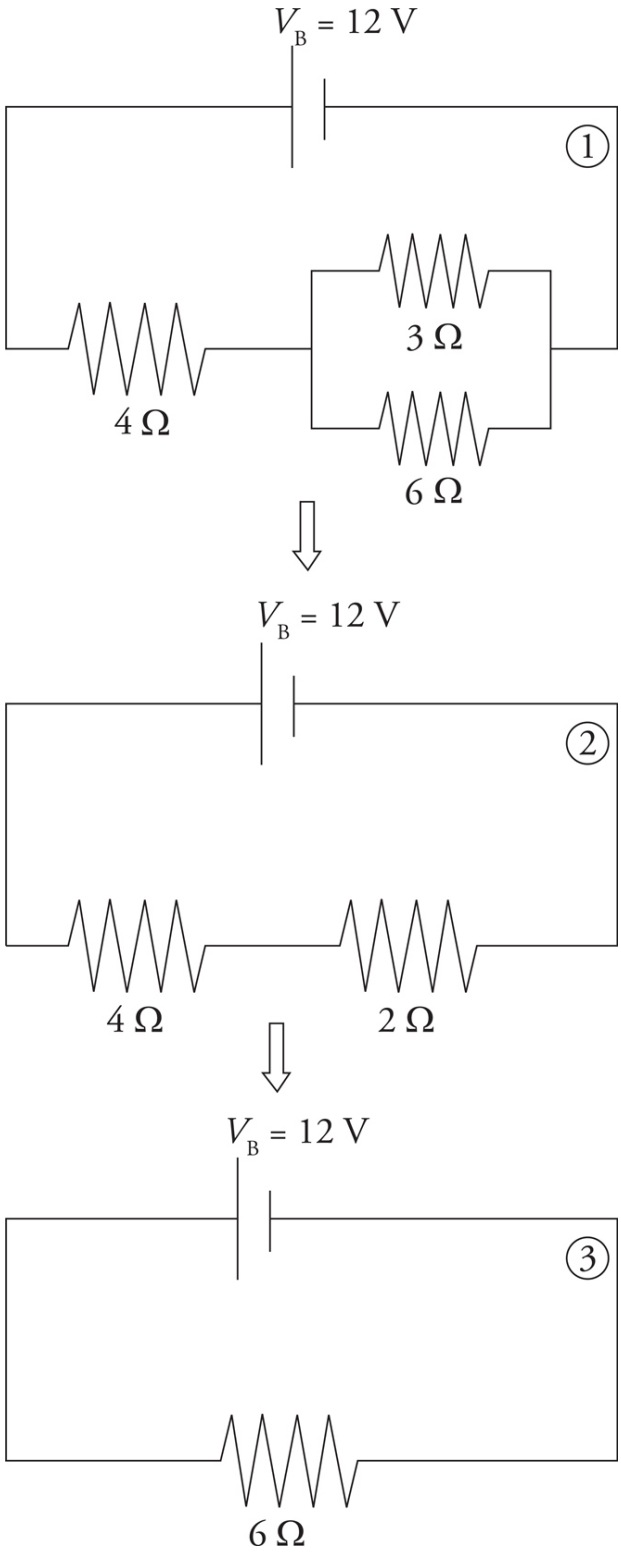
Example 3 Determine the current through each resistor, the voltage drop across each resistor, and the power given off (dissipated) as heat in each resistor of this circuit:



Make It Simple, Then Work Backward

The typical method to working a circuit problem is to make the circuit provided into a simple circuit with a battery and an equivalent resistor. Then to solve for individual values on each resistor, we work backward and build the circuit back to its original form. In the diagram to the right, we made the circuit simpler from diagram 1 to 2 to 3. Then, to solve for the resistors, we work backward from 3 to 2 to 1.

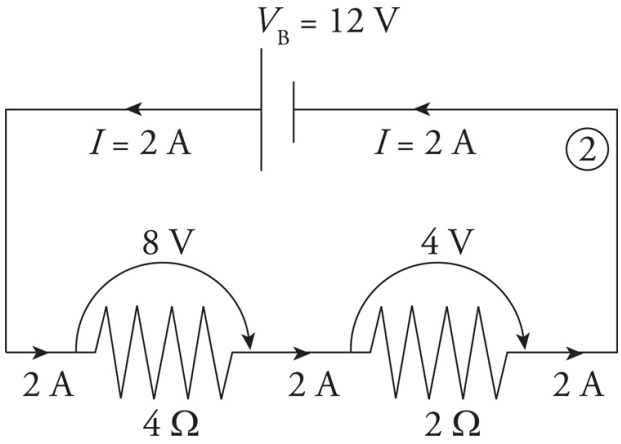
Solution. You might want to redraw the circuit each time we replace a combination of resistors by its equivalent resistance. From our work in the preceding example, we have



From diagram ③, which has just one resistor, we can figure out the current:

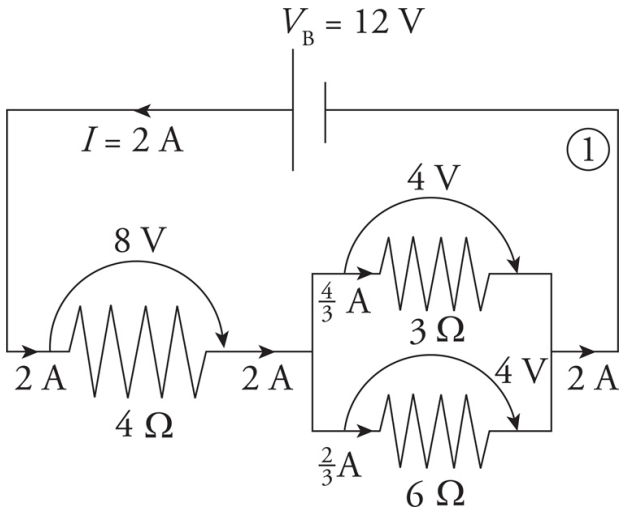
$$I = \frac{V_B}{R} = \frac{12\ \text{V}}{6\ \Omega} = 2\ \text{A}$$

Now we can work our way back to the original circuit (diagram ①). In going from ③ to ②, we are going back to a series combination, and what do resistors in series share? That's right, the same current. So, we take the current, I = 2 A, back to diagram ②. The current through each resistor in diagram ② is 2 A.



Since we know the current through each resistor, we can figure out the voltage drop across each resistor using the equation $V = IR$. The voltage drop across the 4 Ω resistor is (2 A)(4 Ω) = 8 V, and the voltage drop across the 2 Ω resistor is (2 A) (2 Ω) = 4 V. Notice that the total voltage drop across the two resistors is 8 V + 4 V = 12 V, which matches the emf of the battery.

Now for the last step; going from diagram ② back to diagram ①. Nothing needs to be done with the 4 Ω resistor; nothing about it changes in going from diagram ② to ①, but the 2 Ω resistor in diagram ② goes back to the parallel combination. And what do resistors in parallel share? The same voltage drop. So we take the voltage drop, V = 4 V, back to diagram ①. The voltage drop across each of the two parallel resistors in diagram ① is 4 V.



Since we know the voltage drop across each resistor, we can figure out the current through each resistor by using the equation $I = V/R$. The current through the 3 Ω resistor is (4 V)/(3 Ω) = $\frac{4}{3}$ A, and the current through the 6 Ω resistor is (4 V)/(6 Ω) = $\frac{2}{3}$ A. Note that the current entering the parallel combination (2 A) equals the total current passing through the individual resistor $\left(\frac{4}{3}\text{ A} + \frac{2}{3}\text{ A}\right)$. Again, this was expected.

Finally, we will calculate the power dissipated as heat by each resistor. We can use any of the equivalent formulas: $P = IV$, $P = I^2R$, *or* $P = V^2/R$.

For the 4 Ω resistor: $P = IV = (2\text{ A})(8\text{ V}) = 16\text{ W}$

For the 3 Ω resistor: $P = IV = (\frac{4}{3}\text{A})(4\text{ V}) = \frac{16}{3}\text{ W}$

For the 6 Ω resistor: $P = IV = (\frac{2}{3}\text{A})(4\text{ V}) = \frac{8}{3}\text{ W}$

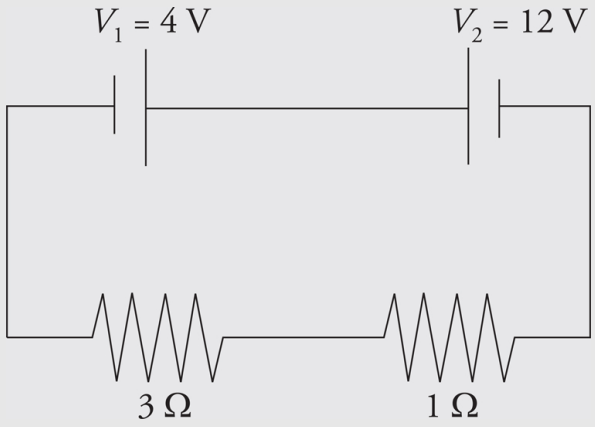
So, the resistors are dissipating a total of

$$16\text{ W} + \frac{16}{3}\text{ W} + \frac{8}{3}\text{ W} = 24\text{ W}$$

If the resistors are dissipating a total of 24 J every second, then they must be provided with that much power. This is easy to check: $P = IV$ = (2 A)(12 V) = 24 W.

Example 4 For the following circuit,

(a) In which direction will current flow and why?
(b) What's the overall emf?
(c) What's the current in the circuit?
(d) At what rate is energy consumed by, and provided to, this circuit?



Solution.

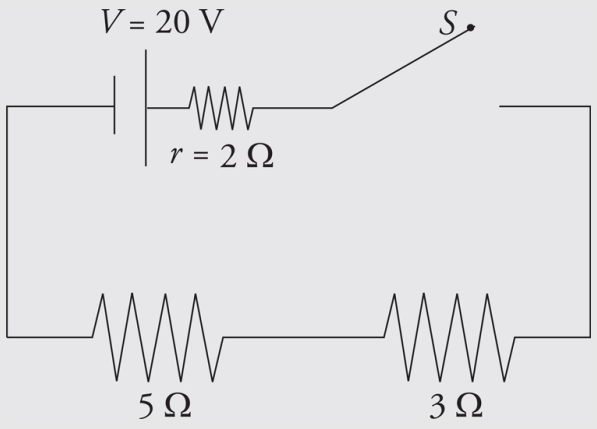
(a) The battery V_1 wants to send current clockwise, while the battery V_2 wants to send current counterclockwise. Since $V_2 > V_1$, the battery whose emf is V_2 is the more powerful battery, so the current will flow counterclockwise.

(b) Charges forced through V_1 will lose, rather than gain, 4 V of potential, so the overall emf of this circuit is $V_2 - V_1 = 8$ V.

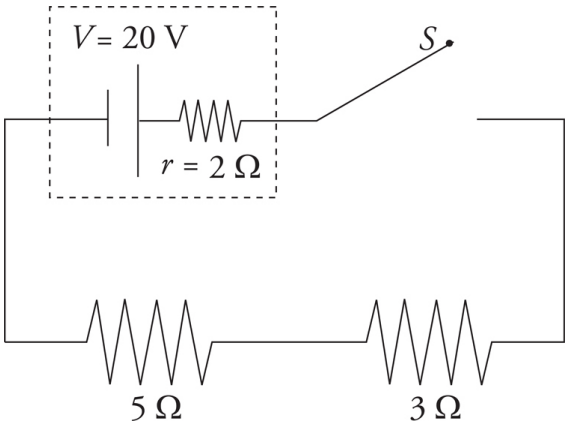
(c) Since the total resistance is 3 Ω + 1 Ω = 4 Ω, the current will be I = (8 V)/(4 Ω) = 2 A. Note that we use the overall emf, and not either of the individual battery emfs.

(d) V_2 will provide energy at a rate of $P_2 = IV_2$ = (2 A)(12 V) = 24 W, while V_1 will absorb at a rate of $P_1 = IV_1$ = (2 A)(4 V) = 8 W. Finally, energy will be dissipated in these resistors at a rate of $I^2R_1 + I^2R_2$ = (2 A)^2(3 Ω) + (2 A)^2(1 Ω) = 16 W. Once again, energy is conserved; the power delivered (24 W) equals the power taken (8 W + 16 W = 24 W).

Example 5 All real batteries contain **internal resistance,** r. Determine the current in the following circuit when the switch S is closed:



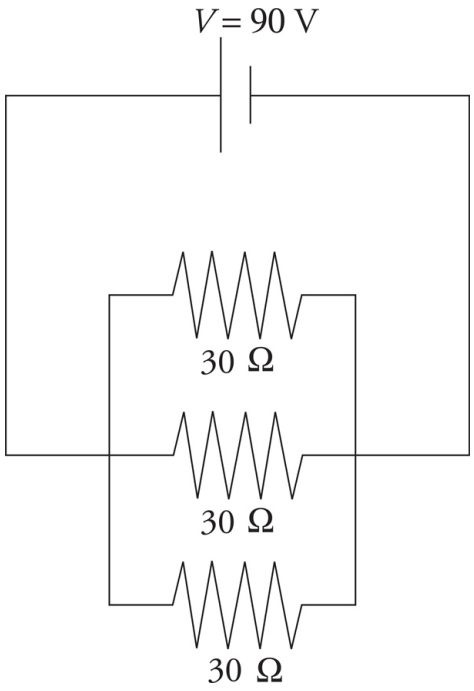
Solution. Before the switch is closed, there is no complete conducting pathway from the positive terminal of the battery to the negative terminal, so no current flows through the resistors. However, once the switch is closed, the resistance of the circuit is 2 Ω + 3 Ω + 5 Ω = 10 Ω, so the current in the circuit is I = (20 V)/(10 Ω) = 2 A. Often the battery and its internal resistance are enclosed in a dashed box:



In this case, a distinction can be made between the emf of the battery and the actual voltage it provides once the current has begun. Since I = 2 A, the voltage drop across the internal resistance is Ir = (2 A)(2 Ω) = 4 V, so the effective voltage provided by the battery to the rest of the circuit—called the **terminal voltage**—is lower than the ideal voltage. It is $V = V_B - Ir$ = 20 V − 4 V = 16 V.

Example 6 A student has three 30 Ω resistors and an ideal 90 V battery. (A battery is *ideal* if it has a negligible internal resistance.) Compare the current drawn from—and the power supplied by—the battery when the resistors are arranged in parallel versus in series.

Solution. Resistors in series always provide an equivalent resistance greater than any of the individual resistances, and resistors in parallel always provide an equivalent resistance smaller than their individual resistances. So, hooking up the resistors in parallel will create the smallest resistance and draw the greatest total current:



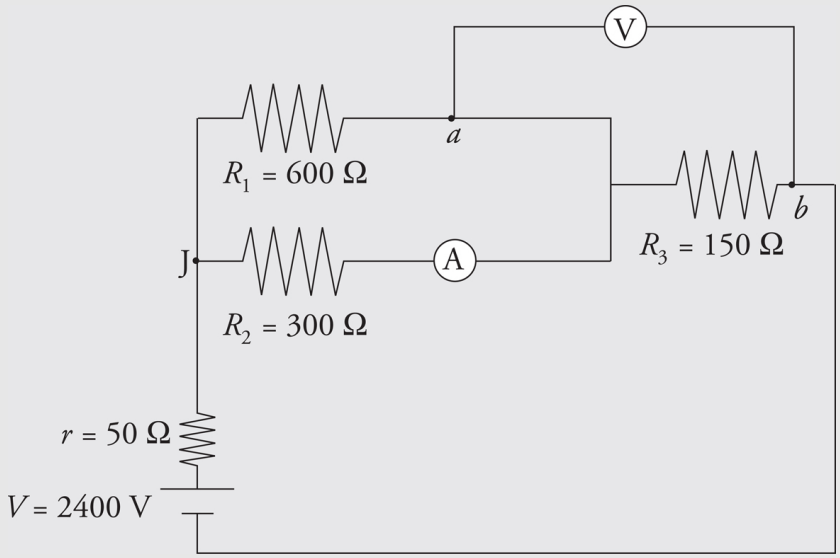
In this case, the equivalent resistance is

$$\frac{1}{R_P} = \frac{1}{30\ \Omega} + \frac{1}{30\ \Omega} + \frac{1}{30\ \Omega} \quad \Rightarrow \quad \frac{1}{R_P} = \frac{1}{10\ \Omega} \quad \Rightarrow \quad R_p = 10\ \Omega$$

and the total current is $I = V/R_P$ = (90 V)/(10 Ω) = 9 A. (You could verify that 3 A of current would flow in each of the three branches of the combination.) The power supplied by the battery will be $P = IV$ = (9 A)(90 V) = 810 W.

If the resistors are in series, the equivalent resistance is R_S = 30 Ω + 30 Ω + 30 Ω = 90 Ω, and the current drawn is only $I = V/R_S$ = (90 V)/(90 Ω) = 1 A. The power supplied by the battery in this case is just $P = IV$ = (1 A)(90 V) = 90 W.

Example 7 A **voltmeter** is a device used to measure the voltage between two points in a circuit. An **ammeter** is used to measure current. Determine the readings on the voltmeter (denoted —Ⓥ—) and the ammeter (denoted —Ⓐ—) in the circuit below.



Ammeters

Ammeters are connected in series because they measure current, and current stays the same in series.

Voltmeters

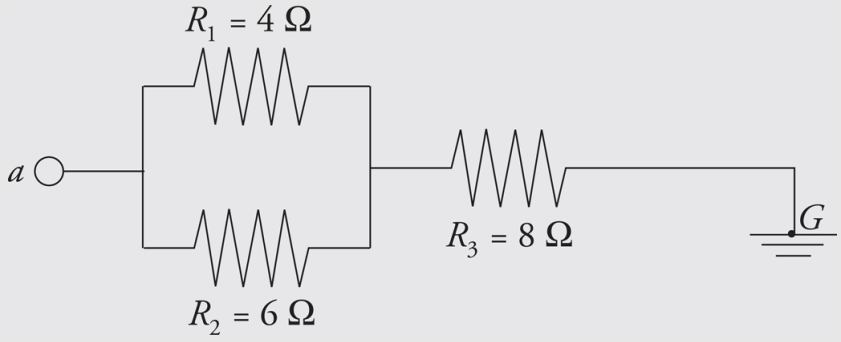
Voltmeters are connected in parallel because they measure voltage, and voltage stays the same in parallel.

Solution. We consider the ammeter to be ideal; this means it has negligible resistance, so it doesn't alter the current that it's trying to measure. Similarly, we consider the voltmeter to have an extremely high resistance, so it draws negligible current away from the circuit.

Our first goal is to find the equivalent resistance in the circuit. The 600 Ω and 300 Ω resistors are in parallel; they're equivalent to a single 200 Ω resistor. This is in series with the battery's internal resistance, r, and R_3. The overall equivalent resistance is therefore R = 50 Ω + 200 Ω + 150 Ω = 400 Ω, so the current supplied by the battery is $I = V/R$ (2400 V)/(400 Ω) = 6 A. At the junction marked J, this current splits. Since R_1 is twice R_2, half as much current will flow through R_1 as through R_2; the current through R_1 is I_1 = 2 A, and the current through R_2 is I_2 = 4 A. The voltage drop across each of these resistors is $I_1R_1 = I_2R_2$ = 1,200 V (matching voltages verify the values of currents I_1 and I_2). Since the ammeter is in the branch that contains R_2, it will read I_2 = 4 A.

The voltmeter will read the voltage drop across R_3, which is $V_3 = IR_3$ = (6 A) (150 Ω) = 900 V. So the potential at point b is 900 V lower than at point a.

Example 8 The diagram below shows a point a at potential V = 20 V connected by a combination of resistors to a point (denoted G) that is **grounded**. *The **ground** is considered to be at potential zero.* If the potential at point a is maintained at 20 V, what is the current through R_3?



Solution. R_1 and R_2 are in parallel; their equivalent resistance is R_P, where

$$\frac{1}{R_P} = \frac{1}{4\ \Omega} + \frac{1}{6\ \Omega} \quad \Rightarrow \quad R_p = \frac{24}{10}\ \Omega = 2.4\ \Omega$$

R_P is in series with R_3, so the equivalent resistance is

$$R = R_P + R_3 = (2.4\ \Omega) + (8\ \Omega) = 10.4\ \Omega$$

and the current that flows through R_3 is

$$I_3 = \frac{V}{R} = \frac{20\ \text{V}}{10.4\ \Omega} = 1.9\ \text{A}$$

Kirchhoff's Rules

When the resistors in a circuit cannot be classified as either in series or in parallel, we need another method for analyzing the circuit. The rules of Gustav Kirchhoff (pronounced "Keer koff") can be applied to any circuit:

The Loop Rule. The sum of the potential differences (positive and negative) that traverse any closed loop in a circuit must be zero.

The Junction Rule. The total current that enters a junction must equal the total current that leaves the junction. (This is also known as the **Node Rule**.)

The Loop Rule just says that, starting at any point, by the time we get back to that same point by following any closed loop, we have to be back to the same potential. Therefore, the total drop in potential must equal the total rise in potential. Put another way, the Loop Rule says that all the decreases in electrical potential energy (for example, caused by resistors in the direction of the current) must be balanced by all the increases in electrical potential energy (for example, caused

by a source of emf from the negative to positive terminal). So the Loop Rule is basically a re-statement of the Law of Conservation of Energy.

What Goes In Must Come Out

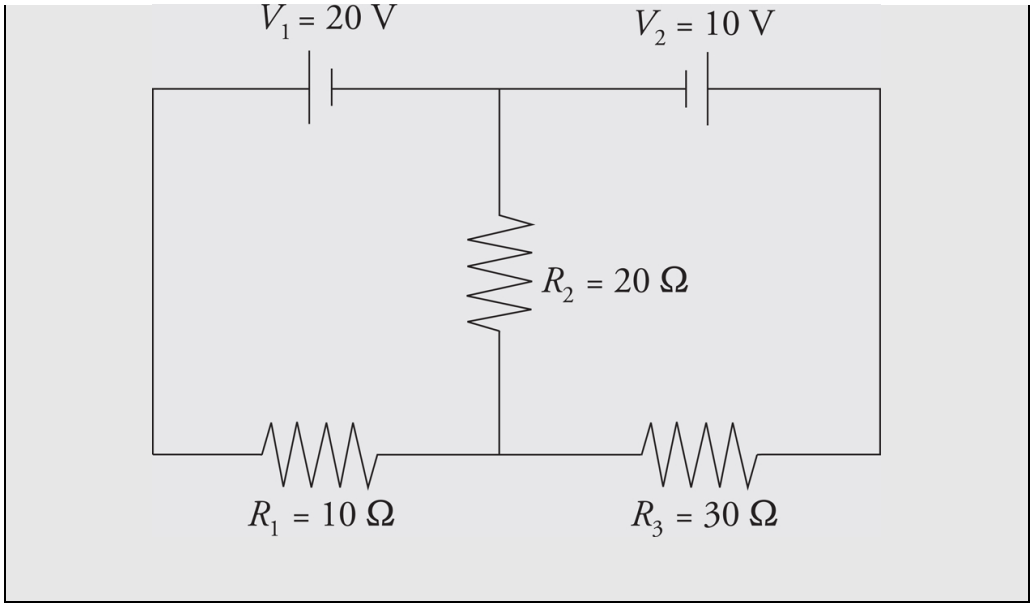
Knowing that the total drop must equal the total rise in potential is a good way to keep yourself in check when doing circuit problems. If 60 V came out of a battery by the end, no more and no less must be used.

Similarly, the Junction Rule simply says that the charge (per unit time) that goes into a junction must equal the charge (per unit time) that comes out. This is basically a statement of the Law of Conservation of Charge. In practice, the Junction Rule is straightforward to apply.

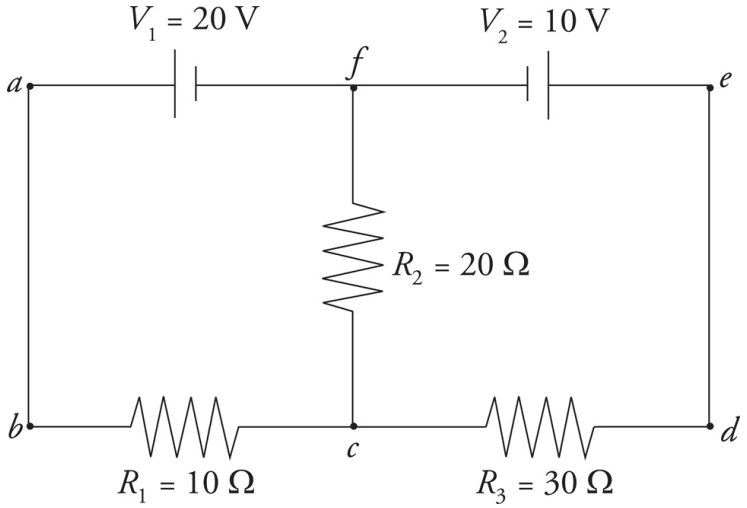
The most important things to remember about the Loop Rule can be summarized as follows:

- First, you choose a direction for the loop—either clockwise or counterclockwise—it doesn't matter.
- When the loop goes across a resistor in the *same* direction as the current, the potential *drops* by *IR*.
- When the loop goes across a resistor in the *opposite* direction from the current, the potential *increases* by *IR*.
- When the loop goes from the negative to the positive terminal of a source of emf, the potential *increases* by *V*.
- When the loop goes from the positive to the negative terminal of a source of emf, the potential *decreases* by *V*.

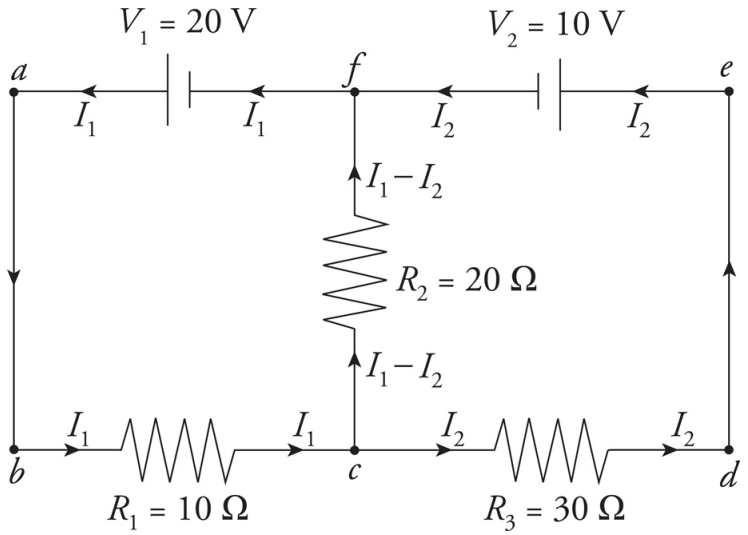
Example 9 Use Kirchhoff's Rules to determine the current through R_2 in the following circuit:



Solution. First, let's label some points in the circuit.



The points *c* and *f* are junctions (nodes). We have two nodes and three branches: one branch is *fabc*, another branch is *cdef*, and the third branch is *cf*. Each branch has one current throughout. If we label the current in *fabc* I_1 and the current in branch *cdef* I_2 (with the directions as shown in the diagram below), then the current in branch *cf* must be $I_1 - I_2$, by the Junction Rule: I_1 comes into *c*, and a total of $I_2 + (I_1 - I_2) = I_1$ comes out.



Now pick a loop, say, *abcfa*, in the counterclockwise direction. Starting at *a*, we go to b, then across R_1 in the direction of the current, so the potential drops by I_1R_1. Then we move to *c*, then up through R_2 in the direction of the current, so the potential drops by $(I_1 - I_2)R_2$. Then we reach *f*, turn left and travel through V_1 from the negative to the positive terminal, so the potential increases by V_1. We now find ourselves back at *a*. By the Loop Rule, the total change in potential around this closed loop must be zero, and

$$-I_1 R_1 - (I_1 - I_2)R_2 + V_1 = 0$$

Since we have two unknowns (I_1 and I_2), we need two equations, so now pick another loop; let's choose *cdefc*, again in the counterclockwise direction. From *c* to *d*, we travel across the resistor in the direction of the current, so the potential drops by I_2R_3. From *e* to *f*, we travel through V_2 from the positive to the negative terminal, so the potential *drops* by V_2. Heading down from *f* to *c*, we travel across R_2 but in the direction *opposite* to the current, so the potential *increases* by $(I_1 - I_2)\,R_2$. At c, our loop is completed, so

$$-I_2R_3 - V_2 + (I_1 - I_2)R_2 = 0$$

Substituting in the given numerical values for R_1, R_2, R_3, V_1, and V_2, and simplifying, these two equations become

$$3I_1 - 2I_2 = 2$$

$$2I_1 - 5I_2 = 1$$

Solving this pair of simultaneous equations, we get

$$I_1 = \frac{8}{11}\,\text{A} = 0.73\,\text{A} \quad \text{and} \quad I_2 = \frac{1}{11}\,\text{A} = 0.09\,\text{A}$$

So the current through R_2 is $I_1 - I_2 = \frac{7}{11}\,\text{A} = 0.64\,\text{A}$.

The choice of directions of the currents at the beginning of the solution was arbitrary. Don't worry about trying to guess the actual direction of the current in a particular branch. Just pick a direction, stick with it, and obey the Junction Rule. At the end, when you solve for the values of the branch current, a negative value will alert you that the direction of the current is actually opposite to the direction you originally chose for it in your diagram.

CAPACITORS IN CIRCUITS

In Chapter 6, you learned about capacitors from the perspective of the electric field they store. Capacitors are frequently used as elements in circuits. They are devices that store charges when a potential difference, which you now know is a voltage, is applied to them.

CAPACITORS IN STEADY STATE

In Chapter 6, we began looking at capacitors by assuming that the two conducting plates stored charges $+Q$ and $-Q$. When capacitors are used as circuit elements, this assumption is not true all of the time. A typical capacitor in a circuit will be in one of three states:

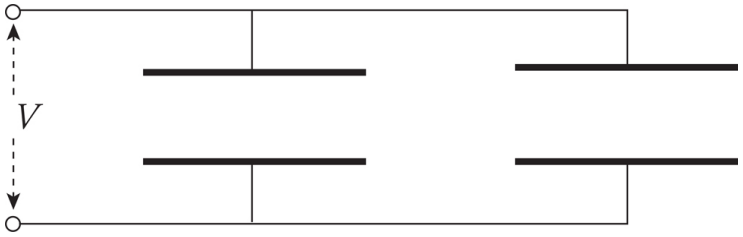
1. Complete uncharged, so that the charge on each plate is 0 C and the capacitor acts like a closed switch with zero resistance to charge.
2. Charging, so that the plates each hold charge q where $0 < q < Q$ and the capacitor acts like a resistor with a continually increasing but nonlinear resistance.
3. Charged, so that one plate holds charge $+Q$ and the other has $-Q$ and the capacitor can store no more charge for the supplied voltage. It acts like an open switch with infinite resistance.

The mathematical description of the value of q during state 2 requires solving a first order differential equation, and is omitted from the AP Physics 2 Exam. Practically, a capacitor on the exam will either be described as "uncharged," corresponding to state 1 where it is treated like a wire with a resistance of zero, or described as "steady state," corresponding to state 3. Sometimes instead of "steady state," the phrase "has been connected for a long time" will be used to indicate that the capacitor has had sufficient time to become fully charged, when it will be treated as an open switch. This will be discussed more fully in the section on RC circuits.

COMBINATIONS OF CAPACITORS

Previously we saw that resistors are often not used alone in circuits, but are found in various parallel and series arrangements. Multiple capacitors are often utilized in parallel or series combinations as well, resulting in an *equivalent capacitance* for the group.

A collection of capacitors are said to be in **parallel** if they all share the same potential difference. The following diagram shows two capacitors wired in parallel:



The top plates are connected by a wire and form a single equipotential; the same is true for the bottom plates. Therefore, the potential difference across one capacitor is the same as the potential difference across the other capacitor.

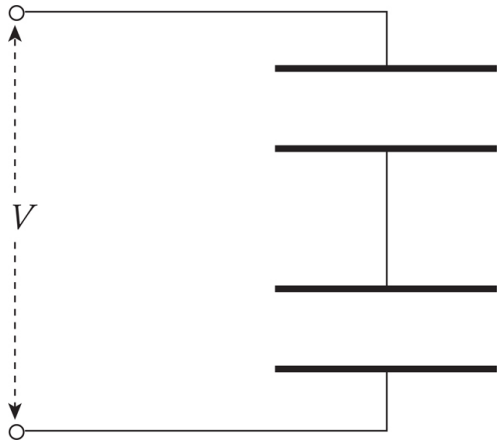
We want to find the capacitance of a *single* capacitor that would perform the same function as this combination. If the capacitances are C_1 and C_2, then the charge on the first capacitor is $Q_1 = C_1V$ and the charge on the second capacitor is $Q_2 = C_2V$. The total charge on the combination is $Q_1 + Q_2$, so the equivalent capacitance, C_P, must be

Equation Sheet

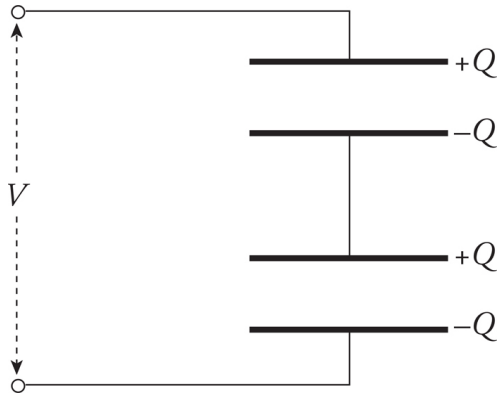
$$C_P = \frac{Q}{V} = \frac{Q_1 + Q_2}{V} = \frac{Q_1}{V} + \frac{Q_2}{V} = C_1 + C_2$$

So the **equivalent** capacitance of a collection of capacitors in parallel is found by adding the individual capacitances.

Exactly like resistors, two capacitors are said to be in **series** when they share the same current. However, current is the amount of charge per second that flows through a point in a circuit, capacitors which share the same current for the same amount of time must share the same amount of charge on each plate. The following diagram shows two capacitors wired in series:



When a potential difference is applied, as shown, negative charge will be deposited on the bottom plate of the bottom capacitor; this will push an equal amount of negative charge away from the top plate of the bottom capacitor toward the bottom plate of the top capacitor. When the system has reached equilibrium, the charges on all the plates will have the same magnitude:



If the top and bottom capacitors have capacitances of C_1 and C_2, respectively, then the potential difference across the top capacitor is $V_1 = Q/C_1$, and the potential difference across the bottom capacitor is $V_2 = Q/C_2$. The total potential difference across the combination is $V_1 + V_2$, which must equal V. Therefore, the equivalent capacitance, C_S, must be

$$C_S = \frac{Q}{V} = \frac{Q}{V_1 + V_2} = \frac{Q}{\frac{Q}{C_1} + \frac{Q}{C_2}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

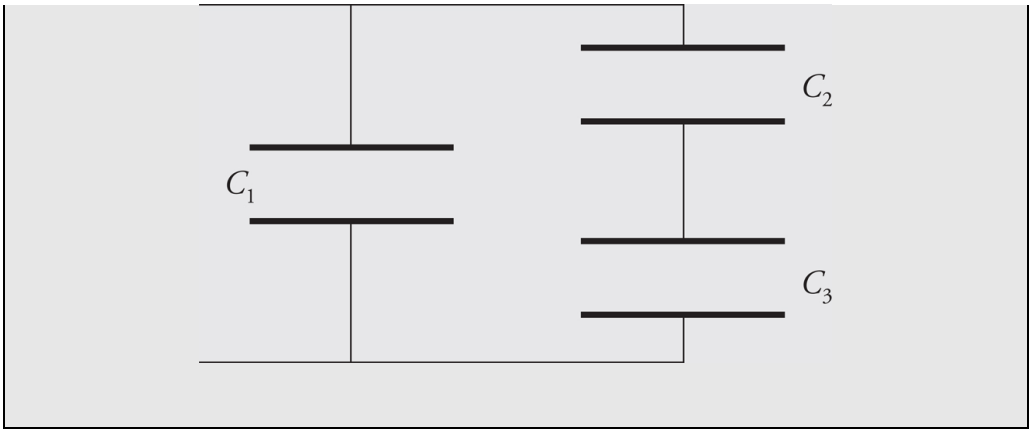
We can write this in another form:

Equation Sheet

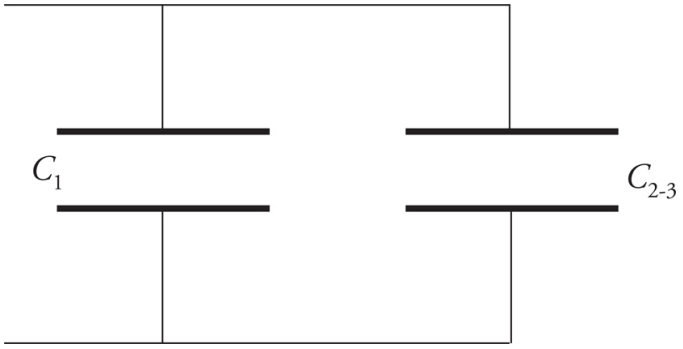
$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2}$$

In words, the *reciprocal* of the capacitance of a collection of capacitors in series is found by adding the reciprocals of the individual capacitances.

Example 10 Given that $C_1 = 2\ \mu F$, $C_2 = 4\ \mu F$, and $C_3 = 6\ \mu F$, calculate the equivalent capacitance for the following combination:



Solution. Notice that C_2 and C_3 are in series, and they are in parallel with C_1. That is, the capacitor equivalent to the series combination of C_2 and C_3 (which we'll call $C_{2\text{-}3}$) is in parallel with C_1. We can represent this as follows:



So, the first step is to find $C_{2\text{-}3}$:

$$\frac{1}{C_{2\text{-}3}} = \frac{1}{C_2} + \frac{1}{C_3} \quad \Rightarrow \quad C_{2\text{-}3} = \frac{C_2 C_3}{C_2 + C_3}$$

Now this is in parallel with C_1, so the overall equivalent capacitance ($C_{1\text{-}2\text{-}3}$) is

$$C_{1\text{-}2\text{-}3} = C_1 + C_{2\text{-}3} = C_1 + \frac{C_2 C_3}{C_2 + C_3}$$

Substituting in the given numerical values, we get

$$C_{1\text{-}2\text{-}3} = (2\mu\ \text{F}) + \frac{(4\mu\ \text{F})(6\mu\ \text{F})}{(4\mu\ \text{F}) + (6\mu\ \text{F})} = 4.4\mu\ \text{F}$$

ALTERING A CAPACITOR'S CAPACITANCE

Recall that for a capacitor, the capacitance is given by

$$C = \kappa\varepsilon_0 A/d$$

Three things may be done to alter the capacitance of a capacitor: a dielectric may be inserted (κ), the area of the plates may charge (A), or the spacing between the plates may charge (d).

There are two scenarios for making these changes to a charged capacitor in a circuit:

1. The battery may remain connected, causing the potential difference to remain constant.
2. The battery may be disconnected, causing the charge on each plate to remain constant.

Which of these scenarios unfolds will influence an analysis of the capacitor in terms of its charge on each plate, potential difference, stored electric field, and stored energy.

Let's look at what happens when the capacitance increases as a result of either adding a dielectric (κ increases), increasing plate area (A increases), or decreasing plate spacing (d decreases).

Capacitance is increased with:

Battery connected. V is constant.

V	Constant
$Q = VC$	Increases
$E = V/d$	Constant/Decreases*
$U_C = \frac{1}{2}QV$	Increases

Capacitance is increased with:

Battery removed. Q is constant.

$V = Q/C$	Decreases
Q	Constant
$E = V/d$	Decreases/Constant*
$U_C = \frac{1}{2}QV$	Decreases

*Note: If the capacitance changes because plate separation (d) is changed, then the term d appears twice in the equations: once in C and again in E. In this case, the electric field would decrease for the constant voltage case and remain constant for the constant charge case.

If the capacitance were decreased instead of increased, the same analysis would apply. The results would be as follows.

Capacitance is increased with:

Battery connected. V is constant.

V	Constant
$Q = VC$	Decreases
$E = V/d$	Constant/Increases*
$U_C = \frac{1}{2}QV$	Decreases

Capacitance is increased with:

Battery removed. Q is constant.

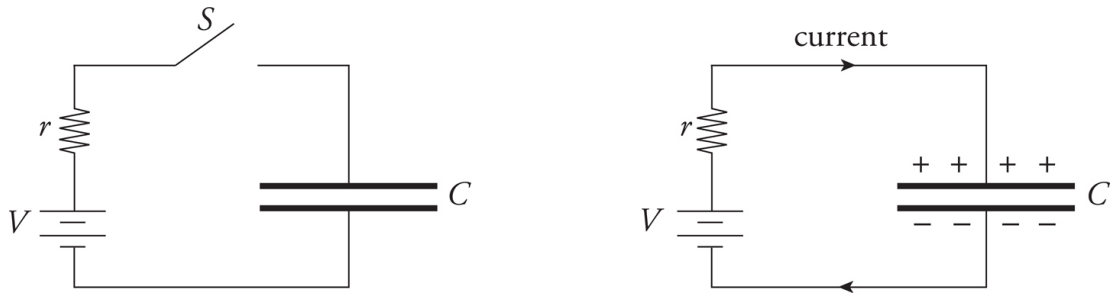
$V = Q/C$	Increases
Q	Constant
$E = V/d$	Increases/Constant*
$U_C = \frac{1}{2}QV$	Increases

Current diminishes over time

The current will initially be $I = V/r$ when the capacitor is empty, and over time it will diminish until the "steady state" is achieved and then $I = 0$. Between these states, the current exponentially decays, but that detail is beyond the scope of the AP Physics 2 Exam.

RESISTANCE-CAPACITANCE (RC) CIRCUITS

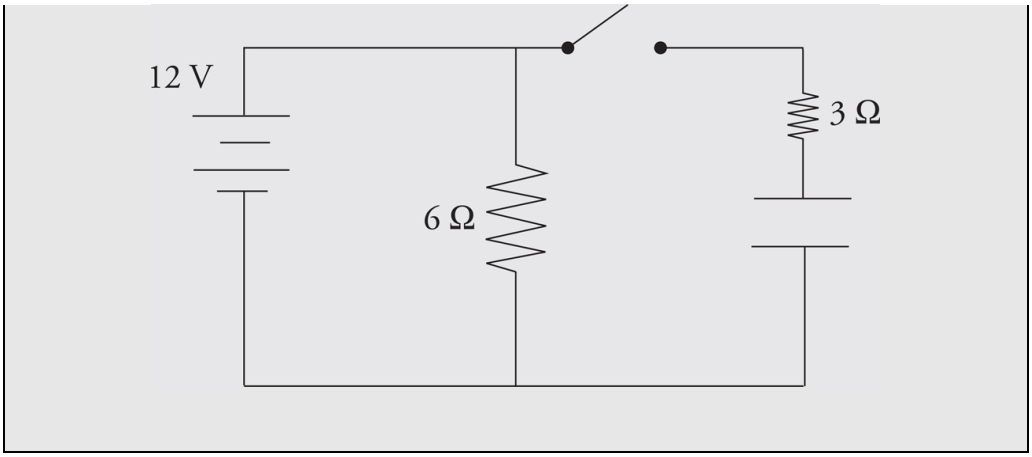
Capacitors are typically charged by batteries. Once the switch in the diagram below is closed, electrons are attracted to the positive terminal of the battery and leave the top plate of the capacitor. Electrons also accumulate on the bottom plate of the capacitor, and this continues until the voltage across the capacitor plates matches the voltage of the battery. When this condition is reached, the current stops and the capacitor is fully charged. Unlike resistors, which have a current flow across them when they experience a potential difference (given by Ohm's Law), a capacitor will be fully charged by a voltage and will prevent any current flow.



Example 11 Determine the current through and the voltage across each electrical device in the following circuit when

(a) the switch has just been closed

(b) the switch has been thrown for a long period of time



Solution.

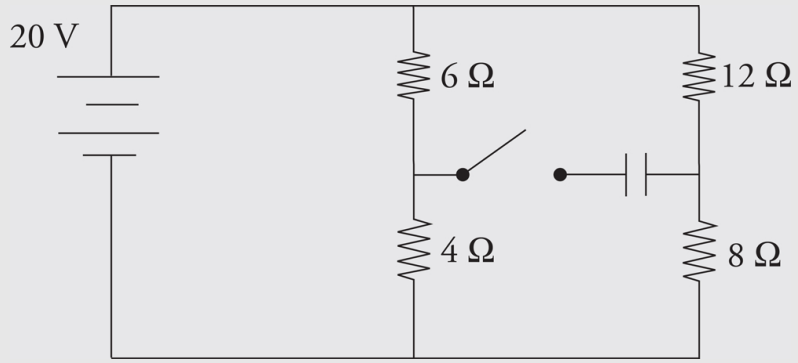
(a) When the switch has just been closed, the capacitor is treated like a wire with no resistance. The circuit behaves like a simple parallel circuit. There are 12 V across the 6 Ω resistor, so 2 amps of current flow through it. There are 12 V across the 3 Ω resistor, so 4 amps flow through it. By Kirchhoff's Law, the currents add 6 amps through the battery, and the equivalent resistance of the circuit is 2 Ω.

(b) After the switch has been closed for a long period of time, the capacitor behaves like an infinite resistor. There are still 12 V across the 6 Ω resistor, so 2 amps of current still flow through it. If the capacitor behaves like an infinite resistor, no current can flow through that branch. There will be no current through the 3 Ω resistor. The capacitor will have 12 V across it, and there will be no voltage across the 3 Ω resistor (otherwise current would be flowing through it). The current will be 2 amps through the battery, and the equivalent resistance of the circuit is 6 Ω.

Example 12 Determine the current through and the voltage across each electrical device in the following circuit when

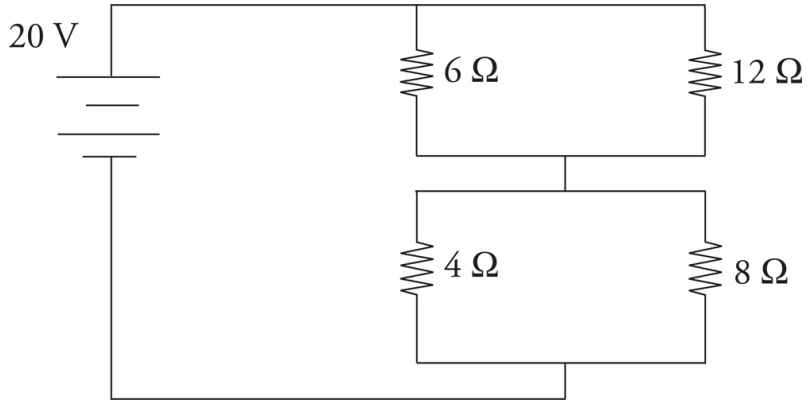
(a) the switch has just been closed

(b) the switch has been thrown for a long period of time



Solution.

(a) Immediately after one throws the switch, the capacitor exhibits behavior similar to that of a wire with no resistance through it. The circuit can be redrawn and thought of as



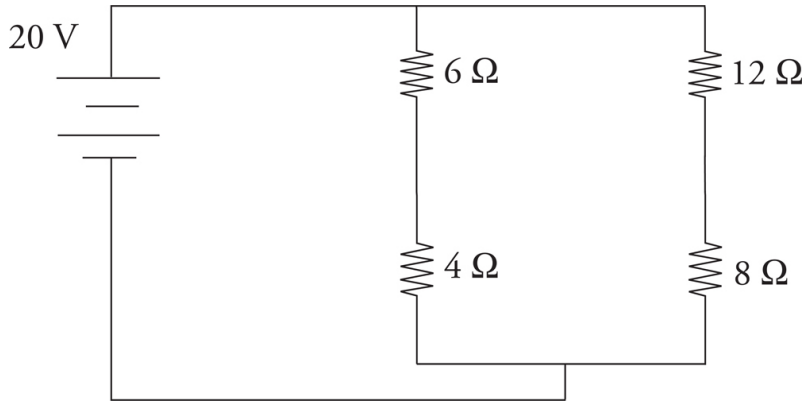
The 6 Ω and 12 Ω resistors are in parallel and can be thought of as 4 Ω (from $1/R_{eq} = 1/R_1 + 1/R_2$). Similarly, the 4 Ω and 8 Ω resistors are in parallel and can be thought of as 2.67 Ω (again, from $1/R_{eq} = 1/R_1 + 1/R_2$). The equivalent resistance of these two branches, which are in series with each other, is 6.67 Ω (from $R_{eq} = R_1 + R_2$).

The current through the battery is therefore 3 amps (from Ohm's Law, $V = IR$).

The current through the 6 Ω and 12 Ω (top two resistors) is 3 amps traveling through an effective resistance of 4 Ω. The voltage drop across the top branch is 12 V (from Ohm's Law). The currents through the 6 Ω and 12 Ω resistors are 2 amps and 1 amp, respectively (again from Ohm's Law).

The current through the 4 Ω and 8 Ω (bottom two resistors) is also 3 amps through an effective resistance of 2.67 Ω. The voltage drop across the branch is 8 V (from Ohm's Law or from the fact that the voltage across each series section must sum to the total voltage across the battery). The currents through the 4 Ω and 8 Ω resistors are 2 amps and 1 amp, respectively (again from Ohm's Law).

(b) When the switch has been closed for a long period of time, the capacitor behaves just like an infinite resistor. The circuit can be redrawn and thought of as



The 6 Ω and 4 Ω resistors are in series and can be thought of as a 10 Ω resistor (from $R_{eq} = R_1 + R_2$). Similarly, the 12 Ω and 8 Ω resistors can be thought of as 20 Ω (again, from $R_{eq} = R_1 + R_2$). Since the branches are in parallel, the voltage is 20 volts across each path and the current can be calculated by Ohm's Law ($V = IR$); 2 amps go through the left branch (the 6 Ω and 4 Ω resistor) and 1 amp goes through the right branch (the 12 Ω and 8 Ω resistor).

The current through the battery is therefore 3 amps (Kirchhoff's Node Rule).

The voltage across each of the 6 Ω, 4 Ω, 12 Ω, and 8 Ω resistors can be obtained from Ohm's Law. They are 12 V, 8 V, 12 V, and 8 V, respectively.

Chapter 7 Review Questions

Click here to download the PDF of Chapter 7 Review Questions.

Solutions can be found in Chapter 12.

Section I: Multiple Choice

1. A wire made of brass and a wire made of silver have the same length, but the diameter of the brass wire is 4 times the diameter of the silver wire. The resistivity of brass is 5 times greater than the resistivity of silver. If R_B denotes the resistance of the brass wire and R_S denotes the resistance of the silver wire, which of the following is true?

(A) $R_B = \frac{5}{16} R_S$

(B) $R_B = \frac{4}{5} R_S$

(C) $R_B = \frac{5}{4} R_S$

(D) $R_B = \frac{5}{2} R_S$

2. For an ohmic conductor, doubling the voltage without changing the resistance will cause the current to

(A) decrease by a factor of 4
(B) decrease by a factor of 2
(C) increase by a factor of 2
(D) increase by a factor of 4

3. A capacitor, battery, and two resistors are to be arranged in a circuit. Which configurations allow there to be a current that is initially through one resistor and finally through the other resistor?

(A) The capacitor should be in series with the battery, and the resistors should be in parallel.

(B) The capacitor should be in parallel with the battery, and the resistors should be in series.

(C) One resistor should be in series with the battery, and the other should be in parallel with the capacitor.

4. A student wants to determine the resistivity of copper. She has a voltmeter reading for a copper wire of known length. What other information will she need?

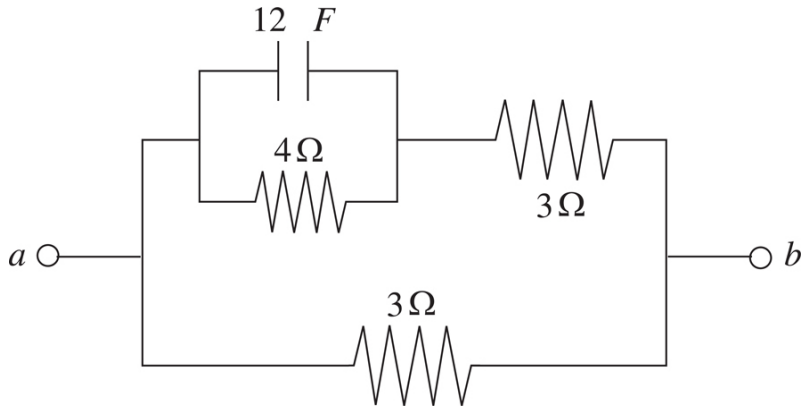
(A) An ammeter reading

(B) The diameter of the wire

(C) Both an ammeter reading and the diameter of the wire

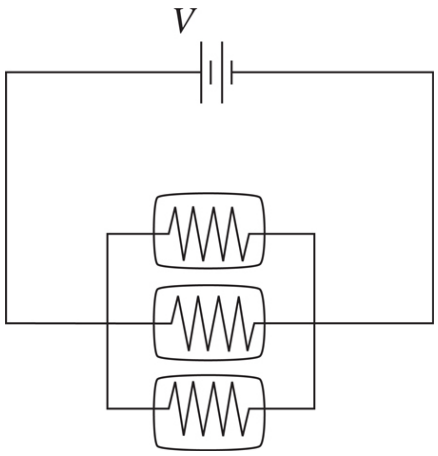
(D) Neither an ammeter reading nor the diameter of the wire

(D) One resistor should be in parallel with the battery, and the other should be in series with the capacitor.



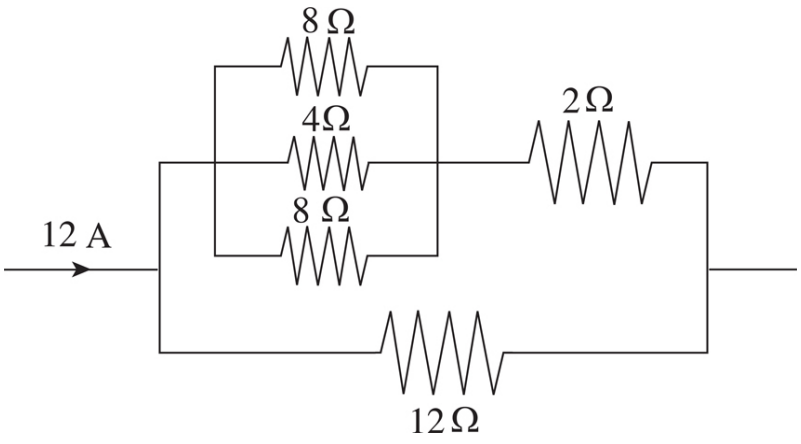
5. The circuit shown above has a constant voltage between points *a* and *b*. The voltage has been flowing for a long time. What is the resistance of the circuit?

(A) 0.47 Ω
(B) 1.5 Ω
(C) 2.1 Ω
(D) 10 Ω



6. Three identical light bulbs are connected to a source of emf, as shown in the diagram above. What will happen if the middle bulb burns out?

(A) The light intensity of the other two bulbs will decrease (but they won't go out).
(B) The light intensity of the other two bulbs will increase.
(C) The light intensity of the other two bulbs will remain the same.
(D) More current will be drawn from the source of emf.



7. What is the voltage drop across the 12 Ω resistor in the portion of the circuit shown above?

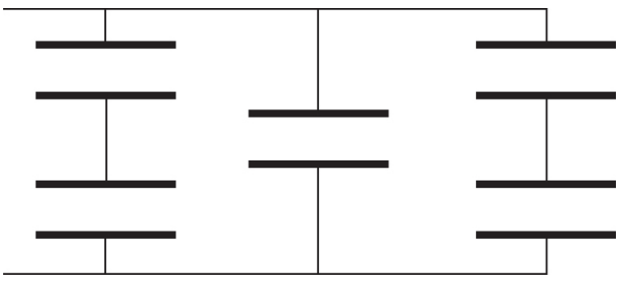
(A) 24 V
(B) 36 V
(C) 48 V
(D) 72 V

8. A simple DC circuit with a single Ohmic resistor is set up. A graph is produced of the voltage drop across the resistor versus current. For an ideal battery, the graph is directly proportional. If the battery has some internal resistance, how, if at all, would such a graph change?

(A) The graph would remain directly proportional.
(B) The graph would be linear, but would have a negative *y*-intercept.
(C) The graph would be linear, but would have a positive *y*-intercept.
(D) The graph would be nonlinear.

9. How much energy is dissipated as heat in 20 s by a 100 Ω resistor that carries a current of 0.5 A?

(A) 50 J
(B) 100 J
(C) 250 J
(D) 500 J

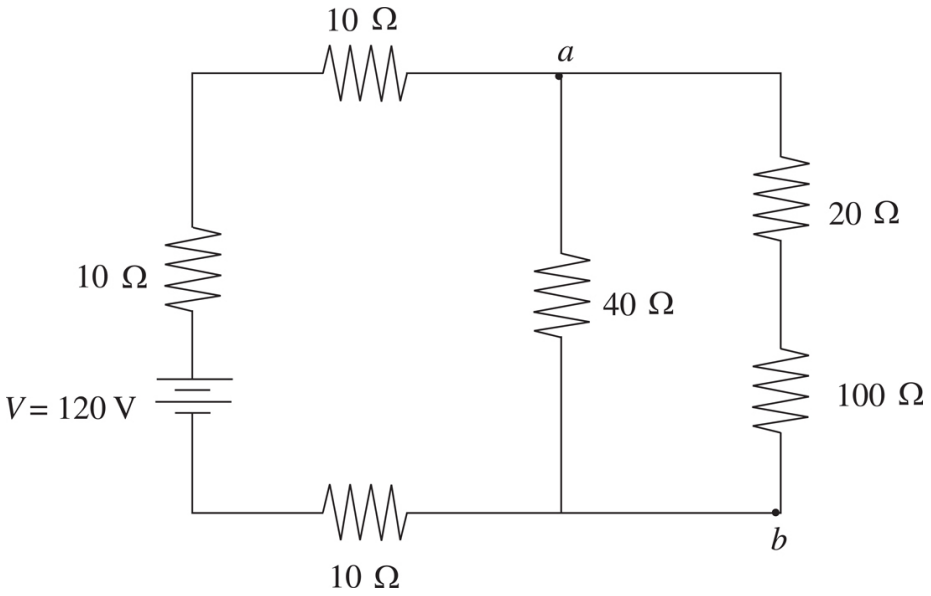


10. If each of the capacitors in the array shown above is C, what is the capacitance of the entire combination?

(A) $C/2$
(B) $2C/3$
(C) $5C/6$
(D) $2C$

Section II: Free Response

1. Consider the following circuit:



(a) At what rate does the battery deliver energy to the circuit?

(b) Find the current through the 40 Ω resistor.

(c)

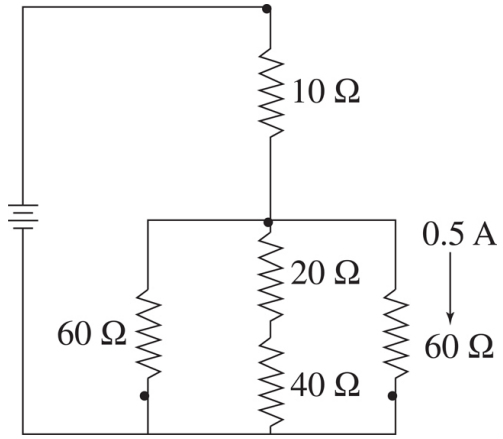
i. Determine the potential difference between points a and b.
ii. At which of these two points is the potential higher?

(d) Given that the 100 Ω resistor is a solid cylinder that's 4 cm long, composed of a material whose resistivity is 0.45 Ω • m,

determine its radius.

(e) Which resistor or resistors, if any, could be replaced with a capacitor so that when the capacitor is fully charged, no current flows out of the battery? Explain your answer.

2. Consider the following circuit:



(a) What is the current through each resistor?

(b) What is the potential difference across each resistor?

(c) What is the equivalent resistance of the circuit?

(d) The rightmost 60 Ω resistor is replaced with a capacitor.

i. When the capacitor is empty and just beginning to fill, will the current in the leftmost 60 Ω resistor be greater than, equal to, or less than the 0.5 A it was before the capacitor was inserted into the circuit?

ii. When the capacitor is fully charged, will the current in the leftmost 60 Ω resistor be greater than, equal to, or less than the 0.5 A it was before the capacitor was inserted into the circuit?

Summary

- The resistance of an object can be determined by $R = \frac{\rho \ell}{A}$ where ρ is the resistivity (a property of the material), ℓ is the length, and A is the cross-sectional area.

- The current is the rate at which charge is transferred and given by $I_{AVG} = \frac{\Delta Q}{\Delta t}$.

- Many objects obey Ohm's Law, which is given by $V = IR$.

- The electrical power in a circuit is given by $P = IV$ or $P = I^2R$ or $P = \frac{V^2}{R}$.

  This is the same power we've encountered in our discussion of energy $P = \frac{W}{t}$.

- In a series circuit

  $V_B = V_1 + V_2 + \ldots$
  $I_B = I_1 = I_2 = \ldots$
  $R_{eq} = R_1 + R_2 + \ldots$

  $R_S = \sum_i R_i$

  In a parallel circuit
  $V_B = V_1 = V_2 = \ldots$

$I_B = I_1 + I_2 + \ldots$

$1/R_{eq} = 1/R_1 + 1/R_2 + \ldots$

$$\frac{1}{R_P} = \sum_i \frac{1}{R_i}$$

- Kirchhoff's Loop Rule tells us that the sum of the potential differences in any closed loop in a circuit must be zero.
- Kirchhoff's Junction Rule (Node Rule) tells us the total current that enters a junction must equal the total current that leaves the junction.
- To find the combined capacitance of capacitors in parallel (side by side), simply add their capacitances.

$$C_p = C_1 + C_2 + \ldots \text{or } C_P = \sum_i C_i$$

- To find the capacitance of capacitors in series (one after another), add their inverses:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots \text{ or } \frac{1}{C_S} = \sum_i \frac{1}{C_i}$$

- When a capacitor is in a circuit and is completely discharged, it will act like a closed switch (equivalent to a resistor with $R = 0$ Ω). When the capacitor is fully charged, it acts like an open switch (equivalent to an infinite resistor).

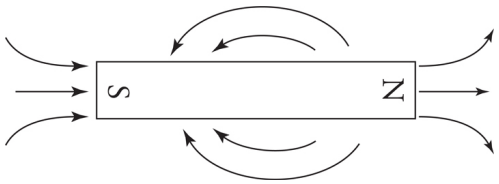
Chapter 8
Magnetic Forces and Fields

INTRODUCTION

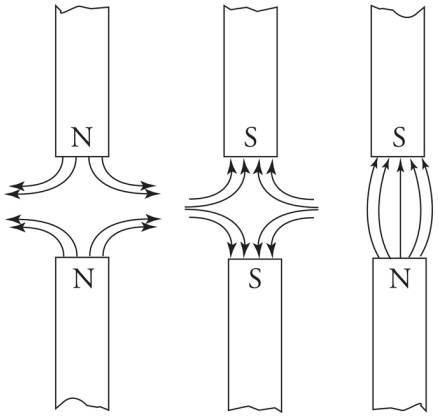
In Chapter 5, we learned that electric charges are the sources of electric fields and that other charges experience an electric force in those fields. The charges generating the field were assumed to be at rest, because if they weren't, then another force field would have been generated in addition to the electric field. Electric charges *that move* are the sources of **magnetic fields,** and other charges that move can experience a magnetic force in these fields.

MAGNETIC FIELDS

Similar to our discussion about electric fields, the space surrounding a magnet is permeated by a magnetic field. The direction of the magnetic field is defined as pointing out of the north end of a magnet and into the south end of a magnet as illustrated below.

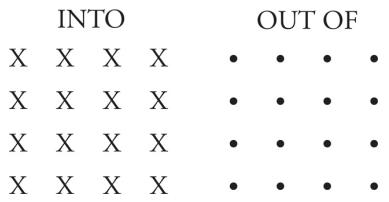


When two magnets get near each other the magnetic fields interfere with each other and can be drawn as follows. Note that, for simplicity's sake, only the field lines closest to the poles are shown.



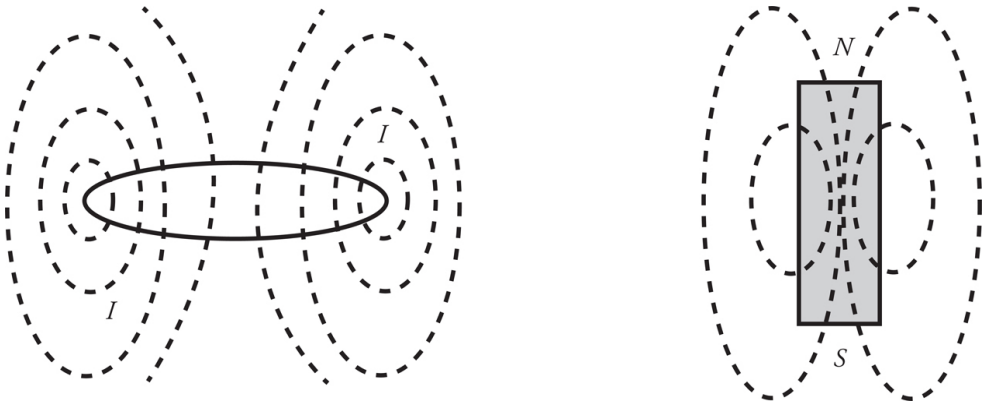
Notice there is a curve to the above fields. We call the field uniform if the field lines are parallel and of equal strength. It is easy to recognize a uniform magnetic field to the right, left, top of the page, or bottom of the page. But you will also see a field going into or out of the page. A field into the page looks as if there were a north pole of a magnet above the page pointing down at the south pole of a magnet that is below the page. It is represented by an area with X's going into the page. A field coming out of the page looks as if there were a north pole of a magnet below the page pointing up at the south pole

of a magnet that is above the page. It is represented by an area with dots (•) coming out of the page.



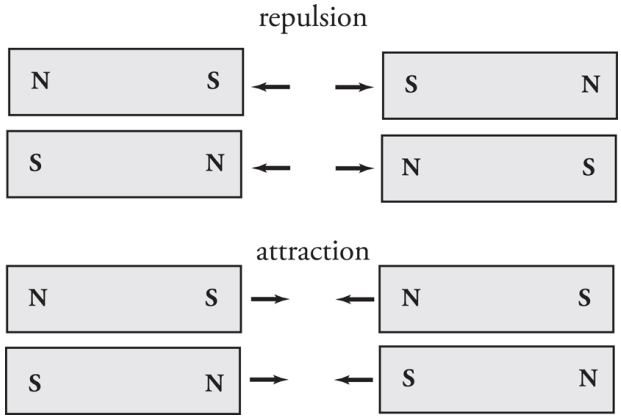
BAR MAGNETS

A permanent bar magnet creates a magnetic field that closely resembles the magnetic field produced by a circular loop of current-carrying wire:



By convention, the magnetic field lines emanate from the end of the magnet designated the north pole (N) and then curl around and re-enter the magnet at the end designated the south pole (S). The magnetic field decreases in strength as distance from the magnet increases. The magnetic field created by a permanent bar magnet is due to the electrons, which have an intrinsic spin as they orbit the nuclei. They are literally charges in motion, which is the ultimate source of any and all magnetic fields. If a piece of iron is placed in an external magnetic field (for example, one created by a current-carrying solenoid), the individual magnetic dipole moments of the electrons will be forced to more or less line up. Because iron is ferromagnetic, these now-aligned magnetic dipole moments tend to retain this configuration, thus permanently magnetizing the bar and causing it to produce its own magnetic field.

As with electric charges, like magnetic poles repel each other, while opposite magnetic poles attract each other.



However, while you can have a positive electric charge all by itself, you can't have a single magnetic pole all by itself: the existence of a lone magnetic pole has never been confirmed. That is, there are no magnetic monopoles; magnetic poles always exist in pairs. If you break a bar magnet into two pieces, it does not produce one piece with just an N and another with just an S; it produces two separate, complete magnets, each with an N-S pair.

THE MAGNETIC FORCE ON A MOVING CHARGE

If a particle with charge q moves with velocity v through a magnetic field **B**, it will experience a magnetic force, $\mathbf{F}_B$:

Equation Sheet

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

with magnitude:

Equation Sheet

$$F_B = |q| vB \sin\theta$$

where θ is the angle between **v** and **B**. From this equation, we can see that if the charge is at rest, then $v = 0$ immediately gives us $F_B = 0$. This tells us that magnetic forces only act on moving charges. Also, if **v** is parallel (or antiparallel) to **B**, then $F_B = 0$ since, in either of these cases, $\sin\theta = 0$. So, only charges that cut across the magnetic field lines will experience a magnetic force. Furthermore, the magnetic force is maximized when **v** is perpendicular to **B**, since if $\theta = 90°$, then $\sin\theta$ is equal to 1, its maximum value.

The direction of $\mathbf{F}_B$ is always perpendicular to both v and B and depends on the sign of the charge q and the direction of $\mathbf{v} \times \mathbf{B}$ (which can be found by using the right-hand rule).

If q is *positive*, use your *right* hand and the *right-hand* rule.

If q is *negative*, you need your thumb to point in the direction of the product of q and v, not just the direction of the velocity when you apply the right-hand rule.

0 or 180

If the velocity v and the magnetic field **B** are parallel or anti-parallel, which means oriented so the angle between the vectors is 180 degrees, the magnetic force $\mathbf{F}_B = 0$.

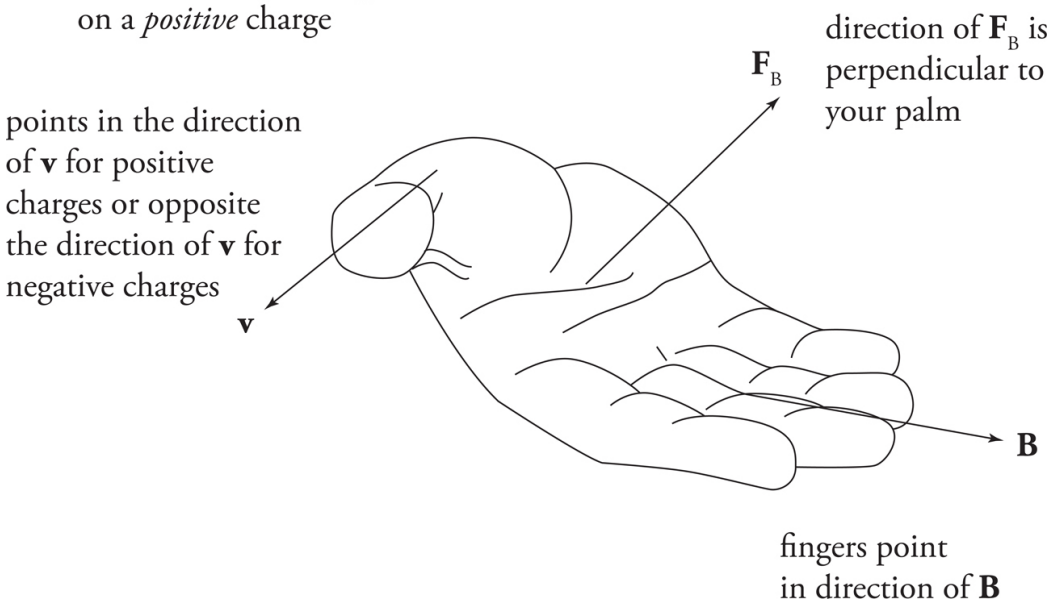
Whenever you use the right-hand rule, follow these steps:

1. Orient your hand so that your thumb points in the direction of the velocity **v**. If the charge is negative, turn your thumb by 180 degrees.
2. Point your fingers in the direction of **B**.
3. The direction of $\mathbf{F}_B$ will then be perpendicular to your palm.

Think of your palm pushing with the force $\mathbf{F}_B$; the direction it pushes is the direction of $\mathbf{F}_B$.

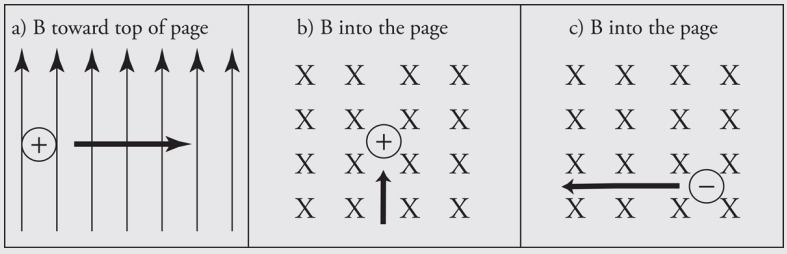
Right-Hand Rule:

For determining the direction of the magnetic force, $\mathbf{F}_B$, on a *positive* charge



Note that there are fundamental differences between the electric force and magnetic force on a charge. First, a magnetic force acts on a charge only if the charge is moving; the electric force acts on a charge whether it moves or not. Second, the direction of the magnetic force is always perpendicular to the magnetic field, while the electric force is always parallel (or antiparallel) to the electric field.

Example 1 For each of the following charged particles moving through a magnetic field, determine the direction of the force acting on the charge.



Solution.

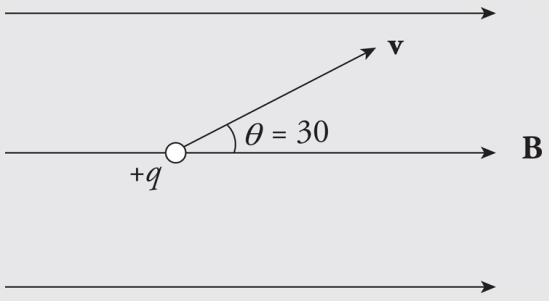
a) If you point your fingers to the top of the page and thumb to the right of the page, your palm should point out of the page. The force is out of the page.

b) If you point your fingers into the page and thumb to the top of the page, your palm should point to the left of the page. The force is to the left of the page.

c) If you point your fingers into the page and thumb to the right of the page (remember, when the charge is negative, your thumb points in the opposite direction from the velocity), your palm should point to the bottom of the page. The force of a positive charge would be toward the bottom of the page, but because this is a negative charge, the force points up.

The SI unit for the magnetic field is the **tesla** (abbreviated **T**) which is one newton per ampere-meter.

Example 2 A charge $+q = +6 \times 10^{-6}$ C moves with speed $v = 4 \times 10^5$ m/s through a magnetic field of strength $B = 0.4$ T, as shown in the figure below. What is the magnetic force experienced by q?

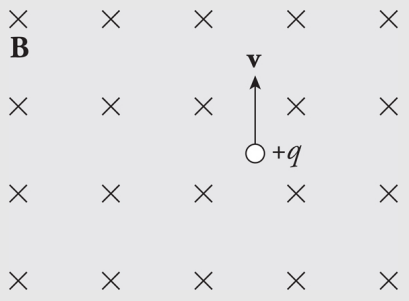


Solution. The magnitude of F_B is

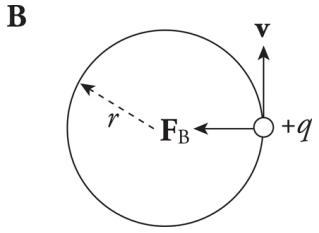
$$F_B = qvB \sin\theta = (6 \times 10^{-6}\ \text{C})(4 \times 10^5\ \text{m/s})(0.4\ \text{T}) \sin 30^\circ = 0.48\ \text{N}$$

By the right-hand rule, the direction is into the plane of the page, which is symbolized by ×.

Example 3 A particle of mass m and charge $+q$ is projected with velocity **v** (in the plane of the page) into a uniform magnetic field **B** that points into the page. How will the particle move?



Solution. Since v is perpendicular to B, the particle will feel a magnetic force of strength qvB, which will be directed perpendicular to v (and to B) as shown:



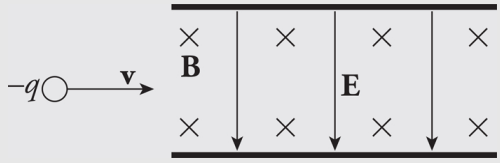
Since $\mathbf{F}_B$ is always perpendicular to **v**, the particle will undergo uniform circular motion; $\mathbf{F}_B$ will provide the centripetal force. Notice that, because $\mathbf{F}_B$ is always perpendicular to v, the magnitude of v will not change, just its direction. *Magnetic forces alone cannot change the speed of a charged particle, they can only change its direction of motion*. The radius of the particle's circular path is found from the equation $F_B = F_C$:

$$qvB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{qB}$$

Magnetic Fields

$\mathbf{F}_B$ is always perpendicular to both **v** and **B**. Magnetic forces cannot change the speed of an object, only its direction. The magnetic field does no work on any charges.

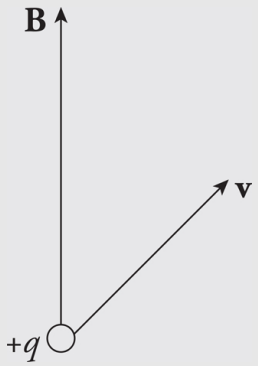
Example 4 A particle of charge $-q$ is shot into a region that contains an electric field, **E**, crossed with a perpendicular magnetic field, **B**. If $E = 2 \times 10^4$ N/C and $B = 0.5$ T, what must be the speed of the particle if it is to cross this region without being deflected?



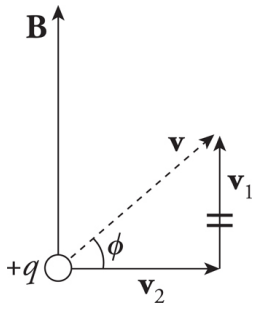
Solution. If the particle is to pass through undeflected, the electric force it feels has to be canceled by the magnetic force. In the diagram above, the electric force on the particle is directed upward (since the charge is negative and E is downward), and the magnetic force is directed downward by the right-hand rule. So F_E and F_B point in opposite directions, and in order for their magnitudes to balance, qE must equal qvB, so v must equal E/B, which in this case gives

$$v = \frac{E}{B} = \frac{2 \times 10^4 \text{ N/C}}{0.5 \text{ T}} = 4 \times 10^4 \text{ m/s}$$

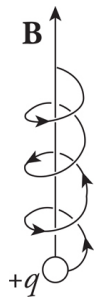
Example 5 A particle with charge $+q$, traveling with velocity **v**, enters a uniform magnetic field **B**, as shown below. Describe the particle's subsequent motion.



Solution. If the particle's velocity were parallel to **B**, then it would be unaffected by **B**. If **v** were perpendicular to **B**, then it would undergo uniform circular motion (as we saw in Example 2). In this case, **v** is neither purely parallel nor perpendicular to **B**. It has a component ($\mathbf{v}_1$) that's parallel to **B** and a component ($\mathbf{v}_2$) that's perpendicular to **B**.



Component $\mathbf{v}_1$ will not be changed by **B**, so the particle will continue upward in the direction of B. However, the presence of $\mathbf{v}_2$ will create circular motion. The superposition of these two types of motion will cause the particle's trajectory to be a *helix*; it will spin in circular motion while traveling upward with the speed $v_1 = v \sin \phi$:


B
+q

THE MAGNETIC FORCE ON A CURRENT-CARRYING WIRE

Since magnetic fields affect moving charges, they should also affect current-carrying wires. After all, a wire that contains a current contains charges that move. Remember, a current is the flow of positive charges.

Let a wire of length ℓ be immersed in magnetic field **B**. If the wire carries a current I, then the force on the wire is

Equation Sheet

$$\mathbf{F}_B = I\boldsymbol{\ell} \times \mathbf{B}$$

with magnitude

Equation Sheet

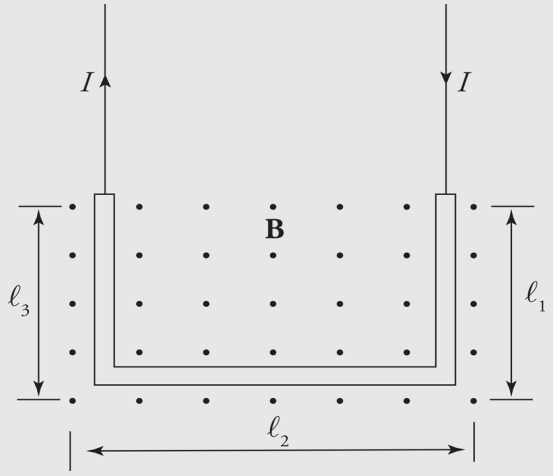
$$F_B = BI\ell \sin\theta$$

where θ is the angle between ℓ and **B**. Here, the direction of ℓ is the direction of the current, I. The direction of $\mathbf{F}_B$ can be found using the right-hand rule and by letting your thumb point in the direction in which the current flows. Remember, a current is the flow of positive charges.

Another Way of Seeing It

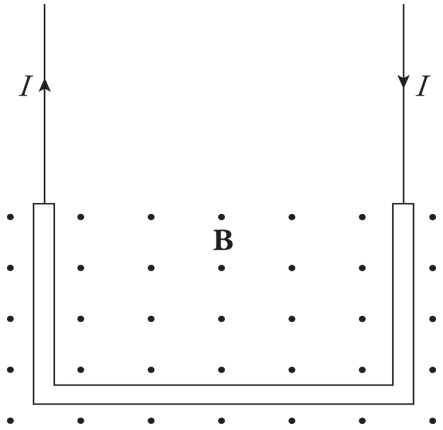
Current I is charge over time (q/t) and length of a wire I is a distance d. Hence, Iℓ = q (d/t). This gives us the same formula as before: $F_B = qv\text{bsin}\,\theta$ = IℓBsin θ

Example 6 A U-shaped wire of mass m is lowered into a magnetic field B that points out of the plane of the page. How much current I must pass through the wire in order to cause the net force on the wire to be zero?



Solution. The total magnetic force on the wire is equal to the sum of the magnetic forces on each of the three sections of wire. The force on the first section (the right, vertical one), $\mathbf{F}_{B1}$, is directed to the left (applying the right-hand rule, and the force on the third piece (the left, vertical one), $\mathbf{F}_{B3}$, is directed to the right. Since these pieces are the same length, these two oppositely directed forces have the same magnitude, $I\ell_1 B = I\ell_3 B$, and they cancel. So the net magnetic force on the wire is the magnetic force on the middle piece. Since I points

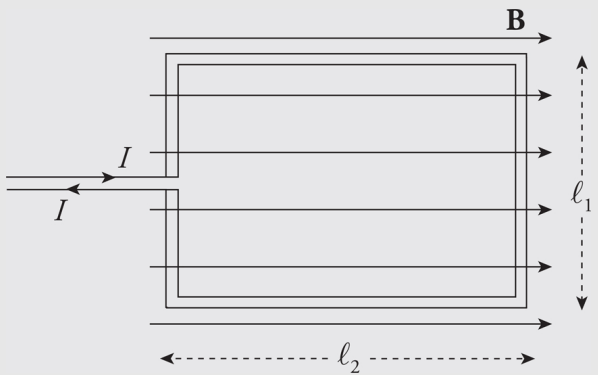
to the left and **B** is out of the page, the right-hand rule tells us the force is upward.



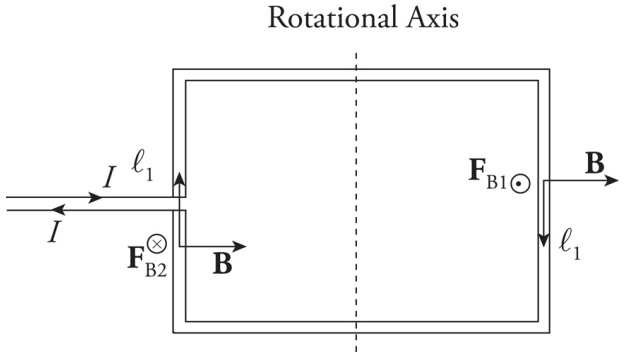
Since the magnetic force on the wire is $I\ell_2 B$, directed upward, the amount of current must create an upward magnetic force that exactly balances the downward gravitational force on the wire. Because the total mass of the wire is m, the resultant force (magnetic + gravitational) will be zero if

$$I\ell_2 \mathrm{B} = mg \quad \Rightarrow \quad I = \frac{mg}{\ell_2 \mathrm{B}}$$

Example 7 A rectangular loop of wire that carries a current I is placed in a uniform magnetic field, **B**, as shown in the diagram below and is free to rotate. What torque does it experience?



Solution. Ignoring the tiny gap in the vertical left-hand wire, we have two wires of length ℓ_1 and two of length ℓ_2. There is no magnetic force on either of the sides of the loop of length ℓ_2, because the current in the top side is parallel to **B** and the current in the bottom side is antiparallel to **B**. The magnetic force on the right-hand side points out of the plane of the page, while the magnetic force on the left-hand side points into the plane of the page.



Each of these two forces exerts a torque that tends to turn the loop in such a way that the right-hand side rises out of the plane of the page and the left-hand side rotates into the page. Relative to the axis shown above (which cuts the loop in half), the torque of $\mathbf{F}_{B1}$ is

$$\tau_1 = rF_{B1} \sin\theta = \left(\frac{1}{2}\ell_2\right)(I\ell_1 B)\sin 90^\circ = \frac{1}{2}I\ell_1\ell_2 B$$

and the torque of $\mathbf{F}_{B2}$ is

$$\tau_2 = rF_{B2} \sin\theta = \left(\frac{1}{2}\ell_2\right)(I\ell_1 B)\sin 90^\circ = \frac{1}{2}I\ell_1\ell_2 B$$

Since both these torques rotate the loop in the same direction, the net torque on the loop is

$$\tau_1 + \tau_2 = I\ell_1\ell_2 B$$

MAGNETIC FIELDS CREATED BY CURRENT-CARRYING WIRES

In the previous section, we examined the force that a current-carrying wire experiences when it is subjected to an external magnetic field. The source of the magnetic field in the previous section was unspecified. As we said at the beginning of this chapter, the sources of magnetic fields are electric charges that move; they may spin, circulate, move through space, or flow down a wire. For example, consider a long, straight wire that carries a current I. The current generates a magnetic field in the surrounding space, of magnitude

Equation Sheet

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

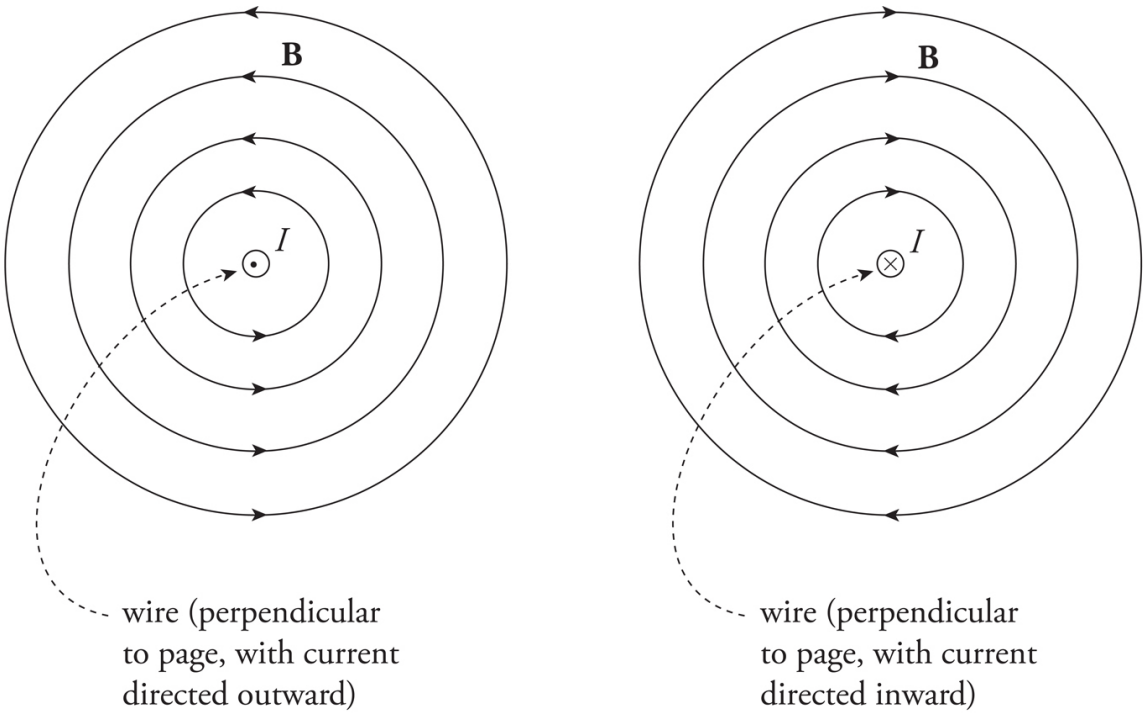
where r is the distance from the wire. The symbol μ_0 denotes a fundamental constant called the permeability of free space. Its value is

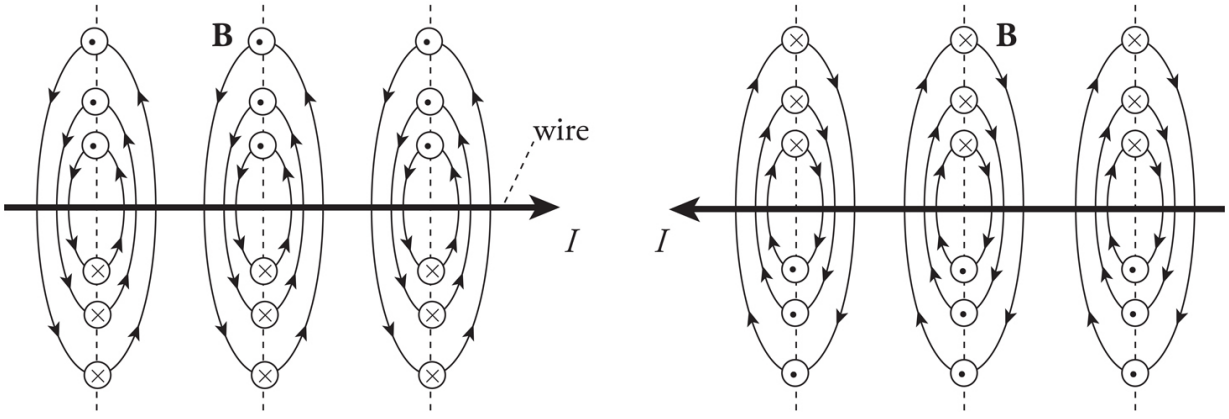
$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2} = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

The magnetic field lines are actually circles whose centers are on the wire. The direction of these circles is determined by a variation of the right-hand rule. Imagine grabbing the wire in your right hand with your thumb pointing in the direction of the current. Then the direction in which your fingers curl around the wire gives the direction of the magnetic field line.

Magnetic Fields vs. Electric Fields

Unlike electric fields that start at a point charge and end at another point charge, magnetic fields are unending loops. Electric charges can be positive or negative and exist by themselves. There is no such thing as a monopole for magnets.



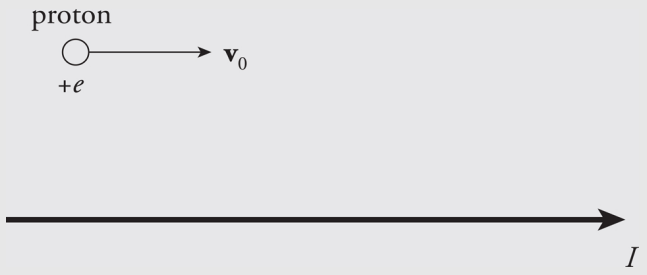




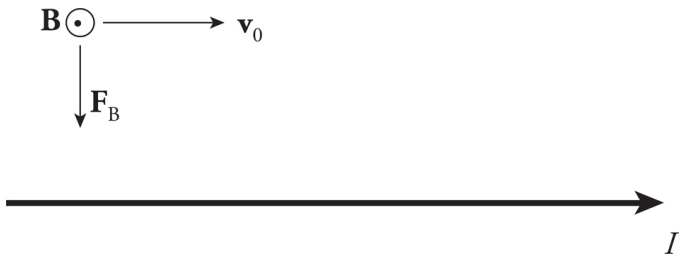
Right-Hand Rule for the Magnetic Field Created by a Current-Carrying Wire:

1. Put your thumb in direction of current or in direction of a positive traveling charge.
2. Grab the wire/path.
3. As the fingers curl around your thumb, it represents the magnetic field going around the wire/path.

Example 8 The diagram below shows a proton moving with a speed of 2×10^5 m/s, initially parallel to, and 4 cm from, a long, straight wire. If the current in the wire is 20 A, what's the magnetic force on the proton?



Solution. Above the wire (where the proton is), the magnetic field lines generated by the current-carrying wire point out of the plane of the page, so $\mathbf{v}_0 \times \mathbf{B}$ points downward. Since the proton's charge is positive, the magnetic force $\mathbf{F}_B = q(\mathbf{v}_0 \times \mathbf{B})$ is also directed down, toward the wire.



The strength of the magnetic force on the proton is

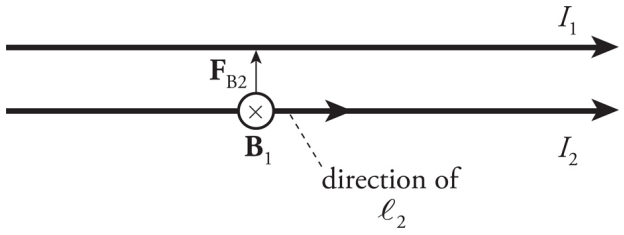
$$F_B = qv_0 B = ev_0 \frac{\mu_0}{2\pi}\frac{I}{r} = (1.6\times10^{-19}\ \text{C})(2\times10^{5}\ \text{m/s})\frac{4\pi\times10^{-7}\ \text{N/A}^2}{2\pi}\frac{20\ \text{A}}{0.04\ \text{m}}$$

$$= 3.2\times10^{-18}\ \text{N}$$

Example 9 The diagram below shows a pair of long, straight, parallel wires, separated by a small distance, r. If currents I_1 and I_2 are established in the wires, what is the magnetic force per unit length they exert on each other?



Solution. To find the force on Wire 2, consider the current in Wire 1 as the source of the magnetic field. Below Wire 1, the magnetic field lines generated by Wire 1 point into the plane of the page. Therefore, the force on Wire 2, as given by the equation $\mathbf{F}_{B2} = I_2(\boldsymbol{\ell}_2 \times \mathbf{B}_1)$, points upward.



The magnitude of the magnetic force per unit length felt by Wire 2, due to the magnetic field generated by Wire 1, is found this way:

$$F_{B2} = I_2\ell_2 B_1 = I_2\ell_2\frac{\mu_0}{2\pi}\frac{I_1}{r} \quad\Rightarrow\quad \frac{F_{B2}}{\ell_2} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$$

By Newton's Third Law, this is the same force that Wire 1 feels due to the magnetic field generated by Wire 2. The force is attractive because the currents point in the same direction; if one of the currents were reversed, then the force between the wires would be repulsive.

SOLENOIDS CREATE UNIFORM FIELDS

The parallel plate capacitor that was discussed in Chapter 6 appears frequently on the AP Physics 2 Exam because it generates a constant electric field in the region between the plates. Knowing that a current carrying wire creates a magnetic field, if you look back at the section on Bar Magnets, you see that the magnetic field produced by a bar magnet closely resembles that produced by a single closed loop of current carrying wire.

A device called a solenoid is constructed by a series of coaxial wires through which a continuous current flows. The magnetic fields from each of the individual loops of wire add together and within the region inside the solenoid, the magnetic field becomes uniform. If the solenoid is very long in comparison to its diameter and the coils are tightly packed together, the field inside a solenoid created in a laboratory is very nearly uniform in both direction and strength.

Unlike a bar magnet, a solenoid is an electromagnet. When there is a current flowing through the wires of the solenoid, the solenoid creates a magnetic field. When there is no current flowing through the wire, there is not a magnetic field. The bar magnet, on the other hand, always produced a magnetic field.

Chapter 8 Review Questions

Click here to download the PDF of Chapter 8 Review Questions.

Solutions can be found in Chapter 12.

Section I: Multiple Choice

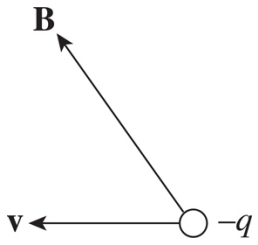
1. Which of the following is/are true concerning magnetic forces and fields? Select two answers.

   (A) The magnetic field lines due to a current-carrying wire radiate away from the wire.
   (B) The kinetic energy of a charged particle can be increased by a magnetic force.
   (C) A charged particle can move through a magnetic field without feeling a magnetic force.
   (D) A moving charged particle generates a magnetic field.

2. Two long wires carry a non-zero current *I*. At the location midway between the two wires, there will be a magnetic field strength of 0 T only if which of the following is true?

   (A) The wires are perpendicular to one another.
   (B) The wires must be coiled to create solenoids.
   (C) The wires must be parallel and the current must flow in the same direction.
   (D) The wires must be parallel and the current must flow in opposite directions.

3. A particle of charge $-q$ moves to the left at speed v through a uniform magnetic field **B** which points into the second quadrant, as shown below.



A second experiment is performed which results in the same magnitude and same direction of the magnetic force on the particle. Which of the following could NOT be the conditions for the new experiment?

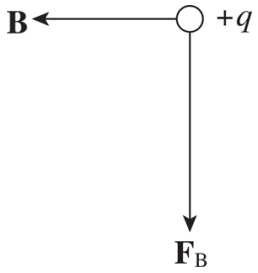
(A) **B** is unchanged, but q is now positive and **v** is directed to the right.

(B) The charge is still negative, but **v** and **B** are rotated 90° into the plane of the paper so that **v** is directed along the $-z$-axis.

(C) The charge is still negative, but **v** and **B** are rotated 90° counterclockwise so that **v** is directed along the $-y$-axis.

(D) **V** is unchanged, but q is now positive and **B** is rotated 180° to point into the fourth quadrant.

4. In the figure below, what must be the direction of the particle's velocity, **v**?



(A) Downward, in the plane of the page

(B) Upward, in the plane of the page

(C) Out of the plane of the page

(D) Into the plane of the page

5. An experiment is performed with a long current carrying wire in a region free from any other magnetic fields. The current is varied and the field strength is recorded. Which of the following statements about a graph of **B** versus **I** is true?

(A) The slope of the graph is directly proportional to the square of the distance the magnetic field strength was measured from the wire.

(B) The slope of the graph is directly proportional to the distance the magnetic field strength was measured from the wire.

(C) The slope of the graph is inversely proportional to the square of the distance the magnetic field strength was measured from the wire.

(D) The slope of the graph is inversely proportional to the distance the magnetic field strength was measured from the wire.

6. Which of the following situations would result in the largest magnetic field?

(A) Measuring at a distance r from a wire carrying a current of I.

(B) Measuring at a distance $2r$ from a wire carrying a current of $I/2$.

(C) Measuring at a distance $r/2$ from a wire carrying a current of $2I$.

(D) All four measurements would result in the same magnetic field.

7. Two long, straight wires are hanging parallel to each other and are 1 cm apart. The current in Wire 1 is 5 A, and the current in Wire 2 is 10 A, in the same direction. Which of the following best describes the magnetic force per unit length felt by the wires?

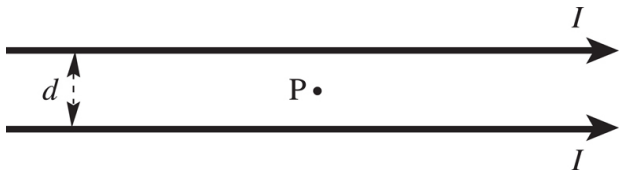
(A) The force per unit length on Wire 1 is twice the force per unit length on Wire 2.

(B) The force per unit length on Wire 2 is twice the force per unit length on Wire 1.

(C) The force per unit length on Wire 1 is 0.0003 N/m, away from Wire 2.

(D) The force per unit length on Wire 1 is 0.001 N/m, toward Wire 2.

8. In the figure below, what is the magnetic field at the Point P, which is midway between the two wires?



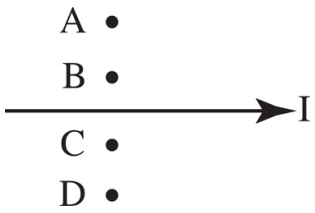
(A) $2\mu_0 I/(\pi d)$, into the plane of the page

(B) $\mu_0 I/(2\pi d)$, out of the plane of the page

(C) $\mu_0 I/(2\pi d)$, into the plane of the page

(D) Zero

9. Here is a section of a wire with a current moving to the right. Where is the magnetic field strongest and pointing INTO the page?



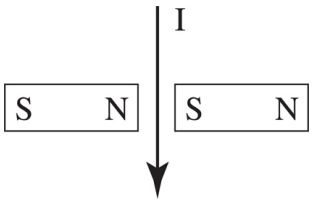
(A) A

(B) B

(C) C

(D) D

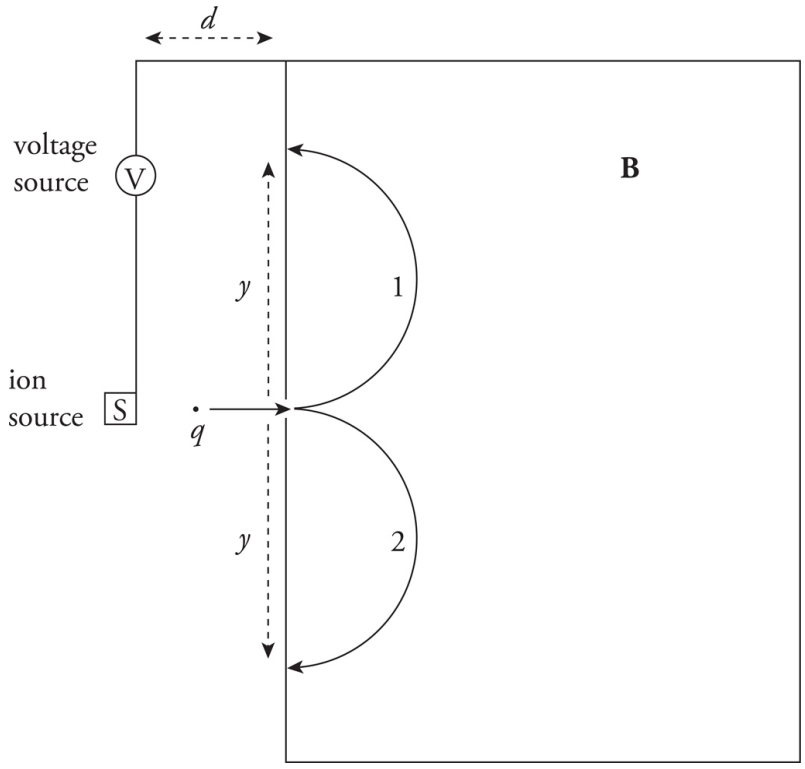
10. What is the direction of force acting on the current-carrying wire as shown below?



(A) To the bottom of the page
(B) Into the page
(C) Out of the page
(D) To the right of the page

Section II: Free Response

1. The diagram below shows a simple mass spectrograph. It consists of a source of ions (charged atoms) that are accelerated (essentially from rest) by the voltage V and enter a region containing a uniform magnetic field, **B**. The polarity of V may be reversed so that both positively charged ions (cations) and negatively charged ions (anions) can be accelerated. Once the ions enter the magnetic field, they follow a semicircular path and strike the front wall of the spectrograph, on which photographic plates are constructed to record the impact. Assume that the ions have mass m.



(a) What is the acceleration of an ion of charge q just before it enters the magnetic field?

(b) Find the speed with which an ion of charge q enters the magnetic field.

(c)

i. Which semicircular path, 1 or 2, would a cation follow?

ii. Which semicircular path, 1 or 2, would an anion follow?

(d) Determine the mass of a cation entering the apparatus in terms of y, q, $\mathbf{B}$, and V.

(e) Once a cation of charge q enters the magnetic field, how long does it take to strike the photographic plate?

(f) What is the work done by the magnetic force in the spectrograph on a cation of charge q?

2. A region of uniform magnetic field directed into the page (in the $-z$ direction) is generated. A positively charged particle travelling in the +x direction at a speed v enters the region of the magnetic field, as shown below.



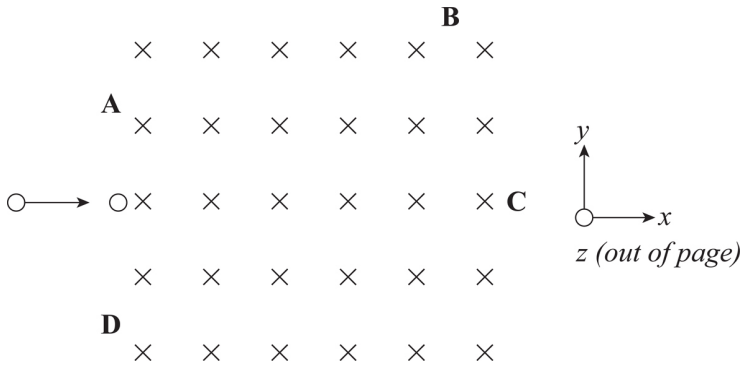
(a) What direction is the magnetic force on the particle at the instant when it enters the magnetic field region?

(b) Some time passes while the particle continues to move in the region of the magnetic field under the influence of the magnetic force.

i. How will the new speed v_{new} compare to the initial speed v?

ii. How will new velocity $\overrightarrow{v_{new}}$ compare to the initial velocity $\vec{v}$? Describe both the magnitude of the velocity as well as its components in comparing it to the initial velocity.

(c) More time passes and the particle leaves the region of the magnetic field. For each position A, B, C, and D on the diagram below, state whether that position could be the position where the particle leaves the region of the magnetic field or not. For each position that you state is a possible exit position, draw the path on the diagram below that the particle would take from the dot where it enters the magnetic field to the letter where it exits.



Summary

- Charges moving through a magnetic field experience a force whose magnitude is given by $F_B = qv\mathbf{B}\sin\theta$ and whose direction is given by the right-hand rule.
- Because the force is always perpendicular to the direction of velocity, the charge may experience uniform circular motion. It would then follow all the appropriate circular motion relationships and orbit in a radius given by $r = \frac{mv}{q\mathbf{B}}$.
- Because wires have charges moving though them, a wire will experience a force if placed in a magnetic field. This is expressed by $F_{\mathrm{B}} = \mathbf{B}I\ell\sin\theta$. Also, a current-carrying wire will produce a magnetic field whose strength is given by $\mathbf{B} = \frac{\mu_o}{2\pi}\frac{I}{r}$, where μ_0 = $4\pi \times 10^{-7}$ T·m/A, I is the current through the wire, and r is the radial distance from the wire.

Chapter 9
Electromagnetic Induction

INTRODUCTION

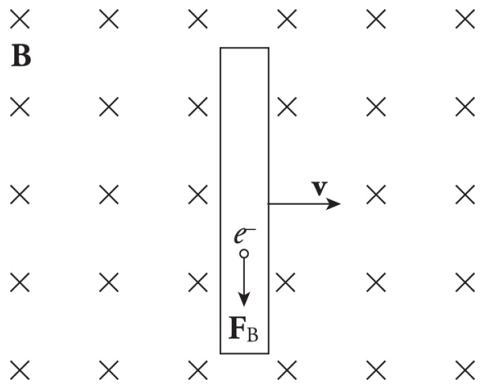
In Chapter 8, we learned that electric currents generate magnetic fields, and we will now see how magnetism can generate electric currents.

Vice-Versa

In the previous chapter, we learned moving charges generate magnetic fields. If we think about this backward, magnetic fields then can generate moving charges or current.

MOTIONAL EMF

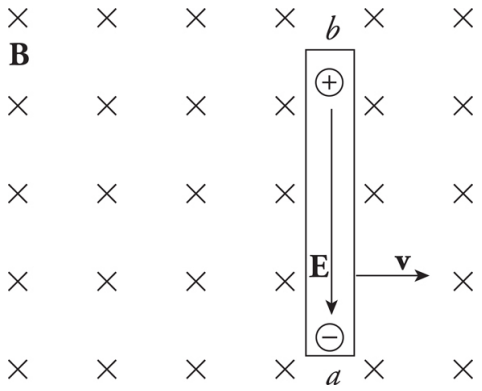
The simple act of moving a conducting rod in the presence of an external magnetic field *creates an electric field* within the rod. The figure below shows a conducting wire of length, ℓ, moving with constant velocity v in the plane of the page through a uniform magnetic field $\boldsymbol{B}$ that's perpendicular to the page. The presence of a moving conductor (recall there are many electrons present in the wire) inside magnetic field results in the creation of an electric field that points in the same direction as the magnetic force on the electrons. (We know that the magnetic force is not directly responsible for the movement of the electrons in the rod because the magnetic force is *always* perpendicular to direction of the motion of charges and therefore does no work. The magnetic field cannot be the source of the energy that causes the charges to move.) We can find the direction of the magnetic field using the right-hand rule the instant the charges begin to move. The charges are negative and moving to the right, so $q\boldsymbol{v}$ points to the left (thumb of your right hand). $\boldsymbol{B}$ points into the page (fingers of your right hand). Your palm shows the direction of the magnetic force is downward. Therefore, the motion of the rod through the external magnetic field generates an electric field within the rod that causes electrons to move toward the bottom of the rod. This *induced electric field* points downward.



Right-Hand Rules

You will have to use a combination of your right-hand rules in order to find the direction of current. Just pay attention to orientation.

As long as the rod continues to move at velocity $\boldsymbol{v}$, the electric field will be maintained within the rod. Electric fields are generated by charge separations, and therefore there will be a surfeit of negative charges on the bottom of the rod and a deficit of electron at the top of the rod.



A charge q in the wire feels two forces: an electric force, $\mathbf{F}_E = q\mathbf{E}$, and a magnetic force, $F_B = qvB\sin\theta = qvB$, because $\theta = 90°$.

If q is negative, $\mathbf{F}_E$ is upward and $\mathbf{F}_B$ is downward; if q is positive, $\mathbf{F}_E$ is downward and $\mathbf{F}_B$ is upward. So, in both cases, the forces act in opposite directions. Once the magnitude of $\mathbf{F}_E$ equals the magnitude of $\mathbf{F}_B$, the charges in the wire are in electromagnetic equilibrium. This occurs when $qE = qvB$; that is, when $E = vB$.

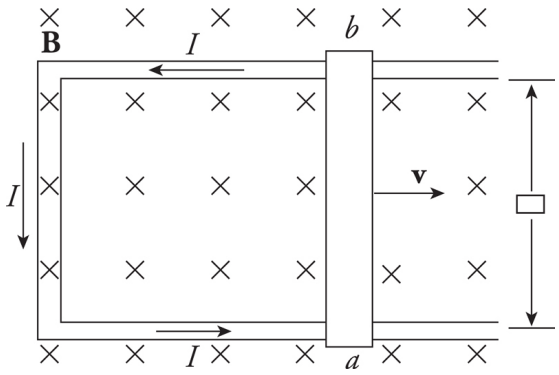
The presence of the electric field requires a potential difference between the ends of the rod. Since negative charge accumulates at the lower end (which we'll call point a) and positive charge accumulates at the upper end (point b), point b is at a higher electric potential. The potential difference V_{ba} is equal to $E\ell$ and, since $E = vB$, the potential difference can be written as $vB\ell$.

Now, imagine that the rod is sliding along a pair of conducting rails connected at the left by a stationary bar. The sliding rod now completes a rectangular circuit, and the potential difference V_{ba} causes current to flow.

Induced Current

An induced current can be created three different ways:

1. Changing the area of the loop of wire in a stationary magnetic field
2. Changing the magnetic field strength through a stationary circuit
3. Changing the angle between the magnetic field and the wire loop



Each of these three changes causes a change in the flux, as we will see in the next section. The motion of the sliding rod through the magnetic field creates an electromotive force, called **motional emf:**

Equation Sheet

$$\varepsilon = B \ell v$$

The existence of a current in the sliding rod causes the magnetic field to exert a force on it. Using the formula $F_B = BI\ell$, the fact that ℓ points upward (in the direction of the current) and **B** is into the page, tells

us that the direction of $\mathbf{F}_B$ on the rod is to the left. An external agent must provide this same amount of force to the right to maintain the rod's constant velocity and keep the current flowing. The power that the external agent must supply is $P = Fv = I\ell\ Bv$, and the electrical power delivered to the circuit is $P = IV_{ba} = I\varepsilon = IvB\ell$. Notice that these two expressions are identical. The energy provided by the external agent is transformed first into electrical energy and then thermal energy as the conductors making up the circuit dissipate heat.

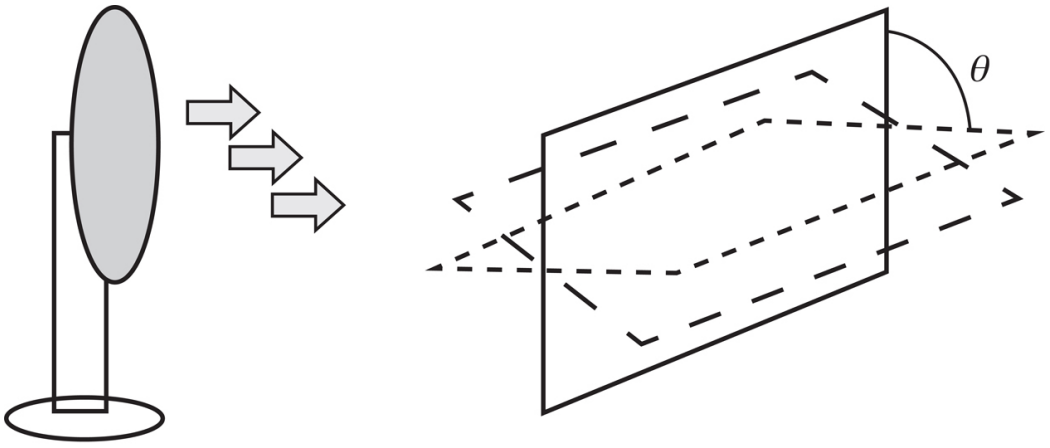
This relationship between current in a coil of wire and magnetic fields sets the basis for Faraday's Law.

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Electromotive force can be created by the motion of a conducting wire through a magnetic field, but there is another way to generate an emf from a magnetic field.

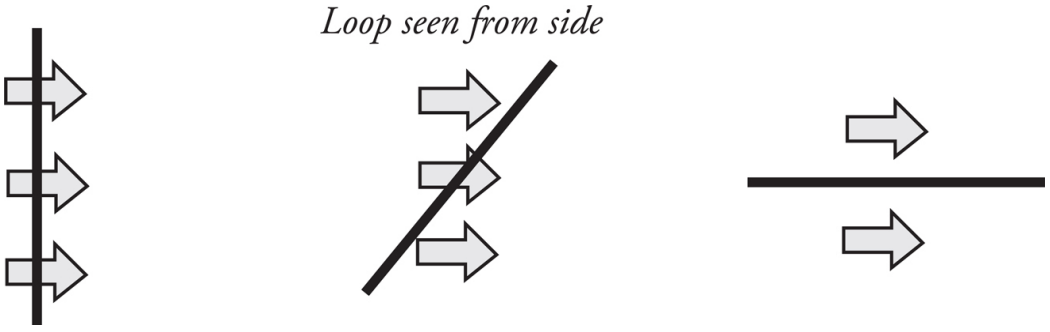
Faraday's discovery found that a current is induced when the magnetic flux passing through the coil or loop of wire changes. Magnetic flux helps us define what that amount of field passing into the loop means.

Imagine holding a loop of wire in front of a fan as shown:



Fan

The amount of air that flows through the loop depends on the area of the loop as well as its orientation (tilt angle θ).



The most effective airflow is when the loop is completely perpendicular, as in the situation to the left. The least effective is when the airflow and loop are in the situation to the right.

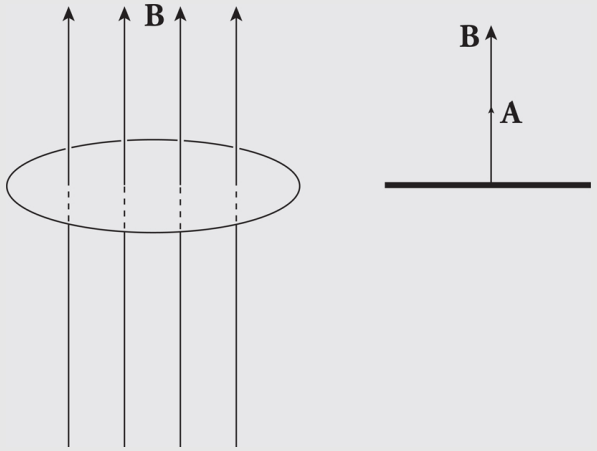
We can apply this idea to a magnetic field passing through a loop. The magnetic flux, Φ_B, through an area A is equal to the product of A and the magnetic field parallel to the area vector. The area vector points normal to the surface.

Equation Sheet

$$\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA \cos \theta$$

Magnetic flux measures the density of magnetic field lines that cross through an area. (Note that the direction of A is taken to be perpendicular to the plane of the loop.)

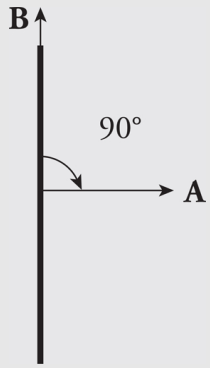
Example 1 The figure below shows two views of a circular loop of radius 3 cm placed within a uniform magnetic field, **B** (magnitude 0.2 T).



(a) What's the magnetic flux through the loop?

(b) What would be the magnetic flux through the loop if the loop were rotated 45°?

(c) What would be the magnetic flux through the loop if the loop were rotated 90°?



Magnetic Flux Units

The SI unit for Magnetic Flux is called the weber (Wb), which is equivalent to one Tesla meter-squared (Tm^2).

Solution.

(a) Since **B** is parallel to **A**, the magnetic flux is equal to BA:

$$\Phi_B = BA = B \cdot \pi r^2 = (0.2\ \text{T}) \cdot \pi(0.03\ \text{m})^2 = 5.7 \times 10^{-4}\ \text{T}\cdot\text{m}^2$$

The SI unit for magnetic flux, the tesla·meter^2, is called a **weber** (abbreviated **Wb**). So $\Phi_B = 5.7 \times 10^{-4}$ Wb.

(b) Since the angle between **B** and **A** is 45°, the magnetic flux through the loop is

$$\Phi_B = BA \cos 45^\circ = B \cdot \pi r^2 \cos 45^\circ = (0.2\ \text{T}) \cdot \pi(0.03\ \text{m})^2 \cos 45^\circ = 4.0 \times 10^{-4}\ \text{Wb}$$

(c) If the angle between **B** and **A** is 90°, the magnetic flux through the loop is zero, since cos 90° = 0.

The concept of magnetic flux is crucial, because changes in magnetic flux induce emf. According to **Faraday's Law of Electromagnetic Induction,** the magnitude of the emf induced in a circuit is equal to the rate of change of the magnetic flux through the circuit. This can be written mathematically in the form

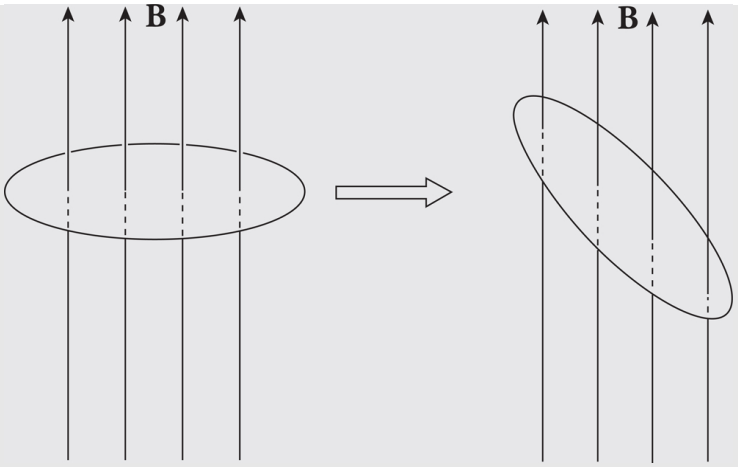
$$\left|\mathcal{E}_{\text{avg}}\right| = \left|\frac{\Delta\Phi_{\text{B}}}{\Delta t}\right|$$

This induced emf can produce a current, which will then create its own magnetic field. The direction of the induced current is determined by the polarity of the induced emf and is given by **Lenz's Law:** the induced current will always flow in the direction that opposes the change in magnetic flux that produced it. If this were not so, then the magnetic flux created by the induced current would magnify the change that produced it, and energy would not be conserved. Lenz's Law can be included mathematically with Faraday's Law by the introduction of a minus sign; this leads to a single equation that expresses both results:

Equation Sheet

$$\mathcal{E}_{\text{avg}} = -\frac{\Delta\Phi_{\text{B}}}{\Delta t}$$

Example 2 The circular loop of Example 1 rotates at a constant angular speed through 45° in 0.5 s.



(a) What's the induced emf in the loop?

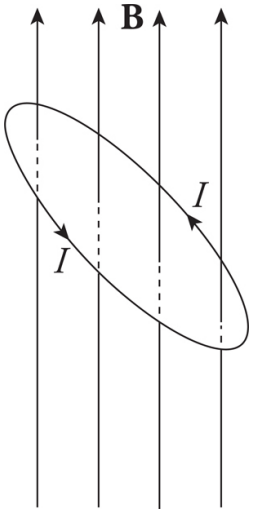
(b) In which direction will current be induced to flow?

Solution.

(a) As we found in Example 1, the magnetic flux through the loop changes when the loop rotates. Using the values we determined earlier, Faraday's Law gives

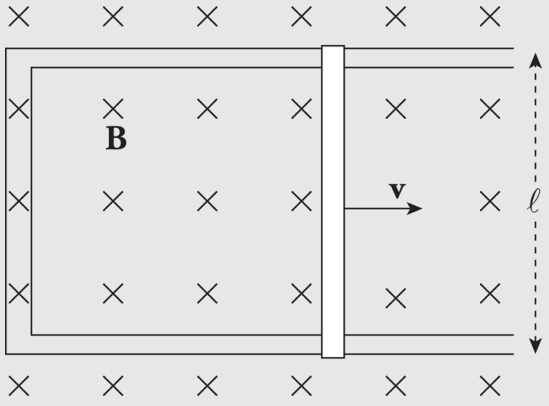
$$\mathcal{E}_{\text{avg}} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{(4.0\times10^{-4}\ \text{Wb}) - (5.7\times10^{-4}\ \text{Wb})}{0.5\ \text{s}} = 3.4\times10^{-4}\ \text{V}$$

(b) The original magnetic flux was 5.7×10^{-4} Wb upward, and was decreased to 4.0×10^{-4} Wb. So the change in magnetic flux is -1.7×10^{-4} Wb upward, or, equivalently, $\Delta\Phi_B = 1.7 \times 10^{-4}$ Wb, downward. To oppose this change we would need to create some magnetic flux upward. The current would be induced in the counterclockwise direction (looking down on the loop), because the right-hand rule tells us that then the current would produce a magnetic field that would point up.



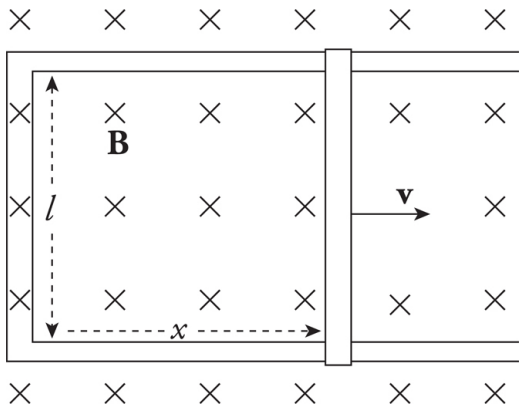
The current will flow only while the loop rotates, because emf is induced only when magnetic flux is changing. If the loop rotates 45° and then stops, the current will disappear.

Example 3 Again consider the conducting rod that's moving with constant velocity **v** along a pair of parallel conducting rails (separated by a distance ℓ), within a uniform magnetic field directed into the page, **B**:



Find the induced emf and the direction of the induced current in the rectangular circuit.

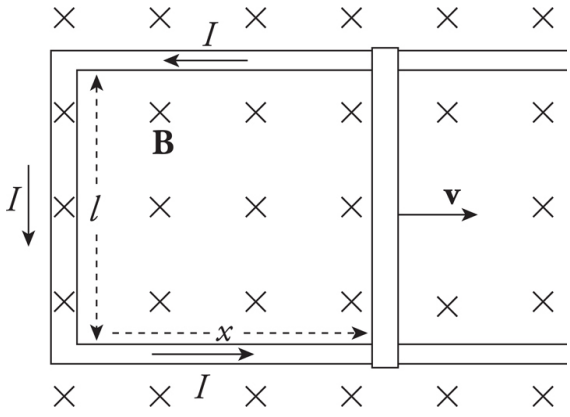
Solution. The area of the rectangular loop is $\ell\ x$, where x is the distance from the left-hand bar to the moving rod:



Because the area is changing, the magnetic flux through the loop is changing, which means that an emf will be induced in the loop. To calculate the induced emf, we first write $\Phi_B = BA = B\ \ell\ x$, then since $\Delta x/\Delta t = v$, we get

$$\mathcal{E}_{avg} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{\Delta(B\ell x)}{\Delta t} = -B\ell\frac{\Delta x}{\Delta t} = -B\ell v$$

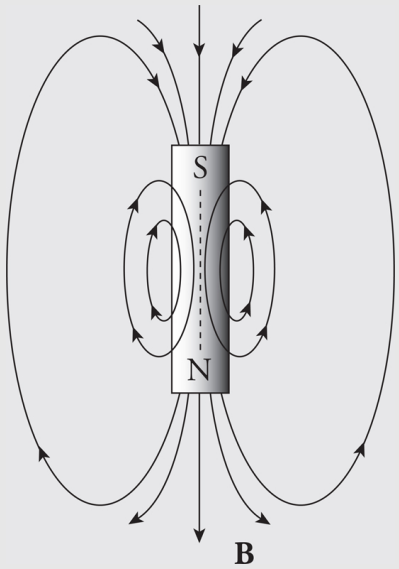
We can figure out the direction of the induced current from Lenz's Law. As the rod slides to the right, the magnetic flux into the page increases. How do we oppose an increasing into-the-page flux? By producing out-of-the-page flux. In order for the induced current to generate a magnetic field that points out of the plane of the page, the current must be directed counterclockwise (according to the right-hand rule).



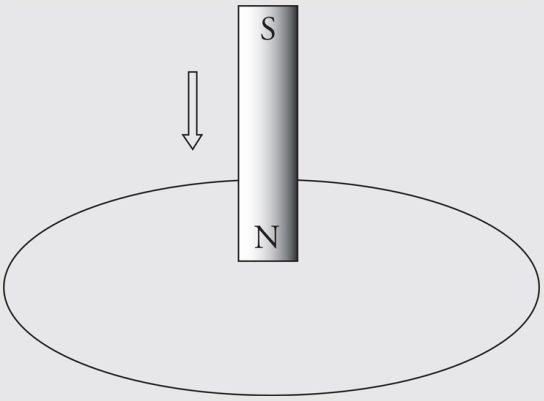
Note that the magnitude of the induced emf and the direction of the current agree with the results we derived earlier, in the section on motional emf.

This example also shows how a violation of Lenz's Law would lead directly to a violation of the Law of Conservation of Energy. The current in the sliding rod is directed upward, as given by Lenz's Law, so the conduction electrons are drifting downward. The force on these drifting electrons—and thus, the rod itself—is directed to the left, opposing the force that's pulling the rod to the right. If the current were directed downward, in violation of Lenz's Law, then the magnetic force on the rod would be to the right, causing the rod to accelerate to the right with ever-increasing speed and kinetic energy, without the input of an equal amount of energy from an external agent.

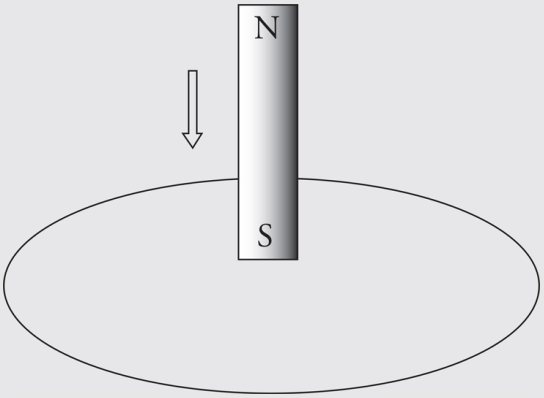
Example 4 A permanent magnet creates a magnetic field in the surrounding space. The end of the magnet at which the field lines emerge is designated the **north pole** (N), and the other end is the **south pole** (S):



(a) The figure below shows a bar magnet moving down, through a circular loop of wire. What will be the direction of the induced current in the wire?



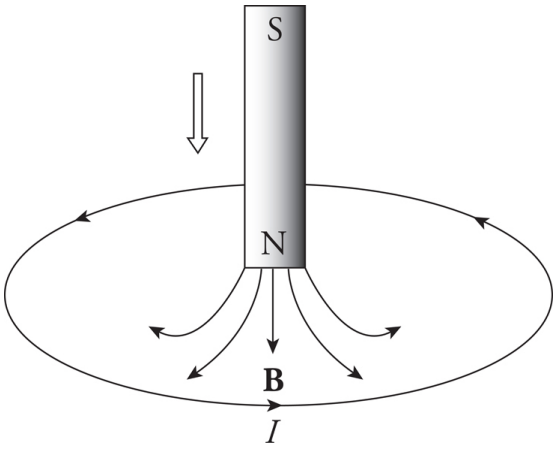
(b) What will be the direction of the induced current in the wire if the magnet is moved as shown in the following diagram?



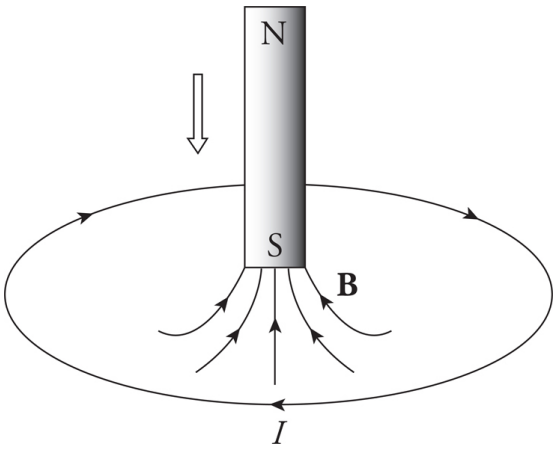
Solution.

(a) The magnetic flux down, through the loop, increases as the magnet is moved. By Lenz's Law, the induced emf will generate a current that opposes this change. How do we oppose a change of *more flux downward*? By creating flux *upward*. So, according to the right-hand rule, the induced current must flow

counterclockwise (because this current will generate an upward-pointing magnetic field):

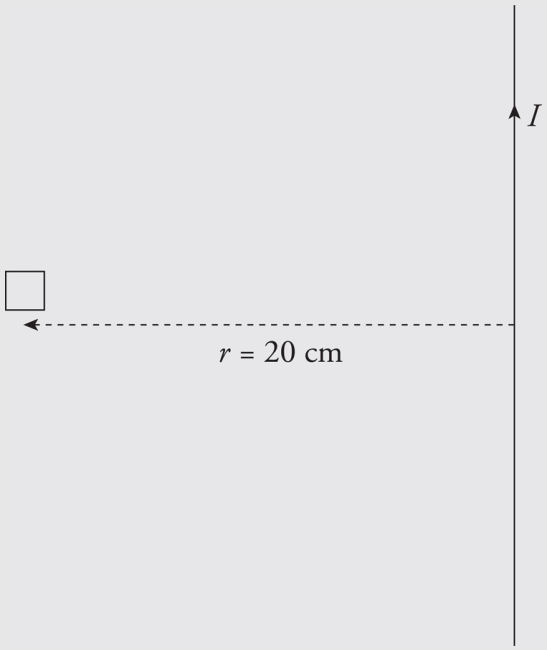


(b) In this case, the magnetic flux through the loop is upward and, as the south pole moves closer to the loop, the magnetic field strength increases so the magnetic flux through the loop increases upward. How do we oppose a change of *more flux upward*? By creating flux *downward*. Therefore, in accordance with the right-hand rule, the induced current will flow clockwise (because this current will generate a downward-pointing magnetic field):



Example 5 A square loop of wire 2 cm on each side contains 5 tight turns and has a total resistance of 0.0002 Ω. It is placed 20 cm from a long, straight,

current-carrying wire. If the current in the straight wire is increased at a steady rate from 20 A to 50 A in 2 s, determine the magnitude and direction of the current induced in the square loop. (Because the square loop is at such a great distance from the straight wire, assume that the magnetic field through the loop is uniform and equal to the magnetic field at its center.)



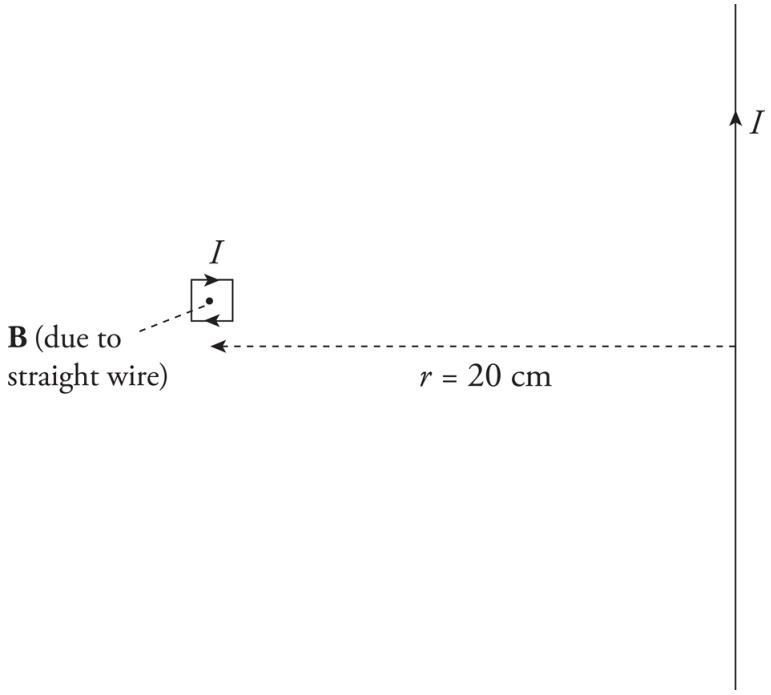
Solution. At the position of the square loop, the magnetic field due to the straight wire is directed out of the plane of the page and its strength is given by the equation $B = (\mu_0/2\pi)(I/r)$. As the current in the straight wire increases, the magnetic flux through the turns of the square loop changes, inducing an emf and current. There are $N = 5$ turns; each loop contributes the same flux, so the total flux becomes the number of loops N times the flux of each individual loop, Φ_B. Faraday's Law becomes $\varepsilon_{avg} = -N(\Delta\Phi_B/\Delta t)$, and

$$\mathcal{E}_{\text{avg}} = -N\frac{\Delta\Phi_B}{\Delta t} = -N\frac{\Delta(BA)}{\Delta t} = -NA\frac{\Delta B}{\Delta t} = -NA\frac{\mu_0}{2\pi r}\frac{\Delta I}{\Delta t}$$

Substituting the given numerical values, we get

$$\begin{aligned}\mathcal{E}_{\text{avg}} &= -NA\frac{\mu_0}{2\pi r}\frac{\Delta I}{\Delta t} \\ &= -(5)(0.02\text{ m})^2\frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{2\pi(0.20\text{ m})}\frac{(50\text{ A}-20\text{ A})}{2\text{ s}} \\ &= -3\times10^{-8}\text{ V}\end{aligned}$$

The magnetic flux through the loop is out of the page and increases as the current in the straight wire increases. To oppose an increasing out-of-the-page flux, the direction of the induced current should be clockwise, thereby generating an into-the-page magnetic field (and flux).



The value of the current in the loop will be

$$I = \frac{\mathcal{E}}{R} = \frac{3\times10^{-8}\text{ V}}{0.0002\ \Omega} = 1.5\times10^{-4}\text{ A}$$

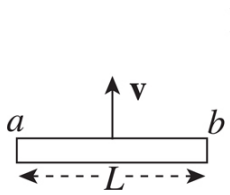
Chapter 9 Review Questions

Click here to download the PDF of Chapter 9 Review Questions.

Solutions can be found in Chapter 12.

Section I: Multiple Choice

1. A metal rod of length L is pulled upward with constant velocity **v** through a uniform magnetic field **B** that points out of the plane of the page.



What is the potential difference between points a and b?

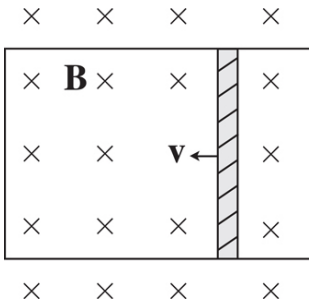
(A) 0

(B) $\frac{1}{2}vBL$, with point b at the higher potential

(C) vBL, with point a at the higher potential

(D) vBL, with point b at the higher potential

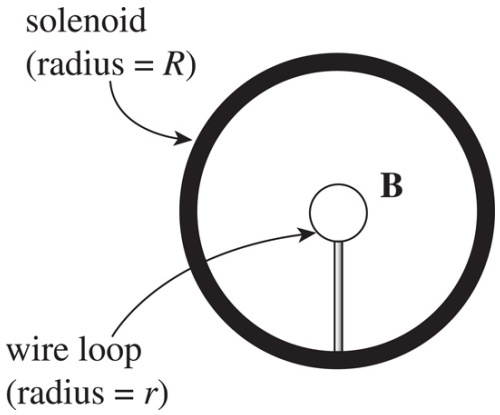
2. A conducting rod of length 0.2 m and resistance 10 ohms between its endpoints slides without friction along a U-shaped conductor in a uniform magnetic field B of magnitude 0.5 T perpendicular to the plane of the conductor, as shown in the diagram below.



If the rod is moving with velocity **v** = 3 m/s to the left, what is the magnitude and direction of the current induced in the rod?

	Current	Direction
(A)	0.03 A	down
(B)	0.03 A	up
(C)	0.3 A	down
(D)	0.3 A	up

3. In the figure below, a small, circular loop of wire (radius r) is placed on an insulating stand inside a hollow solenoid of radius R. The solenoid has n turns per unit length and carries a counterclockwise current I. If the current in the solenoid is decreased at a steady rate of a amps/s, determine the induced emf, ε, and the direction of the induced current in the loop. Note that $B = \mu_0 n I$ for a solenoid.



(A) $\varepsilon = \mu_0 \pi n r^2 a$; induced current is clockwise

(B) $\varepsilon = \mu_0 \pi n r^2 a$; induced current is counterclockwise

(C) $\varepsilon = \mu_0 \pi n R^2 a$; induced current is clockwise

(D) $\varepsilon = \mu_0 \pi n R^2 a$; induced current is counterclockwise

4. As shown in the figures below, a bar magnet is moved as a constant speed through a loop of wire. Figure A shows the bar magnet when it is as a position below the loop of wire and figure B shows the loop of wire after it has passed completely through the loop.

Figure A

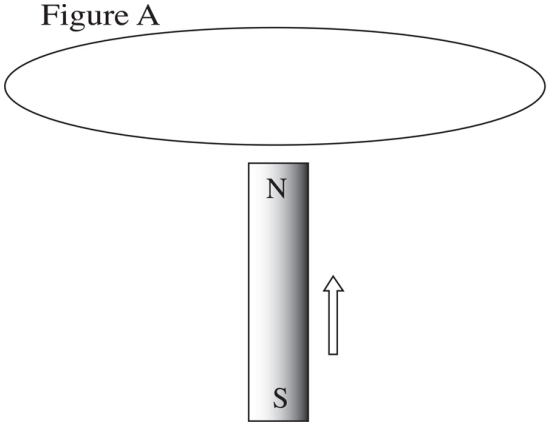
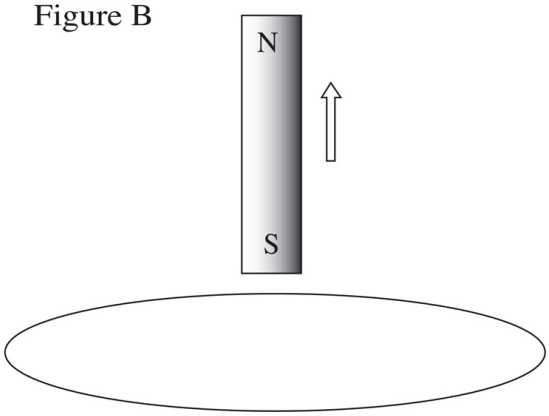


Figure B



Which of the following best describes the direction or directions of the current induced in the loop when the loop is looked at from above? Note that when looking at the loop from above, the bar magnet will be moving toward the viewer.

(A) Always clockwise

(B) Always counterclockwise

(C) First clockwise, then counterclockwise

(D) First counterclockwise, then clockwise

5. Which of the following statements about induced emf is true?

(A) An emf will be induced when moving the loop of wire perpendicularly through the magnetic field.

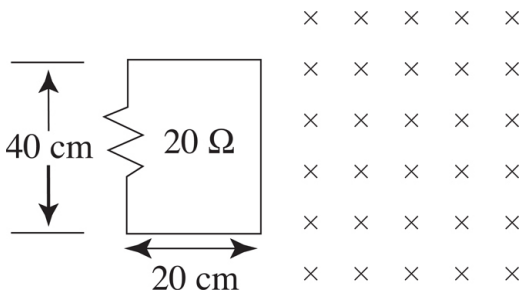
(B) Rotating the wire in the presence of a magnetic field always induces an emf.

(C) Shrinking the size of a loop of wire in constant magnetic field will induce an emf in the wire.

(D) A time varying magnetic field is required to induce an emf.

Section II: Free Response

1. A rectangular wire is pulled through a uniform magnetic field of 2 T going into the page as shown. The resistor has a resistance of 20 Ω.



(a) What is the voltage across the resistor as the wire is pulled horizontally at a velocity of 1 m/s and it just enters the field?

(b) What is the current through the circuit in the above case and in what direction does it flow?

(c) The region containing the magnetic field is 1 meter long in the direction the loop is being pulled. If the loop is pulled at the constant velocity of 1 m/s, describe the current in the loop

from a time before the loop encounters the left edge of the region of the magnetic field until the time that the right edge of the loop completely passes out of the magnetic field region. Indicate the values and the direction of the current.

(d) The loop of wire is rotated 90° clockwise so that the 20 cm side is vertical and the 40 cm side is horizontal at the top. Explain, without relying solely on equations, whether the loop would have to be pulled faster than 1/s, at exactly 1 m/s, or slower than 1 m/s for the current to have the same magnitude as in part (b).

Summary

- An electromotive force is produced as a conducting wire's position changes or as the magnetic field changes. This idea is summarized by $\varepsilon = B\ell v$.

- The flux (ϕ) tells the amount of something (for this chapter the magnetic field) that goes through a surface. The magnetic flux depends on the strength of the magnetic field, the surface area through which the field passes, and the angle between the two. This idea is summed up by $\phi_B = BA\cos\theta$.

- Faraday's Law of Induction says that if the wire is formed in a loop, an electromagnetic force is produced if the magnetic flux changes with time. This idea can be summarized by $\varepsilon_{avg} = -\frac{\Delta\phi_B}{\Delta t}$.

Chapter 10
Optics

INTRODUCTION

Light (or visible light) makes up only a small part of the entire spectrum of electromagnetic waves, which ranges from radio waves to gamma rays. Most waves require a material medium for their transmission, but electromagnetic waves can propagate through empty space. Electromagnetic waves consist of time-varying electric and magnetic fields that oscillate perpendicular to each other and to the direction of propagation of the wave. Through a vacuum, all electromagnetic waves travel at a fixed speed:

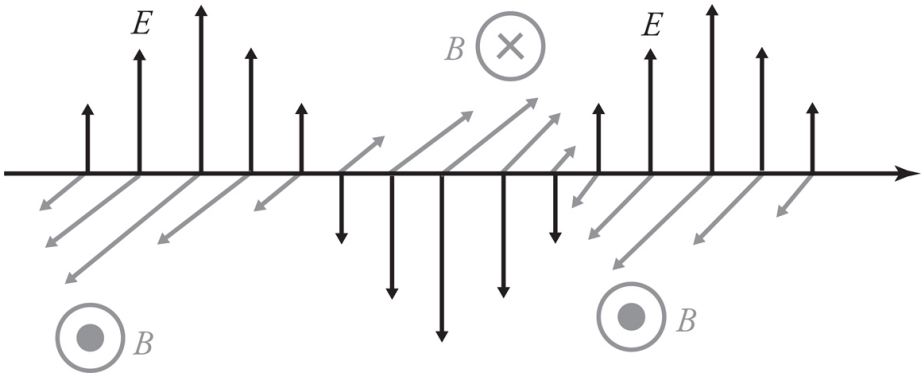
$$c = 3.00 \times 10^8 \text{ m/s}$$

regardless of their frequency. Like all the waves, we can say $v = f\lambda$.

ELECTROMAGNETIC WAVES

Back in AP Physics 1, we saw that if we oscillate one end of a long rope, we generate a wave that travels down the rope and has the same frequency as that of the oscillation.

You can think of an electromagnetic wave in a similar way: an oscillating electric charge generates an electromagnetic (EM) wave, which is composed of oscillating electric and magnetic fields. These fields oscillate with the same frequency at which the electric charge that created the wave oscillated. The fields oscillate in phase with each other, perpendicular to each other and the direction of propagation. For this reason, electromagnetic waves are transverse waves. The direction in which the wave's electric field oscillates is called the direction of polarization of the wave.



Unlike waves on a rope or sound waves, electromagnetic waves do not require a material medium to propagate; they can travel through empty space (vacuum). When an EM wave travels through a vacuum, its speed is constant, $c = 3 \times 10^8$ m/s.

EM waves obey the classic wave equation:

Equation Sheet

$$v = \lambda f$$

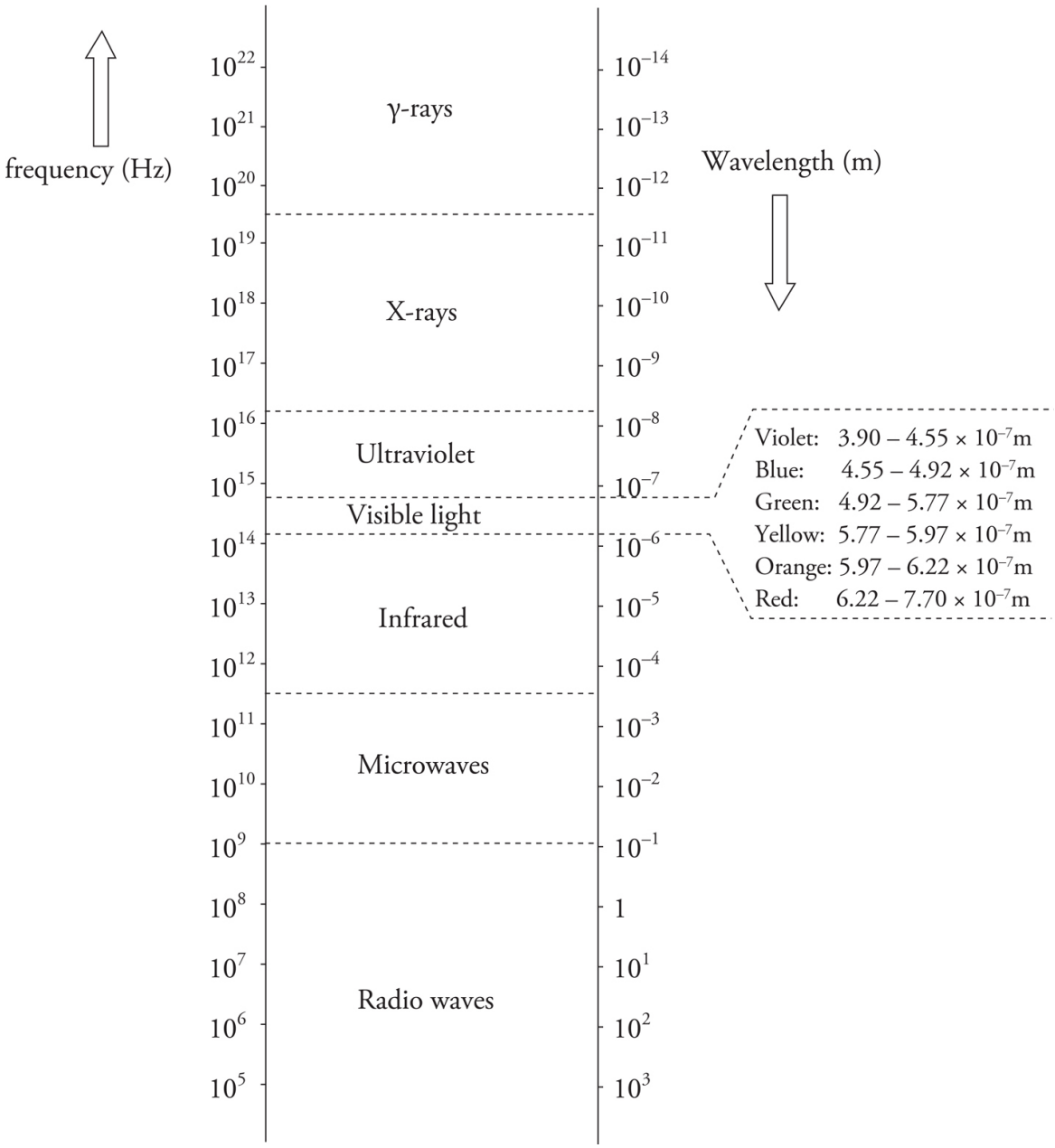
THE ELECTROMAGNETIC SPECTRUM

Electromagnetic waves can be categorized by their frequency (or wavelength); the full range of waves is called the **electromagnetic** (or **EM**) **spectrum**. Types of waves include **radio waves, microwaves, infrared, visible light, ultraviolet, X-rays**, and **γ-rays** (**gamma rays**) and, although they've been delineated in the spectrum below, there's no universal agreement on all the boundaries, so many of these bands overlap. You should be familiar with the names of the major categories, and, in particular, memorize the order of the colors within the visible spectrum (which, as you can see, accounts for only a tiny sliver of the full EM spectrum). In order of increasing wave frequency, the colors are red, orange, yellow, green, blue, and violet, which is commonly remembered as ROYGBV ("roy-gee-biv"). The wavelengths of the colors in the visible spectrum are usually expressed in nanometers. For example, electromagnetic waves whose wavelengths are between 577 nm and 597 nm are seen as yellow light.

Electromagnetic Wave Speed

All electromagnetic waves, regardless of frequency, travel through a vacuum at this speed. The most important equation for waves $v = \lambda f$ is also true for electromagnetic waves. For EM waves traveling through a vacuum, $v = c$, so the equation becomes $\lambda f = c$.

ELECTROMAGNETIC SPECTRUM



Example 1 What's the frequency range for green light?

Solution. According to the spectrum, light is green if its wavelength is between 4.92×10^{-7} m and 5.77×10^{-7} m. Using the equation $v = f\lambda$, we find that the upper end of this wavelength range corresponds to a frequency of

$$f_1 = \frac{v}{\lambda_1} = \frac{3.00 \times 10^8 \text{ m/s}}{5.77 \times 10^{-7} \text{ m}} = 5.20 \times 10^{14} \text{ Hz}$$

while the lower end corresponds to

$$f_2 = \frac{v}{\lambda_2} = \frac{3.00 \times 10^8 \text{ m/s}}{4.92 \times 10^{-7} \text{ m}} = 6.10 \times 10^{14} \text{ Hz}$$

So the frequency range for green light is

$$5.20 \times 10^{14} \text{ Hz} \le f_{\text{green}} \le 6.10 \times 10^{14} \text{ Hz}$$

Example 2 How would you classify electromagnetic radiation that has a wavelength of 1 cm?

Solution. According to the electromagnetic spectrum presented above, electromagnetic waves with $\lambda = 10^{-2}$ m are microwaves.

INTERFERENCE AND DIFFRACTION

As we learned in AP Physics 1, waves experience interference when they meet, and whether they interfere constructively or destructively depends on their relative phase. If they meet *in phase* (crest meets crest), they combine constructively, but if they meet *out of phase* (crest meets trough), they combine destructively. The key to the interference patterns we'll study in the next section rests on this observation. In particular, if waves that have the same wavelength meet, then the difference in the distances they've traveled determine whether the waves are in phase. Assuming that the waves are **coherent** (which means that their phase difference remains constant over time and does not vary), if the difference in their path lengths, $\Delta\ell$, is a whole number of wavelengths—0, ±λ, ±2λ, etc.—they'll arrive in phase at the meeting point. On the other hand, if this difference is a whole number plus one-half a wavelength—$\pm\frac{1}{2}$ λ, ±(1 + $\frac{1}{2}$)λ, ±(2 + $\frac{1}{2}$)λ, etc.—then they'll arrive exactly out of phase. That is,

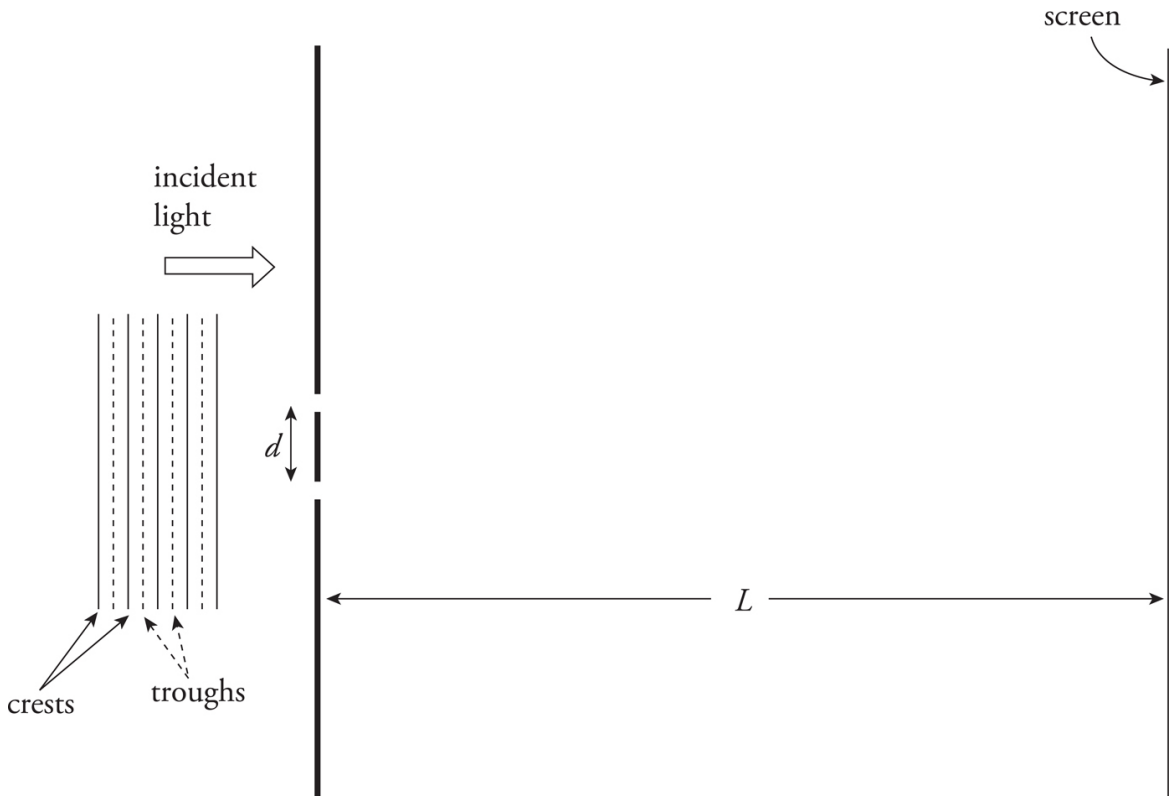
constructive interference: $\Delta\ell = m\lambda$

destructive interference: $\Delta\ell = (m + \frac{1}{2})\lambda$

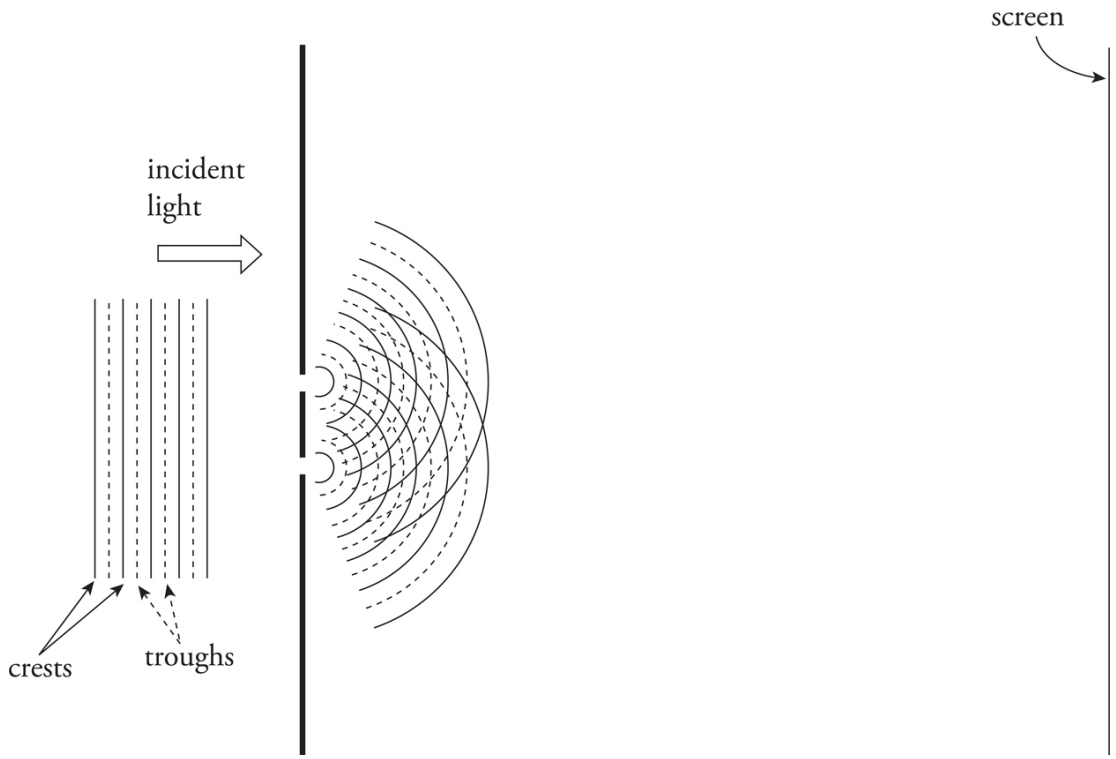
} where m = 0, 1, 2...

Young's Double-Slit Interference Experiment

The following figure shows incident light on a barrier that contains two narrow slits (perpendicular to the plane of the page), separated by a distance *d*. On the right is a screen whose distance from the barrier, *L*, is much greater than *d*. The question is, what will we see on the screen? You might expect that we'll just see two bright narrow strips of light, directly opposite the slits in the barrier. As reasonable as this may sound, it doesn't take into account the wave nature of light.



When a wave encounters an aperture whose width is comparable to its wavelength, the wave will fan out after it passes through. The alteration in the straight-line propagation of a wave when it encounters a barrier is called **diffraction**. In the setup above, the waves will diffract through the slits, and spread out and interfere as they travel toward the screen.

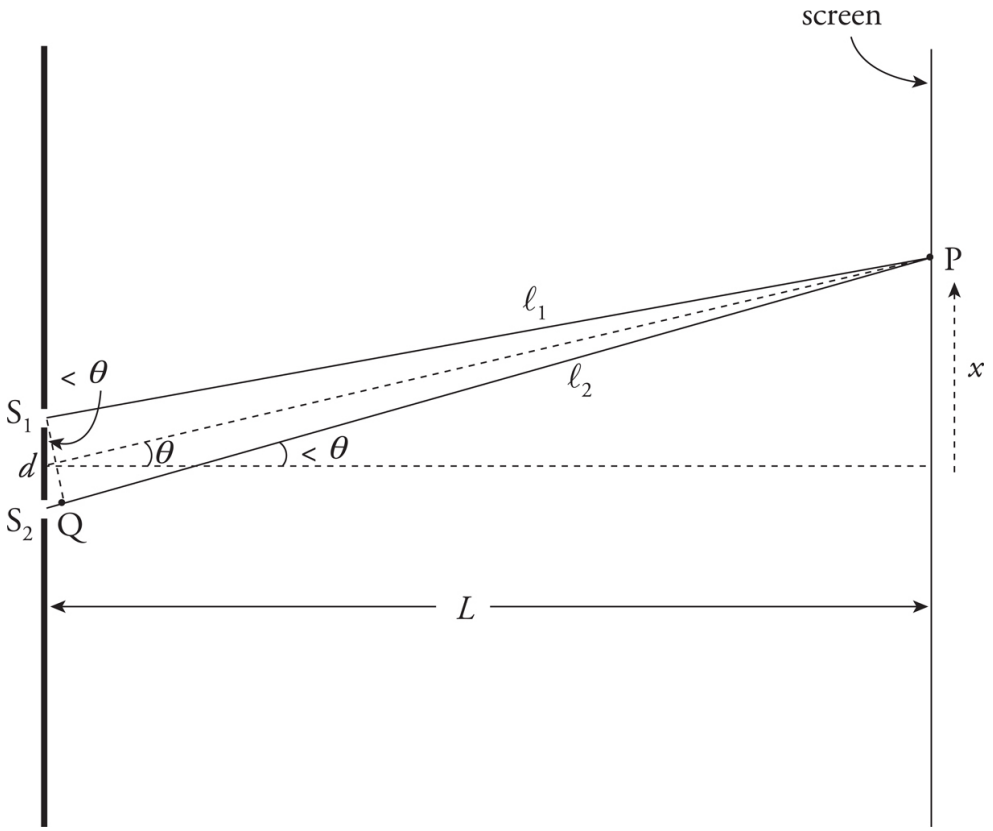


You can clearly see points of interference in the figure above. Look for solid lines intersecting other solid lines—these are points of constructive interference. Look for solid lines intersecting dashed lines—these are points of destructive interference.

The screen will show the results of this interference: There will be bright bands (bright **fringes**) centered at those points at which the waves interfere constructively, alternating with dark fringes, where the waves interfere destructively. Let's determine the locations of these fringes.

In the figure below, we've labeled the slits S_1 and S_2. A point P on the screen is selected, the path lengths from S_1 and S_2 to P are called ℓ_1 and ℓ_2, respectively, and the angle that the line from the midpoint of the slits to P makes with the horizontal is θ. Segment S_1Q is perpendicular to line S_2P. Because L is so much larger than d, the angle that line S_2P makes with the horizontal is also approximately θ,

which tells us that $\angle S_2S_1Q$ is approximately θ and that the path difference, $\Delta\ell = \ell_2 - \ell_1$, is nearly equal to S_2Q.



Because $S_2Q = d \sin \theta$, we get $\Delta\ell = d \sin \theta$. Now, using what we learned earlier about how constructive or destructive interference depends on $\Delta\ell$, we can write:

Equation Sheet

constructive interference (intensity maximum bright fringe on screen): $d \sin\theta = m\lambda$

where $m = 0, 1, 2...$

destructive interference (intensity minimum dark fringe on screen): $d \sin\theta = (m + \frac{1}{2})\lambda$

To locate the positions of, say, the bright fringes on the screen, we use the fact that $x = L \tan \theta$. If θ is small, then $\tan \theta \approx \sin \theta$, so we can write $x = L \sin \theta$ (we can tell this from the figure). Since $\sin \theta = m\lambda/d$ for bright fringes, we get

$$x_m = \frac{m\lambda L}{d}$$

Also, the intensity of the bright fringes decreases as m increases in magnitude. The bright fringe directly opposite the midpoint of the slits—the **central maximum**—will have the greatest intensity when $m = 0$. The bright fringes with $m = 1$ will have a lower intensity, those with $m = 2$ will be fainter still, and so on. If more than two slits are cut in the barrier, the interference pattern becomes sharper, and the distinction between dark and bright fringes becomes more pronounced. Barriers containing thousands of tiny slits per centimeter—called **diffraction gratings**—are used precisely for this purpose.

Example 3 For the experimental set-up we've been studying, assume that $d = 1.5$ mm, $L = 6.0$ m, and that the light used has a wavelength of 589 nm.

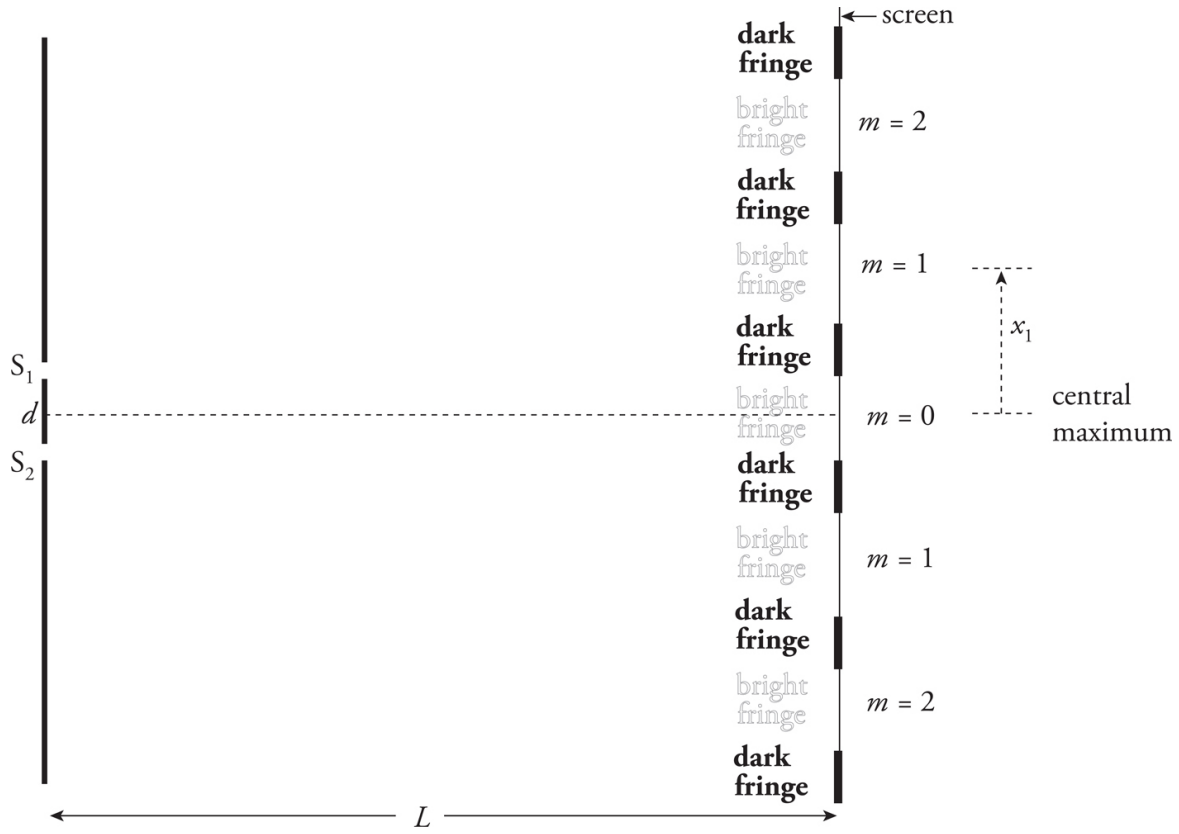
(a) How far above the center of the screen will the second brightest maximum appear?

(b) How far below the center of the screen is the third dark fringe?

(c) What would happen to the interference pattern if the slits were moved closer together?

Solution.

(a) The central maximum corresponds to $m = 0$ ($x_0 = 0$). The first maximum above the central one is labeled x_1 (since $m = 1$). The other bright fringes on the screen are labeled accordingly.



The value of x_1 is

$$x_1 = \frac{1 \cdot \lambda L}{d} = \frac{(589 \times 10^{-9}\ \text{m})(6.0\ \text{m})}{1.5 \times 10^{-3}\ \text{m}} = 2.4 \times 10^{-3}\ \text{m} = 2.4\ \text{mm}$$

(b) The first dark fringe occurs when the path difference is 0.5λ, the second dark fringe occurs when the path difference is 1.5λ, and the third dark fringe occurs when the path difference is 2.5λ, so

$$x_{\substack{\text{3rd minimum} \\ \text{below central max}}} = \frac{(2 + \frac{1}{2})\lambda L}{d} = \frac{(2 + \frac{1}{2})(589 \times 10^{-9}\ \text{m})(6.0\ \text{m})}{1.5 \times 10^{-3}\ \text{m}} = -5.9 \times 10^{-3}\ \text{m} = -5.9\ \text{mm}$$

(c) Since $x_m = m\lambda L/d$, a decrease in d would cause an increase in x_m. That is, the fringes would become larger; the interference pattern would be more spread out.

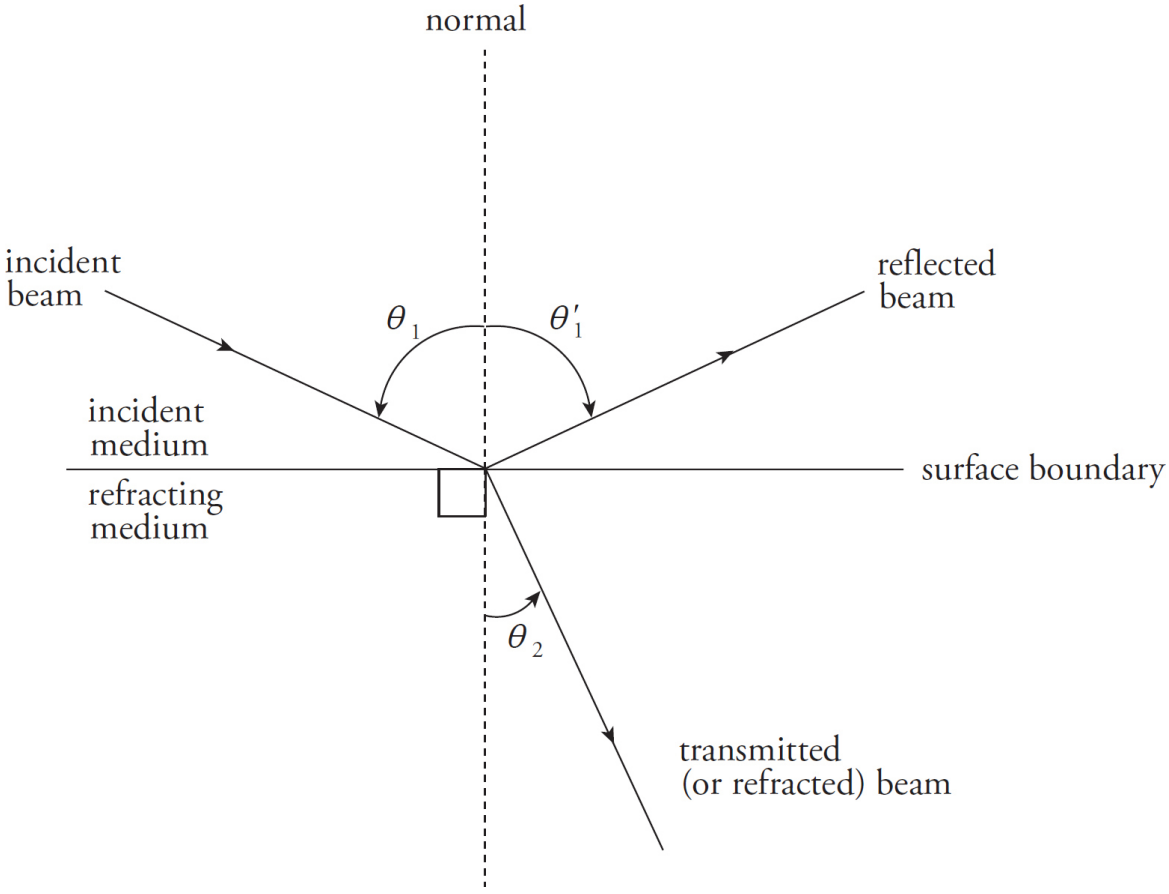
Single-Slit Diffraction

A diffraction pattern will also form on the screen if the barrier contains only one slit. The central maximum will be very pronounced, but lower-intensity maxima will also be seen because of interference from waves arriving from different locations within the slit itself. The width of the central maximum will become wider as the width of the slit is decreased.

For a circular pinhole aperture, the diffraction pattern will consist of a central, bright circular disk surrounded by rings of decreasing intensity.

REFLECTION AND REFRACTION

Imagine a beam of light directed toward a smooth transparent surface. When it hits this surface, some of its energy will be reflected off the surface and some will be transmitted into the new medium. We can figure out the directions of the reflected and transmitted beams by calculating the angles that the beams make with the normal to the interface. In the following figure, an incident beam strikes the boundary of another medium; it could be a beam of light in air striking a piece of glass. Notice all angles are measured from the normal.



Tricky Problems!

Remember that the normal is always perpendicular to the surface. When a question asks for the angle of incidence or reflection, it wants the angle formed with the normal, not the angle to the horizontal surface. Make sure you know if you're solving for the angle of incidence, angle of reflection, or the angle it forms with the mirror.

The angle that the **incident beam** makes with the normal is called the **angle of incidence**, or θ_1. The angle that the **reflected beam** makes with the normal is called the **angle of reflection,** θ'_1, and the angle that the **transmitted beam** makes with the normal is called the **angle of refraction**, θ_2. The incident, reflected, and transmitted beams of light all lie in the same plane.

The relationship between θ_1 and θ'_1 is pretty easy; it is called the **Law of Reflection:**

$$\theta_1 = \theta'_1$$

The Law of Reflection basically states the Angle of Reflection is equal to the Angle of Incidence. In order to describe how θ_1 and θ_2 are related, we first need to talk about a medium's index of refraction.

When light travels through empty space (vacuum), its speed is $c = 3.00 \times 10^8$ m/s; this is one of the fundamental constants of nature. But when light travels through a material medium (such as water or glass), it's constantly being absorbed and re-emitted by the atoms that compose the material and, as a result, its apparent speed, v, is some fraction of c. The reciprocal of this fraction,

Equation Sheet

$$n = \frac{c}{v}$$

is called the medium's **index of refraction.** For example, since the speed of light in water is $v = 2.25 \times 10^8$ m/s, the index of refraction of water is

$$n = \frac{3.00 \times 10^8 \text{ m/s}}{2.25 \times 10^8 \text{ m/s}} = 1.33$$

Note that n has no units; it's also never less than 1, because light always travels slower in a medium than in a vacuum.

***n* of air**

The index of refraction of air is 1.00029, which is so close to 1 that the norm is to consider it to just be 1.

The equation that relates θ_1 and θ_2 involves the index of refraction of the incident medium (n_1) and the index of refraction of the refracting medium (n_2); it's called **Snell's Law:**

Equation Sheet

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

If $n_2 > n_1$, then Snell's Law tells us that $\theta_2 < \theta_1$; that is, the beam will bend (**refract**) *toward* the normal as it enters the medium. On the

other hand, if $n_2 < n_1$, then $\theta_2 > \theta_1$, and the beam will bend *away* from the normal.

Example 4 A beam of light in air is incident upon a piece of glass, striking the surface at an angle of 30°. If the index of refraction of the glass is 1.5, what are the angles of reflection and refraction?

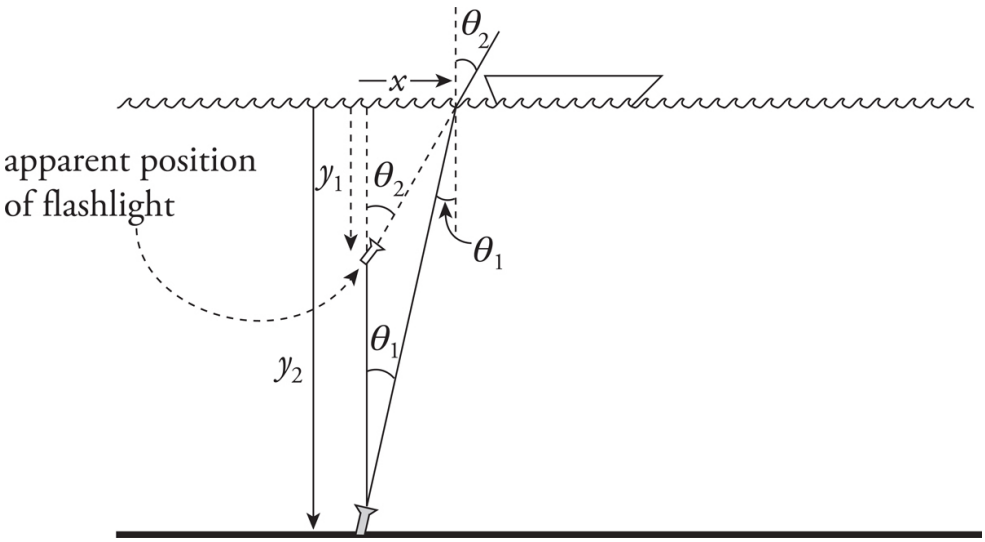
Solution. If the light beam makes an angle of 30° with the surface, then it makes an angle of 60° with the normal; this is the angle of incidence. By the Law of Reflection, then, the angle of reflection is also 60°. We use Snell's Law to find the angle of refraction. The index of refraction of air is close to 1, so we can say that $n = 1$ for air.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$(1)\ \sin 60° = 1.5 \sin\theta_2$$
$$\sin\theta_2 = 0.5774$$
$$\theta_2 = 35°$$

Note that $\theta_2 < \theta_1$, as we would expect since the refracting medium (glass) has a greater index than does the incident medium (air).

Example 5 A fisherman drops a flashlight into a lake that's 10 m deep. The flashlight sinks to the bottom where its emerging light beam is directed almost vertically upward toward the surface of the lake, making a small angle (θ_1) with the normal. How deep will the flashlight appear to be to the fisherman? (Use the fact that $\tan\theta$ is almost equal to $\sin\theta$ if θ is small.)

Solution. Take a look at the figure below.



Since the refracting medium (air) has a lower index than the incident medium (water), the beam of light will bend away from the normal as it emerges from the water. As a result, the fisherman will think that the flashlight is at a depth of only y_1, rather than its actual depth of y_2 = 10 m. By simple trigonometry, we know that

$$\tan \theta_1 = \frac{x}{y_2} \qquad \text{and} \qquad \tan \theta_2 = \frac{x}{y_1}$$

So,

$$\frac{y_1}{y_2} = \frac{\tan \theta_1}{\tan \theta_2}$$

Snell's Law tells us that $n_1 \sin \theta_1 = \sin \theta_2$ (since $n_2 = n_{\text{air}} = 1$), so

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{1}{n_1}$$

Using the approximations $\sin \theta_1 \approx \tan \theta_1$ and $\sin \theta_2 \approx \tan \theta_2$, we can write

$$\frac{\tan \theta_1}{\tan \theta_2} \approx \frac{\sin \theta_1}{\sin \theta_2}$$

which means

$$\frac{y_1}{y_2} = \frac{1}{n_1}$$

Because $y_2 = 10$ m and $n_1 = n_{\text{water}} = 1.33$,

$$y_1 = \frac{y_2}{n_1} = \frac{10 \text{ m}}{1.33} = 7.5 \text{ m}$$

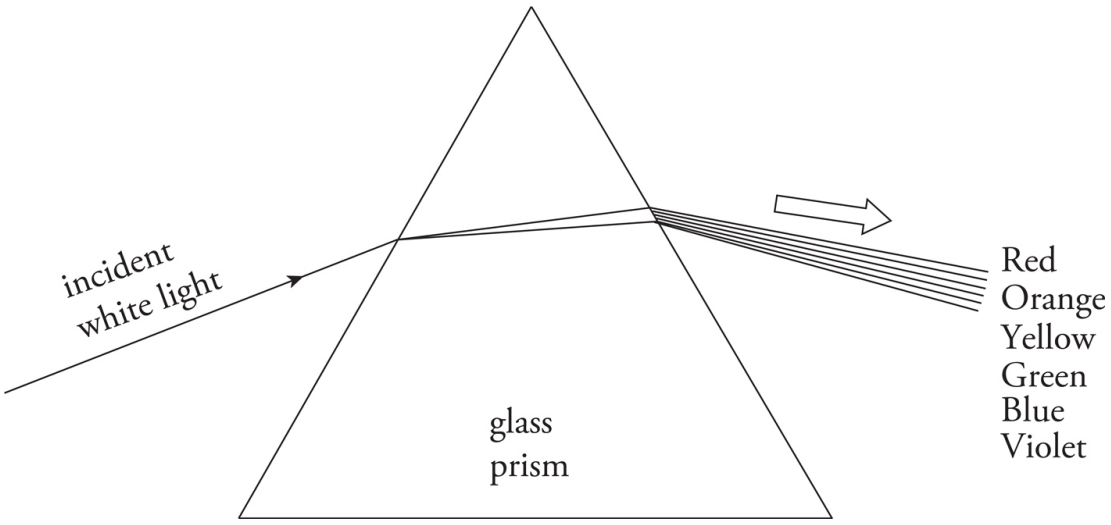
DISPERSION OF LIGHT

One thing we learned when we studied waves is that wave speed is independent of frequency. For a given medium, different frequencies give rise to different wavelengths, because the equation $v = f\lambda$ must always be satisfied and v doesn't vary. But when light travels through a material medium, it displays **dispersion**, which is a variation in wave speed with frequency (or wavelength). So, the definition of the index of refraction, $n = c/v$, should be accompanied by a statement of the frequency of the light used to measure v, since different frequencies have different speeds and different indices. A piece of glass may have the following indices for visible light:

	red	orange	yellow	green	blue	violet
$n =$	1.502	1.506	1.511	1.517	1.523	1.530

Note that as the wavelength decreases, refractive index increases. In general, higher frequency waves have higher indices of refraction. Most lists of refractive index values are tabulated using yellow light of wavelength 589 nm (frequency 5.1×10^{14} Hz).

Although the variation in the values of the refractive index across the visible spectrum is pretty small, when white light (which is a combination of all the colors of the visible spectrum) hits a glass prism, the beam is split into its component colors.



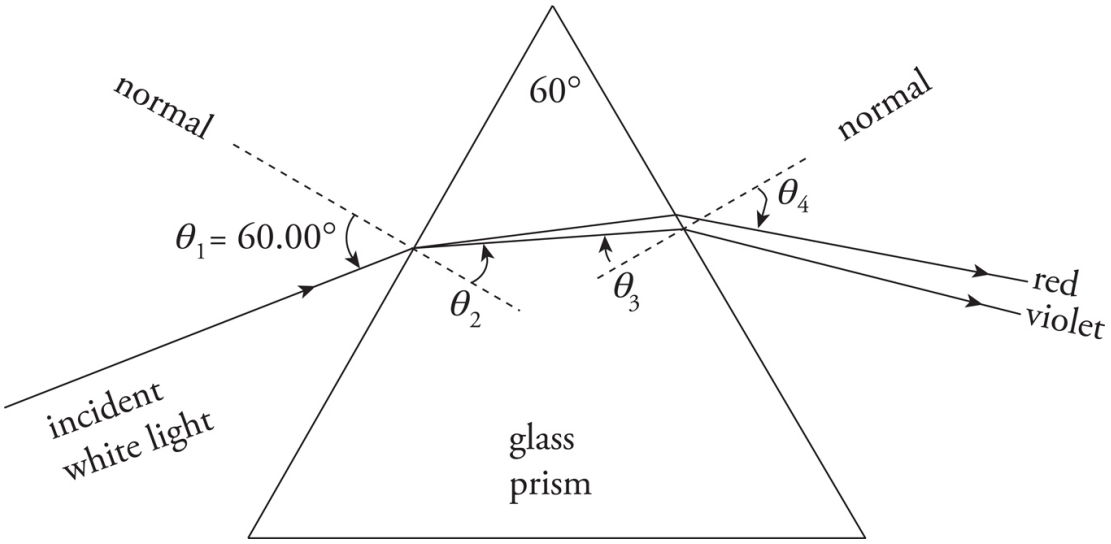
Why? Because each color has its own index, Snell's Law tells us that each color will have its own angle of refraction. Therefore, each color emerges from the prism at a slightly different angle, so the light disperses into its component colors.

Visible Spectrum

White light is not colorless. White light is actually composed of multiple colors of the visible spectrum. Dispersion happens through a prism because each of these colors has different wavelengths and frequencies.

Example 6 In the figure above, assume that the incident beam strikes the prism with an angle of incidence of 60.00°. If the top angle of the prism has a measure of 60.00°, what are the angles of refraction of red light and violet light when the beam emerges from the other side? (Use n = 1.502 for red light and 1.530 for violet light.)

Solution. The beam refracts twice, once when it enters the prism and again when it exits.



Snell's Law tells us that

$$\sin 60.00° = n \sin \theta_2 \text{ and } n \sin \theta_3 = \sin \theta_4$$

where n is the index of the glass. We'll ignore the minuscule variation in n for light in air, and just use $n_{\text{air}} = 1$ for all colors. The relationship between θ_2 and θ_3 follows from the fact that the sum of the measures of the angles in any triangle is 180°, so

$$(90° - \theta_2) + 60° + (90° - \theta_3) = 180°$$

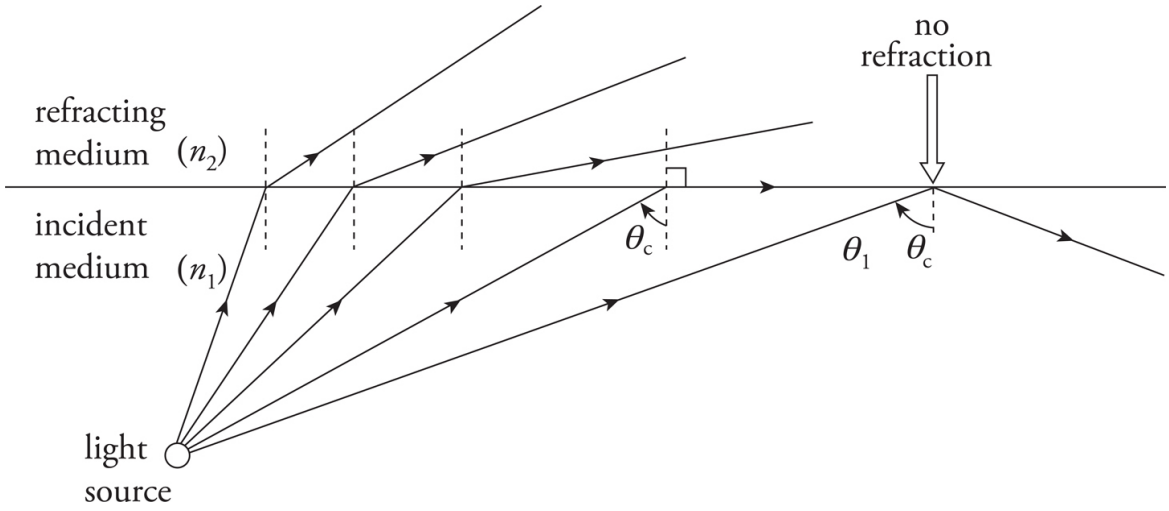
which implies that $\theta_3 = 60° - \theta_2$. We can now make the following table of values:

color	n	θ_2	θ_3	θ_4
red	1.502	35.210°	24.790°	39.03°
violet	1.530	34.474°	25.526°	41.25°

In general, the differences in n are so small that, unless you are specifically addressing dispersion, n will be given as a single number, regardless of color.

TOTAL INTERNAL REFLECTION

When a beam of light strikes the boundary to a medium that has a lower index of refraction, the beam bends away from the normal. As the angle of incidence increases, the angle of refraction becomes larger. At some point, when the angle of incidence reaches a **critical angle**, θ_c, the angle of refraction becomes 90°, which means the refracted beam is directed along the surface.



For angles of incidence that are greater than θ_c, there is *no* angle of refraction; the entire beam is reflected back into the original medium. This phenomenon is called **total internal reflection** (sometimes abbreviated **TIR**).

Total internal reflection occurs when:

1) $n_1 > n_2$

and

2) $\theta_1 > \theta_c$, where $\theta_c = \sin^{-1}(n_2/n_1)$

Subscripts
Always think of n_1 or θ_1 as the incidence ray, meaning where the source comes from. Treat n_2 or θ_2 as the resulting ray.

Notice that total internal reflection cannot occur if $n_1 < n_2$. This is because the largest output of sin θ is 1, so the largest input of $\sin^{-1}(x)$ is 1. If $n_1 > n_2$, then total internal reflection is a possibility; it will occur if the angle of incidence is large enough, that is, if it's greater than the critical angle, θ_c.

Example 7 What is the critical angle for total internal reflection between air and water? In which of these media must light be incident for total internal reflection to occur?

Solution. First, total internal reflection can only occur when the light is incident in the medium that has the greater refractive index and strikes the boundary to a medium that has a lower index. So, in this case, total internal reflection can occur only when the light source is in the water and the light is incident upon the water/air surface. The critical angle is found as follows:

$$\sin\theta_c = \frac{n_2}{n_1} \quad \Rightarrow \quad \sin\theta_c = \frac{n_{\text{air}}}{n_{\text{water}}} = \frac{1}{1.33} \quad \Rightarrow \quad \sin\theta_c = 0.75 \quad \Rightarrow \quad \theta_c = 49°$$

Total internal reflection will occur if the light from the water strikes the water/air boundary at an angle of incidence greater than 49°.

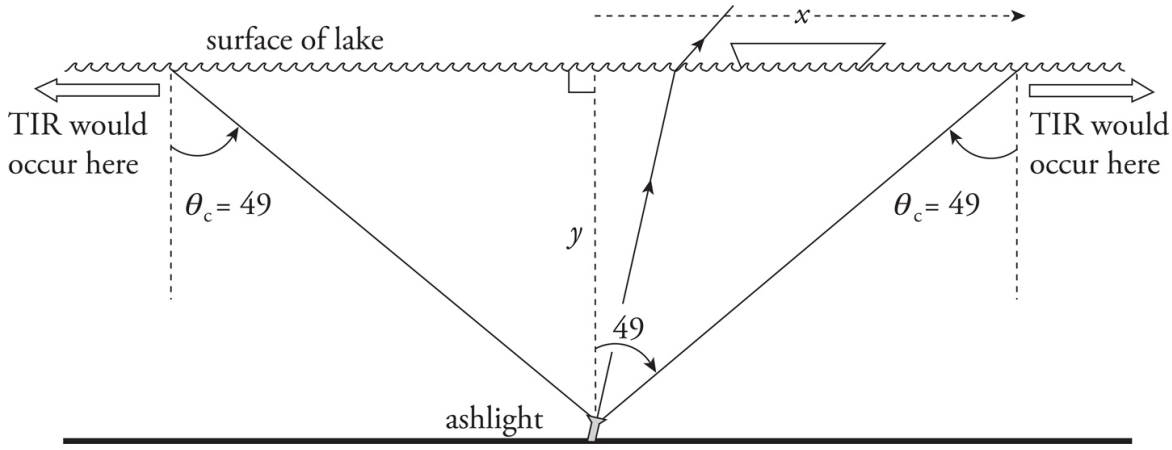
Example 8 How close must the fisherman be to the flashlight in Example 5 in order to see the light it

emits?

Remember to Check

For optics problems, you need to always remember to check if the refracted light ray undergoes total internal reflection. If it undergoes total internal reflection, there will be no refracted ray. Hence, if a trick problem asks for an angle of refraction for a situation that undergoes total internal reflection, there is no angle of refraction.

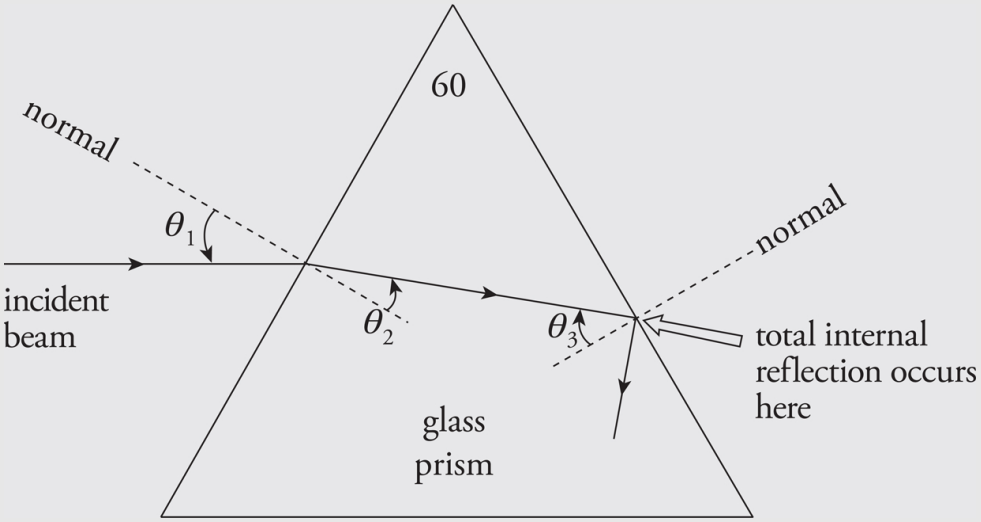
Solution. In order for the fisherman to see the light, the light must be transmitted into the air from the water; that is, it cannot undergo total internal reflection. The figure below shows that, within a circle of radius x, light from the flashlight will emerge from the water. Outside this circle, the angle of incidence is greater than the critical angle, and the light would be reflected back into the water, rendering it undetectable by the fisherman above.



Because the critical angle for total internal reflection at a water/air interface is 49° (as we found in the preceding example), we can solve for x:

$$\tan 49° = \frac{x}{y} \quad \Rightarrow \quad x = y \tan 49° = (10 \text{ m}) \tan 49° = 11.5 \text{ m}$$

Example 9 The refractive index for the glass prism shown below is 1.55. In order for a beam of light to experience total internal reflection at the right-hand face of the prism, the angle θ_1 must be smaller than what value?



Solution. Total internal reflection will occur at the glass/air boundary if θ_3 is greater than the critical angle, θ_c, which we can calculate this way:

$$\sin\theta_c = \frac{n_{\text{air}}}{n_{\text{glass}}} = \frac{1}{1.55} \quad \Rightarrow \quad \theta_c = 40°$$

Because $\theta_3 = 60° - \theta_2$, total internal reflection will take place if θ_2 is smaller than 20°. Now, by Snell's Law, $\theta_2 = 20°$ if

$$\begin{aligned} n_{\text{air}} \sin\theta_1 &= n_{\text{glass}} \sin\theta_2 \\ \sin\theta_1 &= 1.55 \sin 20° \\ \theta_1 &= 32° \end{aligned}$$

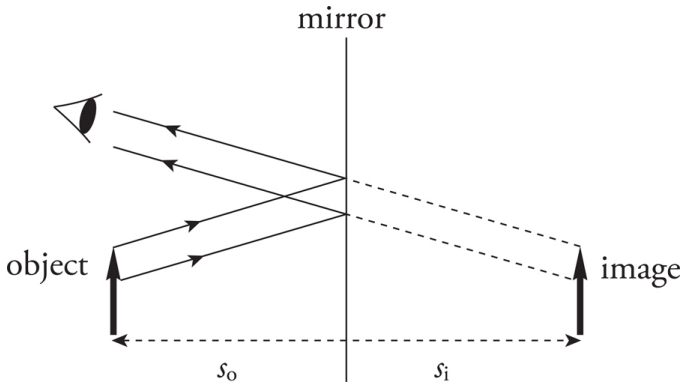
Therefore, total internal reflection will occur at the right-hand face of the prism if θ_1 is smaller than 32°.

MIRRORS

A **mirror** is an optical device that forms an image by reflecting light. We've all looked into a mirror and seen images of nearby objects, and the purpose of this section is to analyze these images mathematically. We begin with a plane mirror, which is flat, and is the simplest type of mirror. Then we'll examine curved mirrors; we'll have to use geometrical methods or algebraic equations to analyze the patterns of reflection from these.

Plane Mirrors

The figure below shows an object (denoted by a vertical, bold arrow) in front of a flat mirror. Light that's reflected off of the object strikes the mirror and is reflected back to our eyes. The directions of the rays reflected off the mirror determine where we perceive the image to be.



There are four questions we'll answer about the image formed by a mirror:

(1) Where is the image?

(2) Is the image real or is it virtual?

(3) Is the image upright or is it inverted?

(4) What is the height of the image (compared with that of the object)?

When we look at ourselves in a mirror, it seems like our image is *behind* the mirror, and, if we take a step back, our image also takes a step back. The Law of Reflection can be used to show that the image seems as far behind the mirror as the object is in front of the mirror. This answers question (1).

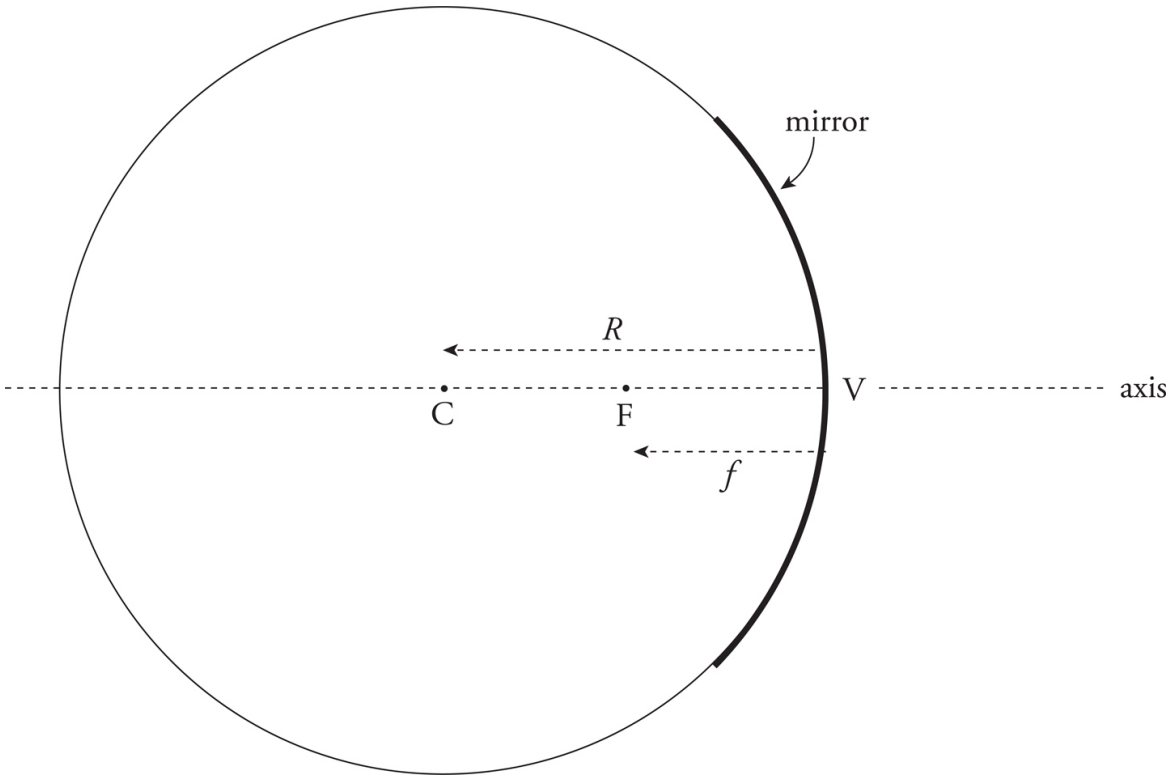
An image is said to be **real** if light rays actually focus at the image. A real image can be projected onto a screen. For a flat mirror, light rays bounce off the front of the mirror; so, of course, no light focuses behind it. Therefore, the images produced by a flat mirror are not real; they are **virtual**. This answers question (2).

When we look into a flat mirror, our image isn't upside down; flat mirrors produce upright images, and question (3) is answered.

Finally, the image formed by a flat mirror is neither magnified nor diminished (minified) relative to the size of the object. This answers question (4).

Spherical Mirrors

A **spherical mirror** is a mirror that's curved in such a way that its surface forms part of a sphere.

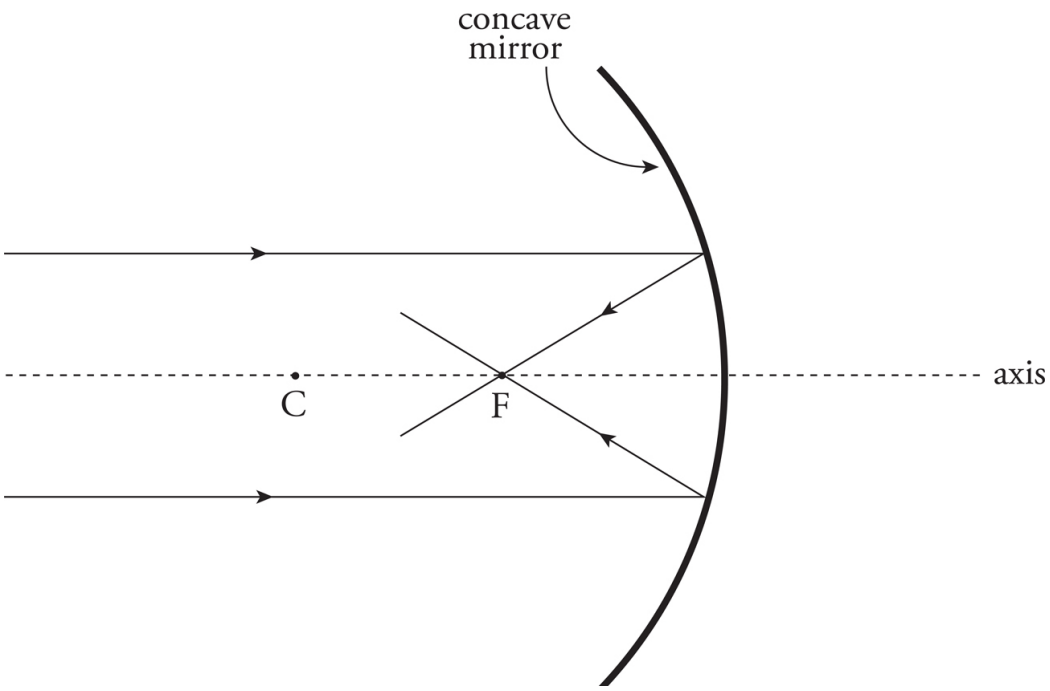


The center of this imaginary sphere is the mirror's **center of curvature,** and the radius of the sphere is called the mirror's **radius of curvature**, *R*. Halfway between the mirror and the center of curvature, C, is the **focus** (or **focal point**), F. The intersection of the mirror's optic **axis** (its axis of symmetry) with the mirror itself is called the **vertex**, V, and the distance from V to F is called the **focal length**, *f*, equal to one-half of the radius of curvature:

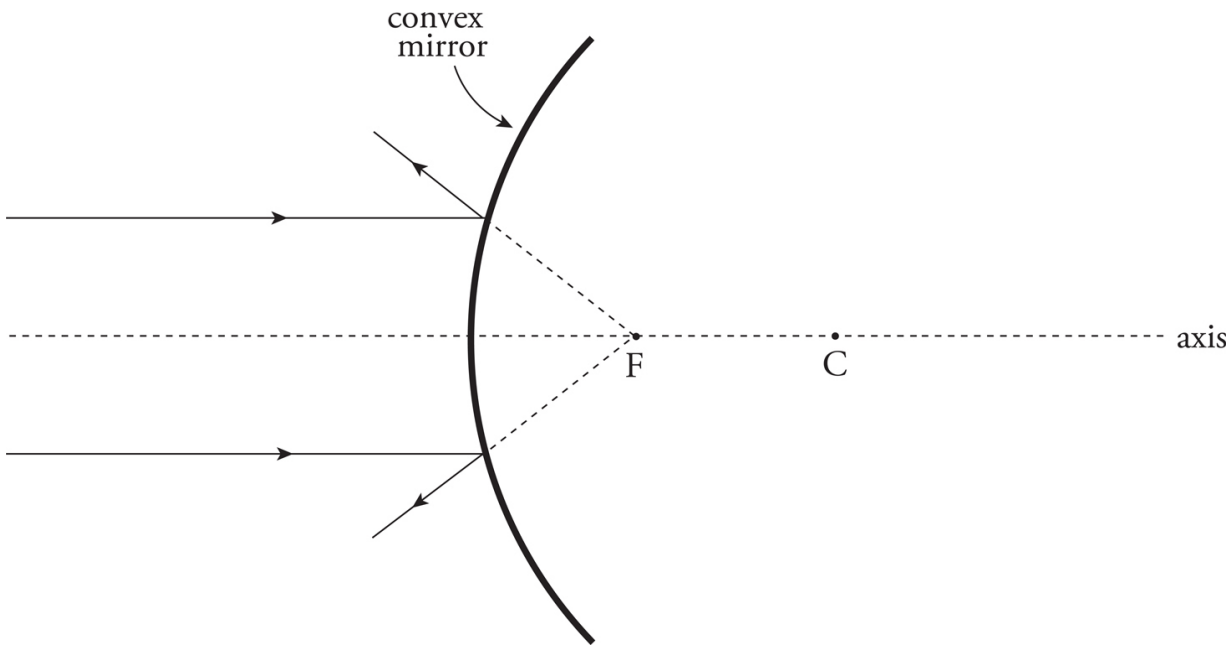
$$f = \frac{R}{2}$$

If the mirror had a parabolic cross-section, then any ray parallel to the axis would be reflected by the mirror through the focal point. Spherical mirrors do this for incident light rays near the axis (**paraxial rays**) because, in the region of the mirror that's close to

the axis, the shapes of a parabolic mirror and a spherical mirror are nearly identical.



The previous two figures illustrate a **concave mirror,** a mirror whose reflective side is *caved in* toward the center of curvature. The following figure illustrates the **convex mirror,** which has a reflective side curving away from the center of curvature. We will call F the **virtual focus** in this case.


convex
mirror
F
C
axis

RAY TRACING FOR MIRRORS

One method of answering the four questions listed above concerning the image formed by a mirror involves a geometric approach called **ray tracing**. Representative rays of light are sketched in a diagram that depicts the object and the mirror; the point at which the reflected rays intersect (or appear to intersect) is the location of the image. Some rules governing rays are as follows:

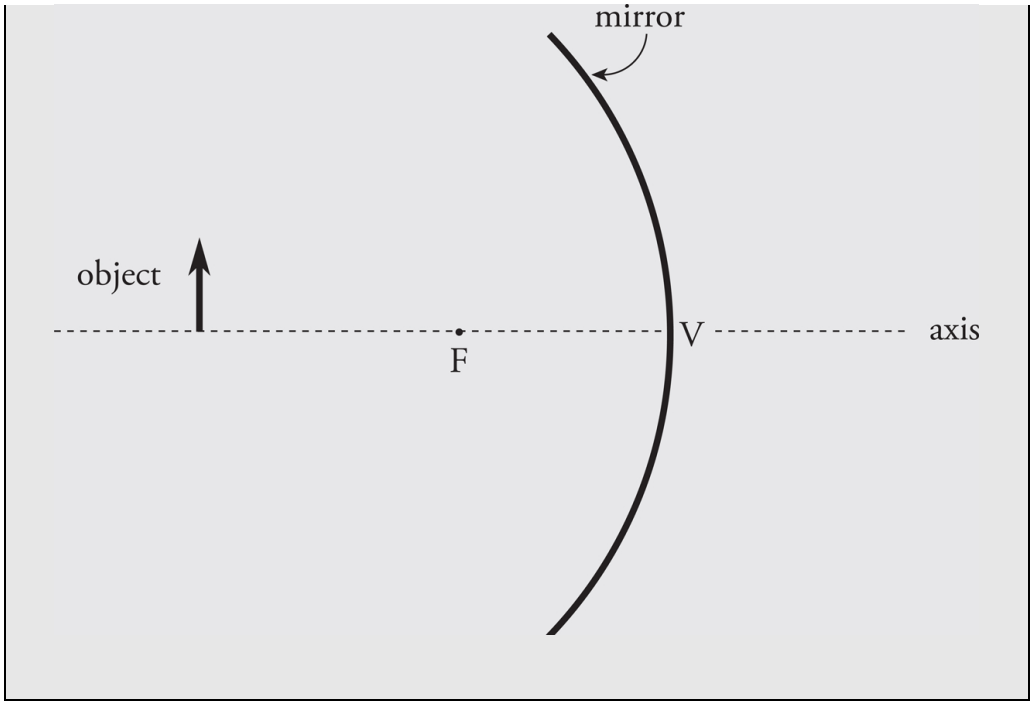
Concave Mirrors

- An incident ray parallel to the axis is reflected through the focus.
- An incident ray that passes through the focus is reflected parallel to the axis.
- An incident ray that strikes the vertex is reflected at an equal angle to the axis.

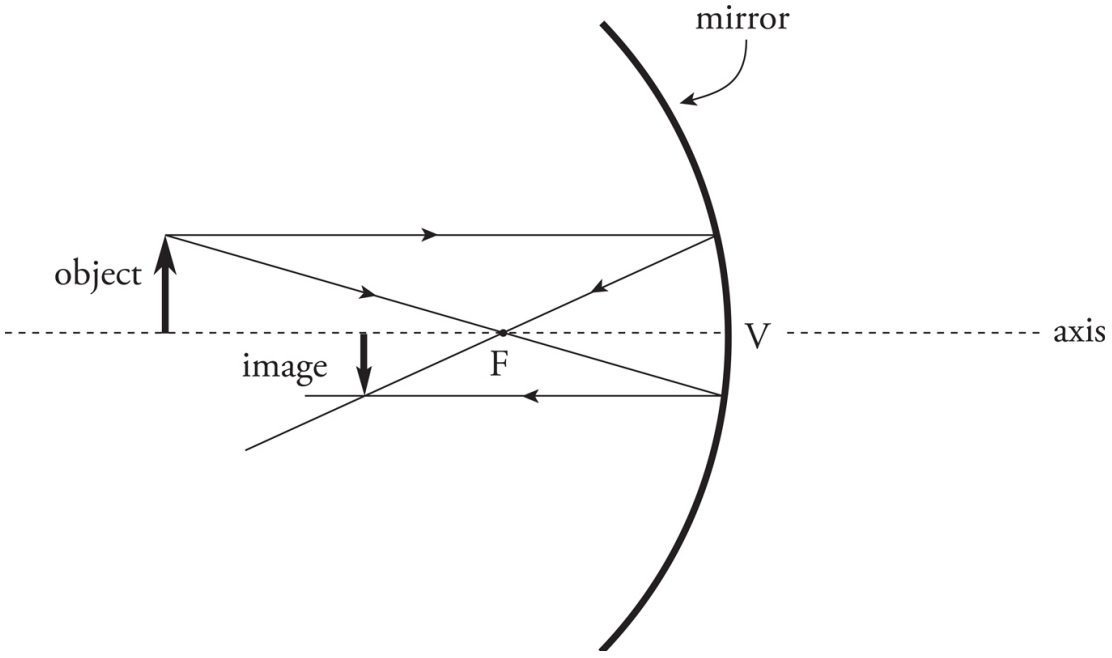
Convex Mirrors

- An incident ray parallel to the axis is reflected away from the virtual focus.
- An incident ray directed toward the virtual focus is reflected parallel to the axis.
- An incident ray that strikes the vertex is reflected at an equal angle to the axis.

Example 10 The figure below shows a concave mirror and an object (the bold arrow). Use a ray diagram to locate the image of the object.

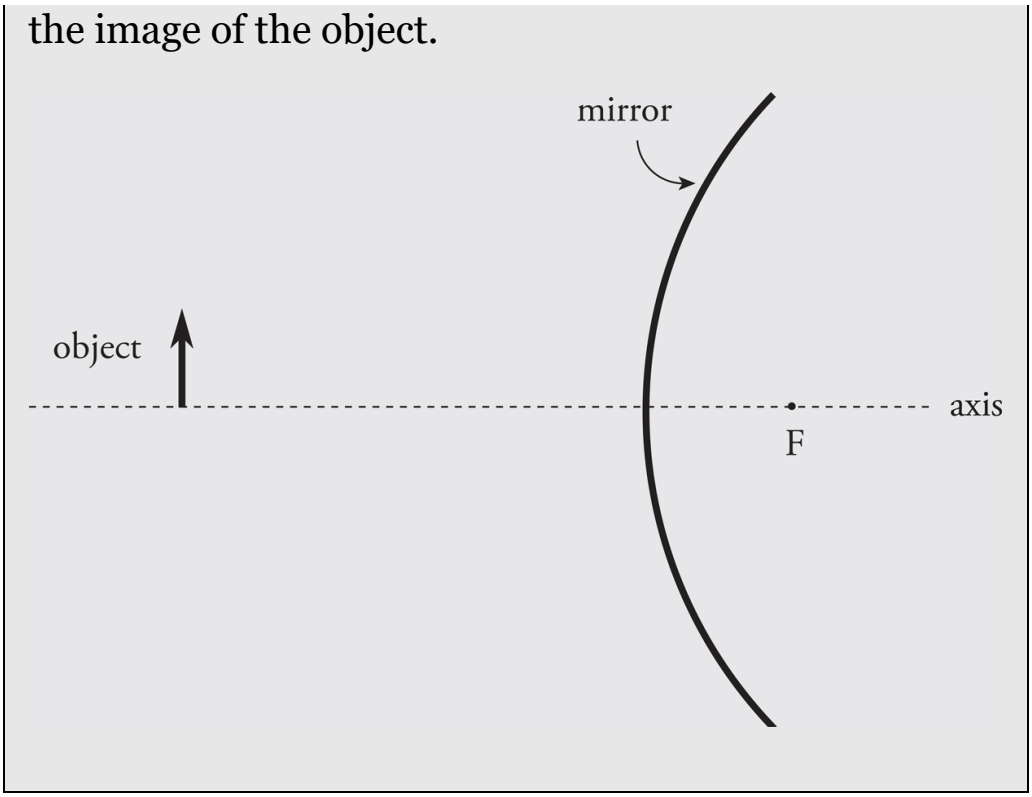


Solution. It only takes two distinct rays to locate the image:

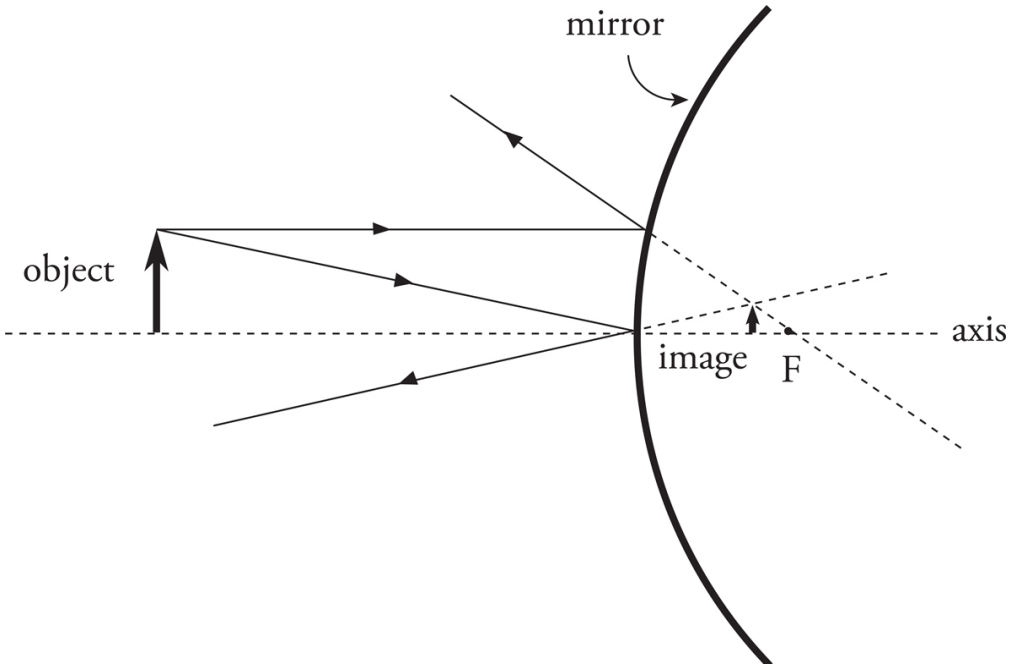


Example 11 The figure below shows a convex mirror and an object (the arrow). Use a ray diagram to locate

the image of the object.



Solution.



The ray diagrams of the preceding examples can be used to determine the location, orientation, and size of the image. The nature of the image—that is, whether it's real or virtual—can be determined by seeing on which side of the mirror the image is formed. If the image is formed on the same side of the mirror as the object, then the image is real, but if the image is formed on the opposite side of the mirror, it's virtual. Another way of looking at this is: if you had to trace lines back to form an image, that image is virtual. Therefore, the image in Example 10 is real, and the image in Example 11 is virtual.

Using Equations to Answer Questions About the Image

While ray diagrams can answer our questions about images completely, the fastest and easiest way to get information about an image is to use two equations and some simple conventions. The first equation, called the **mirror equation,** is

Equation Sheet

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

where s_o is the object's distance from the mirror, s_i is the image's distance from the mirror, and f is the focal length of the mirror. The value of s_o is *always* positive for a real object, but s_i can be positive or negative. The sign of s_i tells us whether the image is real or virtual: If s_i is positive, the image is real; and if s_i is negative, the image is virtual.

The second equation is called the **magnification equation**:

Equation Sheet

$$M = \frac{h_i}{h_o} = \frac{-s_i}{s_o}$$

This gives the magnification; the height of the image, h_i, is $|M|$ times the height of the object, h_o. If M is positive, then the image is upright relative to the object; if M is negative, it's inverted relative to the object. Because s_o is always positive, we can come to two important conclusions. If s_i is positive, then M is negative, so real images are always inverted, and, if s_i is negative, then M is positive, so virtual images are always upright.

Which is the Positive Side?

We will always treat the object's position s_o as being the positive side. For mirrors, if the image is produced on the same side as the object it is considered a positive distance. If an image is produced opposite the side of the object it is considered a negative distance. Concave mirrors will always have a positive focal length (meaning the focal point is on the same side of the object). Convex mirrors will always have a negative focal length (meaning the focal point is on the opposite side of the object).

Finally, to distinguish *mathematically* between concave and convex mirrors, we always write the focal length f as a positive value for concave mirrors and a negative value for convex mirrors. With these two equations and their accompanying conventions, all four questions about an image can be answered.

MIRRORS

converging—concave $\Leftrightarrow$ f positive

diverging—convex $\Leftrightarrow$ f negative

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

- s_o always positive (real object)
- s_i positive $\Rightarrow$ image is real (located on the *same* side of mirror as object)
- s_i negative $\Rightarrow$ image is virtual (located on *opposite* side of mirror from object)

$$M = \frac{h_i}{h_o} = \frac{-s_i}{s_o}$$

given h_o is positive

- h_i and M positive $\Rightarrow$ image is upright
- h_i and M negative $\Rightarrow$ image is inverted

Real or Virtual?

Remember that all real images are inverted and all virtual images are upright.

Example 12 An object of height 4 cm is placed 30 cm in front of a concave mirror whose focal length is 10 cm.

(a) Where's the image?

(b) Is it real or virtual?

(c) Is it upright or inverted?

(d) What's the height of the image?

Solution.

(a) With $s_o = 30$ cm and $f = 10$ cm, the mirror equation gives

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f} \Rightarrow \frac{1}{30\text{ cm}}+\frac{1}{s_i}=\frac{1}{10\text{ cm}} \Rightarrow \frac{1}{s_i}=\frac{1}{15\text{ cm}} \Rightarrow s_i = 15\text{ cm}$$

The image is located 15 cm in front of the mirror.

(b) Because s_i is positive, the image is real.

(c) Real images are inverted.

(d) $$\frac{h_i}{h_o}=\frac{-s_i}{s_o} \Rightarrow h_i=\frac{-s_i h_o}{s_o} \Rightarrow h_i=\frac{(-15\text{ cm})(4\text{ cm})}{30\text{ cm}}=-2\text{ cm}$$

The −2 cm also confirms the image is inverted.

Example 13 An object of height 4 cm is placed 20 cm in front of a convex mirror whose focal length is −30 cm.

(a) Where's the image?
(b) Is it real or virtual?
(c) Is it upright or inverted?
(d) What's the height of the image?

What Produces What?

Only concave mirrors can produce both real images (if $s_0 > f$) and virtual images (if $s_0 < f$). Convex mirrors and plane mirrors can only produce virtual images.

Solution.

(a) With $s_o = 20$ cm and $f = -30$ cm, the mirror equation gives us

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{20\text{ cm}}+\frac{1}{s_i}=\frac{1}{-30\text{ cm}} \quad \Rightarrow \quad \frac{1}{s_i}=-\frac{1}{12\text{ cm}} \quad \Rightarrow \quad s_i=-12\text{ cm}$$

So, the image is located 12 cm behind the mirror.

(b) Because s_i is negative, the image is virtual.

(c) Virtual images are upright.

(d) $$\frac{h_i}{h_o}=\frac{-s_i}{s_o} \Rightarrow h_i=\frac{-s_i h_o}{s_o} \Rightarrow h_i=\frac{-(-12\text{ cm})(4\text{ cm})}{20\text{ cm}}=2.4\text{ cm}$$

[The fact that the magnification is positive tells us that the image is upright, as we said in part (c).]

Example 14 Show how the statements made earlier about plane mirrors can be derived from the mirror and magnification equations.

Solution. A plane mirror can be considered a spherical mirror with an infinite radius of curvature (and an infinite focal length). If $f = \infty$, then $1/f = 0$, so the mirror equation becomes

$$\frac{1}{s_o}+\frac{1}{s_i}=0 \quad \Rightarrow \quad s_i=-s_o$$

So, the image is as far behind the mirror as the object is in front. Also, since s_o is always positive, s_i is negative, so the image is virtual. The magnification is

$$M=-\frac{s_i}{s_o}=-\frac{-s_o}{s_o}=1$$

and the image is upright and has the same height as the object. The mirror and magnification equations confirm our description of images formed by plane mirrors.

Example 15 Show why convex mirrors can only form virtual images.

Solution. Because f is negative and s_o is positive, the mirror equation

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

immediately tells us that s_i cannot be positive (if it were, the left-hand side would be the sum of two positive numbers, while the right-hand side would be negative). Since s_i must be negative, the image must be virtual.

Example 16 An object placed 60 cm in front of a spherical mirror forms a real image at a distance of 30 cm from the mirror.

(a) Is the mirror concave or convex?

(b) What's the mirror's focal length?

(c) Is the image taller or shorter than the object?

Solution.

(a) The fact that the image is real tells us that the mirror cannot be convex, since convex mirrors form only virtual images. The mirror is concave.

(b) With s_o = 60 cm and s_i. = 30 cm (s_i is positive since the image is real), the mirror equation tells us that

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{60\text{ cm}}+\frac{1}{30\text{ cm}}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{f}=\frac{1}{20\text{ cm}} \quad \Rightarrow \quad f = 20\text{ cm}$$

Note that f is positive, which confirms that the mirror is concave.

(c) The magnification is

$$M=-\frac{s_i}{s_o}=-\frac{30\text{ cm}}{60\text{ cm}}=-\frac{1}{2}$$

Since the absolute value of M is less than 1, the mirror makes the object look smaller (minifies the height of the object). The image is only half as tall as the object (and is upside down, because M is negative).

So Many Equations!

The purpose of a mirror is to reflect an image. The purpose of a lens is to refract an image (or see through it). Fortunately, the equations for mirrors and lenses will be the same with a minor difference on the focal length.

Example 17 A concave mirror with a focal length of 25 cm is used to create a real image that has twice the height of the object. How far is the image from the mirror?

Solution. Since h_i (the height of the image) is twice h_o (the height of the object) the value of the magnification is either +2 or −2. To figure

out which, we just notice that the image is real; real images are inverted, so the magnification, M, must be negative. Therefore, $M = -2$, so

$$-\frac{s_\text{i}}{s_\text{o}} = -2 \quad \Rightarrow \quad s_\text{o} = \frac{1}{2} s_\text{i}$$

Substituting this into the mirror equation gives us

$$\frac{1}{s_\text{o}} + \frac{1}{s_\text{i}} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{\frac{1}{2}s_\text{i}} + \frac{1}{s_\text{i}} = \frac{1}{f} \quad \Rightarrow \quad \frac{3}{s_\text{i}} = \frac{1}{f} \quad \Rightarrow \quad s_\text{i} = 3f = 3(25\text{ cm}) = 75\text{ cm}$$

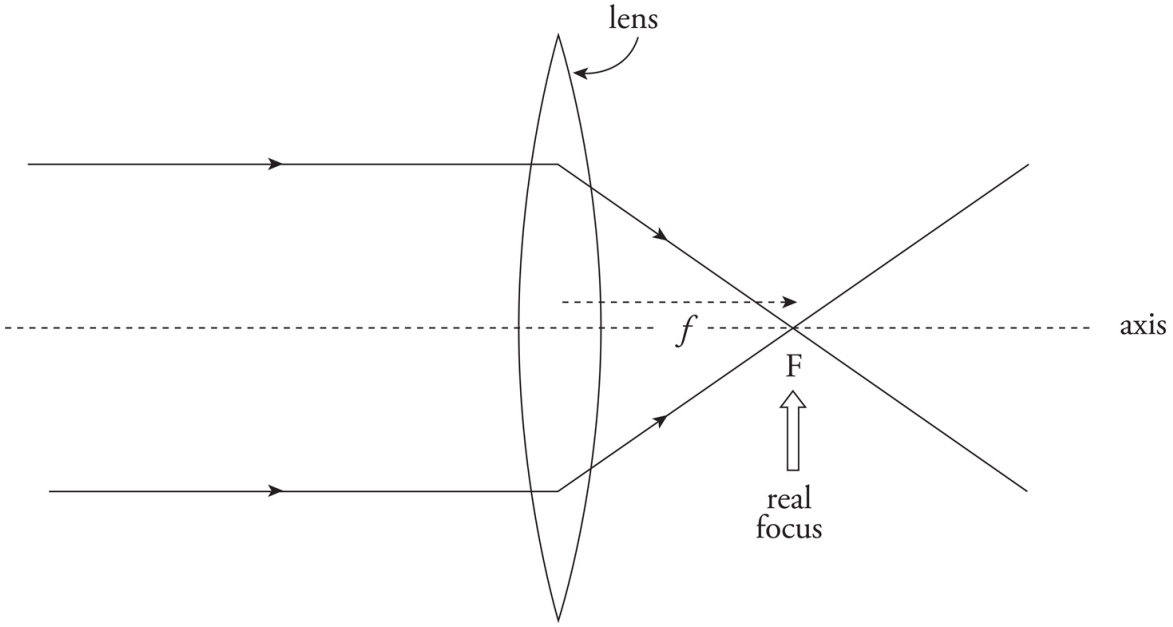
THIN LENSES

A lens is an optical device that forms an image by *refracting* light. We'll now talk about the equations and conventions that are used to analyze images formed by the two major categories of lenses: converging and diverging.

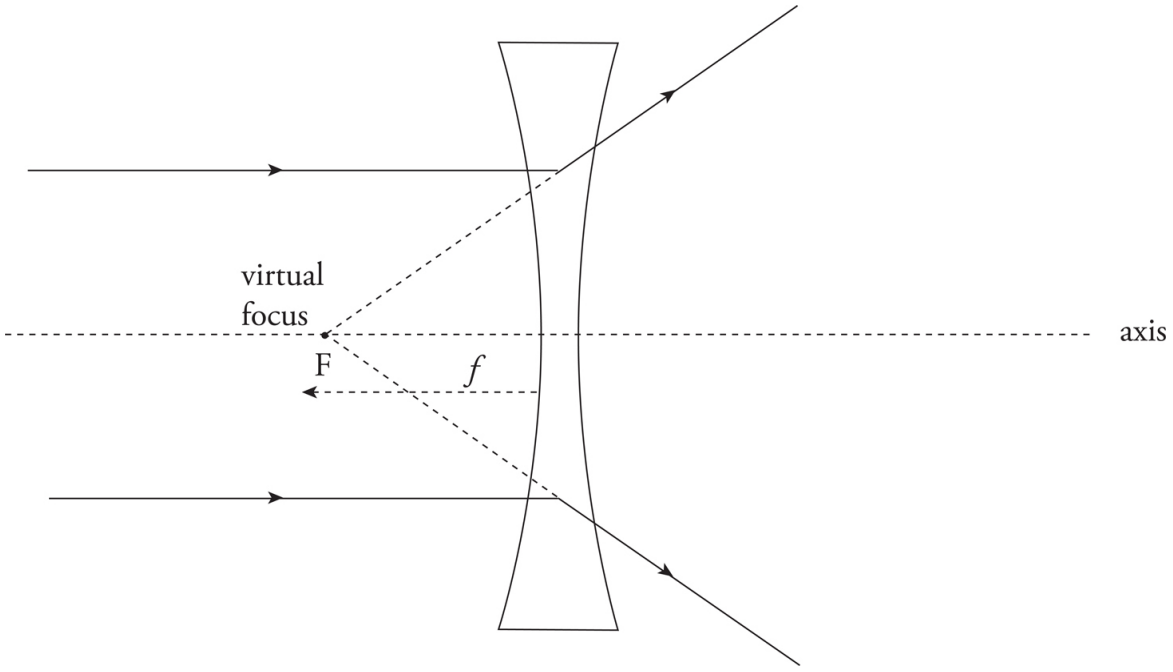
Mirrors vs. Lenses

The equations are exactly the same. The only difference is we switch the locations of the focus. A positive focal length lens will have a focus on the opposite side of the lens from the object. If the focal point is located on the same side of the object then it is a negative focal length.

A **converging lens**—like the bi-convex one shown below—converges parallel paraxial rays of light to a focal point on the far side. (This lens is *bi-convex*; both of its faces are convex. Converging lenses all have at least one convex face.) Because parallel light rays actually focus at F, this point is called a **real focus.** Its distance from the lens is the focal length, *f*.



A **diverging lens**—like the *bi-concave* one shown below—causes parallel paraxial rays of light to diverge away from a **virtual focus**, F, on the same side as the incident rays. (Diverging lenses all have at least one concave face.)



Don't Mix It Up

Converging Lens = Bi-Convex Lens
Diverging Lens = Bi-Concave Lens

RAY TRACING FOR LENSES

Just as is the case with mirrors, representative rays of light can be sketched in a diagram along with the object and the lens; the point at which the reflected rays intersect (or appear to intersect) is the location of the image. The rules that govern these rays are as follows:

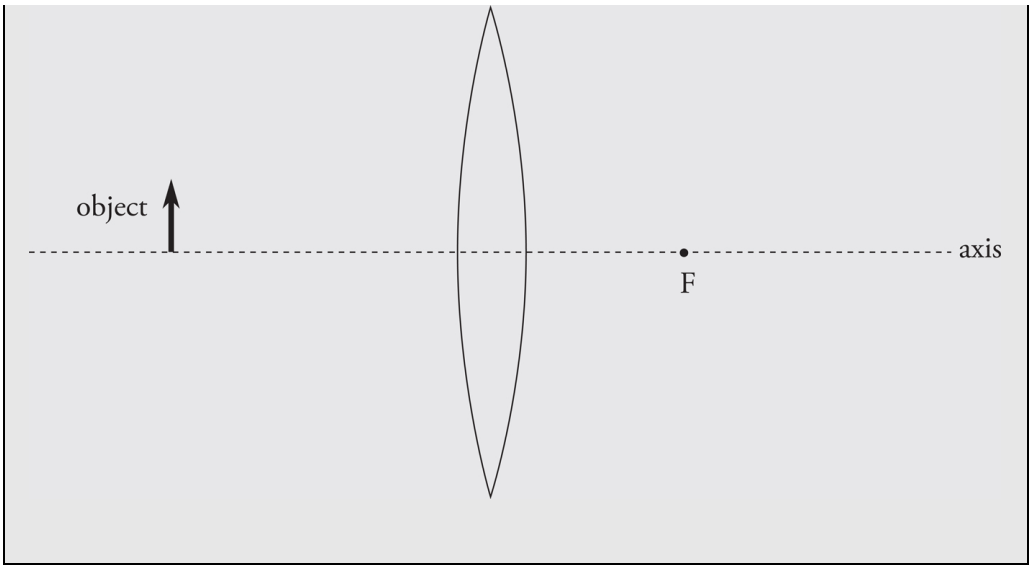
Converging Lenses

- An incident ray parallel to the axis is refracted through the real focus.
- Incident rays pass undeflected through the **optical center**, O (the central point within the lens where the axis intersects the lens).

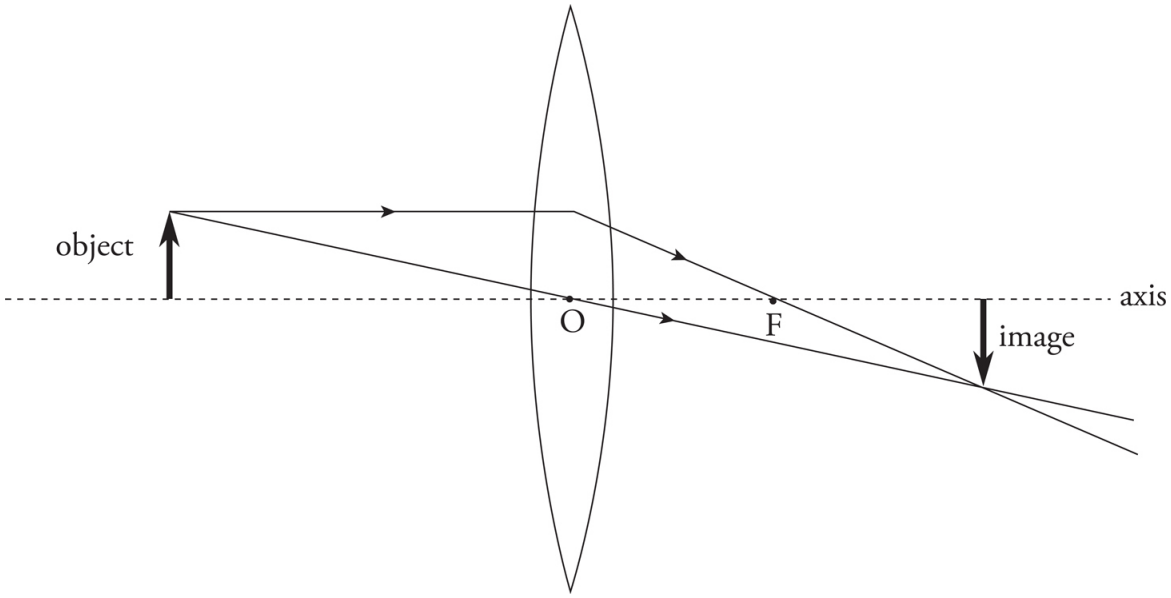
Diverging Lenses

- An incident ray parallel to the axis is refracted away from the virtual focus.
- Incident rays pass undeflected through the optical center, O.

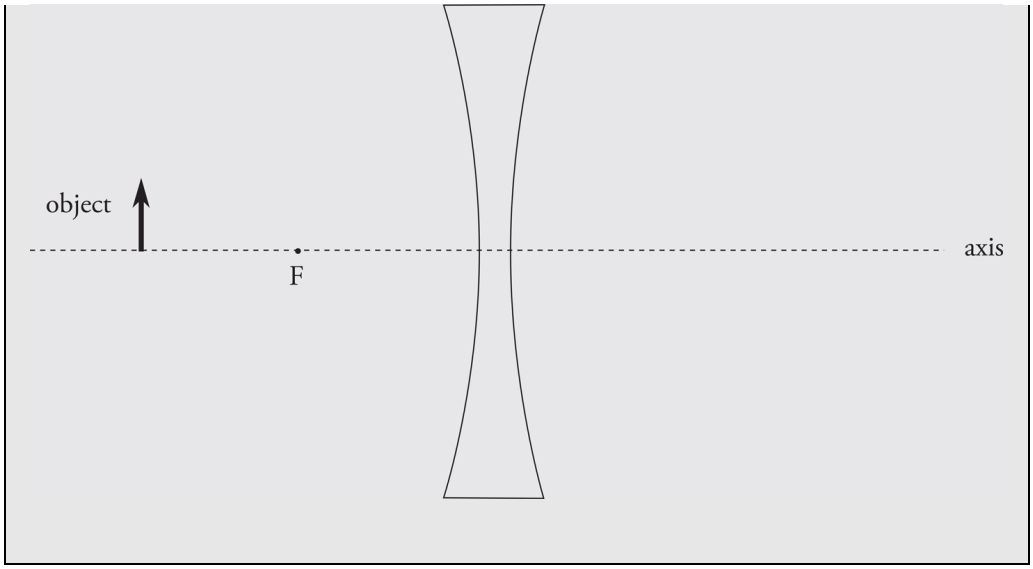
Example 18 The figure below shows a converging lens and an object (denoted by the bold arrow). Use a ray diagram to locate the image of the object.



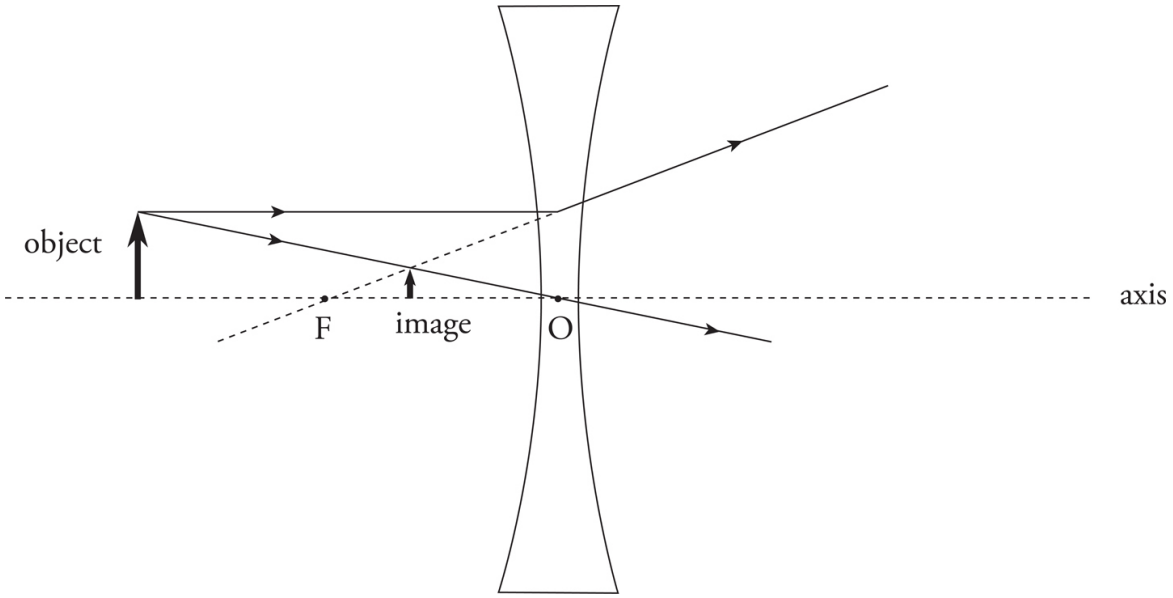
Solution.



Example 19 The figure below shows a diverging lens and an object. Use a ray diagram to locate the image of the object.



Solution.



The nature of the image—that is, whether it's real or virtual—is determined by the side of the lens upon which the image is formed. If the image is formed on the side of the lens that's opposite the object, then the image is real, and if the image is formed on the same side of the lens as the object, then it's virtual. Another way of looking at this is: If you had to trace lines back to form an image, that image is

virtual. Therefore, the image in Example 18 is real, while the image in Example 19 is virtual.

USING EQUATIONS TO ANSWER QUESTIONS ABOUT THE IMAGE

Lenses and mirrors use the same equations, notation, and sign conventions, with the following note. Converging optical devices ($+f$) are concave mirrors and convex lenses. Diverging optical devices ($-f$) are convex mirrors and concave lenses.

LENSES

convex lens—converging $\Leftrightarrow$ f positive
concave lens—diverging $\Leftrightarrow$ f negative

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

- s_o always positive (real object)
- s_i positive $\Rightarrow$ image is real (located on *opposite* side of lens from object)
- s_i negative $\Rightarrow$ image is virtual (located on *same* side of lens as object)

$$M = \frac{h_i}{h_o} = \frac{-s_i}{s_o}$$

- given h_o is positive
- h_i and M positive $\Rightarrow$ image is upright
- h_i and M negative $\Rightarrow$ image is inverted

Example 20 An object of height 11 cm is placed 44 cm in front of a converging lens with a focal length of 24 cm.

(a) Where's the image?
(b) Is it real or virtual?
(c) Is it upright or inverted?
(d) What's the height of the image?

Solution.

(a) With $s_o = 44$ cm and $f = 24$ cm, the lens equation gives us

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{44\text{ cm}} + \frac{1}{s_i} = \frac{1}{24\text{ cm}} \quad \Rightarrow \quad \frac{1}{s_i} = 0.0189\text{ cm}^{-1} \quad \Rightarrow \quad s_i = 53\text{ cm}$$

So, the image is located 53 cm from the lens, on the opposite side from the object.

(b) Because s_i is positive, the image is real.

(c) Real images are inverted.

(d) $$\frac{h_i}{h_o} = \frac{-s_i}{s_o} \Rightarrow h_i = \frac{-s_i h_o}{s_o} \Rightarrow h_i = \frac{(-53\text{ cm})(11\text{ cm})}{44\text{ cm}} = -13\text{ cm}$$

The negative h_i reaffirms that we have an inverted image.

Example 21 An object of height 11 cm is placed 48 cm in front of a diverging lens with a focal length of −24.5 cm.

(a) Where's the image?
(b) Is it real or virtual?
(c) Is it upright or inverted?
(d) What's the height of the image?

Solution.

(a) With $s_o = 48$ cm and $f = -24.5$ cm, the lens equation gives us

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{48\text{ cm}}+\frac{1}{s_i}=\frac{1}{-24.5\text{ cm}} \quad \Rightarrow \quad \frac{1}{s_i}=-0.0616\text{ cm}^{-1} \quad \Rightarrow \quad s_i=-16\text{ cm}$$

The image is 16 cm from the lens, on the same side as the object.

(b) Because s_i is negative, the image is virtual.

(c) Virtual images are upright.

(d) $$\frac{h_i}{h_o}=\frac{-s_i}{s_o} \Rightarrow h_i=\frac{-s_i h_o}{s_o} \Rightarrow h_i=\frac{-(-16\text{ cm})(11\text{ cm})}{48\text{ cm}}=3.7\text{ cm}$$

What Produces What?

Only converging lenses can produce both real images (if $s_0 > f$) and virtual images (if $s_0 < f$). Diverging lenses can only produce virtual images.

These results are illustrated in Example 19.

Example 22 The **power** of a lens, P, is defined as $1/f$, where f is the lens's focal length (in meters). Lens power is expressed in **diopters** (abbreviated D), where 1 D = 1 m^{-1}. When an object is placed 10 cm from a converging lens, the real image formed is twice the height of the object. What's the power of this lens?

Solution. Since h_i, the height of the image, is twice h_o, the height of the object, the value of the magnification is either +2 or −2. To decide, we simply notice that the image is real; real images are inverted, so the magnification, m, must be negative. Since $m = -2$, we find that

$$-\frac{s_i}{s_o} = -2 \quad \Rightarrow \quad s_o = \frac{1}{2} s_i$$

Because s_o = 10 cm, it must be true that s_i = 20 cm, so the lens equation gives us

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{10\text{ cm}} + \frac{1}{20\text{ cm}} = \frac{1}{f} \quad \Rightarrow \quad f = \frac{20}{3}\text{ cm}$$

Expressing the focal length in meters and then taking the reciprocal gives the lens power in diopters:

$$f = \frac{20}{3}\text{ cm} \times \frac{1\text{ m}}{100\text{ cm}} = \frac{1}{15}\text{ m} \quad \Rightarrow \quad P = \frac{1}{f} = \frac{1}{\frac{1}{15}\text{ m}} = 15\text{ D}$$

Chapter 10 Review Questions

Click here to download the PDF of Chapter 10 Review Questions.

Solutions can be found in Chapter 12.

Section I: Multiple Choice

1. An X-ray with a frequency of 1.0×10^{18} *Hz* is an example of an electromagnetic wave. The equation for the electric field of such an X-ray could be given by $E = A\cos(\omega t)$ with...

   (A) A measured in N/C and $\omega = 1.0 \times 10^{18}$ *Hz*.
   (B) A measured in $N \cdot s/C$ and $\omega = 1.0 \times 10^{18}$ *Hz*.
   (C) A measured in N/C and $\omega = 2.0\pi \times 10^{18}$ *Hz*.
   (D) A measured in $N \cdot s/C$ and $\omega = 2.0\pi \times 10^{18}$ *Hz*.

2. In Young's double-slit interference experiment, what is the difference in path length of the light waves from the two slits at the center of the first bright fringe above the central maximum?

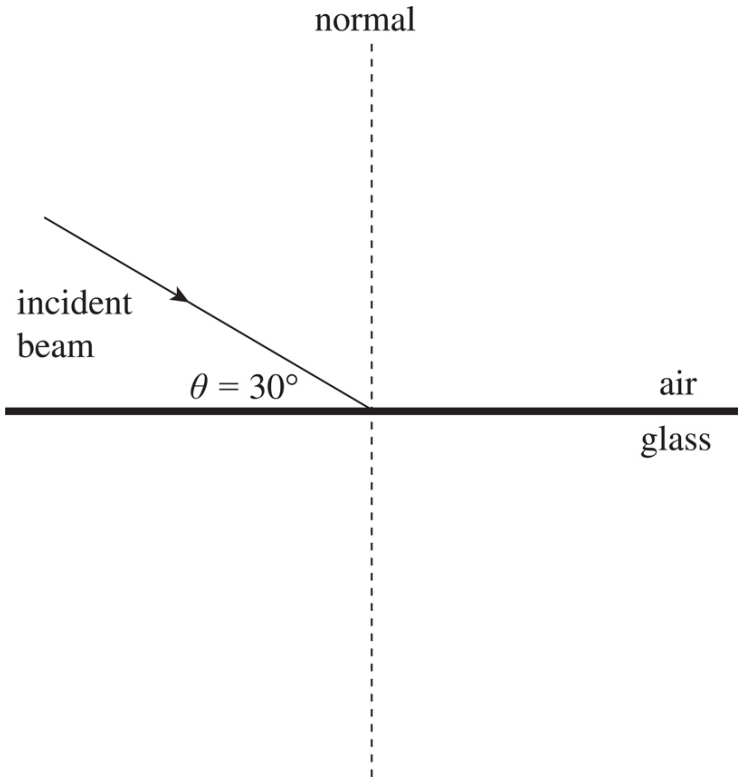
   (A $\frac{1}{4}\lambda$

   (B) $\frac{1}{2}\lambda$

   (C) λ

   (D) $\frac{3}{2}\lambda$

3. A beam of light in air is incident upon the smooth surface of a piece of flint glass, as shown:



As the incident angle is increased toward $\theta = 90°$, what observation is made of the refracted ray? All angle references are relative to the surface as shown for both rays.

(A) The refracted ray angle increases as the incident angle increases, but the value of the refracted angle is always smaller than the incident angle.

(B) The refracted ray angle increases as the incident angle increases, but the value of the refracted angle is always larger than the incident angle.

(C) The refracted ray angle increases as the incident angle increases until at some angle total internal reflection begins to occur.

(D) The refracted ray angle decreases as the incident angle increases, but the value of the refracted angle is always smaller than the incident angle.

4. A convex lens constructed of glass makes a real image of an object when it is in air. When the object is located d_o in front of the lens, the image appears in air at a distance d_i behind the lens. What occurs if the object is still at d_o, but the object and the lens are submerged in water with an index of refraction between that of air and the glass of the lens?

(A) The image is still at d_i and is still real.

(B) The image is at a position closer to the lens than d_i and is real.

(C) The image is at a position farther from the lens than d_i and is real.

(D) The image becomes virtual.

5. A beam of light traveling in Medium 1 strikes the interface to another transparent medium with a lower index of refraction, Medium 2. If the intensity of light is measured to be less in Medium 2 than in Medium 1, then it can be concluded that

(A) The decrease in intensity was caused by the change in the speed of light in the different media.

(B) Total internal reflection occurred at the interface.

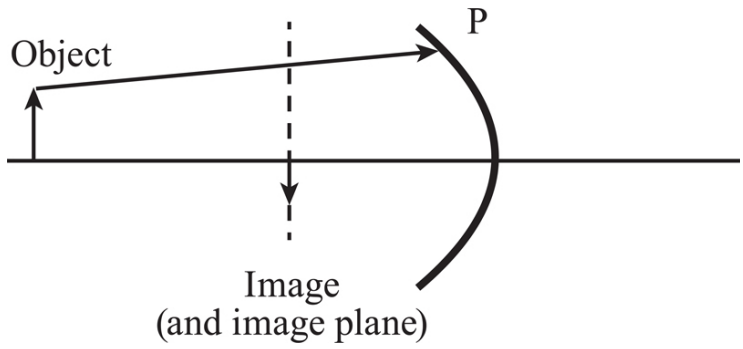
(C) The angle the light travels relative to the normal in Medium 2 will be the same as the angle the light had traveled relative to the normal in Medium 1.

(D) Some part of the light was reflected and/or absorbed at the interface.

6. If a clear liquid has a refractive index of 1.45 and a transparent solid has an index of 2.90 then, for total internal reflection to occur at the interface between these two media, which of the following must be true?

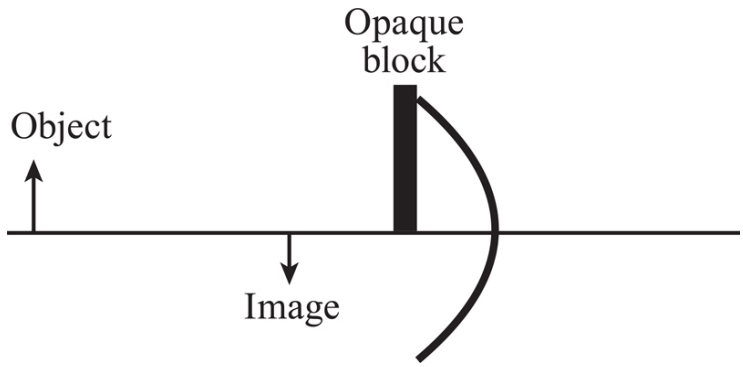
<u>incident beam originates</u> <u>at an angle of incidence greater</u>

	in	than
(A)	The solid	30°
(B)	The liquid	30°
(C)	The liquid	60°
(D)	Total internal reflection cannot occur.	



7. The above picture shows a converging mirror and an object, as well as the image, which is formed from standard ray tracing. Also shown, a dotted vertical line indicates the image place for this configuration. Also shown in a ray that travels from the top of the image to the point P on the curved mirror. The point P lies above the line which is parallel to the optic axis from the tip of the image. Where will the reflected ray from point P intersect the image plane?

(A) On the optic axis
(B) Below the optic axis but above the tip of the image
(C) At the tip of the image
(D) Farther from the optic axis than the tip of the image



8. The above picture shows a converging mirror and an object, as well as the image, which is formed from standard ray tracing. After the image is formed, an opaque block, also shown in the picture, is inserted to block the top half of the mirror. What changes, if any, will be observed in the image?

(A) The image will remain complete but will be diminished in brightness.

(B) Part of the image will be absent, but the rest will be as bright was it was before the insertion of the block.

(C) Part of the image will be absent and the part which remains will be diminished in brightness.

(D) No change will be observed.

9. You are tasked with creating a real image using a diverging lens as your imaging system. Which of the following criteria are true about both the image and the object?

(A) A real image can be created only if the object is farther away from the lens than the focal length.

(B) A real image can be created only if the object is closer to the lens than the focal length.

(C) A real image can be created regardless of whether the object is farther away from the lens or closer to the lens than the focal length.

(D) A real image cannot be created using only a diverging lens.

10. Three double slit experiments are conducted and an interference pattern is observed in each. The second experiment has an identical set up to the original except a shorter wavelength of light is used. The third experiment has an identical set up to the original except the slits are closer together. What is true about the observed interference patterns for the experiments?

(A) Both the second and third experiment have closer fringes than the original.

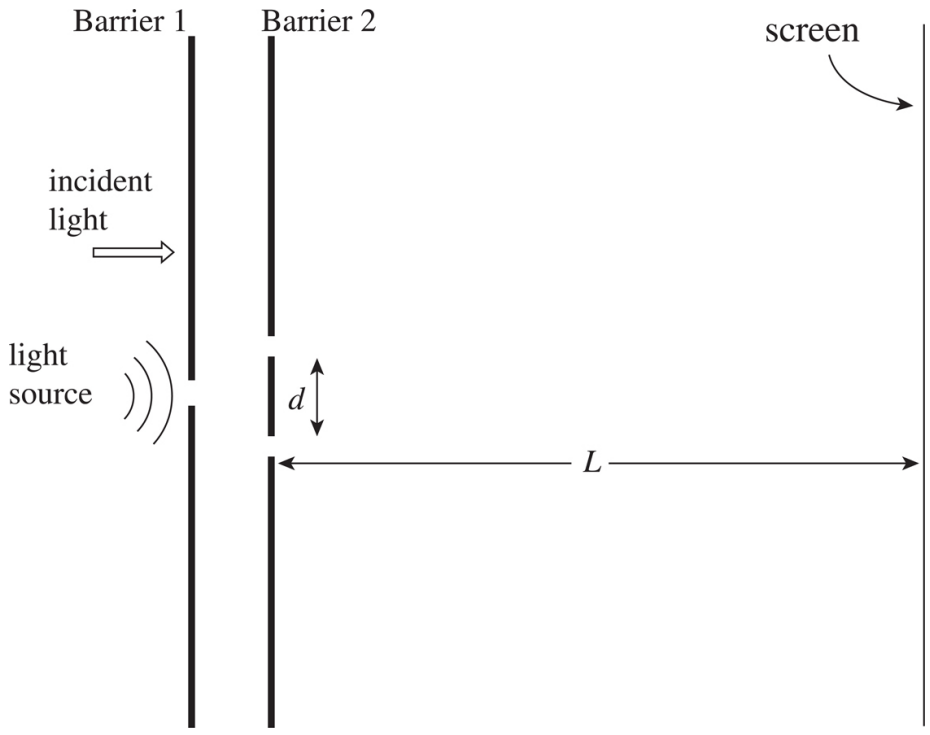
(B) The second experiment has closer fringes than the original and the third has farther fringes.

(C) The second experiment has farther fringes than the original and the third has closer fringes.

(D) Both the second and third experiment have farther fringes than the original.

Section II: Free Response

1. Two trials of a double-slit interference experiment are set up as follows. The slit separation is d = 0.50 mm, and the distance to the screen, L, is 4.0 m.



(a) What is the purpose of the first (single-slit) barrier? Why not use two light sources, one at each slit at the second barrier? Explain briefly.

In the first trial, white light is used.

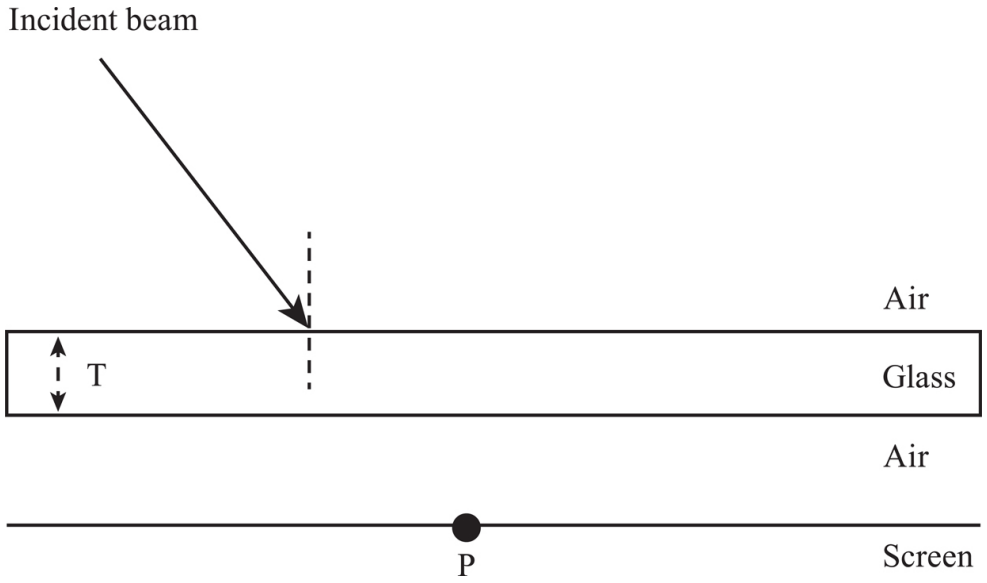
(b) What is the vertical separation on the screen (in mm) between the first-order maxima for red light (λ = 750 nm) and violet light (λ = 400 nm)?

(c) Locate the nearest point to the central maximum where an intensity maximum for violet light (λ = 400 nm) coincides with an intensity maximum for orange-yellow light (λ = 600 nm).

In the second trial, the entire region between the double-slit barrier and the screen is filled with a large slab of glass of refractive index n = 1.5, and monochromatic green light (λ = 500 nm in air) is used.

(d) What is the separation between adjacent bright fringes on the screen?

2. A beam of light shines on a screen with only air between the light source and the screen. The spot where the light strikes the screen is marked and called point P. Then, as the image below shows, a piece of glass with thickness T is placed in the path of the beam. The glass is surrounded on top and bottom by air. The normal to the glass slab at the point where the incident beam strikes the glass is drawn as a dashed line.



(a) Two students are discussing what observations will be made in this new arrangement.

Student 1: The spot on the screen will still be at point P. According to Snell's Law, the light will bend at the first interface and then unbend at the second interface and that it is travelling parallel to its original path. This will cause the spot on the screen will be at the same location.

Student 2: The beam after the glass cannot be parallel to the beam before the glass. Snell's Law is $n_{in} \sin(\theta_{in}) = n_{out} \sin(\theta_{out})$, and because θ_{out} is different from θ_{in} at the first interface, the angles have to be different at the second interface as well.

i. Which parts, if any, of Student 1's reasoning is correct?

ii. Which parts, if any, of Student 2's reasoning is correct?

iii. Which parts, if any, of Student 1's reasoning is incorrect?

iv. Which parts, if any, of Student 2's reasoning is incorrect?

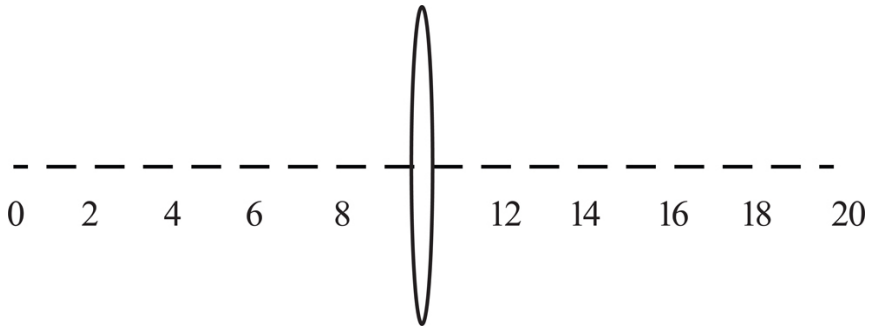
(b) Use ray tracing to draw the path of the beam from where it is incident on the glass to where it strikes the screen. Use appropriate labels to indicate which angles are the same and which are different.

(c) In a paragraph length explanation without relying solely on equations, explain how your answer in part (b) would be different if the area between the screen and the glass were filled with water (whose index of refraction is between that of air and that of glass).

3. An experiment is set up with an object, a single converging lens, and a screen to make an image appear on the screen. The lens has a focal length of 3 cm. The arrangement is configured so

that the image which appears on the screen is the same size as the object. The lens is placed at a position labeled 10 cm on the optic axis.

(a) Draw a diagram of the experimental set up along the optic axis below. Justify your answer.



(b) If the converging lens were replaced with a convex mirror of the same focal length, explain how (if at all) your experimental set-up would have to change to produce an image of the same height as the object.

(c) In a clear, coherent paragraph length response, explain what a virtual image is. Explain what steps would have to be taken to experimentally verify the existence of a virtual image.

Summary

- All electromagnetic waves travel at the speed of light ($c = 3 \times 10^8$ m/s). They obey the wave equation $v = f\lambda$. When monochromatic coherent light goes through double slits (or a diffraction grating), there will be constructive interference when the path difference is given by $PD = m\lambda$ or $d \sin\theta = m\lambda$ where $m = 0, 1, 2\ldots$. There will be destructive interference for $PD = (m + \frac{1}{2})\lambda$ or $d \sin\theta = (m + \frac{1}{2})\lambda$. For small angles, we can say $x_m = \frac{m\lambda L}{d}$, where x_m is the distance the mth maximum is located from the central maximum and shows where constructive interference occurs.

- Geometric optics always measures angles from the normal line. Topics include the fact that the angle of incidence is equal to the angle of reflection and that these angles lie in the same plane.

- Snell's Law states when light enters a new medium (material) it changes speeds and may change directions. This is stated in the formula

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

where n is the index of refraction, which is a ratio of the speed of light in a vacuum to the speed of light in the substance ($n = \frac{c}{v}$). The index of refraction is always greater than 1 and has no units.

When going from a high index of refraction to a lower index of refraction, light may have total internal reflection if the angle is larger than the critical angle ($\sin \theta_c = \frac{n_2}{n_1}$).

- For both thin lenses and curved mirrors, you can use the thin lens equation: $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$.

  Where f is positive for a convex lens or a concave mirror and f is negative for a concave lens or a convex mirror; s_o is always positive.

- For a lens, if s_i is negative, the lens image is virtual and located on the same side of the lens as the object. If s_i is positive, the lens image is real and located on the opposite side of the lens from the object.

- For a mirror, if s_i is negative, the image is virtual and located on the opposite side of the mirror from the object. If s_i is positive, the image is real and located on the same side of the mirror as the object.

- The magnification of a lens is given by $M = \frac{h_i}{h_o} = \frac{-s_i}{s_o}$.

- A **diverging optical device** could be either a convex mirror or a concave lens. The thing these two have in common is that no matter where the object is located they form images that are virtual and upright. The sign of h_i will always be positive and the sign of s_i will always be negative. The sign of the magnification will be positive and its value will be less than one. The image will always be located between the focal length and the optical device.

- A **converging optical device** could be either a concave mirror or a convex lens. The thing these two have in common is that when s_o is *outside* the focal length they form images that are real and inverted. The sign of h_i will be negative and the sign of s_i will be positive. The sign of the magnification will be negative and its absolute value can be any number greater than zero. The other thing they have in common is that when s_o is *inside* the focal length they form images that are virtual and upright. The sign of h_i will be positive and the sign of s_i will be negative. The sign of the magnification will be positive and its value will be greater than one.

Chapter 11
Modern Physics

INTRODUCTION

The subject matter of the previous chapters was developed in the 17th, 18th, and 19th centuries, but as we delve into the physics of the very small, we enter the 20th century.

PHOTONS AND THE PHOTOELECTRIC EFFECT

Max Planck first proposed the idea of light being emitted as individual packets of constant energy, called **quanta**. This is where the name quantum mechanics comes from. The particle nature of light was pioneered by Einstein in 1905, when he showed light transferred energy like a particle, and Arthur Compton in 1923, when he showed light has momentum and can undergo elastic collisions. A quantum of electromagnetic energy is known as a **photon**. Light behaves like a stream of photons, and this is illustrated by the **photoelectric effect.**

When a piece of metal is illuminated by electromagnetic radiation (specifically visible light, ultraviolet light, or X-rays), the energy absorbed by electrons near the surface of the metal can liberate them from their bound state, and these electrons can fly off. The released electrons are known as **photoelectrons.** In this case, the classical, wave-only theory of light would predict three results:

(1) There would be a significant time delay between the moment of illumination and the ejection of photoelectrons. This is because the metal would have to heat up to the point that the thermal energy was enough to overcome the binding energy.

(2) Increasing the intensity of the light would cause the electrons to leave the metal surface with greater kinetic energy. This is because the energy carried by a wave is related to its intensity.

(3) Photoelectrons would be emitted regardless of the frequency of the incident energy, as long as the intensity was high enough.

Surprisingly, none of these predictions was observed. Photoelectrons were ejected within just a few billionths of a second after illumination, disproving prediction (1). Secondly, increasing the

intensity of the light did not cause photoelectrons to leave the metal surface with greater kinetic energy. Although more electrons were ejected as the intensity was increased, there was a maximum photoelectron kinetic energy; prediction (2) was false. And, for each metal, there was a certain **threshold frequency**, f_0: If light of frequency lower than f_0 were used to illuminate the metal surface, *no* photoelectrons were ejected, regardless of how intense the incident radiation was; prediction (3) was also false. Clearly, something was wrong with the wave-only theory of light.

Einstein explained these observations by borrowing from Planck's idea that light came in individual quanta of energy. He called the quanta photons. The energy of a photon is proportional to the frequency of the wave,

Equation Sheet

$$E = hf$$

where h is **Planck's constant** (about 6.63×10^{-34} J·s). A certain amount of energy had to be imparted to an electron on the metal surface in order to liberate it; this was known as the metal's **work function,** or ϕ. If an electron absorbed a photon whose energy E was greater than ϕ, it would leave the metal with a maximum kinetic energy equal to $E - \phi$. This process could occur *very* quickly, which accounts for the rapidity with which photoelectrons are produced after illumination. This explains why prediction (1) was incorrect.

In this view, increasing the intensity (and therefore the energy) just means increasing the number of photons and results in the ejection of more photoelectrons—but since the energy of each incident photon is fixed by the equation $E = hf$, the value of K_{max} will still be $E - \phi$. This can be expressed as

Equation Sheet

$$K_{\max} = hf - \phi$$

This accounts for the observation that disproved prediction (2).

Finally, if the incoming photon's energy was less than phi (or, $E = hf < \phi$), the photon energy would not be enough to liberate electrons. Blasting the metal surface with more photons (that is, increasing the intensity of the incident beam) would also do nothing; none of the photons would have enough energy to eject electrons. This accounts for the observation of a threshold frequency, which we now know is ϕ/h. This can be expressed as

Equation Sheet

$$f_0 = \phi/h$$

and is why prediction (3) was incorrect. Before we get to some examples, it's worthwhile to introduce a new unit for energy. The SI unit for energy is the joule, but it's too large to be convenient in the domains we're studying now. We'll use a much smaller unit, the **electronvolt** (abbreviated **eV**). The eV is equal to the energy gained (or lost) by an electron accelerated through a potential difference of one volt. Using the equation $\Delta U_E = qV$, we find that

$$1 \text{ eV} = (1 \text{ e})(1 \text{ V}) = (1.6 \times 10^{-19} \text{ C})(1 \text{ V}) = 1.6 \times 10^{-19} \text{ J}$$

In terms of electronvolts, the value of Planck's constant is 4.14×10^{-15} eV·s.

Example 1 The work function, ϕ, for aluminum is 4.08 eV.

(a) What is the threshold frequency required to produce photoelectrons from aluminum?

(b) Classify the electromagnetic radiation that can produce photoelectrons.

(c) If light of frequency $f = 4.00 \times 10^{15}$ Hz is used to illuminate a piece of aluminum,

(i) what is K_{max}, the maximum kinetic energy of ejected photoelectrons?

(ii) what's the maximum speed of the photoelectrons? (Electron mass = 9.11×10^{-31} kg.)

(d) If the light described in part (b) were increased by a factor of 2 in intensity, what would happen to the value of K_{max}?

Solution.

(a) We know from the statement of the question that, in order for a photon to be successful in liberating an electron from the surface of the aluminum, its energy cannot be less than 4.08 eV. Therefore, the minimum frequency of the incident light—the threshold frequency—must be

$$f_0 = \frac{\phi}{h} = \frac{4.08 \text{ eV}}{4.14 \times 10^{-15} \text{ eV} \cdot \text{s}} = 9.86 \times 10^{14} \text{ Hz}$$

(b) Based on the electromagnetic spectrum given in the previous chapter, the electromagnetic radiation used to produce

photoelectrons from aluminum must be at least in the ultraviolet region of the EM spectrum.

(c)

(i) The maximum kinetic energy of photoelectrons is found from the equation

$$\begin{aligned} K_{\max} &= hf - \phi \\ &= (4.14\times10^{-15}\ \text{eV}\cdot\text{s})(4.00\times10^{15}\ \text{Hz}) - (4.08\ \text{eV}) \\ &= 12.5\ \text{eV} \end{aligned}$$

(ii) Using the above result and $K = \frac{1}{2}mv^2$, we can find $v_{\max}$:

$$\begin{aligned} v_{\max} &= \sqrt{\frac{2}{m_e}K_{\max}} \\ &= \sqrt{\frac{2}{9.11\times10^{-31}\ \text{kg}}\left(12.5\ \text{eV}\cdot\frac{1.6\times10^{-19}\ \text{J}}{1\ \text{eV}}\right)} \\ &= 2.1\times10^{6}\ \text{m/s} \end{aligned}$$

(d) Nothing will happen. Increasing the intensity of the illuminating radiation will cause more photons to impinge on the metal surface, thereby ejecting more photoelectrons, but their maximum kinetic energy will remain the same. The only way to increase $K_{\max}$ would be to increase the frequency of the incident energy.

Chemistry

Modern Physics begins to explore atoms, which bridges into chemistry. Fortunately, you will not need to know much about chemistry for the AP Physics Exam—just a basic understanding of some topics.

THE BOHR MODEL OF THE ATOM

In the years immediately following Rutherford's announcement of his nuclear model of the atom, a young physicist, Niels Bohr, would add an important piece to the atomic puzzle. Rutherford told us where the positive charge of the atom was located; Bohr would tell us about the electrons.

For fifty years it had been known that atoms in a gas discharge tube emit and absorb light only at specific wavelengths. The light from a glowing gas, passed through a prism to disperse the beam into its component wavelengths, produces patterns of sharp lines called **atomic spectra**. The visible wavelengths that appear in the emission spectrum of hydrogen had been summarized by the *Balmer formula*

$$\frac{1}{\lambda_n} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

where R is the *Rydberg constant* (about $1.1 \times 10^7\ \text{m}^{-1}$). The formula worked—that is, it fit the observational data—but it had no theoretical basis. So, the question was, *why* do atoms emit (or absorb) radiation only at certain discrete wavelengths?

Bohr's model of the atom explains the spectroscopists' observations. Using the simplest atom, hydrogen (with only one electron), Bohr postulated that the electron would orbit the nucleus only at certain discrete radii. When the electron is in one of these special orbits, it does not radiate away energy (as the classical theory would predict). However, if the electron absorbs a certain amount of energy, it is **excited** to a higher orbit, one with a greater radius. After a short time in this excited state, it returns to a lower orbit, emitting a photon in the process. Since each allowed orbit—or **energy level**—has a specific radius (and corresponding energy), the photons emitted in each jump have only specific wavelengths.

When an excited electron drops from energy level $n = j$ to a lower one, $n = i$, the transition causes a photon of energy to be emitted, and the energy of the photon is the difference between the two energy levels:

$$E_{\text{emitted photon}} = |\Delta E| = E_j - E_i$$

The wavelength of this photon is

$$\lambda = \frac{c}{f} = \frac{c}{|E_{\text{photon}}|/h} = \frac{hc}{|E_j - E_i|}$$

Speed of Light
Photons travel at the speed of light.

Example 2

The first five energy levels of an atom are shown in the diagram below:

–3 eV _____ $n = 5$
–4 eV _____ $n = 4$

–7 eV _____ $n = 3$

–15 eV _____ $n = 2$

–62 eV _____ $n = 1$ ground state

(a) If the atom begins in the $n = 3$ level, what photon energies could be emitted as it returns

to its ground state?

(b) What could happen if this atom, while in an undetermined energy state, were bombarded with a photon of energy 10 eV?

Solution.

(a) If the atom is in the $n = 3$ state, it could return to ground state by a transition from $3 \rightarrow 1$, or from $3 \rightarrow 2$ and then $2 \rightarrow 1$. The energy emitted in each of these transitions is simply the difference between the energies of the corresponding levels:

$$E_{3\rightarrow 1} = E_3 - E_1 = (-7 \text{ eV}) - (-62 \text{ eV}) = 55 \text{ eV}$$

$$E_{3\rightarrow 2} = E_3 - E_2 = (-7 \text{ eV}) - (-15 \text{ eV}) = 8 \text{ eV}$$

$$E_{2\rightarrow 1} = E_2 - E_1 = (-15 \text{ eV}) - (-62 \text{ eV}) = 47 \text{ Ev}$$

(b) Since no two energy levels in this atom are separated by 10 eV, the atom could not absorb a 10 eV photon, and as a result, nothing would happen. This atom would be transparent to light of energy 10 eV.

Example 3

(a) Using the energy level diagram for hydrogen below, how much energy must a ground-state electron in a hydrogen atom absorb to be excited to the $n = 4$ energy level?

(b) With the electron in the $n = 4$ level, what wavelengths are possible for the photon emitted when the electron drops to a lower

energy level? In what regions of the EM spectrum do these photons lie?

Solution.

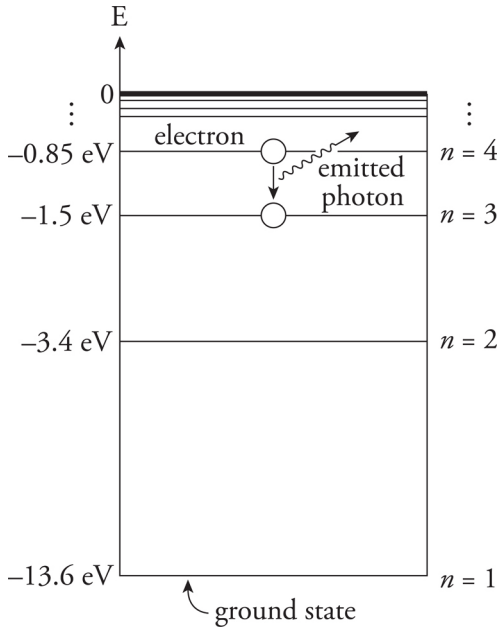
(a) The ground-state energy level ($n = 1$) for hydrogen is −13.6 eV, and $E_4 = -0.85\ \text{eV}$.

Therefore, in order for an electron to make the transition from E_1 to E_4, it must absorb energy in the amount $E_4 - E_1 = (-0.85\ \text{eV}) - (-13.6\ \text{eV}) = 12.8\ \text{eV}$.

(b) An electron in the $n = 4$ energy level can make several different transitions: It can drop to $n = 3$, $n = 2$, or all the way down to the ground state, $n = 1$. The energies of the $n = 2$ and $n = 3$ levels are

$$E_2 = \frac{1}{2^2}(-13.6\ \text{eV}) = -3.4\ \text{eV} \qquad \text{and} \qquad E_3 = \frac{1}{3^2}(-13.6\ \text{eV}) = -1.5\ \text{eV}$$

The following diagram shows the electron dropping from $n = 4$ to $n = 3$:



There are three possible values for the energy of the emitted photon, $E_{4\to3}$, $E_{4\to2}$, or $E_{4\to1}$:

$$E_{4\to3} = E_4 - E_3 = (-0.85\text{ eV}) - (-1.5\text{ eV}) = 0.65\text{ eV}$$
$$E_{4\to2} = E_4 - E_2 = (-0.85\text{ eV}) - (-3.4\text{ eV}) = 2.55\text{ eV}$$
$$E_{4\to1} = E_4 - E_1 = (-0.85\text{ eV}) - (-13.6\text{ eV}) = 12.8\text{ eV}$$

From the equation $E = hf = hc/\lambda$, we get $\lambda = hc/E$, so

$$\lambda_{4\to3} = \frac{hc}{E_{4\to3}} = \frac{(4.14\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{0.65\text{ eV}} = 1{,}910\text{ nm}$$
$$\lambda_{4\to2} = \frac{hc}{E_{4\to2}} = \frac{(4.14\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{2.55\text{ eV}} = 487\text{ nm}$$
$$\lambda_{4\to1} = \frac{hc}{E_{4\to1}} = \frac{(4.14\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{12.8\text{ eV}} = 97\text{ nm}$$

Note that $\lambda_{4\to2}$ is in the visible spectrum; this wavelength corresponds to the color blue-green; $\lambda_{4\to1}$ is an ultraviolet wavelength, and $\lambda_{4\to3}$ is infrared.

WAVE-PARTICLE DUALITY

Light and other electromagnetic waves exhibit wave-like characteristics through interference and diffraction. However, as we saw in the photoelectric effect, light also behaves as if its energy were granular, composed of particles. This is **wave-particle duality:** electromagnetic radiation propagates like a wave but exchanges energy like a particle.

Since an electromagnetic wave can behave like a particle, can a particle of matter behave like a wave? In 1924, the French physicist Louis de Broglie proposed that the answer is "yes." His conjecture, which has since been supported by experiment, is that a particle of mass m and speed v—and thus, linear momentum p—has an associated wavelength, which is called its **de Broglie wavelength:**

Equation Sheet

$$\lambda = \frac{h}{p}$$

Particles in motion can display wave characteristics and behave as if they had a wavelength.

Since the value of h is so small, ordinary macroscopic objects do not display wave-like behavior. For example, a baseball (mass = 0.15 kg) thrown at a speed of 40 m/s has a de Broglie wavelength of

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(0.15\ \text{kg})(40\ \text{m/s})} = 1.1\times10^{-34}\ \text{m}$$

This is much too small to measure. However, with subatomic particles, the wave nature is clearly evident. The 1937 Nobel prize in physics was awarded for experiments by C. J. Davisson and G. P.

Thomson that revealed that a stream of electrons exhibited diffraction patterns when scattered by crystals—a behavior that's characteristic of waves.

Example 4 Electrons in a diffraction experiment are accelerated through a potential difference of 200 V. What is the de Broglie wavelength of these electrons?

Solution. By definition, the kinetic energy of these electrons is 200 eV. Since the relationship between linear momentum and kinetic energy is $p = \sqrt{2mK}$,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{\sqrt{2(9.11 \times 10^{-31}\ \text{kg})\ \ 200\ \text{eV} \cdot \dfrac{1.6 \times 10^{-19}\ \text{J}}{1\ \text{eV}}}} = 8.7 \times 10^{-11}\ \text{m} = 0.087\ \text{nm}$$

This wavelength is characteristic of X-rays.

THE WAVE FUNCTION

There are several major differences between classical physics and modern physics. One of the most important is that the mathematics that underpin modern physics (which are too complicated for treatment here) no longer specify a definite position at a definite time for a particle such as an electron. Instead, that mathematics only give the *probability* that a particle will be measured to be at a particular position when the position is measured. That probability is called the *wave function*, Ψ.

In one interpretation of that mathematics, the act of measuring the position changes the wave function, Ψ, so that it has a single position with a probability of 100%, and that location corresponds to the result of the measurement (this is referred to as "collapsing the wave-function"). However, before the position is measured (or after a measurement has taken place and some time has passed so the position is no longer certain), there are a range of probable locations that can be represented as a graph of Ψ vs x. To interpret such a graph, the position with the largest absolute value is the most probable location for the result of a measurement. Any position where the graph has a height of 0, it is certain that the position will not be observed.

Interpreting graphs

Wave-function graphs will often have values that are positive as well as negative. When interpreting the graph, only the absolute value is important in ranking relative likelihoods of experimental outcomes (technically the likelihood is proportional to Ψ^2, but the absolute value will give correct interpretations and is easier to read from a graph).

RELATIVITY

Another important idea in modern physics is the theory of relativity. Relativity has only two postulates:

1. The results of physical experiments will be the same in any non-accelerating reference frames.
2. The speed of light is constant.

For the AP Exam, you are not required to know any of the mathematical framework surrounding relativity, but only have a qualitative understanding of the consequences of these postulates.

The major consequence of relativity is that time is not absolute, but that space and time are linked together. To see why this must be the case, imagine a car driving down the highway at 65 mph. In non-relativistic physics, the light from the headlight would be moving at the speed of light plus 65 mph. However, the second postulate of relativity states that this cannot be true, and the light from a headlight of a moving car must also be exactly the speed of light.

As the name indicates, relativity describes the consequences of performing experiments in different reference frames and measuring the same results.

When a clock placed on a fast-moving airplane is compared to a clock at rest on the ground, the clock in the airplane shows that less time has passed than the time recorded by the clock on the ground. This phenomenon is known as **time dilation,** and it has been demonstrated experimentally using synchronized atomic clocks.

The speed of light is constant according to the theory of relativity. However, time dilation shows there are disagreements about the amount of time that passes in different reference frames. To be consistent with time dilation, there must also be disagreement about distances. This is known as **length contraction.** A scientist moving

along with the object he measures will observe the object to have a different length than a scientist in a different reference frame.

NUCLEAR PHYSICS

The nucleus of the atom is composed of particles called **protons** and **neutrons**, which are collectively called **nucleons.** The number of protons in a given nucleus is called the atom's **atomic number,** and is denoted Z, and the number of neutrons (the **neutron number**) is denoted N. The total number of nucleons, $Z + N$, is called the **mass number** (or **nucleon number**), and is denoted A. The number of protons in the nucleus of an atom defines the element. For example, the element chlorine (abbreviated Cl) is characterized by the fact that the nucleus of every chlorine atom contains 17 protons, so the atomic number of chlorine is 17; but, different chlorine atoms may contain different numbers of neutrons. In fact, about three-fourths of all naturally occurring chlorine atoms have 18 neutrons in their nuclei (mass number = 35), and most of the remaining one-fourth contain 20 neutrons (mass number = 37).

Nuclei that contain the same numbers of protons but different numbers of neutrons are called **isotopes.** The notation for a **nuclide**—the term for a nucleus with specific numbers of protons and neutrons—is to write Z and A before the chemical symbol of the element:

$$^{A}_{Z}\mathrm{X}$$

The isotopes of chlorine mentioned above would be written as follows:

$$^{35}_{17}\mathrm{Cl} \quad \text{and} \quad ^{37}_{17}\mathrm{Cl}$$

Example 5 How many protons and neutrons are contained in the nuclide $^{63}_{29}\mathrm{Cu}$?

Solution. The subscript (the atomic number, Z) gives the number of protons, which is 29. The superscript (the mass number, A) gives the total number of nucleons. Since $A = 63 = Z + N$, we find that $N = 63 - 29 = 34$.

Example 6 The element neon (abbreviated Ne, atomic number 10) has several isotopes. The most abundant isotope contains 10 neutrons, and two others contain 11 and 12. Write symbols for these three nuclides.

Solution. The mass numbers of these isotopes are 10 + 10 = 20, 10 + 11 = 21, and 10 + 12 = 22. So, we'd write them as follows:

$$^{20}_{10}\mathrm{Ne}\,,\quad ^{21}_{10}\mathrm{Ne}\,,\quad \text{and}\quad ^{22}_{10}\mathrm{Ne}$$

Another common notation—which we also use—is to write the mass number after the name of the element. These three isotopes of neon would be written as neon-20, neon-21, and neon-22.

The Nuclear Force

Why wouldn't any nucleus that has more than one proton be unstable? After all, protons are positively charged and would therefore experience a repulsive Coulomb force from each other. Why don't these nuclei explode? And what holds neutrons—which have no electric charge—in the nucleus? These issues are resolved by the presence of another fundamental force, the **strong nuclear force,** which binds neutrons and protons together to form nuclei. Although the strength of the Coulomb force can be expressed by a simple mathematical formula (it's inversely proportional to the square of their separation), the nuclear force is much more complicated; no simple formula can be written for the strength of the nuclear force.

Binding Energy

The masses of the proton and neutron are listed below.

$$\text{proton: } m_p = 1.6726 \times 10^{-27} \text{ kg}$$

$$\text{neutron: } m_n = 1.6749 \times 10^{-27} \text{ kg}$$

Because these masses are so tiny, a much smaller mass unit is used. With the most abundant isotope of carbon (carbon-12) as a reference, the **atomic mass unit** (abbreviated **amu** or simply **u**) is defined as 1/12 the mass of a ^{12}C atom. The conversion between kg and u is $1 \text{ u} = 1.6605 \times 10^{-27}$ kg. In terms of atomic mass units,

$$m_p = 1.00728 \text{ up}$$

$$m_n = 1.00867 \text{ u}$$

Now consider the **deuteron,** the nucleus of **deuterium,** an isotope of hydrogen that contains 1 proton and 1 neutron. The mass of a deuteron is 2.01356 u, which is a little *less* than the sum of the individual masses of the proton and neutron. The difference between the mass of any bound nucleus and the sum of the masses of its constituent nucleons is called the **mass defect,** Δm. In the case of the deuteron (symbolized **d**), the mass defect is

$$\begin{aligned} \Delta m &= (m_p + m_n) - m_d \\ &= (1.00728 \text{ u} + 1.00867 \text{ u}) - (2.01356 \text{ u}) \\ &= 0.00239 \text{ u} \end{aligned}$$

What happened to this missing mass? It was converted to energy when the deuteron was formed. In 1905, Einstein gave us the famous equation:

Equation Sheet

$$E = mc^2$$

which tells us how much energy mass contains. Because of this, the mass-difference gives us the amount of energy needed to break the deuteron into a separate proton and neutron. Since this tells us how strongly the nucleus is bound, it is called the **binding energy** of the nucleus.

$$E_B = (\Delta m)c^2$$

Using $E = mc^2$, the energy equivalent of 1 atomic mass unit is

$$\begin{aligned} E &= (1.6605\times10^{-27}\ \text{kg})(2.9979\times10^{8}\ \text{m/s})^2 \\ &= 1.4924\times10^{-10}\ \text{J} \\ &= 1.4924\times10^{-10}\ \text{J}\cdot\frac{1\ \text{eV}}{1.6022\times10^{-19}\ \text{J}} \\ &= 9.31\times10^{8}\ \text{eV} \\ &= 931\ \text{MeV} \end{aligned}$$

In terms of electronvolts, then, the binding energy of the deuteron is

$$E_B\ (\text{deuteron}) = 0.00239\ \text{u}\times\frac{931\ \text{MeV}}{1\ \text{u}} = 2.23\ \text{MeV}$$

Since the deuteron contains 2 nucleons, the **binding-energy-per-nucleon** is

$$\frac{2.23\ \text{MeV}}{2\ \text{nucleons}} = 1.12\ \text{MeV/nucleon}$$

This is the lowest value of all nuclides. The highest, 8.8 MeV/nucleon, is for an isotope of nickel, ^{62}Ni. Typically, when nuclei smaller than nickel are fused to form a single nucleus, the binding energy per nucleon increases, which tells us that energy is released in the process. On the other hand, when nuclei *larger* than nickel are *split*, binding energy per nucleon again increases, releasing energy. This is the basis of nuclear fission.

Example 7 What is the maximum wavelength of EM radiation that could be used to photodisintegrate a deuteron?

Solution. The binding energy of the deuteron is 2.23 MeV, so a photon would need to have at least this much energy to break the deuteron into a proton and neutron. Since $E = hf$ and $f = c/\lambda$,

$$E = \frac{hc}{\lambda} \longrightarrow \lambda_{max} = \frac{hc}{E_{min}} = \frac{(4.14 \times 10^{-15}\ \text{eV} \cdot \text{s})(3.00 \times 10^{8}\ \text{m/s})}{2.23 \times 10^{6}\ \text{eV}} = 5.57 \times 10^{-13}\ \text{m}$$

Example 8 The atomic mass $^{27}_{13}$Al is 26.9815 u. What is its nuclear binding energy per nucleon? (Mass of electron = 0.0005486 u.)

Solution. The nuclear mass $^{27}_{13}$Al is equal to its atomic mass minus the mass of its electrons. Since an aluminum atom has 13 protons, it must also have 13 electrons. So,

$$\begin{aligned}\text{nuclear mass of } {}^{27}_{13}\text{Al} &= (\text{atomic mass of } {}^{27}_{13}\text{Al}) - 13m_e \\ &= 26.9815 \text{ u} - 13(0.0005486 \text{ u}) \\ &= 26.9744 \text{ u}\end{aligned}$$

Now, the nucleus contains 13 protons and 27 − 13 = 14 neutrons, so the total mass of the individual nucleons is

$$\begin{aligned}M &= 13m_p + 14m_n \\ &= 13(1.00728 \text{ u}) + 14(1.00867 \text{ u}) \\ &= 27.2160 \text{ u}\end{aligned}$$

and, the mass defect of the aluminum nucleus is

$$\Delta m = M - m = 27.2160 \text{ u} - 26.9744 \text{ u} = 0.2416 \text{ u}$$

Converting this mass to energy, we can see that

$$E_B = 0.2416 \text{ u} \times \frac{931 \text{ MeV}}{1 \text{ u}} = 225 \text{ MeV}$$

so the binding energy per nucleon is

$$\frac{225 \text{ MeV}}{27} = 8.3 \text{ MeV/nucleon}$$

NUCLEAR REACTIONS

Natural radioactive decay provides one example of a nuclear reaction. Other examples of nuclear reactions include the bombardment of target nuclei with subatomic particles to artificially induce radioactivity, such as the emission of a particle or the splitting of the nucleus (this is **nuclear fission**), and the **nuclear fusion** of small nuclei at extremely high temperatures. In all cases of nuclear reactions that we'll study, nucleon number and charge must be conserved. In order to balance nuclear reactions, we write ${}_{1}^{1}\mathrm{p}$ or ${}_{1}^{1}\mathrm{H}$ for a proton and ${}_{0}^{1}\mathrm{n}$ for a neutron. Gamma-ray photons can also be produced in nuclear reactions; they have no charge or nucleon number and are represented as ${}_{0}^{0}\gamma$.

Matter Cannot be Created nor Destroyed

But it *can* undergo other forms—which does not break this rule. The quickest way to determine whether something undergoes alpha, beta, or gamma decay, is to find the amount of nucleons that have escaped and then to determine which kind of decay happened.

Alpha Decay

When a nucleus undergoes alpha decay, it emits an alpha particle, which consists of two protons and two neutrons and is the same as the nucleus of a helium-4 atom. An alpha particle can be represented as

$$\alpha \text{ , } {}^{4}_{2}\alpha \text{ , or } {}^{4}_{2}\mathrm{He}$$

Very large nuclei can shed nucleons quickly by emitting one or more alpha particles, for example, radon-222 ($^{222}_{86}\mathrm{Rn}$) is radioactive and undergoes alpha decay.

$$^{222}_{86}\mathrm{Rn} \rightarrow {}^{218}_{84}\mathrm{Po} + {}^{4}_{2}\alpha$$

This reaction illustrates two important features of a nuclear reaction.

(1) Mass number is conserved.
(2) Charge is conserved.

The decaying nuclide is known as the **parent**, and the resulting nuclide is known as the **daughter.** (Here, radon-222 is the parent nuclide and polonium-218 is the daughter.) Alpha decay decreases the mass number by 4 and the atomic number by 2. Therefore, alpha decay looks like the following:

$$ {}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{X}' + {}^{4}_{2}\alpha $$

Beta Decay

There are three subcategories of **beta** (β) decay, called β^-, β^+, and **electron capture (EC).**

β^- Decay When the neutron-to-proton ratio is too large, the nucleus undergoes β^- decay, which is the most common form of beta decay. β^- decay occurs when a neutron transforms into a proton and releases an electron. The expelled electron is called a **beta particle.** The transformation of a neutron into a proton and an electron (and another particle, the **electron-antineutrino,** $\bar{\nu}_e$) is caused by the action of the **weak nuclear force,** another of nature's fundamental forces. A common example of a nuclide that undergoes β^- decay is carbon-14, which is used to date archaeological artifacts.

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + e^- + \bar{\nu}_e$$

The reaction is balanced, since 14 = 14 + 0 and 6 = 7 + (−1).

β^+ Decay When the neutron-to-proton ratio is too small, the nucleus will undergo β^+ decay. In this form of beta decay, a proton is transformed into a neutron and a **positron,** $^{0}_{+1}e$ (the electron's **antiparticle**), plus another particle, the **electron-neutrino,** ν_e, which are then both ejected from the nucleus. An example of a positron emitter is fluorine-17.

$$^{17}_{9}\mathrm{F} \rightarrow {}^{17}_{8}\mathrm{O} + e^+ + \nu_e$$

Electron Capture Another way in which a nucleus can increase its neutron-to-proton ratio is to capture an orbiting electron and then cause the transformation of a proton into a neutron. Beryllium-7 undergoes this process.

$$^{7}_{4}\mathrm{Be} + e^- \rightarrow {}^{7}_{3}\mathrm{Li} + \nu_e$$

Gamma Decay

In each of the decay processes defined above, the daughter was a different element than the parent. Radon becomes polonium as a result of α decay, carbon becomes nitrogen as a result of β^- decay, fluorine becomes oxygen from β^+ decay, and beryllium becomes lithium from electron capture. By contrast, gamma decay does not alter the identity of the nucleus; it just allows the nucleus to relax and shed energy. Imagine that potassium-42 undergoes β^- decay to form calcium-42.

$$^{42}_{19}\mathrm{K} \rightarrow {}^{42}_{20}\mathrm{Ca}^{*} + e^{-} + \bar{\nu}_{e}$$

The asterisk indicates that the daughter calcium nucleus is left in a high-energy, excited state. For this excited nucleus to drop to its ground state, it must emit a photon of energy, a **gamma ray**, symbolized by γ.

$$^{42}_{20}\mathrm{Ca}^{*} \rightarrow {}^{42}_{20}\mathrm{Ca} + \gamma$$

Let's sum up the three types of radiation:

Type of Radiation	What Happens?	Charge
Alpha	Particle—a helium nucleus, containing 2 protons and 2 neutrons, is released from the nucleus	Positive 2+
Beta	Particle—a highly energetic electron is released from the nucleus. It has negligible mass. When a beta particle is produced, a neutron divides into a proton and an electron. The electron is ejected from the nucleus. That ejected electron is the beta particle.	Negative 1–
Gamma	Wave—a photon of energy is emitted	No

		Charge

Example 9 A mercury-198 nucleus is bombarded by a neutron, which causes a nuclear reaction:

$$^{1}_{0}\mathrm{n} + {}^{198}_{80}\mathrm{Hg} \rightarrow {}^{197}_{79}\mathrm{Au} + {}^{?}_{?}\mathrm{X}$$

What's the unknown product particle, X?

Solution. In order to balance the superscripts, we must have 1 + 198 = 197 + A, so A = 2, and the subscripts are balanced if 0 + 80 = 79 + Z, so Z = 1:

$$^{1}_{0}\mathrm{n} + {}^{198}_{80}\mathrm{Hg} \rightarrow {}^{197}_{79}\mathrm{Au} + {}^{2}_{1}\mathrm{X}$$

Therefore, X must be a deuteron, $^{2}_{1}\mathrm{H}$ (or just d).

DISINTEGRATION ENERGY

Nuclear reactions not only produce new nuclei and other subatomic product particles, they also involve the absorption or emission of energy. Nuclear reactions must conserve total energy, so changes in mass are accompanied by changes in energy according to Einstein's equation

$$\Delta E = (\Delta m)c^2$$

A general nuclear reaction is written

$$A + B \rightarrow C + D + Q$$

where Q denotes the **disintegration energy**. If Q is positive, the reaction is **exothermic** (or **exoergic**) and the reaction can occur spontaneously; if Q is negative, the reaction is **endothermic** (or **endoergic**) and the reaction cannot occur spontaneously. The energy Q is calculated as follows:

$$Q = [(m_A + m_B) - (m_C + m_D)]c^2 = (\Delta m)c^2$$

For spontaneous reactions—ones that liberate energy—most of the energy is revealed as kinetic energy of the least massive product nuclei.

Example 10 The process that powers the Sun—and upon which all life on Earth is dependent—is the fusion reaction:

$$4\,{}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\alpha + 2\mathrm{e}^{+} + 2\nu_{\mathrm{e}} + \gamma$$

(a) Show that this reaction releases energy.

(b) How much energy is released per proton?

Use the fact that $m_\alpha = 4.0015$ u and ignore the mass of the electron-neutrino, ν_e.

Solution.

(a) We need to find the mass difference between the reactants and products:

$$\begin{aligned}\Delta m &= 4m_p - (m_\alpha + 2m_e) \\ &= 4(1.00728\ \text{u}) - [4.0015\ \text{u} + 2(0.0005486\ \text{u})] \\ &= 0.02652\ \text{u}\end{aligned}$$

Since Δm is positive, the reaction is exothermic: Energy is released.

(b) Converting the mass difference to energy gives

$$Q = 0.02652\ \text{u} \times \frac{931\ \text{MeV}}{1\ \text{u}} = 24.7\ \text{MeV}$$

Since four protons went into the reaction, the energy liberated per proton is

$$\frac{24.7\ \text{MeV}}{4\ \text{p}} = 6.2\ \text{MeV/proton}$$

Example 11 Can the following nuclear reaction occur spontaneously?

$$^4_2\alpha + {}^{14}_7\text{N} \rightarrow {}^{17}_8\text{O} + {}^1_1\text{H}$$

(The mass of the nitrogen nucleus is 13.9992 u, and the mass of the oxygen nucleus is 16.9947 u.)

Solution. We first figure out the mass equivalent of the disintegration energy:

$$\begin{aligned}\Delta m &= (m_\alpha + m_{\mathrm{N}}) - (m_{\mathrm{O}} + m_{\mathrm{p}}) \\ &= (4.0015\ \mathrm{u} + 13.9992\ \mathrm{u}) - (16.9947\ \mathrm{u} + 1.00728\ \mathrm{u}) \\ &= -0.00128\ \mathrm{u}\end{aligned}$$

Since Δm is negative, this reaction is nonspontaneous; energy must be *supplied* in order for this reaction to proceed. But how much?

$$|Q| = 0.00128\ \mathrm{u} \times \frac{931\ \mathrm{MeV}}{1\ \mathrm{u}} = 1.19\ \mathrm{MeV}$$

Chapter 11 Review Questions

Click here to download the PDF of Chapter 11 Review Questions.

Solutions can be found in Chapter 12.

Section I: Multiple Choice

1. According to the theory put forth by Louis de Broglie, all matter has wave-like properties such as interference, but these properties are only seen at a microscopic scale. Why are these properties not typically observed at a macroscopic scale?

   (A) The wavelength of matter is typically too large to observe this interference.

   (B) Interference of matter only occurs when these waves interact with other objects comparable to their wavelength and those things are microscopic.

   (C) There are no energy level transitions available to allow for this interference to be observed.

   (D) At the macroscopic scale, the interference is always destructive so it cannot be observed.

2. An experiment is conducted of the photoelectric effect. A metal whose work function is 6.0 eV is struck with a beam light with power 1.0 mW of a frequency 7.2×10^{15} Hz. A photoelectron is ejected from the surface of the metal and is found to require a stopping potential of 24 eV. Which of the following changes could be made so that a photoelectron was not ejected from the surface of the metal? (Choose two.)

   (A) Alter the work function of the metal

   (B) Alter the power of the light

   (C) Alter the frequency of the light

   (D) Alter the stopping potential

3. An atom with one electron has an ionization energy of 25.0 eV. An electron in this atom makes a transition from an excited energy level, where $E = -16.0$ eV, to the ground state. What is the wavelength of the emitted photon from this transition?

(A) 138 nm
(B) 112 nm
(C) 77.5 nm
(D) 49.6 nm

4. The single electron in an atom has an energy of −40 eV when it's in the ground state, and the first excited state for the electron is at −10 eV. What will happen to this electron if the atom is struck by a stream of photons, each of energy 15 eV?

(A) The electron will absorb the energy of one photon and become excited halfway to the first excited state, then quickly return to the ground state, emitting a 15 eV photon in the process.
(B) The electron will absorb the energy of one photon and become excited halfway to the first excited state, then quickly absorb the energy of another photon to reach the first excited state.
(C) The electron will absorb two photons and be excited to the first excited state.
(D) Nothing will happen.

5. The products of several radioactive decays are being studied. Each particle starts with the same speed and enters into a region with a uniform magnetic field directed perpendicular to the initial velocity of the particles. Which observation could be made?

(A) A neutron and an electron are deflected in the same direction, but with the electron turning with a larger radius.
(B) An alpha particle and an electron are deflected in the same direction, but with the alpha particle turning with a larger radius.
(C) An alpha particle and a neutron are both undeflected.

(D) An alpha particle is deflected and a neutron is undeflected.

6. A partial energy-level diagram for an atom is shown below. What photon energies could this atom emit if it begins in the $n = 3$ state?

–3 eV _____ $n = 4$

–5 eV _____ $n = 3$

–8 eV _____ $n = 2$

–12 eV _____ $n = 1$ ground state

(A) 5 eV only
(B) 3 eV or 7 eV only
(C) 2 eV, 3 eV, or 7 eV
(D) 3 eV, 4 eV, or 7 eV

7. Which of the following transitions between energy levels results in emission of the shortest wavelength photon?

(A) A large energy transition to a higher energy level
(B) A small energy transition to a higher energy level
(C) A large energy transition to a lower energy level
(D) A small energy transition to a lower energy level

8. The de Broglie hypothesis, that $\lambda = h/p$, was experimentally confirmed by Davisson, Germer, and Thompson. Which of the following describes this hypothesis?

(A) A photon carries a momentum that depends on the wavelength of that photon.

(B) Momentum will be conserved during quantum mechanical processes.

(C) Photons may behave like particles under certain circumstances.

(D) Electrons will undergo diffraction in certain circumstances.

9. In nuclear reactions, all of the following are true about mass and mass defect EXCEPT

(A) The mass defect is directly proportional to the binding energy.

(B) The mass of the unbound nucleons will be less than their mass when combined.

(C) The energy associated with the mass defect is significantly greater than the energy levels for electrons in the atom.

(D) The mass defect is significantly less than the rest mass of the nucleons.

10. What's the missing particle in the following nuclear reaction?

$$^{2}_{1}\mathrm{H} + {}^{63}_{29}\mathrm{Cu} \longrightarrow {}^{64}_{30}\mathrm{Zn} + (?)$$

(A) Proton

(B) Neutron

(C) Electron

(D) Positron

11. A particular isotope of platinum $^{175}_{78}\mathrm{Pt}$ has a halflife of just over 2.5 seconds. It can decay either via alpha decay or beta(+) decay. What are the daughter nuclei from each of these processes?

(A) Alpha decay results in $^{171}_{76}\mathrm{Os}$ and beta(+) decay results in $^{175}_{75}\mathrm{Ir}$.

(B) Alpha decay results in $^{171}_{76}\mathrm{Os}$ and beta(+) decay results in $^{175}_{79}\mathrm{Au}$.

(C) Alpha decay results in $^{179}_{80}\mathrm{Hg}$ and beta(+) decay results in $^{175}_{75}\mathrm{Ir}$.

(D) Alpha decay results in $^{179}_{80}\mathrm{Hg}$ and beta(+) decay results in $^{175}_{79}\mathrm{Au}$.

Section II: Free Response

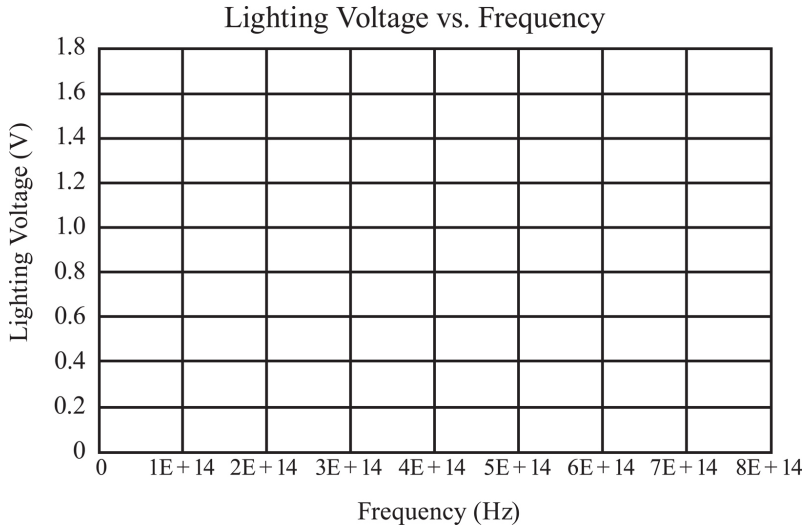
1. An experiment is carried out with a series of different colored LED lights (which glow by emitting photoelectrons) which are connected one at a time to a variable voltage supply. When the voltage supply is set to 0 V, none of the LED bulbs light. As the voltage is increased, each LED turns on at a different voltage setting. As the voltage is further increased, the brightness of the LED increases.

Data is collected of the color wavelength (and corresponding frequency) for each LED and the smallest value of the voltage where the light is illuminated.

Color	Wavelength	Frequency	Lighting Voltage
Red	650	4.62	0.51
Yellow	580	5.17	0.74
Green	532	5.64	0.93
Blue	400	7.50	1.71

(a) Explain why an increase of $\Delta V = 0.1$ V in the voltage after the light is illuminated causes the brightness to increase, but the bulb remains dark for any of the LEDs with the same voltage increase $\Delta V = 0.1$ V from 0 V to 0.1 V.

(b) Plot the data on the axes below. Then calculate the slope and the y-intercept of the plot.



(c)

i. Explain how, if possible, this experiment could be repeated to get a plot with the same slope but a different intercept. If this is not possible, explain why.

ii. Explain how, if possible, this experiment could be repeated to get a plot with a different slope but the same intercept. If this is not possible, explain why.

Summary

- The energy available to liberate electrons near the surface of a metal (the photoelectric effect) is proportional to the frequency of the incident photon. This idea is expressed in $E = hf$, where h is Planck's constant ($h = 6.63 \times 10^{-34}$ J·s).
- The work function (ϕ) indicates the amount of energy needed to liberate the electron. There is a minimum frequency needed to liberate the electrons: $f_0 = \frac{\phi}{h}$. The kinetic energy of the emitted electron is $K_{\max} = hf - \phi$
- Particles in motion have wavelike properties: $\lambda = \frac{h}{p}$, where p is the momentum of the particle. Combining this with $E = hf$ and $c = f\lambda$ yields $E = pc$.
- Standard notation for an element is ${}^{A}_{Z}X$, where A is the mass number, Z is the number of protons in the nucleus, and $A = Z + N$, where N is the number of neutrons in the nucleus.
- A nuclear reaction produces new nuclei, other subatomic particles, and the absorption or emission of energy. The change in mass between the reactants and the products tells how much energy is released (exothermic or + Q) or how much energy is needed to produce the reaction (endothermic or $-Q$) in the general equation $\mathrm{A} + \mathrm{B} \rightarrow \mathrm{C} + \mathrm{D} + Q$, where A and B are reactants

and C and D are products. This energy released or absorbed is given by $\Delta E = (\Delta m)c^2$.

Chapter 12

Solutions to Chapter Review Questions

CHAPTER 3 REVIEW QUESTIONS

Section I: Multiple Choice

1. **D**

Since Point X is 5 m below the surface of the water, the pressure due to the water at X, P_X, is $\rho g h_X = \rho g(5\text{ m})$, where ρ is the density of water. Because Point Y is 4 m below the surface of the water, the pressure due to the water at Y, p_Y, is $\rho g(4\text{ m})$. Therefore,

$$\frac{P_X}{P_Y} = \frac{\rho g(5\text{ m})}{\rho g(4\text{ m})} = \frac{5}{4} \quad \Rightarrow \quad 4P_X = 5P_Y$$

2. **C**

Because the top of the box is at a depth of $D - z$ below the surface of the liquid, the pressure on the top of the box is

$$P = P_{atm} + \rho g h = P_{atm} + \rho g(D - z)$$

The area of the top of the box is $A = xy$, so the force on the top of the box is

$$F = PA = [P_{atm} + \rho g(D - z)]xy$$

3. **B**

The scale reading is the difference between the weight (which is constant) and the buoyant force. The buoyant force is calculated from $F_B = \rho V g$ and will increase as the volume submerged is increased. As the amount the cube is lowered increases, the scale reading will drop at a constant rate, since V is directly proportional to h for a cube.

4. **B**

The independent variable and dependent variable are not changed because those are determined by the experimental setup, so you can eliminate (C) and (D) as answers. Changing the cube to a sphere makes V from $F_B = \rho Vg$ no longer directly proportional to h, so the graphs will not be the same, so you can eliminate (A).

5. **B**

The buoyant force on the Styrofoam block is $F_{\text{buoy}} = \rho_{\text{L}} Vg$, and the weight of the block is $F_{\text{g}} = m_{\text{S}} g = \rho_{\text{S}} Vg$. Because $\rho_{\text{L}} > \rho_{\text{S}}$. the net force on the block is upward and has magnitude $F_{\text{net}} = F_{\text{buoy}} - F_{\text{g}} = (\rho_{\text{L}} - \rho_{\text{S}})Vg$

Therefore, by Newton's Second Law, you have

$$a = \frac{F_{\text{net}}}{m} = \frac{(\rho_{\text{L}} - \rho_{\text{S}})Vg}{\rho_{\text{S}} V} = \left(\frac{\rho_{\text{L}}}{\rho_{\text{S}}} - 1 \right) g$$

6. **D**

The upward force on the ball is the buoyant force. The downward forces are gravity and tension. Gravity will remain constant, so you can eliminate (C). The change in tension results from a change in the buoyant force and not because of pressure, so you can eliminate (A) and (B).

7. **A**

If the object weighs 100 N less when completely submerged in water, the buoyant force must be 100 N; therefore,

$$F_{\text{buoy}} = \rho_{\text{water}} V_{\text{sub}} g = \rho_{\text{water}} Vg = 100 \text{ N} \quad \Rightarrow \quad V = \frac{100 \text{ N}}{\rho_{\text{water}} g} = \frac{100 \text{ N}}{(1000 \frac{\text{kg}}{\text{m}^3})(10 \frac{\text{N}}{\text{kg}})} = 10^{-2} \text{ m}^3$$

Now that you know the volume of the object, you can figure out its weight:

$$F_g = mg = \rho_{object} Vg = (2000 \frac{kg}{m^3})(10^{-2}\ m^3)(10 \frac{N}{kg}) = 200\ N$$

8. **A**

The cross-sectional diameter at Y is 3 times the cross-sectional diameter at X, so the cross-sectional area at Y is $3^2 = 9$ times that at X. The Continuity Equation states that the flow speed, v, is inversely proportional to the cross-sectional area, A. So, if A is 9 times greater at Point Y than it is at X, then the flow speed at Y is 1/9 the flow speed at X; that is, $v_Y = (1/9)v_X = (1/9)\ (6\ m/s) = 2/3\ m/s$.

9. **D**

Each side of the rectangle at the bottom of the conduit is 1/4 the length of the corresponding side at the top. Therefore, the cross-sectional area at the bottom is $(1/4)^2 = 1/16$ the cross-sectional area at the top. The Continuity Equation states that the flow speed, v, is inversely proportional to the cross-sectional area, A. So, if A at the bottom is 1/16 the value of A at the top, then the flow speed at the bottom is 16 times the flow speed at the top.

10. **D**

Apply Bernoulli's Equation to a point at the pump (Point 1) and at the nozzle (the exit point, Point 2). Choose the level of Point 1 as the horizontal reference level; this makes $y_1 = 0$ and $y_2 = 1$ m. Now, because the cross-sectional diameter decreases by a factor of 10 between Points 1 and 2, the cross-sectional area decreases by a factor of $10^2 = 100$, so flow speed must increase by a factor of 100; that is, $v_2 =$

$100v_1 = 100(0.4 \text{ m/s}) = 40 \text{ m/s}$. Because Point 2 is exposed to the air, the pressure there is P_{atm}. Bernoulli's Equation becomes

$$P_1 + \frac{1}{2}\rho v_1^2 = P_{\text{atm}} + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

Therefore,

$$\begin{aligned}
P_1 - P_{\text{atm}} &= \rho g y_2 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 \\
&= (1{,}000\frac{\text{kg}}{\text{m}^3})(10 \text{ m/s}^2)(1 \text{ m}) + \frac{1}{2}(1{,}000\frac{\text{kg}}{\text{m}^3})(40 \text{ m/s})^2 - \frac{1}{2}(1{,}000\frac{\text{kg}}{\text{m}^3})(0.4 \text{ m/s})^2 \\
&\approx (1{,}000\frac{\text{kg}}{\text{m}^3})(10\text{m/s}^2)(1 \text{ m}) + \frac{1}{2}(1{,}000\frac{\text{kg}}{\text{m}^3})(40 \text{ m/s})^2 \\
&= (10{,}000 \text{ Pa}) + (800{,}000 \text{ Pa}) \\
&= 810{,}000 \text{ Pa} \\
&= 810 \text{ kPa}
\end{aligned}$$

Section II: Free Response

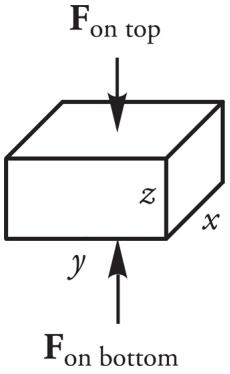
1. (a)

The pressure at the top surface of the block is $P_{\text{top}} = P_{\text{atm}} + \rho_{\text{L}} gh$. Since the area of the top of the block is $A = xy$, the force on the top of the block has magnitude

$$F_{\text{top}} = P_{\text{bottom}} A = \left(P_{\text{atm}} + \rho_{\text{L}} gh\right)xy$$

The pressure at the bottom of the block is $P_{\text{bottom}} = P_{\text{atm}} + \rho_{\text{L}} g\left(h + z\right)$. Since the area of the bottom face of the block is also $A = xy$, the force on the bottom surface of the block has magnitude $F_{\text{bottom}} = P_{\text{bottom}} A = \left[P_{\text{atm}} + \rho_{\text{L}} g\left(h + z\right)\right]xy$

These forces are sketched below:



(b)

Each of the other four faces of the block (left and right, front and back) is at an average depth of $h + \frac{1}{2}z$, so the average pressure on each of these four sides is

$$\overline{P}_{\text{sides}} = P_{\text{atm}} + \rho_{\text{L}} g(h + \frac{1}{2}z)$$

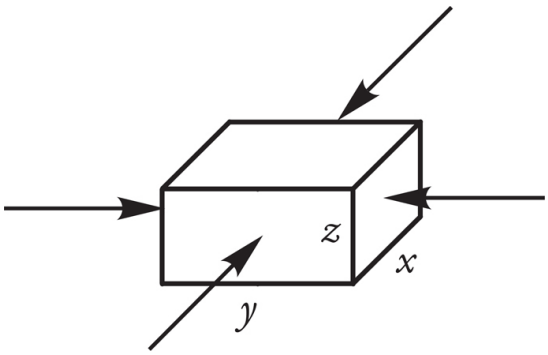
The left and right faces each have area $A = xz$, so the magnitude of the average force on this pair of faces is

$$\overline{F}_{\text{left and right}} = \overline{P}_{\text{sides}} A = [P_{\text{atm}} + \rho_{\text{L}} g(h + \frac{1}{2} z)]xz$$

The front and back faces each have area $A = yz$, so the magnitude of the average force on this pair of faces is

$$\overline{F}_{\text{front and back}} = \overline{P}_{\text{sides}} A = [P_{\text{atm}} + \rho_{\text{L}} g(h + \frac{1}{2} z)]yz$$

These forces are sketched below:



(c)

The four forces sketched in part (b) add up to zero, so the total force on the block due to the pressure is the sum of $F_{\text{on top}}$ and $F_{\text{on bottom}}$; because $F_{\text{on bottom}} > F_{\text{on top}}$, this total force points upward and its magnitude is

$$F_{\text{on bottom}} - F_{\text{on top}} = [P_{\text{atm}} + \rho_{\text{L}} g(h + z)]xy - (P_{\text{atm}} + \rho_{\text{L}} gh)xy = \rho_{\text{L}} gxyz$$

(d)

By Archimedes' Principle, the buoyant force on the block is upward and has magnitude

$$F_{\text{buoy}} = \rho_{\text{L}} V_{\text{sub}} g = \rho_{\text{L}} Vg = \rho_{\text{L}} xyzg$$

This is the same as the result you found in part (c).

(e)

The weight of the block is

$$F_g = mg = \rho_B V g = \rho_B xyzg$$

If F_T is the tension in the string, then the total upward force on the block, $F_T + F_{buoy}$, must balance the downward force, F_g; that is, $F_T + F_{buoy} = F_g$, so

$$F_T = F_g - F_{buoy} = \rho_B xyzg - \rho_L xyzg = xyzg(\rho_B - \rho_L)$$

2. (a)

See the section on Torricelli's Theorem for the derivation of the efflux speed from the hole; applying Bernoulli's Equation to a point on the surface of the water in the tank (Point 1) and a point at the hole (Point 2), the assumption that $v_1 \approx 0$ leads to the result

$$v = \sqrt{2gh}$$

(b)

The initial velocity of the water, as it emerges from the hole, is horizontal. Since there's no initial vertical velocity, the time t required to drop the distance $y = D - h$ to the ground is found as follows:

$$y = \frac{1}{2}gt^2 \quad \Rightarrow \quad D - h = \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \sqrt{\frac{2(D-h)}{g}}$$

Therefore, the horizontal distance the water travels is

$$x = v_x t = \sqrt{2gh} \cdot \sqrt{\frac{2(D-h)}{g}} = 2\sqrt{h(D-h)}$$

(c)

The second hole would be at a depth of $h/2$ below the surface of the water, so the horizontal distance it travels—from the edge of the tank to the point where it hits the ground—is given by the same formula you found in part (b) except it will have $h/2$ in place of h; that is,

$$x_2 = 2\sqrt{\frac{1}{2}h(D-\frac{1}{2}h)}$$

If both streams land at the same point, then the value of x from part (b) is the same as x_2 :

$$\begin{aligned}
2\sqrt{h(D-h)} &= 2\sqrt{\frac{1}{2}h(D-\frac{1}{2}h)} \\
h(D-h) &= \frac{1}{2}h(D-\frac{1}{2}h) \\
D-h &= \frac{1}{2}(D-\frac{1}{2}h) \\
4D-4h &= 2D-h \\
-3h &= -2D \\
h &= \frac{2}{3}D
\end{aligned}$$

(d)

Once again, apply Bernoulli's Equation to a point on the surface of the water in the tank (Point 1) and to a point at the hole (Point 2). Choose the ground level as the horizontal reference level; then $y_1 = D$ and $y_2 = D - h$. If v_1 is the flow speed of Point 1—that is, the speed with which

the water level in the tank drops—and v_2 is the efflux speed from the hole, then, by the Continuity Equation, $A_1v_1 = A_2v_2$, where A_1 and A_2 are the cross-sectional areas at Points 1 and 2, respectively. Therefore, $v_1 = (A_2/A_1)v_2$. Bernoulli's Equation then becomes

$$P_1 + \rho gD + \frac{1}{2}\rho v_1^2 = P_2 + \rho g(D-h) + \frac{1}{2}\rho v_2^2$$

Since $P_1 = P_2 = P_{\text{atm}}$, these terms cancel out; and substituting $v_1 = (A_2/A_1)v_2$, you have

$$\rho gD + \frac{1}{2}\rho\left(\frac{A_2}{A_1}v_2\right)^2 = \rho g(D-h) + \frac{1}{2}\rho v_2^2$$

$$\frac{1}{2}\rho v_2^2\left(\frac{A_2}{A_1}\right)^2 - 1 = -\rho gh$$

$$v_2 = \sqrt{\frac{2gh}{1-\left(\frac{A_2}{A_1}\right)^2}}$$

Now, since $A_1 = \pi R^2$ and $A_2 = \pi r^2$, this final equation can be written as

$$v_2 = \sqrt{\frac{2gh}{1-\left(\frac{r}{R}\right)^4}}$$

[Note that if $r << R$, then $(r/R)^4 \approx 0$, and the equation above reduces to $v_2 = \sqrt{2gh}$, as in part (a).]

3. (a)

Point X is at a depth of h_1 below Point 1, where the pressure is P_1. Therefore, the hydrostatic pressure at X is $P_X = P_1 + \rho_F g h_1$.

(b)

Point Y is at a depth of $h_2 + d$ below Point 2, where the pressure is P_2. The column of static fluid above Point Y contains fluid of density of depth h_2 and fluid of density ρ_F of depth d. Therefore, the hydrostatic pressure at Y is $P_Y = P_2 + \rho_F g h_2 + \rho_V g d$.

(c)

First, notice that Points 1 and 2 are at the same horizontal level; therefore, the heights y_1 and y_2 are the same, and the terms $\rho_F g y_1$ and $\rho_F g y_2$ will cancel out of the equation. Bernoulli's Equation then becomes

$$P_1 + \frac{1}{2}\rho_F v_1^2 = P_2 + \frac{1}{2}\rho_F v_2^2$$

By the Continuity Equation, you have $A_1 v_1 = A_2 v_2$, so $v_1 = (A_2/A_1)v_2$. Therefore,

$$
\begin{aligned}
P_1 - P_2 &= \frac{1}{2}\rho_F v_2^2 - \frac{1}{2}\rho_F v_1^2 \\
&= \frac{1}{2}\rho_F v_2^2 - \frac{1}{2}\rho_F \left(\frac{A_2}{A_1} v_2\right)^2 \\
&= \frac{1}{2}\rho_F v_2^2 \left[1 - \left(\frac{A_2}{A_1}\right)^2\right]
\end{aligned}
$$

(d)

In parts (a) and (b) above, you found that $P_X = P_1 + \rho_F g h_1$ and $P_Y = P_2 + \rho_F g h_2 + \rho_v g d$. Since $P_X = P_Y$, you have

$$P_1 + \rho_F g h_1 = P_2 + \rho_F g h_2 + \rho_V g d$$

so

$$
\begin{aligned}
P_1 - P_2 &= \rho_F g(h_2 - h_1) + \rho_V g d \\
&= \rho_F g(-d) + \rho_V g d \\
&= (\rho_V - \rho_F) g d
\end{aligned}
$$

(e)

In parts (c) and (d), you found two expressions for $P_1 - P_2$. Setting them equal to each other gives

$$\frac{1}{2}\rho_F v_2^2 \left[1-\left(\frac{A_2}{A_1}\right)^2\right] = (\rho_V - \rho_F)gd$$

$$v_2^2 = \frac{\rho_V - \rho_F}{\rho_F} \cdot \frac{2gd}{1-\left(\frac{A_2}{A_1}\right)^2}$$

$$v_2 = \sqrt{\frac{2gd\left(\frac{\rho_V}{\rho_F}-1\right)}{1-\left(\frac{A_2}{A_1}\right)^2}}$$

The flow rate in the pipe is

$$f = A_2 v_2 = A_2 \sqrt{\frac{2gd\left(\frac{\rho_V}{\rho_F}-1\right)}{1-\left(\frac{A_2}{A_1}\right)^2}}$$

Since

$$f = A_2 \sqrt{\frac{2gd\left(\frac{\rho_V}{\rho_F}-1\right)}{1-\left(\frac{A_2}{A_1}\right)^2}} \cdot \sqrt{d}$$

you see that f is proportional to $\sqrt{d}$, as desired.

CHAPTER 4 REVIEW QUESTIONS

Section I: Multiple Choice

1. **B**

   Because the average kinetic energy of a molecule of gas is directly proportional to the temperature of the sample, the fact that the gases are at the same temperature—since they're in the same container at thermal equilibrium—tells you that the molecules have the same average kinetic energy. The ratio of their kinetic energies is therefore equal to 1.

2. **A**

   P–V diagrams have pressure on the vertical axis and volume on the horizontal axis. You know volume is changing, so the graph will have to show a change in the horizontal axis and you can eliminate (B). Then, from the Ideal Gas Law, $PV = nRT$, you see that pressure is unchanged if the volume and temperature are both doubled. The vertical value of the P–V diagram remains constant, resulting in a horizontal line on the graph.

3. **A**

   The work done on the gas during a thermodynamic process is equal to the area of the region in the P–V diagram above the V-axis and below the path the system takes from its initial state to its final state. Since the area below path 1 is the greatest, the work done on the gas during the transformation along path 1 is the greatest.

4. **C**

   During an isothermal change, ΔU is always zero.

5. **B**

There is no way to measure a pressure of 0 Pa, so you can eliminate (A) and (C) as answers. The Ideal Gas Law states $PV = nRT$, so a graph of P vs. n graph will be linear when V and T are both held constant, so you can eliminate (D).

6. **C**

In order for the temperature to remain constant, the entire container needs to be kept in thermal equilibrium with the surroundings. This allows heat to flow into and out of the system and keeps the temperature of the container constant.

7. **A**

By convention, work done *on* the gas sample is designated as positive, so in the First Law of Thermodynamics, $\Delta U = Q + W$, you must write $W = +320$ J. Therefore, $Q = \Delta U - W = 560$ J $- 320$ J $= +240$ J. Positive Q denotes heat *in*.

8. **C**

No work is done during the step from state *a* to state *b* because the volume doesn't change. Therefore, the work done from *a* to *c* is equal to the work done from *b* to *c*. Since the pressure remains constant (this step is isobaric), find that

$W = -P\Delta V = -(3.0 \times 10^5 \text{ Pa})[(10 - 25) \times 10^{-3} \text{ m}^3] = 4{,}500$ J

9. **C**

Choice (A) is wrong because "no heat exchanged between the gas and its surroundings" is the definition of *adiabatic*,

not *isothermal*. Choice (B) cannot be correct since the step described in the question is isothermal; by definition, the temperature does not change. This also eliminates (D) and supports (C). If the sample could be brought back to its initial state *and* have a 100% conversion of heat to work, *that* would violate the Second Law. The Second Law states that heat cannot be completely converted to work with no other change taking place. In this case, there are changes taking place: the pressure decreases and the volume increases.

10. **C**

The Second Law of Thermodynamics indicates that energy will flow from a hot object to a cool object, making the final temperature of the hot object cooler and the cold object warmer.

Section II: Free Response

1. (a)

First, calculate ΔU_{acb}. Using path *acb*, the question tells you that $Q = +70$ J and $W = -30$ J (W is negative here because it is the *system* that does the work). The First Law, $\Delta U = Q + W$, tells you that $\Delta U_{acb} = +40$ J. Because $\Delta U_{a \to b}$ does not depend on the path taken from a to b, you must have $\Delta U_{ab} = +40$ J, and $\Delta U_{ba} = -\Delta U_{ab} = -40$ J. Thus, -40 J $= Q_{ba} + W_{ba}$, where Q_{ba} and W_{ba} are the values along the curved path from b to a. Since $Q_{ba} = -60$ J, it follows that $W_{ba} = +20$ J. Therefore, the surroundings do 20 J of work on the system.

(b)

Again, using the fact that $\Delta U_{a \to b}$ does not depend on the path taken from a to b, you know that $\Delta U_{adb} = +40$ J, as computed above. Writing $\Delta U_{adb} = QW_{adb} + W_{adb}$, if $W_{adb} = -10$ J, it follows that $Q_{adb} = +50$ J. That is, the system absorbs 50 J of heat.

(c)

For the process *db*, there is no change in volume, so $W_{db} = 0$. Therefore, $\Delta U_{db} = Q_{db} + W_{db} = Q_{db}$. Now, since $U_{ab} = +40$ J, the fact that $U_a = 0$ J implies that $U_b = 40$ J, so $\Delta U_{ab} = U_b - U_4 = 40$ J $- 30$ J $= 10$ J. Thus, $Q_{db} = 10$ J. Now let's consider the process *ad*. Since $W_{abd} = W_{ad} + W_{db} = W_{ad} + 0 = W_{ad}$, the fact that $W_{adb} = -10$ J [computed in part (b)] tells you that $W_{ad} = -10$ J. Because $\Delta U_{ad} = U_d - U_a = 30$ J,

it follows from $\Delta U_{ad} = Q_{ad} + W_{ad}$ that $Q_{ad} = \Delta U_{ad} - W_{ad} = 30\text{ J} - (-10\text{ J}) = 40\text{ J}$.

(d)

The process *adbca* is cyclic, so ΔU is zero. Because this cyclic process is traversed *counterclockwise* in the *P–V* diagram, you know that *W* is *positive*. Then, since $\Delta U = Q + W$, it follows that *Q* must be negative.

2. (a)

(i)

Use the Ideal Gas Law:

$$T_a = \frac{P_a V_a}{nR} = \frac{(2.4\times10^5\text{ Pa})(12\times10^{-3}\text{ m}^3)}{(0.4\text{ mol})(8.31\text{ J/mol}\cdot\text{K})} = 870\text{ K}$$

(ii)

Since state *b* is on the isotherm with state *a*, the temperature of state *b* must also be 870 K.

(iii) Use the Ideal Gas Law:

$$T_c = \frac{P_c V_c}{nR} = \frac{(0.6\times10^5\text{ Pa})(12\times10^{-3}\text{ m}^3)}{(0.4\text{ mol})(8.31\text{ J/mol}\cdot\text{K})} = 220\text{ K}$$

(b)

There is heat flow into the system during the step *ab*. As the volume expands, the system does work on the surroundings (uses up some of its internal energy). At a

result, the system would drop in temperature if no heat flowed into the gas to make up for this use of energy. During an isothermal process, $\Delta T = 0$ J, so the change internal energy of the gas, $\Delta U = \frac{3}{2} N k_b \Delta T$ is zero. From the First Law of Thermodynamics, $\Delta U = W + Q$, any energy that is used by expanding (work done by the system on the surroundings) must be replaced by heat flowing into the system.

(c)

Using the equation given, find that

$$W_{ab} = -nRt \cdot \ln \frac{V_b}{V_a} = -(0.4 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(870 \text{ K}) \cdot \ln \frac{48 \times 10^{-3} \text{ m}^3}{12 \times 10^{-3} \text{ m}^3} = -4{,}000 \text{ J}$$

(d)

The total work done over the cycle is equal to the sum of the values of the work done over each step:

$$\begin{aligned} W_{\text{cycle}} &= W_{ab} + W_{bc} + W_{ca} \\ &= W_{ab} + W_{bc} \\ &= (-4{,}000 \text{ J}) + (2{,}200 \text{ J}) \\ &= -1{,}800 \text{ J} \end{aligned}$$

CHAPTER 5 REVIEW QUESTIONS

Section I: Multiple Choice

1. **D**

Electrostatic force obeys an inverse-square law: $F_E \propto 1/r^2$. Therefore, a plot should be made of F verses the inverse square of r to get a line.

2. **C**

The strength of the electric force is given by kq^2/r^2, and the strength of the gravitational force is Gm^2/r^2. Since both of these quantities have r^2 in the denominator, you simply need to compare the numerical values of kq^2 and Gm^2. There's no contest. Since

$kq^2 = (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1\ \text{C})^2 = 9 \times 10^9\ \text{N}\cdot\text{m}^2$

and

$Gm^2 = (6.7 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1\ \text{kg})^2 = 6.7 \times 10^{-11}\ \text{N}\cdot\text{m}^2$

you can see that $kq^2 >> Gm^2$, so F_E is much stronger than F_G.

3. **C**

If the net electric force on the center charge is zero, the electrical repulsion by the $+2q$ charge must balance the electrical repulsion by the $+3q$ charge:

$$\frac{1}{4\pi\varepsilon_0}\frac{(2q)(q)}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{(3q)(q)}{y^2} \quad \Rightarrow \quad \frac{2}{x^2} = \frac{3}{y^2} \quad \Rightarrow \quad \frac{y^2}{x^2} = \frac{3}{2} \quad \Rightarrow \quad \frac{y}{x} = \sqrt{\frac{3}{2}}a = \frac{F_E}{m} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{mr^2}$$

4. **D**

Since P is equidistant from the two charges, and the magnitudes of the charges are identical, the strength of the electric field at P due to $+Q$ is the same as the strength of the electric field at P due to $-Q$. The electric field vector at P due to $+Q$ points away from $+Q$, and the electric field vector at P due to $-Q$ points toward $-Q$. Since these vectors point in the same direction, the net electric field at P is (E to the right) + (E to the right) = ($2E$ to the right).

5. **C**

The acceleration of the small sphere is

$$a = \frac{F_E}{m} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{mr^2}$$

As r increases (that is, as the small sphere is pushed away), a decreases. However, since a is always positive, the small sphere's speed, v, is always increasing.

6. **B**

Since $\mathbf{F}_E$ (on q) = $q\mathbf{E}$, it must be true that $\mathbf{F}_E$ (on $-2q$) = $-2q\mathbf{E} = -2\mathbf{F}_E$.

7. **C**

All excess electric charge on a conductor resides on the outer surface.

Section II: Free Response

1. (a)

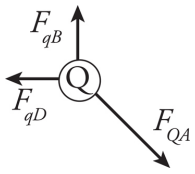
(i)

The force from the charge at A will be attractive and must act at a 45° angle because the force acts along the bisector of the square. The balancing forces from the charges q will be at right angles to one another, and so each must balance a component for the force from Q alone. Since $\sin(45°) = \cos(45°)$, the forces from each q must be equal, and therefore the charges must also be equal.

(ii)

In order for the force on C to be 0, the repulsive force from charge A must be balanced by an attractive force from each q, so the charges at positions B and D must have a negative sign.

A free body diagram of the charge at point C is drawn below.



The distance between points opposite vertices of a square of side length s is $\sqrt{2}s$. You can look at either the x- or y-components because the equations each direction yields

are identical. From the x direction, $F_Q \sin(45°) = F_{qD}$.

Therefore, $F_{qD} = k\dfrac{Q^2}{(\sqrt{2}s)^2} \times \dfrac{\sqrt{2}}{2}$.

$$F_{qD} = k\frac{\sqrt{2}Q^2}{4s^2}$$

Now that you know the force from the charge at D, use Coulomb's Law to determine q.

$$F_{qD} = k\frac{Qq}{s^2} = k\frac{\sqrt{2}Q^2}{4s^2}$$

$$Qq = \frac{\sqrt{2}Q^2}{4}$$

$$q = \frac{\sqrt{2}Q}{4}$$

(b)

Within the square, the field from the charge Q at position A will point into the fourth quadrant (the x-component will be positive and the y-component will be negative). With the charges positive, within the square the field from D will point into the first quadrant (the x-component will be positive and the y-component will be positive) and from B will point into the third quadrant (the x-component will be negative and the y-component will be negative). To cancel the fields, the x-coordinate would have to be closer to point B than point D and the y-coordinate would have to be closer to D than B. Some point which satisfies these two requirements will have a field of 0 *N/C*.

(c)

At the center of the square, the net electric field is the vector sum of the contributions from the four charges. The field from the charges at A and C will always balance one another as long as those charges have the same sign, regardless of that sign. First, the charges are equidistant from the center of the square, so *r* will be equal regardless of the sign or magnitude of the charges. If they are both positive, the field from A will be in the fourth quadrant and the field from C will be in the second quadrant. If both charges are negative, then the field from C will be in the fourth quadrant and the field from A will be in the second quadrant. Therefore, regardless of the magnitude of sign of the charges, the contributions from the charges *Q* will cancel one another. An identical argument can be made for the charges *q* showing that their fields will also cancel with one another. Therefore, the net field at the center of the square must be 0 *N/C* regardless of the signs or magnitudes of *Q* or *q*.

2. (a)

The electric force on charge 1 is $F_1 = k\dfrac{Q \cdot 2Q}{(a + 2a)^2} = k\ \dfrac{2Q^2}{9a^2}$. The force between the charges is repulsive, so the charge at 1 is pushed upward.

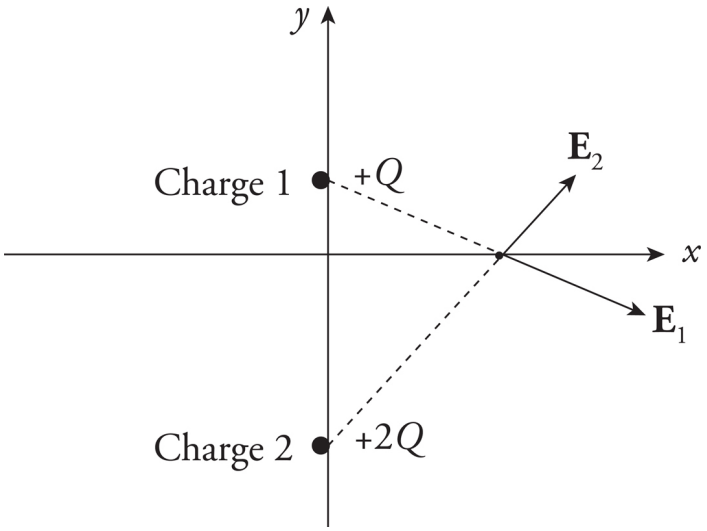
(b)

$\overrightarrow{E_{net}} = \overrightarrow{E_1} + \overrightarrow{E_2} = k\dfrac{Q}{a^2}\,\text{down} + k\dfrac{2Q}{(2a)^2}\,\text{up}$. Making upward positive results in $E_{net} = k\dfrac{Q}{2a^2} - k\dfrac{Q}{a^2} = -k\dfrac{Q}{a^2}$ up.

The net field is $k\dfrac{Q}{2a}$ in the downward direction.

(c)

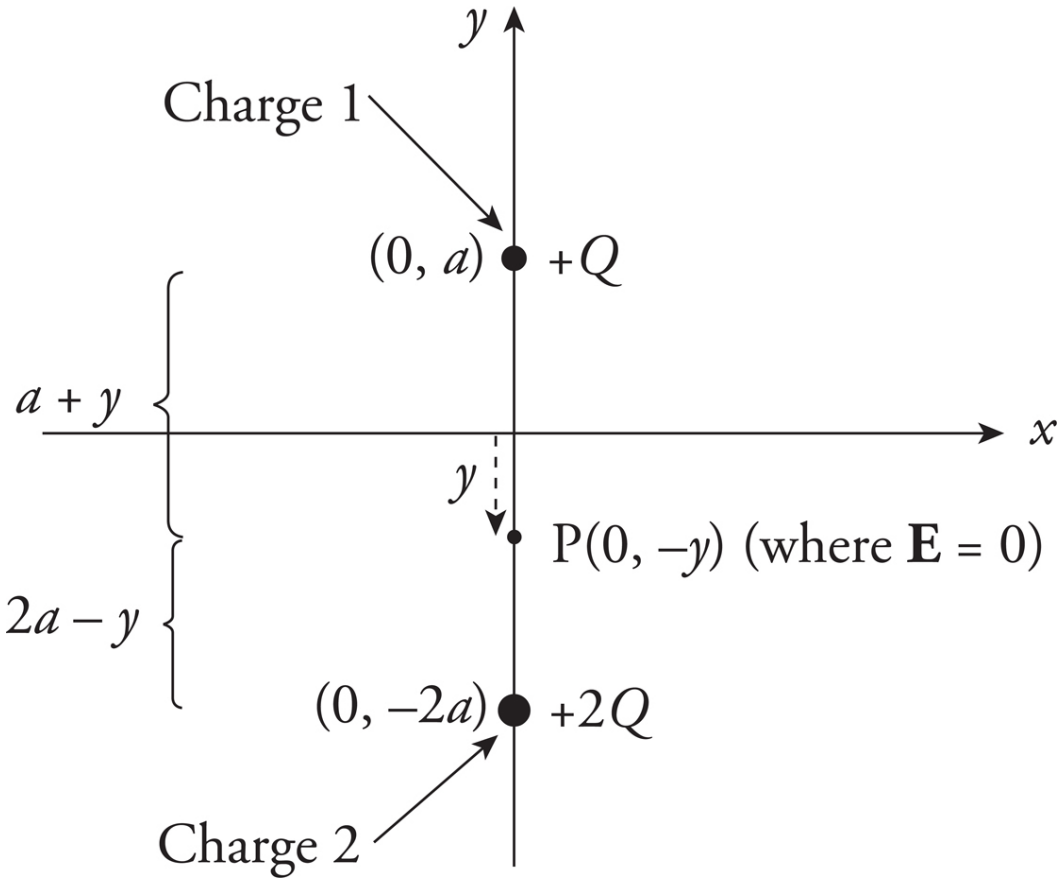
No. The only point on the *x*-axis where the individual electric field vectors due to each of the two charges point in exactly opposite directions is the origin (0, 0). But at that point, the two vectors are not equal and thus do not cancel.



Therefore, at no point on the *x*-axis could the total electric field be zero.

(d)

Yes. There will be a Point P on the *y*-axis between the two charges,



where the electric fields due to the individual charges will cancel each other out.

$$
\begin{aligned}
E_1 &= E_2 \\
\frac{1}{4\pi\varepsilon_0}\frac{Q}{(a+y)^2} &= \frac{1}{4\pi\varepsilon_0}\frac{2Q}{(2a-y)^2} \\
\frac{1}{(a+y)^2} &= \frac{2}{(2a-y)^2} \\
(2a-y)^2 &= 2(a+y)^2 \\
0 &= y^2 + 8ay - 2a^2 \\
y &= \frac{-8a \pm \sqrt{(8a)^2 - 4(-2a^2)}}{2} \\
&= (-4 \pm 3\sqrt{2})a
\end{aligned}
$$

Disregarding the value $y = (-4 - 3\sqrt{2})a$ (because it would place the point P below Charge 2 on the y-axis, where the

electric field vectors do not point in opposite directions), you find that **E** = **o** at the point P = (0, $-y$) = (0, $(4-3\sqrt{2})a$).

CHAPTER 6 REVIEW QUESTIONS

Section I: Multiple Choice

1. **B**

For a point particle, which a uniform sphere will behave like, the relationship between the electric potential and the distance is $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$, so with V on the vertical axis and $\frac{1}{r}$ on the horizontal axis, a graph will be produced which is a straight line whose slope is $\frac{1}{4\pi\varepsilon_0}q$.

2. **D**

Equipotential curves for a parallel plate capacitor are horizontal lines. Both (A) and (B) are at the same potential as (A), so their potential difference is 0 V. As one moves closer to the $+Q$ plate, the potential will increase, so (D) is correct.

3. **B**

Use the definition $\Delta V = -W_E/q$. If an electric field accelerates a negative charge doing positive work on it, then $W_E > 0$. If $q < 0$, then $-W_E/q$ is positive. Therefore, ΔV is positive, which implies that V increases.

4. **D**

By definition,

$V_{A\to B} = \Delta U_E/q$, so $V_B - V_A = \Delta U_E/q$

5. **B**

Because E is uniform, the potential varies linearly with distance from either plate ($\Delta V = Ed$). Since Points 2 and 4 are at the same distance from the plates, they lie on the same equipotential. (The equipotentials in this case are planes parallel to the capacitor plates.)

6. **B**

The charge Q must be positive because the potential decreases as the distance from the charge increases. A negative charge will be attracted to a positive charge. This means that the electric field of Q would do positive work if q were brought closer, and it would do negative work if q were moved farther away.

Section II: Free Response

1. (a)

Greatest A and B A A, B, and C All four Least

The work required to move a charge through an existing electric field can be related to the voltage of that field at the initial and final positions of the charge. All voltages are defined to be 0 when the charges are very far away from one another, so V_i is always 0 V. Charge A is placed first and there is no electric field that charge A is moved through, so the charge distribution has no energy. At location B, in the presence of charge A, there is an electric field and we can calculate the voltage at point B as $V_B = \frac{+kQ}{s}$ Moving the charge to point B adds $W = \frac{kQ^2}{s}$ to the charge distribution. At location C, in the presence of charges at A

and B, calculate the voltage as the sum of the contributions from A and B as $V_C = \frac{kQ}{\sqrt{2}s} + \frac{kQ}{s}$. Charge C is negative, so putting it at position C removed $W = \frac{kQ^2}{s} + \frac{kQ^2}{\sqrt{2}s}$ making the energy of the distribution $-\frac{kQ^2}{\sqrt{2}s} = -\frac{\sqrt{2}kQ^2}{2s}$. Finally, the voltage as position D from the charges at A, B, and C, respectively, is $V_D = \frac{kQ}{s} + \frac{kQ}{\sqrt{2}s} - \frac{kQ}{s} = \frac{kQ}{\sqrt{2}s}$. Charge D is negative, so putting it at position D removed $W = \frac{kQ^2}{\sqrt{2}s}$ from the distribution. The energy in the distribution of all four charges is $U_E = -\frac{2kQ^2}{\sqrt{2}s} = -\frac{2\sqrt{2}kQ^2}{2s}$.

(b)

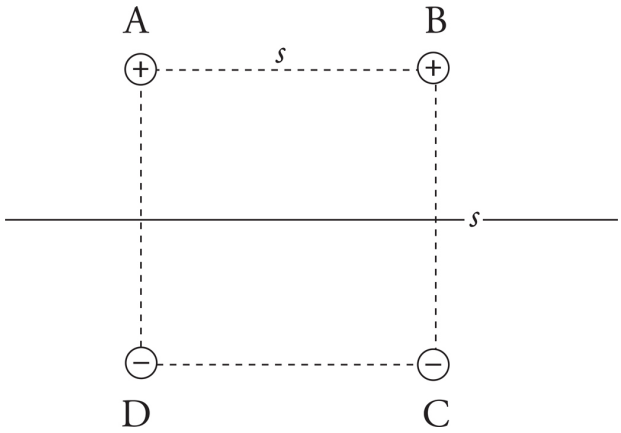
At the exact center of the square, this distance from each charge is *d*, where from the Pythagorean Theorem, $d^2 = \left(\frac{s}{2}\right)^2 + \left(\frac{s}{2}\right)^2$, so $d = \frac{\sqrt{2}s}{2}$. The voltage at the center is then

the sum of the voltages from each charge:

$$V_{center} = V_A + V_B + V_C + V_D = \frac{+kQ}{d} + \frac{+kQ}{d} + \frac{-kQ}{d} + \frac{-kQ}{d} = 0 \text{ V}.$$

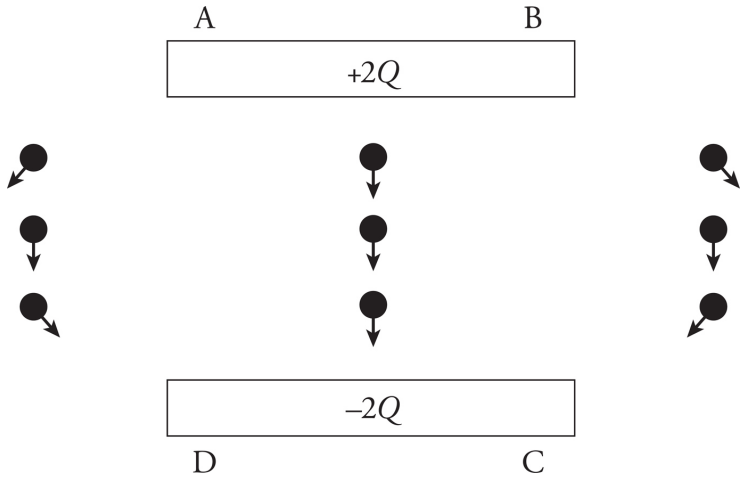
(c)

The equipotential will be described as always the same distance from the pair at AD as well as the same distance from the pair at BD. Thus, the equipotential line is a horizontal line through the square.

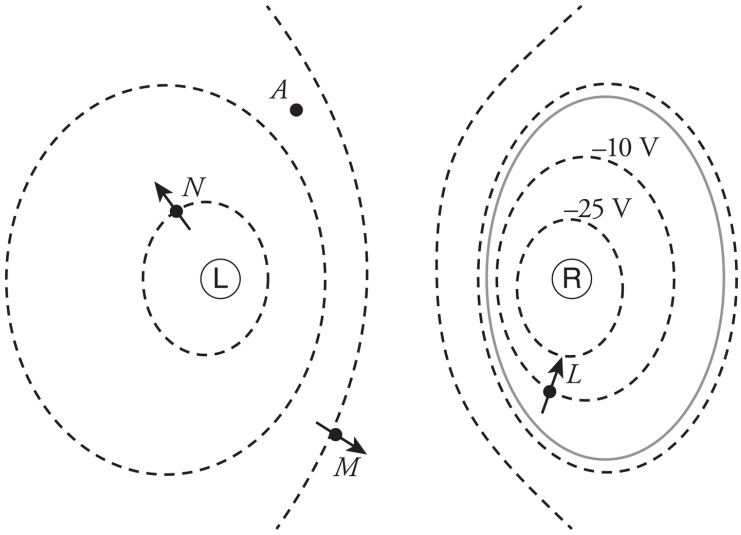


(d)

The electric field along the center line will be uniform because any diminishment from the influence of the top plate is counteracted by an increased influence from the lower plate. However, at a position not between the plates, this is no longer true, and so for the bottom dots, the lower plate is more influential. This causes the electric field at these locations to tend to point toward the negatively charged plate.



2.



(a)

Because the potential difference between adjacent lines is $\Delta V = 15$ V and the potential is going from −25 V to −10 V on the two lines closest to R, the 0 V isoline must be in between the line labeled −10 V and the next line in the image. The light gray solid line is the line where the potential is 0 V.

(b)

42.5 V.

(c)

(i)

The potential is increasing as the isolines get father away from R, so R must be negative and L must be positive.

(ii)

R carries the greater magnitude of charge as the isolines are closer together in the neighborhood of R, indicating a stronger magnitude of electric field.

(d)

(i)

See image above.

(ii)

L > M > N. At L the isolines are closest together, and at N they are farthest apart.

(e)

L carries positive charge, and potential is larger (relative to positions near negative charges) at positions near positive charges. Making L negatively charged would therefore make the potential at point N lower than it is when L is positive.

3. (a)

Outside the sphere, the sphere behaves as if all the charge were concentrated at the center. Inside the sphere, the electrostatic field is zero:

$$E(r) = \begin{cases} 0 & (r < a) \\ \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2} & (r > a) \end{cases}$$

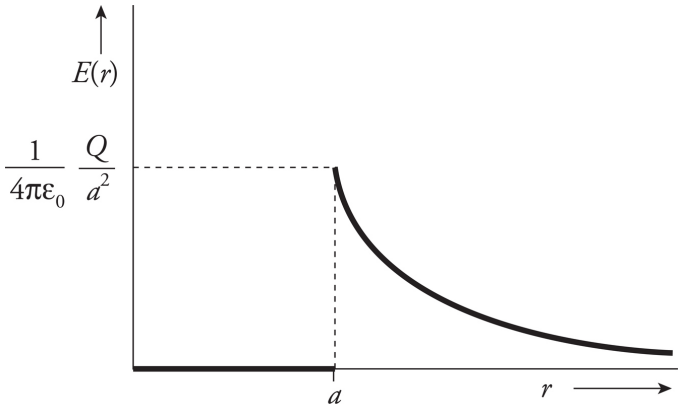
(b)

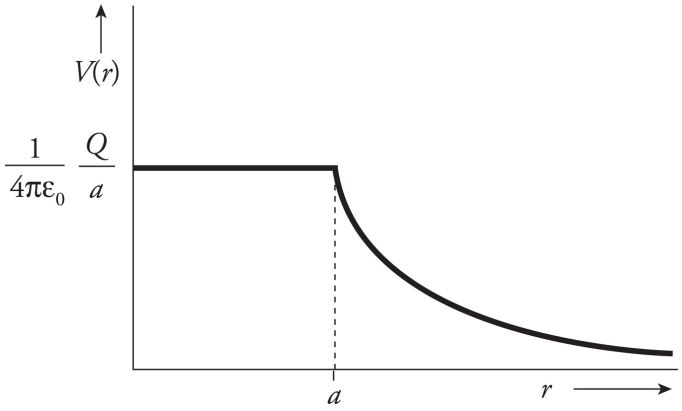
On the surface and outside the sphere, the electric potential is $(\frac{1}{4}\pi\varepsilon_0)(Q/r)$. Within the sphere, V is constant (because E = 0) and equal to the value on the surface. Therefore,

$$V(r) = \begin{cases} \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{a} & (r \le a) \\ \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r} & (r > a) \end{cases}$$

(c)

See diagrams:


E(r)
1/4πε0 Q/a2
a
r


V(r)
1/4πε0 Q/a
a
r

CHAPTER 7 REVIEW QUESTIONS

Section I: Multiple Choice

1. **A**

Let ρ_S denote the resistivity of silver and let A_S denote the cross-sectional area of the silver wire. Then

$$R_B = \frac{\rho_B L}{A_B} = \frac{(5\rho_S)L}{4^2 A_S} = \frac{5}{16}\frac{\rho_S L}{A_S} = \frac{5}{16}R_S$$

2. **C**

The equation $I = V/R$ implies that increasing V by a factor of 2 will cause I to increase by a factor of 2.

3. **D**

In order for one resistor to have current flowing through it immediately when the circuit is connected, it must be in series with the capacitor as all of the current will flow to the capacitor immediately upon constructing the circuit. In order for there to be current in the other resistor after a long time, the second resistor should be in parallel to the branch with the capacitor and the first resistor.

4. **C**

The resistivity ρ is found from the resistance $R = \rho L/A$. To find the resistance, Ohm's Law $V = IR$ is needed. In addition to V and L, you need to find both I and A. The ammeter will yield I, and the diameter of the wire will allow you to determine A.

5. **C**

Because the voltage has been established for a long time, the capacitor will be full, so it behaves like an open switch. This puts 4 Ω resistor and the top 3 Ω resistor in series with one another, making an equivalent resistor of 7 Ω. That combination in parallel with the other 3 Ω resistor, so

$$\frac{1}{Req} = \frac{1}{7\,\Omega} + \frac{1}{3\,\Omega} \rightarrow \frac{1}{R_{eq}} = \frac{0.48}{\Omega} \rightarrow R_{eq} = 2.1\,\Omega.$$

6. **C**

If each of the identical bulbs has resistance R, then the current through each bulb is ε/R. This is unchanged if the middle branch is taken out of the parallel circuit. (What *will* change is the total amount of current provided by the battery.)

7. **B**

The three parallel resistors are equivalent to a single 2 Ω resistor, because $\frac{1}{8\,\Omega} + \frac{1}{4\,\Omega} + \frac{1}{8\,\Omega} = \frac{1}{2\,\Omega}$. This 2 Ω resistance is in series with the given 2 Ω resistor, so their equivalent resistance is 2 Ω + 2 Ω = 4 Ω. Therefore, three times as much current will flow through this equivalent 4 Ω resistance in the top branch as through the parallel 12 Ω

resistor in the bottom branch, which implies that the current through the bottom branch is 3 A, and the current through the top branch is 9 A. The voltage drop across the 12 Ω resistor is therefore $V = IR$ = (3 A)(12 Ω) = 36 V.

8. **B**

Ohm's Law states that $V = IR$, but with an internal resistance in the battery, the total resistance in the circuit is $R_{resistor} + R_{internal}$. Therefore, V_{total}. = $I(R_{resistor} + R_{internal})$. The voltage drop across the resistor is $IR_{resistor}$, so when you isolate the voltage drop across the resistor, you get a line with a negative intercept of $IR_{internal}$.

9. **D**

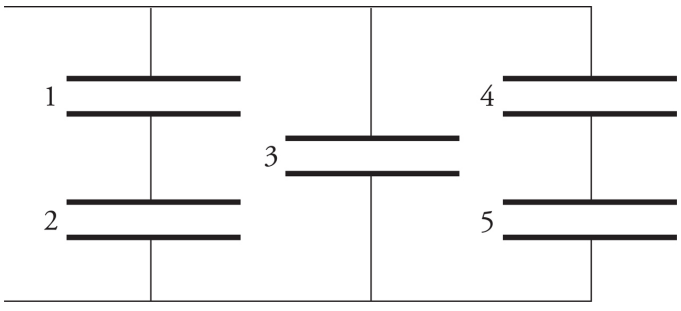
The equation $P = I^2R$ gives

P = (0.5 A)^2(100 Ω) = 25 W = 25 J/s

Therefore, in 20 s, the energy dissipated as heat is

$E = Pt$ = (25 J/s)(20 s) = 500 J

10. **D**



Capacitors 1 and 2 are in series, so their equivalent capacitance is $C_{1\text{-}2} = C/2$. (This is obtained from the equation $1/C_{1\text{-}2} = 1/C_1 + 1/C_2 = 1/C + 1/C = 2/C$.) Capacitors 4 and 5 are also in series, so their equivalent capacitance is $C_{4\text{-}5} = C/2$. The capacitances $C_{1\text{-}2}$, C_3, and $C_{4\text{-}5}$ are in parallel, so the overall equivalent capacitance is $(C/2) + C + (C/2) = 2C$.

Section II: Free Response

1. (a)

The two parallel branches, the one containing the 40 Ω resistor and the other a total of 120 Ω, is equivalent to a single 30 Ω resistance. This 30 Ω resistance is in series with the three 10 Ω resistors, giving an overall equivalent circuit resistance of 10 Ω + 10 Ω + 30 Ω + 10 Ω = 60 Ω. Therefore, the current supplied by the battery is $I = V/R$ = (120V)/(60) = 2 A, so it must supply energy at a rate of $P = IV$= (2 A)(120 V) = 240 W.

(b)

Since three times as much current will flow through the 40 Ω resistor as through the branch containing 120 Ω of resistance, the current through the 40 Ω resistor must be 1.5 A.

(c)

(i)

$V_a - V_b = IR_{20} + IR_{100}$ = (0.5 A)(20 Ω) + (0.5 A)(100 Ω) = 60 V.

(ii)

Point *a* is at the higher potential (current flows from high to low potential).

(d)

Using the equation $R = \rho L/A$, with $A = \pi r^2$, you get

$$R = \frac{\rho L}{\pi r^2} \quad \Rightarrow \quad r = \sqrt{\frac{\rho L}{\pi R}} = \sqrt{\frac{(0.45\ \Omega \cdot \text{m})(0.04\ \text{m})}{\pi (100\ \Omega)}} = 0.0076\ \text{m} = 7.6\ \text{mm}$$

(e)

In order for there to be no current flowing out of the battery after the capacitor is fully charged, the capacitor would have to be in series with the battery. Then, once the capacitor is full, it will have the same potential drop as the battery supplies and the current in the circuit will go to 0 A. This happens if you replace any of the 10 Ω resistors with a capacitor.

2. (a)

There are many ways to solve this problem. If you notice that each of the three branches of the parallel section is 60 Ω, then they must all have the same current flowing through them. The currents through the 20 Ω, 40 Ω, and other 60 Ω resistors are all 0.5 A.

If you had not noticed this, you would have used Ohm's Law to determine the voltage across the resistors and proceeded from there (see part b below). Because 0.5 A goes through each of the three pathways, Kirchhoff's Junction Rule states that the current that must have come though the 10 Ω resistor is 1.5 A.

(b)

The voltage across the 60 Ω resistor is given by Ohm's Law. Because $V = IR$, V = (0.5 A)(60 Ω) = 30 V. All three parallel branches must have the same voltage across them, so the other 60 Ω resistor also has 30 V across it, and the combination of the 20 Ω and 60 Ω resistor must also be 30 V. To determine the voltage across the 20 Ω and 40 Ω resistor, you can rely on the previously solved currents of

0.5 A and Ohm's Law to yield $V = IR$ = (0.5 A)(20 Ω) = 10 V and V = (0.5 A)(40 Ω) = 20 V.

You also could have used the ratio of the resistors. That is, you know the two voltages must sum to 30 V and the voltage drop across the 40 Ω must be twice the amount across the 20 Ω. The voltage across the 10 Ω resistor can be found using Ohm's Law: V = (1.5 A)(10 Ω) = 15 V.

(c)

The equivalent resistance of the circuit can be solved either by adding the resistances or by using Ohm's Law.

If you want to add resistances, start by summing the 20 Ω

and 40 Ω resistors to get 60 Ω. Then add the three parallel

branches usin $\frac{1}{R_P} = \sum_i \frac{1}{R_i}$ or $\frac{1}{R_P} = \sum \frac{1}{60\ \Omega} + \frac{1}{60\ \Omega} + \frac{1}{60\ \Omega}$,

which becomes 20 Ω. Then adding this section in series to

the 10 Ω resistor to get

$R_p = R_1 + R_2$ = 10 Ω + 20 Ω = 30 Ω.

You could have also realized that the total voltage drop

across the battery is 45 V (15 V across the 10 Ω resistor and

30 V across the parallel branch). Using Ohm's Law again,

$R_{\text{eq}} = \frac{V_B}{I_B}$ or $R_{\text{eq}} = \frac{45\,\text{V}}{1.5\text{A}}$ = 30 Ω.

(d)

(i)

The current will be less than 0.5A. The capacitor will draw all the current from the parallel branches the instant it begins to fill. Thus, the remaining 60 Ω resistor will have a current of 0 A when this happens.

(ii)

The current will be greater than 0.5A. The full capacitor acts like a resistor with infinite resistance, which essentially removes a parallel branch from the circuit. Removing a parallel branch lowers the current throughout the circuit, so it lowers the voltage drop across the 10 Ω resistor and increases the voltage drop across the remaining parallel branches. Thus, it increases the current through the remaining 60 Ω resistor.

CHAPTER 8 REVIEW QUESTIONS

Section I: Multiple Choice

1. **C, D**

   Choice (A) is false. The magnetic field lines due to a current-carrying wire encircle the wire in closed loops. Choice (B) is false, since the magnetic force is always perpendicular to the charged particle's velocity vector, it cannot do work on the charged particle; therefore, it cannot change the particle's kinetic energy. Choice (C) is true. If the charged particle's velocity is parallel (or anti-parallel) to the magnetic field lines, then the particle will feel no magnetic force. Choice (D) is also true. Magnetic fields are created by moving charges, $F = qvB \sin \theta$.

2. **C**

   Current carrying wires produce fields which are circles around the wire. The direction of the field is found from the right-hand rule. Choice (A) is false because perpendicular wires will produce perpendicular fields, which will not be able to cancel out. It is not necessary to make solenoids, so (B) is also false. To get the fields to cancel out at a location between the wires, they must carry current in the same direction, so (C) is correct.

3. **B**

   All of the new experimental set-ups have the same magnitude of force as the original. In order to have the new experiment result in the same magnitude of the force, $F_B = |q||v||B| \sin(\theta)$ must be unchanged. Changing the signs of q, v, or B do not change the strength of the force. Rotating v and B by the same amount leaves θ unchanged, and rotating B by 180° results in the same value for $\sin(\theta)$. Thus, the invalid experiment must have a different

direction of the magnetic force. Using the right-hand rule, rotating your thumb and fingers by 90° into the plane of the paper causes the magnetic force to rotate from the original direction of into the plane (recalling q is negative) to finally point to the right.

4. **C**

Since $\mathbf{F}_B$ is always perpendicular to **v**, **v** cannot be upward or downward in the plane of the page; this eliminates (A) and (B). Because the charge is positive, the direction of $\mathbf{F}_B$ will be the same as the direction of $\mathbf{v} \times \mathbf{B}$. In order for $\mathbf{v} \times \mathbf{B}$ to be downward in the plane of the page, the right-hand rule implies that **v** must be out of the plane of the page.

5. **D**

The magnetic field strength for a current carrying wire is given by $B = \frac{\mu_0}{2\pi}\frac{I}{r}$. A graph of B versus I will give a line with a slope of $\frac{\mu_0}{2\pi}\frac{1}{r}$.

6. **C**

Relative to a current carrying wire, the measured value of the magnetic field is directly proportional to the current and inversely proportional to this distance the measurement is taken. Therefore, a large current at a small distance will result in the largest measured magnetic field value.

7. **D**

By Newton's Third Law, neither (A) nor (B) can be correct.

Also, as you learned in Chapter 8, Example 9, if two parallel wires carry current in the same direction, the magnetic force between them is attractive; this eliminates (C). Therefore, the answer must be (D). The strength of the magnetic field at a distance r from a long, straight wire carrying a current I_1 is given by the equation $B_1 = (\mu_0/2\pi)(I_1/r)$. The magnetic force on a wire of length ℓ carrying a current I through a magnetic field $\mathbf{B}$ is $I(\ell \times \mathbf{B})$, so the force on Wire #2 (F_{B2}) due to the magnetic field of Wire #1 (B_1) is $F_{B2} = I_2 \ell B_1 = I_2 \ell \frac{\mu_0}{2\pi}\frac{I_1}{r}$, which implies

$$\frac{F_{B2}}{\ell} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r} = \frac{(4\pi \times 10^{-7}\ \text{N/A}^2)}{2\pi}\frac{(5\ \text{A})(10\ \text{A})}{0.01\ \text{m}} = 0.001\ \text{N/m}$$

8. **D**

The strength of the magnetic field at a distance r from a long, straight wire carrying a current I is given by the equation $B = (\mu_0/2\pi)(I/r)$. Therefore, the strength of the

magnetic field at Point P due to either wire is $B = (\mu_0/2\pi)(I/\frac{1}{2}d)$. By the right-hand rule, the direction of the magnetic field at P due to the top wire is into the plane of the page and the direction of the magnetic field at P due to the bottom wire is out of the plane of the page. Since the two magnetic field vectors at P have the same magnitude and opposite directions, the net magnetic field at Point P is zero.

9. **C**

Use the right-hand rule for wires. If you point your thumb to the right and wrap your fingers along the wire, you will note that the magnetic field goes into the page when you are below the wire and comes out of the page above the wire. This allows you to eliminate (A) and (B). Because $B = \frac{\mu_o I}{2\pi r}$, the closer you are to the wire, the stronger the magnetic field. Choice (C) is closer, so it is the correct answer.

10. **C**

Magnetic fields point from north to south. Therefore, the magnetic field between the two magnets is toward the right of the page. Use the right-hand rule. Because the *B* field is to the right and the charges through the wire flow to the bottom of the page, the force must be out of the page.

Section II: Free Response

1. (a)

The acceleration of an ion of charge q is equal to F_E/m. The electric force is equal to qE, where $E = V/d$. Therefore, $a = qV/(dm)$.

(b) Using $a = qV/(dm)$ and the equation $v^2 = v_0^2 + 2ad = 2ad$, you get

$$v^2 = 2\frac{qV}{dm}d \quad \Rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

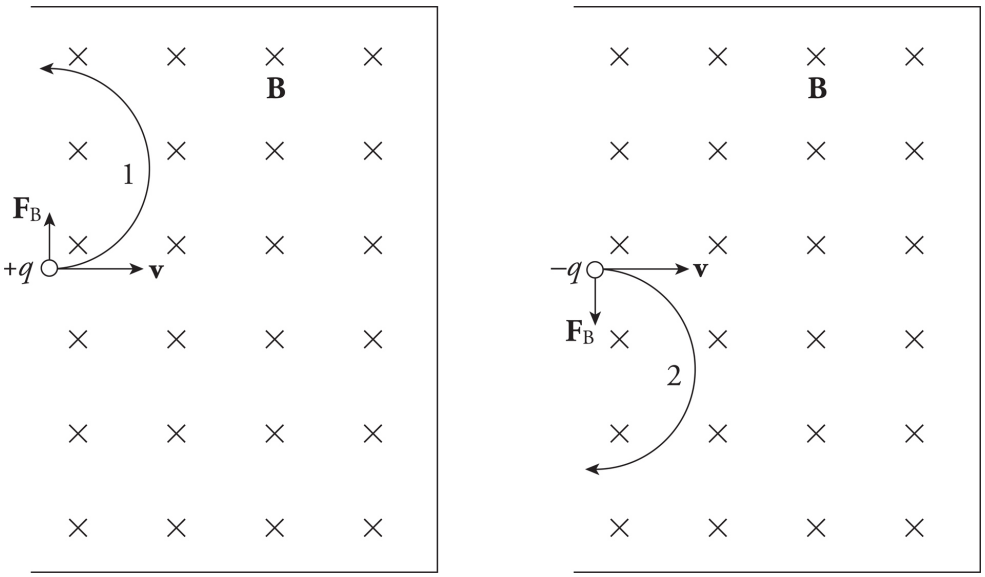
As an alternate solution, notice that the change in the electrical potential energy of the ion from the source S to the entrance to the magnetic-field region is equal to qV; this is equal to the gain in the particle's kinetic energy.

Therefore,

$$qV = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

(c)

(i) and (ii) Use the right-hand rule. Since **v** points to the right and **B** is into the plane of the page, the direction of **v** × **B** is upward. Therefore, the magnetic force on a positively charged particle (cation) will be upward, and the magnetic force on a negatively charged particle (anion) will be downward. The magnetic force provides the centripetal force that causes the ion to travel in a circular path. Therefore, a cation would follow Path 1 and an anion would follow Path 2.



(d)

Since the magnetic force on the ion provides the centripetal force,

$$qvB = \frac{mv^2}{r} \quad \Rightarrow \quad qvB = \frac{mv^2}{\frac{1}{2}y} \quad \Rightarrow \quad m = \frac{qBy}{2v}$$

Now, by the result of part (b),

$$m = \frac{qBy}{2\sqrt{\frac{2qV}{m}}} \quad \Rightarrow \quad m^2 = \frac{q^2B^2y^2}{\frac{8qV}{m}} \quad \Rightarrow \quad m^2 = \frac{mq^2B^2y^2}{8qV} \quad \Rightarrow \quad m = \frac{qB^2y^2}{8V}$$

(e)

Since the magnetic force cannot change the speed of a charged particle, the time required for the ion to hit the photographic plate is equal to the distance traveled (the length of the semicircle) divided by the speed computed in part (b):

$$t = \frac{s}{v} = \frac{\pi \cdot \frac{1}{2} y}{\sqrt{\frac{2qV}{m}}} = \frac{1}{2} \pi y \sqrt{\frac{m}{2qV}}$$

(f)

Since the magnetic force $\mathbf{F}_B$ is always perpendicular to a charged particle's velocity vector $\mathbf{v}$, it can do no work on the particle. Thus, the answer is zero.

2. (a)

The force is directed in the $+y$ direction. The particle is charged positively and is moving in the $+x$ direction, so the thumb points to the right. With your fingers facing inward toward the paper, your palm is toward the top of the paper.

(b)

i)

The new speed is equal to the old speed because the magnetic force cannot do work, and therefore is unable to change the kinetic energy, and thus the speed, of the particle.

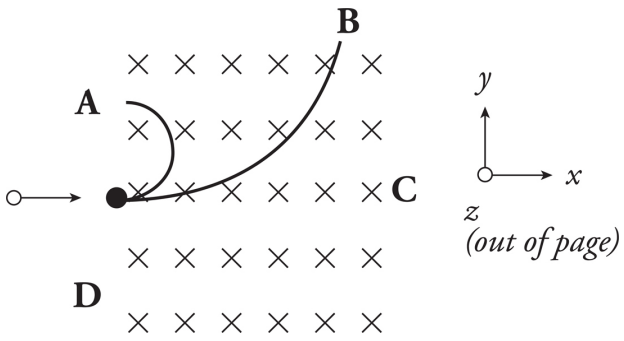
ii)

The new velocity is different from the old velocity as the direction has changed. The particle is traveling along a circular arc, and at any instant the velocity direction will be tangential to that arc. The new velocity will have the same magnitude as the initial velocity, but its x-component will be smaller as the y-component becomes larger. If the particle passes through 90° of the circular arc, the y-

component will again begin to shrink as the x-component grows.

(c)

Both A and B could be exit points from the magnetic field region.



CHAPTER 9 REVIEW QUESTIONS

Section I: Multiple Choice

1. **D**

Since **v** is upward and **B** is out of the page, the direction of **v** × **B** is to the right. Therefore, free electrons in the wire will be pushed to the left, leaving an excess of positive charge at the right. Therefore, the potential at point *b* will be higher than at point *a*, by $\varepsilon = vBL$ (motional emf).

2. **A**

As we discussed in this chapter, the magnitude of the emf induced between the ends of the rod is $\varepsilon = BLv = (0.5\ \text{T})(0.2\ \text{m})(3\ \text{m/s}) = 0.3\ \text{V}$. Since the resistance is 10 Ω, the current induced will be $I = V/R = (0.3\ \text{V})/(10\ \Omega) = 0.03\ \text{A}$. To determine the direction of the current, note that since positive charges in the rod are moving to the left and the magnetic field points into the plane of the page, the right-hand rule states that the magnetic force, $q\mathbf{v} \times \mathbf{B}$, points downward. Since the resulting force on the positive charges in the rod is downward, so is the direction of the induced current.

3. **A**

The magnetic field through the loop is $B = \mu_0 nI$. Since its area is $A = \pi r^2$, the magnetic flux through the loop is $\Phi_B = BA = (\mu_0 nI)(\pi r^2)$. If the current changes (with $\Delta I/\Delta t = -a$),

then the magnetic flux through the loop changes, which, by Faraday's Law, implies that an emf (and a current) will be induced. You get

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{\Delta(\mu_0 nI \cdot \pi r^2)}{\Delta t} = -(\mu_0 n\pi r^2)\frac{\Delta I}{\Delta t} = -\mu_0 \pi nr^2(-a) = \mu_0 \pi nr^2 a$$

Since the magnetic flux into the page is decreasing, the direction of the induced current will be clockwise (opposing a *decreasing into-the-page flux* means that the induced current will create more into-the-page flux).

4. **C**

First, you can eliminate (A) and (B). By definition, magnetic field lines emerge from the north pole and enter at the south pole. The magnetic field from the bar magnet will always point toward a viewer who is looking down at the loop from above. As the north pole gets closer to the loop, the field at the loop grows in strength, but as the south pole recedes from the loop, the magnetic field strength shrinks. Because of this, the flux changes from growing to shrinking as the magnet passes through the loop and the induced current must also change directions. Therefore, (A) and (B) are wrong. To determine whether (C) or (D) is correct, look at the first half of the motion. As the north pole get closer to the loop, the magnetic flux increases. To oppose an increasing flux, the direction of the magnetic field generated by induced current must be downward. Looking from above, a clockwise-induced current generates a downward magnetic field. Therefore, (C) is correct.

5. **C**

An induced emf requires that the magnetic flux through a loop of wire changes over time. Changing the area of the loop of wire will always cause the flux to change. Choice (B) can cause an emf, but will not necessarily depending on the axis of rotation. Choice (D) is also incorrect because while a time varying field will cause an emf, but it is not necessary to have such a field to produce an emf.

Section II: Free Response

1. (a)

$\varepsilon = B\ell v \Rightarrow (2\text{ T})(0.4\text{ m})(1\text{ m/s})$ or 0.8 V.

(b)

$V = IR$ becomes $I = \frac{V}{R}$ or $I = \frac{0.8\text{ V}}{20\ \Omega}$ = 0.04 A. The direction is given by Lenz's Law. Current flows in order to oppose the change in the magnetic flux. Because there is suddenly a new flux "in," current flows to produce an outward flux. This would be in the counterclockwise direction.

(c)

As the edge of the loop first encounters the magnetic field region, the induced current is established at 0.04 A, counterclockwise (found in part b). This continues as long as the flux is changing, which occurs until the left edge of the loop enters the magnetic field region. During the time the entire loop is within the region of the magnetic field, the flux is constant and the emf and current are both 0. Once the loop begins to leave the region of the magnetic field, the flux begins to change again and an induced current is established at 0.04 A. As the loop leaves the region of the magnetic field, the flux is shrinking instead of growing, so the current must be clockwise as the loop exits the magnetic field region.

(d)

The only thing that has changed is that the length of the wire at the leading edge has decreased. Thus, to maintain the same induced current, the speed at which the wire is pulled would have to increase. Moreover, as the length is halved, the speed would need to double in order for the induced current to remain constant.

CHAPTER 10 REVIEW QUESTIONS

Section I: Multiple Choice

1. **C**

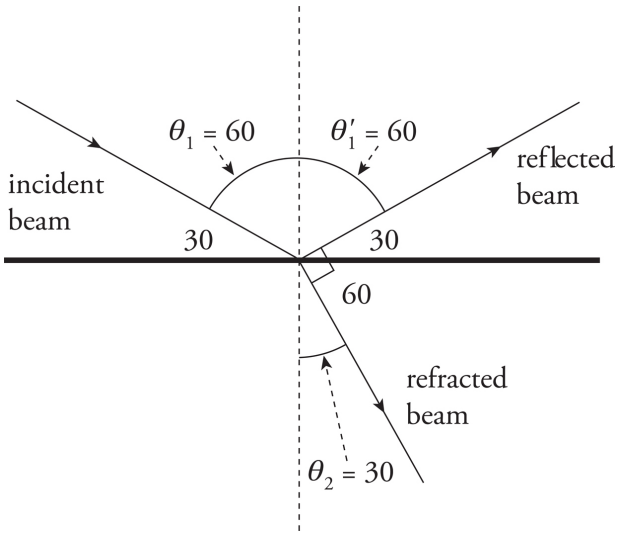
In the equation for a wave, $x = A\cos(\omega t)$, the amplitude A will always have the same units as the wave itself. This is because the value of $\cos(\omega t)$ is a number between -1 and 1 and has no units. An electric field, whether stationary or oscillating, is always measured in N/C (or equivalently V/m), so (B) and (D) are incorrect. In $\cos(\omega t)$, ω is the angular frequency, found using $\omega = 2\pi f$.

2. **C**

Since the fringe is bright, the waves must interfere constructively. This implies that the difference in path lengths must be a whole number times the wavelength, eliminating (A) and (B). The central maximum is equidistant from the two slits, so $\Delta\ell = 0$ there. At the first bright fringe above the central maximum, you have $\Delta\ell = \lambda$.

3. **B**

As light travels into an optically dense medium, it will refract in toward the normal and away from the surface. So the light in the glass will always be at a greater angle from the surface than the light in the air, so (B) is correct. Note that total internal reflection, (C), will not occur in this situation because it only happens with the initial medium being more optically dense.



4. **C**

Any convex lens made of a material with an index of refraction larger than the surroundings will create a real image, so (D) is incorrect. Because the glass to air index difference is greater than the water to air index difference, the glass to air transition result in a greater angle of refraction, so the light in air bends the exiting rays more than the water will. As a result, in point where the rays intersect will be farther from the lends than d_i.

5. **D**

Choice (A) is false because a change in speed at an interface does not cause intensity to change. Choice (B) is false because had there been Total Internal Reflection, the light would have remained in Medium 1. Choice (C) is false because the problem states that Medium 2 has a lower index of refraction, and from Snell's Law, $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$, if $n_2 < n_1$, then $\sin(\theta_2) > \sin(\theta_1)$ and the angles are not equal. Choice (D) is true because when light arrives at an interface, it must be absorbed, reflected, or transmitted. Therefore, any intensity not detected as transmitted into

Medium 2 was either reflected back into Medium 1 or absorbed.

6. **A**

The critical angle for total internal reflection is computed as follows:

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

$$\sin \theta_c = \frac{n_2}{n_1} = \frac{1.45}{2.90} = \frac{1}{2} \quad \Rightarrow \quad \theta_c = 30^\circ$$

Total internal reflection can happen only if the incident beam originates in the medium with the higher index of refraction, and therefore the beam must originate in the solid. The refracted ray must strike the interface of the other medium at an angle of incidence greater than the critical angle.

7. **C**

Every ray, regardless of whether it is a principle ray or not, which originates at the tip of an object and strikes mirror, will converge at the tip of the image.

8. **A**

All of the rays which originate at the tip of the image and travel along paths that remain above the optic axis will be blocked. However, all of the rays which originate at the tip of the image and travel along paths that are below the optic axis at the plane of the block will be unaltered. As a result, fewer rays will converge at the image location, resulting in a less bright image.

9. **D**

Diverging lenses always create virtual images.

10. **C**

In the double slit experiment, $x_m = \frac{m\lambda L}{d}$. Using a shorter wavelength, as in the second experiment, results in an increase in the fringe spacing, so (A) and (B) are false. Using slits which are closer together causes the fringe spacing to decrease, so (D) is false.

Section II: Free Response

1. (a)

Interference effects can be observed only if the light is coherent. Using two independent light sources at the slits in Barrier 2 would generate incoherent light waves. The set-up shown guarantees that the light reaching the two slits will be coherent.

(b)

Maxima are located at positions given by the equation $x_m = m\lambda L/d$, where m is an integer ($y_0 = 0$ is the central maximum). The first-order maximum for red light occurs at

$$x_{1,\text{ red}} = \frac{1 \cdot \lambda_{\text{red}} L}{d} = \frac{1 \cdot (750 \times 10^{-9}\text{ m})(4.0\text{ m})}{0.50 \times 10^{-3}\text{ m}} = 0.006\text{ m} = 6.0\text{ mm}$$

and the first-order maximum for violet light occurs at

$$x_{1,\text{ violet}} = \frac{1 \cdot \lambda_{\text{violet}} L}{d} = \frac{1 \cdot (400 \times 10^{-9}\text{ m})(4.0\text{ m})}{0.50 \times 10^{-3}\text{ m}} = 0.0032\text{ m} = 3.2\text{ mm}$$

Therefore, the vertical separation of these maxima on the screen is $\Delta x = 6.0\text{ mm} - 3.2\text{ mm} = 2.8\text{ mm}$.

(c)

Maxima are located at positions given by the equation $x_m = m\lambda L/d$, where m is an integer. Therefore, you want to solve the equation

$$x_{m,\text{ violet}} = x_{n,\text{ orange-yellow}}$$

$$\frac{m\lambda_{\text{violet}}L}{d} = \frac{n\lambda_{\text{orange-yellow}}L}{d}$$

$$m\lambda_{\text{violet}} = n\lambda_{\text{orange-yellow}}$$

$$m(400\text{ nm}) = n(600\text{ nm})$$

The smallest integers that satisfy this equation are $m = 3$ and $n = 2$. That is, the third-order maximum for violet light coincides with the second-order maximum for orange-yellow light. The position on the screen (relative to the central maximum at $x = 0$) of these maxima is

$$x_{3,\text{ violet}} = \frac{m\lambda_{\text{violet}}L}{d} = \frac{3(400\times10^{-9}\text{ m})(4.0\text{ m})}{0.50\times10^{-3}\text{ m}} = 0.0096\text{ m} = 9.6\text{ mm}$$

(d)

Within the glass, the wavelength is reduced by a factor of $n = 1.5$ from the wavelength in air. Therefore, the difference in path lengths, $d \sin\theta$, must be equal to $m(\lambda/n)$ in order for constructive interference to occur. This implies that the maxima are located at positions given by $x_m = m\lambda L/(nd)$. The distance between adjacent bright fringes is therefore

$$x_{m+1} - x_m = \frac{(m+1)\lambda L}{nd} - \frac{m\lambda L}{nd} = \frac{\lambda L}{nd} = \frac{(500\times10^{-9}\text{ m})(4.0\text{ m})}{1.5(0.50\times10^{-3}\text{ m})} = 0.0027\text{ m} = 2.7\text{ mm}$$

2. (a)

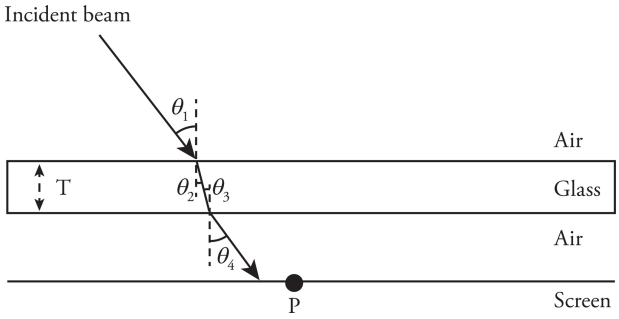
i) Student 1 is correct that “According to Snell’s law, the light will bend at the first interface” and also “unbend at the second interface and that it is travelling parallel to its original path.”

ii) Student 2 is correct that “Snell’s Law is $n_{\text{in}} \sin(\theta_{\text{in}}) = n_{\text{out}} \sin(\theta_{\text{out}})$” and also “$\theta_{\text{out}}$ is different from θ_{in} at the first

interface."

iii) Student 1 is incorrect that "The spot on the screen will still be at point P."

iv) Student 2 is incorrect that "The beam after the glass cannot be parallel to the beam before the glass."



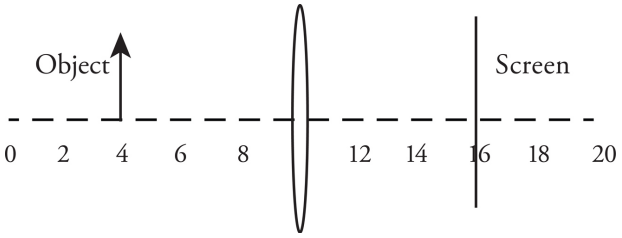
(b)

Here, $\theta_1 = \theta_4$ and $\theta_2 = \theta_3$.

(c)

If the area between the glass and screen were filled with water, it would still be the case that $\theta_2 = \theta_3$ because there is no change to the glass. However, upon leaving the glass and entering the water, the amount that the light would bend away from the normal would be less than in the case of air because the difference in the indices of refraction is smaller with glass-to-water than glass-to-air. As a result, the beams in air and glass would not be parallel and the stop that appeared on the screen would be even farther away from point P than in the original experiment.

3. (a)



In order for the object and image to be the same size, the magnitude of the magnification must be 1, so $|M| = \left|\left(\frac{s_i}{s_o}\right)\right| = 1$ and therefore $|s_o| = |s_i|$. The only way to satisfy the equation $\frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o}$ when the image distance is equal to the object distance is for both to be $2f$. The object should be placed $2f$ before the lens and the screen should be placed $2f$ after the lens. The image will be below the optical axis in this case, so the part of the screen where the image forms needs to also be placed below the optic axis. Note, switching the positions of the object and screen (so that the object is on the right and the screen is on the left) is also correct.

(b)

A converging lens is similar to a convex mirror, but with the mirror the light always stays on one side of the mirror. Both the object and the screen should be placed at the 4 cm mark. The image will be below the optical axis in this case.

(c)

A virtual image is a position in space where the light rays appear to have converged, although no light is actually present at the “image position.” Ray tracing can yield a set of non-converging rays. Tracing those rays “backward” to the point where the lines do converge results in the virtual image position. In order to experimentally verify that a virtual image exists, the virtual image location may be used as the object location for a second imaging system. The second imaging system may then produce a real image on a screen of the virtual image that was its object.

CHAPTER 11 REVIEW QUESTIONS

Section I: Multiple Choice

1. **B**

Waves will interfere with apertures that are approximately the same size as the wave. The wavelengths of matter from the de Broglie theory are on the same order as the size of molecule spacing in solids, and therefore these effects are not visible at macroscopic scales.

2. **A, C**

Photoelectrons will be emitted in the photoelectric effect any time that the energy of the incident light is greater than the work function of the metal. Altering the work function could cause the incident light energy to be below the work function. Altering the incident light frequency (since $E = hf$) could cause the energy of the incident light to be below the work function. Neither the stopping potential nor the power of the incident light have any effect on whether photoelectrons will be emitted during a photoelectric effect experiment.

3. **A**

The ground state of an atom is (−1) times its ionization energy, so the ground state would have to be at an energy level of −25 eV. To transition from −16 eV to −25 eV, a photon with an energy of 9 eV would have to be produced

in order for energy to be conserved. To find the wavelength of the photon, $E = hf = h(c/\lambda) = hc/\lambda$, so $\lambda = hc/E$. From the equation sheet, $hc = 1240 \text{ nm} \cdot \text{eV}$ and

$$\lambda = \frac{1240 \text{ nm} \cdot \text{eV}}{9 \text{ eV}} = 138 \text{ eV}.$$

4. **D**

The gap between the ground-state and the first excited state is

$-10 \text{ eV} - (-40 \text{ eV}) = 30 \text{ eV}$

Therefore, the electron must absorb the energy of a 30 eV photon (at least) in order to move even to the first excited state. Since the incident photons have only 15 eV of energy, the electron will be unaffected.

5. **D**

When a charged particle goes into a magnetic field, it will be deflected, but an uncharged particle will not be deflected. Particles with opposite signs will deflect in opposite directions. An alpha particle is charged and a neutron is not, so (D) is the correct answer.

6. **D**

If the atom begins in the $n = 3$ state, it could lose energy by making any of the following transitions: $3 \rightarrow 2$, $3 \rightarrow 1$, or $3 \rightarrow 2 \rightarrow 1$. The $3 \rightarrow 2$ transition would result in the emission of $-5 \text{ eV} - (-8 \text{ eV}) = 3 \text{ eV}$; the $3 \rightarrow 1$ transition would emit a $-5 \text{ eV} - (-12 \text{ eV}) = 7 \text{ eV}$ photon; and the $2 \rightarrow 1$ transition would result in the emission of $-8 \text{ eV} - (-12 \text{ eV}) = 4 \text{ eV}$.

Therefore, if the atom is initially in the $n = 3$ state, it could emit photons of energy 3 eV, 4 eV, or 7 eV.

7. **C**

To emit a photon, an energy level transition must go to a lower energy level (a transition to a higher level requires absorbing a photon), so (A) and (B) are incorrect. A long wavelength corresponds to a small energy photon, so (D) is also incorrect, and (C) is the correct answer.

8. **D**

At the time that Louis de Broglie made his hypothesis, it was known that light, which often behaves like a wave, also has particle-like behavior. The de Broglie hypothesis is that while electrons (and other matter) most often demonstrate particle-like behaviors, they would also demonstrate wave-like behaviors. Diffraction is a phenomenon associated with wave interference, so de Broglie's hypothesis can be summarized as electrons will diffract in certain circumstances.

9. **B**

The binding energy is given by $E_B = (\Delta m)c^2$, so (A) is true. The binding energy, which is the energy associated with the mass defect, is typically measured in MeV, while typical electronic transitions are measured in eV, so (C) is also true. Mass defect are also typically 4 orders of magnitude smaller than the mass of the involved nucleons, so (D) is true. Choice (B) is false because the binding energy is always positive, so that the mass of the bound nucleons plus the binding energy equals the mass of the constituent nucleons.

10. **B**

In order to balance the mass number (the superscripts), you must have 2 + 63 = 64 + A, so A = 1. In order to balance the charge (the subscripts), you need 1 + 29 = 30 + Z, so Z = 0. A particle with a mass number of 1 and no charge is a neutron, ${}^{1}_{0}\mathrm{n}$.

11. **A**

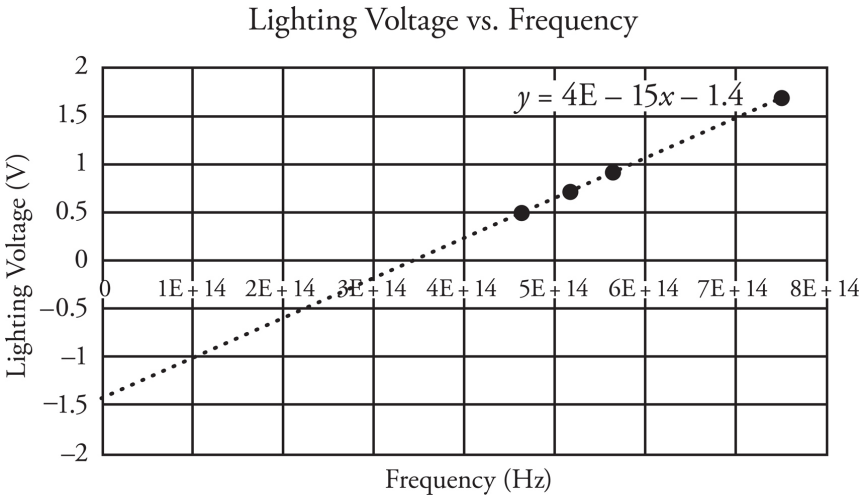
The alpha decay processes produces an alpha particle ${}^{4}_{2}\mathrm{He}$ and a daughter nucleus. The daughter nucleus must have mass number 4 below the original isotope and a charge number 2 below the original isotope, resulting in ${}^{171}_{76}\mathrm{Os}$. The beta(+) decay processes produces a positive beta particle ${}^{0}_{1}e$ and a daughter nucleus. The daughter nucleus must have mass number equal to the original isotope and a charge number 1 below the original isotope, resulting in ${}^{175}_{75}\mathrm{Ir}$.

Section II: Free Response

1. (a)

The work function is the amount of energy, which must be supplied before a photoelectron is released. For any amount of energy supplied by the power supply below the work function of the material that the LED light is made of, there will be no production of photoelectrons. Therefore, the work function of the material must be greater than 0.1 eV since there were no photoelectrons emitted at the voltage of 1 V.

(b)



The slope of the graph is 4×10^{-15}V · s and the y-intercept is –1.4 V. The slope has the value of Planck's constant and the intercept is the work function of the material the LED lights are made of.

(c)

i) To change the intercept, the work function would have to be different, so a supply of LEDs made from a different material would have to be obtained.

ii) It would not be possible to change the slope, as the slope determined from this experiment is Planck's Constant.

Part VI
Practice Test 2

- Practice Test 2
- Practice Test 2: Answers and Explanations

Practice Test 2

Click here to download the PDF of Practice Test 2.

The Exam

AP® Physics 2 Exam

SECTION I: Multiple-Choice Questions

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
90 minutes
Number of Questions
50
Percent of Total Grade
50%
Writing Instrument
Pen required

Instructions

Section I of this examination contains 50 multiple-choice questions. Fill in only the ovals for numbers 1 through 50 on your answer sheet.

CALCULATORS MAY BE USED IN BOTH SECTIONS OF THE EXAMINATION.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding oval on the answer sheet. Give

only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question

Chicago is a
(A) state
(B) city
(C) country
(D) continent

Sample Answer



Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all the multiple-choice questions.

About Guessing

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. Multiple-choice scores are based on the number of questions answered correctly. Points are not deducted for incorrect answers, and no points are awarded for unanswered questions. Because points are not deducted for incorrect answers, you are encouraged to answer all multiple-choice questions. On any questions you do not know the answer to, you should eliminate as many choices as you can, and then select the best answer among the remaining choices.

ADVANCED PLACEMENT PHYSICS 2 TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1\text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_A = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant, $G = 6.67 \times 10^{-11}$ m^3/kg·s^2
Universal gas constant, $R = 8.31$ J/(mol·K)	Acceleration due to gravity at Earth's surface, $g = 9.8$ m/s^2
Boltzmann's constant, $k_B = 1.38 \times 10^{23}$ J/K	

1 unified atomic mass unit,	$1\text{ u} = 1.66 \times 10^{-27}\text{ kg} = 931\text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}\text{ J·s} = 4.14 \times 10^{-15}\text{ eV·s}$
	$hc = 1.99 \times 10^{-25}\text{ J·m} = 1.24 \times 10^{3}\text{ eV·nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\text{ C}^2/\text{N·m}^2$
Coulomb's law constant,	$k = 1/4\pi\varepsilon_0 = 9.0 \times 10^{9}\text{ N·m}^2/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ (T·m)/A
Magnetic constant,	$k' = \mu_0/4\pi = 1 \times 10^{-7}$ (T·m)/A
1 atmosphere pressure,	$1\text{ atm} = 1.0 \times 10^{5}\text{ N/m}^2 = 1.0 \times 10^{5}\text{ Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. The frame of reference of any problem is assumed to be inertial unless otherwise stated.
II. In all situations, positive work is defined as work done on a system.
III. The direction of current is conventional current: the direction in which positive charge would drift.
IV. Assume all batteries and meters are ideal unless otherwise stated.
V. Assume edge effects for the electric field of a parallel plate capacitor unless otherwise stated.
VI. For any isolated electrically charged object, the electric potential is defined as zero at infinite distance from the charged object.

ADVANCED PLACEMENT PHYSICS 2 EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\left|\vec{F}_f\right| \le \mu \left|\vec{F}_n\right|$

$a_c = \frac{v^2}{r}$

$\vec{p} = m\vec{v}$

$\Delta\vec{p} = \vec{F}\,\Delta t$

$K = \frac{1}{2} m v^2$

$\Delta E = W = F_{\|} d = F d \cos\theta$

$P = \frac{\Delta E}{\Delta t}$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega = \omega_0 + \alpha t$

$x = A\cos(\omega t) = A\cos(2\pi f t)$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$\tau = r_\perp F = rF\sin\theta$

$L = I\omega$

$\Delta L = \tau\,\Delta t$

$K = \frac{1}{2} I \omega^2$

$\left|\vec{F}_s\right| = k|\vec{x}|$

a = acceleration
A = amplitude
d = distance
E = energy
F = force
f = frequency
I = rotational inertia
K = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
r = radius or separation
T = period
t = time
U = potential energy
v = speed
W = work done on a system
x = position
y = height
α = angular acceleration
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed

$U_s = \frac{1}{2} k x^2$

$\Delta U_g = mg\,\Delta y$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$

$\vec{g} = \frac{\vec{F}_g}{m}$

$U_G = -\frac{G m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$\left|\vec{F}_E\right| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$

$\Delta U_E = q\Delta V$

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

$\left|\vec{E}\right| = \left|\frac{\Delta V}{\Delta r}\right|$

$\Delta V = \frac{Q}{C}$

$C = \kappa\varepsilon_0 \frac{A}{d}$

$E = \frac{Q}{\varepsilon_0 A}$

$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$

$I = \frac{\Delta Q}{\Delta t}$

$R = \frac{\rho\ell}{A}$

$P = I\,\Delta V$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$B = \frac{\mu_0}{2\pi}\frac{I}{r}$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
ℓ = length
P = power
Q = charge
q = point charge
R = resistance
r = separation
t = time
U = potential (stored) energy
V = electric potential
v = speed
κ = dielectric constant
ρ = resistivity
θ = angle
Φ = flux

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\left|\vec{F}_M\right| = |q\vec{v}||\sin\theta|\left|\vec{B}\right|$

$\vec{F}_M = I\vec{\ell} \times \vec{B}$

$\left|\vec{F}_M\right| = \left|I\vec{\ell}\right||\sin\theta|\left|\vec{B}\right|$

$\Phi_B = \vec{B} \cdot \vec{A}$

$\Phi_B = \left|\vec{B}\right|\cos\theta\left|\vec{A}\right|$

$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t}$

$\mathcal{E} = B\ell v$

ADVANCED PLACEMENT PHYSICS 2 EQUATIONS

FLUID MECHANICS AND THERMAL PHYSICS

$\rho = \frac{m}{V}$

$P = \frac{F}{A}$

$P = P_0 + \rho gh$

$F_b = \rho V g$

$A_1 v_1 = A_2 v_2$

$P_1 + \rho g y_1 + \frac{1}{2}\rho {v_1}^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho {v_2}^2$

$\frac{Q}{\Delta t} = \frac{kA\,\Delta T}{L}$

$PV = nRT = Nk_BT$

$K = \frac{3}{2}k_BT$

$W = -P\Delta V$

$\Delta U = Q + W$

A = area
F = force
h = depth
k = thermal conductivity
K = kinetic energy
L = thickness
m = mass
n = number of moles
N = number of molecules
P = pressure
Q = energy transferred to a system by heating
T = temperature
t = time
U = internal energy
V = volume
v = speed
W = work done on a system
y = height
ρ = density

MODERN PHYSICS

$E = hf$

$K_{\max} = hf - \phi$

$\lambda = \frac{h}{p}$

$E = mc^2$

E = energy
f = frequency
K = kinetic energy
m = mass
p = momentum
λ = wavelength
ϕ = work function

WAVES AND OPTICS

$\lambda = \frac{v}{f}$

$n = \frac{c}{v}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$

$|M| = \left|\frac{h_i}{h_o}\right| = \left|\frac{s_i}{s_o}\right|$

$\Delta L = m\lambda$

$d\sin\theta = m\lambda$

d = separation
f = frequency or focal length
h = height
L = distance
M = magnification
m = an integer
n = index of refraction
s = distance
v = speed
λ = wavelength
θ = angle

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = \frac{1}{2}bh$

Circle
$A = \pi r^2$
$C = 2\pi r$

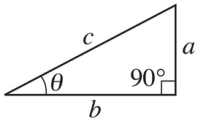
Rectangular solid
$V = \ell wh$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r\ell + 2\pi r^2$

Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

Right triangle
$c^2 = a^2 + b^2$
$\sin\theta = \frac{a}{c}$
$\cos\theta = \frac{b}{c}$
$\tan\theta = \frac{a}{b}$



Section I

PHYSICS 2
SECTION I
Time—90 minutes
50 Questions

Note: To simplify calculations, you may use $g = 10\ \text{m/s}^2$ in all problems.

Directions: Each of the questions or incomplete statements below is followed by four suggested answers or completions. Select the one that is best in each case and mark it on your sheet.

1. A positively charged particle enters a region between the plates of a parallel plate capacitor. The particle is moving initially parallel to the plates. What is the correct description of the trajectory of the particle in the region between the plates?

   (A) The particle will continue straight.
   (B) The particle will move toward the negative plate along a straight-line path.
   (C) The particle will move toward the negative plate along a curved path.
   (D) The particle will move toward the positive plate along a curved path.

2. Which of the following arguments does **not** support the assertion that a parallel plate capacitor held with one plate parallel to the floor is an electric analog of the gravitational field near the surface of the Earth?

   (A) The voltage is constant along horizontal lines.
   (B) The field changes linearly with distance from the lower plate.

(C) Both the gravitational field lines and electric field lines are vertical.

(D) Both the electrical and gravitational force are governed by inverse square laws.

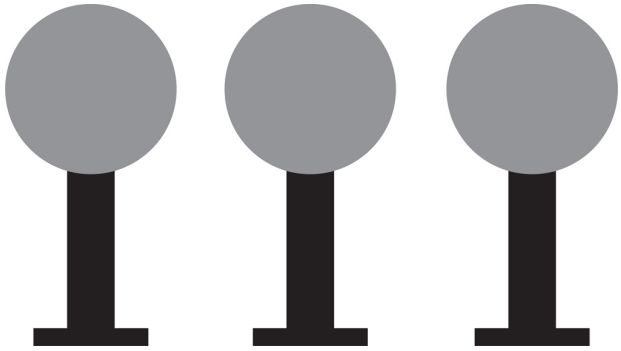
3. An electromagnetic wave moves along the positive z-axis. The wave is polarized so that the magnetic field is confined in the y-axis. Where is there a non-zero electric field from this wave?

(A) The electric field exists everywhere.

(B) The electric field is only along the x-axis.

(C) The electric field is only along the y-axis.

(D) The electric field is only along the z-axis.



4. Three neutral conducting spheres sit on insulating bases. The spheres are separated by a very large distance. The sphere in the center is given a positive charge. The spheres are brought close together, as shown above, but not allowed to come into contact with one another. Which is the correct description of the net charge on each sphere?

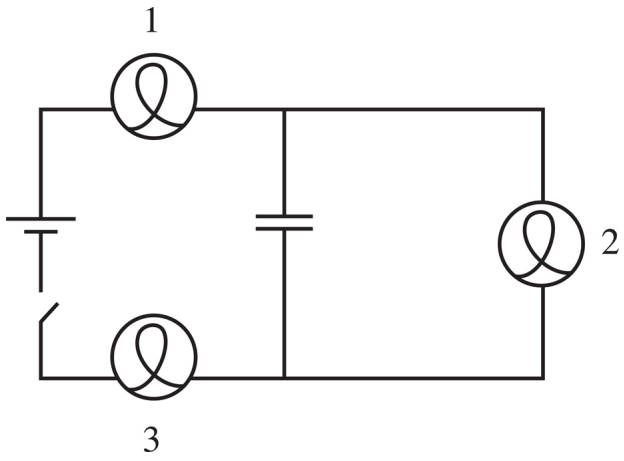
(A) All three spheres are positively charged.

(B) The center sphere is positively charged and the two outer spheres are negatively charged.

(C) The center sphere is positively charged and the other two are neutral.

(D) All three spheres are neutral.

Questions 5–7 refer to the following diagram.



5. The circuit above contains a battery, a switch, three identical light bulbs, and a capacitor. The capacitor is initially uncharged. The instant the switch is closed, which of the bulbs are lit?

(A) All three bulbs are unlit.
(B) Only bulb 1 is lit.
(C) Only bulb 2 is lit.
(D) Bulb 1 and bulb 3 are lit.

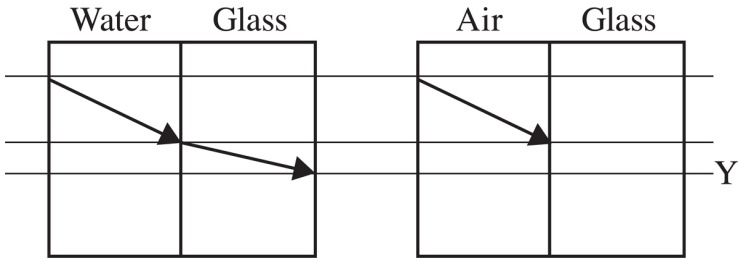
6. After the capacitor is fully charged, which bulbs are lit?

(A) All three bulbs are unlit.
(B) Bulb 1 and bulb 3 are lit.
(C) Only bulb 2 is lit.
(D) All three bulbs are lit.

7. When the capacitor is fully charged, which components have equal voltage drops across them?

(A) The capacitor and the battery
(B) The capacitor and bulb 2
(C) Bulb 1 and bulb 3

(D) All three of the above selections have equal voltage drops across them.



8. A ray of light enters the pair of tanks as shown above. The light rays in the water and glass tank are shown at the left, with the light exiting the pair of tanks at a point Y. The light ray in the air tank is identical to the ray in the water tank, entering at the same height and same angle. Both sets of tanks use the same glass. How will the ray exiting the second glass tank compare to height Y?

(A) The ray will not exit the second tank due to total internal reflection.
(B) The exiting ray will be closer to the top of the tank than Y.
(C) The exiting ray will be at the same height as Y.
(D) The exiting ray will be farther from the top of the tank than Y.

9. A mirror may be used to create a real image under which of the following conditions?

(A) Use a concave mirror and place the object between the mirror and the focal point.
(B) Use a concave mirror and place the object beyond focal point.
(C) Use a convex mirror and place the object between the mirror and the focal point.
(D) Use a convex mirror and place the object beyond the focal point.

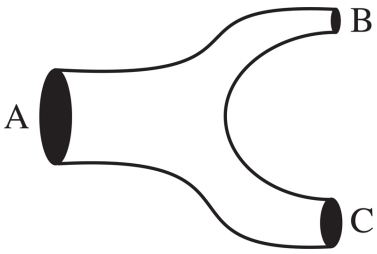
10. The pressure at location A in a pipe is known. The speed of the fluid in the pipe and the pipe diameter are also known at position A. Position B is another location in the pipe. Which pieces of data are required to calculate the speed of the fluid at position B?

(A) The diameter at B only

(B) The diameter at B and the height of B relative to A

(C) The diameter at B, the height of B relative to A and the pressure at B

(D) The diameter at B, the pressure at B, the height of B relative to A and the fluid density



11. The tube shown above carries water. At some point, the tube splits, as shown above. Three points within the tube are labeled A, B, and C. The greatest diameter is at A and the smallest diameter is at B. How do the pressures in the pipes compare at the three points?

(A) $P_A > P_C > P_B$

(B) $P_A = P_B = P_C$

(C) $P_B = P_C > P_A$

(D) $P_B > P_C > P_A$

12. A paper-filled three ring binder sits in a room for a long time. A student touches the metal rings of the binder and the paper. She expects the two objects to feel the same temperature, but she observes that the paper feels warmer than the metal. Which of the following correctly explains her observation?

(A) The objects were in thermal equilibrium, but she must have felt the paper first, as when you feel multiple objects at the same temperature, the ones felt first will feel warmest.

(B) The objects were in thermal equilibrium, but the energy flow occurs more slowly between her fingers and the paper than her fingers and the metal, resulting in the paper feeling warmer.

(C) The objects were in thermal equilibrium and the paper warms up quickly when touched. Her perception is not instantaneous, but is the average temperature over a period of time and higher for the paper.

(D) The paper was warmer than the metal because temperature is related to the kinetic energy of the molecules, and the more massive metal molecules will have less kinetic energy than the paper molecules.

13. A hot cup of water is placed within an insulating container. Later, the water has cooled. Which of the following explains the phenomenon?

(A) The molecules in the water are fast-moving initially and transfer energy in the air in the container, resulting in the air heating up as the water cools.

(B) The molecules in the air are fast-moving initially and transfer energy to the water in the container, resulting in the water cooling down as the air heats up.

(C) As time goes on, the entropy of the system increases, which results in a decrease in energy and the loss of temperature in the cup.

(D) The total heat in the container must remain constant unless there is leakage to the outside surroundings, so the water can only cool down if the surroundings heat up.

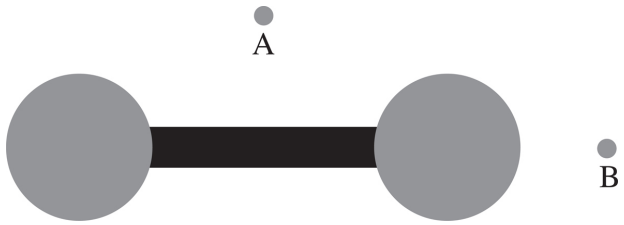
14. A very large sheet of metal has a net negative charge. An electron is placed above the center of the sheet. Which of the

following correctly describes the force on the sheet of charge from the electron?

(A) The force pulls the sheet upward.
(B) The force pushes the sheet downward.
(C) The electron causes a torque on the sheet, but no net upward or downward force exists.
(D) There is no force on the sheet because the electron is a test charge.

15. An experiment is conducted on a circuit consisting of a battery and several wires. The wires are all made of the same metal. It is found that as the cross-sectional area of the wire increases, the current measured coming out of the battery increases linearly. What can be concluded about the wire from this data?

(A) The battery has an internal resistance, which can be found from the slope of the area vs. current graph.
(B) The battery has an internal resistance, which can be found from the y-intercept of the area vs. current graph.
(C) The wires have the same length, which can be found from the slope of the area vs. current graph.
(D) The wires have the same length, which can be found from the y-intercept of the area vs. current graph.



16. An insulating rod separates two conducting spheres as shown above. Point A is midway between the spheres. Point B lies on the axis of the rod. Which of the following arrangements of charges would result in a net torque on the rod?

(A) Charge both spheres negatively and place a positive charge at point A.

(B) Charge one sphere negatively and the other positively and place a positive charge at point A.

(C) Charge both spheres negatively and place a positive charge at point B.

(D) Charge one sphere negatively and the other positively and place a positive charge at point B.

17. The process of charging by induction requires which physical property of a system?

(A) Charge polarization

(B) Uneven charge distribution

(C) Zero net charge

(D) Contact between a conductor and insulator

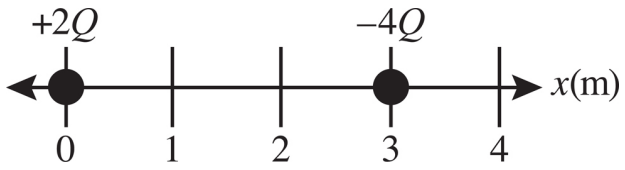
18. To determine the direction of the force on a charge that is moving near a current carrying wire, which data do you need?

(A) Only the direction the charge is moving.

(B) The sign of the charge and its direction of motion.

(C) The sign of the charge, the direction of motion and the direction of the current.

(D) The sign of the charge, the direction of motion, the direction of the current and the position of the charge relative to the wire.

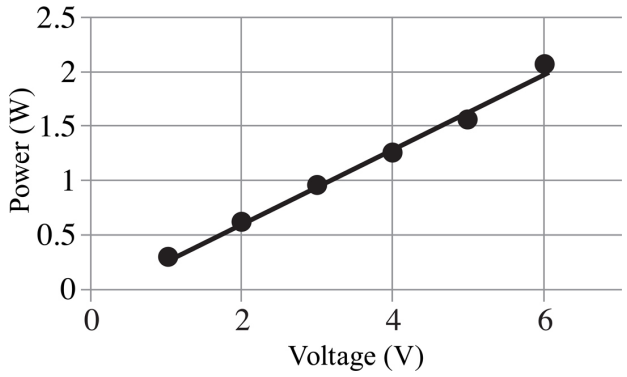


19. A positive charge of 2Q and a negative charge of 4Q are arranged at positions as shown above. What is the correct ranking of the electric potential at points along the *x*-axis?

(A) $V(x = 4\text{ m}) = V(x = 2\text{ m}) > V(x = 1\text{ m})$
(B) $V(x = 4\text{ m}) > V(x = 1\text{ m}) > V(x = 2\text{ m})$
(C) $V(x = 1\text{ m}) > V(x = 2\text{ m}) > V(x = 4\text{ m})$
(D) $V(x = 4\text{ m}) > V(x = 2\text{ m}) > V(x = 1\text{ m})$

20. A large piece of wood has the same weight as a small rock. Both the wood and rock are placed into a pool of liquid. The rock sinks while the wood floats. How do the buoyant forces on the wood and rock compare?

(A) The buoyant forces are the same on each.
(B) The buoyant force on the wood is greater than on the rock.
(C) The buoyant force on the wood is less than on the rock.
(D) The buoyant forces cannot be compared without the density of the liquid.



21. An experiment is conducted to determine the power output from a circuit as various voltages are supplied to the circuit. The circuit is set up so that it draws a constant current. What is the value of the current in the circuit that produced the graph above?

(A) 0.00 A
(B) 0.25 A
(C) 1.0 A
(D) 6.0 A

22. The products of several radioactive decays are being studied. Each particle starts with the same speed and enters into a region with a uniform magnetic field directed perpendicular to the initial velocity of the particles. Which observation could be made?

(A) A positron and an electron both turn in the same direction, but the electron turns with a larger radius.

(B) An alpha particle and an electron both turn in the same direction, but the alpha particle turns with a larger radius.

(C) An alpha particle and an electron both turn in opposite directions, but the alpha particle turns with a larger radius.

(D) A positron and an electron both turn in the opposite directions, but the electron turns with a larger radius.

23. A hypothetical atom contains an undetermined number of energy levels above the ground state. A gas of this atom is entirely in the ground state. Light with a broad spectrum is shined upon the gas and the spectrum of the light is recorded after shining through the gas. Three wavelength of light are observed to have diminished intensity. What can be concluded from this?

(A) There must be exactly two energy levels above the ground state.

(B) There must be exactly three energy levels above the ground state.

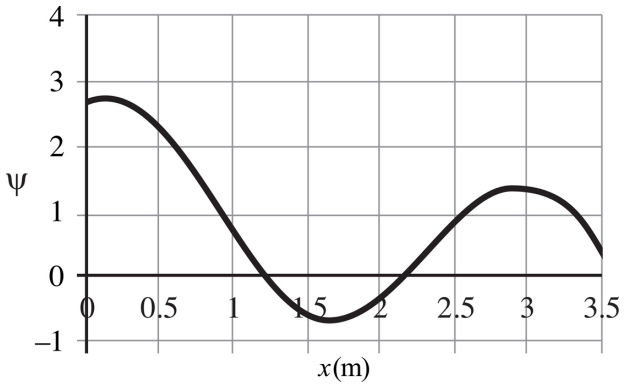
(C) There must be either exactly two or exactly three energy levels above the ground state.

(D) There are an undetermined number of energy levels above the ground state, but there are at least three levels.

24. An unknown nucleus goes through a decay process. Three ${}_{2}^{4}\mathrm{He}$ He alpha particles are emitted, along with two β^{-} particles. The

resulting daughter nucleus is $^{206}_{82}\text{Pb}$. What was the atomic number of the unknown nucleus that started the process?

(A) 84
(B) 86
(C) 88
(D) 90



25. The wave function of a quantum object vs. position is graphed above. Which of the following correctly ranks the probabilities of observing the particle at the listed positions?

(A) $P(x = 0.5\text{ m}) > P(x = 2.25\text{ m}) > P(x = 1.75\text{ m})$
(B) $P(x = 0.5\text{ m}) > P(x = 1.75\text{ m}) > P(x = 1.25\text{ m})$
(C) $P(x = 3.0\text{ m}) > P(x = 1.75\text{ m}) > P(x = 2.0\text{ m})$
(D) $P(x = 3.5\text{ m}) > P(x = 3.0\text{ m}) > P(x = 1.75\text{ m})$

26. Radiothorium-228 decays into Radon-220 through two alpha decays, as shown below.

$$^{228}_{90}\text{Th} \rightarrow {}^{220}_{86}\text{Rn} + {}^{4}_{2}\text{He} + {}^{4}_{2}\text{He}$$

Which equation correctly describes the energy released during this process?

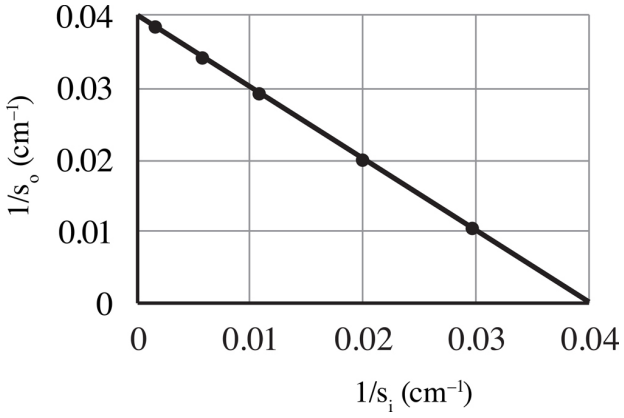
(A) $(m_{\text{Th}} - m_{\text{Rn}})c^2$

(B) $(m_{Th} + 2m_{He} - m_{Rn})c^2$

(C) $(m_{Th} - 2m_{He} - m_{Rn})c^2$

(D) $(m_{Rn} + 2m_{He} - m_{Th})c^2$

Questions 27 and 28 refer to the following graph.



A thin lens is used to make an image for several different object distances. The image and object distances are used to make the above graph.

27. What is the approximate focal length of the lens?

(A) 1 cm

(B) 10 cm

(C) 25 cm

(D) 100 cm

28. In order to achieve a magnification of 1, how far from the lens should an object be placed?

(A) 2 cm

(B) 20 cm

(C) 50 cm

(D) 200 cm

29. A container has a round lid, with mass M, and makes an airtight seal with the body of the container. Some of the air in the container is pumped out. The container is turned over, but the lid stays shut. Which of the following explanations for this phenomenon is correct?

(A) Pumping the air out of the container decreases the gravitational force on the lid.

(B) Pumping the air out creates an upward force that will always balance with the gravitational force.

(C) The force on the lid downward from the molecules inside is less than the force on the lid from the particles in the air surrounding the container.

(D) The force on the lid downward from the molecules inside is less than the difference between the force on the lid from the particles in the air surrounding the container and the gravitational force on the lid.

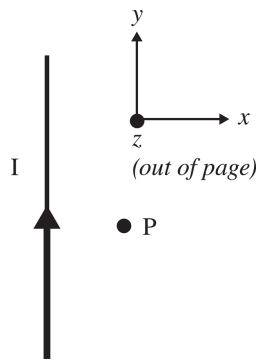
30. A mercury thermometer is placed in a glass of ice-cold water. After some time goes by, the mercury thermometer can be read to determine that the temperature of the water is 0°C. Several minutes later, the reading on the thermometer has not changed. What has happened to the molecules of mercury in the thermometer?

(A) Over time, all the molecules slow down, but the rate of slowing decreases as temperature decreases so it is not noticeable between the two readings.

(B) The mercury molecules stopped moving upon reaching a temperature of 0°C and remained stopped for the second reading.

(C) The mercury molecules reached thermal equilibrium before the first reading, so all their molecules had the same non-zero speed when both readings were taken.

(D) The mercury molecules reached thermal equilibrium before the first reading, but between the readings some of the molecules sped up and others slowed down, resulting in the same reading.



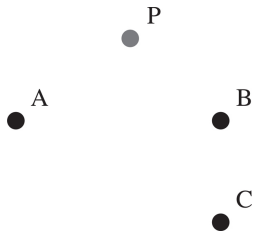
31. A current-carrying wire and coordinate system is shown above. Initially, the wire carries a current I toward the top of the page. The amount of current is steadily decreased until it is 0 A, then steadily increased until it reaches a value of I in the downward direction. This change in current takes time t. The magnetic field versus time at the observation point P from $t = 0$ until t is graphed. Which statement is true concerning the graph if $+z$ is above the horizontal axis and $-z$ is below the horizontal axis?

(A) The graph has a constant slope.

(B) The graph is piecewise linear with a negative slope for the first half and a positive slope for the second half.

(C) The graph is piecewise linear with a positive slope for the first half and a negative slope for the second half.

(D) The graph is a curve showing an inverse relationship.



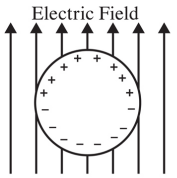
32. Charges are distributed as shown above. At point A is a charge of $+3Q$. At B is a charge of $+1Q$. At C is a charge of $-1Q$. What is the direction of the force on a proton located at point P?

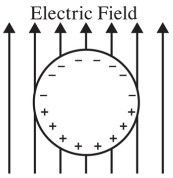
(A) Up and to the left
(B) Down and to the left
(C) Up and to the right
(D) Down and to the right

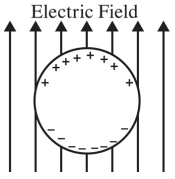
33. All of the following observations of electric field diagrams are correct EXCEPT:

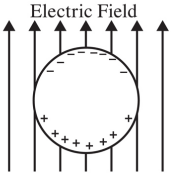
(A) Longer arrows correspond to a greater field magnitude.
(B) Field lines will be larger at positions close to sources or sinks.
(C) Arrow lengths will decrease linearly with distance from sources or sinks.
(D) Arrows point in the direction in which a positive charge would experience a force.

34. A neutral sphere of metal is placed in a region of uniform electric field. The electric field points toward the top of the page. Which of the following diagrams shows the arrangement of the charges on the surface of the sphere once electrostatic equilibrium is reached?

(A) 

(C) 

(B) 

(D) 

35. An electron enters a region of magnetic field created by a long current carrying wire. Along which path could the electron travel without being deflected?

(A) The electron is traveling parallel to the wire and in the same direction that the current is flowing.

(B) The electron is traveling parallel to the wire and in the opposite direction from the direction that the current is flowing.

(C) The electron is traveling straight away from the wire.

(D) The electron is traveling in a circle centered at the wire.

36. Which process will result in charging an uncharged sphere using a charged rod?

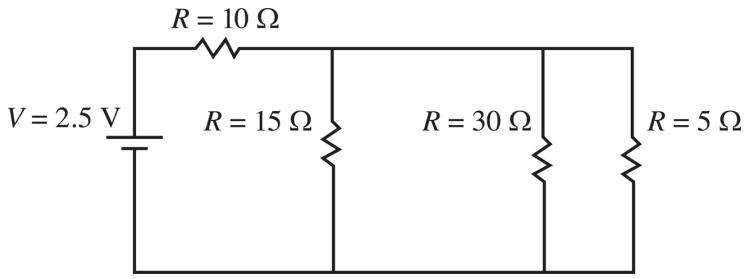
(A) Bringing the rod near the sphere and touching the two objects together, then removing the rod

(B) Grounding the sphere first, then bringing a charged object near the sphere and touching the two objects together, then removing the rod

(C) Bringing the rod near the sphere, but not touching them together, then removing the rod

(D) Grounding the sphere, moving the rod near the sphere, bringing the rod near the sphere but not touching them together, then removing the rod

Questions 37 and 38 refer to the following diagram.

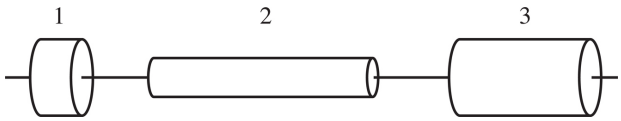


37. The current drawn from the battery is most nearly equal to what value?

(A) 0.042 A
(B) 0.19 A
(C) 0.30 A
(D) 0.68 A

38. A fifth resistor is placed in the circuit. It is parallel beside the 5 Ω resistor. The voltage drop across this resistor is found to be V = 0.50 V. What is the resistance of the additional resistor?

(A) 5 Ω
(B) 10 Ω
(C) 15 Ω
(D) 30 Ω



39. Three cylinders of the same metal act as resistors arranged in series, as shown above. Which of the following correctly ranks the voltage drops across the three resistors?

(A) $V_1 = V_2 = V_3$
(B) $V_3 > V_1 > V_2$
(C) $V_2 > V_3 > V_1$
(D) $V_1 > V_3 > V_2$

40. An electron experiences both electric and magnetic forces. Those forces are balanced. What must be true about the motion of the particle and the alignment of the two fields?

(A) The fields point along the same axis, and the velocity of the electron is parallel to that axis.

(B) The fields point along the same axis, and the velocity of the electron is perpendicular to that axis.

(C) The fields point along the perpendicular axes, and the velocity is parallel to the axis of the electric field.

(D) The fields point along the perpendicular axes, and the velocity along the third perpendicular axis.

41. A pair of electrons are held a fixed distance apart. Which of the following correctly describes the electrical and gravitational forces between the electrons?

(A) The electrical force is much greater than the gravitational force.

(B) The gravitational force is much greater than the electrical force.

(C) The forces are equal as both the gravitational force and electrical force are governed by inverse square laws.

(D) The forces cannot be compared without knowing the value of the distance between the electrons.

42. An electron travels down the center of a solenoid, carrying a current I. Which of the following explanations for the motion of the electron is correct?

(A) The electron accelerates due to the electric force from the charges in the current in the wire.

(B) The electron accelerates due to the magnetic field generated by the current in the wires of the solenoid.

(C) The electron travels at a constant speed but changes direction due to the magnetic field generated by the current in the wires.

(D) The electron travels at a constant velocity because there is no net force on the electron.

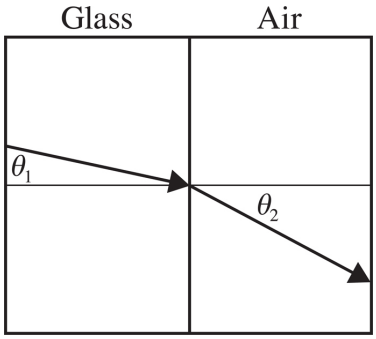
43. A voltage is sent through a tube containing hydrogen gas. The gas emits light. When the light is sent into a spectrometer, several distinct bright lines are seen. Why does this occur?

(A) The voltage causes the gas to heat up to specific temperatures, which are characterized by the lines.

(B) The voltage causes gas molecules to move at specific speeds, which are characterized by the lines.

(C) The voltage sets up standing waves due to the pressure of the gas in the tube, which can be characterized by the lines.

(D) The voltage excites electrons in the gas into specific energy levels, which can be characterized by the lines.



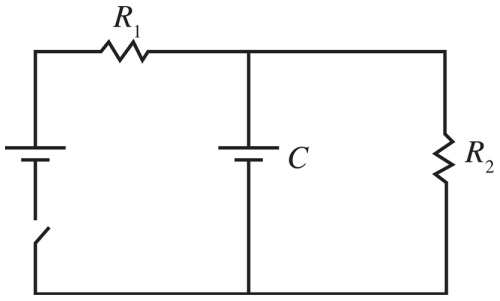
44. An experiment is conducted to determine the critical angle for light going from glass into air, as shown above. A linear plot is made with a vertical axis of sin (θ_1) and a horizontal axis of sin (θ_2). How is the critical angle determined from the graph?

(A) The critical angle can be found from the slope of the line.

(B) The critical angle can be found from the y-intercept of the line.

(C) The critical angle can be found from the horizontal axis value which corresponds to the maximum vertical value.

(D) The critical angle can be found from the vertical axis value which corresponds to the maximum horizontal value.



45. The circuit shown above is set up. The switch is closed and a long time passes. What conditions on the two resistors result in the greatest amount of energy stored in the capacitor?

(A) The energy stored in the capacitor will be greatest if $R_1 > R_2$.

(B) The energy stored in the capacitor will be greatest if $R_1 = R_2$.

(C) The energy stored in the capacitor will be greatest if $R_1 < R_2$.

(D) The energy will be the same regardless of the resistor values.

Directions: For questions 46–50 below, <u>two</u> of the suggested answers will be correct. Select the two answers that are best in each case, and then fill in both of the corresponding circles on the answer sheet.

46. A wire is placed vertically and carries a current in the upward direction. A compass is located directly to the north of the wire. Which of the following observations can be made about the compass needle deflection? Select two answers.

(A) The deflection grows larger as the compass is moved farther from the wire.

(B) The deflection is to the west.

(C) The deflection grows larger as the current is increased.

(D) The deflection changes direction as the compass is moved upward.

47. An ideal gas is confined to a leak-proof box. What type of processes could occur to cause the gas to absorb heat, but have no net work done on the gas? Select two answers.

(A) Isothermal expansion to the original volume, then isobaric compression to the original volume

(B) A doubling of pressure at constant volume

(C) A doubling of pressure at constant volume, then doubling the volume at constant pressure, then halving the pressure at constant volume, and finally halving the volume at constant pressure to return to the initial state

(D) A doubling of pressure at constant volume, then finally halving the pressure at constant volume, then doubling the volume at constant pressure, then finally halving the volume at constant pressure to return to the initial state

48. Which of the following results of the photoelectric effect give support for the particle nature of light? Select two answers.

(A) There is a threshold frequency below in which no electrons are emitted.

(B) Above the threshold frequency, the number of electrons emitted increases with increasing intensity.

(C) The stopping voltage is related only to the maximum kinetic energy electron, not all of the electrons ejected from the metal.

(D) The stopping voltage increases linearly with frequency above the threshold frequency regardless of the light intensity.

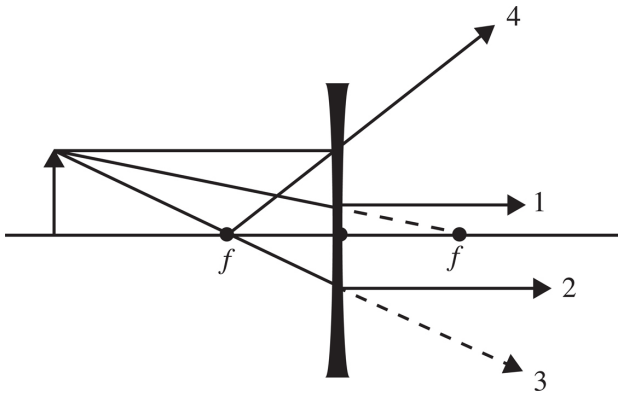
49. A circuit contains four resistors R_1, R_2, R_3, and R_4 and a battery. The first two are in series and the second two are in parallel. How can a fifth resistor, R_5, be added to the circuit to increase the current draw out of the battery? Select two answers.

(A) Place R_5 in parallel with R_1.

(B) Place R_5 in series with R_2.

(C) Place R_5 in parallel with R_3.

(D) Place R_5 in series with R_4.



50. For the diverging lens shown above, which principle rays are correctly drawn? Select two answers.

(A) 1

(B) 2

(C) 3

(D) 4

END OF SECTION I

Section II

PHYSICS 2
SECTION II
Time—90 minutes
4 Questions

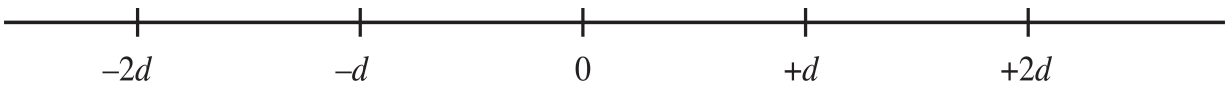
Directions: Questions 1 and 2 are long free-response questions that require about 30 minutes to answer. Questions 3 and 4 are short answer questions that require about 15 minutes to answer. Show your work for each part in the space provided after that part.

1. Three charges are fixed at positions along the x-axis at positions $-d$, 0, and $+d$. The charges at $-d$ and $+d$ are both negative, and the charge a 0 is positive.

   (a) A positively charged object of mass m is placed on the x-axis between 0 and $+d$, close to the position $x = 0$. If the three charges described above do not move as a result of this new charged object, describe the motion of the object after it is released as it moves in the region $0 < x < d$.

   The charge at 0 has a magnitude of $2Q$, while the other two charges have a magnitude of Q.

   (b) On the axes below, sketch the electric field along the x-axis in the vicinity of the charges. An electric field to the right should be graphed as positive and a field pointing left should be graphed as negative.



   (c) Write a mathematical function, $E(x)$, that gives the value of the electric field at any position along the x-axis for $0 < x < d$. Give your answer in terms of Q, d, and fundamental constants.

(d) In order to originally assemble the three charges on the x-axis, some work had to be done. Consider arranging the charges along the x-axis in the following manner: first, bring the $+2Q$ charge to position $x = 0$, then bring the $-Q$ charge to $x = +d$, and finally, bring in the last charge. Bringing the $+2Q$ charge to position 0 required no work. Bringing in the second charge required an amount of work W. Explain whether bringing in the third charge will require more work, less work, or an amount of work equal to W.

2. A student has a convex lens of unknown focal length. He lights a candle in a darkened room and uses the lens and moves a screen until he forms a sharp image. He then records the distance from the candle to the lens and the distance from the lens to the screen. Below is a sketch of his set-up and his data.

S_o (cm)	S_i (cm)
15	61.5
30	19.3
50	15.6
80	13.9
90	13.7
110	13.4

Selected values of the object and image distances.

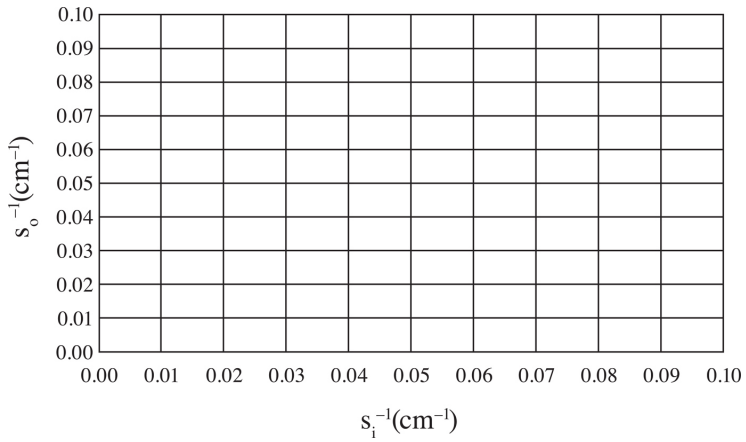
S_o^{-1} (cm^{-1})	S_i^{-1} (cm^{-1})
0.067	0.016
0.050	0.034
0.033	0.052

0.020	0.064
0.017	0.065
0.014	0.068

Selected values of the inverse of the object and inverse of the image distances.

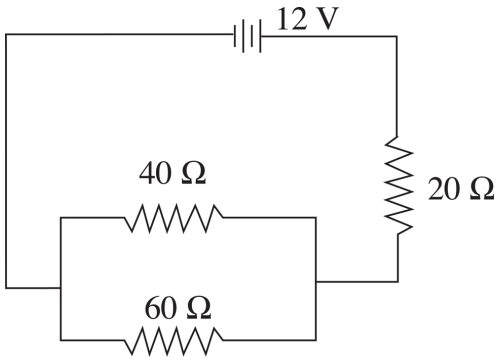
(a) Explain how a graph of $1/s_o$ vs. $1/s_i$ can be used to find the focal length of the lens.

(b) Create a graph of and find the focal length of the lens used in the experiment.



(c) Use ray tracing to make a sketch when the object is 6 cm from the lens.

(d) The top half of the lens is now covered by a sheet of cardboard so that light rays can only strike the bottom half of the lens. Briefly explain what effect this has on the image and how this would affect a ray tracing diagram.



3. The circuit shown is built and the voltage source supplies voltage for 15 minutes before the battery is completely drained. Assume the voltage supplied by the battery is constant at 12 V until the battery is drained, after which the battery supplies 0 V.

(a) What is the equivalent resistance of the circuit?

(b) Two students are discussing the apparatus. Student 1 says, "If the 20 Ω resistor were not present, the overall resistance of the circuit would have been lower and the battery would have lasted longer." Student 2 says, "If the 20 Ω resistor were not present, I think the power output would have been higher and the battery would have drained faster."

i. Use equations to show whether the overall resistance would have been lower without the 20 Ω resistor present.

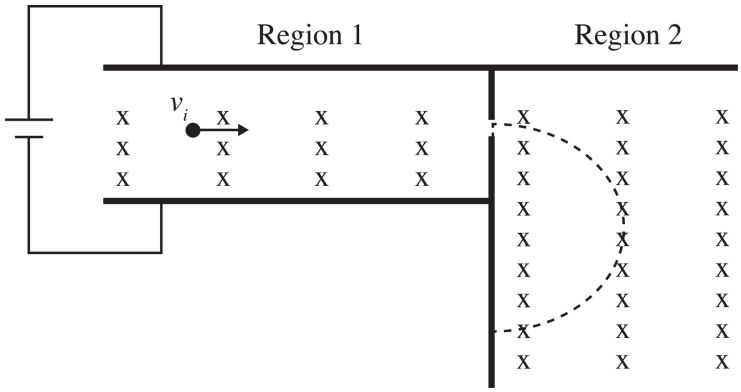
ii. Use equations to determine whether the overall power output would have been higher.

iii. Which student is correct about the battery life?

(c) The 20 Ω resistor is replaced with a capacitor.

i. As soon as the circuit is connected, explain without using equations how the current drawn out of the battery compares between the original circuit and the circuit with the capacitor.

ii. After the capacitor has been connected for a long time, but before the battery is completely drained, how does the current drawn out of the battery compare between the original circuit and the circuit with the capacitor?



4. In both region 1 and region 2, there is a uniform magnetic field, B, directed into the page. There is a uniform electric field, E, in region 1 established by the battery.

(a) A charged particle moves through region 1 undeflected.

i. Explain how this balance occurs between the electric and magnetic forces.

ii. What would happen to the particle if it was moving at a speed $v_2 > v_i$?

iii. How, if at all, would the set up in region 1 have to change if the particle was of the other sign?

(b) A particle at speed v_i travels through region 1 undeflected. It then enters region 2 and follows the dotted path that is indicated.

i. Is this a positively or negatively charged particle?

ii. What distance from the opening in the boundary between region 1 and region 2 does the particle hit the wall? Answer

in terms of m, B, E, v_1, and q.

(c) A similar apparatus is used to determine the percentage of $^{238}_{92}\mathrm{U}$ and $^{235}_{92}\mathrm{U}$ in a gas sample. Explain how such an apparatus can differentiate between the two isotopes.

(d) Another similar apparatus is unable to distinguish between two objects:

The first object has amu 14 and a charge of $-1e$.

The second object has amu 28 and charge $-2e$.

Why can this detector not distinguish between these two objects?

STOP

END OF EXAM

Practice Test 2: Answers and Explanations

PRACTICE TEST 2 ANSWER KEY

1. C
2. D
3. B
4. C
5. D
6. D
7. D
8. B
9. B
10. A
11. D
12. B
13. A
14. B
15. C
16. B
17. B
18. D
19. C
20. B
21. B
22. C
23. D
24. B
25. B
26. D
27. C
28. C
29. D
30. D
31. B
32. C
33. C
34. B
35. D
36. A
37. B
38. B
39. C
40. D
41. A
42. D
43. D
44. C
45. D
46. B, C
47. B, D
48. A, D
49. A, C
50. A, D

SECTION I

1. **C**

   There will be a uniform electric field pointing away from the positive plate and toward the negative plate. A positive particle will move in the direction of the field. Because there will be a constant force on the particle, it will have a curved trajectory.

2. **D**

   All four statements are true. However, when dealing with the gravitational field near the surface of the Earth, the inverse square nature of the gravitational force is not a factor.

3. **B**

   The direction an electromagnetic wave travels must be perpendicular to both the electric and magnetic fields. The electric field will always also be perpendicular to the magnetic field. The wave travels along the z-direction. The magnetic field is along the y-axis. The electric field must be on the x-axis.

4. **C**

   The spheres do not touch, so there cannot be any charging by contact. While the outside sphere will have non-symmetric charge distributions, each will still have a net charge of 0 C.

5. **D**

The uncharged capacitor acts as a wire, shorting bulb 2. A complete loop is made consisting of bulb 1, the capacitor, and bulb 3.

6. **D**

When the capacitor is fully charged, there is still a continuous conducting path that includes the battery and all three lightbulbs. Therefore, all three bulbs will be lit when the capacitor is fully charged.

7. **D**

The capacitor is fully charged when it has the same potential difference as the battery. Bulb 2 is in parallel with the capacitor and must have the same potential difference across it. Bulb 1 and Bulb 3 are in series with each other, and therefore have the same current. They are identical and so they have the same resistance. Ohm's Law says they must therefore have the same voltage drop.

8. **B**

The index of refraction of air is less than the index of refraction of water. According to Snell's Law, $n_1\sin(\theta_1) = n_2\sin(\theta_2)$, a smaller n_1 results in a smaller θ_2. The ray in the glass will be at an angle closer to the normal than the ray in glass from the water/glass apparatus.

9. **B**

Convex mirrors will always create virtual images. A concave mirror will create a virtual image when the object is placed within the focal length of the mirror. The only way to create a real image with a mirror is to use a convex mirror with the object beyond the focal point.

10. **A**

The continuity equation, $A_1v_1 = A_2v_2$, allows one to determine the speed at the second location because the volume flow rate, Av, can be determined at location A, so only the area at location B is needed.

11. **D**

Pressure is highest when speed is slowest. At the widest point in the tube, position A, the speed will be lowest. Because B is narrower than C, the speed at B will be greater than at C, making the pressure highest at point B.

12. **B**

According to the Second Law of Thermodynamics, whenever two objects are in the same environment for a long period of time, they will eventually end up at the same temperature. But even if two objects are the same temperature, one of them will feel cooler when it is able to transfer heat more rapidly than the other. Metal conducts heat better than paper, making the metal feel colder than the paper.

13. **A**

The Second Law of Thermodynamics says that energy will flow from the higher temperature part of a system into the lower energy part of the system.

14. **B**

Newton's Third Law dictates that the electron must exert a force on the sheet, which is opposite to the force the sheet exerts on the electron. The electron will be repelled by the

sheet, so the electron will be pushed up. The sheet must be pushed down.

15. **C**

The problem states that cross-sectional area of the wire is linearly proportional to the current. Ohm's Law, combined with the definition of resistivity, give $V = I\frac{\rho L}{A}$. Algebra gives $A = \left(\frac{\rho}{v}L\right)I$. Because ρ and V are held constant, the slope of the area vs. current graph can be used to find the length of the wires.

16. **B**

By the process of elimination, placing charges at B will result in the lever arm being parallel to the force and will not cause torque, eliminating (C) and (D). The sign of the charges on the two spheres must be different to have a net torque.

17. **B**

Induction requires drawing charge toward one location on a conductor, and then isolating that charge either using a grounding wire or by dividing the conductor.

18. **D**

Force on a moving charge is determined using the right-hand rule. In the right-hand rule, one finger represents the

sign of the charge and its velocity, the next represents the direction of the magnetic field, and the thumb represents the force. To find the direction of the magnetic field from a current carrying wire, you must know the direction the current flows and the location of the moving charge relative to the wire.

19. **C**

In the presence of a positive and a negative point charge, the electric potential will be higher at locations closer to the positive charge and farther from the negative charge.

20. **B**

An object will float when the buoyant force balances with the weight of the object. The buoyant force increases as the volume of the object that is submerged increases; therefore a heavier floating object will have a greater volume submerged than a lighter floating object. An object will sink if, when the entire volume is submerged, the buoyant force is still less than the weight of the object. The buoyant force on the wood equals its weight, while the buoyant force on the rock is less than the weight of the rock.

21. **B**

Power dissipated in a circuit is directly proportional to the product of the current and voltage. The slope of the line of a power vs. voltage graph gives the current.

22. **C**

Charged particles will deflect in a magnetic field. They will move along a circular arc. The direction of the center of the circle can be found using the right-hand rule. The right-hand rule depends on the sign of the charge, so like

charged objects deflect in the same direction. The object with the greater mass to charge ratio will deflect along a larger circle.

23. **D**

Three wavelength of light have been absorbed, indicating that there are three energy differences that correspond to an atomic transition, eliminating (A) and (C). It is possible that there are multiple levels with the same spacing, making (D) correct.

24. **B**

Alpha decay remove 4 nucleons, two of which are protons. The atomic number depends only on the number of protons. Three alpha particles causes the atomic number to decrease by 6. Each β^- decay causes the atomic number to increase by 1. Three alpha particles and two β^- particles cause the atomic number to decrease by 6 then increase by 2. Since the final atomic number is 82, the initial atomic number must have been 86.

25. **B**

The square of the wave function of a quantum object can be interpreted as the likelihood of an observation occurring at the stated position.

26. **D**

Rest-mass energy of a particle is given by $E = mc^2$. The released energy is given by the mass that remains after the decay less the mass that existed before the decay.

27. **C**

The relationship between image distance, object distance and focal length is given by $\frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o}$, which is equivalent to $\frac{1}{s_o} = -\frac{1}{s_i} + \frac{1}{f}$. To find the focal length, you need the reciprocal of the *y*-intercept of a graph of $\frac{1}{s_o}$ vs. $\frac{1}{s_i}$. $\frac{1}{(0.04\ \text{cm}^{-1})} = 25\ \text{cm}$.

28. **C**

A magnification of 1 occurs when $s_o = s_i$. The image distance and the object distance are equal when $\frac{1}{s_o} = 0.02$ on the graph.

29. **D**

Decreasing pressure inside the container decreases the force for the air molecules inside the container compared to the force from the air molecules outside the container. In order for the vertical forces to balance, the total upward force must be equal to the net downward force. The upward force is only from the external air pushing against the lid, while the downward forces are from the air inside the container, the force of gravity on the lid, and a normal force from the sides of the container pushing down on the lid where the seal occurs.

30. **D**

When objects reach thermal equilibrium, their average kinetic energy becomes constant. However, there is a

constant exchange of energy within the system as some molecules speed up and others slow down.

31. **B**

As the current decreases, the magnetic field observed at P will decrease in magnitude. When the current reaches 0, the observed field will be 0. As the current increases, the field magnitude will again increase.

32. **C**

From the charge at point A, the field will be up and to the right. From B, the field will be up and to the left. The right component of A will be stronger than the leftward component of B. From the charge at C, the field will point down and to the right. The upward field lines from either A or B will be greater than the downward field lines from C.

33. **C**

Field lines represent the force a test charge would experience at a given position relative to a charge distribution. For point charges, field strength will decrease as an inverse-square as positions get farther from the sources or sinks.

34. **B**

The positive charges will align with the field. Because the edge of the sphere is curved, there will be a greater concentration of charges near the middle of the curve than at the edges.

35. **D**

The electron will be deflected if there is a non-zero magnetic force. The magnetic force will be zero when the magnetic field is parallel to the direction of travel. The magnetic field around a current carrying wire is crated in concentric circles.

36. **A**

The process of charging by contact involves touching a charged object to an uncharged conductor.

37. **B**

To find the equivalent resistance of the circuit, first

combine the parallel resistors.

$\frac{1}{R_{eq}} = \frac{1}{15} + \frac{1}{30} + \frac{1}{5} \rightarrow R_{eq_{parallel}} = 3.3\ \Omega$. The combination of parallel

resistors is in series with the resistor, so

$R_{eq_{parallel}} = 10 + 3.3 = 13.3\ \Omega$. The current out of the battery is

found from Ohm's Law. $I = \frac{V}{R_{Total}} = \frac{2.5}{13.3}$

38. **B**

The voltage drop in each of the parallel paths must be the same at $V = 0.50$ V. The loop rule indicates that in any closed loop, the voltage drop must equal the voltage of the battery. The 2.5 – 0.5 = 2.0 V voltage drop must occur across the 10 Ω resistor. Ohm's Law indicates that the current through that resistor must be 0.2 A, which is the total current in the circuit.

The junction rule states that this total current must also be the sum of the current in all four branches. The current in the 15 Ω resistor is $I = \frac{0.50}{15} = 0.033$ A, in the 30 Ω resistor is $I = \frac{0.50}{30} = 0.017$ A, and in the 5 Ω resistor is $I = \frac{0.50}{5} = 0.010$. The remaining current must be 0.20 – 0.033 – 0.017 = 0.05 A.

Using Ohm's Law on the unknown resistor,

$$R = \frac{V}{I} = \frac{0.50}{0.05} = 10\ \Omega.$$

39. **C**

The resistance of a wire is directly proportional to the length of the wire and inversely proportional to the cross-sectional area. 1 must have the least resistance and 2 must have the most resistance. Because the resistors are in series, they have the same current through them. Ohm's Law states that the greatest voltage drop is across the greatest resistance value.

40. **D**

The magnetic field must be perpendicular to the direction of motion in order for there to be a magnetic force. The magnetic force will be perpendicular to both the field and the velocity. The electric force is parallel to the electric field. Therefore, the electric field must be perpendicular to

the magnetic field, and both fields must be perpendicular to the velocity.

41. **A**

For electrons, the numerator of the Coulomb force is $9 \times 10^9 * (1.6 \times 10^{-19})^2$ while the numerator of the gravitational force is 6.67×10^{-11} $(9.11 \times 10^{-31})^2$. The denominators are equal for both forces, so the Coulomb force is much greater.

42. **D**

Inside a solenoid, an electron will be equally repelled by any charges within the wires, so there will be a zero net electric force. The velocity of the electron traveling along the axis of the solenoid is parallel to the magnetic field, so there will be no magnetic force.

43. **D**

Electrons are excited into higher energy levels with the input of energy from the voltage source. As the excited electron falls back to lower energy levels, light is emitted at frequencies that characterize the energy differences between levels.

44. **C**

The critical angle occurs at the angle for θ_2 when the value of $\sin(\theta_1) = 1$, which is the maximum value that $\sin(\theta)$ can obtain.

45. **D**

The energy stored in a capacitor depends on the voltage across the capacitor when it is fully charged. The capacitor

will have the same voltage as the battery when fully charged regardless of the resistances of either resistor.

46. **B, C**

The amount and direction of compass needle deflection indicates the strength of the magnetic field and its direction. The field from a current carrying wire is $B = \frac{\mu_0}{2\pi}\frac{I}{r}$, so the field increases as the current increases. The right hand rule with wires says to determine the direction, the thumb points in the direction of current, the index finger points to the place to determine the field and the middle finger points in the direction of the field.

47. **B, D**

Work is done on a gas when volume changes at constant pressure, or it can be found from the area under a pressure-volume curve. Choice (B) involves no change in volume. The first two steps in (D) involve no volume change. The last two steps involve the same amount of work first being done by the gas, then the same amount being done on the gas.

48. **A, D**

The existence of a frequency below which no light is emitted argues for the particle nature of light. If light were

purely a wave, then at sufficiently high intensity, the energy would be available in the wave to transfer to the electron. The linear increase in the stopping potential and frequency, and the independence of the stopping potential on the intensity, also argues for the particle nature of light. A wave explanation would indicate that a higher intensity should give the electrons more energy.

49. **A, C**

The overall current will increase when the overall resistance decreases. Placing an additional resistor in parallel with an existing resistor, whether the existing resistor is part of a series combination or part of a parallel combination, will lower the overall resistance of the circuit.

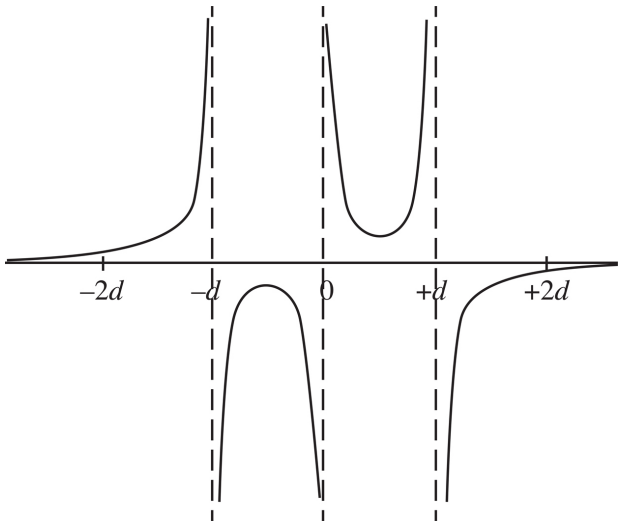
50. **A, D**

For a diverging lens, the “focus” is on the far side of the lens. So (1) shows a light ray traveling into the focus and then bending parallel to the optic axis. (4) shows a light ray traveling parallel to the optic axis, and it leaves on a line connecting the other focal point.

SECTION II

1. (a) A positively charged object will be repelled from the positive charge at position o and attracted to the negative charge as position $+d$, so it will experience a force to the right during the entirety of its motion. The object will move from its release position directly toward the negative charge at position d with a velocity that increases during the entirety of the motion.

(b) The field will be to the right in the two regions $-\infty < x < -d$ and $0 < x < d$, so this is where the plot will be positive. The field magnitude will go toward infinity as the position approaches the charges as $-d$, o, and $+d$. The charge at o has a larger magnitude, so the positions where the charge has the smallest magnitude should be closer to the charges as $+d$ and $-d$.



(c) In the region between o and d, the net charge will depend on the influence of each of the three charges on the x-axis.

$E_{-d}(x) = \dfrac{kQ}{(d + x)^2}$ is directed to the left.

$E_0(x) = \dfrac{k(2Q)}{(x)^2}$ is directed to the right.

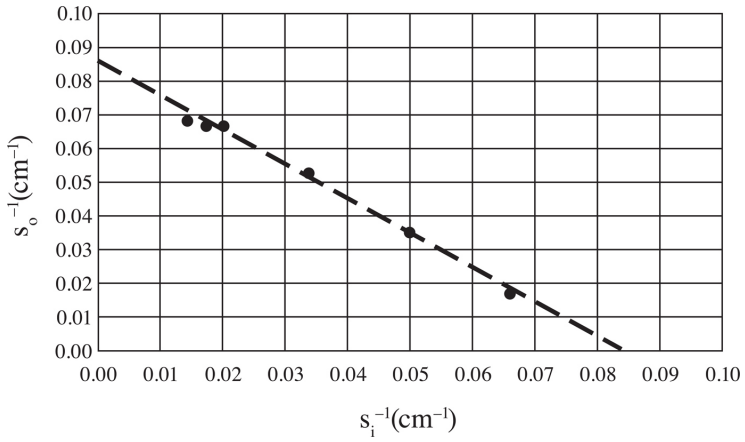
$E_{+d}(x) = \dfrac{kQ}{(d-x)^2}$ is directed to the right.

Thus, $E(x) = \dfrac{kQ}{(d-x)^2} + \dfrac{k(2Q)}{(x)^2} - \dfrac{kQ)}{(d+x)^2}$ directed to the right.

(d) The work required to bring a charge to a position is equal to the electric potential at that position multiplied by the amount of charge that is moved. Once the first two charges have been assembled, the electric potential at position $x = -d$ will be lower than when there was only a positive charge at $x = 0$ because the charge at $x = d$ is negative. Therefore, the work to bring in the third charge will be less than W.

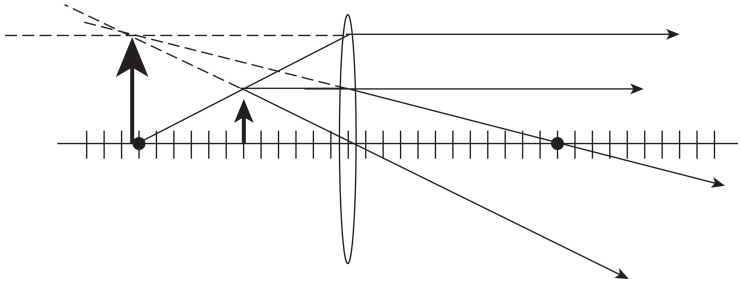
2. (a) $\dfrac{1}{f} = \dfrac{1}{s_i} + \dfrac{1}{s_o}$, so a plot of s_o^{-1} vs. s_i^{-1} will have a y-intercept equal to $\dfrac{1}{f}$.

(b)



The y-intercept is approximately 0.083, so the focal length is $\dfrac{1}{0.083\ \text{cm}^{-1}} = 12$ cm.

(c) The diagram needs to have a focal length of 12 cm and an object distance of 6 cm.



(d) The brightness of the image would decrease but the location of the image would not change. The ray diagram would require using rays that are not the principle rays as all three principle rays are blocked. However, all rays coming from the object converge at the image position, so the image position would not change.

3. (a) The resistors in parallel are combined first.

$$R_{eq} - 1 = 40^{-1} + 60 - 1$$

$$R_{eq} = 24$$

The total equivalent resistance is then $R_{total} = 24 + 20 = 44$ Ω.

(b)

(i) Without the resistor, there would only be the parallel resistor combination.

$$R^{-1}{}_{1000} - = 40^{-1} + 60 - 1$$

$$R_{total} = 24\ \Omega$$

(ii) $P = IV = \left(\frac{V}{R}\right) V$, so a higher resistance results in a lower power output for the same battery.

(iii) A lower power output means that it will take more time for the energy to be used up, so student 1 is correct.

(c)

(i) As soon as the circuit is connected with the capacitor, the capacitor acts as if it had no resistance. This is exactly like (b, i). The lower resistance would cause a greater current to be drawn from the battery.

(ii) After the capacitor is fully charged, no more current can flow in the branch of the circuit with the capacitor. The capacitor is in series with the battery, so there will be no current drawn from the battery.

4. (a)

(i) The electric force is parallel to the electric field in region 1. This points vertical, either up or down, depending on the sign of the charge. The magnetic force is perpendicular to both the velocity and the field, so the magnetic force is also in the vertical dimension. The forces balance when the electric force equals the magnetic force in magnitude.

$$F_E = qE = qvB$$

The value of the charge cancels, so any particle with a speed $v = E/B$ will experience balanced forces.

(ii) A particle moving faster than E/B will experience a greater magnetic force than electric force. It will be deflected and will not go through the hole between region 1 and region 2.

(iii) There would be no change in outcome if the sign of the particle were reversed, but the directions of both the electric and magnetic forces would reverse. They should still balance for $v = \frac{E}{B}$.

(b)

(i) As soon as the particle enters region 2, it has a velocity to the right, a B field into the page, and experiences a downward directed force. From the right hand rule, the force should be upward, so the particle must be negatively charged.

(ii) The magnetic force acts as the centripetal force for the particle in region 2.

$$qvB = \frac{mv^2}{R}$$

$$qB = \frac{mv}{R}$$

$$R = \frac{mv_i}{qB}$$

The particle hits the wall at twice the radius, $2mv_i/qB$.

Note: The answer can also be written substituting $v = \frac{E}{B}$.

(c) Both isotopes have the same charge. Any particle entering region 2 through the hold has the same speed. The isotopes have different masses. Based on the answer to (b, ii), an

isotope with a larger mass will hit the wall at a greater distance from the hold than an isotope with a smaller mass.

(d) When the objects have different charges and masses, the radius from (b, ii) depends on the ratio of mass to charge. Because the ratio for the given example is the same: $-1e$: 14 amu, the apparatus will not distinguish between them.

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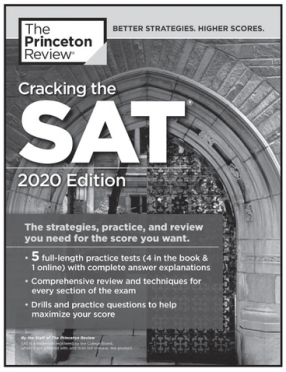

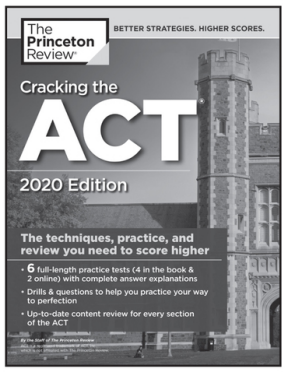



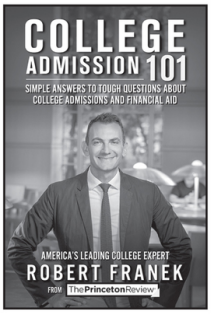

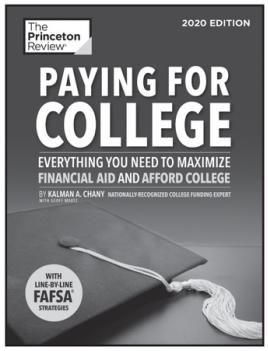

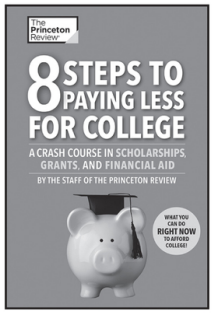

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